

Article

Probabilistic Method to Assess the Impact of Charging of Electric Vehicles on Distribution Grids

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Abstract: This paper describes a grid impact analysis of charging electric vehicles (EV) using charging curves with detailed battery modelling. A probabilistic method using Monte Carlo was applied to a typical Spanish distribution grid, also using mobility patterns of Barcelona. To carry out this analysis, firstly, an IEEE test system was adapted to a typical distribution grid configuration; secondly, the EV and its battery types were modeled taking into account the current vehicle market and the battery characteristics; and, finally, the recharge control strategies were taken into account. Once these main features were established, a statistical probabilistic model for the household electrical demand and for the EV charging parameters was determined. Finally, with these probabilistic models, the Monte Carlo analysis was performed within the established scenario in order to study the lines' and the transformers' loading levels. The results show that an accurate model for the battery gives a more precise estimation about the impact on the grid. Additionally, mobility patterns have been proved to be some of the most important key aspects for these type of studies.

Keywords: electrical vehicles; distribution grids; impact study

1. Introduction

The growing interest in the introduction of fossil fuel alternatives into the transportation and the large number of projects regarding this issue show that, in the future, the presence of the electric vehicle (EV) in the electric system will be significant. Important cities in the world, such as New York, have been the scenario of reference studies about the potential impact of Plug-in Hybrid EV technology on grid operations and electricity system planning [1]. This study illustrates how the proper management of charging patterns can lead to the utilisation of the current grid with any investment in infrastructure. Other studies, like [2], conclude that in cases where Plug-in Hybrid EVs obtain the energy from electricity for 40% of their miles, penetrations up to 50% by the vehicle fleet would not require the increase of electric generation capacity under optimal dispatch rules.

Therefore, it is possible to assume a future with a strong presence of EVs [3]. Under these conditions, the use of wind, solar and renewable energies is generally a potential alternative [4,5] energy source. This situation will represent an opportunity for EVs to be part of an integrated electric system, with the ability to adapt to varying conditions by performing ancillary services like power-frequency, power factor or voltage regulation. However, an EV implementation introduces complexities of this integrated system into the grid. Depending on power level, timing, and duration of the EV connection to the grid, there could be a wide variety of impacts on grid constraints, capacity requirements, fuel types used and emissions generated [6], which must be managed in order to reach an optimal dispatch rule. A control system able to optimally manage all constraints and variables of electrical transportation will lead to moderate investments into the grid and therefore to its implementation.

Simulating the most realistic scenarios is critical in order to assess the grid impact of recharging EVs. The aim of this paper is to model, in the most detailed way, the recharge load curve of EVs using detailed battery models. The most commonly used analysis reported in literature [7-13] uses the constant load model, which, as a conservative approach, presents the worst case, with the disadvantage that the grid impact will be overestimated. In order to proof the grid impact analysis method using detailed battery modelling, a standardised IEEE grid has been adapted to reproduce a typical distribution grid in Barcelona, combined with mobility patterns. The steps to achieve these goals are: modelling the electrical system, establishing the rules for the probabilistic model for household electrical demands and the EV demand (due to its recharge), by implementing the previous conditions in a grid and analysing the results obtained through the Monte Carlo Method.

2. State of the Art

When studying the impact of an EV fleet recharge in a grid, several aspects have to be considered prior to starting the process. Firstly, the electric grid used in the study, taking into account the type of grid, its topology, its voltage level and daily load profile, is important. Secondly, the EV type, its battery

features and the EV fleet modelling should be defined. Finally, the recharging control strategy has to be considered.

When studying the impact of EV recharge on electric grids, most of the authors [7,8,10,11] chose the distribution grid type to perform their simulations. The IEEE has standardised distribution models and available data from real grids corresponding to distribution grids.

Regarding the load profile modelling, [7] used the hourly average household load curve (available from the RELOAD data base used by the Electricity Module of the National Energy Modelling System (NEMS)). A one year period with three different day types (typical weekday, weekend and peak day) and nine load types (space cooling, space heating, water heating, cooking, cloth clothes drying, refrigeration, freezing, lighting and others) were taken into consideration. In [8], a household load profile was extracted from the measurements from [14]. These measurements were taken during over the course of 24 h in 15-minute intervals with a resolution of 15 minutes during on an arbitrary winter day. Random load profiles based on probability density functions were used in [12]. Finally, in [11], grid load profiles were extracted from from the Stockholm Office of Research and Statistics, 2009 [15] (most critical case). Regarding EV modelling, most authors used a generic EV model. However, other authors chose a real model, such as the Chevy Volt [7]. The driving range of electric vehicles are between 32.7 km and 64.37 km. However, regarding the state of charge (SOC) for starting the recharge, the values established for the different types are similar, between 20% and 30% [7,8,10].

Depending on the chosen EV typology, the capacity of its batteries varies drastically. For Plug-in Hybrid EV batteries, the capacity values reported in literature are: 8 kWh [12], 8.2 kWh (2.7 h of recharging time) [11] and 11 kWh (8.8 kWh considering minimum allowable SOC) [8]. In contrast, pure EVs have a capacity of 16 kWh, which almost doubles the capacity of the studied Plug-in Hybrid EVs [7].

For the specific case of the Chevy Volt EV, the main features of its lithium-ion battery are: an energy of 16 kWh, a voltage of 320 V to 350 V, a full recharge time from 6 to 6.5 h using a 110 V outlet and an electric driving range of 64.37 km (40 miles) [7].

Considering vehicle fleet modelling, several authors have taken into account the average travelled distance, the methodology for the fleet modelling and the penetration of EVs into the grid.

The travelled distance indicates the required capacity for the battery and the energy consumed by the proposed EV. The values proposed for the travelled distance per day are 0.01 km/day to 16.1 km/day [12], 16.1 km/day to 32.2 km/day [12] or 32.7 km/day [11].

Regarding the methodology for fleet modelling, in [7], its authors estimate how many vehicles would be in a transformer based on the established EV penetration and also check the transformer loading capacity. Other authors have modelled the vehicle fleet as a single large battery which comprises the sum of all batteries [10]. And finally, other authors estimated the total energy amount for private car transportation for the entire year and then it was divided by 365 in order to obtain the energy for a single day [11].

Finally, for the EV penetration, most authors use values from 0% to 20%–30% of the EV penetration [7,8,10,12,13], although one author makes the hypothesis of reaching 100% of the EV penetration [11].

The EV recharging control strategy determines how the EV obtains energy from the grid. These strategies are based on how and when the recharge is performed. According to the reviewed literature, the recharge can be performed on one of three levels: fast, quick or slow charge. The period of the day when the recharge is performed is usually determined by the peak and off-peak times of the daily load profile.

Four strategies are studied in [7]: slow charging when the vehicles arrive at home (6 pm), slow charging of peak charging, quick-charging (240V/30A) when the vehicles arrive at home (6 pm) and quick-charging (240V/30A) of peak charging. In order to deal with possible overloads in the transformer, the author proposes controlling household loads when charging vehicles through an Advanced Metering Infrastructure (AMI) and performing a stagger charge for the EV's recharge.

