Horizontal segregation of mono-layer granules coordinated by vertical motion

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When agitated, granular mixtures may segregate rather than mix. This phenomenon is not only of interest to scientists involved in fundamental research of non-equilibrium systems [1,2] but also to those developing applications in industrial processing [3]. Different kinds of segregation have been reported [4–8]. For example, when granular grains are vibrated vertically, they segregate according to size due to the void filling effect [9], convective flow [4] or depletion force [5]. In a rotating drum, segregation happens because particles have different repose angles [6]. Shear instability can induce the formation of stripes in a horizontally shaken binary mixture [7]. Segregation caused by gravitational force is found in astronomy [8]. For a granular binary mixture in a confined region, segregation can be identified by same species aggregation. One type of aggregations has the same species congregating in a particular region of space. For example, in the Brazil-nut effect [10], large intruders may rise above the bed particles and congregate on the upper layer of the mixture. Intuitively, the simplest type of granular aggregation is due to attraction between particles so that particles can aggregate at any region [11]. But a particle, though attracted to its own species, may take a very long time to join a cluster if it does not tend to be repelled by other types of particles. That is, attraction alone is not enough to cause segregation and the repelling force between different species, rather than the attractive force among the same species, is the crucial factor for segregation. In social science, Schelling employed the concept of a repelling force to demonstrate race segregation in habitats [12]. In granular systems, the depletion force [5] originating from a geometric constraint in the neighborhood of particles is considered an effective repulsive force between particles of different sizes. An interesting question is: Can an effective repulsive force exist between different particle species in a granular binary mixture, causing the mixture to segregate, in particular when the two particle species are of the same size?

Suppose a mono-layer binary granular system confined within a circular boundary. Particles interact with each other by collision only. Since granular particles lose kinetic energy in inelastic collisions, their ability to repel their neighbors is reduced by collisions. We might say that particles are effectively attracting each other. The inelastic collisions between A-B, A-A, and B-B are respectively characterized by the coefficients of restitution \( \varepsilon_{AB} \), \( \varepsilon_{AA} \), and \( \varepsilon_{BB} \). In general, we have \( \varepsilon_{BB} \leq \varepsilon_{AB} \leq \varepsilon_{AA} < 1 \) and the mixture will not segregate under arbitrary agitation. A possible way to segregate A and B is to create a condition under which there is an effective repulsive force between A and B. This can be achieved in simulations by introducing a superelastic coefficient of restitution between A and B with a value greater than unity [13], \( \varepsilon_{AB} > 1 \). Let us consider two-dimensional particles A and B of same radius \( r = 1 \) and same mass \( m = 1 \) confined in a circle of radius 19 (in units of beads radius). The number of particles A and B is equal, \( N_A = N_B = 100 \). The collisions between particles are modeled by a spring and dashpot as described in refs. [14,15]. Starting from a random distribution for particle positions and velocities, our molecular dynamic (MD) simulations showed that segregations indeed occurred when \( \varepsilon_{AB} > 1 \), \( \varepsilon_{AA} < 1 \) and \( \varepsilon_{BB} < 1 \). Two typical simulation results are shown in fig. 1. Segregation occurs even when particles of the...
same species repel each other (i.e., \(\varepsilon_{AA} > 1\) and \(\varepsilon_{BB} > 1\)) as long as these repulsive forces are small compared to that found between different species. But the kinetic energy of the system is not conserved and the system may not be stable. In the long run, the kinetic energy might become infinitely large if the increase of the kinetic energy following collisions between A and B is not reduced by the losses between A-A and B-B (fig. 1b). Consequently, for a granular binary mixture, a constant repulsive force between different species is not guaranteed to produce a stable segregation. Fortunately, we have found a simple granular system which can automatically tune its repulsive force to result in stable segregation.

