The 2nd HU and SNU Symposium on Mathematics Abstracts

on the occasion of the 7th SNU-HU Joint Symposium

Department of Mathematics Hokkaido Univ.

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HOKKAIDO UNIVERSITY
TECHNICAL REPORT SERIES IN MATHEMATICS

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PDE session

Bochner-Schwartz theorems for generalized functions

Dohan Kim*

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Abstract

Making use of the heat kernel method we define a conditionally positive definite hyperfunction and obtain a Bochner–Schwartz type theorem for hyperfunctions which generalizes both the Bochner–Schwartz type theorem for distributions of Gelfand-Vilenkin and Bochner–Schwartz theorem for (Fourier) hyperfunctions of Chung-Chung-Kim.

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Applications of geometric measure theory to phase separation problems

Yoshihiro Tonegawa

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Abstract

In this talk, I will describe some known results on the singular perturbation problem initially studied by Sternberg, Modica, and others and some recent results on the non-minimal critical points as well as stable critical points via the method of geometric measure theory.

On Global Existence and Blow-up of Solutions of a Weakly Coupled System of Reaction-Diffusion Equations

Noriaki Umeda

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In this lecture, we talk about global existence, uniqueness, asymptotic behavior and blow-up in a finite time for solutions to the initial value problem for a reaction-diffusion system which appears in a chemical reaction theory under interactions of plural kinds of substances. The study of this field was initiated by Fujita (1966) for the scalar equation

$$u_t = \Delta u + u^p \qquad x \in \mathbf{R}^d, t > 0 \tag{1}$$

and was extended by a lot of people so far. Here, we consider nonnegative solutions of the initial value problem for a weakly coupled system

$$\begin{cases} u_{it} = \Delta u_i + u_{i+1}^{p_i}, & x \in \mathbf{R}^d, t > 0, i \in N^*, \\ u_i(x,0) = u_{i,0}(x), & x \in \mathbf{R}^d, i \in N^*, \end{cases} \tag{2}$$

where $N \geq 1$, $N^* = \{1, 2, \dots N\}$, $d \geq 1$, $p_i > 0$ $(i \in N^*)$ and $u_{i,0}$ $(i \in N^*)$ are nonnegative bounded and continuous functions. Throughout this lecture, we mean $u_{N+i} = u_i$, $u_{N+i,0} = u_{i,0}$ and $p_{N+i} = p_i$ for each $i \in \mathbf{Z}$ and we put

$$\alpha_i = \frac{1 + p_i + p_i p_{i+1} + \dots + p_i p_{i+1} \cdots p_{i+N-2}}{|p_1 p_2 \cdots p_N - 1|}.$$

Problem (2) has a nonnegative and bounded solution at least locally in time. For a given initial value u_0, let $T^* = T^*(u_0)$ be the maximal existence time of the solution. If $T^* = \infty$ the solution is global. On the other hand, if $T^* < \infty$ there exists $i \in N^*$ such that

$$\limsup_{t \to T^*} \|u_i(t)\|_\infty = \infty. \tag{3}$$

When (3) holds we say that the solution blows up in a finite time.

Here, we have main results such as following.

(I) *If* $p_1 p_2 \dots p_N \leq 1$, *any solution of* (2) *is global*;

(II) *If* $p_i \geq 1$ *for every* $i \in N^*$, $p_1 p_2 \dots p_N > 0$ *and* $\max_{i \in N^*}\{2\alpha_i\} \geq d$, *then* $T^* < \infty$ *for every nontrivial solution* $u(t)$ *of* (2);

(III) *If* $p_i \geq 1$ *for every* $i \in N^*$, $p_1 p_2 \dots p_N > 0$ *and* $\max_{i \in N^*}\{2\alpha_i\} < d$, *then there exist both non-global solutions and non-trivial global solutions of* (2).

Nonlinear functionals of the Boltzmann equation and uniform stability estimates

Seung-Yeal Ha
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Abstract

In this talk, I will present nonlinear functionals measuring variations and L^1-distance between two classical solutions for the Boltzmann equation with a cut-off inverse power potential. In the case that initial datum is a small, smooth perturbation of vacuum and decays fast enough in the phase space, we show that these functionals satisfy estimates which lead to a uniform BV-type estimate and the L^1-stability of classical solutions.