In [8], two possibilities are suggested: uncoordinated and coordinated charging. Uncoordinated charging is based on the concept in which each individual vehicle starts the recharge at a random time step, while with coordinated charging, the recharge is performed by minimising the grid's power losses through an optimisation problem, using quadratic programming and dynamic programming. In [11], the concept of controlled recharges is also suggested. In the studied paper, unregulated charging establishes that people recharge their cars as soon as they get home if there are no economic incentives; while in the regulated charging through Demand Side Management, the load is shifted from peak to valley periods.

Two strategies of recharge time distribution are suggested in [10]: continuous recharge and timed recharge (22:00–6:00; 21:00–9:00). It also proposes Vehicle to Grid (V2G) for the support of the grid in peak periods.

Finally, [16] introduces the concept of recharging the EV at the workplace in a recharging infrastructure where the recharge is feasible, independent of the number of available outlets and the line's loading. Four scenarios are suggested: a NO-EV scenario where no charging strategy is required, a Plug and Play scenario where no pricing mechanisms are used and where EV users recharge their cars if available outlets exist, an enhanced workplace access scenario where recharge at work is plausible even when the workplace does not offer any outlets, and an off-peak scenario where the recharge is performed between 6am and 8pm.

3. General Model Used for the Study

Evaluating the impact of EV recharge on a grid through a deterministic analysis will only lead to the analysis of a specific scenario. In situations where many possibilities are expected, the most common procedure is to analyse the most critical situation. However, the most critical situation is not always the most feasible one. The Monte Carlo method allows one to ensure that, under specific conditions, the results will be within a range of values with a predefined probability. Because the model will be under deterministic and probabilistic conditions, it is important to have a valid model for both situations.

3.1. Time Horizon

The chosen time horizon for the simulations is the time lapse of 24 h. This time horizon will be divided into one hour periods. The assumption of independence amongst different days is made; therefore, no data relationship is expected between day t - 1 and day t.

3.2. Statistical Variables

When performing Monte Carlo simulations, the first step is to define the statistical variables. Because one of the objectives of this paper is the analysis of the influence of EV recharge, the chosen statistical variables are:

- Regular electric demand
- Demand due to EV recharge

Both statistical variables will be modelled as regular loads, each one independent from the other one. In the same way as regular loads, these two variables have been modelled as an active and a reactive power, with the result of two inputs for each statistical variable; there is, however, just one input, since active and reactive power are related through a fixed power factor which does not vary.

Both regular demand and the demand due to EV recharge are modelled as aggregated load in each node. Equation (1) shows the aggregation of EVs in each node b, for each hour h and each iteration i.

$$P_{b_h_i} = \sum_{j=1}^{n_{EV}} P_{EVj}$$
(1)

3.3. Description of the Algorithm for the Probabilistic Load Flow Implementing the Statistical Model for the Household and EV Consumption

Figure 1 shows the algorithm for the load dispatching. The algorithm is based on the Monte Carlo method. In this diagram two main parts can be observed:

- Sample generation of the statistical variables
- Execution of probabilistic load flows

In the first part of the algorithm, the values for household and EV consumption are generated. Thus, the first step in the process is to obtain the statistical models for these two consumptions. For each different type of consumption, active and reactive power will be determined for each node, hour and iteration. As a result, the active and reactive power are $P_{DD,h,b,i}$ and $Q_{DD,h,b,i}$ for household demand and $P_{EV,h,b,i}$ $Q_{EV,h,b,i}$ for the EV consumption. The subscripts DD and EV stand for a consumption related to the household demand and for the EV recharge, respectively. The subscripts h, b and i correspond to the obtained value for the hour h, at the node b of the grid at the current iteration i. The statistical model for the household consumption, "Statistical model of $P_{DD,h,b,i}$ $Q_{DD,h,b,i}$ ", is obtained by studying Spanish load profiles over the course of one year (Section 4). The statistical model for the EV consumption $Statistical model of P_{EV,h,b,i}$ ", comes from a study where variables, such as urban mobility, technical EV features and the number of EVs in the scenarios, are taken into account [17] (Section 5). The next step in this sample generation is to chose the number of iterations i and define the number of nodes b which comes defined for the study grid. Once these values have been defined, the samples for the household and EV consumption can be generated through their respective statistical models previously obtained.

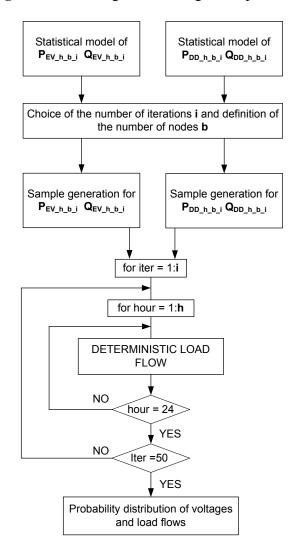


Figure 1. Flow diagram for the general procedure.

In the second part of the algorithm, the probabilistic load flow is run. It consists of an iterative process where a deterministic load flow is performed with the previously generated probabilistic values for each node, each iteration and each hour. The results of this process are the voltages of each node per load flow and iteration, and the line and transformer loadings per load flow and iteration. All these values will allow the determination of the confidence intervals and therefore the tools to compare and evaluate the different generated scenarios.

4. Generating the Probabilistic Model for the Grid's Electrical Demand

Electrical consumption cycles can be classified depending on its their predictable patterns. These patterns can be established either for seasons, for weekdays or for weekend days. For instance, weekdays and weekend days will have different curve shapes, even for the same week. The main factors that determine these patterns are climate and human behaviour. Therefore, it is expected that northern countries have different load patterns than from tropical countries.

A method to obtain the probabilistic model for the electrical demand is presented in this section. This model is obtained through two steps:

- 1. The creation of a statistical model able to generate random values according to previously defined conditions.
- 2. The generation of samples for the performed load flows and for each iteration.

4.1. Data Processing

In order to generate a probabilistic model for the electrical demand, several authors [18,19] have concluded that using a normal distribution is a reasonable approach. As a result, this will be the distribution used in this study.

The data used to build the statistical model has been extracted from the Spanish TSO REE (*Red Eléctrica de España*) and corresponds to Spain's electricity demand from January to September. Figure 2 depicts the daily load profiles of Spain from January to September. Each line represents a day load profile and every month has a different line colour. Different load patterns can be distinguished and since it is necessary to have an accurate model, this data is aggregated according to the similarities of the patterns.

The data corresponding to the months with a high variation of temperature was not considered because of the distortion caused in the model. Through a probability analysis, three patterns were established:

- Winter (January, February)
- Spring (April, May)
- Summer (July, August)

If the consumption profiles are grouped according to the previous classification, it is possible to get a probability distribution for each hour which follows a normal distribution. As an example, Figure 3 shows a winter labour histogram for 24 h of the power demand for Spain (Figure 3(a)) and its standard deviation (Figure 3(b)), both in MW.