In this report, we experimentally examine a granular system that exhibits stable segregation due to the effective repulsive force between different particle species. The system consists of two kinds of spherical beads of the same size placed between two horizontal plates which vibrate sinusoidally in the vertical direction. At certain ranges of driven frequency and amplitude, the regular vertical motion of the beads generates a tunable repulsive force between particles of different species in the horizontal direction in such a way that stable lateral segregation occurs. Wang and Hong have studied this system using computer simulations [13] and showed that the mixture may segregate when both species of beads are in synchronization with the external drive. Melby et al. investigated a similar setup for a mixture of spheres of two sizes and focused on the size effect [5].

The beads we used are aluminum and polymer spheres, both with mass \(m = 0.3\) g and diameter \(d = 6\) mm. In the collision process, the beads can be characterized by their different restitution coefficients, which we measured by the free-falling method. When a bead is released freely from a height \(H\) and falls vertically, the impact velocity is \(\sqrt{2gH}\). If the bouncing height of the bead is \(h\), the reflective velocity will be \(\sqrt{2gh}\). The restitution coefficient \(\varepsilon\), which is defined as the ratio of the relative velocities after and before collisions, is simply given by \(\sqrt{h/H}\). We used 20 samples of beads and performed the experiment with \(H = 10\) cm. The average restitution coefficients are shown in table 1. We determined the coefficients of restitutes between the beads by measuring the velocities of particles before and after head-on collisions. The resultant coefficients between beads have larger uncertainty than that of the coefficients between beads and plate. However, we will see that their precise values are not essential for the occurrence of horizontal segregations.

The beads were confined between two horizontal glass planes and a circular side wall made of acrylic. The distance between the two horizontal planes was \(D = 9.0\) mm, which was 1.5 times of the diameter of the beads. The circular wall had a diameter of 116 mm and could accommodate up to \(N_{\text{max}} \approx 320\) beads in a monolayer. Most of our experiments had \(N = 200\) beads in the container, half type A (aluminum) and half type B (polymer). The acrylic wall of the container and the beads’ surfaces were sprayed with an anti-static material to avoid the accumulation of electrostatic charge. The container was mounted onto a vibration exciter (Model: Bruel & Kjaer 4808) which was driven by a sinusoidal signal. In each experiment we mixed an equal number of these two types of beads and vibrated the container vertically at frequency \(f\) and amplitude \(A\) for 5 minutes. The frequency is read directly from a digital function generator and the amplitude is measured by an optical triangulation method [16] which provides an accuracy of 50 \(\mu\)m. A camera (Canon G10) positioned above the container was used to record the horizontal positions of all beads at 5 seconds intervals. We assigned the bead distribution in each photo a segregation index \(S\) to indi-

| Table 1. Average restitution coefficients of particles used in experiments. |
|-----------------|------------------|------------------|
| \(\varepsilon_{AP}\) (Bead A and plate) | 0.71 ± 0.03     |
| \(\varepsilon_{BP}\) (Bead B and plate) | 0.45 ± 0.03     |
| \(\varepsilon_{AA}\) (Beads A and A)    | 0.72 ± 0.07     |
| \(\varepsilon_{BB}\) (Beads B and B)    | 0.45 ± 0.07     |
| \(\varepsilon_{AB}\) (Beads A and B)    | 0.64 ± 0.1      |

Fig. 1. Two MD simulation results with \(\varepsilon_{AB} > 1\). The segregation index \(S\) (solid curve), which is defined in eq. (1), increases with time. Kinetic energy (diamonds) increases at collisions between particles A and B but decreases at A-A and B-B collisions. Depending on the initial configurations, the average kinetic energy may either stabilize (a) or blow up (b).
Fig. 2. Time evolution of particle distributions and their corresponding segregation indices for two different vibration amplitudes at 40 Hz.

