Realization of Impossible Objects and a New Type of Visual Illusion

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There is a class of line drawings called "pictures of impossible objects", which give us some impression of three-dimensional solid objects but at the same time give us impression that such objects cannot be realized. These pictures are familier in the sense that famous artists such as Escher used these pictures for their artwork. It is usually believed that the solid structures represented in these pictures cannot be realized as three-dimensional objects.

However, some of pictures of impossible objects actually represent three-dimensional solid objects. Those pictures can be identified by the theory of interpretation of line drawing, which has a long history in computer vision.

A typical process of interpretation is the following. First, lines in a given line drawing are classified according to geometric properties of the associated edges in a three-dimensional space; for this purpose a list of possible combination of edge types, which is called a juction dictionary, plays a main role. By this process, a candidate of interpretation of the picture as a solid object is obtained.

The next step is to judge whether the candidate is a correct interpretation. This problem is reduced to the problem of judging the feasibility of a system of linear constraints. Actually, the planarity of each face is represented by a set of linear equations with respect to the depths of the vertices and the coefficient of face equations, while the convexity or concavity of an edge is represented by linear inequality. Therefore, the realizable "impossible objects" can be characterized as the pictures of impossible objects whose system of the associated linear constraints is feasible. This problem can be solved by a standard method for linear programming problems.

Once we find such a realizable "impossible object", we can construct the associated polyhedral object by finding a point in the feasible regions of the associated system of linear constraints. Actually we first construct three-dimensional geometric model, next draw the picture of unfolded surface, and finally construct a solid from the unfolded surface.

The resulting solid object generates a new type of visual illusion in the sense that we are looking at a solid object but at the same time we feel that such an object cannot exist. This illusion arises only when the solid object is seen from a unique viewpoint associated with the object.

The next question is why these pictures give us impression of "impossible object" although the object is realizable. My present answer is that those pictures consist of only three sets of mutually parallel line segments, and we human tends to believe that the faces are connected by the right angle on the surface of the solid represented in those pictures. Actually if we connect adjacent faces by the right angle, we cannot construct the object at all; we can construct it only when we use other than right angles.

In this talk we focus on the way of reducing the realizability of objects to the feasibility of linear constraints, together with some combinatorial aspects of the problem in order to reduce the time complexity as well as to circumvent numerical instability.

We also touch upon a method for realizing other impossible objects by curved surface objects whose surface can be expanded to a plane. By this method we can generate the "impossible object" by cutting and glueing a sheet of paper.

Furthermore, we consider applications of those realizable "impossible objects" to art, architecture, and entertainment.