# Practical expressions for the design of laminated glass

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### Abstract

Due to deformability of the polymeric interlayer, stiffness and strength of laminated glass are usually less than those corresponding to a monolith with same total thickness. A practical design tool consists in the definition of the "effective thickness", i.e., the thickness of an equivalent *monolithic* glass that would correspond to the same deflection and peak stress of the *laminated* glass, under the same constraint and load conditions. Very recently, a new model has been proposed for the evaluation of the effective thickness. Here, a comparison is made with the classical approach by Wölfel-Bennison and the new method is specialized to the most common cases of the design practice, providing synthetic tables for ease of reference and immediate applicability.

**Keywords:** Laminated glass, plate design, effective thickness, strength calculation, composite structures, sandwich structure.

# 1. Introduction.

An effective technique to enhance the post-glass-breakage performance of architectural glazing consists in bonding glass plies together with polymeric interlayers via *lamination* in autoclave at high temperature and pressure. In such a way a laminated glass acquires safety properties because, after breakage, shards remain attached to the polymer and the system maintains a small but significant load bearing capacity, avoiding injuries due to catastrophic collapse.

Stiffness and strength of laminated glass may be considerably less than those of a monolithic glass with the same total thickness, because the interlayer is unable to provide a perfect shear

coupling. As a matter of fact, the response is affected by the shear stiffness of the polymer (in particular by its shear modulus G), that regulates the relative sliding of the constituent glass plies.

Two borderline cases can be recognized: *i*) the *monolithic* limit for  $G \to \infty$ , where the two glass plies are perfectly bonded together (fig. 1a) and the flexural inertia is that corresponding to the total thickness of the laminated glass; *ii*) the *layered* limit for  $G \to 0$ , with free-sliding plies (fig. 1b), for which the flexural inertia is the sum of the inertiae of the isolated plies. In general, the real condition is intermediate between these two borderline cases (fig. 1c).



Figure 1: Laminated glass composed of two plies and one interlayer under flexure. The two limit cases of a) monolithic limit and b) layered limit; c) the intermediate configuration.

Polymers are highly viscoelastic and, consequently, their response depends upon load duration and temperature. In the design practice a full viscoelastic analysis is seldom performed, but rheological effects are taken into account by considering, for the shear modulus G, the secant stiffness at the end of the load history at actual room temperature. The problem is thus simplified and reduced to a case in which all the materials, including the interlayer, are considered linear elastic. Moreover, at least as a first order approximation for a preliminary design, geometric nonlinearities can be neglected when in-plane loads are absent.

In numerical computations, the response of laminated glass could be conveniently modelled by a *layered* shell element that takes into account the competing stiffness between glass and interlayer, but most of the commercial numerical codes do not have such elements in their library. On the other hand, a full three-dimensional analysis is complicated and time consuming. This is why, in the design practice and especially in the preliminary design, it is very useful to consider approximate methods for the calculation of laminated glass. Currently, the most used approach is probably that proposed by Bennison (2009) based upon the theory for composed sandwich beams proposed by Wölfel (1987)0. To illustrate, consider a laminated beam of length l and width b composed of two glass plies of thickness  $h_1$  and  $h_2$  and Young's modulus E, connected by a polymeric interlayer of thickness t and shear modulus G (fig. 2).



Figure 2: Beam composed of two glass plies bonded by a polymeric interlayer. Longitudinal and cross sectional view (not in the same scale).

Let

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$$A_1 = bh_1, A_2 = bh_2, H = t + \frac{h_1 + h_2}{2}, I_1 = \frac{bh_1^3}{12}, I_2 = \frac{bh_2^3}{12}.$$
 (1)

When the *layered limit* is attained (i.e., two free-sliding glass plies), the moment of inertia of the laminated beam equals the sum  $I_1 + I_2$ . In the *monolithic limit*, the moment of inertia reads

$$I_{tot} = I_1 + I_2 + \frac{A_1 A_2}{A_1 + A_2} H,$$
(2)

where  $\frac{A_1A_2}{A_1+A_2}H^2$  represents the baricentrical inertia of the two areas A<sub>1</sub> and A<sub>2</sub>, supposed to be

concentrated in the corresponding centroid.

For intermediate cases, Wölfel (1987) proposed a strong approximation according to which the effective moment of inertia is of the form

$$I_{eq} = I_1 + I_2 + \Gamma \frac{A_1 A_2}{A_1 + A_2} H^2,$$
(3)

where the parameter  $\Gamma$ ,  $0 \le \Gamma \le 1$ , accounts for the capability of the interlayer to transfer shear stress between the glass plies. Wölfel proposed for  $\Gamma$  the expression

$$\Gamma = \frac{1}{1 + \beta \frac{tE}{bl^2} \frac{A_1 A_2}{A_1 + A_2}},\tag{4}$$

where the parameter  $\beta$  depends upon the loading and boundary condition and, for the most common cases, the corresponding values are recorded in (Wölfel (1987)0. Hypothesis (3) is equivalent to assume that the individual bending stiffness of the external layers has no influence on the coupling offered by the central layer: the less the bending stiffness of the external layers, the more accurate is this hypothesis.

Bennison (2009) has adopted Wölfel's approach specifically for the case of laminated glass (Calderone *et al.* 2009). A strong approximation in their proposal consists in using in (3) the universal value  $\beta = 9.6$  although in Wölfel's theory this is associated to one case only, i.e., the case of simply supported beams under uniformly distributed load. From (3), one can easily calculate the *stress-* and the *deflection-effective thickness*, i.e., the (constant) thickness of the homogeneous plate that, under the same boundary and load conditions of the considered problem, has the same maximal stress or maximal deflection, respectively.

Introducing, as per (Bennison 2009), the quantities

$$h_{s;1} = \frac{Hh_1}{h_1 + h_2}, \ h_{s;2} = \frac{Hh_2}{h_1 + h_2}, \ I_s = \frac{1}{E} \frac{A_1 A_2}{A_1 + A_2} H^2 = h_1 h_{s;2}^2 + h_2 h_{s;1}^2,$$
(5)

the deflection-effective thicknesses turns out to be:

$$h_{ef;w} = \sqrt[3]{h_1^3 + h_2^3 + 12\Gamma I_s},\tag{6}$$

whereas the stress-effective thickness for glass plies number 1 and 2 is given by

$$h_{1;ef;\sigma} = \sqrt{\frac{h_{ef;w}^3}{h_1 + 2\Gamma h_{s;2}}}, \quad h_{2;ef;\sigma} = \sqrt{\frac{h_{ef;w}^3}{h_2 + 2\Gamma h_{s;1}}}.$$
(7)

Although these expressions (referred to in the sequel as the Wölfel-Bennison approach) refer to a very particular static scheme, they are commonly used in numerical computations with models of monolithic plates with constant thickness. The stress and strain so calculated are used for structural

verification and, even more so, sometimes also serve to estimate stress concentrations around holes and/or at contact points; but no theoretical basis exists for this procedure.

An alternative formulation has been very recently proposed in (Galuppi and Royer-Cafagni 2012a). This procedure, called *Enhanced Effective Thickness* method, is based upon a variational approach and consists in finding the best approximation for the response of laminated glass among a restricted class of shape functions for the deflection surface through the minimization of the strain energy functional. The main hypotheses for this model are: *i*) the interlayer has no axial or bending stiffness, but only shear stiffness; *ii*) shear deformation of glass is neglected; *iii*) all materials are linear elastic; *iv*) geometric non-linearities are not considered. Remarkably, the method applies to the one-dimensional case of beams under bending (Galuppi and Royer-Cafagni 2012a) 0 but can be naturally extended to the two-dimensional case of plates (Galuppi and Royer-Cafagni 2012b) under the most various load and boundary conditions.

The purpose of this paper is to present the potentiality of this latter approach for the design of laminated glass. Paradigmatic cases are presented where its efficiency is proved by comparison with the results of precise numerical simulations and with the results obtainable with the classical Wölfel-Bennison approach. Tables for the calculation of the relevant coefficients in the most common cases have been added for ease of reference and to facilitate the practical use.

