Inverse Problems in Applied Sciences
− towards breakthrough −

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Hokkaido University, Sapporo, Japan, 3-7 July, 2006

Preface

The purpose of this conference is to establish the collaboration links among the researchers in Asia and worldwide leading researchers in inverse problems. The conference will address both theoretical (Mathematics), applied (Engineering) and development aspects on inverse problems. The proposed conference is intended to nurture Asian-American-European collaborations in this evolving interdisciplinary area. It is envisioned that the conference will lead to a long-term commitment and collaboration among the participated countries and researchers.

Additionally, newcomers to the subject matter will be encouraged to participate through (i) the attendance of tutorial sessions, serial lectures and panel discussion, and (ii) the availability of a practical information on the application of inverse problems to engineering disciplines to enter the study of inverse problem.

In this occasion, the organizers acknowledge the partial supports by the NS PLANNING Inc. company, the 21st century COE program (Mathematics of Nonlinear Structures via Singularities, Department of Mathematics, Hokkaido University, Japan) and several grants in Aids for Scientific Research of Japan Society for the Promotion of Science.

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Invited Speakers

H. Ammari
M. Belishev
M. Burger
J. Cheng
H. Fujiwara
N. Higashimori
O. Imanuvilov
V. Isakov
H. Isozaki
B. Kaltenbacher
S. Kim
M. V. Klibanov
M. Lassas
M. Lim
J. Liu
A. Lorenzi
F. Ma
R. Potthast
T. Sato
K. Shirota
P. Stefanov
Y. Tan
K. Tanuma
G. Uhlmann
Y. Wang
M. Watanabe
E. J. Woo
J. Zou

Electrical Impedance Tomography by Elastic Perturbations

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We propose a new direction for future Electrical Impedance Tomography (EIT) research, for mainly biomedical applications. Our technique is based on simultaneous measure of an electric current and of acoustic vibrations induced by ultrasound waves. This technique can provide high resolutions images, while conserving the most important merits of EIT. This work is joint with E. Bonnetier, Y. Capdeboscq, M. Fink, and M. Tanter.

On a functional model of a class of symmetric operators and its application in inverse problems

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The talk is an attempt to inscribe the BC-method in the scope of model theory that is a branch of functional analysis dealing with representation of the abstractly given operators in the form of the operators acting in functional spaces. We show that determination of the Riemannian manifold Ω from its boundary spectral or dynamical data by the BC-method is equivalent to construction of a canonical functional model of the minimal Beltrami-Laplace operator Δ_0 acting in $L_2(\Omega)$. The basic element of this construction is a metric space $\tilde{\Omega}$ built of the increasing families (nests) of subspaces formed by waves produced by boundary sources. Such nests, parametrized by the action time of sources, play the role of points of the space whereas the distance between two points in $\tilde{\Omega}$ is introduced as the "interaction time" equal to the (doubled) value of the parameter at which the subspaces of two nests begin to intersect. By its construction, the space $\tilde{\Omega}$ turns out to be isometric to the original Ω whereas the corresponding Beltrami-Laplace operator $\tilde{\Delta}_0$ is unitary equivalent to Δ_0. The construction of $\tilde{\Omega}$ can be interpreted in terms of the Spectral Theorem for the von Neumann algebras. Such a philosophy gives a unified look at a rather wide class of inverse problems; in particular, it gives a procedure of time-optimal reconstruction of the Riemannian manifold from dynamical electromagnetic boundary data. The last result generalizes the ones on determination of parameters of the Maxwell system from its response operator (Belishev, Glasman, Isakov, Pestov, Sharafutdinov 1997-2001).

Nonlinear Inverse Scale Space Methods in Imaging and Inverse Problems

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Variational regularization techniques with nonlinear penalty functionals have received growing attention recently since they can yield regularized solutions enhancing certain features. An example of particular importance is the reconstruction of discontinuous solutions with total variation techniques. Unfortunately, such variational techniques have an inherent deficiency, because they tend to overregularize. E.g. in the case of total variation regularization, the variation of the regularized solution is usually smaller than of the exact one, and some important features can be lost due to the regularization. For penalties being the square of some Hilbert space norms, it is well-known that iterative regularization methods (constructed in these Hilbert spaces) can at least in part resolve these deficiencies and yield regularization methods with improved properties (higher qualification, easier implementation, multiscale properties, ...).

In this talk we shall discuss a novel approach to the construction of iterative schemes based on nonlinear penalties, which works in a rather general setup. The original idea is based on constructing a sequence of nonlinear variational problems with appropriate signal-noise decompositions. It turns out that the approach can be reformulated and analyzed in terms of *Bregman distances* associated to the regularization functional. Moreover, the resulting iterative regularization techniques involve a multiscale structure, so that large-scale features are reconstructed before small-scale features. In a natural small-parameter limit one obtains a continuous flow from the initial value to the regularized solution, which generalizes the concept of inverse scale space methods introduced by Scherzer and Groetsch.

The derivation of the methods as well as the analysis of convergence, regularizing, and multiscale properties will be discussed in detail. Moreover, we discuss discretization techniques and implementation aspects. Finally, we show applications to various imaging tasks and to the reconstruction of piecewise constant parameters in systems of partial differential equations.

The results discussed in this talk are based on joint work with D.Goldfarb, G.Gilboa, S.Osher, E.Resmerita, J.Xu, W.Yin.

Numerical Computation of Ill-Posed Problems by Spectral Element Method

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We make direct approach in numerical computation of some ill-posed problems using spectral discretization and multiple-precision arithmetic.

Ill-posedness of inverse problems causes a difficulty in their numerical simulations. We focus on stability in ill-posedness in the research. A discretization scheme of an ill-posed problem is numerically unstable in most cases. Numerical instability leads that computational errors grow rapidly in numerical process then computation fails. Stabilization such as Tikhonov regularization is one of the most effective methods for numerical analysis of ill-posed problems, however strong stabilization for control of computational errors sometimes regularize characteristics of solutions. We also remark that numerical computation of a regularized problem is not always numerically stable since discretization of a mathematically stable problem does not always lead a numerically stable scheme.

We propose direct numerical computation of numerically unstable problems to control computational errors. Two different kinds of computational errors are treated in the research: discretization errors which depend on approximation of differential operators or integral operators, and rounding errors which come from approximation of real numbers and arithmetic on digital computers. We apply spectral discretization to reduce discretization errors with a domain decomposition. And multiple-precision arithmetic is used in computation of the spectral discretization scheme to reduce rounding errors.

A multiple-precision arithmetic software, which is designed and implemented by the authors for fast and large scale scientific numerical computations, is also introduced.

Identification of a domain by the Lamé system

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We consider an inverse problem of identifying the shape of an elastic body by a boundary measurement of an elastostatic field. The elastostatic field is assumed to satisfy the Lamé system with an inhomogeneous term corresponding to the action of the gravity on the earth. The main result is a conditional stability estimate of single-logarithmic type under suitable a priori information. The stability estimate is a consequence of the unique continuation property for solutions to the Lamé system.

Notation. For $x = (x_1, x_2, x_3)^T \in \mathbb{R}^3$, we denote $x' = (x_1, x_2, 0)^T$, where ^{T} means transposition. Let Ω be a cylinder and Γ be an open part of $\partial\Omega$ given by

$$\Omega := \left\{x \in \mathbb{R}^3 \mid 0 < x_3 < 1, |x'| < 1\right\}, \qquad \Gamma := \{x \in \mathbb{R}^3 \mid x_3 = 0, |x'| < 1\},$$

where $|\cdot|$ is the Euclidean norm. We consider that Ω is made of an elastic material with Lamé coefficients λ, $\mu \in C^\infty(\overline{\Omega})$ satisfying $\mu > 0$, $2\mu + 3\lambda > 0$ on $\overline{\Omega}$.

Let $\boldsymbol{u} = (u_1, u_2, u_3)^T$ (column vector) be a displacement vector field defined on (a subdomain of) Ω. We define the strain tensor field $\boldsymbol{\sigma}(\boldsymbol{u})$ and the action of the Lamé operator P by

$$\boldsymbol{\sigma}(\boldsymbol{u}) = \lambda(x)\,(\operatorname{div}\boldsymbol{u})I_3 + \mu(x)\,(\nabla\boldsymbol{u}^T + \nabla\boldsymbol{u}), \qquad P\boldsymbol{u} = \mathbf{div}\,\boldsymbol{\sigma}(\boldsymbol{u}),$$

where I_3 is the identity matrix of order 3, and $\nabla\boldsymbol{u}$ is the Jacobian matrix for the mapping $\boldsymbol{u}$. For a $\mathbb{R}^{3\times3}$-valued function $T = (t_{ij})$, $\mathbf{div}\,T$ denotes the $\mathbb{R}^3$-valued function whose i-th component is the divergence of the i-th row of T: $\mathbf{div}\,T = (\partial_1 t_{i1} + \partial_2 t_{i2} + \partial_3 t_{i3})_{i\downarrow}$ (column vector).

In our inverse problem, we assume that the elastic body Ω is damaged at the surface so that it has the form

$$\Omega_F := \{x \in \Omega \mid 0 < x_3 < F(x'),\, x' \in \Gamma\}$$

for some $F : \overline{\Gamma} \to [0, 1]$. We consider that the shape of the damaged part $\gamma_F := \{x \in \mathbb{R}^3 \mid x_3 = F(x'),\, x' \in \overline{\Gamma}\}$ is unknown and it must be identified. Our inverse problem is to identify the function F by using the boundary values (on Γ) of an elastostatic field defined on Ω_F.

We consider two such domains Ω_{F_1}, Ω_{F_2} and displacement fields $\boldsymbol{u}_j$ on Ω_{F_j} $(j = 1, 2)$.

A priori information. Let $0 < a$, $b < 1$, $0 < \kappa < 1$, $L > 0$, $M > 0$, and $\rho_0 > 0$ be fixed numbers.

(1) Assume that F_j's satisfy

$$F_j \in C^{1,\kappa}(\overline{\Gamma}), \qquad \|\operatorname{grad} F_j\|_{C^0(\overline{\Gamma})} \le L,$$
$$a \le F_j(x') \le 1 \quad (x' \in \overline{\Gamma}), \qquad F_j(x') = 1 \quad (b < |x'| < 1).$$

(2) Assume that $\boldsymbol{u}_j$'s satisfy

$$\boldsymbol{u}_j \in H^2(\Omega_j; \mathbb{R}^3) \cap C^{1,\kappa}(\overline{\Omega_j}; \mathbb{R}^3), \qquad \|\boldsymbol{u}_j\|_{C^{1,\kappa}(\overline{\Omega_j};\mathbb{R}^3)} \le M,$$
$$P\boldsymbol{u}_j = \big(0, 0, -g\rho(x)\big)^T \text{ in } \Omega_j, \qquad \boldsymbol{\sigma}(\boldsymbol{u}_p)\boldsymbol{\nu} = \mathbf{0} \text{ on } \gamma_j,$$

where $\boldsymbol{\nu}$ is the outer unit normal vector on the boundary, g is the gravity acceleration, and $\rho(x)$ is the mass density of the material at the point x satisfying $\rho \ge \rho_0$ on $\overline{\Omega}$.

Theorem. Let a, b, L, M, κ, λ, μ, and ρ_0 be as above. Then there exist $K > 0$ and $0 < \delta < 1$ such that the following holds: Assume that F_1, F_2, u_1, and u_2 satisfy the a priori information, and put $\epsilon := \|\boldsymbol{u}_1 - \boldsymbol{u}_2\|_{H^{3/2}(\Gamma;\mathbb{R}^3)} + \|\boldsymbol{\sigma}(\boldsymbol{u}_1) - \boldsymbol{\sigma}(\boldsymbol{u}_2)\|_{H^{1/2}(\Gamma;\mathbb{R}^3)}$. If $\epsilon < 1$, then we have

$$\|F_1 - F_2\|_{C^0(\overline{\Gamma})} \le \frac{K}{|\ln\epsilon|^\delta}.$$

On some inverse problems associated with the isotropic Lamé system

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We consider the isotropic Lamé system with free stress boundary conditions or zero Dirichlet boundary conditions. We discuss the problem of uniqueness and stability in determining spatially varying density and two Lamé coefficients by a single measurement of the displacement over $(0, T) \times \omega$. The machinery is based on the Carleman estimates.

The inverse conductivity problem with limited data and applications.

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We discuss uniqueness and stability of recovery of the conductivity coefficient a in the elliptic equation

$$div(a\nabla u) = 0 \text{ in } \Omega$$

from single and many boundary measurements. Here Ω is a bounded domain in $\mathbf{R}^n, n = 2, 3$ with Lipschitz boundary.

In case of single measurement we assume that $a = 1 + k\chi_D$ where D is an unknown domain and k is a known constant. We precribe special Dirichlet and Neumann data on parts Γ_0 and Γ_1 of the boundary of Ω and we are given the additional Neumann data on an open part of Γ_0. Under some natural geometrical (of convexity type) assumptions we demonstrate uniqueness of D. Proofs use some modifications of the Novikov's orthogonality method. We give applications to detecting so called p-n junction (∂D) in semiconductor devices.

In case of many measurements we prescribe zero Dirichlet (or Neumann data) for u on very special Γ_0 and the Dirichlet-to Neumann map on Γ_1 and show uniqueness of $C^2(\overline{\Omega})$-smooth positive a. In this case proofs are appropriate versions of methods of complex geometrical optics.

Inverse boundary value problems and hyperbolic spaces

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We present a new approach to the inverse boundary problem based on the structure of hyperbolic space. The starting point is the following observation. Given a boundary value problem $(-\Delta + q)u = 0$ in $\Omega \subset \mathbf{R}^n$, let $v = x_n^{(2-n)/2}u$. Then v satisfies $(-\Delta_g + V)v = 0$, where $V = x_n^2 q - n(n-2)/4$ and $\Delta_g = x_n^2\partial_n^2 - (n-2)x_n\partial_n + x_n^2\Delta_x$, which is just the Laplace-Beltrami operator on $\mathbf{H}^n$.

(1) Horosphere boundary value problem and Barber-Brown algorithm. Consider a boundary value problem for the Schrödinger operator $-\Delta+q(x)$ in a ball $\Omega : (x_1+R)^2+x_2^2+(x_3-r)^2 < r^2$, whose boundary we regard as a horosphere in the hyperbolic space $\mathbf{H}^3$ realized in the upper half space. Let $S = \{|x| = R, x_3 > 0\}$ be a hemisphere, which is generated by a family of geodesics in $\mathbf{H}^3$. By imposing a suitable boundary condition on $\partial\Omega$ in terms of a pseudo-differential operator, we compute the integral mean of $q(x)$ over $S \cap \Omega$ from the associated (generalized) Robin-to-Dirichlet map for $-\Delta + q(x)$. The potential $q(x)$ is then reconstructed by virtue of the inverse Radon transform on hyperbolic space. This justifies the well-known Barber-Brown algorithm in electrical impedance tomography.

(2) Detection of inclusions - Applications to numerical computation. This hyperbolic space approach can also be used to detect the location of non-smooth part of conductivity $\gamma(x)$ of a body Ω in $\mathbf{R}^d$, $d = 2, 3$. Suppose for the sake of simplicity that we know the DN map Λ_0 for the case that γ is a constant, and that the conductivity is different from this constant on a subset $\Omega_1 \subset \Omega$. Take x_0 from outsied of the convex hull of Ω and let $S^\epsilon_{out} = \{x \in \partial\Omega; |x - x_0| > R + \epsilon\}, S^\epsilon_{in} = \{x \in \partial\Omega; |x - x_0| < R - \epsilon\}$. Then one can construct the boundary data $f_\tau(x)$ depending on a large paramter $\tau > 0$ having the following properties : On S^ϵ_{out} (S^ϵ_{in}), $f_\tau(x)$ is exponentially decreasing (increasing) in τ. Let Λ be the DN map for γ. If $R < dis(x_0, \Omega_1)$, then $0 \le (\Lambda - \Lambda_0)f_\tau, f_\tau) < Ce^{-\delta\tau}$, and if $R > dis(x_0, \partial\Omega_1)$, then $((\Lambda - \Lambda_0)f_\tau, f_\tau) > C'e^{\delta\tau}$. This means that one can detect the location of inclusions from the boundary data which are essentially localized on a part of the boundary.

Characterization of nonlinear material behaviour: Parameter identification problems in nonlinear PDEs and their regularized solution

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In this talk we adress the topic of identifying parameters of smart materials as used in a large variety of sensor and actor applications. This leads to parameter identification problems in partial differential equations. A both practically relevant and mathematically challenging situation arises in case of large excitations leading to nonlinear material behaviour.

Here we firstly discuss the identification of parameter curves appearing as coefficients on (nonlinear) PDE models. In this context instability arises, hence we consider regularization issues. Secondly, identification of a more complex model that also takes into account memory effects via hysteresis operators is adressed.

Focusing on piezoelectricity and electromagnetism as application examples, we will present solution techniques and numerical results for both cases.

Inversion Problem of the Gravity Potential

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The inverse gravimetry problem is an ill-posed problem of determining the density variation σ and gravity source Ω from measured surface gravity data. If we are interested in finding a specific material, oil(0.9×10^3 kg m^{-3}), copper(8.9×10^3 kg m^{-3}) or volcanic rock(2.5×10^3 kg m^{-3}) for example, buried in earth, its density σ can be supposed to be a known constant and we want to reconstruct the gravity anomaly from gravity measured on the earth surface.

In this paper, we consider mathematical fundamental questions such as uniqueness and stability within some polygonal anomalies, and introduce a new reconstruction algorithm which makes only use of point gravity measurements.

Global Uniqueness Theorems, Stability Estimates and Numerical Methods for Some Coefficient and Ill-Posed Cauchy Problems

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The key tool of all results to be presented is the method of Carleman estimates, which was first introduced in the field of coefficient inverse problems simultaneously and independently by A.L. Bukhgeim and M.V. Klibanov in 1981 in [1,2]. These results, along with more recent ones, including the newly developed numerical application of those ideas, are described in the recently published book of Klibanov and Timonov [3]. Twenty five years later this remains the single method so far enabling for proofs of global uniqueness and stability results for a wide class of multidimensional coefficient inverse problems with a single measurement. This idea was extended recently by M.V. Klibanov and A.Timonov to the construction of a globally convergent numerical method called *convexification,* see [3]. The notion of global convergence is quite important for coefficient inverse problems, because the vast majority of numerical methods is convergent locally. The latter means that solutions are unreliable, unless it is given a priori that the first guess is located in a small neighborhood of the correct solution, which is only rarely realized in applications.

The presentation will consist of three parts:

Part 1. Global uniqueness and stability theorems for some coefficient inverse problems with single measurement data.

Part 2. Stability estimates for ill-posed Cauchy problems for hyperbolic and parabolic equations both in finite and infinite domains with the lateral Cauchy data.

Part 3. A globally convergent numerical method called *convexification* [3]. The convexification is applicable to a broad class of coefficient inverse problems for both hyperbolic and parabolic Partial Differential equations.

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Also see http://www.math.uncc.edu/people/research/mklibanv.php3 for a copy.

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Inverse problems with imperfectly known boundary

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In many inverse problems the inaccurate model of the boundary causes severe errors for the reconstructions. This has a crucial impact in medical imaging as in practical measurements one usually lacks the exact knowledge of the boundary. In this talk we review recent methods for solving inverse problems with inaccurately modeled boundary. In particular, we consider a method developed together with Ville Kolehmainen (Univ. of Kuopio) and Petri Ola (Univ. of Helsinki) to eliminate the error caused by an incorrectly modeled boundary in electrical impedance tomography (EIT). Using an algorithm based on Teichmuller mappings and optimization methods we can find a unique minimally anisotropic conductivity in the inaccurately modeled domain that agrees with the boundary measurements. Using this conductivity we can also obtain a deformed image of the original isotropic conductivity.

Gradient Estimates for Solutions to the Conductivity Problem

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We establish upper and lower bounds on the gradient of solutions to the conductivity problem in the case where two circular conductivity inclusions in two dimensions, or spherical inclusions in three dimensional case, are very close but not touching. These bounds depend on the conductivities of the inclusions, their radii, and the distance between them. Their novelty is that they give very specific information about the blow up of the gradient as the conductivities of the inclusions degenerate.

On the mathematical issues and numerical implementations of MREIT problem

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Magnetic Resonance Electrical Impedance Tomography (MREIT) is a new medical imaging technique that aims to provide electrical conductivity images with sufficiently high spatial resolution and accuracy. Traditional Electrical Impedance Tomography (EIT) applies the boundary measurement of voltage and current to reconstruct the interior parameter of media, which is of severe ill-posedness, whereas the MREIT technique takes the interior induced electrical current as inversion input data to recover the property of media. The advantage of this new inversion method is its relatively weak ill-posedness due to the application of interior measurement information. Physically, the internal induced electrical current is obtained indirectly from the measurement of magnetic flux density in terms of Maxwell relation.

This practical model gives some restrictions on the inversion schemes and some problems are still open. Firstly the magnetic flux density is obtained only along one direction, which is of the unavoidable noise. Secondly, some mathematical issues of recently developed MREIT-based algorithms such as convergence analysis and denoising technique should be considered. Finally, the numerical implementations of the inversion algorithms using the actual measurement data should also be tested.

The talk will focus on the above topics and present some numerical results.

This is a joint-work with J.K.Seo and E.J.Woo at the Impedance Imaging Research Center (IIRC) of Korea.

Solved and open identification problems related to differential and integro - differential equations

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The talk will be concerned with a few inverse problems chosen among the following ones:

1. recovering unknown kernels depending on time only in integro-differential delay equations;
2. recovering unknown kernels depending on time only in phase transition problems with memory;
3. recovering the 2D-spatial part in unknown kernels in viscoelastic problems;
4. recovering unknown kernels depending on time only in transmission problems related to thermal materials with memory;
5. recovering unknown constants in parabolic equations.

The subjects under 3, 4, 5 are at present under investigation.

Inverse electromagnetic scattering problems with chiral obstacle

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Let us consider the electromagnetic fields which are governed by the time-harmonic Maxwell equations:

$$\nabla \times E - i\omega B = 0,$$

$$\nabla \times H + i\omega D = 0,$$

where E, H, D and B denote the electric field, the magnetic field, the electric and magnetic displacement vectors in R^3, respectively.

