ON NORMAL STABILITY FOR NONLINEAR PARABOLIC EQUATIONS

Gieri Simonett
Department of Mathematics, Vanderbilt University
Nashville, TN 37240, USA

In this lecture we study convergence of solutions for quasilinear and fully nonlinear parabolic equations towards equilibria in situations where the set of equilibria is non-discrete but forms a C^1-manifold.

Our main result can be summarized as follows: suppose that for a nonlinear evolution equation we have a C^1-*manifold of equilibria* $\mathcal{E}$ such that at a point $u_* \in \mathcal{E}$, the kernel $N(A)$ of the linearization A is isomorphic to the tangent space of $\mathcal{E}$ at u_*, the eigenvalue 0 of A is semi-simple, and the remaining spectral part of the linearization A is stable. Then solutions starting nearby u_* exist globally and converge to some point on $\mathcal{E}$. We call this situation the *generalized principle of linearized stability*, and the equilibrium u_* is then termed *normally stable*.

A typical example for this situation to occur is the case where the equations under consideration involve symmetries, i.e. are invariant under the action of a Lie-group.

The situation where the set of equlibria forms a C^1-manifold occurs for instance in phase transitions, geometric evolution equations, free boundary problems in fluid dynamics, stability of traveling waves, and models of tumor growth, to mention just a few. (Joint work with J. Prüss and R. Zacher).

References

[1] J. Prüss and G. Simonett, *Stability of equilibria for the Stefan problem with surface tension*, SIAM J. Math. Anal., **40** (2008), 675–698.
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[3] J. Prüss, G. Simonett, and R. Zacher, *On normal stability for nonlinear parabolic equations*, arXiv:0811.1784v1. Submitted.