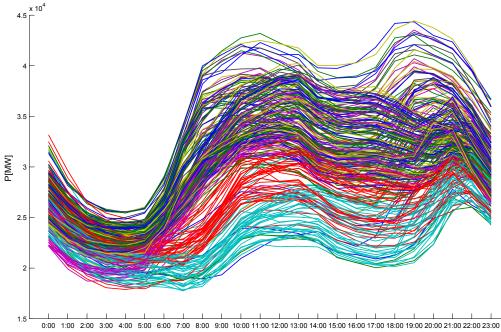
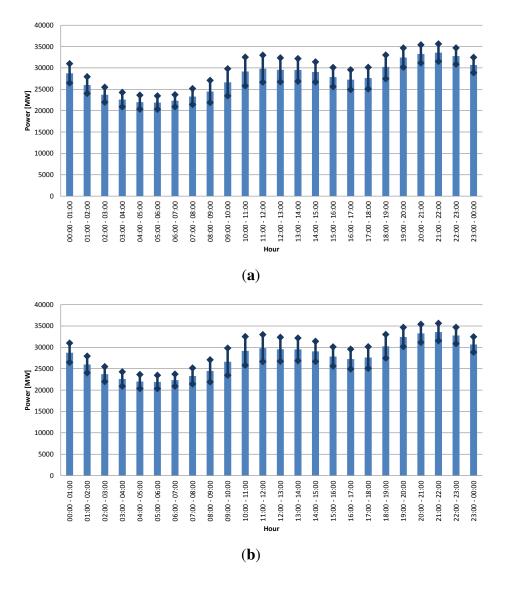


Figure 2. Daily load curves of Spain from January to September.

t [h]

Figure 3. Variation of the overall demand on a labour winter day of Spain. (a) Mean, maximum and minimum value of the power demand; (b) Standard deviation of the power demand.



4.2. Adapting National Demand to a Standard Distribution Grid

Because the studied grid is a distribution grid for a specific region, it is necessary to adapt the load patterns extracted for the entire country. Therefore, the national load curve has been adapted to the study grid. The hypothesis that the placed transformers will be able to supply energy to all the types of loads (household, commercial and industrial) without exceeding the allowed loading levels when no EVs are connected to the grid is supported.

The process begins with the national load curve reduced to a unitary curve, and then a base change is performed in order to adapt the power to the capacity of each transformer station. In order to generate active and reactive power, the hypothesis of a constant power factor of 0.9 was made in order to comply with the worst case considered in the Spanish legislation [20].

5. Generation of the Probabilistic Model for EV Demand

The objective of this section is to model the consumption generated by the EV recharge in the distribution grid in order to evaluate its impact together with the household demand. The EV consumption is difficult to model in a deterministic way because it depends on factors such as urban mobility, the EVs' technical features and the number of EVs. Therefore, it appears to be necessary to create a methodology able to take into account all these factors and also able to generate a valid statistical variable for the EVs' consumption.

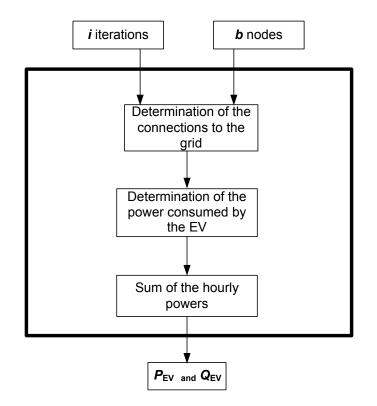
The main difficulty when creating the EV statistical variable is the lack of available data. Because of this, the statistical variable which represents the EV consumption cannot be obtained from historical data, but will have to be generated from a determined number of random variables. Once these are generated, the Monte Carlo method is applied. These random variables are:

- Distribution of the EV along the grid nodes
- Connection time of the EV
- Load curve of the battery of the EV
- Energy consumed by the EV before its connection

In general terms, the two main stages of the methodology of the creation of the EVs' consumption are (Figure 4):

- 1. Generation of the EVs' number of connections for each hour, node and iteration
- 2. Transformation of the number of connections to its corresponding consumed power aggregated per node

Figure 4. Flow diagram of the process "Sample generation for EV active and reactive power $(P_{EV} \text{ and } Q_{EV})$ ".



5.1. Distribution of the EVs along the Grid

In order to set how the EVs are distributed along the nodes of the grid to recharge their batteries, the following variables will be taken into account:

- Number of EVs in the scenario to study
- Distribution of the connection along the nodes of the grid
- Connection time of the EV

5.1.1. Number of EV in the Studied Scenario

Since each EV can travel several times a day, the number of EVs available does not represent the number of EVs that will recharge. Thus, the decisive variable is the number of trips per node, hour and iteration (Figure 5). Because the EVs' penetration depends on the chosen scenario, the first step is to estimate the number of trips for both combustion engine vehicles and for EVs, then establish a fraction of these as electrical. For the simulations in this paper, the values for the EVs' penetration are 20% and 40% of the full number of vehicles.

Figure 5. Matrix N_{trips}.

| | | | N _{trav_1_1_i} | N _{trav_1_2_i} | | N _{trav_1_b_i} |
|-------------------------|------------------|------------------|-------------------------|-------------------------|-----|----------------------------|
| | | ••• | ••• | ••• | | $N_{trav_2_b_i}$ |
| | $N_{trav_1_1_2}$ | $N_{trav_1_2_2}$ | | $N_{trav_1_b_2}$ | ••• | |
| N _{trav_1_1_1} | $N_{trav_1_2_1}$ | | $N_{trav_1_b_1}$ | $N_{trav_2_b_2}$ | ••• | $N_{\text{trav}_h_b_i}$ |
| N _{trav_2_1_1} | $N_{trav_2_2_1}$ | | $N_{trav_2_b_1}$ | | | |
| | | | | $N_{trav_h_b_2}$ | | |
| $N_{trav_h_1_1}$ | $N_{trav_h_2_1}$ | | $N_{trav_h_b_1}$ | | | |

In order to determine the number of EVs, it is necessary to set [17]:

- Number of houses (*n*_{houses})
- Number of vehicles per house(*n_{veh/houses}*)
- Mean number of trips made for each vehicle in one day $(n_{trips/day})$
- Ratio of how many of these trips are made through the private vehicle (F_{trips})

With these values it is possible to determine the trips in a day n_{trips_day} through Equation (2). The number of houses is calculated by [17]

$$n_{trips_day} = n_{houses} \cdot n_{veh/houses} \cdot n_{trips/day} \cdot F_{trips}$$
⁽²⁾

Because the studied grid is an urban distribution grid, the hypothesis that 80% of the grid power is for household use is made. It is also important to consider that the contracted power in urban areas is five times the grid's capacity [17]. Finally, we assume that the contracted power for each house is 4.4 kW [17].

$$n_{house} = \frac{P_{Grid} \cdot 5 \cdot 0.8}{4.4} \tag{3}$$

5.1.2. Distribution of the EVs along the Nodes of the Grid

The hypotheses made for the distribution of the EVs along the nodes of the grid are as follows:

- 1. Each node has the same probability.
- 2. The number of trips in the studied area remains constant. It is assumed that the number of trips going outside the studied area is the same as the number of trips going inside the studied area.

