Too Much R&D?
– Vertical Differentiation in a Model of Monopolistic Competition

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Abstract

This paper discusses a model of vertical and horizontal product differentiation within the Dixit-Stiglitz framework of monopolistic competition. Firms do not only compete in prices and horizontal attributes of their products, but also in the quality that can be controlled by R&D activities. Based upon the results of a general equilibrium model, intra-sectoral trade and the welfare implications of public intervention in terms of research promotion are considered. The analysis is completed by a numerical application to ten basic European industries.

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JEL classifications: D43, F12, L13, L16

1 Introduction

The theoretical concept of monopolistic competition enjoys a great popularity since the seminal work of Dixit and Stiglitz (1977) – the beginning of the second monopolistic revolution, as it is referred to by Brakman and Heijdra (2004). Nowadays, this idea has penetrated different fields of research, not only Microeconomics, but also *Endogenous Growth Theory* or *New Trade Theory*. An essential attribute in models of monopolistic competition is product differentiation. Following the classification of Sutton (1991), Schmalensee (1992) distinguishes Type 1 and Type 2 industries. While a Type 1 is characterized by horizontally differentiated (or homogenous) products, Type 2 firms do not only compete in prices and horizontal product attributes, but also in (perceived) quality, what is described as vertical differentiation. In this context, quality is influenced by R&D (or advertising) expenditures, so that a firm may increase its market share by increasing the quality of its product.

Beyond the oligopolistic market structures and the game-theoretical perspective of this field of Industrial Organization, this paper aims to establish a basic model of vertical product differentiation for a further implementation into international trade. The main assumptions underlying this paper are referenced to the so-called *New Economic Geography* (NEG) that, initially introduced by Krugman (1991), aims to explain industrial agglomeration using the framework of monopolistic competition à la Dixit Stiglitz. The model here incorporates explicitly R&D activities, beyond the "anonymous" consideration within fixed factor, usually exercised in the NEG literature.[1]

Furthermore, we include endogenous quality in the seminal model of Dixit and Stiglitz (1977) and analyze both, vertical as well as horizontal product differentiation. In order to consider the impact of R&D from a macroeconomic point of view, we design a general equilibrium model, where households offer (unskilled) labor and research personnel. Detached from an analysis of particular product markets, with the focus on the mechanisms in aggregates, we choose monopolistic competition, rather than oligopolistic market structures, that are fairly discussed in the literature of Industrial Organization. What is new in this paper is the possibility to model an explicit R&D sector and the analysis of economic policy instruments in terms of research promotion. Finally, we analyze the allocation outcome in presence of vertical linkages by introducing a simple input-output relationship between firms in the manufacturing sector.

The paper is structured as follows. In Section 2, we build up the basic model with one manufacturing industry. After a partial analysis, we extend the model by en-

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[1]See for instance the Footloose Entrepreneur Model of Ottaviano (1996) and Forslid (1999), where the fixed factor, named as human capital, is inter-regionally mobile.

dogenous wages and income, as well as a separate research sector. In section 3, we introduce intermediate trade within the manufacturing sector and consider the comparative statics of market concentration and quality with respect to fixed costs. Section 4 discusses three policy instruments and their implications for social welfare: i) minimum quality standards; ii) the control of research costs; and iii) the optimal supply in the market for R&D services. A numerical application of the model to real economic data follows in section 5: For ten European basic industries we compute quality, marginal research costs, and research elasticities. Finally, the paper closes with a concluding discussions of the main findings and an outlook on future work in section 6 .

2   The Model

Private Demand

Private households consume two types of goods: i) a homogenous good A produced by a Walrasian, constant return sector (often referred to as agricultural sector), and ii) differentiated industrial products provided by a manufacturing sector.[2] Consumer preferences follow a nested utility function of the form:

$$(1) \quad U = M^\mu A^{1-\mu},$$

where M denotes a concave subutility from the consumption of the continuum of n (potential) industrial goods:

$$(2) \quad M = \left[\sum_{i=1}^{n} (u_i)^{1/\sigma} (x_i)^{(\sigma-1)/\sigma} \right]^{\frac{\sigma}{\sigma-1}}, \ \ \sigma > 1 \ , u_i > 0.$$

While x_i is the quantity consumed of variety i, u_i denotes a product specific utility parameter, henceforth denominated as *product quality*, σ is the constant substitution elasticity between varieties.[3] Applying *Two-Stage Budgeting*, we obtain the demand function for a representative industrial product sort:

$$(3) \quad x^D = \mu Y u p^{-\sigma} P^{\sigma-1},$$

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[2] Henceforth, the traditional sector is treated as the numeraire.

[3] The functional form of the subutility is based upon the numerical example of Sutton (1991), p. 48 et sqq..

where μY represents the share in household income for industrial products, p the market price, and P is the price index and defined to be:

$$(4) \quad P = \left[\sum_{i=1}^{n} u_i \left(p_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} .$$

As it is easy to reproduce, the demand elasticity in terms of quantity is constant at σ, and with respect to quality 1. Interestingly, the price index contains information about the product qualities. This results from its property as being the minimum cost for a given subutility M. The higher the quality at constant prices, the lower are consumer costs for reaching a certain level of satisfaction. The demand is increasing (linearly) with respect to a rising product quality, what results from the constant substitution elasticity. Assuming a representative variety, substitution of (4) in (3) reveals a demand that depends upon market size, price and firm number only, but that is independent from product quality.

