Estimating the Optimal Release Conditions
for World Record Holders in Discus

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Abstract

The purpose of the study is to estimate the optimal release conditions for male and female world discus record holders of 1998 under no wind and in the wind. The estimation was achieved by using numerical methods to deal with the lift and drag coefficients in Ganslen's wind tunnel tests of discus. The results showed that under no wind, when the release height of both male and female holders was supposed to be 1.8m , then the male holder could break the world record of 74.08 m at the release angle of 36.50°, the attack angle of –10.25°, and the minimal release speed of 26.662 m/s. The female holder could break the world record of 76.80m at the release angle of 32.75°, the attack angle of –9.25°, and the minimal release speed of 27.044 m/s. Simulating the discus throw in the head wind under the condition of the wind speed ranging from –21m/s to 12 m/s, the researcher found that both male and female holders could gain better performance by throwing the discus at a lower release angle and attitude angle. However, in the tail wind, a better performance could be obtained by throwing the discus at a higher release angle and attitude angle.

Keywords: Release angle, Attack angle, Attitude angle, Release speed

Introduction

Powerful Today, most participants in discus competitions throw the discus in a rotational way. This is mainly because higher release speed can be obtained by a rotational throw. However, analyzing the throwing distances and release parameters of the top three discus-throwing winners in the Olympic Game held in Los Angels in 1984, Gregor et al [1] discovered that their longest throwing distance was not reached at the fastest release speed. Apparently, besides release speed, other factors influence the performance in discus throw. According to Barlett [2] and Soong [3], the factors include release position, speed, release angle, attack angle, attitude angle, drag, and lift. Barlett [2] once raised the following question: what are the optimal conditions for throwers to obtain the best performance?

To explore the optimal conditions, Tutjowitsch [4], Fronlich [5] and Soong [3] respectively simulated the release height, speed, release angle, attack angle, and attitude angle on different subjects. However, since the research methods and the release parameters adopted by the three researchers are different, it is unlikely to conclude which one

is correct. Fortunately enough, Barlett [3] collected from several researchers the data of lift and drag which the flying discus endured in wind tunnel tests, thus contributing a lot to the simulation of discus throw with a computer.

In this study, the researcher took some steps to enhance the credibility of the computer simulation, including making some modifications to the mathematic model set up by Soong [4], quoting the data from Ganslen [6] on the lift and drag coefficients of discus throw in the wind tunnel tests, using Spline curve and numerical methods. Besides, this study adopted C^{++} programming to handle the numerical arithmetic. The attempt of the researcher is to estimate the optimal conditions for the male and female world discus record holders of 1998 in still air and in the wind.

Nomenclature

G	gravity vector
V_s	release speed
α	roll orientation angle
φ_0	initial attack angle
λ	wind speed
T_0	initial relative velocity vector
t_k	time point in the flying process
φ_k	attack angle in t_k
V_k	velocity vector in t_k
F_k	force vector in t_k
Vr_k	relative air speed in t_k

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UL_k	unit vector in the lift direction
Q	position vector in landing position
Cl_k	lift coefficients in t_k
As	the area measure of the side drag
W	wind velocity vector
β	release angle
δ	attitude angle
P_0	release position vector
V_0	release velocity vector in the x-axis of the coordinate system oxyz
Δt	time step between two neighboring time points
P_k	position vector of mass center in t_k
A_k	acceleration vector of mass center in t_k
T_k	relative air velocity vector in t_k
Ud_k	unit vector in the drag direction
Uy	unit vector in the y-axis of the coordinate system oxyz