Convergence property of variational problems related to nonlinear Schrödinger equations and its applications

Reika Fukuizumi (Hokkaido University)

This talk is partly based on a joint work with Masahito Ohta (Saitama University). We study the orbital stability of standing wave solutions $e^{i\omega t}\phi_\omega(x)$ for the following nonlinear Schrödinger equations

$$i\partial_t u = -\Delta u - V(x)|u|^{p-1}u, \quad (t,x) \in \mathbb{R}^{1+n}, \tag{1}$$

which can model beam propagation in an inhomogeneous medium where $V(x)$ is proportional to the electron density. $\omega > 0$, $\phi_\omega(x)$ is a ground state of the following stationary problem:

$$\begin{cases} -\Delta\phi + \omega\phi - V(x)|\phi|^{p-1}\phi = 0, & x \in \mathbb{R}^n, \\ \phi \not\equiv 0. \end{cases} \tag{2}$$

The stability and instability of the standing wave solution for the case $V(x) \equiv 1$ has been well-studied in 1980's, namely, if $p < 1 + 4/n$ then the standing wave solution is stable for any $\omega > 0$. On the other hand, it was shown that if $p \geq 1 + 4/n$ then the standing wave solution is unstable for any $\omega > 0$ (see, e.g., [1] [2]).

Then, the question we address regarding the stability and instability of a standing wave solution of (1) for the case $V(x) \not\equiv 1$ is the following:

–Does $V(x)$ have an effect on the stability or instability range of p ?

We answer this question for the following type of $V(x)$, assuming that $n \geq 3$, $0 < b < 2$ and $1 < p < 1 + (4 - 2b)/(n - 2)$.

(V1) $V(x) \geq 0, \quad V(x) \not\equiv 0, \quad V(x) \in C^2(\mathbb{R}^n, \mathbb{R})$,

(V2) There exist $C > 0$ and $a > \{(n+2) - (n-2)p\}/2 > b$ such that

$$\left| x^\alpha \partial_x^\alpha \left(V(x) - \frac{1}{|x|^b} \right) \right| \leq \frac{C}{|x|^a}$$

for $|x| \geq 1$ and $|\alpha| \leq 2$.

The main purpose in this talk is to observe that under the above assumptions on $V(x)$, the standing wave solution of (1) is stable (resp. unstable) if $p < 1 + (4 - 2b)/n$ (resp. $p > 1 + (4 - 2b)/n$) for sufficiently small $\omega > 0$. Due to the inhomogeneous medium, the unstable effect occurs in the region $1 + (4-2b)/n < p < 1 + 4/n$ which is the stable region in the case where $V(x) \equiv 1$.

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[2] M. Grillakis, J. Shatah and W. Strauss, Stability theory of solitary waves in the presence of symmetry I, J. Funct. Anal. **74** (1987) 160–197.

Cauchy Problems of the Gauged Sigma Model

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Abstract

We study the gauged sigma model. In $\mathbb{R}^{1+1}$ the existence of the global smooth solution will be proved. Furthermore we show that the global weak solutions exist in $\mathbb{R}^{n+1}(n = 2, 3)$.
keywords: gauged sigma model, weak solution, gauge condition.

A criterion of linearized stability of stationary solutions for surface diffusion flow equation*

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Abstract

The geometrical evolution law

$$V = -\Delta\kappa$$

was derived by Mullins to describe the motion of interfaces which is governed purely by mass diffusion within the interfaces (for simplicity we set the diffusion constant to 1). Here V is the normal velocity of the evolving interface, Δ is the Laplace-Beltrami operator and κ is the mean curvature of the interface where we use the sign convention that a sphere with the normal pointing to the inside has positive curvature.

In this talk we study the following problem. The unknown interface $\Gamma(t)$, which is included in a bounded domain $\Omega \subset \mathbb{R}^2$ and touches the boundary of the domain, evolves with respect to time. The motion of the interface $\Gamma(t)$ is described by surface diffusion flow equation, which is denoted by

$$V = -\kappa_{ss}$$

where a subscript s denotes the differentiation with respect to the arc-length parameter along the interface. As the boundary conditions at $\Gamma(t) \cap \partial\Omega$ we take

$$\begin{cases} \Gamma(t) \perp \partial\Omega, \\ \kappa_s = 0. \quad \text{(no-flux condition)} \end{cases}$$

The fundamental features of this model are H^{-1}-gradient flow structure for the length of the interface $\Gamma(t)$ and also the area-preserving property. In addition, we can easily observe that the line segments and the circular arcs are stationary curves for surface diffusion flow equation, since the curvature of their curves is constant.