Industrial Supply
Turning to the supply side of this model, the production of a particular variety requires labor as the only input. The corresponding factor requirement is characterized by a fixed and variable cost:

$$(5) \quad l^M = F + ax,$$

where M is mnemonic for manufacturing. Because of economies of scale and consumer preference for diversity, it is profitable for firms to produce each only one differentiated variety, so that firm number is equal to the number of available product sorts. Beside production, firms have capacity for undertaking research activities. Following Sutton (1991), firms can control their product quality by research investments. Compared to the original Dixit-Stiglitz model, producers have an additional degree of freedom in building up a monopolistic scope beside horizontal product differentiation. Attaining and maintaining a certain level of quality requires research expenditures as given in equation (6).

$$(6) \quad R\left(u\right) = \frac{r}{\gamma} u^{\gamma} \ , \ \gamma > 1.$$

The parameter, r, represents a constant cost rate and γ the research elasticity. The research expenditure function shows a deterministic relation. Furthermore, it is convex what implies that increasing product quality requires more and more investments. After all, research is essential – otherwise, the product quality and

simultaneously demand become zero.[4] The profit function of a manufacturing firms is given by:

$$(7) \quad \pi = px - R - wF - wax,$$

where w denotes the exogenous wage rate. From profit maximization follows the price setting rule:

$$(8) \quad p^* = \left(\frac{\sigma}{\sigma - 1}\right) aw,$$

where the term in brackets is the monopolistic price mark-up, atop the marginal production cost. Normalizing the variable production coefficient, a, by $(\sigma - 1)/\sigma$, the profit maximizing price becomes w.[5]

The optimum research policy can be derived from the first derivative of the profit function with respect to quality:

$$(9) \quad \mu Y u p^{-\sigma} P^{\sigma - 1} (p - wa) = r u^{\gamma}.$$

The term on the right hand side of (9) represents the marginal research cost of increasing product quality, what can also be expressed as γR. The left hand side shows the increase of the operating profit (profit less research costs), in response to a change in quality. From (9) follows the optimum quality of a particular firm:

$$(10) \quad u^* = \left(\frac{\mu Y w^{1-\sigma} P^{\sigma-1}}{\sigma r}\right)^{\frac{1}{\gamma - 1}}.$$

The choice of quality depends upon two factors: the research cost, and the degree of competition. The higher the cost rate, r, the lower is the product quality due to the optimum rule in (9). A decreasing competitive pressure, through an increase of market size, a lower substitution elasticity, or a higher profit maximizing price, makes firms compete in quality rather than prices. In other words, firms expand their research activities with a decreasing degree of competition. The behavior of the other firms, in terms of firm number and quality, effects the demand via the price index. An increase in firm number or competitive quality reduces the price index and, thus, the demand of the particular firm. Finally, the firm is not in funds

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[4]Sutton (1991) assumes a minimum product quality of 1 that is guaranteed even if no research is undertaken. For analytical convenience, we simplify this proposition.

[5]See Fujita et al. (1999), p.54 et sq..

to finance research expenditure, what can be seen by substituting the price index into (10):

$$(11) \quad u^* = \left(\frac{\mu Y}{\sigma r n} \right)^{\frac{1}{\gamma}}.$$

Equation (11) provides a central information: the interdependency between market concentration (measured in the number of firms) and the product quality. The corresponding research expenditures respond positively to market concentration:

$$(12) \quad R^* = \frac{\mu Y}{\sigma \gamma n}.$$

Long Run Equilibrium

The long run equilibrium is characterized by free market entry and exit and thus, a variable firm number. From the zero profit condition, we obtain the equilibrium output of each firm:

$$(13) \quad x^* = \frac{\mu Y}{\gamma w n} + \sigma F.$$

Compared to the original Dixit-Stiglitz outcome, the equilibrium output does not only depend upon exogenous parameters, but also upon the research expenditure. Therefore, the firm size is determined by fixed cost of production, as well as by research expenditure and market size, respectively. From (13), we can derive the equilibrium labor input:

$$(14) \quad \left(l^M \right)^* = F + a x^* = \sigma F + \left(\frac{\sigma - 1}{\sigma} \right) \frac{\mu Y}{\gamma w n}.$$

Because of the partial-analytic attribute of this model, the labor market is in equilibrium at the wage rate, w, so that labor supply is equal to the labor requirement of firms: $L^M = n l^M$. From this identity, the firm number can be derived using equation (14):[6]

$$(15) \quad n^* = \frac{L^M}{l^M} = \frac{1}{\sigma F} \left[L^M - \left(\frac{\sigma - 1}{\sigma} \right) \frac{\mu Y}{\gamma w} \right].$$

General Equilibrium

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[6]From (15) follows the non-negativity condition: $L^M > \left(\frac{\sigma-1}{\sigma} \right) \frac{\mu Y}{\gamma w}$.

For considering the model from macroeconomic point of view, we adopt a simple general equilibrium framework. To internalize wages and income, we need to introduce a separate R&D sector receiving the corresponding expenditures of the manufacturing industry. We assume a linear, constant return technology, where one unit R&D requires one unit of scientific input, research staff for instance.[7] The traditional sector uses (unskilled) labor as an input within a linear technology, where one unit labor generates one unit output, again. Because the homogenous good is the numeraire, the corresponding price is set to 1.

The GDP of the economy consists of the labor income, the agricultural revenues, as well as the earnings of the R&D sector:

(16) $\quad Y = wL^M + L^A + nR.$

Following Fujita et al. (1999), we normalize the manufacturing workforce L^M with λ, and L^A, the agricultural workforce, with $1-\lambda$, so that the total supply of unqualified labor is given by: $L = L^M + L^A$. Hence, the income becomes: $Y = w + nR$. We assume an inelastic labor supply so that wages come from the so-called wage equation that determines the wage at which firms break even:

(17) $\quad x^S = \mu Y u p^{-\sigma} P^{\sigma-1}.$

This equality provides two essential conditions: the instant and simultaneous clearing of the factor market as well as of the product market. We solve this expression for the price and wage, respectively:

(18) $\quad w^* = \left(\dfrac{\mu Y u P^{\sigma-1}}{x^*} \right)^{\frac{1}{\sigma}}.$

We allow intersectoral labor allocation, so that the equilibrium wage rate is equal to 1. Turning to the R&D sector, the cost rate, r, results from the market equilibrium of research services: $rL^R = nR$. Using equation (12) and setting the total supply of R&D services, L^R, equal to 1, the research cost rate is given by:

(19) $\quad r = \dfrac{\mu Y}{\sigma \gamma}.$

Equation (19) implies several important results: i) the cost rate increases with an increasing market size and decreasing homogeneity of downstream products; ii) it

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[7]In fact, instead of considering an autonomous sector, it may be possible to regard R&D as an in-house process of the manufacturing industry that is staffed from a particular labor market.

decreases with a rising research elasticity. The first comes from from the firm's quality policy, where the research expenditure rise with a lower degree of competition. The second is the implicit argument of the marginal research cost. Finally, the income can be expressed as:

$$(20) \quad Y = \frac{\sigma\gamma}{\sigma\gamma - \mu}.$$

Substituting this expression, in combination with the price index as well as the equilibrium output (13), into the wage equation (18), and solving for n, we obtain for the firm number:

$$(21) \quad n^* = \frac{\mu}{F}\left(\frac{\gamma - 1}{\sigma\gamma - \mu}\right).$$