Assume that $\widetilde{D} = \{x \in R^3 | (x_1, x_2) \in D, x_3 \in R\}$ is an infinitely long cylinder parallel to the x_3-direction. Here D is the cross-section of $\widetilde{D}$ in the x_1-x_2-plane, bounded domain in R^2 with $C^{2,\alpha}$ boundary ∂D, $\alpha \in (0, 1)$.

In chiral media $\widetilde{D}$, the electric and magnetic fields are coupled which can be characterized by the Drude-Born-Fedorov constitutive equations::

$$D = \varepsilon(x)(E + \beta(x)\nabla \times E),$$

$$B = \mu(x)(H + \beta(x)\nabla \times H),$$

where ε is the electric permittivity, μ is the magnetic permeability, and β is the chirality admittance for chiral media.

In $R^3 \setminus \widetilde{D}$, the total exterior fields $E = E^i + E^s, H = H^i + H^s$ satisfy the Maxwell's equation: $\nabla \times E = ikH,\ \nabla \times H = -ikE$. Here $k = \omega\sqrt{\varepsilon\mu}$, and the incident field $E^i = E^i(x, d, p) = ik(d \times p) \times de^{ikx\cdot d}$, $H^i = H^i(x, d, p) = ik(d\times p)e^{ikx\cdot d},\ |d| = |p| = 1$ denote incident plane electromagnetic fields with polarization p and incident direction d.

In our paper, we study the inverse scattering problem: for given far fields of scattering fields $E^s(x, d, p)$ and $H^s(x, d, p)$, reconstruct the boundary of chiral obstacle $\widetilde{D}$.

we present the theoretical analysis and numerical methods of this inverse scattering problem.

Magnetic Tomography: from new algorithms to industrial applications

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Magnetic tomography is of importance in several applications, from biomedical imaging to nondestructive testing. We will review recent work on the theory of magnetic tomography, analyzing the null-space of the Biot-Savart integral operator. Then, we will discuss the application of new sampling and probe methods to the reconstruction of defects. Finally, we will investigate the applicability of magnetic tomography for the investigation of fuel cells in an industrial setting. Numerical and real data reconstructions will be shown.

A fast imaging for UWB pulse radars

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A UWB pulse radar is a promising candidate as an environment measurement technique for a variety of applications including robots. The radar imaging is known as one of the ill-posed inverse problems, for which many algorithms have been studied. However, they require a long calculation time, which is not acceptable for realtime operations for robotics. In order to solve this problem, we have developed a fast imaging algorithm for UWB pulse radars, SEABED algorithm, which utilizes the reversible transform between the real space and the data space. This transform directly gives the target image without iterative methods, which is the reason why SEABED algorithm works so quickly. Although this transform is valid only for targets with clear boundaries, this condition is naturally satisfied for most of indoor objects.

We assume a mono-static radar system, where an omni-directional antenna is scanned on the x-y plane. A strong echo is received from (x, y, z), a point on a target boundary, for the antenna position $(X, Y, 0)$ with a delay $Z = ct/2$ where t is the time delay, c is the speed of radiowave. The forward transform, BST (Boundary Scattering Transform) is expressed as

$$\left\{ \begin{array}{lcc} X & = & x + z\partial z/\partial x, \\ Y & = & y + z\partial z/\partial y, \\ Z & = & z\sqrt{1 + (\partial z/\partial x)^2 + (\partial z/\partial y)^2}. \end{array} \right. \tag{1}$$

We have clarified that the inverse transform of BST, IBST (Inverse BST) is expressed as

$$\left\{ \begin{array}{lcc} x & = & X - Z\partial Z/\partial X, \\ y & = & Y - Z\partial Z/\partial Y, \\ z & = & Z\sqrt{1 - (\partial Z/\partial X)^2 - (\partial Z/\partial Y)^2}. \end{array} \right. \tag{2}$$

SEABED algorithm deals with the transform IBST from the data space (X, Y, Z) to the real space (x, y, z), which corresponds to the imaging procedure. First, a set of points (X, Y, Z) in the data space are extracted as equiphase-surfaces from the received signals. Next, IBST is applied to the extracted surfaces (X, Y, Z) to obtain the reconstructed image in the real space. Fig. 1 shows an application example of SEABED algorithm, where the left image is the true target shape, and the right image is the estimated image. The calculation time to obtain the entire image is 0.1sec with a single Xeon 2.8GHz processor.

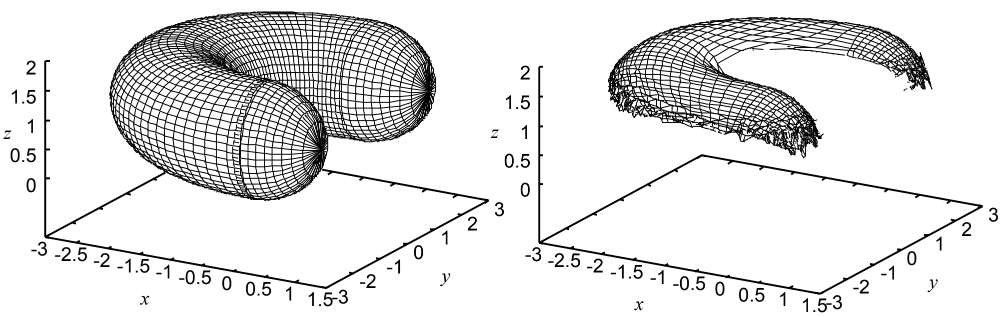

Figure 1: The estimated image by SEABED algorithm (Calculation within 0.1sec).

Numerical method for an inverse dynamical problem for composite beams

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In this talk, we present a method of numerical reconstruction for an inverse dynamical problem of composite beam. The direct problem is to find the displacement vector $w = (u_1(x,\,t), u_2(x,\,t),\, v_1(x,\,t),\, v_2(x,\,t))^T$ of steel-concrete composite beams with length L satisfied the following form:

$$\begin{cases} Cw_{,tt} - A_{k,\mu}w = 0 & \text{in } (0,\,L)\times(0,\,T)\,, \\ w|_{t=0} = 0\,, \quad w_{,t}|_{t=0} = 0 & \text{in } (0,\,L)\,, \\ Dw|_{x=0} = \overline{U}\,, \quad Dw|_{x=L} = 0 & \text{on } (0,\,T)\,, \end{cases}$$

where C is the 4×4 diagonal matrix $C = \operatorname{diag}(\rho_1,\,\rho_2,\,\rho_1,\,\rho_2)$. $A_{k,\,\mu}$ is the spatial differential operator defined by

$$A_{k,\,\mu}w = \begin{pmatrix} (a_1u_{1,x})_{,x} + k(u_2 - u_1 + v_{2,x}e_s) \\ (a_2u_{2,x})_{,x} - k(u_2 - u_1 + v_{2,x}e_s) \\ -(j_1v_{1,xx})_{,xx} + \left(\dfrac{ke_c^2}{6}(2v_{1,x} + v_{2,x})\right)_{,x} - \mu(v_1 - v_2) \\ -(j_2v_{2,xx})_{,xx} + \left(\dfrac{ke_c^2}{6}(2v_{2,x} + v_{1,x})\right)_{,x} + (k(u_2 - u_1 + v_{2,x}e_s)e_s)_{,x} + \mu(v_1 - v_2) \end{pmatrix}$$

and D is the operator given by $Dw = (u_1,\,u_2,\,v_1,\,v_2,\,v_{1,x},\,v_{2,x})^T$. Here we denote by ρ_i the linear mass density of i-th beam, by a_i and j_i the section's flexural stiffness and axial stiffness of i-th beam respectively. The coefficients k and μ are, respectively, the shearing and the axial stiffness for unit length of the connection of two beams. The coefficients ρ_i, a_i, j_i, and the Dirichlet type boundary data $\overline{U}$ are assumed to be given.

Our inverse problem is to determine two coefficients k and μ from the knowledge of Neumann type boundary data $\overline{Q} = (-\overline{N}_1,\,-\overline{N}_2,\,-\overline{T}_1,\,-\overline{T}_2,\,-\overline{M}_1,\,-\overline{M}_2)^T$ at $L = 0$ and the inner measurements $\overline{v}_i = v_i|_I$ $(i = 1, 2)$. Here $\overline{N}_i$ means the axial force of i-th beam. $\overline{T}_i$ and $\overline{M}_i$ are the shear force and the bending moment respectively. I is given open interval such that $I \subseteq [0,\,L]$.

The purpose of this talk is to present an numerical method for the identification of unknown coefficients k and μ. To identify numerically these coefficients, we make use of the variational method. We introduce a cost functional of two variables by using the measurements, and the unknown coefficients are identified by finding a minimum of this functional. We make use of the projected gradient method to find the minimum of functional. We show theoretically the existence of the derivatives under appropriate assumptions. Moreover we show the effectiveness of our method from some numerical experiments.

Geodesic tensor tomography for a class of non-simple manifolds with boundary

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On a compact manifold with boundary $(M, \partial M, g)$, we study the geodesic X-ray transform

$$If(\gamma) = \int_\gamma \langle f, \dot\gamma^2 \rangle \, dt,$$

where, in local coordinates, $\langle f, \dot\gamma^2 \rangle = f_{ij}\dot\gamma^i\dot\gamma^j$, and f is a symmetric 2-tensor. We study the question of s-injectivity of I_Γ defined as I restricted to a certain open set $\Gamma \ni \gamma$. S-injectivity means that $I_\Gamma f = 0$ implies that $f = dv$, for some 1-form v vanishing on ∂M, where d is the symmetric differential. The main assumption is that for each $(x, \xi) \in T^*M$, there is a geodesic $\gamma \in \Gamma$ through x normal to ξ with endpoints outside M (we extend M and g near M) and no conjugate points. Some topological assumptions are imposed but no convexity of the boundary is assumed. There might be geodesics with conjugate points, or even periodic or trapped ones. The main results are:

- S-injectivity for analytic metrics,
- Recovery of singularities,
- Stability estimate,
- Generic s-injectivity for a family of metrics g, and geodesic sets Γ_g.

Some Computational Aspects for Geophysical Inverse Problems

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By use of the least square technique, many geophysical inverse problems can be formulated into optimization problems. The optimizing iteration processes for those problems are usually time consuming since the cost functions are with a great number of variables and a great times of computation for numerically solving the direct problems should be carried out while we evaluate the cost functions and their derivatives. Therefore it is necessary to accelerate both the numerical solution for direct problems and the optimization algorithm. In this paper, some parallel, multi-scaling techniques and comparison are used to accelerate either the numerical solution of the direct problems or the optimizing iteration by noticing special features of some geophysical problems.

Perturbation of Rayleigh-wave velocity caused by a fully anisotropic term

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The elastic wave equations for dynamic deformations of the homogeneous elastic medium are

$$\sum_{j,k,l=1}^{3} C_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} = \rho \frac{\partial^2}{\partial t^2} u_i, \qquad i = 1, 2, 3. \tag{1}$$

Here (x_1, x_2, x_3) is the Cartesian coordinates, $\mathbf{C} = \big(C_{ijkl}\big)_{i,j,k,l=1,2,3}$ is the elasticity tensor, which has the physically natural symmetries

$$C_{ijkl} = C_{jikl} = C_{klij}, \qquad i, j, k, l = 1, 2, 3,$$

and satisfies the strong convexity condition, $\boldsymbol{u} = \boldsymbol{u}(\mathbf{x}, t) = (u_1, u_2, u_3)$ is the displacement of $\mathbf{x} = (x_1, x_2, x_3)$ at the time t, and ρ is the uniform mass density.

Suppose that the elasticity tensor $\mathbf{C} = \big(C_{ijkl}\big)_{i,j,k,l=1,2,3}$ has the form

$$\mathbf{C} = \mathbf{C}^{\mathrm{Iso}} + \mathbf{A}, \tag{2}$$

where $\mathbf{C}^{\mathrm{Iso}}$ is the isotropic part of $\mathbf{C}$,

$$\mathbf{C}^{\mathrm{Iso}} = \big(C^{\mathrm{Iso}}_{ijkl}\big)_{i,j,k,l=1,2,3}, \qquad C^{\mathrm{Iso}}_{ijkl} = \lambda\, \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{kj})$$

with the Lamé moduli λ and μ, and $\mathbf{A}$ is the fully anisotropic part of $\mathbf{C}$,

$$\mathbf{A} = \big(a_{ijkl}\big)_{i,j,k,l=1,2,3}.$$

Rayleigh waves are elastic surface waves which propagate along the traction-free surface with the phase velocity in the subsonic range and whose amplitude decays exponentially with depth below that surface. These waves can be discribed by the surface-wave solutions to (1).

In an isotropic half-space, where $\mathbf{C} = \mathbf{C}^{\mathrm{Iso}}$ (i.e., $\mathbf{A} = \mathbf{O}$), the phase velocity v_R^{Iso} of Rayleigh waves which propagate along the surface is the unique solution to the following bicubic equation in v in the subsonic range $0 < v < \sqrt{\mu/\rho}$

$$(\rho\, v^2)^3 - 8\mu\,(\rho\, v^2)^2 + \frac{8\mu^2(3\lambda + 4\mu)}{\lambda + 2\mu}\,\rho\, v^2 - \frac{16\mu^3(\lambda + \mu)}{\lambda + 2\mu} = 0.$$

In this presentation we investigate the perturbation of the phase velocity v_R of Rayleigh waves, i.e., the shift in v_R from its isotropic value, caused by the anisotropic part $\mathbf{A}$. To determine anisotropy of the elastic medium from observations of Rayleigh-wave velocity is a classical and important inverse problem in nondestructive testing. As a first step, we give the formula for v_R which is correct to within terms linear in the components of $\mathbf{A}$.

Therorem In a perturbed anisotropic elastic medium whose elasticity tensor $\mathbf{C}$ is given by (2), the phase velocity of Rayleigh waves which propagate along the surface of the half-space $x_3 \le 0$ in the direction of the 2-axis can be written, to within terms linear in the anisotropic part $\mathbf{A} = \big(a_{ijkl}\big)_{i,j,k,l=1,2,3}$, as

$$v_R \;=\; v_R^{\mathrm{Iso}} - \frac{1}{2\rho\, v_R^{\mathrm{Iso}}} \cdot \Big[\gamma_1(v_R^{\mathrm{Iso}})\, a_{2222} + \gamma_2(v_R^{\mathrm{Iso}})\, a_{2323} + \gamma_3(v_R^{\mathrm{Iso}})\, a_{2233} + \gamma_4(v_R^{\mathrm{Iso}})\, a_{3333}\Big], \tag{3}$$

where $\gamma_i(v), i = 1, 2, 3, 4$ are the functions of v which can be written explicitly in terms of λ, μ and ρ.

Remark Only four components $a_{2222}, a_{2323}, a_{2233}$ and a_{3333} of the anisotropic part $\mathbf{A}$ can influence the first order perturbation of the phase velocity v_R.

Inverse Boundary Problems with Incomplete Data

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We consider several inverse boundary problems where we have only incomplete data. This information is encoded in the Dirichlet-to-Neumann (DN) map measured in open subsets of the boundary. We survey some recent results for several inverse problems, including Calderón's inverse conductivity problem, the DN map associated to the Schrödinger equation with a magnetic and electrical potential, and the DN map associated isotropic elasticity system. We consider also applications to determining inclusions and cavities from incomplete data in all of these cases.

Detection of irregular points by regularization in numerical differentiation and an application to the edge detection

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The numerical differentiation is a typical ill-posed problem which can be treated by the Tikhonov regularization. In this paper, we prove that the L^2-norms of the second order derivatives of the regularized solutions blow up in any small interval I where the exact solution is not in $H^2(I)$. This generalizes the previous results by Wang, Jia and Cheng where the interval I is assumed to be whole interval. One application in the image edge detection is presented.

Inverse scattering problem for time dependent Hartree-Fock equation

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The following non-linear Schrödinger equation is called time dependent Hartree-Fock equation (TDHF):

$$i\frac{\partial \boldsymbol{u}}{\partial t} = H_0\boldsymbol{u} + \int_{\mathbf{R}^n} Q(x,y,t)\bar{\boldsymbol{u}}(y,t)dy,$$

where $\boldsymbol{u} = \boldsymbol{u}(x,t) = {}^t(u_1(x,t)\cdots,u_N(x,t))$, $(x,t) \in \mathbf{R}^n \times \mathbf{R}$ and $H_0 = -\Delta$. Q is the $N \times N$ matrix in the following form

$$\begin{aligned} Q(x,y,t) &= (V_{jk}(x,y,t))_{1\le j,k\le N} \\ V_{jk}(x,y,t) &= v_{jk}(x-y)\{u_j(x,t)u_k(y,t) - u_k(x,t)u_j(y,t)\}, \end{aligned}$$

where $v_{jk}(x)$ are interactions acting on j-th and k-th particles. TDHF is derived in order to obtain an approximate solution of linear N-body Schrödinger equation based on Pauli's exclusion principle.

Denote by $\{\mathcal{S}[\boldsymbol{\phi}](x), \boldsymbol{\phi}(x)\}$, $\boldsymbol{\phi}(x) = {}^t(\phi_1(x)\cdots,\phi_N(x))$ the scattering data for (TDHF). If all functions $v_{jk}(x)$ tends to zero sufficiently fast for $|x| \to \infty$, then $\mathcal{S}[\boldsymbol{\phi}](x)$ is represented as follows.

$$\begin{aligned} \mathcal{S}[\boldsymbol{\phi}](x) &= \boldsymbol{\phi}(x) + \frac{1}{i}\int_{\mathbf{R}} e^{itH_0}F(\boldsymbol{u}(t))dt, \qquad (1) \\ F(\boldsymbol{u}(t)) &= \int_{\mathbf{R}^n} Q(x,y,t)\bar{\boldsymbol{u}}(y,t)dy. \end{aligned}$$

Inverse scattering problem is: given the scattering data, find $v_{jk}(x)$, $j,k = 1,\cdots,N$.

This problem is mathematically restated as follows. Given functions $\mathcal{S}[\boldsymbol{\phi}](x)$ and $\boldsymbol{\phi}(x)$, solve the integral equation (1) with respect to unknown functions $v_{jk}(x)$, $j,k = 1,\cdots,N$.

In this talk, it will be shown that interactions, which is highly specific, are reconstructed from the scattering data in the case of three particles $N = 3$.

Theorem Let $N = 3$ and interactions be the following form:

$$v_{jk}(x) = \lambda_j|x|^{-\sigma_j}, \qquad \lambda_j \in \mathbf{R}, \quad 2 \le \sigma_j \le 4 \text{ and } \sigma_j < n, \quad j = 1,2,3.$$

Then there exist the scattering data such that we reconstruct $v_{jk}(x)$, that is, λ_j and σ_j, $j = 1,2,3$.

The proof of this theorem is based on the linearization by using the small amplitude limit of the scattering data. Reconstruction formulae derived in this theorem do not hold obviously for linearly dependent ϕ_j, $j = 1,2,3$. The difficult point of the proof is to characterize $\boldsymbol{\phi}(x)$ satisfying these reconstruction formulae. By controlling supports of functions ϕ_j theorem is proved.

Conductivity imaging using EIT and MREIT techniques: experimental results

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When we inject current into an electrically conducting object such as the human body, it induces voltage, current density, and magnetic flux density distributions. These distributions are determined by the geometry, electrode configuration, and conductivity distribution of the object. Information about the internal conductivity distribution is of significant importance in many biomedical applications.

Conductivity image reconstruction has been the active research goal of Electrical Impedance Tomography (EIT) since early 1980s. EIT utilizes measured current-voltage data on the boundary to provide cross-sectional images of internal conductivity distribution. Even though it has a limited spatial resolution due to the ill-posed nature of the corresponding inverse problem, it has significant advantages of high temporal resolution and portability. For all potential clinical applications of EIT, very accurate boundary current-voltage measurements must be provided to any EIT image reconstruction algorithm. In this talk, we describe the development of a new multi-frequency EIT system. With the operating frequency range of 10Hz to 500kHz, the EIT system can produce time-difference and also frequency-difference images in real time. Numerous performance indices including signal-to-noise ratio, reciprocity error, and distinguishability will be presented together with reconstructed EIT images of several conductivity phantoms and human subjects. Clinical applicability of the EIT system and its future improvements in both hardware and software will be discussed.

Magnetic Resonance Electrical Impedance Tomography (MREIT) has been lately suggested to overcome the ill-posedness of the image reconstruction problem in EIT. The key idea is to utilize internal magnetic flux density data that can be measured by using an MRI scanner. After discussing the measurement techniques and image reconstruction algorithms in MREIT, we will present experimental data and reconstructed conductivity images of several tissue phantoms and animal subjects. We will show that the spatial resolution of MREIT images is comparable to that of conventional MR images as long as we inject enough current. Summarizing latest outcomes of the MREIT research, we will suggest future research directions to make MREIT a new clinically applicable conductivity and current density imaging technique.

Nonlinear Multigrid Gradient Methods for Parameter Identifications

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In this talk, we shall discuss the nonlinear multigrid gradient method for efficiently identifying physical parameters in nonlinear inverse systems. We will address the motivation and correct formulation of the method and their detailed performance in numerical identifications. This is a join work with Jingzhi Li (Department of Mathematics, CUHK).

Lectures and Tutorials

H. W. Engl

C. Groetsch

R. Kress

N. Tosaka

Nonlinear Inverse Problems: Functional Analytic Theory, Numerical Methods, Applications

Heinz W. Engl
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Since our task is to give a survey indicating future lines of research, we first present the functional analytic theory of nonlinear inverse problems and their regularization by Tikhonov's method and by iteration. This includes questions of convergence, convergence rates and implementable parameter choice and stopping rules for such procedures. We give some numerical examples from parameter identification in partial differential equations.

The (by now) classical theory usually works in Hilbert spaces, which is quite restrictive. In many applications, one needs a regularization term which does not live in a Hilbert space involving, e.g., the total variation seminorm or entropy-like terms. Some convergence theory (based on Bregman distances) is available, but there are also open questions.