5.1.3. Connection Time of the EVs

The massive recharge of EVs during a specific time period could lead to an excessive demand of power that the system cannot deal with and to too low voltages that will break the current standards. Therefore, a realistic model is required to avoid this non-feasible situation. This model is achieved by implementing two scenarios:

- 1. The EV recharges its batteries after every trip.
- 2. The EV recharges its batteries at the end of the day when all trips are completed.

In both scenarios, the following hypotheses are assumed:

- An EV must have its battery partially or completely discharged in order to start a recharge.
- An EV can only recharge its battery if there are available outlets.
- It is assumed that EV drivers show a cautious attitude towards the possibility of ending up with a depleted battery and will therefore arrive at their destination before the battery is depleted.

The connection time of the EVs is obtained from the mobility curve of the private transport in Barcelona [21]. Because the mean trip time is 21.11 minutes on weekdays and 28.34 minutes at weekends, it is assumed that vehicles will start their recharge in the same hour when the trip is started. As a consequence, the hourly recharge curve of the EVs will have the same pattern as the mobility curve for private transport (Figure 6).

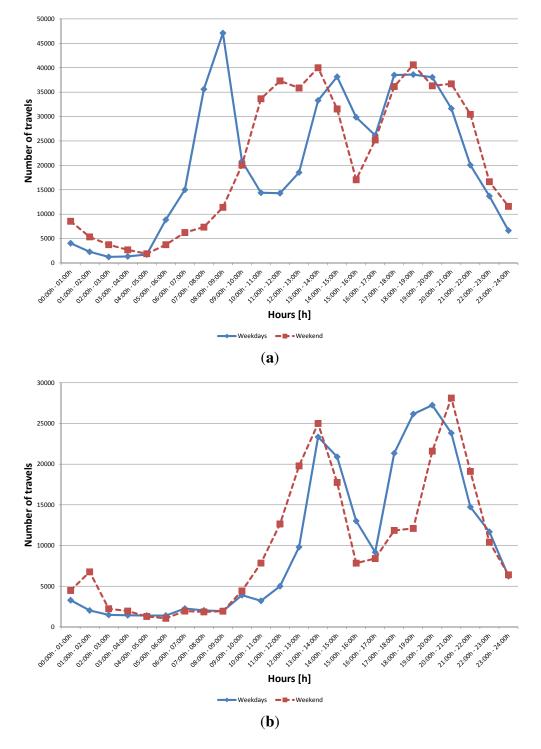


Figure 6. Mobility curves of the metropolitan area of Barcelona. (a) Mobility curve in thousands of travels; (b) Mobility curve of the travels back to home in thousands of travels.

Although the mobility patterns in the entire area of Barcelona are known, it is necessary to adapt these to our area of study. Equation (4) shows how to adapt the number of travels to the proposed study grid with a specific power (N_{trips_hour}), where $n_{trips'_day}$ are all trips in the metropolitan area of Barcelona

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during the day, n_{trips_day} are all trips in our system (with a specific power) during the day and $N_{trips'_hour}$ are trips in Barcelona for the hour h.

$$N_{trips_hour} = \frac{n_{trips_day}}{n_{trips'_day}} \cdot N_{trips'_hour} = \begin{bmatrix} n_{trips_1} \\ n_{trips_2} \\ \vdots \\ n_{trips_h} \end{bmatrix}$$
(4)

г

In order to model the number of vehicles recharging at a specific time of the day as a non-deterministic variable, it can be assumed that the arrivals will follow a Poisson distribution [22]. The Poisson distribution establishes the probability that a number k of events could happen in a determined period of time if they occur with a known frequency and are independent of the time of the last event. In some situations, in which the expected value is bigger than 5, it is possible to transform the Poisson distribution into a normal function [23] [Equation (5)], where $N_{\mu,trips,h}$ stands for the vector with average values and $N_{\sigma_{trips}h}$ stands for the vector with the deviations for each hour. Then, the values can be determined for each hour, node and iteration (Figure 5).

$$N_{\mu_trips_h} = \begin{bmatrix} n_{trips_1} \\ n_{trips_2} \\ \vdots \\ n_{trips_h} \end{bmatrix} \quad and \quad N_{\sigma_trips_h} = \begin{bmatrix} \sqrt{n_{trips_1}} \\ \sqrt{n_{trips_2}} \\ \vdots \\ \sqrt{n_{trips_h}} \end{bmatrix}$$
(5)

5.2. Consumed Power Associated with EV Consumption

Once the number of trips has been determined, it is possible to associate each travel with an EV's recharge. In order to define each recharge, it is necessary to have previously defined the duration of the recharge, the amount of energy to be recharged and the battery's load curve. In summary, the analogy of the matrix active power of the EV's recharge for each hour, node and iteration will be obtained from the matrix number of trips.

5.2.1. Consumed Power by EV Recharge, $P_{EV_{-}h_{-}b_{-}i}$

Figure 7 describes the process of creating the vector for the consumed power associated with the EV's consumption. As can be seen from the figure, the needed inputs are: the Number of trips matrix $(n_{trips_h_b_i})$, the probability distribution of the consumed energy and the probability distribution of the battery's capacity.

The Number of trips matrix $(n_{trips_h_b_i})$ establishes the number of vehicles recharging for each hour, node and iteration (its obtainment process is described in Section 5.1).

It is important to highlight that even if two vehicles have travelled the same distance using the same battery, the process of recharging the battery does not necessarily have to be the same, since it directly depends on its SOC. Therefore, a statistical model for the total energy to be recharged has been generated from a list of batteries (Table 1). The total energy to be recharged is defined by Equation (6), where C_e is the specific consumption. In this case it is 0.158 kWh/km, obtained as the mean from Table 1, and D is the travelled distance which follows a log normal distribution with a shape value of 1.929 and a scale factor of 1.508 [17,21].

$$E_{cons_EV} = C_e \cdot D \tag{6}$$

Figure 7. Flow diagram of the consumed power by the EV's recharge.

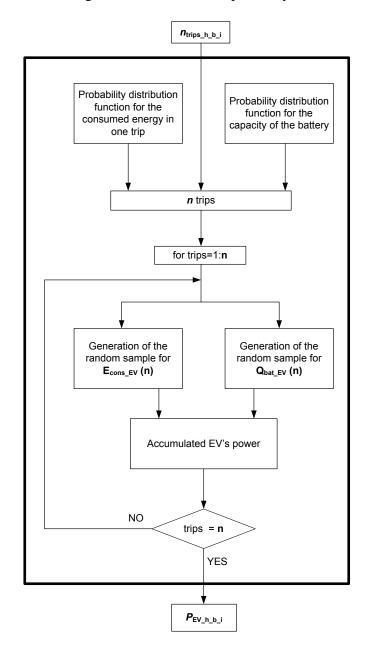


Table 1. EV and Plug-in Hybrid EV Models, Battery Type and Capacity, and Consumption.

| Electric Vehicle (EV) | Capacity [kWh] | Consumption [kWh/km] | Technology |
|-----------------------|-------------------|-------------------------|------------|
| BYD F3DM | 16.5 | 0.160 | Li-Ion |
| BYD e6 (75kW) | 59.4 | 0.160 | Li-Ion |
| BYD e6 (200kW) | 59.4 | 0.175 | Li-Ion |