Materials and Methods

First, as Figure 1 (a) demonstrated, the coordinate system $OXYZ$ was the base coordinate system and $OX^1Y^1Z^1$ was the transformation coordinate system. The initial point of $OX^1Y^1Z^1$ was supposed to be the release position of discus, P_0 was the release position vector, and Z_0 was the release height. Symbol α controls the horizontal angle of discus, β was the release angle of discus, δ was the attitude angle, and the release speed vector V_0 went along the axis x of the new coordinate system oxyz. Besides, in Figure 1 (a), W meant the wind velocity vector, and T_0 was the relative velocity vector. In Figure 1 (b) and (c), ϕ_0 represents the initial attack angle. In the Figure 1 (b), where the lift produced by T_0 was upward, the angle of attack ϕ_0 was defined as positive. In the Figure 1 (c), where the lift produced by T_0 was downward, the attack angle ϕ_0 was defined as negative.
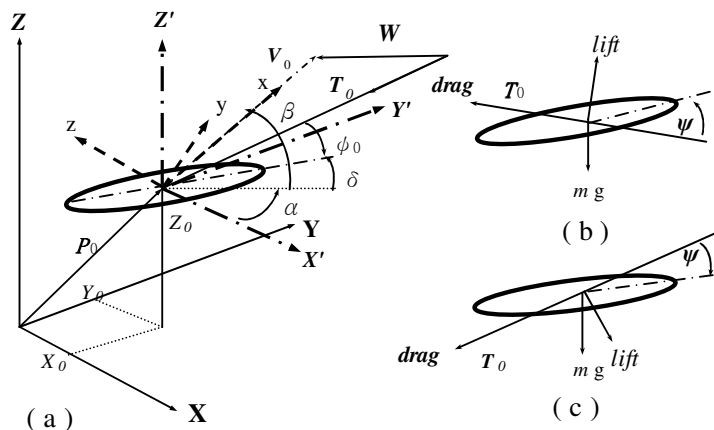

Figure 1. (a) represents the reference coordinate systems of discus releasing moment and the release parameters. Figures (b) and (c) represent the drag, lift and gravity received by discus at different attack angles.

Numerical model

This study adopted Euler integration [7] to estimate the position, velocity and acceleration vector of the flying discus. First, t_k (k=0, 1, ..., n) was used to represent a particular time point in the continuous period starting from the release of discus until its landing. In the base coordinate system of Figure 1 (a), $P_k = [X_k, Y_k, Z_k]^T$ was the position vector of mass center in t_k. In the discus-flying process, two neighboring time points, t_k and t_{k+1}, were randomly selected, and the two points were very close to each other, making the time step $\Delta t = t_{k+1} - t_k$ very tiny, then the position vector of mass center moving from t_k to t_{k+1} could be viewed as a movement of constant acceleration. Thus, P_{k+1}, the position vector of t_{k+1}, and V_{k+1}, the velocity vector, could be written as

$$P_{k+1} = P_k + V_k \Delta t + \frac{1}{2}(A_k \Delta t^2) \tag{1}$$

$$V_{k+1} = V_k + A_k \Delta t \tag{2}$$

In equations (1) and (2), V_k was the velocity of mass center in t_k, and A_k was the acceleration vector. $A_k = F_k / m$, m was the mass of the discus. F_k was the forces imposed upon the flying discus, which included gravity, drag, and lift [4, 8-9]. Thus, for F_k in t_k, the equation could be written as

$$F_k = \frac{1}{2}\rho As(Cd_k Vr_k^2 Ud_k + Cl_k Vr_k^2 Ul_k) + mG \tag{3}$$

So In equation (3), ρ was the mass density, G was the gravitational acceleration vector, and $G = [0, 0, -g]^T$, g was the gravitational constant 9.81m/s2. When $t_0 = 0$, the release position vector $P_0 = [X_0, Y_0, Z_0]^T$, with X_0 and Y_0 being the horizontal positions of the release, and Z_0 being the release height. The initial release velocity vector V_0 went

along the x-axis of the new coordinate system oxyz. Therefore, V_0 could be written as

$$V_0 = Azy(\alpha, \beta)[Vs, 0, 0]^T \qquad (4)$$

$$Azy(\alpha, \beta) = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha & -\cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & -\sin\alpha\sin\beta \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad (5)$$