A further restriction of the classical theory is that the error concept is deterministic, bounds for data noise are given in terms of norms in function spaces. This gives rise to worst-case error estimates for regularized solutions. Such concepts neglect statistical error concepts. We present a recently developed theory for convergence in distribution of regularized solutions of stochastic ill-posed problems; the tool used is the Prokhorov distance of probability measures. We also indicate relations to others stochastic approaches to inverse problems including Bayesian methods.

Application fields which will present new challenges to our community include mathematical finance and systems biology. We present some examples from these fields, e.g., identification of two-factor interest rate models in financial derivatives and of metabolic and genetic networks.

The results we present have many coauthors from our group, whom we will acknowledge in talk.

Integral Equations of the First Kind, Inverse Problems and Regularization: A Crash Course

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This paper is an expository survey of the basic theory of regularization for Fredholm integral equations of the first kind and related background material on inverse problems. We begin with an historical introduction to the field of integral equations of the first kind, with special emphasis on model inverse problems that lead to such equations. The basic theory of linear Fredholm equations of the first kind, paying particular attention to E. Schmidt's singular function analysis, Picard's existence criterion, and the Moore-Penrose theory of generalized inverses is outlined. The fundamentals of the theory of regularization are then treated and a collection of exercises and a bibliography are provided.

Uniqueness in inverse obstacle scattering

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The inverse problem we consider in this tutorial is to determine the shape of an obstacle from the knowledge of the far field pattern for the scattering of time-harmonic acoustic waves. We will concentrate on uniqueness issues, i.e., we will investigate under what conditions an obstacle and its boundary condition can be identified from a knowledge of its far field patterns for incident plane waves. We will review some classical and some recent results and draw attention to open problems.

Numerical methods in inverse obstacle scattering

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For the approximate solution of the inverse obstacle scattering problem to reconstruct the boundary of an impenetrable obstacle from the knowledge of the far field pattern for the scattering of time-harmonic acoustic waves, roughly speaking, one can distinguish three groups of methods. **Iterative methods** interpret the inverse obstacle scattering problem as a nonlinear ill-posed operator equation and apply iterative schemes such as regularized Newton methods or Landweber iterations for its solution. **Decomposition methods**, in principle, separate the inverse problem into an ill-posed linear problem to reconstruct the scattered wave from its far field pattern and the subsequent determination of the boundary of the scatterer from the boundary condition. Finally, the third group consists of the more recently developed **sampling and probe methods**. In principle, these methods are based on criteria in terms of an indicator function that decides whether a point lies inside or outside the scatterer. The tutorial will give a survey by describing one or two representatives of each group including a discussion on the various advantages and disadvantages.

Inverse analysis with use of filter theory

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There exist many kinds of inverse problems in applied science and engineering which are govened by differential equatin(s). These can be formulated as inverse problems of differential equation(s), and these are very difficut to solve analytically or numerically. However, the impotance of inverse analysis has been recognized in various fields. Then, an effective solution procedure to solve inverse problems is required. In practical applications of inverse analysis, the measured data obtained through field obsevations usually contain noise. Consequenently, the mathematical model for the inverse problem must be formulated with the stochastic consideration. In this tutorial talk a solution procedure in order to solve the above-mentioned mathematical model is developed and explained the details.

The mathematical model of discretized inverse problems is formulated as follows:

the observation equation,

$$\mathbf{y} = \mathbf{m}(\mathbf{z}) + \mathbf{n}$$

the restoration eqation,

$$\tilde{\mathbf{z}} = \mathbf{B}(\mathbf{y})$$

the estimation criterion,

$$J = E_{\mathbf{n}}(\| \tilde{\mathbf{z}} - \mathbf{z} \|^2)$$

in which $\mathbf{z}$ is the original vector, $\tilde{\mathbf{z}}$ is the estimated vector, $\mathbf{y}$ is the observation vector, $\mathbf{n}$ is the noise vector, $\mathbf{m}$ is the observation mapping and $\mathbf{B}$ is the restoration mapping. Some solution procedures corresponding to three kinds of filter by applying the filter theory to the above stochastic model are constructed in on-line identification.

The applicability and effectveness of the method is demonstrated by using numerical performances on the defect identification and the damage identification problems in engineering fields.

General Speakers

P. D. Alain
A. Amirov
F. Bauer
A. Benaddallah
M. Cristo
M. Cristofol
Y. Daido
H. Fang
J. Foukzon
C. Groetsch
T. Hohage
Y. C. Hon
S. Huang
H. Igarashi
M. Ikehata
K. Ito
X. Jia
Y. Kamimura
H. Kang
H. Kawakami
M. Kawashita
S. Kubo
P. Kuegler
K − M. Lee
K − H. Leem
D. Lesnic
G. Li
S. Li
X. Li
C − L. Lin
M. Lukas
X. Luo
Y. Ma
S. Nagayasu
T. Nara
W. Ning
T. Ohe
H. Okano
H. K. Pikkarainen
J. L. Rousseau
S. Saitoh
M. Salo
H. Sasaki
A. Satoda
T. Shigeta
S. Shiota
M. Sini
T. Takeuchi
T. Takiguch
H. Takuwa
I. Trooshin
Q − F. Wang
Y. Wei
T. Yamazaki
K. Yoneda
G. Yuan
B. Zakhariev

Non homogeneous Heat Equation: Identification and Regularization for the Inhomogeneous Term

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We study the nonhogeneous heat equation under the form: $u_t - u_{xx} = \varphi(t)f(x)$, where the unknown is the pair of functions (u, f). Under various assumptions about the the function φ and the final value in $t = 1$ i.e. g(x), we propose different regularizations on this ill-posed problem based on the Fourier transform associated with a Lebesgue measure. For $\varphi \not\equiv 0$ the solution is unique [1].

Numerical results are given.

In two dimension this problem can be formulated in the following way: let Q be a heat conduction body and let $\varphi = \varphi(t)$ be given. We then consider the problem of finding a two-dimensional heat source having the form $\varphi(t)f(x, y)$ in Q. The problem is ill-posed. Assuming ∂Q is insulated and $\varphi \not\equiv 0$, we show that the heat source is defined uniquely by the temperature history on ∂Q and the temperature distribution in Q at the initial time $t = 0$ and at the final time $t = 1$. Using the method of truncated integration and the Fourier transform, we construct regularized solutions and derive explicitly error estimate [2]. In both cases the regularization is obtained by troncature on the domain and not on the integrand in the various integral forms as in [3] and [4].

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Unique Continuation and an Inverse Problem for Hyperbolic Equations Across a General Hypersurface

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For a hyperbolic equation $p(x,t)\partial_t^2 u(x,t) = \Delta u(x,t) + \sum_{j=1}^n q_j(x,t)\partial_j u + q_{n+1}(x,t)\partial_t u + r(x,t)u$ in $\mathbb{R}^n \times \mathbb{R}$ with $p \in C^1$ and $q_1, \ldots, q_{n+1}, r \in L^\infty$, we consider the unique continuation and an inverse problem across a non-convex hypersurface Γ. Let Γ be a part of the boundary of a domain and let $\nu(x)$ be the inward unit normal vector to Γ at x. Then we prove the unique continuation near a point x_0 across Γ if $\nabla p(x_0,t) \cdot \nu(x_0) < 0$. Moreover we establish the conditional stability in the continuation. Next we prove the conditional stability in the inverse problem of determining a coefficient $r(x)$ from Cauchy data on Γ over a time interval. The key is a Carleman estimate in level sets of paraboloid shapes.

Choosing the Regularization Parameter with Very Limited Information on the Noise Level

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Introduction

We will consider the following general ill-posed problem:

$$Ax = y$$

where A is a linear compact operator mapping between two Hilbert spaces $\mathcal{X}$ and $\mathcal{Y}$. Instead of y can just measure the noisy version $y^\delta = y + \delta\xi$ where ξ is a kind of normalized error vector.

For solving inverse problems of such kind a number of different approaches have been developed (E.g. Tikhonov, TSVD and many more). All of these approaches somehow balance between the unreliable input data y^δ and some kind of assumptions one thinks to previously know concerning the solution x. Normally this balance is expressed in terms of a regularization parameter. The key to successfully regularizing is getting the regularization parameter with the least possible knowledge on x and the noise $\delta\xi$.

Overview of choosing the regularization parameter

Classically in numerical mathematics one assumes $\|\xi\| \leq 1$ which allows to obtain optimal convergence rates w.r.t. δ with fast methods like Morozov's discrepancy principle. The Bakushinskii veto tells that without the knowledge of δ we can always construct problems which do not converge at any rate.

Furthermore looking at the problem from a more stochastical viewpoint we can assume that ξ is a Gaussian random vector (or a similar quantity) which is much more realistic; the classical one is just a subcase of that. Having δ at hand we can still regularize in expectation in an (almost) rate optimal way, e.g. using a Lepskij-type balancing principle for choosing the regularization parameter. (Almost rate optimal here always means that we are loosing a logarithmic factor).

On the other hand from a completely stochastical viewpoint where we also assume some kind of stochastical prior on the distribution of x one can regularize (almost rate optimal) even without having the exact δ at hand. One possibility is generalized cross validation for searching a proper regularization parameter. However these methods are in comparison to ones like the discrepancy principle rather slow. Furthermore quite often the priors assumed are not at all similar to the ones normally used in numerical mathematics.

Our new approach

We will assume that ξ is again a Gaussian random vector and x is fulfilling some very general distribution assumptions oriented at the notion of Hilbert scales. However we will not need in what space along the Hilbert scale x is actually situated.

In the case of truncated singular value decomposition (TSVD) with cut-off point n we will construct a functional $f(y^\delta, A, n)$ which in expectation has its minimum at the optimal regularization parameter. The construction of this functional is comparably fast to Morozov's discrepancy principle.

Out of this functional we are able to prove that we get (almost) rate optimal convergence with respect to δ. Into f enter just quantities we know, therefore we get a completely data driven method which just generates the knowledge about δ it needs out of y^δ.

Please note that this is not a contradiction to the Bakushinskii veto because this method just works in expectation, i.e. the bad cases though existent are so rare that they do not contribute.

Numerical experiments

In the end we will present numerical results comparing this new method with other choosing strategies for the regularization parameters; not just for TSVD but also for Tikhonov regularization.

Surprisingly, for a considerable number of examples this new method produces better results without knowing δ than other methods which additionally have the knowledge of the noise level.

Inverse problems for the heat equation with discontinuous diffusion coefficients

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The question of the identification of a diffusion coefficient, c, is studied for the heat transfer problem in a bounded domain, with the main particularity that c is discontinuous. Such regularity can be encountered in the case of embedded materials. Let $\Omega \subset \mathbb{R}^n$ be a bounded connected open set. The set $\overline{\Omega}$ is assumed to be a $\mathcal{C}^2$ submanifold with boundary in $\mathbb{R}^n$. Let Ω_0 and Ω_1 be two non-empty open subsets of Ω such that

$$\Omega_0 \Subset \Omega, \text{ and } \Omega_1 = \Omega \setminus \overline{\Omega}_0.$$

We denote by $S = \overline{\Omega}_0 \cap \overline{\Omega}_1$ the interface, which is assumed to be $\mathcal{C}^2$ and we denote by n the outward unit normal to Ω_1 on S and also the outward unit normal to Ω on Γ. Let S_0 (resp. S_1) be the side of the interface S corresponding to the positive (resp. negative) direction of the normal n.
Let $T > 0$. We shall use the following notations $\Omega' = \Omega_0 \cup \Omega_1$, $Q' = \Omega' \times (0,T)$, and $\Sigma = \Gamma \times (0,T)$.
We consider the following transmission problem for the heat equation:

$$\begin{cases} \partial_t y - \nabla \cdot (c \nabla y) = 0 & \text{in } Q', \\ y(x,t) = h(t,x) & \text{in } \Sigma, \\ \text{transmission conditions (TC1)} & \text{on } S \times [0,T], \\ y(0,x) = y_0(x), & \text{in } \Omega, \end{cases} \tag{1}$$

with

$$\begin{cases} y_{|S_0 \times [0,T]} = y_{|S_1 \times [0,T]}, \\ c_0 \partial_n y_{|S_0 \times [0,T]} = c_1 \partial_n y_{|S_1 \times [0,T]}. \end{cases} \tag{TC1}$$

We assume a monotonicity on the coefficients c in connection to the observation location: the observation zone has to be located in the region where the coefficient is the smallest. Let y be the solution of (1) associated to c and $\widetilde{y}$ the solution of (1) associated to another coefficient $\widetilde{c}$. We assume that we can measure both the normal flux $\partial_n \partial_t y$ on $\gamma \subset \partial\Omega$ on the time interval (t_0, T) for some $t_0 \in (0,T)$ and Δy in Ω at time $T' \in (t_0, T)$. The interior of γ is non empty with respect to the topology on Γ induced by the Euclidian topology on $\mathbb{R}^n$. In the case of piecewise *constant* diffusion coefficients, i.e. $c_{|\Omega_i}$ and $\widetilde{c}_{|\Omega_i}$, $i = 0, 1$, are constant, our main results are
(i) the injectivity of the map

$$\begin{gathered} L^\infty(\Omega) \times L^2(\Omega) \to L^2((t_0,T) \times \gamma) \times L^2(\Omega), \\ (c, y_0) \mapsto (\partial_n \partial_t y, \Delta y(T')); \end{gathered}$$

(ii) the stability for the diffusion coefficient, c: there exists $C > 0$ such that

$$|c - \widetilde{c}|^2_{L^\infty(\Omega)} \le C |\partial_n(\partial_t y - \partial_t \widetilde{y})|^2_{L^2((0,T)\times\gamma)} + C|\Delta y(T', .) - \Delta \widetilde{y}(T', .)|^2_{L^2(\Omega')};$$

(iii) the stability for the initial condition, y_0: there exists $C > 0$ such that

$$|y_0 - \widetilde{y}_0|_{L^2(\Omega)} \le C / \ln \Big(|(y - \widetilde{y})(T')|_{H^2(\Omega')} + |\partial_n(\partial_t y - \partial_t \widetilde{y})|^2_{L^2((0,T)\times\gamma)} \Big).$$

The key ingredient to these stability results is a *global* Carleman estimate for the operator $\partial_t - \nabla \cdot (c\nabla(.))$ and the open set Ω. To obtain a Carleman estimate we have to introduce a geometric condition on Ω. The use of Carleman estimates to achieve uniqueness and stability results in inverse problems is now well-established. Some authors make use of *local* Carleman inequalities and deduce uniqueness and hölder estimates. Others make use of *global* Carleman inequalities and deduce Lipschitz stability results (and hence uniqueness results). We shall follow this second approach.

Stability properties of inverse parabolic problems with unknown boundaries

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We treat the stability issue for inverse problems arising from nondestructive evaluation by thermal imaging. We consider the determination of an unknown portion of the boundary of a thermic conducting body by overdetermined boundary data for a parabolic initial-boundary value problem.

Inverse Problems for a two by two reaction-diffusion system using Carleman estimate with one observation

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The problem of the identification of one or several coefficients in a reaction-diffusion system is studied in this paper. The main difficulty is to use as few observations as possible. We obtain a Carleman estimate with one control force and deduce a stability result for one (or two) coefficients using four (or five) localized observations. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain of $\mathbb{R}^n$. Let $T > 0$ and $t_0 \in (0, T)$. We consider the following reaction-diffusion system which arises in mathematical biology:

$$\begin{cases} \partial_t u = \Delta u + au + bv & \text{in } \Omega \times (t_0, T), \\ \partial_t v = \Delta v + cu + dv & \text{in } \Omega \times (t_0, T), \\ u(t,x) = h(t,x), \;\; v(t,x) = g(t,x) & \text{on } \partial\Omega \times (t_0, T), \\ u(t_0, x) = u_0 \text{ and } v(t_0,x) = v_0 & \text{in } \Omega, \end{cases} \tag{1}$$

where a, b, c, and d in $L^\infty(\Omega)$. The aim is to prove a stability result for the coefficient b (or a) or both coefficients a and b under additive positivity assumptions on the coefficients. Our paper is based on earlier works on controllability of phase-field systems and parabolic systems.
We obtain the following Lipschitz stability results

(i) Let ω be a subdomain of an open set Ω of $\mathbb{R}^n$. Let (u, v), (resp. $(\widetilde{u}, \widetilde{v})$) be solutions to (1) associated to (a, b, c, d, u_0, v_0) (resp. to $(a, \widetilde{b}, c, d, \widetilde{u_0}, \widetilde{v_0})$) satisfying regularity properties and an additive positivity hypothesis:
(h_1) Let $r > 0$, $b \geq 0$, $c \geq c_0 > 0$, $c + dr \geq 0$, $\widetilde{u}_0 > 0$, $\widetilde{v}_0 \geq r$, $h \geq r$ and $g \geq r$,
(h_2) $\widetilde{u}_0$ and $\widetilde{v}_0$ belong to $H^m(\Omega)$ for $m > \frac{n}{2} - 2$.
The first assumption allows us to give a maximum principle for $\widetilde{v}$:

$$r > 0, \; |\widetilde{v}(T', \cdot)| \geq r > 0 \quad \text{for} \quad T' = \frac{t_0 + T}{2} \quad \text{on } \Omega.$$

The second assumption gives regularity properties for $\widetilde{u}$ and $\widetilde{v}$ using classical Sobolev imbeding, $\widetilde{u}$ and $\widetilde{v}$ belong to $L^\infty(\Omega)$. Then there exists a constant $C = C(\Omega, \omega, c_0, t_0, T, r)$ such that

$$|b - \tilde{b}|^2_{L^2(\Omega)} \leq C|\partial_t v - \partial_t \tilde{v}|^2_{L^2((t_0,T)\times\omega)} + C|\Delta u(T', \cdot) - \Delta\tilde{u}(T', \cdot)|^2_{L^2(\Omega)} \\ + C|u(T', \cdot) - \tilde{u}(T', \cdot)|^2_{L^2(\Omega)} + C|v(T', \cdot) - \tilde{v}(T', \cdot)|^2_{L^2(\Omega)}.$$

To obtain a stability result for the coefficient a, we have to replace (h_1) by
(h_1') Let $r > 0$, $a \geq 0$, $c > 0$, $c + dr \geq 0$, $\widetilde{u}_0 \geq r$, $\widetilde{v}_0 > 0$, $h \geq r$ and $g \geq r$.

(ii)If (u, v), (resp. $(\widetilde{u}, \widetilde{v})$) are solutions to (1) associated to (a, b, c, d, u_0, v_0) (resp. to $(\widetilde{a}, \widetilde{b}, c, d, \widetilde{u_0}, \widetilde{v_0})$). Assume that assumption (h_2) of (i) is fulfilled and furthermore
(h_1'') Let $r > 0$, $ar + b \geq 0$, $c > 0$, $c + dr \geq 0$, $\widetilde{u}_0 \geq r$, $\widetilde{v}_0 \geq r$, $h \geq r$ and $g \geq r$, ,
(h_3) $(b - \widetilde{b}) \times (a - \widetilde{a}) \geq 0$,
then there exists a constant $C' = C'(\Omega, \omega, c_0, t_0, T, r)$ such that

$$|a - \widetilde{a}|^2_{L^2(\Omega)} + |b - \widetilde{b}|^2_{L^2(\Omega)} \leq C'|\partial_t u - \partial_t \widetilde{u}|^2_{L^2((t_0,T)\times\omega)} + C'|\partial_t v - \partial_t \widetilde{v}|^2_{L^2((t_0,T)\times\omega)} \\ + C'|\Delta u(T', \cdot) - \Delta\widetilde{u}(T', \cdot)|^2_{L^2(\Omega)} + C'|u(T', \cdot) - \widetilde{u}(T', \cdot)|^2_{L^2(\Omega)} + C'|v(T', \cdot) - \widetilde{v}(T', \cdot)|^2_{L^2(\Omega)}.$$

The key ingredient to these stability results is a *global* Carleman estimate for the system with one control force. The use of Carleman estimates to achieve stability and uniqueness results in inverse problems is now well-established.

Reconstruction of Inclusion for the Inverse Boundary Value Problem of Non-stationary Heat Equation

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Let $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ be a bounded domain. Γ which is boundary of Ω is C^2 if $n \geq 2$. Ω is considered as an isotropic heat conductive medium with heat conductivity:

$$\gamma(x,t) = 1 + (k-1)\chi_{D(t)}$$

for each $0 \leq t \leq T$ with $0 < T < \infty$. Here $k > 0$ is a constant such that $k \neq 1$, $D(t)$ is a bounded domain with C^2 boundary $\partial D(t)$ such that $\overline{D(t)} \subset \Omega$, $\Omega \setminus \overline{D(t)}$ is connected, the dependency of $\partial D(t)$ on $t \in [0,T]$ is C^1 and $\chi_{D(t)}$ is the characteristic function of $D(t)$. We set $D := \bigcup_{0 \leq t \leq T} D(t) \times \{t\}$.

Now, we consider the boundary value problem:

$$(MP) \quad \begin{cases} \big(P_D u\big)(x,t) := \partial_t u(x,t) - \mathrm{div}\big(\gamma(x,t)\nabla_x u(x,t)\big) = 0 \quad \text{in } \Omega_T \\ \partial_\nu u(x,t) = f(x,t) \quad \text{on } \Gamma_T, \quad u(x,0) = 0\,. \end{cases}$$

where $\cdot_T := \cdot \times (0,T)$. The physical meaning of u and f are the temperature and heat flux, respectively.

Theorem(Unique Solvability) For given $f \in H^{-\frac{1}{2},0}(\Gamma_T)$, there exists a unique solution $u = u(f) \in W(\Omega_T) := \{u \in H^{1,0}(\Omega_T) : \partial_t u \in L^2((0,T) : (H^1(\Omega))^*)\}$ to (MP).

Next, we define the Neumann to Dirichlet map Λ_D as follow.

Definition(Neumann-to-Dirichlet map) Let $u(f)$ be the solution to (MP). Define $\Lambda_D : H^{-\frac{1}{2},0}(\Gamma_T) \to H^{\frac{1}{2},0}(\Gamma_T)$ by

$$\Lambda_D(f) := u(f) \quad \text{on } \Gamma_T.$$

Now, we consider the inverse problems:
(IP) **Suppose k, D are unknown. Reconstruct D from Λ_D.**

Our main theorem is the following.