# 2. Enhanced effective thickness approach

The *enhanced effective thickness* (EET) method defines the equivalent moment of inertia  $I_R$  as the weighted harmonic mean of the moments of inertia corresponding to the layered and monolithic limit. This is a substantial difference with respect to (3) that uses the weighted arithmetic mean. This approach can be applied to the most various static schemes and load conditions.

### 2.1. The one-dimensional case. Laminated glass beams.

When applied to the same case of Figure 2, using the same notation of Section 1 the strain energy of the laminated beam can be written as a function of the vertical displacement v(x), the same for the two glass components, and the horizontal displacements  $u_1(x)$  and  $u_2(x)$  of the centroid of the upper and lower glass ply, respectively. Under the hypothesis that strains are small and the rotations moderate, the minimization of the strain energy leads to differential equilibrium equations with appropriate boundary conditions, that can be hardly solved without the use of a numerical procedure.

In order to define simple expressions for the *equivalent thickness*, the problem is simplified by introducing convenient shape functions for v(x),  $u_1(x)$  and  $u_2(x)$  that are compatible with the qualitative properties of the solution. It is natural to consider as the shape function for v(x) the form of the elastic curve g(x) of a monolithic beam with constant cross section under the same loading and boundary conditions of the problem at hand. In particular, we set

$$v(x) = \frac{g(x)}{EI_R},\tag{8}$$

where  $I_R$  is an unknown parameter representing the moment of the inertia of the laminated glass beam. We further assume that  $I_R$  is the weighted harmonic mean of  $I_{tot}$  (the monolithic limit) and  $I_1 + I_2$  (the layered limit), that is

$$\frac{1}{I_R} = \frac{\eta}{I_{tot}} + \frac{1 - \eta}{I_1 + I_2},\tag{9}$$

where the non-dimensional weight parameter  $\eta$  plays a role analogous to that of  $\Gamma$  in (3), because it tunes the response from the layered limit ( $\eta = 0$ ) to the monolithic limit ( $\eta = 1$ ). As illustrated in (Galuppi and Royer-Cafagni 2012a), minimization of the strain energy allows to determine the best value of  $\eta$  in the form

$$\eta = \frac{1}{1 + \frac{Et}{Gb} \frac{I_1 + I_2}{I_{tot}} \frac{A_1 + A_2}{A_1 A_2} \Psi},$$
(10)

where, by denoting by  $\Omega$  the one-dimensional domain representative of the reference configuration of the beam, the quantity  $\Psi$  is defined as

$$\Psi = \frac{\int_{\Omega} p(x)g(x)dx}{\int_{\Omega} g'(x)^2 dx},$$
(11)

where p(x) is associated with the distributed load (see Figure 2).

Clearly,  $\Psi$  depends upon the boundary and load conditions and its values are recorded in Sect. 3 for the cases of most practical relevance. Notice as well that  $\eta$  depends upon the mechanical and geometrical properties of the laminated beam, and one can show (Galuppi and Royer-Cafagni 2012) that when  $G \to \infty$  then  $\eta \to 1$  and when  $G \to 0$ , then  $\eta \to 1$ . From (9), the deflectioneffective thickness  $\hat{h}_w$  then turns out to be

$$\hat{h}_{w} = \sqrt{\frac{1}{\frac{\eta}{h_{1}^{3} + h_{2}^{3} + 12I_{s}} + \frac{1 - \eta}{h_{1}^{3} + h_{2}^{3}}}.$$
(12)

Recalling the definitions (7) of  $h_{s;1}$  and  $h_{s;2}$ , one also finds the following expressions for the stress-effective thickness:

$$\hat{h}_{1;\sigma} = \sqrt{\frac{h_1^3 + h_2^3 + 12I_s}{2\eta h_{s;2}} + \frac{\hat{h}_w^3}{h_1}}, \quad \hat{h}_{2;\sigma} = \sqrt{\frac{h_1^3 + h_2^3 + 12I_s}{2\eta h_{s;1}} + \frac{\hat{h}_w^3}{h_2}}.$$
 (13)

The Enhanced Effective Thickness approach presents no additional difficulty with respect to the Wölfel-Bennison formulations, giving compact formulas (12) and (13) for laminated glass design. Moreover, it can be readily extended to the two-dimensional case.

## 2.2. The two-dimensional case. Laminated glass plates.

When considering the laminated glass plate identified by the x - y domain  $\Omega$  (see Figure 3) under distributed load p(x,y), the strain energy can be written as a function of the vertical displacement w(x, y), the same for the two glass plies, and the horizontal x and y components of displacements of the middle plane of the upper and lower glass plate. Minimization leads to a system of partial differential equations with appropriate boundary conditions. In order to simplify the problem, we again introduce a convenient shape functions for the displacement components.



Figure 3: Plate composed of two glass plies bonded by a polymeric interlayer. Overall and cross sectional view (not in the same scale).

Defining the flexural rigidity of each glass ply as  $D_1 = \frac{Eh_1^3}{12(1-v^2)}$  and  $D_2 = \frac{Eh_2^3}{12(1-v^2)}$ , it can be demonstrated that the flexural rigidity for the monolithic limit reads (Galuppi and Royer-Cafagni 2012b)

$$D_{tot} = D_1 + D_2 + 12 \frac{D_1 D_2}{D_1 h_2^2 + D_2 h_1^2} H^2,$$
(14)

Then, the shape function for w(x,y) can be selected as the elastic deformed surface of a monolithic plate with constant thickness under the same loading and boundary conditions. In analogy with (5), we set

$$w(x,y) = \frac{g(x,y)}{D_R},$$
(15)

where  $D_R$  is the equivalent rigidity and the shape function g(x,y) is uniquely determined by the shape of the laminated glass plate in x - y plane, by the external load p(x, y) and by the geometric boundary conditions.

Assuming, in analogy with (9),

$$\frac{1}{D_R} = \frac{\eta}{D_{tot}} + \frac{1 \cdot \eta}{D_1 + D_2},$$
(16)

minimization of the strain energy allows to determine the counter part of (10) for the two dimensional case in the form

$$\eta = \frac{1}{1 + \frac{t}{G} \frac{D_1 + D_2}{D_{tot}} \frac{12D_1D_2}{D_1h_2^2 + D_2h_1^2} \Psi}, \qquad (17)$$

where now

$$\Psi = \frac{\int_{\Omega} p(x, y)g(x, y)dx \, dy}{\int_{\Omega} [g_{,x}(x, y)^2 + g_{,y}(x, y)^2]dxdy},$$
(18)

depends upon the plate shape, the load distribution p(x, y) and the boundary conditions. The stressand deflection-effective thicknesses may be readily calculated and take expressions analogous to (12) and (13), respectively.

It is important to note that the only "difficulty" of the proposed method consists in calculating  $\Psi$  from (18), because all the other formulas are simple analytical expressions. In the following we will report tables with values of  $\Psi$  that refer to the most common cases of the design practice.

## 3. Examples

The results obtainable with the EET approach are now compared with those proposed by Bennison (2009) and with the numerical experiments performed by means of the finite element software SJ-Mepla, specifically conceived of for laminated glass (SJ MEPLA 2011).

## 3.1. One-dimensional examples. Various constraint and load conditions.

Table 1 summarizes the values of  $\Psi$  evaluated through equation (11) as a function of the beam length l for various constraint and load condition. Such a coefficient allows to simply evaluate  $\eta$  through equation (10).

LOADING AND BOUNDARY CONDITIONS	Ψ	LOADING AND BOUNDARY CONDITIONS	Ψ
	$\frac{168}{17 l^2}$		$\frac{42}{l^2}$
	$\frac{15}{l^2+2ab}$		$\frac{14}{5l^2}$
	$\frac{10}{l^2}$		$\frac{5}{2l^2}$
	$\frac{10}{l^2}$		$\frac{45}{14l^2}$
	$\frac{21}{l^2}$		$\frac{21}{l^2}$

Table 1 : Laminated glass beams under different boundary and load conditions; values of coefficient  $\Psi$  for different boundary and load conditions.

