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The optimum performance characteristics of a four-temperature-level irreversible absorption refrigerator at maximum specific cooling load

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Abstract. An irreversible cycle model of an absorption refrigerator operating between four temperature levels is established and used to analyse the performance of the refrigerator affected by the irreversibility of finite-rate heat transfer and the internal irreversibilities of the working substance. The specific cooling load of the refrigerator is taken as an objective function for optimization. The maximum specific cooling load is determined. Other main parameters of the refrigerator at maximum specific cooling load are calculated. The optimal distribution relation of the heat-transfer areas of the heat exchangers is derived. Several special cases are discussed in detail so that some important conclusions relative to absorption refrigerators in the literature may be derived directly from the present paper. Moreover, the influence of the internal irreversibilities on the performance of the system is analysed. The results obtained here can describe the optimal performance of a four-temperature-level absorption refrigerator affected simultaneously by the internal and external irreversibilities and provide the theoretical bases for the optimal design and operation of real absorption refrigerators operating between four temperature levels.

1. Introduction

Since the early 1970s there has been increasing concern about the very damaging effects of the emissions of chlorofluorocarbons on the stratospheric ozone layer [1]. Many countries have been searching more environment-friendly refrigerants and new advanced refrigeration cycles. The absorption refrigerators using H_2O/LiBr, NH_3/H_2O, etc as the working substance conform to all of the ozone-preserving regulations because there are no chlorofluorocarbons. On the other hand, the absorption refrigerators [2–7] can be driven by ‘low-grade’ heat energy such as waste heat in industries, solar energy and geological heat, and have a large potential for decreasing the consumption of primary energy sources and for reducing the heat pollution for the environment. Thus, absorption refrigerators for industrial and domestic use are generating renewed interest throughout the world [8–13].

A single-stage absorption refrigerator is the simplest form of absorption refrigeration cycles. It consists primarily of a generator, an absorber, a condenser, and an evaporator. It normally transfers heat between three temperature levels, but very often between four temperature levels [14–16]. According to the theory of classical thermodynamics, the coefficient of performance of a reversible absorption refrigerator operating between four temperature levels is given by

$$\varepsilon_r = \frac{[(1/T_a) - (1/T_e)] + n[1/T_c - 1/T_g]}{[(1/T_a) - (1/T_e)] + n[(1/T_c) - (1/T_g)]}$$  (1)

where $n = q_c/q_a$; $q_a$ and $q_c$ are, respectively, the rates of heat transfer from the absorber and condenser to two heat reservoirs at different temperatures $T_a$ and $T_c$; $T_g$ is the temperature of the heat source; and $T_e$ is the temperature of the cooled space. When $T_a = T_e = T_0$, equation (1) may be written simply as

$$\varepsilon_r = \frac{T_e - T_0}{T_e - T_0}$$  (2)

which is independent of $n$. Equation (2) just gives the coefficient of performance of a three-temperature-level reversible absorption refrigerator.
The reversible coefficient of performance \( \epsilon_r \) is important in theory, but it is invariably far more than the coefficient of performance of real absorption refrigerators and hence is of very limited practical value. Real absorption refrigerators usually suffer from a series of irreversibilities. For example, there exists not only the irreversibility of the finite-rate heat transfer, but also the internal irreversibilities resulting from the friction, eddies, mixing and other irreversible effects inside the working substance, each of which can in principle decrease the coefficient of performance of the refrigerators. Although many authors have investigated the influence of various key irreversibilities on the performance of absorption refrigerators operating between three temperature levels and obtained many significant results, the performance of absorption refrigerators operating between four temperature levels has rarely been analysed. In particular, the comprehensive influence of several major irreversibilities on the performance of such refrigeration systems has not yet been studied, so that it is necessary to develop the new theory of four-temperature-level absorption refrigerators further.

A four-temperature-level absorption refrigerator is a complex device and its performance is affected by many irreversibilities. If all of the individual details of the device were taken into account, it would obscure the physical content and make calculations very difficult or impossible. In order to obtain some significant analytical solutions of the key performance parameters it is necessary to establish a simplified cycle model which can reveal the main performance characteristics of a four-temperature-level absorption refrigerator. In this paper we will consider, simultaneously, the influence of the irreversibility of the finite-rate heat transfer and the internal irreversibilities of the working substance on the performance of a four-temperature-level absorption refrigerator.