Theorem
If $n = 1$, there is a reconstruction procedure for the inverse problem for (IP) under some additional condition. The details of the reconstruction procedure will be given in my talk.

The uniqueness and stability of the identification are known. See [1] and [2], respectively. However, the reconstruction has not been known. For the reconstruction, we tied to develop the analogue of probe method known for elliptic inverse problem. This is the first attempt to study the reconstruction for the inverse boundary value problem for non-stationary heat equation.

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The Retrieval theory of GPS Dropsonde wind-finding system

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Wind measurement accuracy has been demonstrated to be a significant factor in airdrop accuracyThe key technology of GPS-dropsonde wind-finding system is to build reasonable and well-posed retrieved arithmetic. in physics and mathematics This paper study the following three questions:

(1) The research of direct problem. Based on the principle of fluid mechanics, a hydrodynamic equation for the GPS-dropsonde moving in the wind is proposed in this paper.. The responding errors of No.120 and No.20 balloon are analyzed in quantitative method; and we would proposed the reasonable arithmetic to avoid such errors.

(2) Inverse problem study. The improved theoretical framework of variational adjoint theory is developed based on the optimal theory, regularization idea and traditional adjoint methods. and this theory would be used to 3-D GPS-dropsonde retrieval model depends on the discrete.

(3) Assimilation study. From the view of information technology, time and space tendency of observations are assimilated to improve the results and the discrepancy between observations and model solution are treated in H1 space rather than L2 space. At the same time, How the wind retrieved from dropsonde was assimilated to Numerical prediction model was studied.

Inverse problem for Navier-stokes equation with external periodical imposed azimuthal magnetic field

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Consider a viscous electrically conducting incompressible fluid between two rotating infinite cylinders in the presence of an azimuthal external periodical magnetic field. The cylindrical system of coordinates (R, ϕ, z) is used. The equations govern the problem are

$$\begin{aligned} &\frac{\partial u_R}{\partial t} + (\mathbf{u}\nabla)\, u_R - \frac{u_\phi^2}{R} + \frac{1}{4\pi\rho}\left[(\mathbf{B}\nabla)\, B_R - \frac{B_\phi^2}{R}\right] \\ &= -\frac{1}{\rho}\frac{\partial}{\partial R}\left(P + \frac{B^2}{8\pi}\right) + \nu\left[\Delta u_R - \frac{2}{R^2}\frac{\partial u_\phi}{\partial \phi} - \frac{u_R}{R^2}\right] + \Psi_R + \varepsilon\frac{\partial W_{1,R}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial u_\phi}{\partial t} + (\mathbf{u}\nabla)\, u_\phi + \frac{u_\phi u_R}{R} + \frac{1}{4\pi\rho}\left[(\mathbf{B}\nabla)\, B_\phi - \frac{B_\phi B_R}{R}\right] \\ &= -\frac{1}{\rho R}\frac{\partial}{\partial \phi}\left(P + \frac{B^2}{8\pi}\right) + \nu\left[\Delta u_\phi + \frac{2}{R^2}\frac{\partial u_R}{\partial \phi} - \frac{u_\phi}{R^2}\right] + \Psi_\phi + \varepsilon\frac{\partial W_{1,\phi}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \end{aligned} \tag{2}$$

$$\frac{\partial u_z}{\partial t} + (\mathbf{u}\nabla)\, u_z + \frac{1}{4\pi\rho}(\mathbf{B}\nabla)\, B_z = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(P + \frac{B^2}{8\pi}\right) + \Psi_z + \varepsilon\frac{\partial W_{1,z}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \tag{3}$$

$$\frac{\partial u_R}{\partial R} + \frac{u_R}{R} + \frac{1}{R}\frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z} = 0, \tag{4}$$

$$\frac{\partial B_R}{\partial t} + (\mathbf{u}\nabla)\, u_R - (\mathbf{B}\nabla)\, u_R = \eta\left[\Delta B_R - \frac{2}{R^2}\frac{\partial B_\phi}{\partial \phi} - \frac{B_\phi}{R^2}\right] + \varepsilon\frac{\partial W_{2,R}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \tag{5}$$

$$\frac{\partial B_\phi}{\partial t} + (\mathbf{u}\nabla)\, u_\phi - (\mathbf{B}\nabla)\, u_\phi = \eta\left[\Delta B_\phi + \frac{2}{R^2}\frac{\partial B_R}{\partial \phi} - \frac{B_\phi}{R^2}\right] + \varepsilon\frac{\partial W_{2,R}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \tag{6}$$

$$\frac{\partial B_z}{\partial t} + (\mathbf{u}\nabla)\, u_z - (\mathbf{B}\nabla)\, u_z = \eta\Delta B_z + \varepsilon\frac{\partial W_{2,R}(R,\phi,z,t)}{\partial t\partial R\partial\phi\partial z}, \tag{7}$$

$$\frac{\partial B_R}{\partial R} + \frac{B_R}{R} + \frac{1}{R}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z} = 0, \tag{8}$$

where g is the gravity, ρ is the density, $\nu = \mu/\rho$ is the kinematic viscosity, η is the magnetic diffusivity, $\mathbf{u}$ is the velocity, $\mathbf{B}$ is the magnetic field, $\Psi = (\Psi_R, \Psi_\phi, \Psi_z)$ is the external periodical force, $W_1 = (W_{1,R}, W_{1,\phi}, W_{1,z})$ and $W_2 = (W_{2,R}, W_{2,\phi}, W_{2,z})$ is a standard Wiener processes, $\varepsilon \ll 1$.

We studied Cauchy problem and corresponding inverse problem for SPDE (1)-(8). For Cauchy problem the advanced numerical-analytical method analogous to method [1], [2], [3] is proposed. For corresponding inverse problem the linear operator equation is obtained.

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Integral Equation Models for the Inverse Problem of Biological Ion Channel Distributions

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Olfactory celia are thin hair-like filaments that extend from olfactory receptor neurons into the nasal mucus. Transduction of an odor into an electrical signal is accomplished by a depolarizing influx of ions through cyclic nucleotide gated channels in the membrane that forms the lateral surface of the celium. In an experimental procedure developed by S. Kleene a celium is detached at its base and drawn into a recording pipette. The celium base is then immersed in a bath of a channel activating agent (cAMP) which is allowed to diffuse into the celium interior opening channels as it goes and initiating a transmembrane current. The total current is recorded as a function of time and serves as data for a nonlinear integral equation of the first kind modeling the spatial distribution of ion channels along the length of the celium. We discuss some linear Fredholm integral equations that result from simplifications of this model. A numerical procedure is proposed for a class of integral equations suggested by this simplified model and numerical results using simulated and laboratory data are presented.

Characterization of the eigenvalues of the far–field operator

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The far–field operator of a scattering problem maps the density of an incident Herglotz wave function to the far–field pattern of the corresponding scattered field. It contains all information which can be observed from an object by scattering experiments at a fixed frequency in the far–field regime. Without absorption it is known that the far–field operator is compact and normal and that its eigenvalues lie on a circle through the origin. Recently, the far–field operator has been used as an operator in the linear sampling and the factorization method to reconstruct the scatterer, and a key step of the latter method is an eigenvalue decomposition of the far–field operator.

We will show two characterizations of eigenvalues of the far–field operator involving an eigenvalue equation in the exterior domain and on the boundary, respectively. In the following we only consider acoustic obstacle scattering problems, but similar results also hold true for electromagnetic and for medium scattering problems.

Let Ω_D and Ω_N be smooth, compact, disjoint, and simply connected subsets of $\mathbb{R}^3$, and denote by $\Omega_\mathrm{ext} := \mathbb{R}^3 \setminus (\Omega_\mathrm{D} \cup \Omega_\mathrm{N})$ the exterior domain. Moreover, we define the trace operator $\gamma u := (u|_{\partial\Omega_\mathrm{D}}, \frac{\partial u}{\partial n}|_{\Omega_\mathrm{N}})$ with values in $X_1 := H^{1/2}(\partial\Omega_\mathrm{D}) \oplus H^{-1/2}(\partial\Omega_\mathrm{N})$. The scattered field u_s corresponding to the incident Herglotz wave function $u_i(x) := \int_{S^2} e^{ikx\cdot\xi} g(\xi)\, ds_\xi$ satisfies

$$\begin{aligned}
&\Delta u + k^2 u = 0 \qquad \text{in } \Omega_\mathrm{ext},\\
&\gamma u = 0 \qquad \text{on } \partial\Omega_\mathrm{ext},\\
&r\left\{\frac{\partial u}{\partial r} - iku\right\} \to 0 \qquad \text{as } r = |x| \to \infty
\end{aligned}$$

where $u = u_s + u_i$ denotes the total field. The far–field operator $F : L^2(S^2) \to L^2(S^2)$ maps the density g to the far–field pattern u_∞ of u_s. We further introduce the complementary trace operator $\gamma_2 u := (\frac{\partial u}{\partial n}|_{\Omega_\mathrm{D}}, u|_{\partial\Omega_\mathrm{N}})$ with values in $X_2 := H^{-1/2}(\partial\Omega_\mathrm{D}) \oplus H^{1/2}(\partial\Omega_\mathrm{N})$ and the boundary integral operator $B : X_2 \to X_1$, $\varphi \mapsto \gamma v$ where

$$v(x) := \int_{\partial\Omega_\mathrm{D}} \Phi(x-y)\varphi(y)\, ds_y + \int_{\partial\Omega_\mathrm{N}} \frac{\partial\Phi(x-y)}{\partial n(y)}\varphi(y)\, ds_y, \qquad x \in \Omega_\mathrm{ext}.$$

Here $\Phi(x) := \frac{e^{ik|x|}}{4\pi|x|}$ is the fundamental solution to the Helmholtz equation. Moreover, we will denote by $\Im(B)$ the operator with Φ replaced by its imaginary part $\Im(\Phi)$ in the definition of B. Then the following holds true:

Theorem: *Assume that k^2 is not an eigenvalue of $-\Delta$ with Dirichlet boundary conditions on $\partial\Omega_\mathrm{D}$ and Neumann boundary conditions on $\partial\Omega_\mathrm{N}$, and let $z \in \mathbb{C}$. Then the following statements are equivalent:*

1. *z is an eigenvalue of F.*
2. *There exists a solution $u = u_i + u_s$ to the scattering problem such that*

$$\begin{aligned}
&\frac{4\pi}{k}\Im(u_s) = z u_i \qquad \textit{in } \Omega_\mathrm{ext},\\
&\Im(\gamma_2 u) = 0 \qquad \textit{on } \partial\Omega_\mathrm{ext}.
\end{aligned}$$

3. *There exists a real–valued solution $\varphi \in X_2$ to the generalized eigenvalue problem*

$$-\frac{4\pi}{k}\Im(B)\varphi = zB\varphi.$$

A Fundamental Solution Method for Inverse Heat Conduction Problems

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Abstract: In this talk a new meshless and integration-free numerical scheme for solving an inverse heat conduction problem will be discussed. The numerical scheme is developed based on the use of the fundamental solution as a radial basis function. To regularize the resultant ill-conditioned linear system of equations, we apply successfully both the Tikhonov regularization technique and the L-curve method to obtain a stable numerical approximation to the solution. The approach is readily extendable to solve high-dimensional problems under irregular domain and inverse source identification problems.

Inverse Problems in GPS Positioning and numerical compution

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In the new absolute positioning regularization algorithm for standalone GPS, Firstly, To make a decision about the approximate GPS receiver position with the help of Bancroft method, which can offer the initial value for linearization of the observation equation. Because the number of observation satellites is always over than four, in order to make full use of these information, and now solve it with the LS method in china and abroad. Because the solution of LS method not be unique and when observation data have noise, the solution is ill-posed. In order to solve the problem, we introduce into the regularization method, which take into account the characteristic of the observation data and the optimum choice of regularization parameter and compute the error covariance matrix using the method of the Galilo data processing software. The experiment result indicate that the horizontal positioning precision of the method using C/A code is about three meters, better than five to ten meters precision offered by the traditional standard positioning service.

In the GPS static positioning, Using the classical Kalamn method. The correction to approximate GPS receiver position which can offered by the Bancroft method is as the filtering state vector. We take into account the state of GPS receiver in the static positioning and choosing the error covariance matrix. The experiment result indicate that the positioning precision is very good.

In the GPS dynamic positioning, A new nonlinear model including errors computed using navigation messages (acquired from Satellite transmitted data) for position estimation is developed. A new GPS positioning method based on an iterative algorithm, and the new model is proposed. The method uses an algorithm that iteratively uses Unscented Kalman tilters and smoothers to compute the position estimates. The first experimental results, and comparison of results obtained with different algorithms are also presented. First results show that our approaches provide better estimation than other solutions. Future research directions are also discussed.

Topology Optimization Using an Immune-based Algorithm

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Unlike classical optimization methods, topology optimization (TO) algorithms do not require any predefined shape or topology as a starting point for the design process. The design domain is divided in a number of cells or elements, and the material properties of each cell are considered as optimization variables. Thus, any shape or topology can be generated by these methods, which makes TO algorithms the best suited for inverse problems[1].

Stochastic algorithms for optimization (e.g., genetic algorithms) do not rely on derivative information for performing the search for optimal solutions, and present the ability of escaping local attractors on and finding the global optimum for multimodal objective functions. For these reasons, they have been widely used in applications where the behavior of the objective function is unknown, or the cost of the gradient evaluation is too high. Natural systems have been traditionally a powerful inspiration for the development of such algorithms. In this talk a new topology optimization algorithm based on the natural process of Clonal Selection (CS) and Affinity Maturation (AM) in immune cells will be presented. These processes are responsible by the evolution of protein shapes in cells of the immune system, for dealing with harmful intruders in the body. By using an analogy with this system, it is possible to evolve arbitrary shapes to match virtually any input signal.

The proposed algorithm is coupled with the Multigrid (MG) method[2], a technique for allowing multiresolution analysis using the Finite Element Method (FEM). This coupling allows the TO algorithm to first search for the overall distribution of material within the design space (by optimizing a coarse mesh), and then successively refining the solutions until an optimal topology is achieved. An example of topology evolution using this scheme is the pattern-matching problem ilustrated in Figure 1.

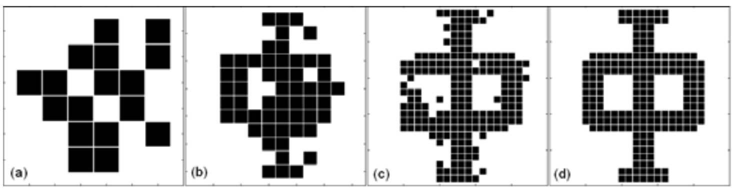

Figure 1: Topology evolution starting from an initial random coarse grid (a), following successive refinements in the mesh (b,c) until a final configuration is achieved (d).

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An inverse source problem for the heat equation and the enclosure method

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An inverse source problem for the heat equation is considered.
Let Ω be a bounded domain of n-dimensions ($n = 1, 2$) with smooth boundary. Let T be an arbitrary positive number. Let $u = u(x, t)$ satisfy the heat equation with a source term in $\Omega\times\,]0,\,T[$:

$$u_t = \triangle u + f(x, t).$$

The problem is
Inverse Problem. Assume that there exist a non negative number T_0 less than T and point $x_0 \in \Omega$ such that $f(x_0, T_0) \neq 0$ and $f(x, t) = 0$ for all $0 < t < T_0$ and all $x \in \Omega$.
Extract T_0 and information about the set $\{x \in \Omega \,|\, f(x, T_0) \neq 0\}$ from the *data* $u|_{\partial\Omega\times]0,\,T[}$, $\partial u/\partial\nu|_{\partial\Omega\times]0,\,T[}$ and $u(\,\cdot\,, 0)$.
The number T_0 and the set $\{x \in \Omega \,|\, f(x, T_0) \neq 0\}$ are the time and position when and where the heat source $f(x, t)$ firstly appeared.

It is shown that the idea of Ikehata's *enclosure method* to the present problem which was introduced for inverse boundary value problems for elliptic equations yields two types of extraction formulae of the information. It is based on the new roles of the plane progressive wave solutions or their complex versions for the *backward heat equation*.

Error estimate for non-characteristic Cauchy problem of the 2-D heat equation

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In this paper, we get a Hölder type estimate for the 2-D heat equation and weight function method is used to prove the result.Also,Hölder type continuous dependence results for discrete solutions of the heat equation are proved and numerical example is given.

An inverse problem in advection-diffusion

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Motivated by a class of ocean circulation inverse problems, we consider the following

Problem Given real-valued functions $f(y)$, $g(y)$, determine a real-valued, continuous function $v(x) \in L^1(0,\infty)$ so that the (overspecified) elliptic system

$$\begin{cases} -\Delta\phi + v(x)\frac{\partial\phi}{\partial y} = 0 & \text{in } \Omega, \\ \phi(0,y) = f(y) & \text{on } \partial\Omega, \\ -\frac{\partial\phi}{\partial x}(0,y) = g(y) & \text{on } \partial\Omega, \\ \lim\limits_{x+|y|\to\infty} \phi(x,y) = 0 & \end{cases}$$

admits a classical solution $\phi(x,y)$. Here $\Omega = (0,\infty)\times(-\infty,\infty)$ and Δ denotes the two-dimensional Laplace opertator.

Let $f(y) \not\equiv 0$ and we suppose that the Dirichlet data $f(y)$ is the Fourier image of a function $\hat{f}(\lambda)$ with $(1+|\lambda|)\hat{f}(\lambda) \in L^1(R)$ and the Neumann data $g(y)$ is the image of a function $\hat{g}(\lambda) \in L^1(R)$. Then our problem is reduced to the inverse scattering problem for the energy dependent Schrödinger equation

$$E'' - \lambda^2 E - i\lambda\, v(x)\, E = 0 \qquad \left({}'' = \frac{d^2}{dx^2}, \quad 0 < x < \infty\right).$$

The Jost solution $e(x,\lambda)$ is defined for each λ in $\operatorname{Re}\lambda \le 0$ as the solution of this equation with the asymptotic behavior $e(x,\lambda) = e^{\lambda x}[1+o(1)]$ as $x \to \infty$. By means of the transformation representation

$$e(x,\lambda) = e^{\lambda x} - \lambda \int_x^\infty K(x,t)e^{\lambda t}dt \qquad (\operatorname{Re}\lambda < 0),$$

we can establish the following procedure for the reconsruction of $v(x)$.

Therorem If our problem has a solution $v(x) \in L^1(0,\infty)$ then $v(x)$ can be reconstructed from f, g in the following three steps:

Step 1. The function $\dfrac{e(0,\lambda)}{\overline{e(0,\lambda)}}$ on the imaginary axis can be determined uniquely from the data f, g, and is represented as

$$\frac{e(0,\lambda)}{\overline{e(0,\lambda)}} = C + \int_{-\infty}^\infty F(t)e^{\lambda t}dt \qquad (\operatorname{Re}\lambda = 0)$$

in terms of a function $F(t) \in L^1(R)$.

Step 2. The integral equation

$$K(x,t) + \int_x^\infty \overline{K(x,r)}F(r+t)dr + \int_x^\infty F(r+t)dr = 0 \qquad (x \le t < \infty)$$

with the function $F(t)$ is solved uniquely in the space of bounded, continuous functions on the interval $[x,\infty)$ for each $x \ge 0$, and the transformation kernel $K(x,t)$ is obtained.

Step 3. The function $v(x)$ is determined from $K(x,x)$ by

$$v(x) = -2i\,\frac{d}{dx}\log(1+K(x,x)).$$

Complete Solutions to Conjectures of Polya-Szegö and Eshelby in Two Dimensions

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Eshelby conjectured in 1961 that if for a given uniform loading the field inside an elastic inclusion is uniform, then the inclusion must be an ellipse or an ellipsoid. On the other hand, Pólya and Szegö conjectured in 1951 that if the polarization tensor associated with an inclusion has the minimal trace, then the inclusion must be a disk or a ball. We prove both conjectures in two dimensions. We show that if the polarization tensor has the minimal trace, then the field inside the inclusion must be uniform. We then show that if the (elastic or electric) field inside the inclusion is uniform, then the inclusion must be an ellipse.

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Estimation Problem for the Shape of a Domain based on Parabolic Equations

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In this talk we are concerned with the inverse problem determining the shape of some unknown portion of the boundary of a domain based on a parabolic equation on the domain. Such a problem has been treated by "Kurt Bryan and Lester F. Caudill Jr., *Stability and reconstruction for an inverse problem for the heat equation*, Inverse Problems **14** (1998) 1429–1453" in multi-dimensional case; we treat the problem in somewhat different and more general situation. Let B be an $n-1$ dimensional bounded Lipschitz domain and put $\Omega := B\times(0,1)$. We suppose that the back surface $B\times\{0\}$ is deformed and its shape is given by a Lipschitz function $x_n = S(t,x')$, where $t\in[0,T]$ and $x'\in B$. Our aim is to determine the function S from measurements of the temperature on the front surface $B\times\{1\}$. Put $\Omega_{(t)} := \{x=(x',x_n) :\ x'\in B,\ S(t,x')<x_n<1\}$ for $t\geq 0$ and $\tilde{Q}_T := \left\{(t,x) :\ t\in(0,T), x\in\Omega_{(t)}\right\}$. We consider that the temperature $u(t,x)$ on $\tilde{Q}_T$ is given by the following parabolic equation:

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) - \mathcal{P}u(t,x) = 0 & \text{on} \quad \tilde{Q}_T \\ u(t,x) = 0 & \text{on} \quad \{(t,x) :\ t\in(0,T), x'\in B, x_n = S(t,x')\} \\ \dfrac{\partial u}{\partial \mathcal{N}}(t,x) = \psi(t,x) & \text{on} \quad \{(t,x) :\ t\in(0,T), x'\in B, x_n = 1\} \\ \dfrac{\partial u}{\partial \mathcal{N}}(t,x) = 0 & \text{on} \quad \{(t,x) :\ t\in(0,T), x'\in \partial B\} \\ u(0,x) = h(x) & \text{on} \quad \Omega_{(0)}, \end{cases}$$

where $\mathcal{P}$ is an elliptic operator:

$$\mathcal{P}u(t,x) = \nabla_x \bullet (A(t,x)\nabla_x u(t,x)) - \boldsymbol{b}(t,x)\bullet\nabla_x u(t,x) - a(t,x)u(t,x)$$

and $\partial/\partial\mathcal{N}$ denotes the conormal derivative relative to $\mathcal{P}$. As in the paper of Bryan and Caudill, we also consider the inverse problem through an appropriate linearized problem. The linearized problem is derived in a systematic way by using the weak form and it approximates the original inverse problem for small deformation of the back surface. Denote by $d(t,x')$ the temperature on the front surface of the deformed domain corresponding to $S(t,x')$. Then we have the following to the linearized problem:

Theorem 1. Suppose that one of the following conditions is fulfilled:
(i) The shape S is independent of the time variable t, and $h\not\equiv 0$ or $\psi\not\equiv 0$.
(ii) For every open interval $I\subset(0,T)$, $\psi\not\equiv 0$ on $I\times(B\times\{1\})$.
Then, $d(t,x')$ on $(0,T)\times B$ determines $S(t,x')$ on $(0,T)\times B$ uniquely.

