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Optimal performance analysis of irreversible cycles used as heat pumps and refrigerators

Jincan Chen
CCAST (World Laboratory), PO Box 8730, Beijing 100080, People’s Republic of China and Department of Physics, Xiamen University, Xiamen 361005, People’s Republic of China

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Abstract. A general irreversible cycle model is used to investigate the optimal performance of a class of heat-driven pumps affected by the three main irreversibilities, which are finite-rate heat transfer between the working fluid and the external heat reservoirs, internal dissipation due to the working fluid and heat leakage between heat reservoirs. The coefficient of performance of the cycle system is taken as an objective function for optimization. Some equivalent parameters are introduced so that the relevant calculation is simplified. The coefficient of performance versus dimensionless heating load curves describing the general performance characteristics of heat-driven heat pumps are plotted. The optimal temperatures of the working fluid at the maximum coefficient of performance are determined. Moreover, it is expounded that the irreversible cycle model is very useful. The optimal performance not only of a class of irreversible heat-driven refrigerators but also of an irreversible Carnot heat pump and refrigerator may be directly analysed by using the cycle model. The results obtained may provide some new theoretical bases for the optimal design and operation of two classes of real heat pump and refrigerator systems driven by ‘low-grade’ heat energy and ‘high-grade’ work.

1. Introduction

Like reversible models in classical thermodynamics, endoreversible models have played an important role in the development of finite-time thermodynamics. In recent decades, endoreversible cycle models [1–17] have been used successfully to analyse the influence of finite-rate heat transfer on the performance of heat engines, refrigerators, heat pumps and other thermodynamic systems. However, for real thermodynamic systems, there exist other sources of irreversibility besides the irreversibility of finite-rate heat transfer. The quest for new models which can include the several main irreversibilities usually existing in real thermodynamic systems is becoming one of the important subjects in finite-time thermodynamics. Some new models of Carnot cycles including several irreversibilities have recently been established [18–25] and some significant conclusions concerning irreversible Carnot cycles have been deduced. Yet, few efforts have been made to investigate the influence of multi-irreversibilities on the performance of a class of heat pumps and refrigerators driven by heat energy.

In the present paper, a general irreversible cycle model which includes finite-rate heat transfer, heat leakage and other irreversibilities due to the internal dissipation of the working fluid will be established and used to analyse the optimal performance of a class of irreversible heat pumps and refrigerators driven by heat energy. At the same time, it is pointed out that the optimal performance of an irreversible Carnot heat pump and refrigerator can be directly deduced from the results obtained in this paper.

2. A general irreversible cycle model

It is well known that real heat-driven heat pumps are not as efficient as is a reversible three-heat-source heat pump, which consists of three reversible isothermal and three reversible adiabatic processes and whose coefficient of performance is given by [26]

$$\psi_r = \frac{T_h - T_c}{T_h} \frac{T_p}{T_p - T_c} > 1$$  (1)

where $T_h$, $T_p$ and $T_c$ are the temperatures of the heat source, heated space and heat sink, respectively.

Real heat-driven heat pumps are complex devices and suffer from a series of irreversibilities. Besides the irreversibility of finite-rate heat transfer which is considered in the endoreversible cycle models [27], there also exist other sources of irreversibility. For example,
heat leakage from the heated space and internal dissipation of the working fluid may be two other main sources of irreversibility, each of which can decrease the coefficient of performance and the heating load of heat pumps. When the influence of these main irreversibilities on the performance of a heat-driven heat pump is considered, it may be assumed that the cycle of the working fluid in the heat pump still consists of three isothermal and three adiabatic processes, but these processes are irreversible. Figure 1 is a schematic diagram of an irreversible heat-driven heat pump, in which \( T_1, T_2 \) and \( T_3 \) are, respectively, the temperatures of the working fluid in three isothermal processes; \( k_1, k_2 \) and \( k_3 \) are, respectively, the heat conductances between the working fluid and the three external heat reservoirs at temperatures \( T_h, T_p \) and \( T_c \); \( k_L \) is the heat loss coefficient of the heated space; \( Q_1 \) and \( Q_3 \), respectively, the heat absorbed from the heat source at temperature \( T_h \) and from the heat sink at temperature \( T_c \) by the working fluid per cycle; \( Q_2 \) is the heat pumped to the heated space at temperature \( T_p \) per cycle; and \( Q_L \) is the heat leakage from the heated space during a cycle [28, 29]. Then, the net heats \( Q_p \) and \( Q_c \) transferred to the heated space and released from the heat sink, respectively, during a cycle are

\[
Q_p = Q_2 - Q_L \quad (2)
\]

\[
Q_c = Q_3 - Q_L. \quad (3)
\]

The irreversible cycle model mentioned above is obviously more general than an endoreversible cycle model, because it includes the three main irreversibilities usually existing in real heat pumps.