calculate the degree of segregation between A and B. $S$ was calculated by the following formula:

$$S = \frac{1}{N} \sum_{i} \frac{n_{ii}}{n_{ii} + n_{ij}}, \quad (1)$$

where $n_{ii}$ is the number of neighbors of bead $i$ that are of the same type as $i$, and $n_{ij}$ is the number of neighbors of bead $i$ that are of different type from $i$. A bead is a neighbor of bead $i$ if its distance to $i$ is smaller than a given value $r$. For a randomly mixed configuration of equal number of A and B beads, $S$ is close to 0.5. When types A and B are clustered separately with a boundary between them, $S$ will be very close to 1. In our experiments, the value of $S$ is always between 0.5 and 1. Configurations with $S$ below 0.5 are possible when the bead positions are specially arranged, but this never occurred during vibration of beads starting with a random configuration. To appropriately define the neighborhood of a bead, one must choose $r$ based on the filling fraction of the beads in the container. When the filling fraction is 1, i.e., when the maximal number of beads is arranged in a compact configuration in the container, a bead’s neighbors can be simply defined as those beads with which it is in direct contact. In this case, $r$ is simply the diameter of the beads or, equivalently, the distance between two beads in direct contact. For filling fractions below 1, we set $r$ as the distance between any pair of closest beads when the beads are uniformly distributed on the plane. For our experimental setup with $N = 200$, $r$ is equal to 8.2 mm, or 1.37 times the bead diameter.

Figure 2 shows the evolution of the segregation index $S$ of $N = 200$ beads for two different vibration amplitudes at frequency $f = 40$ Hz. At $A = 0.2 \text{ mm}$, $S$ fluctuates between 0.5 and 0.6. The bead configuration stays random at all times. At a larger amplitude $A = 0.45 \text{ mm}$, the same type of beads gradually aggregates. $S$ increases constantly and reaches a value close to 0.95 in about two minutes. By that time, the two types of beads are well segregated into two distinct clusters. When we vibrated the system with a smaller frequency, $f = 30$ Hz, the beads mixture did not segregate at either $A = 0.2 \text{ mm}$ or $A = 0.45 \text{ mm}$ amplitudes. After 5 minutes of vibration, their final $S$ values were less than 0.6. When different values of $N$, ranging from 170 to 230, are used the results are qualitatively the same: beads segregate ($S > 0.75$) in the case of $f = 40$ Hz and $A = 0.45 \text{ mm}$, but remain randomly distributed in the other three cases. We have also checked that the final segregation indices do not depend on the initial configurations. As a demonstration, we show in fig. 3 that a segregated state ($S > 0.75$) obtained under a segregation-favored vibration with $f = 40$ Hz and $A = 0.3 \text{ mm}$ is changed gradually to a random state ($S < 0.75$) when
vibration is switched to be unfavorable to segregation (see the phase diagram of segregation in fig. 7).

We first investigated the vertical velocity of the beads as a function of vibration amplitude. A smaller system was used for this purpose. Twelve beads (6 aluminum and 6 polymer) were placed in a parallelepiped 14.4 cm in length and 9 mm both in height and width. A fast camera (1000 fps, 320 × 228 resolution) was used to record the vertical motion of each bead for 3 seconds. We tracked the brightest spot on the surface of each bead by a tracking program which gave the position of each bead in a precision of 0.3 mm. Given the continuous position of each bead, we could easily calculate the average vertical speed for type-A and type-B beads, respectively: $u_A$ and $u_B$, and the average height difference between pairs AA, BB, and AB: $h_{AA}$, $h_{BB}$, and $h_{AB}$. Figure 4 plots $u_A$, $u_B$, $h_{AA}$, $h_{BB}$, and $h_{AB}$ as a function of vibration amplitude at frequency 40 Hz. At small vibration amplitudes, neither type of bead has enough momentum to reach the top plane, so they gain energy only from the bottom plane at the random vibration phase. When the vibration amplitude increases to above a critical value, the beads would collide with both the bottom and the top planes. The average bead’s speed increases significantly at the critical amplitude. Same-type beads actually move in synchronization (i.e., they collide with the top plane at a fixed vibration phase) above the critical amplitude. The inset of fig. 4 shows the trajectories of two beads at amplitude 0.3 mm.