For the sake of comparison, in the present section, four paradigmatic cases are analyzed in detail. With the same notation of Figure 2, assumed geometrical and structural parameters are l = 3150 mm, b = 1000 mm,  $h_1 = h_2 = 10$  mm, t = 0.76 mm, E = 70 GPa, while the shear modulus *G* of the polymeric interlayer is varied to evaluate its influence on the shear-coupling of the glass plies. The distributed pressure on the beam is taken equal to 0.75 kN/m<sup>2</sup> so that, with b = 1000 mm, the distributed load per unit length becomes p = 0.75N/m. For the case of concentrated force, we take F = 1 kN.

In the following graphs, the stress- and deflection-effective thicknesses, calculated through (12) and (13), are plotted as function of *G* with a continuous line, whereas the effective thicknesses

calculated with the Wölfel-Bennison's is represented with a dashed curve. Results of numerical experiments are indicated with dots.

The cases considered here are: *i*) simply supported beam under uniformly distributed load (

Figure 4); *ii*) simply supported beam under concentrated load (Figure 5); *iii*) beam with three supports under distributed load (Figure 6); *iv*) double clamped beam under uniformly distributed load (Figure 7). For case ii), the value  $\beta = 12$ , recorded in the original Wölfel paper (Wölfel 1987) has been used.



Figure 4: Simply supported beam under uniform load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.



Figure 5: Simply supported beam under concentrated load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach with  $\beta = 12$ ; the enhanced effective thickness (EET) approach; the numerical simulations.



Figure 6: Beam with three supports under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.



Figure 7: Double clamped beam under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.

In the case of simply supported beams under uniform load the models give results that in practice coincide, a finding that is not surprising because this is the simplest case upon which the Wölfel approach is calibrated. Numerical results confirm the good approximation that is achieved. Also for the case of simply supported beam under concentrated load, the two approaches give results that practically coincide. However, it is evident from Figure 5 that the agreement with the numerical simulations is good for the deflection effective thickness, whereas the stress-effective thickness is qualitatively different, especially in those branches close to the monolithic limit.

In the case of beam with three supports and of clamped beam, there is a substantial deviation between the EET and W-B approaches especially for the lowest values of G, but the numerical experiments are in favor of the EET approach. Observe that W-B is not on the side of safeness, because it predicts effective thicknesses greater than in reality and, consequently, underestimates deflection and stress.

#### 3.2. Two-dimensional examples. Plates under various constraint and load conditions

In the present section, several cases of practical importance for rectangular plates are analysed. Apart from uniformly distributed pressure, we have also considered the action of a (pseudo-) concentrated load whose imprint, according to the indication of most structural standards. is supposed to be a 100 mm  $\times$  100 mm square.

Tables 2.1 and 2.2 collect values of the coefficient  $\Psi \text{ [mm}^{-2} \cdot 10^6 \text{]}$  that are necessary to evaluate  $\eta$  as *per* (17), calculated according to equation (18) as a function of the plate length *a* [mm] and of the aspect ratio  $\lambda = b/a$ .

The shape functions g(x,y) for w(x,y), introduced in (15), can be found in (Timoshenko 1970) and (Batista 2010) in the form of trigonometric and hyperbolic series. In the calculation of  $\Psi$  as per (18), we have considered only the first term in the series (first order approximation) for the cases in which the load is distributed; it can be directly verified that higher order approximations, obtained by considering more terms of the series, do not substantially increase the level of accuracy. On the other hand, when the plate is loaded on a small area (pseudo-concentrated load), the use of higher-order terms of the series increases notably the precision of the deflection- and stress-effective thickness. In Table 2.1, the values of  $\Psi$  for plates under pseudo-concentrated load have been obtained by using a third order approximation.

It should also be remarked that for the case of plates with one edge built in, the deformed shape under a uniformly distributed load is cylindrical in type and, consequently, the coefficient  $\Psi$ ,

and hence the coefficient  $\eta$  and the deflection- and stress-effective thickness, turns out to be independent upon the width *b*.