Using this expression, the equilibrium firm size is given by:

$$(22) \quad x^* = \sigma F\left(\frac{\gamma}{\gamma - 1}\right).$$

For the equilibrium rate of research services, we obtain:

$$(23) \quad r^* = \frac{\mu}{\sigma\gamma - \mu},$$

so that product quality and research expenditures become:

$$(24) \quad u^* = \left[\frac{F}{\mu}\left(\frac{\gamma(\sigma\gamma - \mu)}{\gamma - 1}\right)\right]^{\frac{1}{\gamma}}$$

$$(25) \quad R^* = \frac{F}{\gamma - 1}.$$

From (22) one can see, that firm size depends upon exogenous parameters, but is times the term in brackets higher than the firm size of the original Dixit-Stiglitz model. In addition, the higher γ, what implies the more expensive is a quality improvement, the lower the firm size as a result of a higher price competition, and the lower the quality. The relation between quality and market concentration is described by:

$$(26) \quad u = \left(\frac{\gamma}{n}\right)^{\frac{1}{\gamma}}.$$

The lower the firm number the higher are research expenditures and product quality. The reason is straightforward: a decreasing firm number increases demand and profits due to the price index. Because research is financed by sales revenues, the capacity for R&D investments expands, what, in consequence, increases product quality. The opposite relation can be derived from the market clearing condition $npx = \mu Y$. The firm number with respect to quality is given by:

$$(27) \quad n = \frac{\mu\gamma\left(\gamma - 1\right)\left(\sigma\gamma - \mu\right)}{\gamma^2\sigma F\left(\sigma\gamma - \mu\right) - \mu^2\left(\gamma - 1\right)u^\gamma}.$$

The firm number increases in product quality, what can be traced back to the simple market size argument: the higher the quality the higher R&D expenditures and thus the corresponding household income, what increases the market size and makes new firms enter.[8] The overall mechanism between quality and firm number complies with the results of Sutton (1998), where an increasing market concentration goes along with a high R&D intensity:

$$(28) \quad \frac{R}{px} = \frac{\mu\left(\gamma - 1\right)}{\sigma F\gamma\left(\sigma\gamma - \mu\right)n} = \frac{1}{\sigma\gamma}.$$

As apparent in equation (28), the R&D intensity is negatively correlated to the firm number. In the equilibrium, it is dependent upon the substitution and research elasticity only, what implies that the R&D intensity declines with an increasing homogeneity of products and increasing research effort.

Stability of Equilibrium

We assume free market entry and exit in response to firm profits, following the adjustment process: $\dot{n} = f\left(\pi\right), f(0) = 0, f' > 0.$[9] We consider the case, where an additional firm decides to enter the market. In consequence, the price index decreases, what reduces the demand for a particular variety. Without quality improvement, firms make losses at constant output, what leads to market exits, finally. But because firms have the capacity to improve quality by research investments, they may counteract this development. As we have seen above, a higher firm number reduces the financial resources for research and development via price index. Hence, the product quality as well as demand and profits decrease. This leads to firm exits and

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[8]Function (28) has a pole at $u = \left[\frac{\gamma^2\sigma F(\sigma\gamma - \mu)}{\mu^2(\gamma - 1)}\right]^{1/\gamma}$, that is always below the equilibrium value (24).

[9]See Neary (2001).

a return to the former equilibrium again. This mechanism can be shown by totally differentiating the profit function with respect to price, quantity and quality:[10]

$$(29) \quad d\pi = \frac{p}{\sigma}dx + \left[\frac{\mu(\gamma-1)}{\sigma\gamma-\mu}u^{\gamma-1}\right]\frac{du}{u}.$$

Firm profits respond to changes in demand and quality only, while they are not affected by prices due to the price setting rule. An increase of demand gives always rise to profits and, thus, market entry of new firms. The same applies for a quality improvement. The dependency becomes transparent by expressing the profit function subject to quality only:

$$(30) \quad \pi = \left(\frac{\gamma-1}{\gamma}\right)ru^\gamma - wF.$$

The upper diagram in Figure 1 shows the profit function (30) with parameter settings $\gamma = 2$, $r = 1$, $F = 1$, and $\mu = 0.2$, $\sigma = 2$, for the lower diagram, respectively. According to the total differential (29), an increase in product quality out of the equilibrium makes profits increase due to rising demand, what leads to market entries of new firms. As given by equation (26), a decreasing market concentration is accompanied by lower R&D investments what reduces the product quality, until the equilibrium is reached again.

3   Vertical Linkages

In this section, we extend the model by a simple input-output-structure, where the manufacturing industry uses differentiated intermediate products from an imperfect upstream sector, in accordance to Ethier (1982). Instead of considering two separated sectors, we aggregate them to one manufacturing industry, as practised by Krugman and Venables (1995). By this approach, vertical linkages become horizontal, inter-sectoral allocation intra-sectoral. Main implications are: i) the industry uses a fixed proportion of its output as input again; ii) the technical substitution elasticity for intermediates is identical to σ; iii) firms have the same quality preferences as consumers; and iv) the price index for intermediates is the same as for final products. The corresponding production function is:

$$(31) \quad F + ax = Zl^{1-\alpha}I^\alpha \quad , \quad I = \left[\sum_{i=1}^{n}(u_i)^{1/\sigma}(x_i)^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}},$$
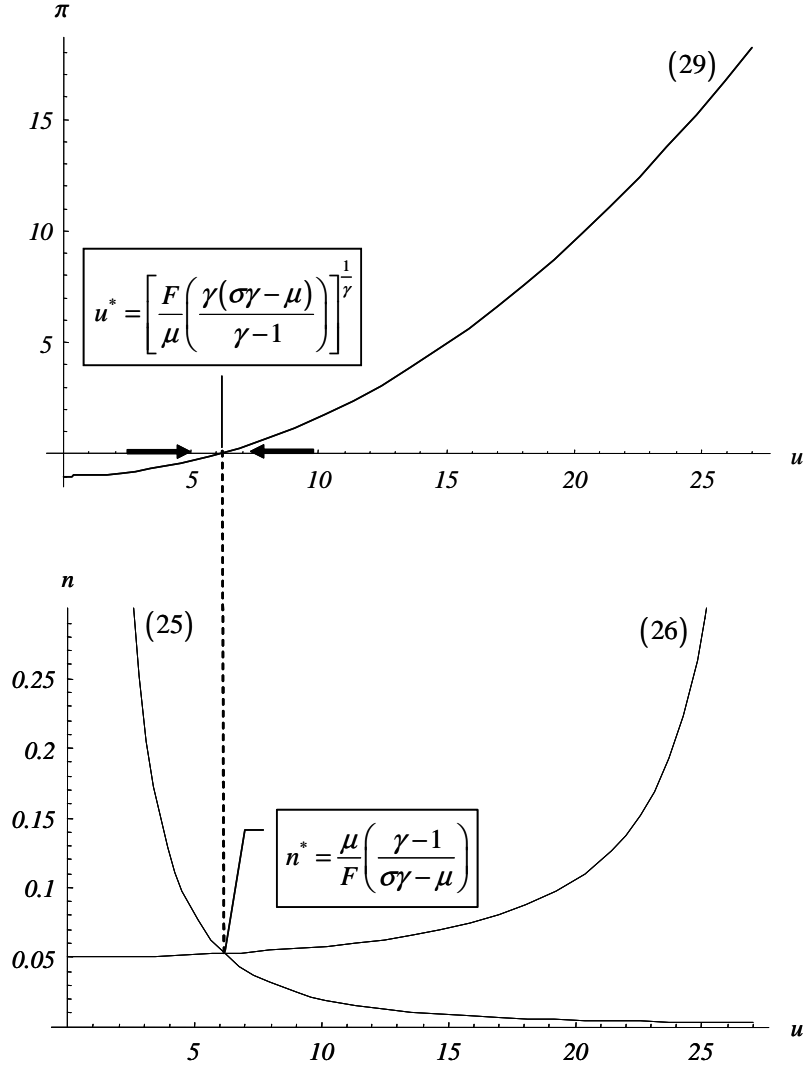
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[10]See Appendix.

Figure 1: *Quality and Firm Number*

where Z represents a level parameter, that is normalized by $(1-\alpha)^{\alpha-1}\alpha^{-\alpha}$, and I an input composite of a continuum of (potential) differentiated products. From two-stage budgeting, we obtain the cost function that is the analogue of the expenditure function of consumers:

$$(32) \quad C = (F+ax)\,w^{1-\alpha}P^{\alpha} + R.$$

The intermediate demand function is:

$$(33) \quad x^{u} = \alpha\,(C-R)\,up^{-\sigma}P^{\sigma-1},$$

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where u denotes *upstream*. The total demand for a particular variety is composed of consumer and intermediate demand, x^d and x^u:

(34) $\quad x^D = x^d + x^u = up^{-\sigma} P^{\sigma-1} \left[\mu Y + n\alpha \left(C - R \right) \right],$

where the term in square brackets represents the total expenditures, E, for industrial products. Equation (34) reflects the so-called forward and backward linkages between firms. The more firms produce in the economy the higher is the intermediate demand what increases firm number. Contrariwise, the more firms, the lower the price index, what implies a decrease of procurement costs for intermediates on one hand, and an increase of competition on the other hand. The relation interaction of these two forces is crucial for the model dynamics in this section.

From profit maximization, we obtain the same price setting rule as in the previous section:

(35) $\quad p^* = w^{1-\alpha} P^{\alpha},$

where the term on the right hand side describes marginal cost as a composite of wage rate and intermediate prices. The optimum product quality is given by:

(36) $\quad u^* = \left(\dfrac{x^D w^{1-\alpha} P^{\alpha}}{\sigma r} \right)^{\frac{1}{\gamma}}.$

The associated research investments are:

(37) $\quad R^* = \dfrac{x^D w^{1-\alpha} P^{\alpha}}{\gamma \sigma}.$

Using this expression, the equilibrium firm size results from the zero-profit-condition:

(38) $\quad x^* = \sigma F \left(\dfrac{\gamma}{\gamma - 1} \right),$

what is the same as in the model without vertical linkages. Turning to the labor market, the equilibrium wage rate follows from the wage equation, again:

(39) $\quad \left(w^{1-\alpha} P^{\alpha} \right)^{\sigma} = \dfrac{u P^{\sigma-1} E}{x}.$

Due to inter-sectoral labor mobility, the wage rate is 1, again. In the research market, the equilibrium price for R&D services can be expressed with equations (37) and (38) as:

(40) $\quad r = \left(\dfrac{1}{\gamma - 1} \right) n F P^{\alpha}.$

A noteworthy result is that the optimum quality (36) becomes with (38) and (40) the simple relationship between firm number and quality, as we have seen in the previous section at (26). For the determination of the equilibrium firm number, we bear in mind that the total expenditures on manufactures are the same as the aggregate turnover of the industry: $E = npx$. Using equations (32)-(40), the firm number with respect to quality is:

$$(41) \quad n^* = \left[\frac{\mu(\gamma - 1)}{F(\sigma\gamma(1-\alpha) + \alpha - \mu)} \right]^{\frac{(1-\sigma)(1-\alpha)}{(1-\sigma)(1-\alpha)+\alpha}} u^{\frac{\alpha}{(\sigma-1)(1-\alpha)-\alpha}}$$

The firm number is unique, positive, if the the denominator of the term in square brackets fulfills the following condition:

$$(42) \quad \sigma\gamma > \frac{\mu - \alpha}{1 - \alpha},$$

that is always valid. Having a closer look on equation (41), it is apparent that the impact of quality on market concentration depends upon the denominator of the corresponding exponent:

$$(43) \quad \frac{\partial n}{\partial u} \gtrless 0 \quad \Rightarrow \quad 1 \gtrless \left(\frac{1}{\sigma - 1} \right) \left(\frac{\alpha}{1 - \alpha} \right)$$

The direction of change in the firm number with respect to a change in quality is not positive definite, as it is in the previous section, characterized in equation (28). In fact, the correlation depends upon the strength of two competing forces. An increasing quality raises R&D investments and simultaneously consumer and intermediate demand. The increasing quality reduces the price index, what dampens the demand reduction, but increases the prices again, via the monopolistic price setting rule. The overall effect implies a net reduction of demand. In contrast, the increasing quality means higher research expenditures, what implies a smaller budget for intermediates. Actually, the production cost $C - R$ can be expressed as: $FP^\alpha \left(\frac{\gamma\sigma - 1}{\gamma - 1} \right)$. As apparent, a decreasing price index causes lower production cost, what decreases the intermediate demand due to the constant cost share α. All in all, the direction of change depends upon the strength of the forward linkage and the direct demand effect.