Our goal in this talk is to derive a criterion of linearized stability of these stationary curves (i.e. line segments or circular arcs) by investigating the sign of eigenvalues for the linearised problem of the above-mentioned nonlinear problem. According to circumstances, the development of these results into the three phase boundary motion by surface diffusion with triple junction will be also mentioned.

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*This is a joint work with Harald Garcke (Universität Regensburg) and Kazuo Ito (Kyushu University)

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INVERSE PEOBLEM session

Application of Oscillating-decaying Solutions for Identifying Cavity

Gen Nakamura

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Abstract

In this paper, we give a better estimate for the oscillating-decaying solution. The estimate is used to show the Polya type identity for identifying a polygonal cavity.

Inverse dynamical problem for connected beams

Mourad Sini

Department of Mathematics
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Abstract

We consider an inverse dynamical problem for a composite beam formed by two connected beams. The vibrations of the composite beam are governed by a differential system where a coupling takes place between longitudinal and bending motions. In this paper, we neglect bending motions and we only deal with the longitudinal motions. These motions are governed by a two-by-two second order system coupled in the lower order terms by the shearing stiffness coefficient, which models the connection between the two beams. We are concerned with the reconstruction of this coefficient from the dynamical Neumann to Dirichlet operator.

Identification of Simple Poles via Boundary Measurements and an Application to EIT *

Hyeonbae Kang† and Hyundae Lee ‡

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Seoul National University, Korea

Abstract

We consider the problem of identifying simple poles of a meromorphic function by means of the value of the function measured on a circle enclosing those poles. We propose an algorithm for this problem in a mathematically rigorous way with a stability estimate. Results of numerical testing are provided to show validity of the algorithm. We then apply the method to an electrical impedance tomography problem to detect diametrically small inclusions of disk shape via boundary measurements.

Mathematics subject classification (MSC2000): 35B30

Keywords: detection of simple poles, boundary measurements, electrical impedance tomography, small inhomogeneities

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Numerical Experiments for Reconstruction Formulas of Conductivity at the Boundary from Localized Dirichlet to Neumann Map

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1. Introduction

In this paper, we present a new numerical method for determining the conductivity and its normal derivative at the boundary of the body from local measurements. Given a strictly positive, real-valued conductivity $\gamma \in L^\infty(\Omega)$, we apply a voltage potential f on the boundary and solve the Dirichlet problem

$$\nabla\cdot(\gamma\nabla)u = 0 \quad \text{in} \quad \Omega, \qquad u|_{\partial\Omega} = f. \tag{0.1}$$

where $u(x)$ is electric voltage potential. The resulting current density distribution is

$$\Lambda_\gamma f = \gamma\frac{\partial u}{\partial\nu}|_{\partial\Omega}.$$

2. Reconstruction formulas

For simplicity, we take $\Omega\subset\mathbb{R}^2$ to be the unit disc.

The reconstruction formulae in weak form:

$$\int_{\partial\Omega}\gamma\eta^2 d\sigma = \lim_{N\to\infty} N^{-1}\int_{\partial\Omega}\overline{\Phi_N}\Lambda_\gamma\Phi_N d\sigma \tag{0.2}$$

$$\int_{\partial\Omega}\frac{\partial\gamma}{\partial\nu}\eta^2 d\sigma = \lim_{N\to\infty}\int_{\partial\Omega}(4\overline{\Phi_N}\Lambda_\gamma\Psi_N - 2\overline{\Phi_N}\Lambda_\gamma\Phi_N) \tag{0.3}$$

where $\eta\in C_0^M(\partial\Omega)$: smooth, compactly supported function, and $\int_{\partial\Omega}\eta^2(\theta)d\theta = 1$.

Functions Φ_N and Ψ_N are given by

$$\Phi_N(\theta) = e^{iN\theta}\eta(\theta), \qquad \Psi_N(\theta) = e^{i\frac{N}{2}\theta}\eta(\theta).$$

Our reconstruction algorithm consist essentially of evaluating the right hand sides of (0.2) and (0.3) with finite N.