In equation (4), V_s was the release speed, and $Azy(\alpha, \beta)$ was the transformation matrix [10]. In Figure 1 (a), the initial relative velocity vector T_0 was the subtraction of the release velocity vector V_0 from the wind velocity vector W, so $T_0 = W - V_0$, and $W_0 = [\lambda\cos\alpha, \lambda\sin\alpha, 0]^T$. λ was positive in tail wind and negative in head tail. In the moment of t_k, $T_k = W - V_k$. Vr_k, the relative air speed in t_k, could be written as

$$Vr_k = \|T_k\| \qquad (6)$$

Therefore, Ud_k, the unit vector in the drag direction, could be written as

$$Ud_k = T_k / \|T_k\| \qquad (7)$$

If the relative velocity vector was presented as $T_k = [Tx_k, Ty_k, Tz_k]^T$, then ψ_k, the angle of attack in t_k, could be written as

$$\psi_k = \delta - \tan^{-1}(-Tz_k / \sqrt{Tx_k^2 + Ty_k^2}) \qquad (8)$$

In Figure 1 (A), the unit vector Uy in the y-axis of the new coordinate system could be presented as

$$Uy = Azy(\alpha, \beta)[0, 1, 0]^T \qquad (9)$$

In equation (3), when the unit vector in the lift direction Ul_k was at three different attack angles, then the formulations were as followed

$$\begin{cases} \text{When} \quad \frac{\pi}{2} > \psi_k > 0 \qquad Ul_k = Uy \times T_k / \| Uy \times T_k \| \\ \\ \text{When} \quad -\frac{\pi}{2} < \psi_k < 0 \qquad Ul_k = T_k \times Uy / \| T_k \times Uy \| \quad (10) \\ \\ \text{When} \quad \psi_k = 0 \qquad \text{Lift Force} = 0 \end{cases}$$

As for the throwing distance, as Figure 2 demonstrated, if the discus touched the ground in t_n, then $P_n = [X_n, Y_n, Z_n]^T$, and Q the position vector in the landing position, could be presented as $Q = [Qx, Qy, Qz]^T$ (Chiu, 2003):

$$\begin{cases} \text{When} \quad \frac{\pi}{2} > \delta > 0 \quad Q = P_n + Azy(\alpha, \delta)[-r, 0, 0]^T \\ \\ \text{When} \quad -\frac{\pi}{2} < \delta < 0 \quad Q = P_n + Azy(\alpha, \delta)[r, 0, 0]^T \quad (11) \\ \\ \text{When} \quad \delta = 0 \quad Q = P_n \end{cases}$$

In equation (11), r was the radius of the discus. When $\pi/2 > \delta > 0$, the landing situation was what Figure 2 (a)

demonstrated, When $-\pi/2 < \delta < 0$, the landing situation was what Figure 2 (b) showed. When $\delta = 0$, the discus landed

$$Azy(\alpha, \delta) = \begin{bmatrix} \cos\alpha\cos\delta & -\sin\alpha & -\cos\alpha\sin\delta \\ \sin\alpha\cos\delta & \cos\alpha & -\sin\alpha\sin\delta \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \qquad (12)$$

on the ground in a horizontal way, which was similar to landing in the position of mass center[9]. Therefore, $Q = P_n$, and the horizontal throwing distance d could be writer as :
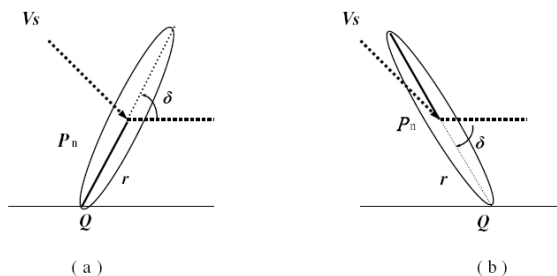

Figure 2. The graphs for different landing positions of the discus .