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¢		λ=b/a 3[mm] 500 600 800 1000 1500	0.14918,343370,581887,991208,10536,820	0.2         1232,69         856,090         481,156         307,557         136,349	0.3           567,067           393,202           220,499           140,793           62,3643	0.4 334,9 232,7 130,7 83,20 36,93	4         0           994         230           176         160           284         90,           878         58,           883         26,	0.5 0,168 0,114 4864 1740 0733	<b>0.6</b> 177,77 125,08 71,980 46,872 21,410	0.7           153,44           10,41           4           65,593           18           43,589           17	0.8           40         148,95           36         110,681           37         68,375           35         46,423           25         22,343	0.9           8         162,5088           13         124,4915           0         79,1656           8         54,3862           0         26,4174	1           193,6331           150,3770           96,0054           65,7299           31,62618		
4		3[mm] 500 600 800 1000 1500 2000	0.1 4918,34 3370,58 1887,99 1208,10 536,820 301,823	0.2         1232,69         856,090         481,156         307,557         136,349         76,5778	0.3 567,067 393,202 220,499 140,793 62,3643 35,0205	0.4 334,9 232,7 130,7 83,22 36,92 20,8	4         0           994         230           176         160           284         90,           878         58,           883         26,           068         14,	<b>).5</b> ),168 ),114 4864 1740 0733 7442	<b>0.6</b> 177,77 125,08 71,980 46,872 21,410 12,230	0.7           153,44           110,41           4           65,553           8           43,583           9           11,846	0.8           40         148,95           36         110,681           37         68,375           95         46,423           25         22,343           87         13,0731	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           .6         15,48895	<b>1</b> 3 193,6331 5 150,3770 96,0054 65,7299 31,62618 5 18,42744		
ţ		x=b/a 500 600 800 1000 1500 2000 2500	0.1         4918,34         3370,58         1887,99         1208,10         536,820         301,823         193,088	0.2           1232,69           856,090           481,156           307,557           136,349           76,5778           48,9613	0.3 567,067 220,499 140,793 62,3643 35,0205 22,3909	0.4 334,1 232,1 130,1 83,2 36,9 20,8 13,3	4         0           994         230           176         160           284         90,           878         58,           883         26,           068         14,           188         9,4	<b>)</b> ,168 <b>)</b> ,114 <b>4</b> 864 <b>1740 0733 7442 6</b> 975	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047	0.7           153,44           110,41           4           65,593           8           43,588           77           20,462           11,846           9           7,7168	0.8           40         148,95           36         110,681           37         68,375           35         46,423           325         22,343           87         13,0731           36         8,5644	0.9           8         162,5088           3         124,4919           0         79,1656           8         54,3862           0         26,4174           .6         15,48895           3         10,15214	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886		
ţ		a[mm] 500 600 800 1000 1500 2500 3000	0.1         4918,34         3370,58         1887,99         1208,100         536,820         301,823         193,088         134,046	0.2 1232,69 856,090 481,156 307,557 136,349 76,5778 48,9613 33,9778	0.3 567,067 220,499 140,793 62,3643 35,0205 22,3909 15,5392	0.4 334,9 232,7 130,7 83,20 36,99 20,80 13,3 9,25	4         0           994         230           176         160           284         90,           878         58,           883         26,           068         14,           188         9,4           103         6,5	<b>1.5</b> 0,168       0,114       4864       1740       0733       7442       6975       9278	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265	0.7           153,44           10,41           4           65,593           8           43,588           7           20,462           9           11,846           9           7,7168           1           5,4230	0.8           40         148,95           36         110,681           37         68,375           35         46,423           25         22,343           87         13,0731           36         8,5644           30         6,0398	0.9           8         162,5088           3         124,4919           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939		
		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm]	0.1 4918,34 3370,58 1887,99 1208,10 536,820 301,823 193,088 134,046 0.2		0.3 567,067 393,202 220,499 140,793 62,3643 35,0205 22,3909 15,5392 0.6	0.4 334,9 232,1 130,1 83,2 36,9 20,8 13,3 9,25	4         0           994         230           176         160           284         90,           878         58,           883         26,           068         14,           188         9,4           103         6,5	0,168 0,114 4864 1740 0733 7442 6975 9278	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265	0.7           153,44           110,41           4           65,593           8           43,589           9           11,846           9           7,7169           1           5,4230           1.25	0.8           148,95           36           110,681           37           68,375           35           46,423           25           22,343           87           36           37,0731           36           8,5644           30           6,0398           1.667	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2.5         2.5	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           5		
ţ		a[mm] 500 600 800 1000 1500 2000 2500 3000 3000 500	0.1       4918,34       3370,58       1887,99       1208,100       536,820       301,823       193,088       134,046       0.2       1232,65	0.2           1232,69           856,090           307,557           136,349           76,5778           33,9778           0.4           0.5           33,9778           0.4           33,9778	0.3       567,067       393,202       220,499       140,793       62,3643       35,0205       22,3909       15,5392       0.6       4	0.4 334,9 232,7 130,7 83,2 36,9 20,8 13,3 9,25	4         0           994         230           176         160           284         90,           878         58,           883         26,6           068         14,           1188         9,4           1103         6,5           0.8         148, 958	0,168 0,114 4864 1740 0733 7442 6975 9278 193,	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265 L 6331 3	0.7           2         153,4/           2         110,41           4         65,593           8         43,583           7         20,463           9         7,7163           10         5,4233           11         5,4233           11,1367         11,1367	0.8           148,95           36           110,681           37           68,375           95           46,423           25           27,343           87           387           395           46,423           25           26,343           87           13,0731           96           8,5644           91           6,0398 <b>1.667</b> 394,6583	0.9           8         162,5088           3         124,4919           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2.5         387,8492	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           571,0113		
		a[mm] 500 600 800 1000 1500 2000 2500 3000 3000 500 600	0.1       4918,34       370,58       1887,99       1208,100       536,820       301,823       193,088       134,046       0.2       1232,65       856,090	0.2           1232,69           856,090           481,156           307,557           136,349           76,5778           48,9613           33,9778           0.4           334,99           232,17	0.3       567,067       393,202       220,499       140,793       62,3643       35,0205       22,3909       15,5392       0.6       4     177,7       6     125,0	0.         334,9         232,1         130,1         83,20         36,90         20,80         13,31         9,251         772         12         772         12         982	4 0 994 230 176 160 284 90, 878 58, 883 26, 068 14, 188 9,4 103 6,5 0.8 0.8 148,958 110,6813	0,168 0,114 4864 1740 0733 7442 6975 9278 193, 150,	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265 L 6331 3 3770 2	0.7           2         153,4/           2         110,41           4         65,593           8         43,583           7         20,463           9         7,7166           9         7,7166           1         5,4230           11,1367         30,8654	0.8           148,95           36           110,681           37           68,375           95           46,423           25           2,343           87           3,0731           96           8,5644           01           6,0398           1.667           394,6583           276,0053	0.9           8         162,5088           3         124,4919           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2.5         387,8492           269,3501	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,6467		
		a[mm]         500           500         600           800         1000           1500         2000           2500         3000           a[mm]         b/a           500         600           800         800	0.1       4918,34       370,58       1887,99       1208,100       536,820       301,823       193,088       134,046       0.2       1232,65       856,090       481,156	0.2           1232,69           856,090           481,156           307,557           136,349           76,5778           48,9613           33,9778           0.4           334,99           232,17           130,28	0.3       567,067       393,202       220,499       140,793       62,3643       35,0205       22,3909       15,5392       4     177,7       6     125,0       4     71,98	0.4         334,9         232,7         130,7         83,23         36,93         20,86         13,37         9,257         772         882         1304	4 0 994 23C 176 16C 284 90, 878 58, 883 26, 068 14, 188 9,4 103 6,5 0.8 0.8 148,958 1148,958 110,5813 68,3750	0,168 0,114 4864 1740 0733 7442 6975 9278 193, 150, 96,0	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265 1 6331 3 3770 2 0054 1	0.7           2         153,4/           2         110,41           4         65,593           8         43,583           7         20,462           9         11,846           9         7,7163           1         5,4233           1.25         11,1367           30,8654         38,2211	0.8           148,95           36           110,681           37           68,375           95           46,423           25           2,343           87           13,0731           96           8,5644           01           6,0398           1.667           394,6583           276,0053           156,1950	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2.5         387,8492           269,3501         151,5144	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,6467           144,9263		