According to the first and second laws of thermodynamics,

\[
Q_1 - Q_2 + Q_3 = Q_1 - Q_p + Q_c = 0 \quad (4)
\]

and we can introduce an irreversibility factor

\[
I = \frac{Q_2/T_2}{Q_1/T_1 + Q_3/T_3} \geq 1 \quad (5)
\]

to describe the irreversibilities due to the internal dissipation of the working fluid. Equation (5) shows clearly that, when \( I = 1 \), the cycle of the working fluid is endoreversible; when \( I > 1 \), the cycle of the working fluid is irreversible. Thus, the optimal performance of an endoreversible heat-drive heat pump [27] may be derived directly from the results obtained in this paper.

When the isothermal processes are endoreversible, which implies that there exist no other irreversibilities in these isothermal processes except finite-rate heat transfer between the working fluid and the external heat reservoirs, the irreversibility factor may be expressed simply as

\[
I = \frac{\Delta S_2}{\Delta S_1 + \Delta S_3} \geq 1. \quad (6)
\]

Here \( \Delta S_1, \Delta S_2 \) and \( \Delta S_3 \) are, respectively, the entropy differences of the working fluid in the three isothermal processes at temperatures \( T_1, T_2 \) and \( T_3 \). These entropy differences are defined to be positive, as shown in figure 2.

When heat transfer obeys a linear law [1–5, 30], one has

\[
Q_1 = k_1(T_h - T_1)t_1 \quad (7)
\]

\[
Q_2 = k_2(T_2 - T_p)t_2 \quad (8)
\]

\[
Q_3 = k_3(T_c - T_3)t_3 \quad (9)
\]

\[
Q_L = k_L(T_p - T_c)t \quad (10)
\]

where \( t_1, t_2 \) and \( t_3 \) are, respectively, the times spent on the three isothermal processes, and \( t \) is the cycle time. Because the adiabatic processes are not affected by thermal resistances, the time spent on the adiabatic processes is small compared with that spent on the isothermal processes and is usually negligible [31–33]. Thus, the cycle time is given approximately by

\[
t = t_1 + t_2 + t_3. \quad (11)
\]

It is important to note that the irreversible cycle model mentioned above has an extensive range of application. It may also be used directly to analyse the optimal performance of a class of irreversible heat-driven refrigerators.
3. The \( \psi \)-II characteristic curves

Using the above equations and definitions of the coefficient of performance \( \psi \) and the heating load \( \pi \) of a heat-driven heat pump, we obtain

\[
\psi = \frac{Q_p}{Q_1} = \frac{Q_2}{Q_1} \left( 1 - \frac{Q_L}{Q_2} \right) = \frac{T_1 - T_3}{T_1} \frac{IT_2}{T_2} - \frac{T_1 - T_3}{T_1} \frac{IT_2}{T_2} \times \left[ 1 - C \left( \frac{1}{k_2(T_2 - T_p)} + \frac{k_1(T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \right] \quad (12)
\]

\[
\pi = \frac{Q_p}{Q_1} = \frac{Q_2}{Q_1} - C = \left( \frac{1}{k_2(T_2 - T_p)} + \frac{T_1(1T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \left[ 1 - C \left( \frac{1}{k_2(T_2 - T_p)} + \frac{k_1(T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \right] - C \quad (13)
\]

where \( C = k_L(T_p - T_e) \). It can be seen from equations (12) and (13) that, as long as some reasonable equivalent parameters

\[
T_{2e} = IT_2 \quad T_{pe} = IT_p \quad k_{2e} = k_2/I \quad (14)
\]

are introduced, equations (5), (8), (12) and (13) may be simplified to

\[
\frac{Q_2}{T_{2e}} = \frac{Q_2}{T_{1}} - \frac{Q_3}{T_{3}} = 0 \quad (15)
\]

\[
Q_2 = k_{2e}(T_{2e} - T_{pe}p)_2 \quad (16)
\]

\[
\psi = \frac{T_1 - T_3}{T_1} \frac{T_{2e} - T_{pe}p}{T_{2e} - T_{3}} \left[ 1 - C \left( \frac{1}{k_2(T_2 - T_{pe})} + \frac{k_1(T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \right] \quad (17)
\]

\[
\pi = \left( \frac{1}{k_2(T_2 - T_{pe})} + \frac{k_1(T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \left[ 1 - C \left( \frac{1}{k_2(T_2 - T_{pe})} + \frac{k_1(T_2 - T_3)}{k_1(T_2 - T_3)(T_2 - T_1)} \right) \right] - C \quad (18)
\]