3. Numerical methods

Transform (x_1, x_2) to new coordinate system (θ, r)

$$x_1 = (1-r)\cos(\theta), \quad x_2 = (1-r)\sin(\theta)$$

Then Ω and $\partial\Omega$ are given by $0 < r \le 1$ and $r = 0$ respectively $(-\pi\le\theta<\pi)$.

Take $\eta(\theta)$ in the form:

$$\eta(\theta) = \begin{cases} c(\epsilon\theta - \frac{\pi}{2})^\alpha(\epsilon\theta + \frac{\pi}{2}\cos(\epsilon\theta)) & -\frac{\pi}{2\epsilon}\le\theta<\frac{\pi}{2\epsilon} \\ 0 & \text{otherwise,} \end{cases}$$

where $\epsilon = 4$ and $\alpha = 4$. The constant c is chosen such that $\int_\Gamma \eta^2(\theta)d\theta = 1$.

For reconstruction, we compute $g_N(\beta) := N^{-1}\int_{\partial\Omega}\overline{\Phi_{N,\beta}}\Lambda_\gamma\Phi_{N,\beta}d\sigma$ and $h_N(\beta) := \int_{\partial\Omega}(4\overline{\Psi_{N,\beta}}\Lambda_\gamma\Psi_{N,\beta} - 2\overline{\Phi_{N,\beta}}\Lambda_\gamma\Phi_{N,\beta})d\sigma$. We consider $g_N(\beta)$ and $h_N(\beta)$as approximation to $(\gamma|_{\partial\Omega}*\eta^2)(\beta)$ and $(\partial\gamma/\partial r|_{\partial\Omega}*\eta^2)(\beta)$.

4. Results

We construct a conductivity distribution inside the unit disc that roughly models a cross-section of the human chest. So our chest phantom conductivity takes the form

$$\gamma(x_1, x_2) = \frac{3}{2} + \frac{2}{10}x_2 + \frac{3}{2}\chi_{D_1} - \chi_{D_2} - \chi_{D_3}.$$

The conductivity was reconstructed from the simulated noisy EIT measurements with 32 or more electrodes. Normal derivative could not be recovered using 32 or 64 electrodes with acceptable accuracy, but reconstruction from 96-electrodes had 14% relative L^2 error.

The results suggest that formula (0.2) and (0.3) can be used for approximately recovering the trace and normal derivative of a smooth conductivity from local EIT measurements with at least 32 and 96 electrodes, respectively.

Inverse Problem for the Nonselfadjoint Schrödinger Operator with Energy Dependent Potential in Two Dimensions

Michiyuki Watanabe *
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COE Researcher

We consider the two dimensional Schrödinger equation of the form

$$-\Delta u(x) + i\sqrt{E}b(x)u(x) = Eu(x), \qquad x \in \mathbf{R}^2 \tag{1}$$

with asymptotic expansion

$$u(x) = e^{i\sqrt{E}\omega\cdot x} + \frac{e^{i\sqrt{E}r}}{r^{1/2}}A(E,\theta,\omega) + o(r^{-1/2}), \quad r \to \infty, \tag{2}$$

where $E > 0$, $r = |x|$, $\theta = x/|x|$ and $\omega \in S^1$. $A(E,\theta,\omega)$ is called the scattering amplitude. The equation (1) is derived from the dissipative wave equation

$$w_{tt}(x,t) - \Delta w(x,t) + b(x)w_t(x,t) = 0$$

by considering the time harmonic solution of the form $w(x,t) = e^{i\sqrt{E}t}u(x)$.

We shall consider the inverse scattering problem of determining $b(x)$ from the scattering data $A(E,\theta,\omega)$ at fixed energy $E > 0$. The uniqueness theorem will be shown without assuming the smallness condition on $b(x)$ under the low energy.

Theorem 1. *Assume that* $b(x)$ *is a function on* $\mathbf{R}^2$ *satisfying*

$$|b(x)| \le C(1+|x|)^{-1-\varepsilon}, \qquad C > 0, \quad \varepsilon > 0.$$

Then there exist $E > 0$ *such that the equation* (1) *has a solution with asymptotic expansion* (2).