Table 1. Cont.

| Electric Vehicle (EV) | Capacity [kWh] | Consumption [kWh/km] | Technology |
|---------------------------------------|-------------------|-------------------------|------------|
| CHANA BENNI | 9.0 | _ | AGM |
| Micro-Vett Fiat 500 | 22.0 | _ | Li-Ion |
| Mitsubishi i MiEV | 16.5 | 0.100 | Li-Ion |
| Smart electric drive Coupe | 14.5 | 0.122 | Li-Ion |
| Subaru Estella | 9.0 | | Li-Ion |
| Tata Indica Vista EV | 26.5 | 0.133 | Li-Ion |
| SEAT Leon Twin Drive Ecomotive | 12.0 | 0.240 | Li-Ion |
| Zytel Gorila | 10.8 | 0.150 | Lead-Acid |
| Opel Ampera | 16.5 | 0.133 | Li-Ion |
| REVA NXR | 13.5 | 0.090 | Li-Ion |
| Micro-Vett Fiorino M1-Fi(LC-EG)-Li | 13.5 | — | Li-Ion |
| Micro-Vett Fiorino M1-Fi(HC-Eg)-Li(S) | 21.0 | 0.240 | Li-Ion |
| Micro-Vett Fiorino M1-Qu(HC-Eg)-Li(S) | 21.0 | 0.240 | Li-Ion |
| Micro-Vett Fiorino M1-Qu(HC-Eg)-Li(L) | 32.0 | 0.240 | Li-Ion |
| Smart electric drive Cabrio | 14.5 | 0.122 | Li-Ion |
| Think City 2010 | 83.0 | 0.144 | NiNa |
| Peugeot ION | 16.5 | 0.125 | Li-Ion |
| Tesla Roadster | 51.5 | 0.231 | Li-Ion |
| TOYOTA Prius Plug-In Hybrid | 5.0 | 0.062 | Li-Ion |
| Citroen C-Zero | 16.5 | _ | Li-Ion |
| Mini-E | 35.0 | 0.130 | Li-Ion |
| Volkswagen Golf | 26.5 | | Li-Ion |
| Nissan Leaf | 24.0 | | Li-Ion |
| Chevrolet Volt | 16.5 | _ | Li-Ion |
| Brusa Spyder | 16.0 | _ | Li-Ion |
| Phoenix SUT | 35.0 | _ | Li-Ion |
| Lumeneo Smera | 10.0 | — | Li-Ion |

The type of battery which defines the load curve was also generated from (Table 1). A log normal function was created from the capacities to be generated for random batteries; and depending on the value, a known load curve from an existent model was assigned, as Figure 8 shows. The abbreviations in Figure 8 stand for location parameter (*Loc*), scale parameter (*Scale*) and sample size (*N*) of the log normal function obtained.

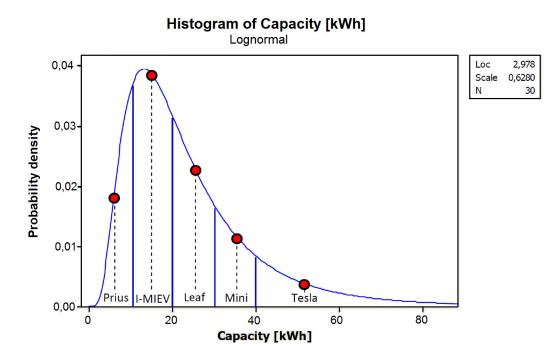


Figure 8. Distribution function of the batteries capacity.

Once the number of the EV's trips is known, the EV's consumed energy per trip as well as the type of battery per trip are then determined. With these two variables, the power consumed during the EV recharge $(P_{EV.h.b.i})$ is calculated. Additionally, this algorithm also considers the fact that the EV's recharge could last for more than one hour in the *Accumulated EV's power* block. As a result, the EV's power that would be in each node would be the power of the EVs that have arrived at the node b and the power of those which are still recharging because they had arrived during the previous hours.

Following this algorithm, the EV's power to recharge in each hour, node and iteration $(P_{EV_h_b_i})$ can be obtained. It is important to note that this algorithm takes into account the random characteristics of the number of EV's that would recharge in each node, the consumed energy for each EV during its trip and the type of battery that the EV would have.

Finally, another conclusion that can be drawn from this process is that the duration of the connection time is a random variable which does not depend on the travel time. The connection time for each EV depends directly on the travelled distance D (which follows a log normal distribution) and the specific consumption C_e which depends on the type of battery (which also is chosen by following a log normal function).

6. Study Grid

In order to evaluate the impact caused by the EV on the grid, it is important to choose the scenario for the simulations, which basically consists of choosing a grid to be used. The type of grid chosen will define the accuracy of the results; thus, the more realistic the grid is, the more reliable the results will be.

When choosing the grid for the impact study, four aspects are important, namely: the real grid, the testing grid, medium voltage and low voltage. A real grid has the main advantage of its practically-oriented results, although it has the disadvantage of obtaining the data. On the other hand, a

testing grid—not even being a real grid—has the advantage of the availability of the data and the certainty of working properly. Regarding the grid voltage level, the extra high voltage grid (transmission grid) has been dismissed because this grid type is intended to deal with high amounts of energy; therefore, the impact of the recharge of EVs will not have a clear effect. However, an MV (Medium Voltage) or LV (Low Voltage) distribution grid with a radial topology will allow a better interpretation of the results. Even though these grids do not have massive generation units, it is possible to find distributed generation units. Table 2 shows the advantages and disadvantages of the different types of grids.

| Voltage | Topology | Benefits | Drawbacks |
|---------|----------|---|--|
| High | Meshed | Introduces the possibility of assessing the impact of large generation units. Lower error in the demand's modeling. | Due to the high short-circuit grid's power and its topology, a reduced impact on the grid is expected. |
| Medium | Radial | Brings the opportunity of introducing distributed generation units and assessing its impact. Lower error in the demand's modeling. | Due to the high short-circuit grid power, a reduced impact on the grid is expected. |
| Low | Radial | Due to its low short-circuit power, a high impact on the grid is expected. | It is not feasible to include any generation unit. Higher error in demand modeling. |

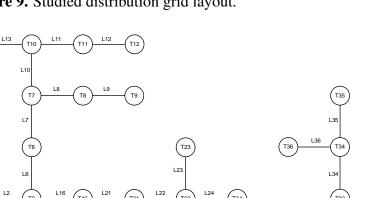
Table 2. Benefits and drawbacks of performing the study with different types of grids.

6.1. Proposed Grid

In order to perform the study, the test system chosen was the IEEE 37 bus test system because of its length [24] (5.5 km) and due to the fact that it is similar to a typical distribution underground grid in Barcelona. Even through the IEEE 37 node test grid has features in common with the desired distribution grid, aspects like voltages and impedances have been adapted. Other grids were also studied, such as the IEEE 13 node and the IEEE 123 node, but were discarded because of their length.