It is interesting to note that the fact that the forms of equations (7), (9) and (15)–(18) are similar to those of the fundamental expressions describing the optimal performance of an endoreversible heat-driven heat pump affected by the same heat leak.

For the sake of convenience, let \( x = T_{2e}/T_1, y = T_{2e}/T_3 \) and \( z = T_{2e} \). Then equations (17) and (18) may be re-written as

\[
\psi = \frac{y - x}{y - 1} \left[ 1 - C \left( \frac{1}{k_2(z - T_{pe})} + \frac{1 - x}{y - 1} + \frac{y - 1}{k_3(T_e - z/y)(y - x)} \right) \right] \quad (19)
\]

\[
\pi = \left( \frac{1}{k_2(z - T_{pe})} + \frac{y - 1}{k_1(T_h - z/x)(y - x)} + \frac{1 - x}{y - 1} + \frac{y - 1}{k_3(T_e - z/y)(y - x)} \right) - C. \quad (20)
\]

In order to determine the optimal coefficient of performance for a given heating load, we introduce the Lagrangian

\[
L = \psi + \lambda \pi = \frac{y - x}{y - 1} \left[ 1 - C \left( \frac{1}{k_2(z - T_{pe})} + \frac{1 - x}{y - 1} + \frac{y - 1}{k_3(T_e - z/y)(y - x)} \right) \right] + \lambda \left[ \frac{1}{k_2(z - T_{pe})} + \frac{k_1(T_h - z/x)(y - x)}{y - 1} \right] \quad (21)
\]

From the Euler–Lagrange equations

\[
\frac{\partial L}{\partial x} = 0 \quad \frac{\partial L}{\partial y} = 0 \quad \frac{\partial L}{\partial z} = 0 \quad (22)
\]

and equation (21), one obtains

\[
\sqrt{k_1(xT_h - z)} = \sqrt{k_{2e}(z - T_{pe})} = \sqrt{k_3(yT_e - z)} \quad (23)
\]

On substituting the solutions of equation (23) into equations (19) and (20), we obtain

\[
\psi = \frac{(B_1T_h - T_e)x + B_2T_{pe} - \pi}{B_1xT_h - T_e + B_2T_{pe} + \pi + C} \quad (24)
\]

\[
\pi = k \sqrt{(xT_h - T_{pe})(B_1T_h - T_e)x + B_2T_{pe}} \quad (25)
\]

where

\[
k = \frac{k_{2e}}{[1 + (k_{2e}/k_3)^{1/2}]^2} \quad B_1 = \frac{1 + (k_{2e}/k_3)^{1/2}}{1 + (k_{2e}/k_3)^{1/2}} \quad B_2 = \frac{(k_{2e}/k_3)^{1/2} - (k_{2e}/k_3)^{1/2}}{1 + (k_{2e}/k_3)^{1/2}}. \quad (26)
\]

On solving equations (24) and (25) we find that the optimal relation between \( \pi \) and \( \psi \) is given by

\[
\pi = k \left[ (T_h - T_e)T_{pe} - \psi(1 + \frac{C}{\psi})T_h(T_{pe} - T_e) \right] \times \left[ T_e + B_1^2 \left( \frac{1}{\psi} + \frac{B_1^2}{1 + \frac{C}{\psi}} \right) T_h \right] \left[ \frac{1}{\psi} + \frac{B_1^2}{1 + \frac{C}{\psi}} \right] \quad (27)
\]

When \( k_L = 0 \), equation (27) may be simplified to

\[
\pi = k \left( \frac{T_h - T_e}{T_h - T_{pe}} \right) \frac{T_{pe}}{T_h} \psi - \psi T_h(T_{pe} - T_e) \quad (28)
\]

Equation (28) shows that, for a heat-driven heat pump affected by the internal dissipation of the working fluid and finite-rate heat transfer, the optimal coefficient of performance is a monotonically decreasing function of the heating load. When \( \pi = 0 \),

\[
\pi = \frac{T_h - T_e}{T_h - T_{pe}} \frac{T_{pe}}{T_c} \Rightarrow \psi_{ri} < \psi_r. \quad (29)
\]

This shows clearly that, even if the heat-leak losses of the heated space are negligible and the heating load is equal
performance is always smaller than the dissipation of the working fluid. Real heat pumps are irreversible heat-driven heat pump should be specified coefficient of performance, it is always desirable which is just the optimal relation between \( \pi_m \) and \( \psi_{\text{max}} \) from equations (24) and (25), we can plot easily the characteristic curves of a heat-driven heat pump, as shown in figure 3, in which the heating load of the heat-driven heat pump.

