I will talk on the Navier–Stokes equations in $D \subset \mathbb{R}^3$:

\[ (NS) \begin{cases} u_t - \Delta u + (u \cdot \nabla)u + \Omega e_3 \times u + \nabla p = 0, & \nabla \cdot u = 0 \quad \text{in } D \times (0, T), \\
 u|_{\partial D} = 0, & u|_{t=0} = u_0, \end{cases} \]

Here $u = u(x, t) = (u_1, u_2, u_3)$ is an unknown velocity field and $p = p(x, t)$ is an unknown pressure. In the equation $(NS)$ $e_3$ is the vertical unit vector $(= (0, 0, 1))$ and the term $\Omega e_3 \times u$ is called the Coriolis force term with the Coriolis parameter $\Omega \in \mathbb{R}$.

The Coriolis force term is obtained in the following way. Consider the standard Navier–Stokes equation in $\mathbb{R}^3$:

\[ v_t - \Delta v + (v \cdot \nabla)v + \nabla q = 0, \quad \nabla \cdot v = 0 \]

with initial velocity

\[ v|_{t=0}(y) = v_0(y) + (\Omega/2)e_3 \times y = v_0(y) + (\Omega/2)Jy \quad \text{with } J = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Note that $\text{rot}v|_{t=0} = \text{rot}v_0 + \Omega e_3$. In this equation we introduce the transform

\[ \begin{cases} u(x, t) = \exp \left\{ -\Omega Jt/2 \right\} v(y, t) - (\Omega/2)Jx \\
p(x, t) = \exp \left\{ -\Omega Jt/2 \right\} q(y, t) + \Omega^2 |y'|^2/8. \end{cases} \]

Here $y = (y', y_3) = \exp \{\Omega Jt/2\} x$. Then $(u, \nabla p)$ satisfies $(NS)$.

From a meteorological point of view, if Coriolis parameter $\Omega$ is sufficiently large, flow will be independent of vertical direction $x_3$ asymptotically, that is 3 dim. flow will close to 2 dim. flow as $|\Omega| \to \infty$. This phenomena is called the Taylor–Proudman theorem. Our main purpose is to study this singular perturbation problem.
In my talk I will discuss the following results.
For a fixed $\Omega$, we obtained several unique existence theorems of the Cauchy problem and boundary value problem ($\mathbb{R}^3_+$) in some function spaces which include periodic functions, almost periodic functions and some $L^\infty$ functions. Furthermore we will give some calculation which impress us that Coriolis term is not minor term.
In the case of the stationary problem in the Poincaré domain we noted some asymptotic behavior of velocity field in $L^2(D)$ as $\Omega \to \infty$.

My talk is based on joint works with Prof. Y. Giga, Prof. K. Inui, Prof. A. Mahalov and Prof. J. Saal:


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