Turning to the second central variable, the equilibrium product quality can be derived from (37), (39), and (41), what leads to equation (26), again. This result, in conjunction with the total differential of the profit function (see Appendix), ensures a unique, globally stable equilibrium at:

$$(44a) \quad u^* = \gamma^{\frac{\alpha}{\gamma(1-\sigma)(1-\alpha)+\gamma\alpha-\alpha}} \left[\frac{\mu(\gamma - 1)}{\gamma F(\sigma\gamma(1-\alpha) + \alpha - \mu)} \right]^{\frac{(1-\sigma)(1-\alpha)}{\alpha-\gamma(1-\sigma)(1-\alpha)-\gamma\alpha+\alpha}}$$

$$(44b)\ n^* = \gamma^{\frac{\alpha - \gamma(1-\sigma)(1-\alpha)}{\alpha - \gamma(1-\sigma)(1-\alpha) - \gamma\alpha + \alpha}} \left[\frac{\mu(\gamma - 1)}{\gamma F (\sigma\gamma(1-\alpha) + \alpha - \mu)} \right]^{\frac{\gamma(\sigma-1)(1-\alpha)}{\gamma(\sigma-1)(1-\alpha) - \alpha(\gamma-1)}}.$$

At this point, we may have a closer look on the effects of changes in the fixed (production) cost, F, on market concentration and quality. On condition of (42), the direction of change depends upon the exponents of the terms in brackets:

$$(45a)\ \frac{\partial u}{\partial F} \lesseqgtr 0 \quad \Rightarrow \quad \left(\frac{\gamma}{\gamma - 1} \right) \lesseqgtr \left(\frac{1}{\sigma - 1} \right) \left(\frac{\alpha}{1 - \alpha} \right)$$

$$(45b)\ \frac{\partial n}{\partial F} \gtreqless 0 \quad \Rightarrow \quad \left(\frac{\gamma}{\gamma - 1} \right) \lesseqgtr \left(\frac{1}{\sigma - 1} \right) \left(\frac{\alpha}{1 - \alpha} \right).$$

This result differs from the single sector model, where the equilibrium market concentration is positively correlated with fixed cost. The reason is straightforward. An increase of fixed cost leads to a decrease of profits and to an accompanying market exit of firms. With vertical linkages, an increase of F, implying a rising factor requirement, causes an increase in the intermediate demand, what gives rise to firm profits and market entries. The relation between the cost effect and the forward linkage determines the response of market concentration on the fixed cost.[11] The response of quality on changing fixed cost is reversed to firm number, as it becomes apparent at the inequality signs. As shown in the previous section and apparent at the quality policy (36), firm number and quality are negatively correlated due to the price index.

Summarizing the outcomes of this section, we can make the following statements: i) There is a unique and definite equilibrium, due to condition (42). ii) In contrast to the single sector model, the market clearing function (41) can also be downward sloping. In this context, a positive (negative) slope implies weak (strong) linkages between manufacturing firms , so that inequality (43) qualifies to be a measure for the classification of industries in terms of their sectoral coherence. iii) The equilibrium shows a different behavior with respect to changes in the exogenous variables, as we have seen at the example of the fixed cost, F. An economic policy has to regard the strength of the sectoral linkages, in order to meet the welfare maximizing objectives. In the next section, we have a closer look on the impact of political instruments. Based on these results, we derive a framework for a research promoting policy.

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[11]In the original Dixit-Stiglitz model with vertical linkages, the condition (45b) is: $\frac{\partial n}{\partial F} \gtreqless 0 \Rightarrow 1 \gtreqless \left(\frac{1}{\sigma-1} \right) \left(\frac{\alpha}{1-\alpha} \right)$.

4 Welfare Analysis

With respect to the allocation outcome in imperfect markets, we consider basic economic policy instruments in this section. We depart from the view of a social planner, that has the capacity to control central macroeconomic variables, more to a practical point of view, where the state has limited possibilities in its instruments. By doing this, we determine the welfare of the market allocation in order to compare it with a situation, where the state disposes of the potential to control i) the quality by minimum standards, ii) the research cost rate, and iii) the supply of R&D activities.

Minimum Quality Standards
At first, we compare the effects of state-controlled quality standards with the unregulated equilibrium in terms of welfare losses. Assuming a given quality $\bar{u}$, the research investments become:

$$(46) \quad R = \frac{r}{\gamma}\bar{u}^{\gamma}.$$

With this expression, the equilibrium output is:

$$(47) \quad x = \sigma\frac{r}{\gamma}\bar{u}^{\gamma} + \sigma F.$$

The firm number can be expressed with (47) and $1 + nR$ for the household income as:

$$(48) \quad n = \frac{\mu}{\frac{r}{\gamma}\bar{u}^{\gamma}(\sigma - \mu) + \sigma F}.$$

From the research market equilibrium, we obtain $1 = \frac{n}{\gamma}\bar{u}^{\gamma}$. In combination with equation (46), the research cost rate is given by:

$$(49) \quad r = \frac{\mu\bar{u}^{\gamma} - \gamma\sigma F}{\bar{u}^{\gamma}(\sigma - \mu)}.$$

Substituting (49) into (48), yields equation (26), finally. For establishing a socially optimal quality, we choose the welfare as a function of consumer utility. From household optimization, we obtain the maximum utility as the real income of households:[12]

$$(50) \quad W = YP^{-\mu}.$$

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[12]We neglect the term $\mu^{\mu}(1 - \mu)^{1-\mu}$.