When \( k_L = 0 \) and \( I = 1 \), equation (27) or (28) may be simplified further to

\[
\pi = \frac{k}{1} \left( \frac{1}{T_c} \right) \left( \frac{1}{T_p} \right) - \psi_{\text{max}} \left( \frac{1}{T_c} \right) \left( \frac{1}{T_p} \right) = \frac{1}{T_c} + \frac{1}{T_p} - \psi_{\text{max}} \left( \frac{1}{T_c} \right) \left( \frac{1}{T_p} \right) = \pi_{\text{max}}
\]

which is just the optimal relation between \( \pi \) and \( \psi \) of an endoreversible heat-driven heat pump. Equation (30) has been used to analyse the optimal performance of an endoreversible heat-driven heat pump [27].

When \( k_L > 0 \), \( \psi \) is not a monotonic function of \( \pi \). There exists an extremal value \( \psi_{\text{max}} \) for \( \psi \). Starting directly from equations (24) and (25), we can plot easily the \( \psi-\pi \) characteristic curves of a heat-driven heat pump, as shown in figure 3, in which \( \Pi = \pi/(kT_c) \) is the dimensionless heating load of the heat-driven heat pump.

It is seen from figure 3 that \( \psi_{\text{max}} \) is an important parameter because it determines the upper bound of the coefficient of performance of irreversible heat pumps. When \( \psi < \psi_{\text{max}} \), there are two different heating loads \( \pi \) for a given coefficient of performance \( \psi \), of which one is smaller than \( \pi_m \) and the other is larger than \( \pi_m \). For a specified coefficient of performance, it is always desirable to obtain a large heating load. Thus, the heating load of an irreversible heat-driven heat pump should be

\[
\pi \geq \pi_m.
\]

This shows that \( \pi_m \) is another important parameter of irreversible heat-driven heat pumps because it determines the lower bound of the heating load.

It is also seen from figure 3 that, no matter whether there exist heat leak losses, the coefficient of performance of heat-driven heat pumps always approaches unity when \( \pi \rightarrow \pi_{\text{max}} \). It has been pointed out that heat pumps operating under such a condition are nonetheless not as efficient as direct conductance of heat [27]. Thus, the heating loads of heat-driven heat pumps should not be near \( \pi_{\text{max}} \).

### 4. Optimal temperatures of three isothermal processes

In order to make a heat-driven heat pump attain its optimal performance, the temperature of the working fluid must be controlled to operate within the reasonable range. Starting from equation (19) and the extremal conditions

\[
\frac{\partial \psi}{\partial x} = 0 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial z} = 0 \quad (32)
\]

we can prove that, when the coefficient of performance attains its maximum \( \psi_{\text{max}}, x, y \) and \( z \) must satisfy the following equations:

\[
x = \frac{T_p}{T_h} \left\{ \frac{1 + a}{k_1} \left( \frac{1 + a}{k_1} + \frac{(k_2r/\psi_{\text{max}})}{k_3} \right)^{1/2} - \frac{1}{2} \right\}
\]

\[
y = \frac{T_p}{T_c} \left\{ \frac{1 + a}{k_1} \right\}
\]

\[
z = \frac{T_p}{T_c} \left\{ \frac{1 + a}{k_1} \right\}
\]

where

\[
d = \frac{C}{(k'T_c)} \quad a = \{d[1 - (1 - d)T_c/T_{pe}]\}^{1/2}
\]

\[
k' = k_2r/[1 + (k_2r/k_3)]^{1/2}.
\]