		a[mm]         500           500         600           800         1000           1500         2000           2500         3000           3000         500           600         800           1000         1000	0.1 4918,34 370,58 1887,99 1208,10 536,820 301,823 193,088 134,046 0.2 1232,65 856,090 481,156 307,557	0.2           1232,69           856,090           481,156           307,557           136,349           76,5778           48,9613           33,9778           0.4           334,99           232,17           130,287           130,287	Joint Control       567,067       393,202       220,499       140,793       62,3643       35,025       22,3693       15,532       4       177,7       6       125,024       4       71,98       8	0.4         334,9         232,7         130,7         83,23         36,9         20,80         13,3         9,25         772         1304         604         6228	4         0           994         230           176         160           284         90,           878         58,           883         26,           0068         14,           1108         9,4           1003         6,5           0.8         148, 958           1148, 958         106,5813           68,3750         46,4238	.5       0,168       0,114       4864       1740       0733       7442       6975       9278       193,       150,       96,0       65,7	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265 L 6331 3 3770 2 054 1 7299 5	0.7           2         153,4/           2         110,41           4         65,59:           8         43,58:           7         20,462           9         11,846           9         7,716:           1         5,4230           1.25         11,1367           30,8654         38,2211           91,0467         20,467	0.8           148,95           36           110,681           37           68,375           95           46,423           25           22,343           87           13,0731           96           8,5644           01           6,0398           1.667           394,6583           276,0053           156,1950           100,2270	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2.5         387,8492           269,3501         151,5144           96,9706         1	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,64677           144,9263           92,7528		
4 4		a[mm] 500 600 800 1000 1500 2000 2500 3000 2500 3000 600 800 1000 1500	0.1 4918,34 370,58 1887,99 1208,10 536,820 301,823 193,088 134,046 0.2 1232,65 856,090 481,156 307,557 136,345	0.2           1232,69           ≥5,090           4¥1,156           307,557           136,349           76,5778           4&,9613           3,9778           0           33,9778           30,32,17           130,28           130,28           33,978	Joint Control       567,067       393,202       220,499       140,793       62,3643       35,025       22,3909       15,532       4       177,7       6       125,0       4       4       71,98       8     46,87       3     21,41	0.           334,           232,           130,           83,22           36,92           20,80           13,33           9,25           5           6           772           2082           1004           6           728           20,07	4         0           994         230           176         160           284         90,           878         58,           883         26,           0068         14,           1108         9,4           1003         6,5           0.8         148, 958           1106,5813         68,3750           46,4238         22,3430	>.5       >,168       >,114       4864       1740       0733       7442       6975       >278       193,       150,       96, (, 65, 7, 31), (, 65, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 7, 13), (, 66, 13), (, 66, 14), (, 14)	0.6 177,77 125,08 71,980 46,872 21,410 12,230 7,9047 5,5265 1 6331 3 3770 2 054 1 7299 9 5262 4	0.7           2         153,4/           2         110,41           4         65,59:           8         43,58:           7         20,462           99         11,846           99         7,716:           11         5,4230           11.1367         30,8654           38,2211         91,0467           91,0467         41,7225	0.8           148,95           36           110,681           37           68,375           95           46,423           25           22,343           87           13,0731           96           8,5644           91           6,0398           1.667           394,6583           276,0053           156,1950           100,2270           44,66439	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2         2.5           3         87,8492           269,3501         151,5144           96,9706         43,09864	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,6467           144,9263           92,7528           41,22347		
t t		a[mm] 500 600 800 1000 1500 2000 2500 3000 2500 3000 600 800 1000 1500 2000	0.1       4918,34       370,58       1887,99       1208,10       536,820       301,823       193,088       134,046       0.2       1232,69       856,090       481,156       307,557       136,349       76,5778	0.2           1232,69           ≥5,090           4×1,156           307,557           136,349           76,5778           4&,9613           3J,9778           0           334,99           232,17           130,28           130,28           336,988           20,808	Joint Control       567,067       393,202       220,499       140,793       62,3643       35,025       22,399       15,532       4       177,7       6       125,02       4       171,73       6       4       171,73       6       125,02       4       171,73       6       125,02       4       121,13       8       46,87       3       21,41       8       12,23	0           334, !.           232, !.           130,           83, 21           36, 91           20, 83           13, 33           9, 25           100, 100           120, 200           120, 200           120, 200           130, 33           9, 25           100           100           100           100           100           100           100           100	4         0           994         230           176         160           284         90,           878         58,           883         26,           0068         14,           1108         9,44           1003         6,55           0.8         1           148,958         1           100,6813         3           68,3750         46,4238           22,3430         13,07316	1,168 ),114 4864 1740 0733 7442 6975 9278 193, 150, 96,€ 65,1, 65,1, 18,4	0.6       177,77       125,08       71,980       46,872       21,410       12,230       7,9047       5,5265       1       6331       3770       2054       17299       5262       2744	0.7           2         153,4/           2         110,41           4         65,59           8         43,589           7         20,462           99         11,846           99         7,7169           11,1367         30,8654           33,2211         91,0467           91,0467         41,7225           3,76545	0.8           148,95           36         110,681           37         68,375           95         46,423           25         22,343           87         13,0731           36         8,5644           31         6,0398           1.667         394,6583           276,0053         156,1950           100,2270         44,66439           25,15044         150,104	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           2         5387,8492           269,3501         151,5144           96,9706         43,09864           24,24312         24312	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,6467           144,9263           92,7528           41,22347           23,18820		
		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] <sup>2</sup> =b/a 500 600 800 1000 1500 2000 2500 2500	0.1       4918,34       3370,58       1887,99       1208,10       536,820       301,823       193,088       134,046       0.2       481,156       307,557       136,349       76,5778       48,9613	0.2           1232,69           ≥5,090           4×1,156           307,557           136,349           76,5778           4&9613           3,9778           0           334,99           232,17           130,28           33,9778           232,17           334,99           232,17           334,99           232,17           334,99           232,17           334,99           36,988           20,806           36,988           20,806           313,318	Joint Control       567,067       393,202       220,499       140,793       62,3643       35,025       22,399       15,532       4       177,7       6       125,02       4       177,7       6       125,02       4       14,198       8       4,687       3       21,411       8       12,238       70,044	0           334,           334,           232,           130,           83, 21           36, 91           20, 88           9, 255           0           0           772           130,           0	4         0           994         230           176         160           284         90,           878         58,           883         26,           0068         14,           1138         9,44           103         6,55           0.8         1           148,958         1           100,68         13           68,37         50           46,42         38           22,3430         13,07316           8,56443         443	>.5       ),168       ),114       4864       1740       0733       7442       6975       9278       193,       150,       96,(       31,6       18,4       12,0	0.6         177,77         125,08         71,980         46,872         21,410         12,230         7,9047         5,5255         1         6331         3770         2         054         17299         2262         22744         2886	0.7           2         153,4/           2         110,41           4         65,59:           8         43,58:           7         20,46:           9         11,846           9         7,716:           1         5,4230           1.25         11,1367           30,8654         38,2211           91,0467         41,7225           3,76545         5,31348	0.8           148,95           36           110,681           37           68,375           95           46,423           25           26           37           387           13,0731           36           8,5644           394,6583           276,0053           156,1950           100,2270           44,66439           25,15044           16,10541	0.9           8         162,5088           3         124,4915           0         79,1656           8         54,3862           0         26,4174           6         15,48895           3         10,15214           1         7,15987           269,3501         151,5144           96,9706         43,09864           24,24312         15,51565	1           193,6331           150,3770           96,0054           65,7299           31,62618           18,42744           12,02886           8,45939           371,0113           257,6467           144,9263           92,7528           41,22347           23,18820           14,84045		