Drag and lift coefficients

Drag and lift coefficients of discus in the flying process came from the data of the wind tunnel tests by Ganslen [6], which were introduced by Bartlett [3]. In this study, the numerical points in the drag coefficient(Cd_i) and lift coefficient(Cl_i) curves relative to the attack angle(ψ_i) were measured in equal intervals, respectively represented by (ψ_i , Cd_i) and (ψ_i , Cl_i), i=0,1, 2, ...,N. Then the two sets of points were processed with Spline curve [11].

The values for the attack angle related to the drag and lift curves, presented in Table 1, were measured by referring to Ganslen's [6] graph for drag and lift curves. Each set of numerical points consisted of nine points (i=0,1,2,….,9), the value of Cd_i ranged from 0.05 to 1.08, and Cl_i ranged from 0.00 to 0.88[9].

Table 1. The measured values for the drag and lift curves adopted from Ganslen

i	ψ_i (°)	Cd_i	Cl_i
0	0	0.05	0.00
1	10	0.16	0.31
2	20	0.35	0.61
3	30	0.55	0.88
4	40	0.65	0.55
5	50	0.77	0.46
6	60	0.88	0.42
7	70	1.00	0.37
8	80	1.02	0.17
9	90	1.08	0.00

Data Analysis

According to the methods in this study, to estimate the throwing distance, we needed to input the release parameters in the computer programs, including the initial release speed Vs, the release height Z_0, the release angle β, the attitude angle δ, the angle of attack ψ_0 and the wind speed λ. But as was shown from the formula, the value of the initial

attack angle ψ_0 could be obtained by the values of the following five parameters: the release speed V_s , release angle β , the attitude angle δ, wind speed λ and the roll orientation angle α . So in fact, the attack angle ψ_0 was not considered to be a release parameter, but as a parameter in the process of calculating. Besides, there were three other parameters: X_0 of P_0 , Y_0 of P_0 and the roll orientation angle α , but they were assumed as zero in calculating .

Results and Discussions

The diameter and weight of discus in the simulation of this study corresponded to the regulations set by IAAF (1985/86)[12], with the minimum weights of discus 2.00kg for male throwers and 1.00kg for female throwers, the minimum diameter of 21.90cm and of 18.0cm respectively. In this study, the air mass density ρ was 1.23kg/m3 under the standard air pressure [8].

As for the attitude angle of the flying discus, according to Soong's [4] study, when the discus was thrown at the release speed of 25.5 m/s , the attitude angle of 35° , the release angle of 35° and the spinning speed of 36.9rev/s, the attitude angle would change from the release angle of 35° to 41.09° when the discus touched the ground. The attitude angle of the flying discus only rotated 6.09°, and the throwing distance was 64.389 m. Soong also reported that the faster the release speed was, the faster the discus would rotate ; when the rotating angle was smaller, the discus wouldn't swag or lose speed.

The subjects of this study were male and female world record holders, the male holder's record was 74.08m and the female holder's record was 76.80m [13]. To achieve such great performances, the release speed must be above 25.5 m/s. At this release speed, the discus' rotating speed will be faster, and the attitude angle will be more stable in the flying process。 Therefore, in estimating the performance of male and female world record holders, the attitude angle δ is assumed as a fixed value.

Comparison of estimated performance

In this study, the estimation of the throwing distance was based both on the data about drag and lift coefficients in Ganslen's [6] wind tunnel tests, and on the application of numerical methods. To confirm the correctness of the estimated throwing distances, this study analyzed the data of several previous researches [3], selected the release parameters for male optimal throw, by which we mean those distances which are above 60m (Table 2), and then made a comparison of these estimated performances.