Thus, when the coefficient of performance is maximum, the temperatures of the working fluid in three isothermal processes are, respectively, determined by

\[
T_1 = \frac{T_h}{1 + (\frac{k_2r}{k_1})^{1/2}} \left\{ \frac{1 + a}{1 - d} + \frac{(k_2r/k_3)^{1/2}}{k_1} \right\} \equiv T_{1m}
\]

\[
T_2 = \frac{T_p}{1 + (\frac{k_2r}{k_1})^{1/2}} \left\{ \frac{1 + a}{1 - d} + \frac{(k_2r/k_3)^{1/2}}{k_1} \right\} \equiv T_{2m}
\]

\[
T_3 = \frac{T_c}{1 + (\frac{k_2r}{k_1})^{1/2}} \left\{ \frac{1 + a}{1 - d} + \frac{(k_2r/k_3)^{1/2}}{k_1} \right\} \equiv T_{3m}.
\]

We can determine further that the optimal temperatures of the working fluid in three isothermal processes should be

\[
T_1 \leq T_{1m} \quad T_2 \geq T_{2m} \quad T_3 \leq T_{3m}.
\]
By substituting equations (33)–(35) into equations (19) and (20), we can obtain the maximum coefficient of performance

\[
\psi_{\text{max}} = \frac{D - [D_1/B_1 - (k_{2e}/k_1)^{1/2}]T_e/T_h}{D - T_c/T_h} \left( 1 - \frac{C}{\pi_m + C} \right)
\]

where

\[
\pi_m = k_{2e} T_{pe} \left( \frac{D_1}{1 + (k_{2e}/k_3)^{1/2}} - 1 \right)
\]

\[
\times \left\{ \frac{D - \left[ \frac{D_1}{B_1 - (k_{2e}/k_1)^{1/2}} \right] T_e}{T_h} \right\}
\]

\[
\times \left[ 1 + \left( \frac{k_{2e}}{k_1} \right)^{1/2} D + \left[ \left( \frac{k_{2e}}{k_1} \right)^{1/2} - \left( \frac{k_{2e}}{k_3} \right)^{1/2} \right]^2 \right]
\]

\[
\times \frac{D_T e}{T_c} - \left[ 1 + \left( \frac{k_{2e}}{k_1} \right)^{1/2} \right] T_c/T_h \right\}^{-1} - C
\]

\[
D = (1 + a)/(1 - d) \quad \text{and} \quad D_1 = D + (k_{2e}/k_3)^{1/2}.
\]

5. The \( \varepsilon-R \) characteristic curves

When the cycle model shown in figure 1 is used to analyze the optimal performance of an irreversible heat-driven refrigerator, \( T_p \) and \( T_e \) are, respectively, the temperatures of the heat sink and the cooled space. According to the definition of the coefficient of performance and the cooling rate of a heat-driven refrigerator, we obtain

\[
\varepsilon = \frac{Q_e}{Q_i} = \psi - 1
\]

\[
\frac{r}{1} = \frac{Q_p}{Q_i} \left( 1 - \frac{Q_i}{Q_p} \right) = \frac{\pi}{\psi} - 1.
\]

On substituting equations (43) and (44) into equations (24), (25) and (27), we obtain

\[
\varepsilon = \frac{(1 - x)T_e}{B_1 x T_h - T_e + B_2 T_{pe} r + C}
\]

\[
r = \frac{k (1 - x) (x T_h - T_e + B_2 T_{pe}) T_e}{B_1 B_2 x^2 T_h + (B_1^2 T_h - T_e + B_2^2 T_{pe}) x + B_1 B_2 T_{pe} - C}
\]

\[
r = k \varepsilon [(T_h - T_{pe}) T_e - \varepsilon (1 + C/r) T_h (T_{pe} - T_e)]
\]

\[
\times \left[ (1 + \varepsilon) T_e + B_1^2 (1 + \varepsilon) \varepsilon (1 + C/r) T_h
\]

\[
- B_2^2 \varepsilon \left( 1 + C/r \right) \left( 1 + \frac{\varepsilon}{1 + \varepsilon} \right) T_{pe}]^{-1}
\]

\[= \varepsilon / (1 + \varepsilon) C.
\]

Equations (45)–(47) are the fundamental optimal relations of an irreversible heat-driven refrigerator. They may be used directly to analyze the influence of different irreversibilities on the performance of a heat-driven refrigerator.