Theorem 2. Take a basis $\{S_i\}_{i=1}^{\infty}$ of $H := H^{1,1}_{,0}((0,T)\times B)$ and denote $d_i(t,x')$ the temperature on the front surface in the case where $S=S_i$. For any fixed $m\in\boldsymbol{N}$, choose a sequence $(\overline{a}_1,\dots,\overline{a}_m)$ by the method of least squares for $\|d-\sum_{i=1}^m a_i d_i\|^2_{L^2((0,T)\times B)}$. Put $S_{(m)}(t,x') := \sum_{i=1}^m \overline{a}_i S_i(t,x')$. Then there is a dense linear subspace $\mathcal{S}$ of H such that $\lim_{m\to\infty} S_{(m)} = S$ in H for every $S\in\mathcal{S}$. Moreover, for given S and $S'\in H$, denote by d and d' the corresponding temperature on the front surface and by $\{\overline{a}_i\}$ and $\{\overline{a}'_i\}$ the coefficients determined by the the method of least squares from d and d' respectively. Then it holds

$$\max_{1\leq i\leq m} |\overline{a}_i - \overline{a}'_i| \leq K_m \|d-d'\|_{L^2((0,T)\times B)}$$

with a positive constant K_m determined by $d_1,\dots,d_m$.

Singularities of the scattering kernel on the channel of the Rayleigh wave

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Let Ω be the half-space in $\mathbf{R}^3$ with bounded perturbation (undulation), and consider the elastic wave equation $(\partial_t^2 - L)u(t,x) = 0$ in $\mathbf{R} \times \Omega$ with the Neumann boudary condition $Nu = 0$ on $\mathbf{R} \times \partial\Omega$, where $L = \sum a_{ij}\partial_{x_i}\partial_{x_j}$ is the isotopic operator with constant coefficients. Then, there exist several kinds of waves, i.e., P-wave, S-wave, the Rayleigh wave, etc. The Rayleigh wave is concentrated near the boundary, and seems to behave with reflection of situations of the boundary. It is known that the scattering theory of the Lax-Phillips type can be constructed with setting of the free space $\mathbf{R}^3_+$ and that the limiting absorption priciple is also obtained containing the part of the Rayleigh wave. Furthermore, the scattering kernel is represented of the Majda type, and is decomposed into some components depending the channels, e.g., the component for the incoming P-wave and the outgoing S-wave, etc.

The purpose in this talk is to describe the representation of the scattering kernel on the channel of the Rayleigh wave and to show that informations of singularities of the kernel can be derived from the representaion. In the free space $\mathbf{R}^3_+ = \{(x', x_3) : x_3 > 0\}$ there is the Rayleigh wave of the form

$$w_0^R(t, x', x_3; \omega) = \sum_{j=1}^{2} C_j \int_{\mathbf{R}} e^{-i\sigma(t - c_R^{-1}x'\omega)} e^{-|\sigma| c_R^{-1}\xi_R^{(j)} x_3} a_R^{(j)}(\sigma, \omega)\, d\sigma, \qquad \omega \in S^1,$$

where c_R is the velocity of the Rayleigh wave, $a_R^{(1)}(\sigma,\omega) = {}^t(\omega, (\mathrm{sgn}\sigma) i\xi_R^{(1)})$, $a_R^{(2)}(\sigma,\omega) = {}^t(\xi_R^{(2)}\omega, (\mathrm{sgn}\sigma)1)$ and $\xi_R^{(j)}$, C_j are some positive constants only depending on the velocities of the waves. This has singlarity on the boundary of the type $\delta(t - c_R^{-1}x'\omega)$. Furthermore, in the perturbed space there exsists the wave $w^R_{+,tot}$ approximately equal to w_0^R as $t \to -\infty$. The scattering kernel $S_{RR}(s,\theta,\omega)$ $(\theta, \omega \in S^1)$ on the channel of the Rayleigh wave is represented by means of w_0^R and $w_+^R = w^R_{+,tot} - w_0^R$ in the following way, which is corresponding to Majda's representation for the d'Alembert equation.

Theorem 1. *We have for some positive constant C_R*

$$\begin{aligned} S_{RR}(s,\theta,\omega) &= C_R \int_{\Omega\cap\mathbf{R}^3_+} \int_{\mathbf{R}} \partial_{s'} w_0^R(s', y; \theta) \cdot (\partial_t^2 - L) w_+^R(s' - s, y; \omega)\, ds'\, dy \\ &+ C_R \int_{\partial(\Omega\cap\mathbf{R}^3_+)} \Big\{ \int_{\mathbf{R}} \partial_{s'} w_0^R(s', y; \theta) \cdot (N w_+^R(s'-s, y;\omega))\, ds' - \int_{\mathbf{R}} N\partial_{s'} w_0^R(s', y;\theta) \cdot w_+^R(s' - s, y; \omega)\, ds' \Big\} dS_y. \end{aligned}$$

Informations of sing supp $[S_{RR}(\cdot, \theta, \omega)]$ are derived from this theorem. The Dirichlet-Neumann operator has a hyperbolic part in the elliptic region of L. The Rayleigh wave comes from this part, and is expressed by means of the scalar-valued wave equation on the boundary, which is equal to $(\partial_t^2 - c_R^2\Delta)u = 0$ outside the region of the perturbation. Let $(q(t, y'; \omega), p(t, y'; \omega))$ be the bicharacterisic curve for this wave equation with $(q(0, y'; \omega), p(0, y'; \omega)) = (y', c_R^{-1}\omega)$. Assume that any of those curves is non-trapping, and set

$$\begin{aligned} M_\omega^+(\theta) &= \{y' \in \partial\mathbf{R}^3_+;\ \lim_{t\to\infty} p(t-s, y';\omega) = c_R^{-1}\theta,\ c_R^{-1}\omega\, y' = s \text{ (for } s \text{ small enough negatively)}\}, \\ s^+(\theta,\omega) &= \sup_{y' \in M_\omega^+(\theta)} \lim_{t\to\infty} (c_R^{-1} q(t-s, y';\omega)\cdot\theta - t), \end{aligned}$$

which are independent of s. Let $M_\omega^+(\theta) \neq \phi$.

Theorem 2. (i) sing supp $[S_{RR}(\cdot,\theta,\omega)] \subset (-\infty, s^+(\theta,\omega)]$.
(ii) *If $M_\omega^+(\theta)$ consists of only one point, $S_{RR}(s,\theta,\omega)$ is singular at $s = s^+(\theta,\omega)$.*
Theorem 2 is proved by constructing the asymptotic solutions for w_+^R, etc. and inserting them into the expression after modifying the represntation in Theorem 1.

Identification of Plural Cracks by the Passive Electric Potential CT Method with Piezoelectric Material

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The present authors proposed the active electric potential CT (computed tomography) method for identification of cracks and defects. In this method cracks and defects are identified from electric potential distribution measured on the surface of a cracked body under electric current application. The present authors also proposed the passive electric potential CT method using piezoelectric material for identification of cracks and defects. The use of piezoelectric material made it possible to obtain electric potential distribution without application of the electric current. The usefulness of the passive electric potential CT method has been examined by numerical simulations and experiments.

In this study the passive electric potential CT method using piezoelectric film was applied to the identification of plural through cracks. For identification of cracks an inverse analysis scheme based on the least residual method was applied, in which square sum of residuals is evaluated between the measured electric potential distributions and those computed by using the finite element method. Akaike information criterion (AIC) was used to estimate the number of cracks. Numerical simulations were carried out on the identification of plural cracks and a single crack. The location and size of these cracks were quantitatively estimated by the present method. The number of cracks was correctly estimated, even when the plural cracks were closely located and the measured electric potential distribution was similar to that for a single crack.

A Nonlinear Operator Approach to Online Parameter Estimation in Nonlinear Dynamical Systems

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Given a nonlinear dynamical system

$$u_{*t} = f(q_*, u_*, t)$$

that describes the evolution of the physical state u_* but involves uncertain parameters q_*, the task of online parameter identification is to estimate q_* simultaneously to the evolution of u_* based on in general partial and noisy observations y_*^δ of the latter. On-line or real-time determination of process parameters plays a central role in adaptive control. It is a self-contained important part of self-tuning regulators but also occurs implicitly in model-reference adaptive controllers. The theory of on-line estimators is rather well developed in the linear and finite dimensional case, i.e., when both u_* and q_* belong to finite dimensional spaces, but to our knowledge the nonlinear and infinite dimensional case, e.g., online estimation of parameter functions or online parameter estimation in PDEs, is only considered in the context of time dependent partial differential equations, then requiring full observations of the state u_* and its spatial derivatives in dependence of the PDE-order, i.e., the exact data take the form $y_*(t) = (u_*(t), \nabla u_*(t), ...)$.

In this talk, we present an approach to the online estimation problem in (possibly) infinite dimensions that is based on an abstract, nonlinear and time-dependent parameter-to-output map. Our approach also allows for partial state observations and - in the PDE case - makes spatial data differentiation redundant. We present both theoretical and numerical results for nonlinear ODE as well as PDE examples and comparisons to existing techniques.

Nonlinear Integral Equations in Inverse Scattering from a Neumann Crack

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The inverse problem of recovering the geometry and the physical properties of a scatterer from the knowledge of the far field pattern of a scattered field is of fundamental importance for example in non-destructive testing or in medical imaging. In this talk we consider a time-harmonic scattering problem from a sound hard crack which is modeled by

Definition 1. (DP)
For an open arc $\Gamma \subset \mathbb{R}^2$

$$\Gamma := \{z(s) : s \in [-1,1], z \in C^3[-1,1] \text{ and } |z'(s)| \neq 0, \forall s \in [-1,1]\}$$

with end points x^*_{-1}, x^*_1*, given an incident plane wave* $u^i(x,d) := e^{ik<x,d>}$ *with a wave number* k *and a unit vector* d *giving the direction of propagation, find a solution* $u := u^i + u^s \in C^2(\mathbb{R}^2 \setminus \Gamma) \cap C(\mathbb{R}^2 \setminus \Gamma_0)$ *to the Helmholtz equation*

$$\Delta u + k^2 u = 0, \qquad \text{in } \mathbb{R}^2 \setminus \Gamma, k > 0. \tag{1}$$

which satisfies the Neumann boundary conditions

$$\frac{\partial u_\pm}{\partial \nu} = 0 \qquad \text{on } \Gamma_0 := \Gamma \setminus \{x^*_{-1}, x^*_1\} \tag{2}$$

and the Sommerfeld radiation condition

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial \nu} - iku^s\right) = 0, \qquad r := |x| \tag{3}$$

uniformly for all directions $\hat{x} := \frac{x}{|x|}$

We shall use the boundary integral equation method for solving the scattering problem. The scattering problem in the unbounded domain is thus converted into a boundary integral equation. The inverse problem we are considering is

Definition 2. (IP)
Determine the scatterer Γ *if the far field pattern* $u_\infty(\cdot, d)$ *is known for all incident directions* d *and for one wave number* $k > 0$.

A common way to handle the inverse scattering problem is via solving the so-called far field equation. Because of the nonlinearity of the far field operator, linearization methods such as Newton method will be used. For the reconstruction of the shape of the crack, we will therefore need the Fréchet derivative of the far field operator which maps the unknown crack to the far field pattern of the scattered field. The computation of the Fréchet derivative of the far field operator is much involved. Follow the new method proposed by Kress and Rundell [1] based on the reciprocity gap functional, we will first derive the equivalence of the inverse problem with a system of two nonlinear integral equations and then use the Newton method for the reconstruction of the unknown crack.

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A preconditioned $(F^*F)^{1/4}$ method in inverse obstacle scattering

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In this work, we present a preconditioned $(F^*F)^{1/4}$ method for inverse obstacle scattering problem for time harmonic plane waves. In particular, appropriate preconditioniner is constructed via the Algebraic Multigrid method and the problem becomes well conditioned due to eigenvalues shifting away from zero. We finally characterize the scattering obstacle using only the spectral data of the preconditioner and not F, and we hence avoid the computation of the regularization constant. In figure 1, $\circ$ and $\times$ represent the eigenvalues of F and AMG preconditioner, respectively. Figure 1 displays the eigenvalues of both operators about the origin is shown. It is easy to see from that the large number of eigenvalues of F are clustered around the origin compared to the ones of AMG preconditioner. The actual image reconstruction using the spectral data of the AMG preconditioner is shown in figure 2. The two objects are not penetrable and are excited by a harmonic incident wave while the data are contaminated by 5% Gaussian noise.

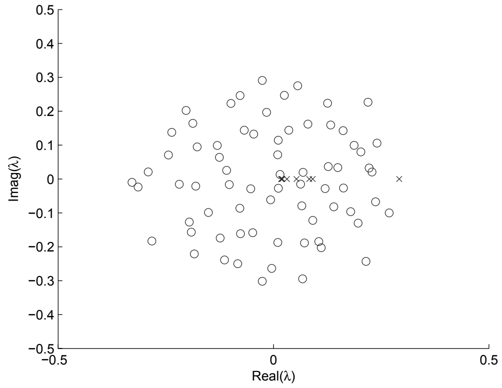

Figure 1: Eigenvalue Distributions

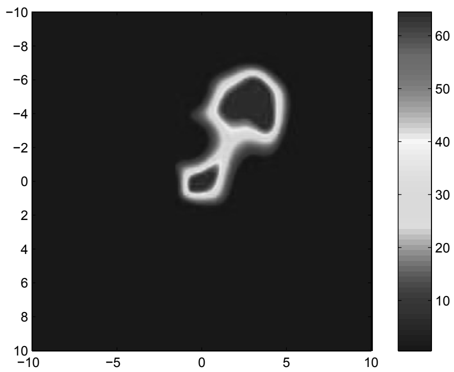

Figure 2: A kite and an oblique ellipse

Iterative methods for the Cauchy problem of Stokes flow

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We consider a steady slow viscous incompressible flow of fluid (creeping flow) which occupies the region Ω between two infinitely long cylinders in 2-D, or between two spheres in 3-D, having outer boundary Γ_0 and inner boundary Γ_1. The inverse problem under investigation requires the reconstruction of the fluid velocity $\underline{u}$ and the pressure p of such a flow from the knowledge of the Cauchy data, i.e. fluid velocity and traction on the part of the boundary Γ_0, i.e.

$$\begin{cases} \Delta\underline{u} - \nabla p = 0, \quad \nabla\cdot\underline{u} = 0 & \textit{in } \Omega \\ \underline{u} = \underline{\phi} \in L^2(\Gamma_0), & \textit{on } \Gamma_0 \\ \underline{t}(\underline{u},p) =: \left(\nabla\underline{u} + (\nabla\underline{u})^T - p\right)\underline{\nu} = \underline{\psi} \in L^2(\Gamma_0) & \textit{on } \Gamma_0 \end{cases}$$

If $\underline{u}$ is divergence free and $(\underline{u},p) \in L^2(\Omega) \times (H^1(\Omega))^*$ satisfies the Stokes equation $\Delta\underline{u} - \nabla p = 0$, then we call $(\underline{u},p)$ a Stokes pair.

The iterative Landweber-Fridman type method for solving this Cauchy problem is as follows:

Step 1. Set $k = 0$ and choose an arbitrary intial guess $\underline{\eta}_0 \in L^2(\Gamma_1)$. The first approximation is the Stokes pair $(\underline{u}_0, p_0)$ subject to $\underline{u}_0 = \underline{\eta}_0$ and $\underline{t}(\underline{u}_0, p_0) = \underline{\psi}$ on Γ_0.

Step 2. Find the Stokes pair $(\underline{v}_0, q_0)$ with $\underline{v}_0 = \underline{0}$ on Γ_1 and $\underline{t}(\underline{v}_0, q_0) = \underline{\phi} - \underline{u}_0$ on Γ_0.

Step 3. For $k \geq 1$, having constructed $(\underline{u}_{k-1}, p_{k-1})$ and $(\underline{v}_{k-1}, q_{k-1})$, the Stokes pair $(\underline{u}_k, p_k)$ satisfies $\underline{u}_k := \underline{\eta}_k = \underline{\eta}_{k-1} - \gamma\underline{t}(\underline{u}_k, p_k) = \underline{\psi}$ on Γ_0, where $\gamma > 0$.

Step 4. Finally, the Stokes pair $(\underline{v}_k, p_k)$ is constructed with $\underline{v}_k = \underline{0}$ on Γ_1 and $\underline{t}(\underline{v}_k, q_k) = \underline{\phi} - \underline{u}_k$ on Γ_0.

The mixed direct problems solved in this procedure are well-posed in $L^2(\Omega) \times (H^1(\Omega))^*$, and the different restrictions to the boundary are well-defined. This and the following theorem can be proved.

Therorem Let $\underline{\phi}$ and $\underline{\psi}$ be given in $\mathrm{L}^2(\Gamma_0)$. Then, if $\gamma > 0$ is sufficiently small, the sequence $(\underline{u}_k, p_k)$ in the above procedure converges to the solution of the Cauchy problem $(\underline{u}, p)$, which is assumed to exist in $L^2(\Omega) \times (H^1(\Omega))^*$, for any initial guess $\underline{\eta}_0 \in L^2(\Gamma_1)$.

Suppose now that instead of $\underline{\phi}$, we have only its approximation, say $\underline{\phi}^\epsilon \in L^2(\Gamma_0)$, satisfying $\|\underline{\phi} - \underline{\phi}^\epsilon\|_{L^2(\Gamma_0)} \leq \epsilon$, where $\epsilon \geq 0$ is an upper bound for the error in the measurements. Then we stop the iteration of the algorithm above according to the discrepancy principle, namely at the smallest index k for which $\|\underline{u}_k^\epsilon - \underline{\phi}^\epsilon\|_{L^2(\Gamma_0)} \approx \epsilon$.

From a numerical point of view, it might be difficult to choose the parameter $\gamma > 0$ in the right interval. However, it is possible to propose parameter-free procedures such as the conjugate gradient and the minimal error methods.

The numerical implementation based on the boundary element method confirm that the iterative procedures produce convergent and stable numerical solutions.

Conditional stability for source parameter identification in multidimensional advection-dispersion equation with final observations

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Consider the following multidimensional advection-dispersion equation with Dirichlet boundary condition:

$$c_t - D_L \triangle c + u\ \nabla \cdot c + q(x)c = 0,\ \ (x,t) \in \Omega_T \tag{1}$$

$$c(x,0) = 0,\ \ x \in \Omega \tag{2}$$

$$c(x,t)|_{\partial\Omega} = g(x,t),\ \ x \in \partial\Omega, 0 \le t \le T \tag{3}$$

where $\Omega_T = \Omega \times (0,T)$, and $\Omega \subset R^N$ be bounded domain, $T > 0$.

Our problem is to determine the source parameter $q = q(x)$ with the following overposed final observations at $t = T$:

$$c(x,T) = c_T(x),\ \ x \in \Omega. \tag{4}$$

Suppose $< c_1, q_1 >$ and $< c_2, q_2 >$ are two pairs of solutions of the inverse problem (1)-(4) correspondingly to the known data (g_1, c_T^1) and (g_2, c_T^2) respectively, Then it follows that

$$\int_{\Omega_T} c_2(q_2 - q_1)\ \varphi\ dxdt = \int_{\Omega} (c_T^1 - c_T^2) v(x) dx + D_L \int_0^T \int_{\partial\Omega} (g_1 - g_2)\varphi_n\ dS\ dt \tag{5}$$

where $\varphi = \varphi(x,t)$ denotes a solution of the following adjoint problem with input data $v = v(x)$:

$$\varphi_t + D_L \triangle \varphi + u \nabla \varphi - q_1 \varphi = 0,$$

$$\varphi|_{\partial\Omega} = 0,$$

$$\varphi(x,T) = v(x).$$

By data compatibility analysis and integral identity method based on the above identity (5), a conditional stability for the inverse problem (1)-(4) can be constructed via a suitable topology.

Keywords: Inverse problem of determining source parameter; multidimensional advection-dispersion equation; integral identity method; conditional stability

Estimation of coefficients in a hyperbolic equation with impulsive inputs

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For the solution to

$$\partial_t^2 u(x,t) - \triangle u(x,t) + q(x)u(x,t) = \delta(x_1)\delta'(t), \quad u|_{t<0} = 0,$$

we consider an inverse problem of determining

$$q(x), \quad x \in \Omega$$

from data

$$f = u|_{S_T} \quad \text{and} \quad g = \frac{\partial u}{\partial \nu}|_{S_T}.$$

Here $\Omega \subset \{(x_1, \ldots, x_n) \in \mathbb{R}^n | x_1 > 0\}$, $n \geq 2$, is a bounded domain,

$$S_T = \{(x,t);\ x \in \partial\Omega, x_1 < t < T + x_1\}$$

and $T > 0$.

For suitable $T > 0$, we prove an $L^2(\Omega)$-size estimation of q:

$$\|q\|_{L^2(\Omega)} \leq C \left\{ \|f\|_{H^1(S_T)} + \|g\|_{L^2(S_T)} \right\},$$

provided that q satisfies a priori uniform boundedness conditions.

We use an inequality of Carleman type in our proof.

Inverse boundary value problem for Stokes equation

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We prove a global identifiability of the viscosity parameter in an incompressible fluid by boundary measurements. This is a joint work with H. Heck and J-N Wang.