where one moves approximately in synchronization with its own kind while the other does not gain enough energy to hit the upper plane (for clarity, the positions of bottom and upper planes in the inset have been shifted by a distance equal to the bead’s radius). The evidence of synchronization is that the value of the average speed above the critical amplitude is in agreement with the theoretical value for the synchronization state, and we can assume that all beads are in synchronization with the external frequency. By neglecting the gravity effect since, in our case, $g/8f^2$ is much smaller than the bead’s flying distance $D - d$, the average bead speed at amplitude $A$ can be determined by the resonance condition to yield[17]

$$v = 2\pi f \times \frac{\pi(D - d) - 2\sqrt{(4\alpha + \pi^2)A^2 - \alpha(D - d)^2}}{4\alpha + \pi^2},$$

where $\alpha = \left(\frac{1 - \varepsilon}{1 + \varepsilon}\right)^2$ and $\varepsilon$ is the restitution coefficient between bead and plane. The value of $v$ calculated by eq. (2), shown by dashed lines in fig. 4, is in very good agreement with the measured experimental speeds. When all beads of the same type are approximately in synchronization, we would expect that the height difference of the average pair will be smaller than when they are out of phase. Also, since the two types of beads have different restitution coefficients, $\varepsilon_{AP}$ and $\varepsilon_{BP}$, the paired height difference between two beads of different types $h_{AP}$ is always larger than $h_{AA}$ or $h_{BB}$ when at least one type of bead has entered the synchronization state. The experimental data shown in fig. 3 are consistent with these expectations.

Presumably, for our system with $N = 200$ beads, bead types A and B have the same average speeds $u_A$ and $u_B$ as measured in the smaller 12 bead system, and the average paired height differences are also given by $h_{AA}$, $h_{BB}$, and $h_{AB}$. If the motion of the beads were perfectly vertical, there would be no segregation since there would be no collisions between beads. System imperfection, due to plane roughness or vibration unbalance, yields small perturbations in the horizontal velocity. Beads with non-vanishing horizontal speed will collide with their neighbors. After colliding, two beads separate horizontally with a relative speed dependent on the collision angle $\theta$ with respect to the horizontal plane (fig. 5). When $\theta$ is large, collisions...
between A and B have a relatively larger $h_{AB}$, causing the beads to gain horizontal energy by transferring momentum from the $z$-component to the horizontal direction [13]. Thus, collisions between beads induce an effective horizontal repulsive force between beads. According to the simulations described above, the horizontal segregation would occur if the repulsive force between A and B is larger than that between A-A and between B-B. Let us define the horizontal repulsive force per unit mass as the average relative horizontal speed after collision divided by the vibration period $T$, to indicate how strongly two beads tend to separate from each other. When beads $i$ and $j$, with respective vertical speeds $u_i$ and $u_j$ (neglecting the horizontal speeds), collide at an angle $\theta_{ij}$, which is determined by the average height differences $h_{ij}$, the horizontal repulsive force $F_{ij}$ between these two beads can be easily found to be

$$F_{ij} = \sin \frac{2\theta_{ij}}{4T} (u_i + u_j) \varepsilon_{ij}, \quad (3)$$

where $\sin \theta_{ij} = \frac{h_{ij}}{d}$ and $\varepsilon_{ij}$ is the restitution coefficient between beads $i$ and $j$. $F_{ij}$ is zero when two beads are in synchronization, $\theta_{ij} = 0$. Given the definition eq. (3) and all measured values of $u_A$, $u_B$, $h_{AA}$, $h_{BB}$, and $h_{AB}$, we calculated $F_{AA}$, $F_{BB}$, and $F_{AB}$ as a function of vibration amplitude at frequency 40 Hz (fig. 6). For amplitudes larger than about 0.25 mm, and given a value approximately equal to the critical amplitude for beads A to move in synchronization (fig. 4), $F_{AB}$ is larger than both $F_{AA}$ and $F_{BB}$, and segregation occurs with a segregation index $S > 0.75$ (fig. 6).