For example, when \( I = 1 \), the cycle of the working fluid in the heat-driven refrigerator is endoreversible. Starting from equations (45)–(47), we can derive easily all the results in [10, 34]. When \( I > 1 \) and \( k_L = 0 \), equation (47) may be simplified to

\[
r = k \varepsilon \left( \frac{T_h - T_{pe}}{1 + \varepsilon} T_e - \varepsilon T_h (T_{pe} - T_e) \right)
\]

\[
(48)
\]

It can be seen from equation (48) that, even if \( r = 0 \), the coefficient of performance of a heat-driven refrigerator affected by finite-rate heat transfer and internal dissipation of the working fluid

\[
\varepsilon = \frac{T_h - T_p}{T_h - T_{pe} - T_p} \approx \varepsilon_{ri}
\]

cannot attain that \( \varepsilon_r \) of a reversible heat-driven refrigerator, given by \( \varepsilon_r = (T_h - T_p) T_e / (T_h (T_{pe} - T_e)) \).

For the general case, equations (45) and (46) may be used to plot the characteristic curves of an irreversible heat-driven refrigerator, as shown in figure 4, in which \( R = r/(k T_e) \) is the dimensionless cooling rate of a refrigerator. It can be seen from figure 4 that the \( \varepsilon-R \) characteristic curve of an irreversible heat-driven refrigerator is divided into three parts by the operating states of \( \varepsilon = 0, \varepsilon = \varepsilon_{\text{max}} \) and \( r = r_{\text{max}} \), whereby the slope of one part is negative and the slope of the others is positive. When a heat-driven refrigerator is operated in those parts of the \( \varepsilon-R \) curve with positive slope, the coefficient of performance decreases as the cooling rate decreases. These regions are not the optimal operating regions. The optimal operating region of a heat-driven refrigerator should be situated in the part of the \( \varepsilon-R \) curve with negative slope. In such a case, the coefficient of performance will increase as the cooling rate decreases and vice versa. Thus, the coefficient of performance and cooling rate should be, respectively, constrained by

\[
\varepsilon_{\text{max}} \geq \varepsilon \geq \varepsilon_m
\]

(50)
corresponds to a work source \[35, 36\]. In this case, \(Q\) adopted in the present paper is that it can include three significant conclusions have been drawn \[29, 37\].

The important feature of the irreversible cycle model of an irreversible Carnot heat pump and refrigerator and the optimal relation between \(\eta\) and \(r\) have been used to analyse the optimal performance and cooling rate, respectively.

\[ r_m \leq r \leq r_{\text{max}} \quad (51) \]

where \(\varepsilon_m\) and \(r_m\) are, respectively, the coefficient of performance at the maximum cooling rate \(r_{\text{max}}\) and the cooling rate at the maximum coefficient of performance \(\varepsilon_{\text{max}}\). This shows that \(\varepsilon_{\text{max}}, r_{\text{max}}, \varepsilon_m\) and \(r_m\) are four important parameters of heat-driven refrigerators, because they determine the upper and lower bounds of the coefficient of performance and cooling rate, respectively.

6. The case of \(T_h \rightarrow \infty\)

When \(T_h \rightarrow \infty\), the cycle system shown in figure 1 is identical to an irreversible Carnot cycle driven by work, because the high-temperature heat source with \(T_h \rightarrow \infty\) corresponds to a work source \[35, 36\]. In this case, \(Q_1\) corresponds to work input \(W\) and figure 1 may be simplified to figure 5. Then, the optimal relation between \(\psi\) and \(r\) of irreversible Carnot heat pumps

\[ \psi = \frac{(k'T_{pe} + C + \pi)^\pi}{(C + \pi)(C + \pi + k'(T_{pe} - T_r))] \quad (52) \]

and the optimal relation between \(\varepsilon\) and \(r\) of irreversible Carnot refrigerators

\[ \varepsilon = \frac{(k'T_e - C - r)r}{(C + r)(C + r + k'(T_{pe} - T_r))] \quad (53) \]

can be derived from equations (27) and (47). Equations (52) and (53) have been used to analyse the optimal performance of an irreversible Carnot heat pump and refrigerator and some significant conclusions have been drawn \[29, 37\].

7. Conclusions

The important feature of the irreversible cycle model adopted in the present paper is that it can include three main irreversibilities often existing in real heat pump and refrigeration systems. It is the further development of the endoreversible cycle models of two classes of heat pump and refrigeration systems driven by ‘low-grade’ heat energy and ‘high-grade’ work and is able to contain many of the simple cycle models that have appeared in the literature in recent years. It is very important that the optimal operating regions of irreversible heat pump and refrigeration systems and some new performance bounds related to the primary parameters be determined. The results obtained will play a more instructive role in the optimal design and operation of real systems than could those derived from the endoreversible cycle models.

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