In Table 2, only the release height of 1.8m by Frohlich (1981)[1] was listed, and those which lacked the data on release height were marked the symbol *. Consequently, to make a comparison of the throwing distances estimated by different researchers, the release height for each estimation was hypothesized to be 1.8m as [1] suggested. On the other hand, the release angles and the attitude angles listed by Cooper et al [14] and Ganslen [6] were not fixed values, but values within a certain range. The minimum values within

each range were adopted as the parameters for estimating. For example, it was found that the release angle β =35° and the attitude angle δ =25° in Cooper et al [14], while the release angle β =36° and the attitude angle δ =23° in Ganslen (1964)[6]. Third, the value of Δt should be taken into consideration. According to numerical integral, the value of Δt can be too large. When the value of Δt is larger, chances of estimation error will be greater. However, if the value is too small, the calculating time will be increased. Fortunately, in estimating the throwing distance, Soong [4] assumed the value of Δt as 0.005sec, and the error of simulation was very small; therefore, referring to Soong, this study assumed that Δt =0.005sec.

Table 2. Comparison of estimated data on the throwing distance between this study and other studies

researchers	Release Parameters					$d*$	D (my data)	A
	Vs	Z_0	β	δ	ψ_0			
	(ms^{-1})	(m)	(°)	(°)	(°)	(m)	(m)	(%)
Cooper et al.(1959)	24.4	*	35-40	25-30	-5 to-10	61.3	61.18	0.95
Ganslen(1964)	24.4	*	36-39	23-27	-9to-16	64.9	62.17	4.21
Frohlich(1981)	25	1.8	37	27	-10	65.4	65.27	0.20
Soong(1982)	25.5	*	35	26	-9	66.1	67.73	2.47

This table included the estimated data on the distance above 60 m for males under no wind. * meant the data were not given by researchers. $d*$ represented the estimated throwing distance by other researchers. D meant the estimated throwing distance by the researcher of this study. A meant the relative error, and the average value of the relative error was 1.96% . release parameters and the data in column $d*$ were adopted from Bartlett .

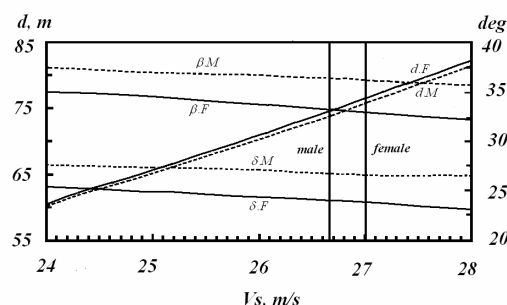### *Estimating the release speed of world record holders*

Figure 3. The release speed, the optimal distance, release angle and attitude under no wind. $d.M$, $\beta.M$, $\delta.M$ represented the curves for the throwing distance , the release angle, and attitude angle of the male holder; Z_0 = 1.8m；Vs = 26.662m/s. $d.F$, $\delta.F$, $\beta.F$ represented the curves for the throwing distance, the release angle ,and attitude angle of female holder ; Z_0 = 1.8m; Vs = 27.044m/s.

Based on the release parameters in Table 2 (λ=0m/s), this study estimated the throwing distance of discus, and made a comparison of estimated data between this study and other studies. It was found that the maximum relative error was 4.21%, the minimum was 0.2%, and the average value was 1.96%. It showed that the estimation results of this study

were reliable, which were based on the data on drag and lift coefficients in Ganslen's [6] wind tunnel tests, and hypothesized the attitude angle δ as a fixed value in the flying process of discus.

Basically, the release height and speed are mainly determined by the discus thrower's shape and his/her ability to spin and accelerate, while the attack angle and the release angle are determined by the throwing skills. How can a coach discover the optimal attack angle and attitude angle to help the trainee throw farther, if the discus coach has already known his/her release height and speed?

Referring to the optimal release parameters indicated by researchers Soong [4] and Bartlett [3], this study set the range for the release angle β and the initial attack angle δ. The calculating interval of angles for β and ψ_0 was 0.25°. Within the range for β and ψ_0, the following items were calculated with computer: the estimated distance in each interval, the farthest distance, and the optimal release and attack angles.