Without external intervention, social welfare is:

$$(51) \quad W^* = \gamma^{\frac{\mu}{\sigma-1}} \left[\frac{\sigma\gamma}{\sigma\gamma - \mu} \right] \left[\frac{\mu}{F} \left(\frac{\gamma - 1}{\gamma \left(\sigma\gamma - \mu \right)} \right) \right]^{\frac{\mu(\gamma-1)}{\gamma(\sigma-1)}}.$$

Having regard to quality regulation, the welfare function with respect to quality becomes with equations (48) and (49):

$$(52) \quad W = \left[\frac{\sigma\bar{u}^\gamma - \gamma\sigma F}{\bar{u}^\gamma \left(\sigma - \mu \right)} \right] \left(\gamma\bar{u}^{1-\gamma} \right)^{\frac{\mu}{\sigma-1}}.$$

The limiting values of the hyperbolic welfare function are $-\infty$ for $u \to 0$, and 0 for $u \to \infty$. The unique maximum value, the target for a quality policy, is given by:

$$(53) \quad \bar{u} = \left[\left(\frac{\gamma}{\gamma - 1} \right) \frac{F}{\mu} \left(\gamma \left(\sigma - 1 \right) + \mu \left(\gamma - 1 \right) \right) \right]^{\frac{1}{\gamma}}.$$

As easily to prove, the socially optimal product quality is always lower as in the model without regulation. Figure 2 shows the welfare function for the parameter constellation as in the previous illustration.
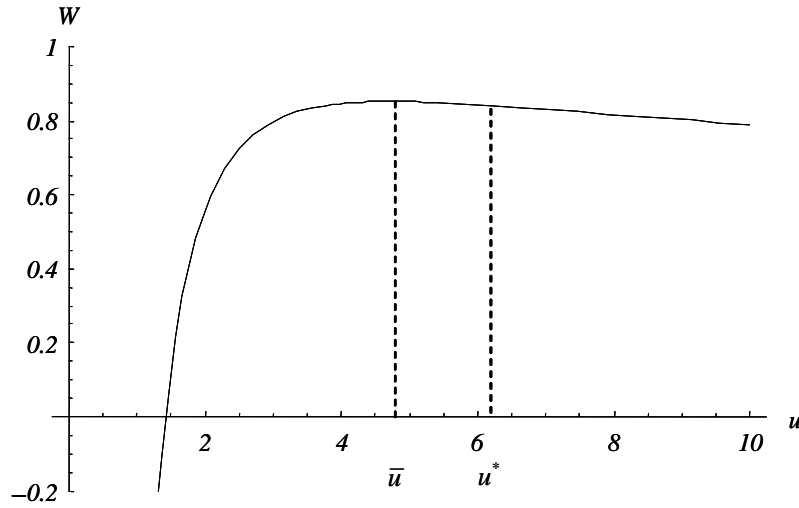

Figure 2: *Quality and Welfare*

The establishment of quality standards implies a welfare gain due to a reduction of market concentration. The research investments and quality, respectively turn out to be too high without regulation and, in consequence, the firm number too low,

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what is a result of market imperfections.[13] These findings cause several problems for real economic policy: i) if quality is too high, minimum standards fail the welfare optimum; ii) in turn, maximum standards are not practicable what implies an indirect control via alternative political instruments; iii) variation in model premises may change these outcomes. Assuming bounded rationality of consumers or information asymmetry, for instance, could lead to systematic underestimation of quality with the cause of a socially too low equilibrium. Setting quality standards means a welfare maximum. In contrast, removing these deficiencies on the demand side results (by public information, for example) in the unregulated quality only, that it too high again. Beside these exceptions, we concentrate on the impact of an indirect quality control by variations of the research cost rate.

Optimal Control of Research Costs

The state can control research cost in two different ways: via subsidization and taxation as well as via a stately-owned or -regulated research sector. The argument for a public intervention is not a failure on the research market itself, but rather in the corresponding downstream sector. The choice of a research cost rate is linked with a excess demand or supply, so that a case differentiation is required for the derivation of the welfare function.

At first, we consider a cost rate above the equilibrium value, so that the demand for R&D becomes the bottleneck. While household income, firm number and firm size remain constant, the quality decreases due to the firm product policy. Although research investments do not change, the employment in the R&D sector declines. The welfare function with respect to the research cost rate can be expressed as:

$$(54) \quad W\left(r > r^*\right) = \left[\frac{\sigma\gamma}{\sigma\gamma - \mu}\right]\left[\left(\frac{F\gamma}{\gamma - 1}\right)\frac{\mu\left(\gamma - 1\right)}{F\left(\sigma\gamma - \mu\right)}\right]^{\frac{\mu}{\sigma - 1}} r^{\frac{\mu}{\gamma(1-\sigma)}}.$$

Because the terms in square brackets are positiv, the welfare decreases monotonically with increasing cost rate, so that a scale up of r leads always to welfare losses.

If the cost rate is set below the equilibrium value, the demand for R&D services is larger than the market capacity. In consequence, the quality becomes:

$$(55) \quad u = \left[\frac{\gamma\sigma F}{\mu - r\left(\sigma - \mu\right)}\right]^{\frac{1}{\gamma}},$$

The welfare function is now:

$$(56) \quad W\left(r < r^*\right) = (1 + r)\,\gamma^{\frac{\mu}{\gamma(\sigma - 1)}}\left[\frac{\mu\left(1 + r\right) - r\sigma}{\sigma F}\right]^{\frac{\mu(\gamma - 1)}{\gamma(\sigma - 1)}}.$$

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[13]This complies with the results of the seminal Dixit-Stiglitz model. See the introduction of Brakman and Heijdra (2004), pages 19 et sqq., for instance.

The limiting values of equation (56) are $\left(\frac{\mu}{\sigma F}\right)^{\frac{\mu(\gamma-1)}{\gamma(\sigma-1)}}$ for $r \to 0$ and $-\infty$ for $r \to \infty$.[14] From (56), the welfare maximizing research cost rate is:

$$(57) \quad r_{max} = \frac{\mu\left[\mu\left(\gamma-1\right)+\sigma-\gamma\right]}{\sigma\left[\gamma\left(\sigma-1\right)-\mu\right]+\mu\left[\gamma-\mu\left(\gamma-1\right)\right]}.$$

Because of possible negative values of (57), the socially optimal research cost rate is defined to be:

$$(58) \quad \bar{r} = \begin{cases} r_{max} & \forall \ \mu\left(\gamma-1\right) > \sigma + \gamma \\ 0 & \forall \ \mu\left(\gamma-1\right) \leq \sigma + \gamma. \end{cases}$$