2. An irreversible cycle model

Figure 1 shows a schematic diagram of a four-temperature-level absorption refrigerator. Heat energy \( q_g \) is supplied from the heat source at a high temperature \( T_g \) to the generator, in which the working fluid is concentrated, in the absorbent, by evaporating the working medium. The weak solution passes through the valve into the absorber. The working medium is then condensed in the condenser and subsequently transferred to the evaporator. In such a process, the amount of heat \( q_e \) is released from the condenser to one heat reservoir at temperature \( T_c \). The liquid working medium is evaporated due to the additional heat \( q_e \) from the cooled space at a low temperature \( T_e \) to the evaporator. The vapourized working medium is transferred to the absorber where it is absorbed by the weak solution and the amount of heat \( q_a \) is released from the absorber to the other heat reservoir at temperature \( T_a \). The strong solution produced in the absorber is pumped to the generator. Work input required by the solution pump in the system is negligible relative to the energy input to the generator and is often neglected for the purpose of analysis [17]. According to the first law of thermodynamics, we have

\[
q_g + q_e = q_a + q_c.
\]  

(3)

Obviously, the performance of a four-temperature-level absorption refrigerator is closely dependent on the irreversible factors to which the refrigeration system is subject. First, it is assumed that the heat transfer between the working substance in the heat exchangers and the external heat reservoirs is carried out under a finite temperature difference and obeys a linear law [18–21], while the working substance flows steadily and continuously. The other parts of the system are well insulated. Second, it is assumed that these heat exchange processes are isothermal and the equations of heat transfer may be written as

\[
q_g = U_g A_g (T_g - T_1) \]

(4)

\[
q_a = U_a A_a (T_2 - T_a) \]

(5)

\[
q_e = U_e A_e (T_e - T_4) \]

(6)

and

\[
q_c = U_c A_c (T_c - T_3) \]

(7)
where $T_1$, $T_2$, $T_3$ and $T_4$ are, respectively, the temperatures of the working substance in the generator, absorber, condenser, and evaporator, $U_e$, $U_a$, $U_c$, and $U_e$ are, respectively, the overall heat-transfer coefficients of the generator, absorber, condenser, and evaporator; and $A_g$, $A_a$, $A_c$, and $A_e$ are, respectively, the heat-transfer area of the generator, absorber, condenser, and evaporator. The total heat-transfer area between the cycle system and the external heat reservoirs is

$$A = A_g + A_a + A_c + A_e.$$  

Lastly, we may introduce an irreversibility factor \[22–24\] to describe summarily the internal irreversibilities of the working substance. In the following analysis, $I$ is assumed to be constant. When the cycle of the working substance is internally reversible, $I = 1$. When the cycle of the working substance is internally irreversible, $I > 1$.

Using equations (1) and (3)–(9), we find that the coefficient of performance and the specific cooling load \[25, 26\] (i.e. the cooling load per unit total heat-transfer area) of a four-temperature-level absorption refrigerator are, respectively, given by

$$\varepsilon = \frac{q_e}{q_s} = \frac{1}{(1/T_2) - (1/T_1)} + n[(1/T_3) - (1/T_2)]$$

and

$$R = \frac{q_e}{A} = \left(\frac{1}{U_e(T_e - T_2)} + \frac{q_e}{q_s}U_e(T_e - T_1)\right)^{-1}$$

respectively, the specific cooling load attains its maximum, i.e.

$$\frac{\partial R}{\partial x} = 0$$

where $x$, $y$, and $z$ are determined by (A7)–(A9). In such a case, the coefficient of performance is given by

$$\varepsilon_m = \left(1 - \frac{1}{d_2 + (1 + b_2)B + n[d_1 + (1 + b_1)B/T_e^* - T_e^*/T_e]}\right)^{-1}.$$  

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$$\varepsilon = \frac{y - x + n(z - x)}{1 - y + n(1 - z)}$$

and

$$R = \left[\frac{1}{U_e(T_e - T_4)} + \frac{1 - y + n(z - x)}{U_e(T_e - T_4 - x)} + \frac{1 - x}{U_e^*((T_e - T_4)/y - T_4^*)} + \frac{n(1 - x)}{U_e^*((T_e - T_4)/z - T_4^*)}\right]^{-1}.$$  