Beads gain horizontal kinetic energy when $\theta$ is large and their horizontal speeds are small prior to collision. However, in our system, beads will not keep a large horizontal speed for long. The horizontal kinetic energy either transfers back into vertical motion or is dissipated in the collisions. That is, the vertical synchronous motion driven by regular external force plays the role of a reservoir which constantly supplies and absorbs horizontal energy through collisions between beads. Our experimental data and previous numerical simulations [18] show that the horizontal speed of a typical bead is small most of the time except when it intermittently assumes a value larger than the vertical speed ($u_A$ or $u_B$). The system is stable in the sense that the horizontal speed will not increase without limit and, since $F_{AB} > F_{AA} > F_{BB}$ for most of the time, it tends to segregate horizontally.

As mentioned above, at frequency 40 Hz our binary system will reach a stable segregation when either one kind of bead or both kinds of beads are in vertical synchronization. It is well known that, at high amplitudes, the motion of the beads can be bifurcated (have two phases) and even enter chaos (have many phases) [13,19]. Wang and Hong have predicted by simulation that segregation might disappear when one kind of beads, say type B, enters a two-phase state [13]. They have also performed an analytic analysis by neglecting gravity and showed that vibration amplitude has a frequency-independent upper bound, above which segregation disappears. We conducted experiments for vibration frequency at $f = 20, 25, 30, 35$ and 40 Hz, and with amplitude $A$ ranging from 0.2 mm to 3 mm, while maintaining the acceleration $\Gamma = 4\pi^2Af^2$ between 1g and 10g, and calculated the segregation index $S$ of the final state. The results are plotted in the frequency-amplitude phase space (fig. 7). If $S > 0.75$, we marked it by a plus showing it is a segregated state [20]. A cross means that the final state is not segregated (i.e., $S < 0.75$). Two dashed curves show the operating range of vibration acceleration 1g $< \Gamma < 10g$. (The vibrator we used has a limited maximum acceleration at 10g.) Our experimental results are in good agreement with Wang and Hong’s analytic results (the shaded region) for frequencies at 30, 35 and 40 Hz. However, due to the capacity of our vibrator, we were not able to produce large enough amplitudes that can destroy the segregation at high frequencies ($f > 30$ Hz). At lower frequencies (20 and 25 Hz), segregation occurs at larger amplitudes than had been predicted. The circles in fig. 5 show the critical amplitudes...
at which beads A began to move synchronously (while beads B enter synchronization at larger amplitudes). Our experiments showed that, approximately at these critical amplitudes, the binary mixture began to segregate.

In summary, we carried out experiments on a monolayer of a granular binary mixture vibrating vertically between two horizontal planes. We observed collisions between beads, presumably because the beads gain horizontal speed due to plane roughness or vibration unbalance. The collisions between beads at an angle with respect to the horizontal plane induce an effectively horizontal repulsive force between beads by transferring momentum from the $z$-component to the horizontal direction. When at least one kind of bead moves in synchronization, the horizontal repulsive force between heterogeneous beads is likely to be larger than that between homogenous beads, resulting in segregation. Meanwhile, non-horizontal collisions can also harness the fast-moving beads by transferring their horizontal kinetic energy back into steady vertical motion so that the final segregation index is stable.

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References

20. Note that it is easy to visually determine if a configuration is well segregated when it has an $S$ value greater than 0.9. In experiments, a segregation mechanism may not work perfectly under different conditions, so a final configuration may have an $S$ value equal to 0.64 or 0.76, for example, making it very difficult to visually determine whether segregation had occurred. We choose a somewhat arbitrary threshold value of 0.75, the average of $S$ values for random (0.5) and complete-segregation (1.0) configurations. A configuration with $S > 0.75$ is segregated; otherwise it is not segregated.