If we complete the welfare function for the whole range of r, we have to consider both equations, (54) and (56). The graphs intersect at their lower and upper limit, respectively: the non-regulated equilibrium r^*. All in all, we obtain a continuous but non-differentiable welfare function. Figure 3 shows the socially optimal and unregulated research cost rate and the corresponding welfare values with the parameters from above. It is a quite noteworthy fact that reducing quality to the optimum
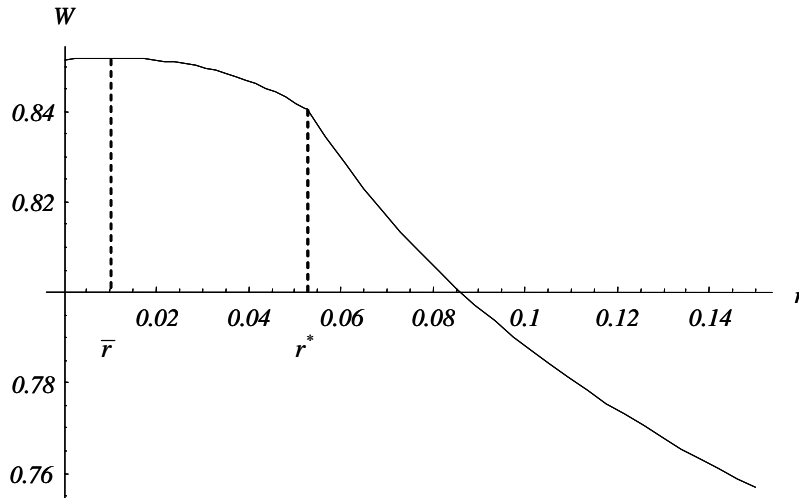

Figure 3: *Research Cost and Welfare*

level, is realizable by a reduction of the research cost rate, only – against intuition and partial analytical results. All in all, this dependency can be traced back to the occurring disequilibrium in the research market and the special characteristics of

[14]If $\left(\frac{\gamma-1}{\gamma}\right) < \left(\frac{\sigma-1}{\mu}\right)$ holds, the codomain of r is $]0, \frac{\mu}{\sigma-\mu}[$ due to a negative root. The upper limit is greater than the equilibrium cost rate without regulation, so that it is not a part of the total (piecewise-defined) welfare function (54) and (56)

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the Dixit-Stiglitz framework. Due to a research price below the equilibrium value, the fixed supply of researchers is rationed to the increased demand of firms. The research investments, R, become r/n. In consequence, these expenditures begin to decrease. This reduces overall fixed costs, and hence, the average costs and break even output, what causes an entry of new firms. The increased firm number and lower research expenditures correspond with a decreasing quality, via equation (26). Finally, we obtain a lower income and quality, but a higher firm number, compared to the unregulated equilibrium. The decrease of household income is more than compensated by the decrease of the price index.[15]

Supply of Research Activities
An alternative policy instrument consists in the control of the supply of R&D services and scientific personnel, respectively. In reality, the range of activities varies from a totally state-controlled research sector, an intensifying of (academic) education in terms of quantity and quality, until the directed promotion, by funding programs, for instance. As mentioned above, we neglect the financing of public market intervention, but rather ask for the impact on allocation and welfare.
In section 2, we set the (price inelastic) supply of R&D equal to 1. Now we relax this restriction and allow L^R to be non-zero positive. In consequence, the equilibrium research cost rate becomes:

$$(59) \quad r^* = \frac{\mu}{L^R \left(\sigma \gamma - \mu \right)},$$

where the income remains constant at (20). The equilibrium quality can now be expressed as:

$$(60) \quad u^* = \left[\frac{F L^R \gamma \left(\sigma \gamma - \mu \right)}{\mu \left(\gamma - 1 \right)} \right]^{\frac{1}{\gamma}}$$

All in all, with increasing research supply, the firms are able to improve quality without raising the research investments, so that market concentration as well as firm size remain unchanged. If the firm number is constant at increasing quality, the price index declines what increases the real income and welfare, finally. These results imply that an increase in R&D supply leads to a better quality at unaffected market concentration. Even though, an economic policy may increase the social welfare, it fails to meet the welfare maximum.

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[15]The derivative of the price index with respect to research cost is: $\left[\left(\frac{\gamma - 1}{\gamma (1 - \sigma)} \right) \left(\frac{\mu - -\sigma}{\mu - r(\sigma - \mu)} \right) \right] P < 0$, where the denominator of the second fraction is always positive (see equation (54)).

5   Numerical Application

In this section, we adopt the modeling results to real economic data and aim to determine quality and research costs for selected European industries. The required data are extracted from the EUROSTAT online data base and contain firm number, turnover, and R&D expenditures (estimated) for 2003. Furthermore, we need the corresponding substitution elasticities. Therefor we take the OLS-estimated values of Hummels (1999), table 4. For the simulation, we need to make several assumptions:

- At first, we assume monopolistic competition for the respective industry. For this case, we have to choose a sufficient degree of aggregation to avoid monopolistic or oligopolistic market structures and corresponding deviations from the model of symmetric and independently acting (one product) firms. On this note, we have to solve a trade-off, where a too high aggregation leads to substitution elasticities that tend to be too low (smaller than 1). In this simulation, we choose two-digit, in one case three-digit, industries.

- Implementing a general equilibrium model requires the consideration of multiple industries. In this case, we choose a particular industry to analyze within the manufacturing sector of the model and adopt the Walrasian sector to the others. Hence, we assume a competitive market structure for the residual economy.

- Through inter-sectoral mobility, we allow workers to move between sectors, what may be critical, depending on type of work and industry.

- Because we simulate a closed economy, international trade relations are excluded. For the definition of an economic area with a high degree of domestic trade, we choose the European Union (EU-25) and neglect its transcontinental trade.

- In the regard to R&D, we assume a 1:1 relationship between research and manufacturing sector and thus, neglect cross-sectoral research activities and potential spill-over effects, just as we do not allow knowledge exchange between firms.

- R&D investments are only employed for quality improvements, what includes also the design of new (better) products. Research activities for cost reduction cannot be separated from the official statistic data and are inevitably integrated in the R&D expenditures.

- Finally, we rule out any public interventions and assume infinitely fast adjustment processes, as well as an instantaneously (deterministic) effect of R&D on quality.

Table 1 in the Appendix shows the simulation output for ten industries. For the computation of the research elasticity, the relation (30) is used. The research cost rate r is determined by equation (23), where the income share for manufactures, μ, is the ratio of the respective industrial turnover and the GDP (2003) earned in the EU-25. With the values for r and γ, the quality can be computed via equation (6). The marginal research costs come from the derivative of the same equation.