Table 2.1: Representative examples of laminated glass plates: values of coefficient  $\Psi[mm^{-2} \cdot 10^6]$  for different load and boundary conditions.

		a[mm]	0.2	0.4	0.6		0.8	1	1.	25	1.667	2.5	5
	<del>* * * * *</del>	500	59,8176	55,7770	51,67	62 4	18,5518	46,3972	44,6	6462	42,9996	41,5644	41,6559
		600	41,5400	38,7340	35,88	62 3	33,7165	32,2203	31,0	0043	29,8609	28,8641	28,5614
	'i ¦ ⊺	800	23,3663	21,7879	9 20,18	60 1	L8,9655	18,1239	17,4	1399	16,7967	16,2361	15,6021
	a	1000	14,9544	13,9442	2 12,91	.90 1	12,1379	11,5993	11,1	1615	10,7499	10,3911	10,4140
→	6 9	1500	6,64640	6,19744	4 5,741	.80 5	5,39464	5,15525	4,96	5069	4,77774	4,61826	4,41235
	<u>i!</u> ↓	2000	3,73860	3,48606	5 3,229	76 3	3,03449	2,89983	2,79	9039	2,68748	2,59777	2,60350
	<mark>∢ b</mark> →	2500	2,39270	2,23108	3 2,067	05 1	L,94207	1,85589	1,78	3585	1,71999	1,66257	1,70853
		3000	1,66160	1,54936	5 1,435	45 1	L,34866	1,28881	1,24	1017	1,19443	1,15457	1,10309
		a[mm] λ=b/a	0.2	0.4	0.6		0.8	1	1.	25	1.667	2.5	5
	<del>* * * * *</del>	500	42,3350	39,0757	7 38,15	57 3	37,7017	37,4193	37,1	1767	36,8990	36,5295	35,7883
r•1		600	29,3993	27,1359	26,49	70 2	26,1818	25,9856	25,8	3171	25,6243	25,3677	24,8530
	! i	800	16,5371	15,2639	9 14,90	46 1	14,7272	14,6169	14,5	5221	14,4137	14,2693	13,9798
	la	1000	10,5837	9,76893	3 9,538	93 9	9,42544	9,35481	9,29	9,29417 9,	9,22476	9,13236	8,94708
	li !!!	1500	4,7039	4,34175	5 4,239	53 4	1,18908	4,15769	4,13074	3074	4,09989	4,05883	3,97648
	<b>└</b>	2000	2,6459	2,44223	3 2,384	73 2	2,35636	2,33870	2,32	2354	2,30619	2,28309	2,23677
	<mark>∢ b</mark> →	2500	1,6934	1,56303	3 1,526	23 1	L,50807	1,49677	1,48	3707 :	1,47596	1,46118	1,43153
		3000	1,1760	1,08544	1,059	88 1	L,04727	1,03942	1,03	3269	1,02497	1,01471	0,99412
		a[mm] λ=b/a	0.1	0.2	0.3	0.4	4 0	.5 0	.6	0.7	0.8	0.9	1
	¥ ¥ ¥ ¥ ¥	a[mm] 500	<b>0.1</b> 61,1205	<b>0.2</b> 59,8176	<b>0.3</b> 57,9303	<b>0.4</b> 55,77	<b>4 0</b> 770 53,6	. <b>5 0</b>	. <b>6</b> 5762 4	<b>0.7</b> 49,9790	<b>0.8</b> 0 48,551	<b>0.9</b> 8 47,3700	<b>1</b> 46,3972
		a[mm] <b>500</b> <b>600</b>	<b>0.1</b> 61,1205 42,4448	0.259,817641,5400	<b>0.3</b> 57,9303 40,2294	<b>0.4</b> 55,77 38,73	4         0           770         53,6           340         37,2	.5 0 5340 51,0 2459 35,3	.6 5762 4 3862 3	<b>0.7</b> 49,9790 34,7077	0.8           0         48,5513           7         33,7163	0.9           8         47,3700           5         32,8958	<b>1</b> 46,3972 32,2203
E		a[mm] 500 600 800	0.161,120542,444823,8752	0.2       59,8176       41,5400       23,3663	<b>0.3</b> 57,9303 40,2294 22,6290	<b>0.4</b> 55,77 38,73 21,78	4         0           770         53,6           340         37,2           879         20,9	.5 0 5340 51, 2459 35, 5508 20,	.6 5762 4 3862 3 1860 3	<b>0.7</b> 49,9790 34,7077 19,5231	0.8           0         48,5513           7         33,7163           1         18,9653	0.9           8         47,3700           5         32,8958           5         18,5039	146,397232,220318,1239
* * *		a[mm] 500 600 800 1000	0.161,120542,444823,875215,2801	0.2       59,8176       41,5400       23,3663       14,9544	<b>0.3</b> 57,9303 40,2294 22,6290 14,4826	<b>0.4</b> 55,77 38,73 21,78 13,94	4         0           770         53,6           340         37,2           879         20,9           442         13,4	.5 0 5340 51, 2459 35, 508 20, 1085 12,	.6 5762 4 3862 5 1860 5 9190 5	<b>0.7</b> 49,9790 34,7077 19,5231 12,4948	0.8           0         48,551           7         33,716           1         18,965           3         12,137	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425	146,397232,220318,123911,5993
* * * * *	a a	λ=b/a a[mm] 500 600 800 1000 1500	0.161,120542,444823,875215,28016,79116	0.259,817641,540023,366314,95446,64640	<b>0.3</b> 57,9303 40,2294 22,6290 14,4826 6,43670	0.4 55,77 38,73 21,78 13,94 6,197	4         0           770         53,6           340         37,2           879         20,9           442         13,4           744         5,95	.5         0           6340         51,1           2459         35,1           9508         20,1           1085         12,5           6934         5,7	.6 5762 4 3862 3 1860 3 9190 3 4180 4	<b>0.7</b> 49,9790 34,7077 19,5231 12,4948 5,55322	0.8           0         48,5513           7         33,7163           1         18,9653           3         12,1373           2         5,39464	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425           4         5,26333	146,397232,220318,123911,59935,15525
*****		a[mm] 500 600 800 1000 1500 2000	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860	0.3         57,9303         40,2294         22,6290         14,4826         6,43670         3,62064	0.4 55,77 38,73 21,78 13,94 6,197 3,486	4         0           770         53,6           340         37,2           879         20,9           442         13,4           744         5,95           606         3,35	.5         0           6340         51,1           2459         35,1           9508         20,1           4085         12,2           5934         5,7           5213         3,2	.6 5762 3862 1860 29190 4180 2976	<b>0.7</b> 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369	0.8           48,5512           7         33,7162           1         18,9652           3         12,1372           2         5,39462           3         3,03442	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425           4         5,26333           9         2,96062	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983
*****		a[mm] 500 600 800 1000 1500 2000 2500	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003       2,44482	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270	0.3         57,9303         40,2294         22,6290         14,4826         6,43670         3,62064         2,31721	0.4 55,77 38,73 21,78 13,94 6,197 3,486 2,231	4         0           770         53,6           340         37,2           379         20,9           442         13,4           744         5,95           506         3,35           108         2,14	.5         0           6340         51,1           2459         35,1           9508         20,1           4085         12,1           6934         5,74           5213         3,22           4536         2,00	.6           5762           3862           1860           1910           4180           2976           5705	<b>0.7</b> 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369 1,99916	0.8           48,551           33,716           18,965           12,137           5,3946           3,0344           19,420	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425           4         5,26333           9         2,96062           7         1,89480	1 46,3972 32,2203 18,1239 11,5993 5,15525 2,89983 1,85589
*****		a[mm] 500 600 800 1000 1500 2000 2500 3000	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003       2,44482       1,69779	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160	0.3         57,9303         40,2294         22,6290         14,4826         6,43670         3,62064         2,31721         1,60918	0.4 55,77 38,73 21,78 13,94 6,197 3,486 2,231 1,549	4         0           770         53,6           340         37,2           379         20,9           442         13,4           744         5,95           506         3,35           108         2,14	.5         0           6340         51,0           2459         35,0           2508         20,0           4085         12,0           5934         5,70           5213         3,22           4536         2,00           4538         2,00           4538         2,00           4538         1,40	.6       5762       3862       1860       2190       180       2976       5705       3545	0.7 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369 1,99916 1,38831	0.8           48,551           33,716           18,965           12,137           2,5,3946           3,0344           5,1,9420           1,9420           1,3486	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425           4         5,26333           9         2,96062           7         1,89480           6         1,31583	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881
*****		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] <sup>λ=b/a</sup>	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003       2,44482       1,69779       0.2	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160       0.4	0.3 57,9303 40,2294 22,6290 14,4826 6,43670 3,62064 2,31721 1,60918 0.6	0.4 55,77 38,73 21,78 13,94 6,197 3,486 2,231 1,549	0           770         53,6           340         37,2           379         20,9           442         13,4           744         5,99           506         3,39           108         2,14           336         1,48	.5         0           6340         51,1           2459         35,3           3508         20,1           4085         12,1           6934         5,73           5213         3,22           4536         2,00           3983         1,43	.6 5762 4 3862 5 1860 5 19190 5 1	0.7 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369 1,99916 1,38831 25	0.8         48,551         33,716         18,965         12,137         2         5,3946         3,0344         5         1,9420         1,3486	0.9           8         47,3700           5         32,8958           5         18,5039           9         11,8425           4         5,26333           9         2,96062           7         1,89480           6         1,31583 <b>2.5</b>	1 46,3972 32,2203 18,1239 11,5993 5,15525 2,89983 1,85589 1,28881 5
*****		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] 500	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003       2,44482       1,69779       0.2       325,393	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160       0.4       114,750	0.3       57,9303       40,2294       22,6290       14,4826       6,43670       3,62064       2,31721       1,60918       0.6       0     74,93	0.4       55,77       38,73       21,78       13,94       6,197       3,486       2,231       1,549       77       6	4         00           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           606         3,33           108         2,14           336         1,48           0.8         60,4550	5         0           6340         51,1           2459         35,2           8508         20,2           9508         12,7           6343         3,2           1535         2,00           1536         2,00           1538         2,00           1538         2,00           1538         2,00           1539         1,47           1535         5,55	.6 → 762 3862 3862 ↓180 ↓180 ↓180 ↓180 ↓2976 ↓ 5705 ↓ 3545 ↓ 48,9	0.7 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369 1,99916 1,38831 25 25	0.8           48,551:           33,716           18,965:           21,137:           22,5,3946:           3,0344:           5,19420:           1,3486           1,3486           1,53585	0.9           47,3700           32,8958           13,5039           11,8425           4           5,26333           2,96062           1,31583           2,5           4,325	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           5,759068
*****		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] 500 600	0.1       61,1205       42,4448       23,8752       15,2801       6,79116       3,82003       2,44482       1,69779       0.2       325,393       225,968	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160       0.4       114,750       79,6875	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,62064           2,31721           1,60918           0           74,93           5           52,044	0.4       55,77       38,73       21,78       13,94       6,197       3,486       2,231       1,549       77       6       00     4	4         00           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           506         3,35           108         2,14           936         1,48           936         1,48           60,4550         4,198           \$41,9827         1	5         0           5340         51,1           2459         35,2           9508         20,           9508         12,7           5034         5,7           5213         3,2           1536         2,00           8983         1,4           53,5051         3,7,1563	.6 5762 4 3862 5 1860 5 190 5 4180 5 5705 5 5705 5 5705 5 1. 