Using (13) and its extremal conditions

$$\frac{\partial R}{\partial x} = 0 \quad \frac{\partial R}{\partial y} = 0 \quad \frac{\partial R}{\partial z} = 0 \quad \frac{\partial R}{\partial T_4} = 0$$

we can prove (a detailed derivation is given in the appendix) that for a given value of $n$, when the temperatures of the working substance in the generator, absorber, condenser, and evaporator, are given by

$$T_1 = T_3 \frac{C - C_1 + B}{d_1 + (1 - b_1)B}$$

$$T_2 = T_4 \frac{C - C_1 + B}{d_2 + (1 + b_2)B}$$

$$T_3 = T_e \frac{C - C_1 + B}{d_3 + (1 + b_3)B}$$

respectively, the specific cooling load attains its maximum, i.e.

$$\frac{\partial R}{\partial x} = 0$$

and

$$R_{\text{max}} = \left[\frac{U_e(T_e(T_e - C) - B)}{C}\right]^{1/2} \left[\left(\frac{1}{d_2 + (1 + b_2)B + n[d_1 + (1 + b_1)B/T_e^* - T_e^*/T_e]}\right)^{-1} \left[\frac{(-1) + n}{x + n(z - y)} \right]^{-1}\right]^{-1}$$

where $x$, $y$, and $z$ are determined by (A7)–(A9). In such a case, the coefficient of performance is given by

$$\varepsilon_m = \left(1 - \frac{1}{d_2 + (1 + b_2)B + n[d_1 + (1 + b_1)B/T_e^* - T_e^*/T_e]}\right)^{-1}.$$  

$R_{\text{max}}$ is one important performance parameter of absorption refrigerators operating between four temperature levels. It determines an upper bound for the specific cooling loads of four-temperature-level absorption refrigerators. $\varepsilon_m$ is another important performance parameter of absorption refrigerators operating between four temperature levels. Similar to the efficiency \[27–29\] of a Carnot heat engine at maximum power output or maximum specific power output \[30\], $\varepsilon_m$ is a valuable acquisition for the further understanding of the performance of absorption refrigerators operating between four temperature levels.
4. Optimization of heat-transfer areas

In order to make a four-temperature-level absorption refrigerator operate in the optimal states, the heat-transfer areas of the absorber, generator, evaporator and condenser cannot be chosen arbitrarily. They must still satisfy a certain relation. Using (3)–(7), (10), (12) and (A5), we obtain

\[
\frac{y q_a}{x q_g} = \left(\frac{U^+}{U^-}\right)^{1/2} \left(\frac{A_g}{A_e}\right) = \frac{y(1-x)}{x(1+y+n(1-z))} \tag{21}
\]

\[
\frac{z q_a}{z q_g} = \left(\frac{U^+}{U^-}\right)^{1/2} \left(\frac{A_e}{A_g}\right) = \frac{n z(1-x)}{x(1+y+n(1-z))} \tag{22}
\]

and

\[
\frac{q_e}{x q_g} = \left(\frac{U^+}{U^-}\right)^{1/2} \left(\frac{A_e}{A_g}\right) = \frac{y(x+n(z-x))}{x(1+y+n(1-z))} \tag{23}
\]

From (21)–(23), we derive a very simple and useful optimal distribution relation of the heat-transfer areas as

\[
\sqrt{U_g} A_g + \sqrt{U_e} A_e = \sqrt{U_g} A_g + \sqrt{U_e} A_e. \tag{24}
\]

From (24) and the relevant expressions mentioned above, one can also derive the relations between the total heat-transfer area \(A\) and the heat-transfer areas \(A_i\) (i = g, a, c and e) of a four-temperature-level absorption refrigerator at maximum specific cooling load. It is clear that using (24) as a theoretical guidance, engineers can design better heat exchangers of real four-temperature-level absorption refrigerators.

5. Several special cases

5.1.

