

# Recent Development of Parabolic Quasi-Variational Inequalities

Nobuyuki Kenmochi

(kenmochi@faculty.chiba-u.jp)

Department of Mathematics, Faculty of Education

Chiba University, Chiba, 263-8522 Japan

In this talk we discuss a class of parabolic quasi-variational inequalities of the form

$$u'(t) + \partial\varphi^t(u; u(t)) \ni f(t), \quad 0 < t < T, \quad \text{in } X^*,$$

$$u(t) = u_0(t) \quad \text{for } -\delta_0 \leq t \leq 0, \quad \text{in } X$$

under the following setting:  $X$  is a real reflexive Banach space compactly embedded in a real Hilbert space  $H$ , so  $X \subset H \subset X^*$ , where  $X^*$  is the dual of  $X$  and  $\delta_0$  and  $T$  are fixed positive numbers;  $\varphi^s(v; z)$ ,  $-\delta_0 \leq s \leq t$ , is a proper l.s.c. convex function in  $z$  on  $X$  for  $v$  belonging to a suitable class of functions from  $[-\delta_0, t]$  into  $X$  in which  $\varphi^s(v; \cdot)$  depends continuously on  $v$  in a non-local way;  $u_0$  is a prescribed initial function given on  $[-\delta_0, 0]$ .

As a simple example shows, the existence question for the above type of quasi-variational inequalities are quite delicate. We shall give sufficient conditions for the existence of a local in time solution of our problem. Moreover, in order to illustrate our conditions we apply our abstract result to a concrete obstacle problem having the obstacle depending on the unknown function.