6.2. Proposed Scenario

With regard to the previous discussion, it has been concluded that the study will be based on an IEEE 37 node standard test feeder (Figure 9), but adapting it to 25 kV and with all the necessary adaptations for its elements (Tables 3–5).



T22

т26

L27

Т2

Т28

T29

Т30

L2

T21

T19

Figure 9. Studied distribution grid layout.

. Т2

. T16

T17

L18

L20 (T20

L17

Table 3. Features of the proposed lines for the studied grid.

L19 (T18

| $A [\mathrm{mm^2}]$ | $m{R}$ [Ohm/km] | X_L [Ohm/km] | I_{max} [A] |
|----------------------|-----------------|----------------|---------------|
| 240 | 0.125 | 0.116 | 415 |
| 400 | 0.0778 | 0.105 | 530 |

Table 4. Features of the transformers used in the studied grid.

| Feature | Assigned Value |
|---|-------------------------------|
| Power | 160-250-400-630 kVA |
| Connection type: 250-400-630 kVA | Dyn11 |
| Voltage of the HV coupling | 25 kV |
| No-load voltage of the LV coupling | 420 V |
| Transformer's tap positions | $-5 - 2.5 \ 0 + 2.5 + 5 + 10$ |
| Capacity of resisting short-circuit events in the LV side | 22.2 I _{nom} |

| Transformer station | Number of transformers | Pnom [kVA] | Line | L [km] | S [mm ²] |
|---------------------|------------------------|-----------------|------|--------|-----------------------------|
| TO | 1 | 25000 | - | - | - |
| T1 | 1 | 160 | L1 | 0.3 | 400 |
| T2 | 3 | 250 + 250 + 400 | L2 | 0.2 | 400 |
| T3 | 1 | 400 | L3 | 0.4 | 240 |

Table 5. Features of the elements used in the studied grid.

| Transformer station | Number of transformers | Pnom [kVA] | Line | L [km] | S [mm ²] |
|---------------------|------------------------|------------|------|--------|-----------------------------|
| T4 | 2 | 160 + 250 | L4 | 0.4 | 240 |
| T5 | 1 | 160 | L5 | 0.3 | 240 |
| T6 | 1 | 400 | L6 | 0.2 | 240 |
| Τ7 | 1 | 250 | L7 | 0.3 | 240 |
| Т8 | 2 | 250 + 250 | L8 | 0.2 | 240 |
| Т9 | 1 | 400 | L9 | 0.3 | 240 |
| T10 | 1 | 160 | L10 | 0.2 | 240 |
| T11 | 2 | 250 | L11 | 0.3 | 240 |
| T12 | 1 | 250 | L12 | 0.3 | 240 |
| T13 | 2 | 250 + 250 | L13 | 0.3 | 240 |
| T14 | 1 | 160 | L14 | 0.3 | 240 |
| T15 | 2 | 250 + 630 | L15 | 0.2 | 240 |
| T16 | 2 | 630 + 250 | L16 | 0.4 | 240 |
| T17 | 2 | 400 + 160 | L17 | 0.4 | 240 |
| T18 | 1 | 250 | L18 | 0.2 | 240 |
| T19 | 1 | 160 | L19 | 0.5 | 240 |
| T20 | 2 | 250 + 160 | L20 | 0.1 | 240 |
| T21 | 1 | 250 | L21 | 0.4 | 240 |
| T22 | 2 | 160 + 250 | L22 | 0.2 | 240 |
| T23 | 2 | 160 + 250 | L23 | 0.3 | 240 |
| T24 | 1 | 630 | L24 | 0.2 | 240 |
| T25 | 2 | 250 + 400 | L25 | 0.5 | 240 |
| T26 | 2 | 400 + 250 | L26 | 0.2 | 240 |
| T27 | 1 | 160 | L27 | 0.3 | 240 |
| T28 | 1 | 400 | L28 | 0.2 | 240 |
| T29 | 1 | 160 | L29 | 0.2 | 240 |
| T30 | 2 | 250 + 250 | L30 | 0.3 | 240 |
| T31 | 1 | 250 | L31 | 0.2 | 240 |
| T32 | 2 | 160 + 250 | L32 | 0.7 | 240 |
| T33 | 1 | 400 | L33 | 0.3 | 240 |
| T34 | 2 | 160 + 250 | L34 | 0.5 | 240 |
| T35 | 2 | 400 + 250 | L35 | 0.3 | 240 |
| T36 | 1 | 400 | L36 | 0.4 | 240 |

Table 5. Cont.

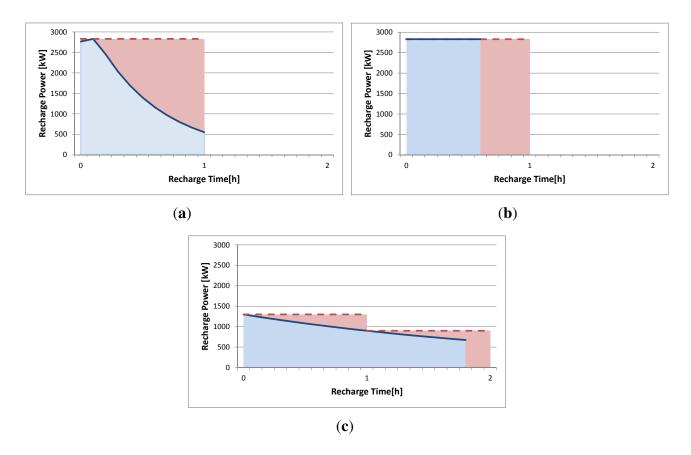
7. Results without the Electrical Demand: Recharge Curve of the EV

Several authors [7,8,10–13] performed the grid impact analysis, using a constant recharge load model. The novelty of this paper is the use of a detailed recharge curve for the impact analysis. Thus, the

objective of this section is to compare the impact of the different models of batteries for the recharge of EVs, regardless of the situation of the grid. Therefore, only the demand created by the EV's recharge is considered in these simulations, which will provide a better idea of the EV's recharge effect. The different models for the EV's battery recharge are:

- Constant Power: The recharge is performed at constant power (Figure 10(b)).
- Variable Power: The recharge is performed based on the battery's recharge model (Figure 10(a)).
- Variable model with average values for the power to recharge: The recharge is performed based on the battery's recharge model but the chosen value for the power is the mean for the established periods (Figure 10(c)).

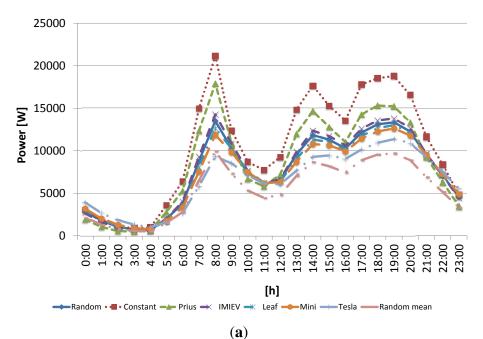
Figure 10. Recharge profile of the recharging process of three different types of batteries. (a) Recharge curve of a Toyota Prius's battery; (b) Recharge curve of a battery at constant power; (c) Recharge curve of a Nissan Leaf's battery.