When \(U_a = U_e\), i.e. the overall heat-transfer coefficient of the absorber is equal to that of the condenser, \(b_1 = b_1 \equiv b\) and the expressions relative to the optimal performance of a four-temperature-level absorption refrigerator may be simplified considerably. For example, (20) may be written as

\[
\varepsilon_m = \left(1 - (1+n)(1+b)(1+n T^+_a/T_c^2) + (1-b_1) B_1\right) \times \left[(1+n)(1-b_1)(T^+_a/T_c) + (1+b) B_1\right] \times \left[1 + (n T^+_a/T_c^2)\right]^{-1} \frac{T^+_a}{T_c} \times \left[\left[(1+n)(1+b)^2(1+n T^+_a/T_c^2)\right]ight]
\]

\[
-(1+n)(1-b_1) T^+_a / T_c T^+_a / T_c \left[(b+b_1)\right] \times [(1+n)(1-b_1)(T^+_a/T_c) + (1+b) B_1] \times [1 + (n T^+_a/T_c^2)]^{-1} - 1 \right)^{-1} \tag{25}
\]

where \(B_1 = [(1+n)(1+n T^+_a/T_c^2) T^+_a/T_c]^1/2\).

5.2.

When the overall heat-transfer coefficients of the heat exchangers are the same, i.e. \(U_g = U_e = U_c = U_r\), the temperatures of the working substance in the generator, absorber, condenser and evaporator, may be written simply as

\[
T_1 = T_g \left[\sqrt{T + \frac{1+(n T^+_a/T_c^2)}{1+n T^+_a/T_c^2}}\right] \left(1 + \sqrt{T}\right)^{-1} \tag{26}
\]

\[
T_2 = T_a \left[1 + \sqrt{T} \left(\frac{1+n T^+_a/T_c^2}{1+n T^+_a/T_c^2}\right)\right] \left(1 + \sqrt{T}\right)^{-1} \tag{27}
\]

\[
T_3 = T_c \left[1 + \sqrt{T} \left(\frac{1+n T^+_a/T_c^2}{1+n T^+_a/T_c^2}\right)\right] \left(1 + \sqrt{T}\right)^{-1} \tag{28}
\]

and

\[
T_4 = T_e \left[\sqrt{T + \frac{1+n T^+_a/T_c^2}{1+n T^+_a/T_c^2}}\right] \left(1 + \sqrt{T}\right)^{-1} \tag{29}
\]

In such a case, the maximum specific cooling load and the corresponding coefficient of performance may be simplified as

\[
R_{max} = \frac{U_e T_g}{(1 + \sqrt{T})^2} \left[1 - \left(\frac{1+n T^+_a/T_c^2}{1+n T^+_a/T_c^2}\right)^2\right] \frac{T_e}{T_r - T_e} \tag{30}
\]

and

\[
\varepsilon_m = \left[1 - \left(\frac{1+n T^+_a/T_c^2}{1+n T^+_a/T_c^2}\right)^{1/2}\right] \times T_e \left[\left(\frac{(1+n) T^+_a T_e}{1+n T^+_a/T_c^2} - T_e\right)^{-1} \right]. \tag{31}
\]

It can be proven from (26) and (29) that if \(T_a < T_c\), then the optimal temperatures \(T_1\) and \(T_2\) of the working substance in the generator and evaporator at maximum specific cooling load will increase as the value of \(n\) increases, and that if \(T_a > T_c\), then \(T_1\) and \(T_2\) will decrease as the value of \(n\) increases. Similarly, it can be proven from (27) and (28) that if \(T_a < T_c\), then the optimal temperatures \(T_3\) and \(T_4\) of the working substance in the absorber and condenser at maximum specific cooling load will decrease as the value of \(n\) increases, and that if \(T_a > T_c\), then \(T_3\) and \(T_4\) will increase as the value of \(n\) increases. If \(T_a = T_c\), then \(T_j(j = 1, 2, 3\) and \(4\)) is independent of the value of \(n\). When \(T_a, T_e, T_c\) and \(I\) are kept constant, figures 2(a)–2(d) give the curves of \(T_j\) against \(n\) for three different temperatures of \(T_a\).