48,9 33,9	0.7 49,9790 34,7077 19,5231 12,4948 5,55322 3,12369 1,99916 1,38831 25 9526 4 9949 1	0.8           2         48,551:           7         33,716           1         18,965:           3         12,137:           2         5,3946           2         3,0344:           5         1,9420           1         1,3486           1.3486         1,3486           45,3585         31,4989	0.9           47,3700           32,8958           13,5039           11,8425           4           5,26333           9           2,96062           7           1,89480           6           1,31583           2.5           42,6990           29,6521	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           5,79068           32,7445
		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] 500 600 800	0.1         61,1205         42,4448         23,8752         15,2801         6,79116         3,82003         2,44482         1,69779         0.2         325,393         225,968         127,1068	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160       0.4       114,750       79,6875       3       41,742	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,62064           2,31721           1,60918           0           74,93           5           52,044           2           2           2           2           2           3           6           2           2           2           2           2           2           2           2           2	0.4       55, 77       38, 73       21, 78       13, 94       6, 197       3, 48       2, 231       1, 549       77     6       00     4       25     2	4         00           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           506         3,35           108         2,14           936         1,48           60,4550         4,48           11,9827         23,6153	.5 0 G340 51, 4450 35, 508 20, 1085 12, 5034 5,7 513 3,2 1,536 2,00 3988 1,4 53,5051 37,1563 20,904	.6 5762 4 3862 5 1860 5 19190 5 4180 9 2976 5 5705 5 3545 5 1.1. 48,9 33,9 19,1	0.7       49,9790       34,7077       19,5231       12,4948       5,55322       3,1236       1,99916       1,38831       25       9526       49949       1221	0.8           48,551:           33,716           18,965:           2,53946           3,0344:           5,19420           1,3486           1,3486           45,3585           31,4989           17,7182	0.9           47,3700           32,8958           18,5039           11,8425           4           5,26333           9           2,96062           7           1,89480           6           4,2,6990           29,6521           16,6793	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           37,9068           32,7445           19,9520
****		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] 500 600 800 1000	0.1         61,1205         42,4448         23,8752         15,2801         6,79116         3,82003         2,44482         1,69779         0.2         325,393         225,968         127,1068         81,3484	0.2       59,8176       41,5400       23,3663       14,9544       6,64640       3,73860       2,39270       1,66160       0.4       114,750       79,6875       44,8242       28,6874	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,52064           2,31721           1,60918           0         74,93           5         52,044           2         29,279           18,7346	0.4       55,77       38,73       21,78       13,94       6,197       3,486       2,231       1,549       77       600       425       2442	4         0           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           506         3,35           108         2,14           936         1,48           0.8         50,4550           11,9827         23,6153           5,11376         5	.5 0 G340 51, 4450 35, 508 20, 1085 12, 9394 5,7 513 3,2 1536 2,0 3988 1,4 53,5051 37,1563 20,9004 13,37€2	.6 5762 4 3862 5 1860 5 190 5 4180 9 2976 5 33545 5 19,1 48,9 33,9 19,1 12,2	0.7         49,9790         34,7077         19,5231         12,4948         5,55322         3,12369         1,99916         1,38831         25         9526         49949         1221         33816	0.8           3         48,551:           3         33,716:           1         18,965:           3         12,137:           2         5,3946:           3         3,0344:           5         1,9420:           1         1,3486:           1.3486:         1,3486:           1.4667         1,3486:           1.7,7182         1,33962	0.9           47,3700           32,8958           18,5039           11,8425           5,26333           2,96062           1,89480           6           1,31583           2,6990           29,6521           16,6793           10,67475	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           37,9068           32,7445           19,9520           9,47670
*****		a[mm] 500 600 800 1000 1500 2000 2500 3000 3000 a[mm] 500 600 800 1000 1500	0.1         61,1205         42,4448         23,8752         15,2801         6,79116         3,82003         2,44482         1,69779         0.2         325,393         225,968         127,1068         81,3484         36,1548	0.2           59,8176         4           41,5400         4           23,3663         4           41,544         4           6,64640         4           3,73860         4           2,39270         4           1,66160         4           114,750         4           414,750         4           414,750         4           414,750         4           44,8242         4           44,8242         4           412,749         4	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,52064           2,31721           1,60918           0         74,93           5         52,044           2         29,27           9         18,734           9         8,326	55,77       38,73       21,78       13,94       6,17       3,48       2,231       1,549       77     6       000     4       25     2       41     6	4         0           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           506         3,35           108         2,14           936         1,48           0.8         -           50,4550         -           41,9827         23,6153           5,11.376         -           5,71723         -	5         0           6340         51,           6340         51,           6450         35,           6508         20,           6343         12,           6344         5,7           6213         3,2           1536         2,0           6383         1,4           53,5         5051           37,1         563           20,9004         13,3           13,3         762           5,9,4         500	.6 5762 4 3862 : 1860 2 190 : 4180 2 2976 : 5705 : 33545 : 48,9 48,9 33,9 19,1 48,9 33,9 19,1 12,2 5,43	0.7       49,9790       34,7077       19,5231       12,4948       5,55322       3,12369       1,38831       1,38831       25       9526       49949       3816       13816       3816	0.8           2         48,551;           7         33,716;           1         18,965;           3         12,137;           2         5,3946;           3         3,0344;           5         1,9420;           1         1,3486;           1,3486;         1,3486;           1,4989;         17,7182;           1,33962;         5,03983;	0.9           47,3700           32,8958           18,5039           11,8425           5,26333           2,96062           1,89480           6           1,31583           2,6990           29,6521           16,6793           10,67475           4,74433	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           37,9068           32,7445           19,9520           9,47670           4,02073
******		a[mm] 500 600 800 1000 1500 2000 2500 3000 3000 a[mm] 500 600 800 1000 1500 2000	0.1         61,1205         42,4448         23,8752         15,2801         6,79116         3,82003         2,44482         1,69779         0.2         325,393         225,968         127,1068         81,3484         36,1548         20,3371	0.2           59,8176         4           41,5400         4           23,3663         4           41,9544         4           6,64640         4           3,73860         2           2,39270         1           1,66160         4           114,750         4           28,6874         4           12,7499         7,17187	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,52064           2,31721           1,60918           0         74,93           5         52,04           2         29,27           9         18,734           9         8,326           7         4,683	0.4 55,77 38,73 21,78 6,197 3,48€ 2,231 1,549 77 6 00 4 25 2 2 442 1 5 41 6 60 3	4         0           770         53,6           340         37,2           3879         20,9           442         13,4           744         5,99           506         3,39           108         2,14           936         1,48           0.8         -           50,4550         -           41,9827         23,6153           5,11.376         -           5,77723         3,77844	5         0           6340         51,1           6340         51,1           6450         35,0           6508         20,0           6340         12,1           6343         12,2           6343         5,7           6213         3,2           1536         2,00           6383         1,42           53,5051         3,7,1563           20,9004         13,3762           5,9,4501         3,3,4407	.6 5762 8862 1860 190 190 190 190 190 190 190 19	0.7         49,9790         34,7077         19,5231         12,4948         5,55322         3,12369         1,99916         1,38831         25         9526       4         9949       3         1221       3         3816       1         3918       9         5954       2	0.8           2         48,551:           7         33,716:           1         18,965:           3         12,137:           2         5,3946:           3         3,0344:           5         1,9420:           1         1,3486:           1.3486:         1,3486:           1.4667         1,3486:           1.7,7182         1,33962:           5,03983         2,83490	0.9           47,3700           32,8958           18,5039           11,8425           4           5,26333           9           2,96062           7           1,89480           6           1,31583           2,66990           29,6521           16,6793           10,67475           4,74433           2,66869	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           37,9068           32,7445           19,9520           9,47670           4,02073           2,36918
******		a[mm] 500 600 800 1000 1500 2000 2500 3000 a[mm] 2=b/a 500 600 800 1000 1500 2000 2500 2500	0.1           61,1205           42,4448           23,8752           15,2801           6,79116           3,82003           2,44482           1,69779           0.2           325,393           225,968           127,1063           81,3484           36,1548           20,3371           13,0157	0.2           59,8176         41,5400           23,3663         14,9544           6,64640         3,73860           2,39270         1,66160           1,66160         414,750           1,4,9544         2,39270           1,66160         44,8242           2,392,70         1,4,750           1,4,750         79,6875           3,44,8242         28,6874           12,7499         7,17187           4,59000         4,59000	0.3           57,9303           40,2294           22,6290           14,4826           6,43670           3,52064           2,31721           1,60918           0         74,93           5         52,04           2         29,27           9         18,734           9         8,326           7         4,683           0         2,997	0.44 55,777 38,73 3,344 6,137 3,348 2,231 1,549 7,7 6 0 0 4 2,23 1,549 2,231 2,2312 2,2312	•         •	5         0           6340         51,           6340         51,           6450         35,           6508         20,           6340         12,           6343         12,           6344         5,7           5213         3,2           1536         2,00           8983         1,42           53,5051         3,7,1563           20,9004         13,3762           5,9,4501         3,3,4407           3,3,4407         2,1402	.6 5762 8862 1860 190 190 190 190 190 190 190 19	0.7         49,9790         34,7077         19,5231         12,494         5,55322         3,12369         1,99916         1,38831         25         9929         33816         1         3918         9954         5954         5954	0.8           2         48,551:           7         33,716:           1         18,965:           3         12,137:           2         5,3946:           3         3,0344:           5         1,9420:           1         1,3486:           1.3486:         1,3486:           1.4667         1,3486:           1.4989         1,4989           1.7,732         1,33962           5,03833         2,83490           1,8434         1	0.9           47,3700           32,8958           18,5039           11,8425           4           5,26333           9           2,96062           7           1,89480           6           1,31583           2,66990           29,6521           16,6793           10,67475           4,74433           2,66869           1,70796	1           46,3972           32,2203           18,1239           11,5993           5,15525           2,89983           1,85589           1,28881           37,9068           32,7445           19,9520           9,47670           4,02073           2,36918           1,90781