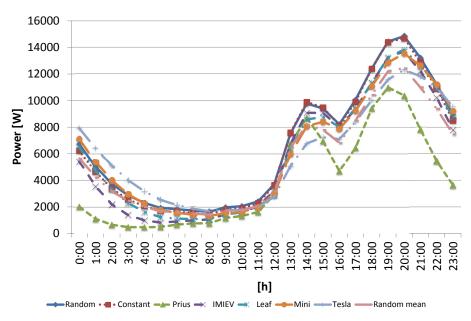


The simulations are performed on the chosen grid (Figure 9), with 1000 EVs during a regular weekday with 50 iterations per hour, using the software *DIgSILENT Power Factory* [25]. *DIgSILENT Power Factory* is a program to model, analyse and simulate generation, transmission, distribution and industrial grids and its interactions. This software enables as well load flow studies to be run using a scripting language called DSL. Both the recharge at the end of each trip and at the end at all day trips have been considered.

Figure 11 shows the results of these simulations. It is important to note the difference between the recharge energies in each simulation. In the case of the recharge after each trip (Figure 11(a)), it can be seen that the energy recharge directly depends on the used battery type. This difference is based on the consideration that in the same hour all vehicles will be connected to the grid, recharging at the maximum battery power. Figure 10 shows the three model types for the battery recharge of 1650 kWh The blue colour stands for the real energy required and red stands for the additional energy assumed in this simulation. Several studies from [17,21] demonstrate that the majority of the population consume less than 4 kWh during each trip.

Figure 11. Recharge profile of 1000 EVs distributed along a 53 node low voltage grid [W].(a) Recharge after each trip; (b) Recharge at the end of all day trips.





Accordingly, battery recharges would take place in the exponential part of the curve. Therefore, in order to achieve a more precise model and not include this unrealistic extra energy, it would be necessary to measure in periods smaller than one hour when the recharge takes place in the exponential part of the model's curve.

For the case of the recharge at the end of all day trips (Figure 11(b)), the error is much smaller, since recharges are deeper and take place in the bulk part of the curve of the recharge model where minimising the error due to the exponential part of the curve.

8. Results with Electrical Demand: Impact into the Distribution Grid

The EV's impact on the study grid (Figure 9) is studied in this section. Prior to the simulations, it had been studied which results can provide a better understanding of the impact of EVs. Because of the topology of the grid and the household load profiles, it has been observed [17] that the greatest impact is in Line 1 (L1), and because the difference is not significant in the transformers' loadings, the value will be the mean of them all (Equation (7)).

$$L_{transf} = \frac{\sum_{i=1}^{N_{transf}} L_{transf_i}}{N_{transf}}$$
(7)

8.1. Comparison of the Constant Power Battery's Recharge Model and Variable Model with Mean Values for the Power to Recharge

The results of the Monte Carlo simulation are depicted below with 0%, 20% and 40% of EV penetration for the most unsuitable scenario (the one with the highest household consumption, which in this case is the "*winter weekday*" case), using the constant power model (Figure 10(a)) and the variable model with mean values (Figure 10(c)).

As Figure 12(a) shows, the maximum load increase occurs in Line 1 (L1) for a penetration of 40% with the constant power model. This maximum load increase takes place when household demand reaches its maximum (20:00), taking the loading level from 60 % (with no EV) to 74%. In the case of the mean value model, the loading level in the same hour is 66%, which corresponds with the expected accuracy of this model (Section 7). It is important to point out that with a 40% EV penetration, the grid has already experienced a violation of the admissible level of 60% at 8:00; with the constant power model and it reaches 68% and with the mean value model it reaches 63%.

Taking into account the conclusions extracted from Section 7, it can be seen that for the recharge at the end of all day trips, the differences between the batteries' recharge models are not as significant as in the previous scenario (Figure 13); the maximum difference between the models is 1%.

Figure 13 shows the loading level for the transformers. Comparing the loading levels of the transformers with the line loading levels, it can be observed that the load pattern remains the same although the transformer case reaches higher loading levels.

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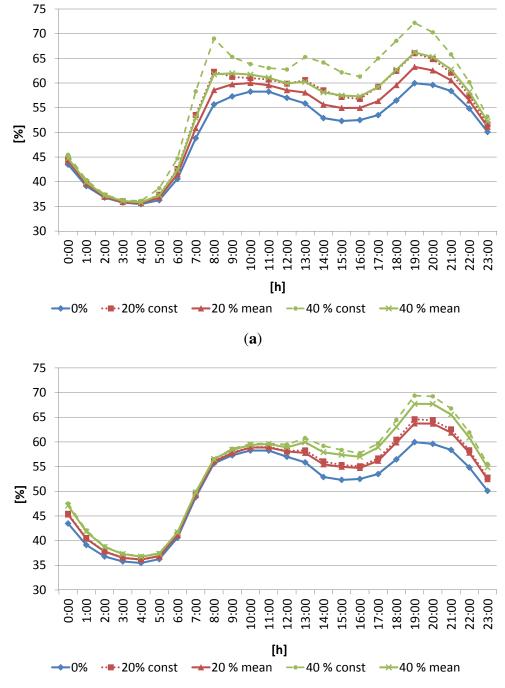


Figure 12. Loading profile of the Line 1 (L1) through a winter weekday. (a) Recharge of the battery at the end of each travel; (b) Recharge of the battery at the end of all day travels.

(b)

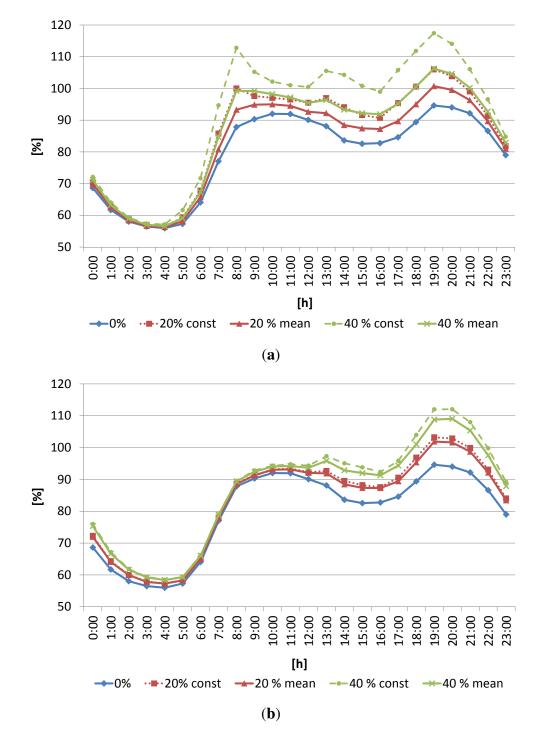


Figure 13. Loading profile of the transformers through a winter weekday. (**a**) Recharge of the battery at the end of each travel; (**b**) Recharge of the battery at the end of all day travels.