According to the above method, the calculating interval was 0.25°for both the release angle and attack angle, and the release height was 1.8m for both male and female holders [1]. On the other hand, the range for release speed was Vs = [25m/s, 28m/s], each calculating step was 0.002m/s, and the estimated results were as Figure 3 and Table 3 demonstrated. It was found that the male holder could reach 74.08m, the distance of world record [13], at the minimum release speed of 26.662m/s, the optimal release angle of 36.50°, and the attack angle of -10.25°. The conditions were similar to the estimated results by Frohlich [1] and Soong [4] in Table 2. As for the female holder, she could reach 76.80m, the distance of the world record [13], at the minimum release speed of 27.044m/s, the optimal release angle of 32.75°, and the attack angle of -9.25°[9].

Table 3.The optimal release and attack angles for male and male world record holders throwing a discus under no wind

	Release Parameters					
	Vs	Z_0	β	δ	ψ_0	*Distance*
Sex	(ms⁻¹)	(m)	(°)	(°)	(°)	(m)
Male	26.662	1.80	36.50	26.25	-10.25	74.08
Female	27.044	1.80	32.75	23.50	-9.25	76.80

It was hard to make a comparison of the optimal conditions for male throwers and female throwers, since there were few related studies in previous literature. Besides, the weights of the discus and the discus' area are different. For example, the weight of discus for females is only half of the weight for males and its area is smaller, the effects caused by the drag, lift, and gravity are different. It was shown in Figure 5 that a better performance for the female world record holder could be achieved[9], when the release speed ranged from 25m/s to 28m/s, the optimal attitude angle ranged from 23°to 25.25°, the optimal release angle ranged from 32.25°to 35°, and the discus was thrown in a lower and flatter way. On the other hand, a better performance for the male world record

holder could be achieved, when the optimal attitude angle ranged from 26.5°to 27.5°[9], the optimal release angle ranged from 35.75°to 37.5°, and the release angle and the optimal attitude angle were higher that the female's.
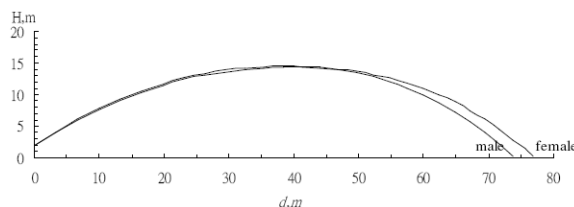
Figure 4. The optimal flying trajectory of discus for male and female world record holders ; H indicates the height; λ =0m/s.
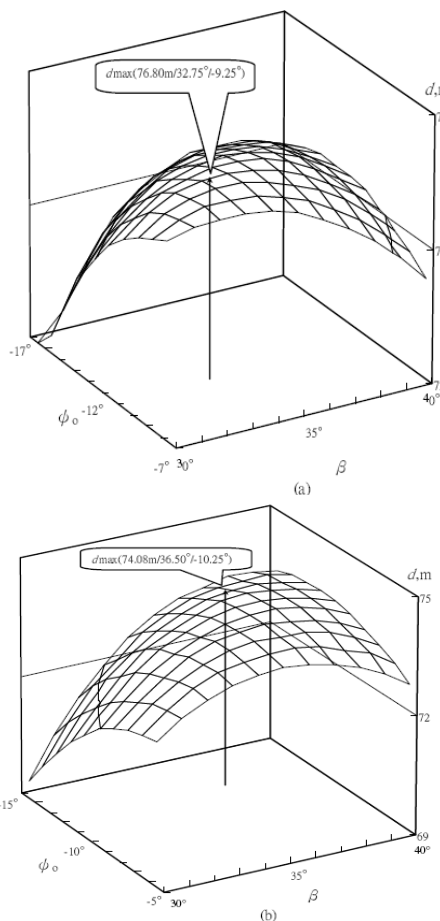
Figure 5. The relationship among the release angle, the attack angle, the performance for the male and female world record holders. Figure 5(a) presented the male's throws; λ =0m/s; Vs =26.662m/s; Z_0 =1.8m. Figure 5(b) presented the female's throws; λ =0m/s; Vs =27.044m/s; Z_0 =1.8 m. The symbol dmax in Figure 5(a) and (b) represented world record/release angle/angle of attack.