Table 2.2: Further examples of laminated glass plates under various boundary and load conditions (same notation of Table 2.1). Value of the coefficient  $\Psi[mm^2 \cdot 10^6]$ .

It is important to note that:

- for plates with the same boundary and loading condition in x and y direction (for example plate supported on four sides) under a constant distributed load, the parameter a denotes the longer edge of the plate (note that, in such cases, Tables 2.1 and 2.2 give  $\lambda = b/a < 1$ );
- for plates with different boundary and loading condition in *x* and *y* direction (for example plate supported on two sides) under a constant distributed load, the identification of the edges is shown in the sketch of Tables 2.1 and 2.2; in such cases Tables give either λ < 1 or λ > 1.

For example, the value of coefficient  $\Psi$  for a plate of dimension 3000 mm x 1800 mm, supported on 3000 mm edge, can be found in table 2.2 by choosing *a*=3000 mm,  $\lambda$ =0.6.

In the sequel, we compare the deflection- and stress-effective thickness calculated according to the proposed EET approach through equations (12) and (13), with the ones calculated with the W-B formulas (3) and (4). Results are also validated by means of numerical analysis performed by the finite element software SJ-Mepla. Assumed structural parameters are the size of the plate a = 3000 mm and b = 2000 mm; the thicknesses of the glass plies  $h_1 = h_2 = 10$  mm; the thickness of the interlayer t = 0.76mm; the elastic parameters for glass E = 70 GPa and v = 0.22. The shear elastic modulus G of the polymeric interlayer is again varied between 0.01MPa and 10MPa. The distributed pressure on the plate is taken equal to  $0.75 \cdot 10^{-3}$  N/mm<sup>2</sup>.

The most frequent case in the design practice is certainly that of a rectangular plate with all the sides simply supported, subject either to a distributed or concentrated load. The graphs of Figure 8 compare the deflection- and stress-effective thickness calculated according to the EET and the W-B approaches to the results of the numerical experiments. It is very evident here that the two formulations give different results at the qualitative level. Again W-B is not on the side of safeness, because it underestimate deflection and stress.



Figure 8: Rectangular plate simply supported on four sides under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.

In such a case, the behavior predicted by the EET approach is close to Wölfel-Bennison's whenever the aspect ratio is such that plate response is similar to the response of a beam  $(\lambda = b/a \gg 1)$ . This is not surprising because the Wölfel-Bennison's model is calibrated on the case of simply supported beams under uniformly distributed load. On the contrary, the greatest differences between the EET and W-B approaches are obtained when the plate is square  $(\lambda = 1)$ , i.e., when the deflections of beam and plate differ the most. This is shown in Figure 9, where the percentage error on the evaluation of the deflection- and stress- effective thicknesses are plotted as a function of the aspect ratio  $\lambda$ .



Figure 9: Error on the evaluation of the deflection- and stress-effective thickness, for different plate lengths *a* and aspect ratio.

In the case of rectangular plates simply supported on four sides under a pseudo-concentrated load, the conclusions about the stress and deflection-effective thicknesses are similar to those for the case of uniformly distributed load. As mentioned above, under load conditions of this type, consideration of just the first-order approximation of the shape function g(x,y) does not give acceptable accuracy. This finding is evidenced in Figures 10 and 11, where a comparison is made between the effective thicknesses evaluated with either first-order approximation or third-order approximation. It is evident from the graphs that the use of third-order terms in the series improves the precision especially for what the calculation of deflection is concerned.

For such cases, the value  $\beta = 9.6$  proposed by Bennison has been used; the value  $\beta = 12$  recorded in the original work by Wölfel for a beam under concentrated load does not lead to better results.



Figure 10 Rectangular plate simply supported on four sides under pseudo-concentrated load acting at the centre of the plate. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach (1<sup>st</sup> and 3<sup>rd</sup> order accuracy); the numerical simulations.



Figure 11 Rectangular plate simply supported on four sides under concentrated load acting at the middle of one edge of the plate. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach (1<sup>st</sup> and 3<sup>rd</sup> order accuracy); the numerical simulations.

Figure 12 compares the EET and W-B results for the case of plate simply supported on three sides (with a free side of length b). In this condition both models give results in good agreement with the numerical experiments.



Figure 12: Rectangular plate simply supported on three sides under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the EET approach; the numerical simulations.

It should also be observed, as discussed at length in (Galuppi and Royer-Cafagni 2012b), that when the deformation of the plate tends to be cylindrical, so that its response is similar to that of a beam, the predictions of W-B and EET tend to coincide. This is the case of a plate simply supported on two opposite sides, to which Figure 13 refers to.



Figure 13: Rectangular plate simply supported on two sides under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the EET approach; the numerical simulations.

The case of rectangular plates point-wise supported at the corners does apply to frameless glazing. It is evident from Figure 14 that the EET and W-B give similar results, in agreement to numerical outcomes.



Figure 14: Rectangular plate simply supported at the four corners under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.

In the case of rectangular plate with two opposite edge simply supported, the third edge built in and the fourth edge free, it is evident from Figure 15 that the Enhanced Effective Thickness model and Wölfel-Bennison approach give substantially different results and that numerical experiments are in favor of EET.



Figure 15: Rectangular plate with two opposite edges simply supported and one edge built in, under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the enhanced effective thickness (EET) approach; the numerical simulations.

Figure 16 shows the comparison the EET and W-B results for the case of rectangular plate with one edge built-in. From this, it is evident that the EET and W-B now give substantially different

results; deflection-effective thickness calculated through EET approach is in agreement with the results of the numerical simulation. The evaluation of the stress-effective thickness is not so precise, because it is affected by stress intensification near the clamped edge.



Figure 16 Rectangular plate with one edge built in under distributed load. Comparison of the effective thicknesses obtained with: Wölfel-Bennison (WB) approach; the EET approach; the numerical simulations.

#### 4. Conclusions

One of the currently most-used simplified approaches for the structural design of laminated glass is that due to Bennison (2009), which is based upon the original work by Wölfel (1987). However, Wölfel's model was primarily conceived of for a sandwich beam with external plies with considerable axial stiffness but negligible bending stiffness and an intermediate layer that can only bear shear stress, with zero axial and flexural strength. Whenever the external layers present considerable bending stiffness, as in the case of laminated glass, Wölfel proposed a very approximate solution that in any case, as we have verified here, gives results in agreement with more accurate (numerical) methods of analysis for the only case in which the load is uniformly distributed and the deformed shape tends to be cylindrical, i.e. case of simply supported beams or rectangular plates simply supported on two opposite sides.

When the load is not uniformly distributed, the standard Wölfel – Bennison approach gives results that are not on the side of safeness. Better approximations can be achieved with the

Enhanced Effective Thickness approach. Here we have recorded the significant parameters necessary for a quick calculation of the effective thickness for the cases of most practical importance, which presents no additional difficulty with respect to the more traditional formulation.

In the two-dimensional case of plates, the results obtained with Wölfel-Bennison are accurate only when the plate is rectangular and simply supported on two opposite edges, i.e., when its deformed shape tends to be cylindrical and its response similar to that of a simply supported beam. When this is not the case, the Enhanced Effective Thickness method gives results that fit more closely the real situation both for the deflection and the stress calculation.

The EET method furnishes compact formulas also for the two-dimensional case and, remarkably, the most relevant expression (12) and (13) are analogous to those corresponding to the one dimensional case. The coupling offered by the interlayer can be readily evaluated by using the values of  $\Psi$  that have been tabulated here for all those cases that are relevant for the design practice. However, using (18), the value of  $\Psi$  can be calculated with no difficulty for any laminated plate under any load condition. The enhanced effective-thickness approach thus seems to represent an accurate and powerful tool for the practical calculation of laminated glass.

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