An expansion theorem for two-dimensional elastic waves and its application

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Let $u(x) \in C^2$ be a solution of the scalar Helmholtz equation $\Delta u + k^2 u = 0$ in the exterior of the ball with radius $a > 0$ and satisfy Sommerfeld's radiation condition. It is a well-known property [1], [4] that u, in the spherical coordinates (r, θ, ϕ), can be expressed as

$$u(r, \theta, \phi) = r^{-1} e^{ikr} \sum_{n=0}^{\infty} f_n(\theta, \phi) r^{-n}, \tag{1}$$

where the series converges for $r > a$ and converges absolutely and uniformly with respect to r, θ, ϕ in the domain $r > a + \varepsilon > a$. The series may be differentiated term by term in all variables. Moreover, the coefficients f_n for $n > 0$, can be constructed recursively from the far-field pattern $f_0(\theta, \phi)$. Similar results for Maxwell's equations and elastic equations in three dimensions were proved by Wilcox [5] and by Dassios [2], respectively. In two dimensions, a convergent expansion theorem for the scalar radiation solution was established by Karp [3]. However, a similar expansion theorem for two-dimensional elastic waves is still missing. The present paper is an attempt to fill this gap.

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Robust generalized cross-validation for choosing the regularization parameter

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Linear Fredholm integral equations of the first kind

$$Kf(x) \equiv \int_a^b k(x,t)f(t)\,dt = g(x)$$

arise in many important applications, often with discrete noisy data $y_i = g(x_i) + \epsilon_i$, $i = 1, \ldots, n$. Since these equations are ill-posed, it is essential to use some form of regularization. The popular method of Tikhonov regularization gives a good estimate f_λ of the solution, provided we make a good choice of the regularization parameter λ. One of the most successful methods for choosing the parameter is generalized cross-validation (GCV). It is known to have favourable asymptotic properties as $n \to \infty$; in particular, the "expected" GCV estimate λ_V is asymptotically optimal with respect to the risk $ER(\lambda) = \sum(Kf_\lambda(x_i) - g(x_i))^2$, meaning that $ER(\lambda_V)/\mathrm{min}ER(\lambda) \to 1$ as $n \to \infty$. However, for small or medium sized n, GCV may not be reliable, sometimes giving a value of λ that is far too small (corresponding to a very noisy f_λ).

We propose a new robust GCV method (RGCV) which chooses λ to be the minimizer of

$$\gamma V(\lambda) + (1-\gamma)F(\lambda),$$

where $V(\lambda)$ is the GCV function, $F(\lambda)$ is a certain average measure of the influence of each data value on f_λ, and $\gamma \in (0,1)$ is a robustness parameter. As with GCV, the method requires no knowledge of the error variance or the smoothness of the solution. We show that because of the properties of $F(\lambda)$, the RGCV method is more reliable than GCV for smaller values of n. We also show that RGCV has good asymptotic properties as $n \to \infty$, including that the "expected" RGCV estimate of λ is asymptotically optimal with respect to the "robust risk" $\gamma ER(\lambda) + (1-\gamma)v(\lambda)$, where $v(\lambda)$ is the variance component of the risk. We will compare RGCV and GCV using numerical simulations for the problem of estimating the second derivative from noisy data.

Dynamical systems method for solving the operator equations of the first kind

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The DSM(dynamical systems method) for solving equation $\mathcal{A}u = f$ consists of solving the Cauchy problem

$$\dot{u}_{\delta,h}(t) = \Phi_{\delta,h}(t, u_{\delta,h}(t)), \quad t > 0, \quad u_{\delta,h}(0) = u_0 \tag{1}$$

where

$$\Phi_{\delta,h}(t, u_{\delta,h}(t)) = -[\mathcal{B}_h u_{\delta,h}(t) + \varepsilon(t) u_{\delta,h}(t) - \mathcal{F}^{\delta,h}], \tag{2}$$

or

$$\Phi_{\delta,h}(t, u_{\delta,h}(t)) = -(\mathcal{B}_h + \varepsilon(t))^{-1} \Big[\mathcal{B}_h u_{\delta,h}(t) + \varepsilon(t) u_{\delta,h}(t) - \mathcal{F}^{\delta}\Big]; \tag{3}$$

with

$$\mathcal{B}_h = \mathcal{A}_h^* \mathcal{A}_h, \quad \mathcal{F}^{\delta,h} = \mathcal{A}_h^* f^\delta, \quad \|\mathcal{A} - \mathcal{A}_h\| \le h, \quad \|f - f^\delta\| \le \delta. \tag{4}$$

We set $r = \sqrt{\delta^2 + h^2}$, y is the unique minimal-norm solution to equation $\mathcal{A}u = f$.

The purpose of this talk is to prove the following theorems:

Therorem Under certain conditions on $\varepsilon(t)$ the solution $u_{\delta,h}$ to (1) at $t = t_{\delta,h}$, will have the property

$$\lim_{r \to 0} \|u_{\delta,h}(t_{\delta,h}) - y\| = 0,$$

this $t_{\delta,h}$ can be chosen as a root of the following equation

$$\sqrt{\varepsilon(t)} = (\delta + h)^b, \quad b \in (0, 1).$$

Let us use Euler's method to solve Cauchy problem (1) with (2), (1) with (3), respectively, numerically.

$$p_{\delta,h}^{n+1} = p_{\delta,h}^n - \omega_n \big[(\mathcal{B}_h + \varepsilon_n) p_{\delta,h}^n - \mathcal{F}_{\delta,h}\big], \quad n = 0, 1, 2 \cdots, \tag{5}$$

$$p_{\delta,h}^0 := u_0, \quad \varepsilon_n := \varepsilon(t_n), \quad t_n := \Sigma_{i=0}^n \omega_i, \quad \omega_i > 0 \tag{6}$$

$$q_{\delta,h}^{n+1} = (1 - \omega_n) q_{\delta,h}^n + \omega_n (\mathcal{B}_h + \varepsilon_n)^{-1} \mathcal{F}_{\delta,h}, \quad n = 0, 1, 2 \cdots, \tag{7}$$

$$q_{\delta,h}^0 := u_0, \quad \varepsilon_n := \varepsilon(t_n), \quad t_n := \Sigma_{i=0}^n \omega_i, \quad \omega_i > 0, \tag{8}$$

we can also get another iterative process by using other methods, such as the implicit Euler method, Runge-Kutta method. In this section it is only proved that under certain conditions the iterative process (7) with (8) converges to y.

Therorem Assume $\mathcal{A}$ is linear, bounded operator in H. Let
1) δ be the level of noise in (7): $\|f - f^\delta\| \le \delta$ and (4) holds;
2) $n = n(\delta, h)$ be chosen in such away that $\lim\limits_{r \to 0} n(\delta, h) = \infty$;
3) $\varepsilon(t) \in C[0, \infty)$, $\varepsilon(t) \searrow 0 (t \to \infty)$, and $\frac{|\dot{\varepsilon}(t)|}{\varepsilon(t)^2} \to 0 (t \to \infty)$;
4) $\sum_{n=1}^\infty \omega_n = \infty$, $0 < \omega_n < 1$, $\lim\limits_{r \to 0} \frac{\delta + h}{\sqrt{\varepsilon_{n(\delta,h)}}} = 0$.
Then

$$\lim_{r \to 0} \|q_{\delta,h}^n - y\| = 0$$

where $n := n(\delta, h)$.

An Algorithm Reconstructing Source Term for Neutron Transport Equation

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Based on the fundamental solution of neutron transport equation, we gave an algorithm for reconstructing the source term of this differential-integral equation and numerical simulation. The numerical results show that this algorithm is reliable and efficient.

An inverse problem for the one-dimensional wave equation in multilayer media

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We consider half-line media which consist of many kinds of substances. Assume that we can observe only the data near the boundary point of the half-line. Our purpose is to guess the situation away from the boundary point by the nondestructive inspections.

Now, we introduce the notations and formulate this problem. Put $h_0 := 0$. Let h_k be a positive constant and $h_k > h_{k-1}$ for $k = 1, \dots, N-1$. We call the interval (h_{k-1}, h_k) Medium k for $k = 1, \dots, N-1$ and the interval (h_{N-1}, ∞) Medium N. Let a_k and b_k be positive constants for $k = 1, \dots, N$. The positive number a_k describes the speed of the waves through Medium k, and b_k the impedance of Medium k. Put $P_k := \partial_t^2 - a_k^2 \partial_x^2$ for $k = 1, \dots, N$. Suppose $0 < y < h_1$. We consider the following equations:

$$
\begin{array}{rl}
(\mathrm{W}.1) & P_1 u(t,x) = \delta(t, x-y), \quad 0 < x < h_1, \\
(\mathrm{W}.k) & P_k u(t,x) = 0, \quad h_{k-1} < x < h_k \qquad (k = 2, \dots, N-1), \\
(\mathrm{W}.N) & P_N u(t,x) = 0, \quad h_{N-1} < x, \\
(\mathrm{B}) & \partial_x u(t,x)|_{x=0+0} = 0, \\
(\mathrm{I}.k) & u(t,x)|_{x=h_k-0} = u(t,x)|_{x=h_k+0} \qquad (k = 1, \dots, N-1), \\
(\mathrm{J}.k) & a_k b_k \partial_x u(t,x)|_{x=h_k-0} = a_{k+1} b_{k+1} \partial_x u(t,x)|_{x=h_k+0} \qquad (k = 1, \dots, N-1).
\end{array}
$$

The equation (B) means the free boundary condition at the point $x = 0$. The equation (I.k) describes the continuity of the displacement of the waves at the point $x = h_k$, and (J.k) the continuity of the stress. The equations (E) $= \{$(W.1)–(W.N), (B), (I/J.1)–(I/J.$N-1$)$\}$ express the situation that the initial data is the delta function at the point y in Medium 1 at the time $t = 0$ with the boundary condition (B) and the interface or transmission conditions (I/J.k).

In this talk, we show the following main result.

Main result. Suppose that the constants a_1, b_1, y are known. Assume $b_j \neq b_{j+1}$ for $j = 1, \dots, N-1$. Assume that the observation data $v(t) := u(t, 0)$ are given on $[0, T)$, where $u(t,x)$ denotes the solution of the equations (E). Then b_{k+1} and $(h_k - h_{k-1})/a_k$ are reconstructed by the following process:

- The first step: Put $v_1(t) := (1/a_1) H(t - y/a_1) - v(t)$, where H is the Heaviside function.
- The $(k+1)$-st step ($k = 1, 2, \dots$): If $v_k(t) \equiv 0$ then the process is finished. If $v_k(t) \not\equiv 0$, then put $t_k := \inf\{t \in [0, T) : v_k(t) \neq 0\}$, reconstruct the constants $(h_k - h_{k-1})/a_k$ and b_{k+1} by

$$
\frac{h_k - h_{k-1}}{a_k} := \frac{1}{2}\left(t_k + \frac{y}{a_1}\right) - \sum_{j=1}^{k-1} \frac{h_j - h_{j-1}}{a_j},
$$

$$
b_{k+1} := \frac{2^{2k-2} \prod_{j=1}^{k-1} (b_j b_{j+1}) + v_k(t_k + 0) a_1 \prod_{j=1}^{k-1} (b_j + b_{j+1})^2}{2^{2k-2} \prod_{j=1}^{k-1} (b_j b_{j+1}) - v_k(t_k + 0) a_1 \prod_{j=1}^{k-1} (b_j + b_{j+1})^2} b_k,
$$

  define $v_{k+1}(t)$ by

$$
v_{k+1}(t) := v_k(t) + \frac{1}{a_1} g^{(k)}\left(t; \frac{y}{a_1}; b_1, \dots, b_{k+1}; \frac{h_1}{a_1}, \frac{h_2 - h_1}{a_2}, \dots, \frac{h_k - h_{k-1}}{a_k}; T\right),
$$

  and go the next step, where $g^{(k)}$ can be expressed explicitly, however we omit the explicit formula here.

Direct Computation of Harmonic Moments for Tomographic Reconstruction

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We present the tomographic reconstruction of a 2D polygonal object D from its projections. Let $f(x, y)$ and $p(r, \theta)$ be a characteristic function of D and its projection

$$p(r, \theta) = \int\int_{\mathbb{C}} f(x, y)\delta(r - x\cos\theta - y\sin\theta)dxdy, \tag{1}$$

respectively. Then, it is shown[1] that the harmonic moments of the object $c_n \equiv \int\int_{\mathbb{C}} f(x, y)(x + iy)^n dxdy$ are related to the vertices $v_k(\in \mathbb{C})$ of D as

$$n(n-1)c_{n-2} = \sum_{k=1}^{N} a_k v_k^n, \tag{2}$$

where N is the number of the vertices of D. Eq. (2), so called the moment problem, can be algebraically solved for v_k from c_{n-2} for $n = 2, 3, \cdots, 2N - 3$[1]. However so far, the harmonic moments have been obtained indirectly via the geometrical moments of the object $\mu_{p,q} \equiv \int\int_{\mathbb{C}} f(x, y)x^p y^q dxdy$, through the following three steps[2]: 1) compute the geometrical moments of the projections $h_n(\theta) \equiv \int_{\mathbb{R}} p(r, \theta)r^n dr$ from projections, 2) solve the simultaneous equations $h_n(\theta) = \sum_{j=0}^{n} C_{n,j} \cos^{n-j}\theta \sin^j \theta \mu_{n-j,j}$ for $\mu_{n-j,j}$, then 3) compute $c_n = \sum_{j=0}^{n} C_{n,j} i^j \mu_{n-j,j}$.

We showed first that the harmonic moments can be computed much more directly and efficiently from projections as follows:

Theorem 1
$$2\pi c_n = \int_0^{2\pi}\int_0^{\infty} p(r, \theta)r^n \mathrm{e}^{in\theta} drd\theta \tag{3}$$

Furthermore, we showed the following theorem holds, which is effective for the real condition where the projection number is finite:

Theorem 2 *The harmonic moments can be strictly computed from the finite number of projections as*

$$\frac{c_n}{2^n} = \frac{1}{M}\sum_{k=0}^{M-1}\int_0^{\infty} p(r, \theta_k)r^n \mathrm{e}^{in\theta_k} dr \tag{4}$$

if the projection number M satisfys

$$M > N \quad \text{when } M \text{ is odd}, \tag{5}$$
$$M > 2N \quad \text{when } M \text{ is even}. \tag{6}$$

From this theorem, we observe an interesting property: in order to reconstruct the object shape from less number of projections, the projection number should be odd. Numerical results will be shown at the conference.

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An inverse initial-boundary value problem for the operator
$$L_p = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - p_1(x)\frac{\partial}{\partial t} - p_2(x)\frac{\partial}{\partial x}$$

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We consider a wave equation with damping coefficient

$$\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) + p_1(x)\frac{\partial u}{\partial t}(x,t) + p_2(x)\frac{\partial u}{\partial x}(x,t),\ 0 < x < 1, -T < t < T,$$
$$u(x,0) = 0, \quad \frac{\partial u}{\partial t}(x,0) = \delta(x), \quad 0 \le x \le 1,$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0, \quad -T \le t \le T$$

where $T \ge 2$, the complex-valued functions $p_1, p_2 \in C^1[0,1]$ and $\delta(x)$ is the Dirac delta function. We discuss the inverse problem of determining simultaneously the coefficients $p_1(x)$ and $p_2(x)$, $0 \le x \le 1$ from observation data $u(0,t), -T \le t \le T$. We prove a reconstruction formula for $p_1(x)$ and $p_2(x)$ from $u(0,t)$ by the inverse spectral theory.

Inverse crack problem and the Mittag-Leffler function

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Let Ω be a two-dimensional simply connected domain with smooth boundary and $\Sigma = \cup_{m=1}^{M}\Sigma_m \subset \Omega$ be a finite union of mutually disjoint perfectly insulated cracks. We assume that each Σ_m is non-self-intersecting smooth curve. Let u be a non constant solution of the Laplace equation in $\Omega \setminus \Sigma$ and satisfy $\partial u/\partial\nu = 0$ on Σ. The so-called *inverse crack problem* is to extract information about unknown cracks Σ from Cauchy data $(f, g) = (u|_{\partial\Omega}, \partial u/\partial\nu|_{\partial\Omega})$ for finitely many or infinitely many u. This problem corresponds to *electrical impedance tomography* that finds the unknown cracks in the conductive material using the electrical potentials and currents on the surface.

For a similar inverse problem for cavities instead of cracks, Ikehata has established an extraction formula of the convex hull of the unknown cavities from (f, g) for a single u under the condition that the cavities are *polygonal* and the diameter is so small compared with the distance between the cavities and $\partial\Omega$. The formula is based on the asymptotic behaviour of the so-called *indicator function* calculated for a single set of Cauchy data as $\tau \longrightarrow \infty$ which involves the harmonic function $v(x;\tau,\omega) = e^{\tau x\cdot(\omega+i\omega^{\perp})}$ with a large parameter τ and two unit vectors ω, $\omega^{\perp}$ perpendicular to each other. Moreover, Ikehata has proven that: if one has g for u with $f = v|_{\partial\Omega}$, then another indicator function calculated from infinitely many set of the Cauchy data (f, g) yields a similar extraction formula of the convex hull of the cavities having general shape. Numerical implementation of the both types of the formulae are done in [Ikehata-Ohe, Inverse Problems, 18(2002), 111-124] and [Ikehata-Siltanen, Inverse Problems, 16(2000), 1043-1052].
In [Ikehata, CONM, 348(2004), 41-52], Ikehata proposed the idea of introducing two new indicator functions by replacing v above with a harmonic function with the same large parameter τ coming from the Mittag-Leffler function which is a generalization of the exponential function. The numerical implementation of the formula with the new indicator function of infinitely many measurement type has been done in [Ikehata-Siltanen, Inverse Problems 20(2004), 1325-1348] for inclusions.

In this talk, we study the asymptotic behaviour of the new indicator functions in the inverse crack problem, and discuss a numerical reconstruction method for cracks.

Calibration of Multi-Phase Flow Function and Quantificaiton of Uncertainty in Petroleum Reservoir Forecast

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Production forecasts for petroleum reservoirs are essentially uncertain due to the lack of data. Firstly, direct measurements of rock and fluid properties are available at only a small number of sparse well locations. Secondly, oil production and pressure data reflect roughly integrated responses over a limited number of time intervals.

As a result, a reservoir engineer needs to calibrate the unknown physical parameters based on insufficient observations which cannot constrain the subsurface properties all over a field. These physical parameters describe flow through a reservoir, and are incorporated into the nonlinear partial differential equations, which are usually solved numerically. The unknown parameters are adjusted so that the simulated profile can match the observed data. This process is an inverse problem called history-matching in the petroleum industry. By nature, inverse problems are ill-posed and may have non-unique solutions. The aim of this talk is to propose a methodology for history-matching and uncertainty quantification in a petroleum reservoir.

This talk addresses two issues: 1) How can we calibrate physical properties in history-matching? 2) How can we predict uncertain oil production based on history-matching? In order to tackle the first question, we parameterise the reservoir properties and adopt a stochastic sampling method called the Neighbourhood Approximation algorithm. It samples parameter space selectively using geometrical properties of Voronoi cells to bias the sampling to regions of parameter space where a good fit is likely. Then, for the second question, we utilise Bayesian framework along with Markov Chain Monte Carlo and Neighbourhood Approximation in parameter space. It allows us to determine a probability distribution of physical parameters rather than to obtain one solution. Then through the flow simulations we can quantify uncertainty in future oil recovery.

The physical properties to be adjusted in this study were relative permeabilities which describe multi-phase flow in the porous media of a rock system. In petroleum reservoirs, multi-phase flow occurs, especially when water or gas is injected into reservoir to enhance oil recovery. For example, oil-water flow strongly depends on the ratio of water volume to the total fluid volume which is called water saturation. Hence the relative permeability is usually represented as a function of fluid saturation.

We parameterised the flow function using flexible B-splines and validated the history-matching result in comparison with the reference profiles and functions. For the numerical experiments, we used a synthetic reservoir model where water was injected into one edge and oil was produced from the other edge. The hypothetical production history was created using the ideal fine-scale model and random noise. Then, the relative permeabilities were calibrated in a coarse-scale model because of the computational cost. Here, the coarse-scale relative permeability should encapsulate the sub-grid features overlooked by this simplification. The result of uncertainty quantification showed that the lack of knowledge of the sub-grid features leads to a substantial amount of uncertainty in reservoir performance forecasting.

State estimation approach to nonstationary inverse problems: discretization error and filtering problem

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We consider the following nonstationary inverse problem. We are interested in the values of the quantity X in a domain D along time. We are not able to perform direct measurements of X but we can observe the quantity Y at direct time instants. The quantity Y depends linearly on X at the given time instant. We have a model for the time evolution of the quantity X. Since we cannot be sure that the evolution model is correct, we have added a source term representing possible modelling errors. The quantity Y cannot be measured exactly. In the measured values of Y there is an additional measurement noise. We want to calculate an estimate for X based on the measured values of Y. To be able to solve the nonstationary inverse problem we view it as a state estimation problem. The state estimation system we are interested in consists of the equations

$$\mathrm{dX(t)} = AX(t)\mathrm{dt} + \mathrm{dW(t)}, \quad \mathrm{t} > 0, \tag{1}$$

$$X(0) = X_0, \tag{2}$$

$$Y(t) = BX(t) + S(t), \quad t > 0. \tag{3}$$

The time evolution of the state of the system X is modeled by a stochastic differential or partial differential equation (1). We assume that A is a densely defined sectorial operator and W is a Hilbert space valued Wiener process. The observation equation (3) is linear with additive measurement noise S. The time discretization of the continuous infinite dimensional state estimation system (1)–(3) is exact since the solution to the state evolution equation (1) is given by an analytic semigroup and the observation equation (3) depends only on the given time instant. For computational reasons the space discretization of the time discrete infinite dimensional state estimation system is performed. The novel contribution of the article is the analysis of the space discretization. The distributions of the discretization errors in the discretized state evolution and observation equations are introduced. The solution to the corresponding finite dimensional filtering problem is presented.