We denote that

$$L = -\Delta + q_E, \qquad q_E = i\sqrt{E}b - E.$$

Let $A_j(E,\theta,\omega)$ be the scattering amplitude corresponding to the potentials $b_j(x), j = 1,2$. We shall denote by $W^{l,p}$ the usual Sobolev space of order l in L^p and denote by B_R the disk with radius $R > 0$.

Theorem 2. *Let* $b_1(x), b_2(x) \in W^{1,p}(\mathbf{R}^2)$ *for some* $p > 2$ *and* $\operatorname{supp} b_1, \operatorname{supp} b_2 \subset B_R$ *for some* $R > 0$. *Assume that* $\|b_j\|_{W^{1,p}(\mathbf{R}^2)} \le M, j = 1,2$ *and* 0 *is not a Dirichlet eigenvalue for* L *in* B_R. *Then there exists a positive number* $N = N(M,R,p)$ *depending on* M,R,p *such that if*

$$A_1(E,\theta,\omega) = A_2(E,\theta,\omega), \qquad \text{for all } \theta,\omega \in S^1$$

holds at fixed $E > 0$ *which satisfies* $E < N$, *then we have*

$$b_1(x) = b_2(x) \qquad \text{in } \mathbf{R}^2.$$

*Supported by 21st Century COE Program, "Mathematics of Non-Linear Structure via Singularity", Department of Mathematics, Hokkaido University.

NUMERICAL ANALYSIS session

Symmetry breaking and symmetry recovery in the models of insects flight

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Abstract

Insects' flight mechanism has not been fully understood because of its strong nonlinearity due to the dynamical interaction between flapping wing and separation vortex from the wing. Without such special interaction, insect can not even sustain their weight during flying. Wing-vortex interactions has been analyzed by real insect, mechanical model, and numerical simulations, and some lift-enhance mechanisms has been proposed. But the detail of such mechanism has not been understood.

We tackle this problem by focusing on the concept of symmetry. Symmetry breaking or symmetry recovery is clear evidence of the property change of the wing-vortex interaction, and knowledge from the result will help us to understand the insects flight. Two models matching on this issue will be reported in this talk.

One is a symmetry-breaking of two-dimensional symmetric flapping model, focusing on the centroidal motion due to wing-vortex interaction. This model does not assume uniform flow, which is indespensable for the flight of airplane, but it assumes the wing flapping, the resulting airflow due to separation vortices, and the centroidal motion. A new type of symmetry-breaking mechanism is observed in this model. The interaction among wing and two coherent vortices formed by separation vortex causes asymmetric centroidal motion. We will clarify the occurrence of symmetry-breaking in terms of bifurcation.

The other is a symmetry-recovery of a heaving wing, focusing on the effect of elasticity on the wing-vortex interaction. Heaving of the symmetric rigid wing in uniform flow can generate an asymmetric wake pattern(wake deflection) when heaving amplitude is larger than a critical value. We will show the wake deflection is symmetrized if the wing is replaced by an elastic one. In that case, the tip of the elastic plate moves like eight-figure. This motion reduces the circulation of the coherent vortices than the rigid plate, which enhances heaving critical amplitude.

Recent progresses on nonconforming finite element methods

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Abstract

It has been well-known that standard simple conforming finite elements of lowest order for (Navier-)Stokes and plane elasticity problems would produce undesirable features such as instability or numerical locking. Contrary to conforming finite elements, the use of simple nonconforming finite elements of lowest order gives numerically stable solutions. Also the use of nonconforming elements draws attention in view of domain decomposition methods, such as mortar methods.

The P_1-nonconforming simplicial elements of Crouzeix-Raviart (1973) is quite a natural choice as the lowest-order elements for simplices. However, in the quadrialteral case, it has been a kind of puzzle to choose a lowest-order nonconforming element, (including the works of Houde (1984), Rannacher and Turek (1992), Douglas et al. (1999),) especially since Arnold, Boffi, and Falk (2000) gave an equivalent condition for a quadrialteral element to have an optimal-order of convergence. In this talk, motivated by the work of Arnold, Boffi, and Falk, we (with Chunjae Park of Seoul National University) introduce a P_1-nonconforming quadrilateral finite element for second-order elliptic problems in two dimension. We will introduce several interesting aspects of this new element. Applications of nonconforming finite element will be also addresses.

