A reformulation of Coombs' Theory of Unidimensional Unfolding by representing attitudes as intervals

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Abstract

An examination of the logical relationships between attitude statements suggests that attitudes can be ordered according to favourability, and can also stand in relationships of implication to one another. The traditional representation of attitudes, as points on a single dimension, is inadequate for representing both these relations but representing attitudes as intervals on a single dimension can incorporate both favourability and implication.

An interval can be parameterised using its two endpoints or alternatively by its midpoint and latitude. Using this latter representation, the midpoint can be understood as the ‘favourability’ of the attitude, while the latitude can be understood as its ‘generality’. It is argued that the generality of an attitude statement is akin to its latitude of acceptance, since a greater semantic range increases the likelihood of agreement.

When Coombs’ Theory of Unidimensional Unfolding is reformulated using the interval representation, the key question is how to measure the distance between two intervals on the dimension. There are innumerable ways to answer this question, but the present study restricts attention to eighteen possible ‘distance’ measures. These measures are based on nine basic distances between intervals on a dimension, as well as two families of models, the Minkowski $r$-metric and the Generalised Hyperbolic Cosine Model (GHCM). Not all of these measures are distances in the strict sense as some of them fail to satisfy all the metric axioms.

To distinguish among these eighteen ‘distance’ measures two empirical tests, the triangle inequality test, and the aligned stimuli test, were developed and tested using two sets of attitude
statements. The subject matter of the sets of statements differed but the underlying structure was the same. It is argued that this structure can be known *a priori* using the logical relationships between the statement’s predicates, and empirical tests confirm the underlying structure and the unidimensionality of the statements used in this study. Consequently, predictions of preference could be ascertained from each model and either confirmed or falsified by subjects’ judgements.

The results indicated that the triangle inequality failed in both stimulus sets. This suggests that the judgement space is not metric, contradicting a common assumption of attitude measurement. This result also falsified eleven of the eighteen ‘distance’ measures because they predicted the satisfaction of the triangle inequality.

The aligned stimuli test used stimuli that were aligned at the endpoint nearest to the ideal interval. The results indicated that subjects preferred the narrower of the two stimuli, contrary to the predictions of six of the measures. Since these six measures all passed the triangle inequality test, only one measure, the GHCM (item), satisfied both tests. However, the GHCM (item) only passes the aligned stimuli tests with additional constraints on its operational function. If it incorporates a strictly log-convex function, such as cosh, the GHCM (item) makes predictions that are satisfied in both tests. This is also evidence that the latitude of acceptance is an item rather than a subject or combined parameter.
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