Carleman estimate and applications for the one-dimensional heat equation with a non-smooth coefficient

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Many inverse problems can be solved by means of Carleman-type estimates. We consider here the following heat equation for $(t,x) \in Q := [0,T] \times (0,1)$:

$$\partial_t q - \partial_x(c\partial_x q) = f,$$

with Dirichlet boundary conditions. In this work, the coefficient c is positive and piecewise C^1 with a finite number of discontinuities, $x_1, \ldots, x_n$.

Consider first a subinterval $\omega := (a,b) \subset\subset \Omega := (0,1)$ and the case when moving from a to 0, resp. from b to 1, the jumps encountered for c are non negative. In this case, the Carleman estimate with an observation on ω (see the term of the r.h.s. of $(*)$) is known ([DOP]). In the case of arbitrary signs for the jumps in the coefficient c, the existence of such an estimate was open. In the one dimensional case, some controllability results have however been proved ([FZ]). In the one-dimensional case, without any monotonicity condition on c, we prove that such a Carleman estimate remains valid. Consequently, in our talk, we will deduce the usual applications : controllability, extension to a class of non-linear heat equations, inverse problems...

The main result is the following

Theorem. *Let an open subset $\omega \subset\subset \Omega, \omega \neq \emptyset$. There exist $\lambda_1 = \lambda_1(\Omega,\omega) > 0$, $s_1 = s_1(\lambda_1, T) > 0$ and a positive constant $C = C(\Omega,\omega)$ so that the following estimate holds*

$$(*) \quad \begin{aligned} &s\lambda^2 \iint_Q e^{-2s\eta}\varphi|c\partial_x q|^2 \, dxdt + s^3\lambda^4 \iint_Q e^{-2s\eta}\varphi^3|q|^2 \, dxdt \\ &\qquad \leq C\left[s^3\lambda^4 \iint_{(0,T)\times\omega} e^{-2s\eta}\varphi^3 \, |q|^2 \, dxdt + \iint_Q e^{-2s\eta} \, |\partial_t q \pm \partial_x(c\partial_x q)|^2 \, dxdt\right], \end{aligned}$$

for $s \geq s_1$, $\lambda \geq \lambda_1$ and for all $q \in \aleph$.

Note that we also obtain the same type of inequality with a boundary observation. We have used the following notations : $q \in \aleph$ if and only if q is continuous on Q, C^2 on $Q' := [0,T] \times (\Omega \setminus \{x_1, \ldots, x_n\})$ and, for each $t \in (0,T)$, $x \to q(t,x)$ belongs to the domain of the selfadjoint operator $\partial_x(c\partial_x)$ in $L^2(\Omega)$ (which implies the usual transmission conditions). In the l.h.s. of $(*)$, we can introduce the quantity $\|M_1(e^{-s\eta}q)\|^2 + \|M_2(e^{-s\eta}q)\|^2$, usual in these Carleman inequalities.

The functions η and φ are positive weight functions on Q given by

$$\varphi(x,t) = \frac{e^{\lambda\beta(x)}}{t(T-t)}, \qquad \eta(x,t) = \frac{e^{\lambda\bar{\beta}} - e^{\lambda\beta(x)}}{t(T-t)},$$

with $\bar{\beta} > \|\beta\|_\infty$ (see e.g. [FI], [DOP]).

In the proof of the Carleman estimate, the construction of the continuous function β is essential [FI]. We show that a modification of the usual requirements on β, enables us to consider the earlier situation, as well as that without monotonicity conditions, and opens other perspectives.

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Applications of the Tikhonov regularization using reproducing kernels to inverse problems

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The Tikhonov regularization is a basically important idea and method in numerical analysis, however, the extremal functions in the Tikhonov functionals are represented by using the associated singular values and singular functions for the case of compact operators. So, their representations are restrictive and abstract in a sense. We gave new representations for the case of general bounded linear operators by using the theory of reproducing kernels and various concrete representations for typical cases. Our representations are both analytical and numerical. We were able to establish a general theory of the Tikhonov regularization using the theory of reproducing kernels containing error estimates and convergence rates. We shall present the general theory and its concrete results for the typical inverse problem for heat conduction with computer graphs as the evidence of the power of our inverse formulas. - In particular, we shall give a much more improved version for [11] by using the Paley-Wiener spaces as reproducing kernel Hilbert spaces.

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Recovering first order terms from boundary measurements

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We consider the problem of recovering nonsmooth first order terms in scalar elliptic equations from boundary measurements, in dimensions three and higher. The main example is to construct a magnetic field from the Dirichlet to Neumann map related to the Schrödinger operator. This extends earlier results on the problem (due to Nakamura-Sun-Uhlmann and others) by relaxing the regularity assumptions on the coefficients, and by giving a constructive method.

More precisely, consider the magnetic Schrödinger operator

$$H = \sum_{j=1}^{n} \Big(\frac{1}{i} \frac{\partial}{\partial x_j} + W_j \Big)^2 + V$$

where $W : \Omega \to \mathbf{R}^n$ is a magnetic potential and $V : \Omega \to \mathbf{R}$ is an electric potential in a bounded domain Ω. The Dirichlet to Neumann map is given by the magnetic normal derivative

$$\Lambda : f \mapsto (\nabla + iW)u \cdot \nu|_{\partial\Omega}$$

where u is the solution of $Hu = 0$ in Ω with boundary values f on $\partial\Omega$. We show that one may construct the magnetic field $\operatorname{curl} W$ and the electric potential V from the boundary measurements Λ.

The inverse scattering problem for Schrödinger and Klein-Gordon equations with a nonlocal nonlinearity

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We consider the inverse scattering problem for the nonlinear Schrödinger equation

$$i\partial_t u + \Delta u = f(u) \tag{1}$$

and for the nonlinear Klein-Gordon equation

$$\partial_t^2 w - \Delta w + w = f(w) \tag{2}$$

in space-time $\mathbf{R} \times \mathbf{R}^n$. The nonlocal nonlinear term $f(v)$ has the form $f(v) = \lambda(x)(|\cdot|^{-\sigma} * |v|^2)v$, where we assume that $\lambda \in C^1(\mathbf{R}^n) \cap W^1_\infty(\mathbf{R}^n)$ and $\lambda(0) \neq 0$.

The purpose of this talk is to determine σ, and to reconstruct λ from the knowledge of the scattering operator:

To state our results, we give some notation. Let $\langle x \rangle = (1 + |x|^2)^{1/2}$. For $s, k \in \mathbf{R}$, let H^s_p and $H^{s,k}$ be the Sobolev space $(1-\Delta)^{-s/2} L_p(\mathbf{R}^n)$ and the weighted Sobolev space $(1-\Delta)^{-s/2} \langle x \rangle^{-k} L_2(\mathbf{R}^n)$, respectively. Especially, H^s denotes H^s_2. For $1 \le r \le \infty$, let $\acute{r}$ be the Hölder conjugate of r. For $\alpha > 0$, $\phi : \mathbf{R}^n \to \mathbf{C}$, we denote $\phi(\alpha^{-1}(x - x_0))$ by $\phi_{\alpha,x_0}(x)$. For (1), we set

$$T[\phi] = \lim_{\varepsilon \downarrow 0} \frac{i}{\varepsilon^3} \langle (S - I)(\varepsilon\phi), \phi \rangle_{L^2(\mathbf{R}^n)},$$

where S is the scattering operator for (1). For (2), we put

$$K[\phi] = \lim_{\varepsilon \downarrow 0} \frac{i}{\varepsilon^3} \left\langle (S - I)(\varepsilon\, {}^t(\phi, 0)),\ {}^t(0, \phi) \right\rangle_{H^1(\mathbf{R}^n) \oplus L_2(\mathbf{R}^n)},$$

where S is the scattering operator for (2).

Therorem 1. Let $n \ge 2$, $1 < \sigma \le 4$, $\sigma < n$. Assume that $\phi \in H^1 \cap H^{0,1}$ and $\phi \neq 0$. For (1), we have the formula for determining σ

$$\sigma = 2n + 2 - \lim_{\alpha \downarrow 0} \ln \frac{|T[\phi_{e\alpha}]|}{|T[\phi_\alpha]| + \alpha^{2n+2}}.$$

Therorem 2. Let $I = (6(n-1)/(3n-5), 2n/(n-2)]$. Suppose that $n \ge 3$, $\max\{n/(n-1), 4/3\} < \sigma \le 4$, $\sigma < n$,

$$\phi \in H^{1,1/3} \cap \bigcap_{r \in I} H^{(n+1)(1/2-1/r)}_{\acute{r}}$$

and $\phi \neq 0$. For (2), we have the formula for determining σ

$$\sigma = 2n + 1 - \lim_{\alpha \downarrow 0} \ln \frac{|K[\phi_{e\alpha}]|}{|K[\phi_\alpha]| + \alpha^{2n+1}}.$$

Remark By using the determined σ, we have the reconstruction formula for λ. For instance, the formula for λ of (1) is given by

$$\lambda(x_0) = \frac{\lim_{\alpha \to 0} \alpha^{-(2n+2-\sigma)} T[\phi_{\alpha,x_0}]}{\int |y|^{-\sigma} |u_0(t, x - y)|^2 |u_0(t, x)|^2 d(t, x, y)}.$$

Here, $\phi_{\alpha,x_0}(x)$ denotes $\phi(\alpha^{-1}(x - x_0))$, and $u_0 = e^{it\Delta}\phi$.

Inverse analysis on circular cylindrical shell subjected to hydrostatic pressure

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The inverse analysis to identify the water level and system rigidity as unknown parameters are performed on circular cylindrical shell subjected to hydrostatic pressure. The displacements of radius direction on some locations are adopted as the observation data in the inverse problem.

Governing equation of the circular cylindrical shell subjected to hydrostatic pressure is given as follows,

$$Da^2 w,_{xxxx} + Etw = -a^2\gamma(h_1 - x_1)$$

where t, a and x_1 are thickness, radius, parameter of axis direction of shell, γ is water density, E is Young's modulus, and D, h_1 are the bending stiffness and water level as unknown parameter that should be identified. Generaly, D is given as

$$D = \frac{Et^3}{12(1-\nu^2)}$$

where ν is Poisson's ratio.

The observation equation should be expressed with nonlinear operator becouse the relation between the bending stiffness and displacement given in the state vector becomes the nonlinear expression. The extended observation equation can be written as follows,

$$\mathbf{Y}_k = \mathbf{m}_k(\mathbf{Z}_k) + \mathbf{v}_k$$

We expand the nonlinear observation equation in the neighbor hood of estimation. Negleoting the higher terms of expansion we can obtain the following observation equation with the sensitivity matrix as result of a linearization of the nonlinear obsavation equation,

$$\mathbf{Y}_k = \mathbf{M}_k\mathbf{Z}_k + \mathbf{v}_k$$

where

$$\mathbf{M}_k = \left(\frac{\partial \mathbf{m}_k(\mathbf{Z}_k)}{\partial \mathbf{Z}_k}\right)_{\mathbf{Z}_k = \hat{\mathbf{Z}}_{k/k-1}}$$

The filter equation can be written as follows,

$$\hat{\mathbf{Z}}_{k+1/k} = \hat{\mathbf{Z}}_{k/k-1} + \mathbf{B}_k(\omega_k - \mathbf{m}_k(\hat{\mathbf{Z}}_k)_{k/k-1})$$

where

$$\mathbf{B}_k = \mathbf{M}_k^T(\mathbf{M}_k\mathbf{M}_k^T + \gamma\mathbf{Q}_k)^{-1}$$

where $\mathbf{B}_k$ is the parametric projection filter as filter gain. The parametric projection filter is including the parameter γ to be regularized the filtering process. The procedure to obtain the adaptive parameter γ in each filtering step is introduced in this inverse analysis.

The natable characteristics of present filtering algorithm in appling the inverse problem are made clear through several numerical caluculations.

Convergence property of the variational method for the Cauchy problem of the Laplace equation

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For a two dimensional annulus domain $\ := \{(x\ y);\ R_{\mathrm{id}}{}^2 < x^2 + y^2 < R_{\mathrm{d}}{}^2\}$ with the outer boundary $\Gamma_{\mathrm{d}} = \{(x\ y);\ x^2 + y^2 = R_{\mathrm{d}}{}^2\}$ and the inner one $\Gamma_{\mathrm{id}} = \{(x\ y);\ x^2 + y^2 = R_{\mathrm{id}}{}^2\}$, we consider the Cauchy problem of the Laplace equation:

Problem For given Cauchy data $(\overline{u}\ \overline{q}) \in H^{1/2}(\Gamma_{\mathrm{d}}) \times \{\ v\ \ n \in H\ ^{1/2}(\Gamma_{\mathrm{d}});\ -\Delta v = 0 \text{ in }\ ,\ v|_{\Gamma_{\mathrm{d}}} = \overline{u},\ v|_{\Gamma_{\mathrm{id}}} = \ ,\ \ \in H^{1/2}(\Gamma_{\mathrm{id}})\}$, nd $u \in H^{1/2}(\Gamma_{\mathrm{id}})$ such that

$$-\Delta u = 0 \quad \text{in}$$
$$u = \overline{u} \qquad \frac{u}{n} = \overline{q} \quad \text{on} \quad \Gamma_{\mathrm{d}}$$

where n denotes the unit outward normal to Γ_{d}.

This problem can be regarded as the following minimization problem:

Problem Find $\ ^* \in H^{1/2}(\Gamma_{\mathrm{id}})$ such that

$$J(\ ^*) = \inf_{\ \in H^{1/2}(\Gamma_{\mathrm{id}})} J(\ ) \qquad J(\ ) := \int_{\Gamma_{\mathrm{d}}} |v(\ ) - \overline{u}|^2 d\Gamma$$

where $v = v(\ ) \in H^1(\ )$ depending on $\ \in H^{1/2}(\Gamma_{\mathrm{id}})$ is the solution of the primary problem:

$$-\Delta v = 0 \quad \text{in}$$
$$\frac{v}{n} = \overline{q} \quad \text{on} \quad \Gamma_{\mathrm{d}}$$
$$v = \quad \text{on} \quad \Gamma_{\mathrm{id}}$$

To nd the minimum of $J(\ )$, we generate a minimizing sequence $\{\ _k\}_{k=0}^{\infty}$ by the steepest descent method: $\ _{k+1} = \ _k - \rho_k J'(\ _k)$ starting with an initial guess $\ _0$, with a suitably chosen numerical sequence $\{\rho_k\}_{k=0}^{\infty}$. The rst variation is explicitly given by $J'(\ ) = -\ \widehat{v}\ \ n|_{\Gamma_{\mathrm{id}}}$, where $\widehat{v} = \widehat{v}(v(\ )) \in H^2(\ )$ depending on the solution $v = v(\ )$ of the primary problem is the solution of the adjoint problem:

$$-\Delta \widehat{v} = 0 \qquad \text{in}$$
$$\frac{\widehat{v}}{n} = 2(v - \overline{u}) \quad \text{on} \quad \Gamma_{\mathrm{d}}$$
$$\widehat{v} = 0 \qquad \text{on} \quad \Gamma_{\mathrm{id}}$$

We obtain a simple theorem and its corollary on a convergence property as follows:

Theorem For the exact $\ ^*$, suppose that the error $\mu_k = \ ^* - \ _k$ can be expanded in the nite Fourier series:

$$\mu_k = \sum_{|j|=M}^{N} a_j^{(k)} e^{ij\theta}$$

with some nonnegative integers $M\ N$ $(M \leq N)$. Then, $\{\ _k\}_{k=0}^{\infty}$ converges to $\ ^*$ for $0 < \rho_k < 2\ C_M$ with $C_j := 8R_{\mathrm{d}}{}^{2|j|+1}R_{\mathrm{id}}{}^{2|j|\ \ 1}\ (R_{\mathrm{id}}{}^{2|j|} + R_{\mathrm{d}}{}^{2|j|})^2$.

Corollary The optimal step size ρ_{opt} in the sense that the compression factor δ in $\|\mu_{k+1}\|_{L^2(\Gamma_{\mathrm{id}})} \leq \delta\|\mu_k\|_{L^2(\Gamma_{\mathrm{id}})}$ is minimized is given by $\rho_{\mathrm{opt}} = 2\ (C_M + C_N)$.

Based on the results above, the Armijo criterion is no longer necessary to choose the step sizes for each iteration. Hence, the cost of numerical computations can be reduced. We con rm the propriety of these theoretical results through some numerical results obtained by the nite element method.

Structural damage identification based on variable parametric projection filter

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The structural damage identification analysis of some kinds of frame structure model are perfomed as a frame work of inverse problem. As the inverse analysis method filtering algorithm based on parametric projection filter are employed in filter equation. In this study the natural frequencies calculated by motion equation of shear-type structure and lateral stiffness of each story on frame structure model are adopted as the observation data and unknown parameter that should be identified, respectively.

The natural frequency equation of the frame model assumed shear deformation is written as follows,

$$|-\omega^2\mathbf{M}+\mathbf{K}|=0$$

where ω is circular natural frequency, $\mathbf{M}$ is mass matrix and $\mathbf{K}$ is lateral stiffness matrix.

The observation equation should be expressed with nonlinear operator becouse the relation between the natural frequencies and lateral stiffness given in the state vector becomes the nonlinear expression. The extended observation equation can be written as follows,

$$\mathbf{Y}_k=\mathbf{m}_k(\mathbf{Z}_k)+\mathbf{v}_k$$

We expand the nonlinear observation equation in the neighbor hood of estimation. Negleoting the higher terms of expansion we can obtain the following observation equation with the sensitivity matrix as result of a linearization of the nonlinear obsavation equation,

$$\mathbf{Y}_k=\mathbf{M}_k\mathbf{Z}_k+\mathbf{v}_k$$

where

$$\mathbf{M}_k=\left(\frac{\partial\mathbf{m}_k(\mathbf{Z}_k)}{\partial\mathbf{Z}_k}\right)_{\mathbf{Z}_k=\hat{\mathbf{Z}}_{k/k-1}}$$

The filter equation can be written as follows,

$$\hat{\mathbf{Z}}_{k+1/k}=\hat{\mathbf{Z}}_{k/k-1}+\mathbf{B}_k(\omega_k-\mathbf{m}_k(\hat{\mathbf{Z}}_k)_{k/k-1})$$

where

$$\mathbf{B}_k=\mathbf{M}_k^T(\mathbf{M}_k\mathbf{M}_k^T+\gamma\mathbf{Q}_k)^{-1}$$

where $\mathbf{B}_k$ is the parametric projection filter as filter gain. The parametric projection filter is including the parameter γ to be regularized the filtering process. The procedure to obtain the adaptive parameter γ in each filtering step is introduced in this inverse analysis.

The natable characteristics of present filtering algorithm in appling the inverse problem are made clear through several numerical caluculations.

Point-wise determination of the surface impedance from scattering data. I. The acoustic case.

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The propagation of time-harmonic acoustic fields in a homogeneous media is governed by the *Helmholtz equation*

$$\Delta u + \kappa^2 u = 0 \quad \text{in} \quad \mathbb{R}^3 \setminus \overline{D} \tag{1}$$

where κ is the real positive *wave number*. At the boundary of the scatterers the total field u satisfies the *impedance boundary condition*

$$\frac{\partial u}{\partial \nu} + i\lambda u = 0 \text{ on } \Gamma_I \tag{2}$$

with some continuous function λ and the *Dirichlet condition*

$$u = 0 \text{ on } \Gamma_D \tag{3}$$

where $\partial D = \Gamma_I \cup \Gamma_D$ with Γ_I and Γ_D disjoint. We assume that $\lambda(x) \geq \lambda_0 > 0$ where λ_0 is a constant.

Given an incident field u^i which satisfies $\Delta u^i + \kappa^2 u^i = 0$, we look for solutions $u := u^i + u^s$ of (1) and (2) where the *scattered field* u^s is assumed to satisfy the Sommerfeld radiation condition

$$\lim_{r \to \infty} r\left(\frac{\partial u^s}{\partial r} - i\kappa u^s\right) = 0, \tag{4}$$

$r = |x|$ and the limit is uniform with respect to all the directions $\theta := \frac{x}{|x|}$. It is well known that this reflected field satisfies the following asymptotic property,

$$u^s(x) = \frac{e^{i\kappa r}}{r} u^\infty(\theta) + O(r^{-2}), \quad r \to \infty, \tag{5}$$

where the function $u^\infty(\cdot)$ defined on the unit sphere $\mathbb{S}$ is called the far-field associated to the incident field u^i. Taking particular incident fields given by the plane waves, $u^i(x,d) := e^{i\kappa d \cdot x}$, $d \in \mathbb{S}$, we define the far-field pattern $u^\infty(\theta, d)$ for $(\theta, d) \in \mathbb{S} \times \mathbb{S}$.

We prove a point-wise formula which gives explicitly the values of this surface impedance as a function of the far field pattern. This formula enables us to distinguish and recognize the coated and the non coated parts of the obstacle.

A reconstruction scheme for identifying source locations in two dimensional heat equations

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We considered the source location determination problem for a two dimensional heat equations with unknown source locations. We propose a numerical reconstruction scheme for recovering the number of unknown sources and all source locations.

Can all measurable plane sets be reconstructed by two projections?

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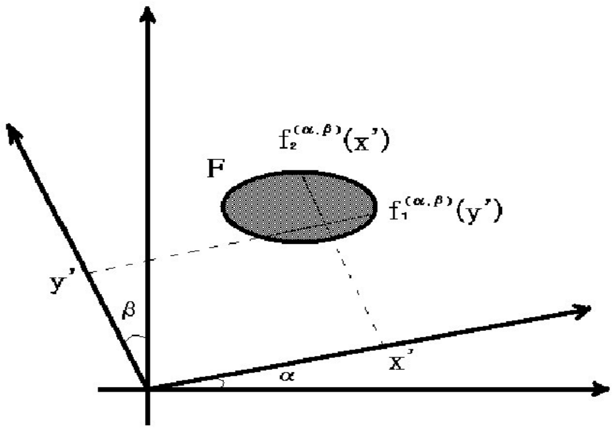
We discuss reconstruction of measurable plane sets from their two projections. Let $F \subset \mathbb{R}^2$ be a measurable plane set such that $\lambda_2(F) < \infty$, where λ_i is the Lebesgue measure on $\mathbb{R}^i$, $i = 1, 2$. Denote by $f(x, y)$ the characteristic function of F. Let $x_{(\alpha)}$ (resp. $y_{(\beta)}$) axis be the x axis rotated by the angle α (resp. the y axis rotated by the angle β).