Using (30) and (31), we may obtain some obvious statements as follows. If \(T_a < T_c\), then the derivatives of both the maximum specific cooling load \(R_{max}\) and the corresponding coefficient of performance \(\varepsilon_m\) with respect to \(n\) are always negative. This implies that \(R_{max}\) and \(\varepsilon_m\) will decrease as the value of \(n\) increases and will reach their asymptotic values when \(n\) tends to infinity. Thus, \(R_{max}\) and \(\varepsilon_m\) will be minimum when all heat \((q_a + q_e)\) is rejected from condenser to the heat reservoir at \(T_c\). If \(T_a > T_c\), then the derivatives of \(R_{max}\) and \(\varepsilon_m\) with respect to \(n\) are always positive. This implies that \(R_{max}\) and \(\varepsilon_m\) will increase.
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Figure 2. The optimal temperatures \( T_j \) ((a) \( j = 1 \), (b) \( j = 2 \), (c) \( j = 3 \) and (d) \( j = 4 \)) of the working substance in the generator, absorber, condenser and evaporator at maximum specific cooling load as a function of \( n \) for three different temperatures \( T_a = 293 \) K, 303 K and 313 K. The dashed (\( I = 1 \)) and full (\( I = 1.05 \)) lines are presented for the temperatures \( T_g = 403 \) K, \( T_c = 303 \) K and \( T_e = 273 \) K.

as the value of \( n \) increases and will reach their asymptotic values when \( n \) tends to infinity. Thus, \( R_{\text{max}} \) and \( \varepsilon_m \) will be maximum when all heat \( (q_a + q_c) \) is rejected from condenser to the heat reservoir at \( T_c \). If \( T_a = T_c \) then \( R_{\text{max}} \) and \( \varepsilon_m \) are independent of the value of \( n \). When \( T_g, T_c, T_e \) and \( I \) are kept constant, the behaviours described above are those shown in figures 3 and 4, respectively, where \( R_{\text{max}} = R_{\text{max}}/(U_eT_g) \) is the dimensionless maximum specific cooling load.

It is of interest to note that the behaviour of \( \varepsilon_m \) varying with \( n \) is similar to that of the reversible coefficient of performance \( \varepsilon_r \) varying with \( n \), which is shown in figure 5. However, it is clearly seen from figures 4 and 5 that the results obtained here are more realistic than those of reversible cycles.

5.3.

When \( T_a = T_e = T_0 \) and \( U_a = U_c \), a four-temperature-level absorption refrigerator becomes an absorption refrigerator operating between three temperature levels. Consequently, the optimal performance of the absorption refrigerator operating between three temperature levels may be derived directly from the above results. For example, we can obtain

\[
R_{\text{max}} = U_e (\sqrt{T_g} - \sqrt{T_0})^2 \times \frac{T_c}{\sqrt{(1 + b)(1 - b_1)}(1 - b_1)^2 - (b + b_1)^2 T_c} 
\]

(32)

and

\[
\varepsilon_m = (1 - \sqrt{T_0/T_g}) \times \frac{T_c}{\sqrt{T_g T_0^* - T_c + [(1 - b_1)/(b + b_1)](\sqrt{T_g T_0^* - T_0^*})}}
\]

(33)

from (19) and (20), where \( T_0^* = IT_0 \). Equations (32) and (33) are just the maximum specific cooling load and the
corresponding coefficient of performance of an irreversible absorption refrigerator operating between three temperature levels. When $I = 1$, the optimal performance of an endoreversible absorption refrigerator operating between three temperature levels [3, 26] may easily be derived from (32) and (33). It is thus clear that the cycle model established in this paper also includes that of a three-temperature-level absorption refrigerator.

6. The influence of $I$

When the influence of the internal irreversibilities of the working substance is negligible, $I = 1$ and the cycle is internally reversible. In such a case we can directly obtain the optimal relation of a four-temperature-level endoreversible absorption refrigerator as long as the parameters $U_a^*$, $U_c^*$, $T_a^*$ and $T_c^*$ in the above equations are replaced by the parameters $U_a$, $U_c$, $T_a$ and $T_c$, respectively. This clearly shows that the cycle model established here is general. It includes the endoreversible cycle model of a four-temperature-level absorption refrigerator. The results obtained in this paper may be used to discuss the optimal performance of a four-temperature-level endoreversible absorption refrigerator and the curves of $T_j$ ($j = 1, 2, 3$ and $4$), $R_{\max}^*$ and $\varepsilon_m$ varying with the values of $n$ are shown by the dashed lines in figures 2–4.