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A numerical method for backward parabolic problems

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Abstract

A parallel method for time-discretization of backward parabolic problems is proposed. The problem is reformulated to a set of Helmholtz type problems with a parameter on a suitably chosen contour in the complex plane, and we solve them by the finite element method. Finally, we obtain a regularized solution cut off high frequency terms by the inverse Laplace transform without requiring the knowledge of the eigenfunctions of the differential operator. Since the regularization is performed without perturbing anything, quality of the solution is much improved than that of other methods and the two proposed discretization schemes have arbitrary high order of accuracy, and the spectral accuracy respectively. Error estimates and some numerical examples are presented.

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A Quadratic Nonconforming Finite Element

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Abstract

A new quadratic nonconforming element on rectangles is introduced. Bases are constructed for second-order elliptic boundary value problems with Dirichlet and Neumann conditions. The local and global interpolation operators are defined. Error estimates of optimal order are derived in both broken energy and $L^2(\Omega)$ norms for second-order of elliptic problems. Brief numerical results are also shown to confirm the optimality of the presented quadratic nonconforming element methods.

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Level Set Based Geometric Optics

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Abstract

We introduce a level set approach for ray tracing and the construction of wavefronts in geometric optics. This is important in a wide variety of applications in wave propagation. Our approach automatically handles the multivalued solutions that appear and automatically resolves the wavefronts. This is achieved through solving for the bicharacteristic strips, whose projection to spatial space gives the wavefronts, in a reduced phase space under a Eulerian and partial differential-equation-based framework. The bicharacteristic strips are represented using a level set approach for handling higher codimensional objects and the partial differential equations responsible for the evolution are reduced forms of the Lioville equations. Results for the two and three dimensional case for constant and variable indices of refraction are shown and compared to those of other current methods in the field.

Periodic Traveling Waves in an Undulating Band Domain and Their Homogenization Limit

Bendong Lou

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(joint work with Hiroshi Matano* and Ken-ichi Nakamura†)

Abstract

We study a curvature-dependent motion of curves in a band domain in $\mathbb{R}^2$ with periodically undulating boundaries. The law of motion is given by $V = \kappa + A$, where V is the normal velocity of the curve, κ is the curvature, and A is a positive constant.

We first establish a necessary and sufficient condition for the existence of periodic traveling waves.

We then study how the average speed of the periodic traveling wave depends on the geometry of the boundary. More specifically, we consider the homogenization problem as the period of the boundary undulation, denoted by ε, tends to zero, and determine the homogenization limit of the avarage speed of periodic traveling waves. Quite surprisingly, this homogenized speed depends only on the maximum opening angle of the domain boundary. Our analysis also shows that, for any small $\varepsilon > 0$, the average speed of the traveling wave is smaller than A, which equals the speed of the planar front. This implies that boundary undulation always lowers the speed of the traveling waves.

Moreover, we analyze the complex boundary behavior. It is shown that the propagation of the traveling wave near the boundaries is divided into three different stages.

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Level set based image segmentation using active contours

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Abstract

Thistalk presents an image segmentation model to detect objects and edges (or boundaries) in a given image, based on techniques of curve evolution, Mumford–Shah functional and level set method. In the level set formulation, the active contour evolves by mean curvature or by the fitting term until this curve stops on the boundaries of object. The fittingterm does not depend on the gradient of the given image, as in the classical active contour models, but is instead based on Mumford–Shah segmentation techniques. And thistalk gives a numerical algorithm using finite difference scheme and we present some experimental results using the model described above.

Reinforcement Learning Model with Short-Term Memory Capability

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Reinforcement learning is one of the learning algorithms in a field of artificial intelligence. An agent changes action selection rules based on interaction with an environment. For a measure of goodness of an agent's behavior, an agent receives reward. An aim of an agent is maximizing a return (sum of future reward).

However, an agent that depends on only immediate sensory inputs cannot decide a right action. Because generally perception ability of an agent is incomplete. Such a situation is called "perceptual aliasing". One of the method to deal with this problem is using internal memory. In particular, effect of redundant memory is argued in a precedence study. In this study, suitable strategy for utilizing redundant memory is considered by focusing on initial prediction of return.

Simple "delayed response task" that needs short-term memory capability is investigated. Agents that have two factors - redundant memory and "optimistic" initial prediction of return - show relatively good learning performance. Performance of agents that have only either one of these factors is inadequate. Therefore, importance of optimistic initial prediction of return for utilizing redundant memory is revealed.

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