There is the Voronoi diagram as one of the most fundamental concepts in computational geometry. The Voronoi diagram is a tessellation of a space into regions, and its algorithms and applications have been studied in a variety of fields, for example, geographic information, mesh generation in the finite element method, pattern recognition, image compression and so on.

The Voronoi diagram has also been generalized in a variety of directions. A typical such direction is the generalization of the distance. The most fundamental Voronoi diagram is defined according to the Euclidean distance, while many generalized Voronoi diagrams are generated by replacing the Euclidean distance with other distances.

In this talk, we introduce a boat-sail distance. Suppose that we want to travel on the surface of water with a boat. If there is no flow of water, the boat can move in any direction at the same maximum speed. If the water flows, on the other hand, the speed of the boat is anisotropic; the boat can move faster in the same direction as the flow, while it move only slowly in the direction opposite to the flow direction. Thus, a boat-sail distance is defined as the smallest time necessary for the boat to move from one point to another. According to this distance, the surface of water is partitioned into regions. This partition is called a boat-sail Voronoi diagram.

It is easy to define the boat-sail distance and the associated Voronoi diagram, but it is difficult to compute the boat-sail distance. If we know the path achieving the smallest time, we can calculate the distance. However, we cannot know it in advance. Hence, it is hard to calculate the distance directly.

In order to overcome this difficulty, we construct a numerical method. For this purpose, we reduce the problem of computing the boat-sail distance to a boundary value problem of a partial differential equation. This idea is the same as the idea for reducing the problem of computing the Euclidean distance to a boundary value problem of the eikonal equation. If we set the speed of the boat to 0, then our proposal equation is identical with eikonal equation. Hence, our formulation can be considered a generalization of the eikonal equation.

Our proposal equation is a kind of static Hamilton-Jacobi equations which depend on the variables of a position and the spatial gradient of a unknown function with respect to the the variables of the position. Furthermore, this type of Hamilton-Jacobi equations can be classified into two types in the view of the numerical computation.

First is the isotropic type, that is, the boat moves in any direction at the same speed. The representative equation is eikonal equation, which represents the Huygen’s principle. For this type, the fast marching method is well known as an efficient and stable computational technique.

Second is the anisotropic type, that is, the speed of the boat changes depending on
the direction. Our partial differential equation belongs to this type. Our equation is the
generalization of eikonal equation, and hence, we would like to apply the first marching
method to our equation. However, it is known that, when the first marching method is
applied to the anisotropic type, the fast marching method does not work well.

In this talk, we propose a new numerical method by extending the scheme of the fast
marching method and show the efficiency and the stableness of the proposal method by
numerical experiments.

We can also consider the boat-sail distance on curved surfaces, and so we touch upon
an efficient method for computing this distance.