For $-\pi/2 < \alpha, \beta < \pi/2$ ($\alpha < \pi/2 + \beta$), we define the projection functions as

$$f_1^{(\alpha,\beta)}(y') := \int_{-\infty}^{\infty} f(-y' \sin\beta + t\cos\alpha, y'\cos\beta + t\sin\alpha)dt,$$

$$f_2^{(\alpha,\beta)}(x') := \int_{-\infty}^{\infty} f(x'\cos\alpha - t\sin\beta, x'\sin\alpha + t\cos\beta)dt,$$

where $x' = (x', \alpha)$, $y' = (y', \pi/2 + \beta)$ in the polar coordinate.



A number of studies are done on the problem to reconstruct F from the given pair of projections $f_1^{(0,0)} =: f_1$ and $f_2^{(0,0)} =: f_2$, where f_1 and f_2 are non-negative and have the same L^1 norm. It was proved by G.G. Lorentz that the answer to this problem (for $\alpha = \beta = 0$) splits into three cases; (i) F is uniquely reconstructed (unique case), (ii) F is non-uniquely reconstructed (non-unique case) and (iii) There is no set having f_1, f_2 as projections (inconsistent case).

For the unique case, A. Kuba A. and A. Volčič gave a reconstruction formula.

L. Huang and the author studied stability in this reconstruction and gave an algorithm for approximation of the reconstruction from projections possibly containing noise and error.

In this talk, we first discuss generalization of the known results for $\alpha = \beta = 0$ in the frame of general α and β. The main purpose in this talk is to study the following problem.

Problem
For any measurable plane set F, are there any angles α and β such that F is uniquely reconstructed from the pair of projections $f_1^{(\alpha,\beta)}$ and $f_2^{(\alpha,\beta)}$?

We give an example to show that this problem is negatively solved.

A note on the construction of the complex geometrical optics solutions for second order elliptic equations

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Let n (≥ 3) be an integer and Ω be a bounded domain in $\mathbb{R}^n$ with C^∞ boundary $\partial\Omega$. We set the second order elliptic operator in $\widetilde{\Omega}$ $(\subset \Omega)$ by $L(x, D_x) = \sum_{j=1}^n D_{x_j}^2 + \sum_{j=1}^n b_j(x) D_{x_j} + q(x)$, where $D_{x_j} = -i\partial/\partial x_j$, $b_j \in C^\infty(\widetilde{\Omega}; \mathbb{C})$ and $q \in L^\infty(\widetilde{\Omega}; \mathbb{C})$.

The problem of this talk is to construct a non-local exact solution $L(x, D_x)(e^{ih^{-1}g(x)} a(x; h)) = 0$ on Ω, where $g \in C^\infty(\widetilde{\Omega}; \mathbb{C})$ and $h > 0$ small enough. The solution $u(x; h) = e^{ih^{-1}g(x)} a(x; h)$ is called a complex geometrical optics solution. This kind of solutions is used in the inverse problems. When $b_j(x) = 0$, it is known that the information of Dirichlet-Neumann map (DN map) gives the identification of the potentials. In the case $b_j(x) = 0$, Bukhgeim-Uhlmann showed some partial data of DN map implies the uniqueness of the potential $q(x)$. Recently Kenig-Sjöstrand-Uhlmann proposed the new approach for this problem.

The purpose of this talk is to review of the result by Kenig-Sjöstrand-Uhlmann in order to apply their method to the case $b_j(x) \neq 0$. We shall prove the next result:

Proposition 1 Let $s = 0, 1$ and φ be a limiting Carleman weight for $|D_x|^2$. For $h > 0$ small enough, $v \in H^{s-1}(\Omega)$, there exists $u \in H^s(\Omega)$ such that

$$\begin{cases} e^{-\varphi(x)/h}(-h^2 L(x, D_x))(e^{\varphi(x)/h} u) = v, \\ h||u||_{H^s(\Omega)} \leq ||v||_{H^{s-1}(\Omega)}. \end{cases}$$

The function $\varphi \in C^\infty(\widetilde{\Omega}; \mathbb{R})$ with $\nabla_x \varphi(x) \neq 0$ is a limiting Carleman weight for $|D_x|^2$ if and only if

$$\{a, b\}(x, \xi) = 0, \quad \text{when} \quad a(x, \xi) = b(x, \xi) = 0,$$

where $a(x, \xi) = |\xi|^2 - |\nabla_x \varphi(x)|^2$ and $b(x, \xi) = 2\nabla_x \varphi(x) \cdot \xi$. The key to prove Proposition 1 is to show Carleman estimate.

Proposition 2

$$h(||e^{\varphi(x)/h} v|| + ||hDe^{\varphi(x)/h} v||) \leq C||e^{\varphi(x)/h}(h^2 \sum_{j=1}^n D_{x_j}^2 + h^2 \sum_{j=1}^n b_j(x) D_{x_j}) v||,$$

for $v \in C_0^\infty(\Omega)$.

Kenig-Sjöstrand-Uhlmann showed this estimate for $b_j(x) = 0$. We consider Proposition 2 from two points of view. One is the expansion of the phase function. The other is a transformation that appears in L^2 well posedness for Schrödinger equation. By considering two viewpoints, we can obtain Proposition 2. The details will be explained in the talk.

Inverse spectral scattering on graphs

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We investigate an inverse spectral scattering problem on noncompact graphs, containing compact edges.

Developed identification of coefficient inverse issue for 2D parabolic model by boundary pointwise observation

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Let Ω be an open set in $\mathbf{R}^2$ and $Q = \Omega \times (0,T)$, $\partial\Omega$ be the boundary of Ω. One interesting issue arising in inverse problems described by parabolic partial differential equations of $u(\mathbf{x},t), \mathbf{x} = (x_1, x_2)$.

$$\begin{cases} u_t - \Delta u_{\mathbf{xx}} + a(\mathbf{x})u = s(\mathbf{x},t), & (\mathbf{x},t) \in Q, \\ u_t(\mathbf{x},0) + ku(\mathbf{x},0) = u_0(\mathbf{x}), & \mathbf{x} \in \partial\Omega. \end{cases} \tag{1}$$

Here in (1), $a(\mathbf{x}) > 0$ is unknown coefficient and k is constant. $s(\mathbf{x},t) = \delta(t)\delta(\mathbf{x}-\mathbf{x}_0)$ is the source function.

Inverse Problems I: Given the boundary pointwise observation $u(\mathbf{x},t) = \delta(t)\sum_{j=1}^{m}\delta(\mathbf{x}-\mathbf{x}'_j),\ \mathbf{x}, \mathbf{x}'_j \in \partial\Omega$ for source function $s(\mathbf{x},t)$ to identify the unknown coefficient $a(\mathbf{x})$ in Ω.

A developed global convergent algorithm is considered to solve the problems I. For experiment setting, Ω is regarded as the rectangle, e.g. $\Omega = (0,l) \times (0,l)$ and l is proper positive constant.

- **Sequential minimization algorithm.**
  - *Transformation.* Using Laplace transformation to convert the equation (1) to boundary problem of integrate equations without unknown $a(\mathbf{x})$.
  - *Approximate.* Constructing the approximate solutions using taylor expand formula for two augments, and assume that involving time functions is quadratic polynomial with undeterminate parameter. For example, let us discrete the rectangle as $0 \le x_1^1 < x_1^2 <, ..., < x_1^{n-1} < x_1^n \le 1$, $0 \le x_2^1 < x_2^2 <, ..., < x_2^{n-1} < x_2^n \le 1$, and let i, j be the nodes number of sub-rectangle, then approximate solution $p(x_1, x_2, s) \approx p_{ij}(x_1, x_2, s)$ at ij-element is approached by

$$\begin{aligned} p_{ij}(x_1,x_2,s) \;=\; & a_i(s)\frac{(x_1 - x_1^{i-1})^2}{2} + b_j(s)\frac{(x_2 - x_2^{j-1})^2}{2} + c_{ij}(s)(x_1 - x_1^{i-1})(x_2 - x_2^{j-1}) \\ & + p'(x_1^{i-1}, x_2^{j-1}, s)(x - x_1^{i-1}) + p'(x_1^{i-1}, x_2^{j-1}, s)(x - x_2^{j-1}) + p_{ij}(x_1^{i-1}, x_2^{j-1}, s). \end{aligned} \tag{2}$$

    Here in (2), the $a_i(s), b_j(s), c_{ij}(s)$ are assumed as $a_i(t) = \alpha_i s^2 + s,\ b_j(t) = \beta_j s^2 + s,\ c_{ij} = \gamma_{ij} s^2 + s$ for $\alpha_i, \beta_j, \gamma_{ij} \in \mathbf{R}^1$.
  - *Numerical solution.* Considering extrapolated boundary condition, implement the iteration of quadratic polynomial p_{ij} on each discrete sub-rectangle to minimize the approximate cost function given by Carleman's weighted functions.
- **Inversion**. Calculating unknown coefficients without calculating the approximate solutions.
- **Numerical demonstration**. Unknown coefficient $a(\mathbf{x})$ is identified for various observation in Fig. 1.

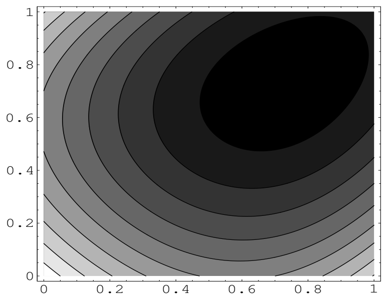

Fig. 1. Contour plot of $a(\mathbf{x})$

It's hope that the new conclusions will be a development for global convergent algorithm to coefficient inverse problems in two dimensional case.

On mixed and componentwise condition numbers for Moore-Penrose inverse and linear least squares problems with applications to Tikhonov regularization

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Classical condition numbers are normwise: they measure the size of both input perturbation and output using some norms. To take into account the relative of each data component, and, in particular, a possible data sparseness, componentwise condition numbers have been increasingly considered. These are mostly of two kinds: mixed and componentwise.

In this talk, we give explicit expressions, computable from the data, for the mixed and componentwise condition numbers for the computation of the Moore-Penrose inverse as well as for the computation of solutions and residues of linear least squares problems. In both case the data matrices have full column (row) rank.

We will apply our new results to the Tikhonov regularization problem, which will improve the known results by several authors.

Inverse problems for n-dimensional Vibrating System

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We consider the following ordinary differential operator A_{P,h_k,H_k}:

$$\begin{cases} A_P : \{L_2(0,1)\}^{2n} \to \{L_2(0,1)\}^{2n} \\ (A_P u)(x) = B_{2n}\frac{du}{dx}(x) + P(x)u(x) \quad (0 \le x \le 1) \\ D(A_P) = \left\{ u \in \{H^1(0,1)\}^{2n} \;\middle|\; u_{l+2}(0) = h_l u_l(0),\ u_{l+2}(1) = H_l u_l(1) \;\; (l = 1,2,\cdots,n) \right\} \end{cases}$$

Here $B_{2n} := \begin{pmatrix} E_n & O \\ O & E_n \end{pmatrix}$ and E_n is $n \times n$-identity matrix, and

$$P(x) := \begin{pmatrix} p_{1,1}(x) & p_{1,2}(x) & \dots & p_{1,2n}(x) \\ p_{2,1}(x) & p_{2,2}(x) & \dots & p_{2,2n}(x) \\ \vdots & \vdots & \ddots & \vdots \\ p_{2n,1}(x) & p_{2n,2}(x) & \dots & p_{2n,2n}(x) \end{pmatrix}$$

where $p_{k,l}(x)$ $(k,l = 1,2,\cdots,2n)$ are real-valued C^1-functions defined on $[0,1]$. h_l, H_l $(l = 1,2,\cdots,n)$ are real values satisfying $|h_l| \ne 1, |H_l| \ne 1$.

Now we consider the following mixed problem:

$$E(P,a)\begin{cases} \frac{\partial u}{\partial t}(t,x) = B_{2n}\frac{\partial u}{\partial x}(t,x) + P(x)u(t,x) & -T \le t \le T, \quad 0 \le x \le 1 \\ u_{n+l}(t,0) = h_l u_l(t,0) \quad 1 \le l \le n, \quad -T \le t \le T \\ u_{n+l}(t,1) = H_l u_l(t,1) \quad 1 \le l \le n, \quad -T \le t \le T \\ u(0,x) = a(x) \end{cases}$$

This system describes proper vibrations for various phenomena such as an electric oscillation in n transmission lines. For the case of the electric oscillation, $u(x)$ means the electric current and the voltage, and $P(x)$ means the characteristic of the transmission lines.

I talk about the uniqueness for the following inverse problem:

"Determine a coefficient matrix $P(x)$ and an initial value $a(x)$ from the boundary values $u(t,0), u(t,1)$."

For the case of $n = 1$, there are results by M. Yamamoto and I. Trooshin. Here, I talk about the case of $n = 2$ for simplicity. In that case, the analysis is more complicated than for the case of $n = 1$. For the case of $n \ge 3$, we can apply the similar methods and obtain the similar results to the results for the case of $n = 2$.

We introduce the following set:

$$M_T(P,a) := \left\{ (Q,b) \in \{C^1[0,1]\}^{20} \;\middle|\; v(t,0) = u(t,0),\ v(t,1) = u(t,1) \;\; (t \in [-T,T]) \right\}$$

where u, v are the solutions to $E(P,a), E(Q,b)$. It is obvious that $(P,a) \in M_T(P,a)$. If we had $M_T(P,a) = \{(P,a)\}$, then the boundary values determine P and a uniquely. Therefore, for the discussion of uniqueness or non-uniqueness in our inverse problem, it is sufficient to determine the set $M_T(P,a)$.

I talk about the necessary and sufficient condition for $(Q,b) \in M_T(P,a)$. Here, we apply the spectral property of the operator A_{P,h_k,H_k}.

A solution method for positive linear inverse problems and its application to trip distribution with inconsistent data

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This talk presents a method for solving *positive linear inverse problems*

$$X\theta \approx y \qquad X : \text{matrix of nonnegative elements} \qquad \theta, y : \text{positive vectors, } \theta \text{ unknown}$$

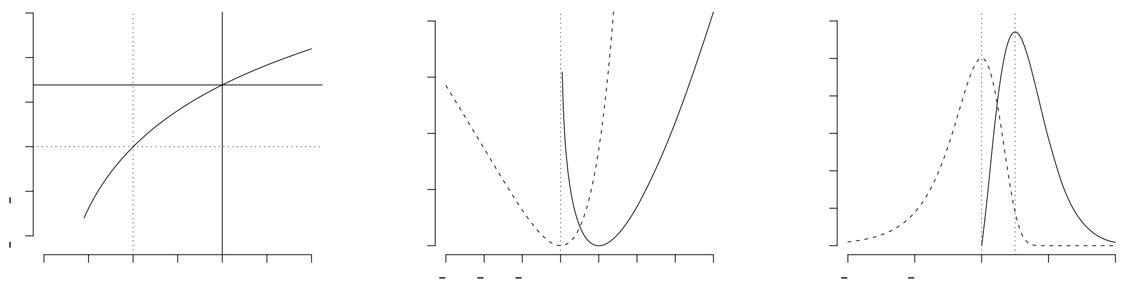
together with its application to trip distribution. The method is a positive parallel to the method of least squares such that guarantees the unique existence of positive solution, permits a quadratically convergent iteration, enables control of the solution stability, and provides a statistical interpretation of the results.

Assume that the first $\dim\theta$ elements of y represent guesses for the elements of θ. Then the upper part of X is an identity matrix; it secures that the system of equations is overdetermined. Weights w are introduced in order to reflect that elements of y may have different accuracies. If $y = X\theta + \varepsilon$ is solved by the method of weighted least squares $\hat{\theta} := \arg\min_\theta \sum_i w_i \mathrm{J}_i(\theta)$, $\mathrm{J}_i(\theta) := (y_i - X_i\,.\theta)^2 = \varepsilon_i^2$, where $X_i\,.$ is the i-th row of X, the solution may well include negative values even if the above nonnegativity and positivity conditions for X and y hold.

The proposed method assumes the errors ξ_i to be multiplicative rather than additive and replaces the least squares by the least *rectangles*:

$$y_i = \xi_i\, X_i\,.\theta \qquad X_i\,. : i\text{-th row of X} \qquad \mathrm{J}_i(\theta) := \frac{y_i - X_i\,.\theta}{X_i\,.\theta} \log \frac{y_i}{X_i\,.\theta} = (\xi_i - 1)\log\xi_i\,.$$

While the least squares minimizes the sum of square areas with the sides ε_i, the least semilogs minimizes the sum of $\xi_i - 1$ by $\log\xi_i$ rectangle areas, as illustrated in Figure (a). A variable transformation renders the



(a) A rectangle to minimize (b) Loss functions (c) Probability distributions

optimization unconstrained as the dashed line in Figure (b). The problem permits a Newton type algorithm which converges quadratically. The gradient and the Hessian have simple closed forms. The method enjoys a straightforward statistical interpretation as a maximum likelihood estimation of the probability distributions in Figure (c), solid and dashed lines corresponding to those in Figure (b).

The method is applied to trip distribution. The basic problem is to fill in the entries of a matrix, called origin-destination (OD) table, given its row sums and column sums. This amounts to solving a *consistent* but underdetermined system of linear equations $\{\sum_j \theta_{i\,j} = y_{i\cdot},\ \sum_i \theta_{i\,j} = y_{\cdot j}\}$. The standard method of trip distribution involves picking a plausible solution based on a principle such as entropy maximization. When the data are *inconsistent*, e.g. $\sum_i y_{i\cdot} \neq \sum_j y_{\cdot j}$ it has been customary to use some *ad hoc* convention such as adjusting the row and the column sums so that both add up to the same amount. Application of the method of least rectangles permits to abolish such conventions facilitating automatic data acquisition.

Inverse problems of determination of principal parts for a parabolic equation

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We consider an initial/boundary value problem for a parabolic equation:

$$\begin{cases} \partial_t y(t,x) - \sum_{i,j=1}^{n} \partial_j(a_{ij}(x)\partial_i y(t,x)) + c(x)y(t,x) = h(t,x), & (t,x) \in Q \equiv (0,T)\times\Omega, \\ y(t,x) = 0, & (t,x) \in \Sigma \equiv (0,T)\times\partial\Omega, \\ y(0,x) = 0, & x \in \Omega. \end{cases} \tag{1}$$

Here $\Omega \subset \mathbb{R}^n$ is a bounded domain whose boundary $\partial\Omega$ is of C^2. The input $h \in C_0^\infty((0,T)\times\omega)$, ω is an arbitrarily fixed sub-domain of Ω.

We assume that $c \in C^6(\overline{\Omega})$, $c > 0$ on $\overline{\Omega}$ and

$$a_{ij} \in C^6(\overline{\Omega}), \quad a_{ij} = a_{ji}, \quad 1 \le i,j \le n,$$

and that the coefficients $\{a_{ij}\} \equiv \{a_{ij}\}_{1\le i,j\le n}$ satisfy the uniform ellipticity: there exists a constant $r > 0$ such that

$$\sum_{i,j=1}^{n} a_{ij}(x)\zeta_i\zeta_j \ge r|\zeta|^2, \quad \forall \zeta \in \mathbb{R}^n, \quad x \in \overline{\Omega}.$$

We consider the following inverse problems:

(1). Determine a_{ij}, $1 \le i,j \le n$, globally.

Inverse problem I. Let $\Gamma_0 \ne \emptyset$ be an arbitrary fixed relative open subset of $\partial\Omega$. Determine $a_{ij}(x)$, $x \in \Omega$, $1 \le i,j \le n$ by boundary measurements on $(0,T)\times\Gamma_0$ of solution y to (1) and measurements on Ω of y at a fixed time $\theta \in (0,T)$.

(2). Determine a_{ij}, $1 \le i,j \le n$, locally.

Let $S \subset \partial\Omega$ be a fixed relative open subset of $\partial\Omega$, Ω_0 be a subdomain of Ω satisfying $\partial\Omega_0 \supset S$.

Inverse problem II. Determine $a_{ij}(x)$, $x \in \Omega_0$, $1 \le i,j \le n$ by boundary measurements on $(0,T)\times S$ of solution y to (1) and measurements in Ω_0 of y at a fixed time $\theta \in (0,T)$.

Our main results:

(1). We obtain uniqueness and Lipschitz stability for Inverse problem I.

(2). We obtain uniqueness and Hölder stability for Inverse problem II.

Submissive Quantum Mechanics: New Status of the Theory in Inverse Problem Approach

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In recent years we have achieved a breakthrough in quantum mechanics and deepening its formalism due to new approach: using inverse problem theory and super-symmetry with computer visualization without which the contemporary graduation even from the best universities of the world is still quantum-defective. Our last achievement is radical improvement of the theory of waves in periodic structures (it opens the "black box" of classical Bloch-Floquet theory). Its generalization to multi-channel formalism gives us complete sets of exactly solvable models taking into account internal excitations. This enriches our quantum intuition (making possible even qualitative solutions 'in mind'. We have recently published our new book with the same title in Russian and put also its English version (still draft) into Internet: http://thsun1.jinr.ru/ zakharev/ (free access). (Springer Verlag referee Reshetikhin, prof. from Chicago, estimated this book as "very good"). The new theory reveals the elementary and universal constituents ("bricks" and building blocks) for construction (at least theoretically) of quantum systems with the given properties as with a "children toy constructor set". This means the most perfect degree of understanding of the subject. The fundamental depth of the discovered algorithms (complete sets of exact models) is combined with the extremely clear presentation. I have also published books "Lessons in Quantum Intuition"; "New ABC of Quantum Mechanics (in pictures)" and a big article in Physical Encyclopedia v.4. Millions of physicists, have studied quantum mechanics but have no notion about inverse problem and SUSY achievements with notion of the elementary and universal constituents ("bricks" and building blocks) for exact and qualitatively clear spectral, decay and scattering control. Quantum mechanics was as a Moon visible only from one side (direct problem). Now it is possible to get much more deep notion about this science using inverse methods.