The optimal throw in the wind

According to the regulations of track and field of IAAF (1985/86)[12], the discus throw is not limited by the wind speed; that is, the world record established under any wind speed will be accepted. Based on the release height and release speed in Table 3, this study estimated the optimal performance,

release angle, and attitude angle for both male and female world record holders when the wind speed ranged from –21 to 12m/s.

It was shown from the flying curve in Figure 6 that the throwing distance in head wind was farther than that in tail wind. This phenomenon corresponded to the estimation of Soong [4]. Furthermore, Figure 6 demonstrated that when the head wind speed was -17m/s, the male holder could throw up to 84.286m, which was 10.206m more than the distance achieved under no wind. When the speed in head wind was over -17m/s，the throwing distance would decrease gradually. On the other hand, when the head wind speed was over -13m/s （d =85.924m, β =26.00°, δ =15.25°）, the female holder's throwing distance would also decrease[9].
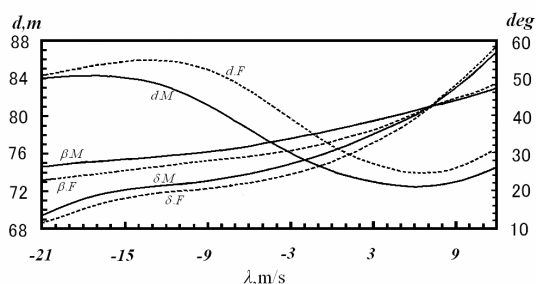

Figure 6. The relationship among the release angle, attitude angle, and performance under different wind speeds. $d.M$, $\beta.M$, $\delta.M$ represented the curves for the male's throwing distance, release angle, and attitude angle; Z_0 =1.8m; Vs =26.662m/s. $d.F$, $\delta.F$, $\beta.F$ represented the curves for the female's throwing distance, release angle, and attitude angle; Z_0 =1.8m; Vs =27.044m/s .

Another finding presented in Figure 6 was that when the tail wind speed increased, the throwing distances for both male and female holders would also decrease. However, when the tail wind speed was over 7m/s, the distance for the male holder would begin to increase （ d =72.282m, β =42.00°, δ =40.00°）, and the distance for the female holder would also increase （ d =73.681m, β =40.25°, δ =36.50°）[9].

The third result, an interesting one, could be found in Figure 6. When the head wind speed was around 8 m/s, and the release angle and attitude angle were the same, both male and female holders could obtain their optimal throwing distance. If the tail wind speed increased, the optimal distance could be obtained only when the attitude angle was larger than the release angle.

Conclusions

Optimal release conditions for the discus world record holders are as followed. The optimal release conditions for male world record holder under no wind required that the male holder throw at the release angle of 36.50° , the attack angle of -10.25° and the release speed faster than 26.662m/s. Under these optimal conditions, he could break the original world record of 74.08m. As for the female holder, the optimal conditions under no wind for her required that she throw the discus at the release angle of 32.75°, the attack angle of -9.25°, and the release speed faster than 27.044m/s. Under these optimal conditions, she could break the original world record of 76.80m.

Second, when the female holder threw the discus under no wind at the release speed ranging from 25m/s to 28m/s, the optimal attitude angle for her would range from 23°to 25.25°, the optimal release angle would range from 32.25°to 35°, and her throwing distance would range from 60.49m to 82.40m. On the other hand, when the male holder threw the discus under no wind at the release speed ranging from 25m/s to 28m/s, the optimal attitude angle for him would range from 26.5°to 27.5°, the optimal release angle would range from 35.75°to 37.5°, and his throwing distance would range from 60.30m to 81.62m.

Third, if the wind speed ranged from -21m/s to 12m/s, a better performance would be achieved if the discus was thrown at low release angle and attitude angle in head wind. Reversely, higher release angle and attitude angle should be adopted in tail wind.

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