It can clearly be seen from (10)–(13) that if the influence due to the internal irreversibilities of the working substance is taken into account, one may summarize as follows. The temperatures of the two heat reservoirs at $T_a$ and $T_c$ are equivalently increased by a factor of $I$, and the overall heat-transfer coefficients of the absorber and condenser are equivalently reduced by a factor of $I$. The greater the value of $I$, then the higher the equivalent temperatures of the two heat reservoirs and the smaller the equivalent overall heat-transfer coefficients of the absorber and condenser. Figures 2–4 clearly show that the influence of $I$ on the performance of a four-temperature-level absorption refrigerator is quite obvious. The maximum specific cooling load and the corresponding coefficient of performance will decrease quickly as $I$ increases. Thus, one should do one’s best to reduce the internal irreversibilities of the working substance.

7. Conclusions

We have established an irreversible cycle model which can describe the optimal performance of a four-temperature-level absorption refrigerator affected by the irreversibility of finite-rate heat transfer and the internal irreversibilities of the working substance. Using the cycle model to analyse the performance of a four-temperature-level absorption refrigerator, we have obtained many realistic and useful results. It is expected that these results may lay a foundation for the deeper investigation of real four-temperature-level absorption refrigerators.
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Appendix

From equations (13) and (14), we obtain

\[ -\frac{T_4(y + nz)^2}{U_4(T_k - T_4^2)} + \frac{1}{U_4^*(T_k/y - T_4^*)} + \frac{n}{U_4^*(T_k/z - T_4^*)} = 0 \] (A1)

and

\[ \frac{1 + n}{U_4(T_k - T_4^2)} + \frac{1}{U_4^*(T_k/y - T_4^*)} - \frac{T_4[y - (1 + n)x]/x^2}{U_4^*(T_k/z - T_4^*)^2} = 0 \] (A2)

From (A1)–(A4), we obtain

\[ \sqrt{U_4}(T_k x - T_4) = \sqrt{U_4^*}(T_k - T_4^*) y = \sqrt{U_4^*}(T_k - T_4^*) z \] (A5)

Solving equations (A1)–(A4) yields

\[ Cu^2 - 2C_1 T_c u + C_2 T_c^2 = 0 \] (A6)

where

\[ b_1 = (U_4/U_4^*)^{1/2} \quad b_2 = (U_4/U_4^*)^{1/2} \]

\[ b_3 = (U_4/U_4^*)^{1/2} \quad u = T_c - T_4 \]

\[ C = (1 + b_2)^2 + n(1 + b_3)^2 T_c^2/T_c^* - (1 + n)(1 - b_1)^2 T_c^2/T_c^* \]

\[ C_1 = 1 + b_2 + n(1 + b_3)^2 T_c^*/T_c^* - (1 + n)(1 - b_1) T_c^*/T_c^* \]

\[ C_2 = 1 + n T_c^2/T_c^* - (1 + n) T_c^2/T_c^* \]

Solving (A5) and (A6) gives

\[ x = \frac{T_c C - (1 - b_1)(C_1 - B)}{T_c^* C} = \frac{T_c d_1 + (1 - b_1)B}{T_c^* C} \] (A7)

\[ y = \frac{T_c C - (1 + b_2)(C_1 - B)}{T_c^* C} = \frac{T_c d_2 + (1 + b_2)B}{T_c^* C} \] (A8)

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\[ z = \frac{T_c C - (1 + b_3)(C_1 - B)}{T_c C} = \frac{T_c d_3 + (1 + b_3)B}{T_c C} \] (A9)

and

\[ u = \frac{T_c C_1 - B}{C} \] (A10)

where

\[ d_1 = (1 + b_2)(b_1 + b_2) + n(1 + b_1)(b_1 + b_3)T_c^*/T_c^* \]

\[ d_2 = n(b_1 + b_3)(b_1 - b_2)T_c^*/T_c^* + (1 + n)(1 - b_1)T_c^*/T_c^* \]

\[ d_3 = (1 + b_2)(b_2 - b_1) + (1 + n)(1 - b_1)(b_1 + b_3)T_c^*/T_c^* \]

\[ B = \frac{-n(b_1 - b_2)^2 T_c^*/T_c^* + (1 + n)(b_1 + b_2)^2 T_c^*/T_c^* + n(1 + b_1 + b_3)^2 T_c^*/T_c^*}{2} \]

From (A7)–(A10), we obtain equations (15)–(18).

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