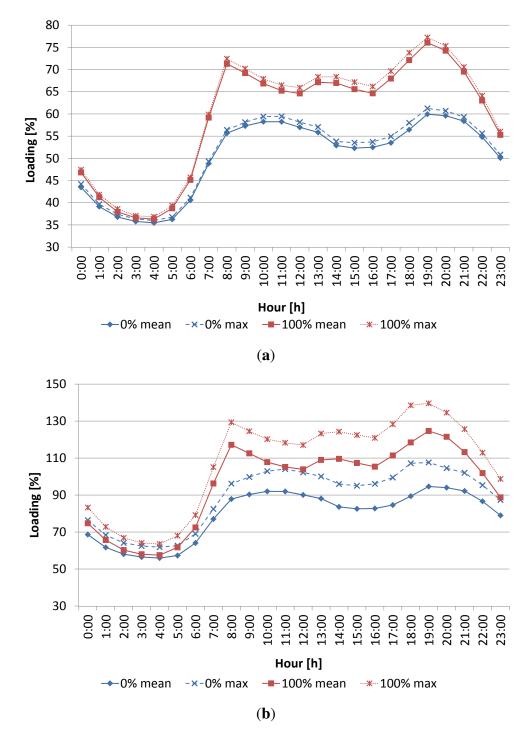
8.2. Extreme Scenario: EV Penetration of 100%

The purpose of this section is to study the effect of a 100% EV penetration in the grid. Two situations will be considered: recharging after every trip and recharging at the end of the day. Results will be compared with the 0% EV penetration case.

Figure 14 depicts the loading profile of L1 and the transformer when the recharge is performed after every trip. As it can be seen in the Figure 14(a), lines do not overload their maximum value.

Nevertheless, depending on the country's legislation, these values can be considered not suitable under normal operating conditions. Regarding the transformer's loading level (Figure 14(b)), it can be seen that overloading is produced all day long except during the off-peak time period.

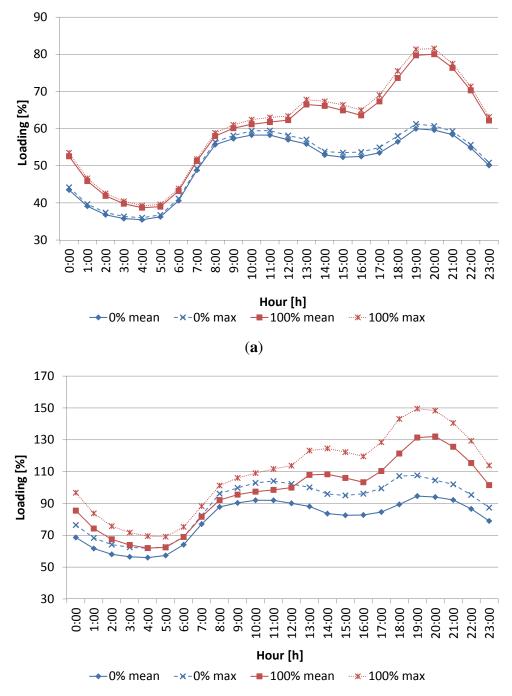
Figure 14. Grid's loading profile with an EV's penetration of 100% recharging at the end of each travel. (a) Loading profile of the line L1; (b) Loading profile of the transformers.



The EV's recharging after all the trips of the days is shown in Figure 15. For the case of the line's loading (Figure 15(a)), as in the previous case, it does not exceed 100% of its capacity but it reaches higher values (the hour when this maximum value is achieved is the same for both scenarios). The

transformer's loading level (Figure 15(b)) presents a decrease of its loading until 17:00. At the time of the maximum loading level (20:00) the loading is approximately 10% above of the "*Recharging after every trip case*" transformer's loading level.

Figure 15. Grid's loading profile with an EV's penetration of 100% recharging at the end of all day travels. (a) Loading profile of the line L1; (b) Loading profile of the transformers.



(b)

9. Conclusions

The objective of this paper is to analyse the impact of the recharge of a fleet of EVs in a standard IEEE test system adopted to an underground distribution grid in Barcelona (Spain), implementing an improved model for the recharge of the batteries. In order to achieve this objective, the following tasks have been performed:

- Obtainment of the probabilistic model for electrical demand. It consists of creating a statistical model able to generate random values according to the available data of the Spanish hourly load profile (rescaled to the study grid). The seasonal patterns of the consumption were obtained in order to generate a normal distribution for each hour of each pattern. Using these normal distributions, the random values for the hourly power due to the electrical demand can be generated. The pattern chosen for the simulation is the winter pattern because the highest consumptions are experienced, which results in the most critical case.
- Battery model creation. The load curves of the different types of batteries have been implemented in order to include their behaviour in the simulations and maintain accuracy.
- Obtainment of the probabilistic model for the electrical demand due to the EV consumption. In order to get the EV consumption, several steps had to be taken:
 - 1. Study of the mobility patterns of Barcelona and adaptation to the studied area in order to get the number of EVs that would be recharging in the area. The distribution of the number of EVs along the nodes has been done considering that each node has the same probability of recharging vehicles.
 - 2. Determination of the consumed power of the EVs for each recharge. The consumed energy per travel is based on the specific consumption of the battery and the distance travelled. In this study the travelled distance follows a log normal distribution with a shape value of 1.929 and a scale factor of 1.508.
 - 3. Obtainment of the type of battery of each EV. The type of battery determines the specific consumption of the previous step. In the study, a log normal function was created from the capacities to be generated for random batteries, and depending on the value, a known load curve from an existent model was assigned (Figure 8).
- One of the main interests of this study is to obtain the impact of EVs recharging in a real distribution grid situation. Thus, an IEEE test system has been adapted to an underground cable system used, like in Barcelona. The main advantage of this new test system is that it also contains mobility-related data that can be used in future studies.
- Simulation using the Monte Carlo method of the possible scenarios using the power system simulator *DIgSILENT*. Once all the consumptions per node, hour and iteration have been determined (EV's consumption and conventional demand), deterministic load flows with these values can be performed in order to get the confidence interval for the result variables.

The results demonstrate that the impact of non-constant recharge battery models on the grid is smaller than the impact of the constant one (Figure 11). This is caused by the significant inaccuracy of the energy consumed during the recharge of the constant power model, due to the following two factors:

- 1. The constant battery model does not take into account that during the last stage of the recharge, consumption decreases in an exponential way (constant voltage stage). This inaccuracy gets accentuated when the recharge is performed during this stage (recharge after each trip).
- 2. The simulation using time periods with one hour resolution results in an imprecision when the recharge lasts less than one hour. This situation results in the recharge of more energy during the simulation than required.

Despite the simulations using a non-constant battery model in order to procure the results where the impact is not the highest, during the days of the year with the highest household consumption, transformer stations could experience an overload due to the combination of the household electrical consumption and the EV recharge consumption (Figure 13). The lines will not have such severe problems (Figure 12), even though it is possible that some lines cannot remain under 60% of admissible loading power established by the utility. The same results can be observed for the critical case of the 100% of EVs penetration (Figure 14 and Figure 15).

Finally, it is essential to highlight that the variables regarding mobility aspects are some of the key factors for these kinds of studies.

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