Results

All in all, we obtain only a rough estimation for the magnitudes of (calibrated) real parameters. Having at first a look at the quality column, we obtain a widely spread distribution. It is obvious that the quality correlates with the R&D intensity, where research intensive branches show large values for quality, such as Pharmaceutics or Computers, for instance.

The research elasticities indicate a strong divergence across the industries. The parameter tends to be high for branches with a low research intensity: Foods, Basic Metals and Metal Manufactures vs. research intensive sectors with a high γ, Pharmaceutics or Computers, for instance. In this context, we can observe a relatively distinctive relation between substitution and research elasticity, where a low σ, what means a high product differentiation, corresponds with a high γ. This indication leads to the assumption that R&D investments are not used for quality improvements only, but also for horizontal product differentiation.

Furthermore, we can observe large differences for the research cost rate, r: the highest values are in the Automotive and Pharmaceutical industry, in contrast to the lowest value in the Metal Production and Metal Manufactures. An obvious reason might be the lower demand for R&D services and personnel in the latter cases.

The column headed with $\partial R/\partial u$ shows the marginal research costs. As we can see, the lowest correspond with the highest research intensities – concerning Pharmaceutics, Computers and Medical Instruments etc., again.

Finally, the last column displays the welfare maximizing research cost rate, in accordance with equations (57) and (58). As apparent, these values are very low, unless 0. Based on these results, an economic policy should decrease the cost rates down to the particular values. This would imply an almost costless research, for reducing quality to the welfare maximum level, a result that is arguable in the context of real markets. This issue is discussed in the next section, beside a summary of the most important outcomes of this paper and outlook on future research.

6 Conclusions

In the course of this paper, several results confirmed the inverse relationship between market concentration and research intensity in monopolistic competitive markets. Equation (26), the zero-profit condition, shows a simple relation between firm number and quality, controlled by the exogenous research elasticity. Furthermore, this paper showed the opposite dependence in (28), where the quality determines market size and, thus, the firm number. The interaction of these two mechanisms generate an equilibrium, where the firm number increases with an increasing research elasticity, a higher degree of horizontal differentiation, and a lower production fixed cost. In contrast, the quality increases with a lower research and a higher substitution elasticity, and increasing fixed cost. In this context, we explored that, in equilibrium, the research intensity is determined by the research and substitution elasticity only.

The implementation of vertical linkages leads to partially different results. Even though we obtain a unique and globally stable equilibrium, and the relation (26) remains unchanged, the outcomes of the comparative statics depend upon the strength of the vertical linkages.

Political intervention is legitimated by a welfare loss due to imperfect markets. We made out that the unregulated quality is above the social optimum, with the implication that minimum quality standards do not have an impact on the allocation outcome. An alternative instrument ist the price control of R&D services. Surprisingly, the research cost rate must be reduced to meet the lower welfare maximizing quality. In consequence, the firm number increases what reduces the price index at constant household income, this, finally, implies a higher real income. An increase in the R&D capacities leads to an increase in welfare but not to the corresponding maximum. Ultimately, a social planner that aims to reach a certain level of (maximized) welfare, is supposed to combine research price instruments with an adapted control of R&D supply.

The numerical application was conceived to quantify the magnitudes of the model outcomes with the imputation of real data – considering the strong restriction of the underlying assumptions. It may be pointed out that an advanced simulation analysis using statistical data requires a detailed econometric foundation. However, the results computed in this paper show a wide spreading in the quality across the corresponding industries. Using the suggested approach, we consider aggregates of industries and products. The results tend to be sufficient to compare differentiated products within one sector, but less for comparison across industries, say cars and computers, for instance. Based on the outcomes of this simulation, the welfare maximizing research cost rates would be zero, or almost zero. In consideration of these

extreme results, we have to keep in mind that the policy aims to correct market failures and not to promote technology development, as an example.

In the face of the underlying assumptions, one may complain two weaknesses of this model: i) the industry-specific R&D sector, and ii) the absence of knowledge spillover effects. Indeed, the model could describe a more realistic picture, if we implement R&D that supplies several sectors, as it is observable in fields of fundamental research. And likewise, it may be interesting to consider internal R&D of one firm that generates externalities for other firms in the corresponding industry. In this context, the quality of a particular firm i is not only depending upon the input of its own research input, $u_i\left(L_i^R\right)$, but also upon the R&D efforts of the whole sector: $u_i\left(L_i^R, \sum_{j=1}^n L_j^R\right)$. However, these issues will be part of future work.

Regarding international trade, for which this paper is intended to be a foundation of implementing vertical product differentiation and R&D, the model opens the possibility to analyze the impact of trade integration on quality and research investments, and, in turn, the effects of R&D on spatial concentration and specialization. Based on the extensions, we can model the implications of factor mobility (even in terms of scientific labor force) and agglomeration by inter-sectoral linkages. Furthermore, on the base of the policy instruments considered in this paper, we are able to determine the impact of quality standards as trade barrier, and to draw conclusions for a location-oriented R&D policy.

7 Technical Appendix

Derivation of Cost Function and Intermediate Demand
The optimization problem on the lower stage is:

$$(61) \quad min. \quad \sum_{i=1}^n p_i x_i^u \quad s.t. \quad I = \left[\sum_{i=1}^n (u_i)^{1/\sigma} (x_i)^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}}$$

The compensated demand for intermediates results from the first order conditions:

$$(62) \quad x^u = u\left(\frac{P}{p}\right)^{\sigma}.$$

The upper stage of optimization is given by:

$$(63) \quad min. \quad C = PI + wl + R \quad s.t. \quad F + ax = Zl^{1-\alpha}I^{\alpha}.$$

From the first order conditions we obtain:

$$(64) \quad l = \left(\frac{1-\alpha}{\alpha}\right)\frac{PI}{w}.$$

Substituting this expression into the general cost function, leads to:

$$(65) \quad I = \frac{\alpha(C-R)}{P}.$$

Equation (65) can now be inserted into the compensated intermediate demand (62), from which the intermediate demand function (33) follows.

Total Differential of the Profit Function

Starting from the profit function:

$$(66) \quad \pi = \mu Y u p^{1-\sigma} P^{\sigma-1} - aw\mu Y u p^{-\sigma} P^{\sigma-1} - wF - \frac{r}{\gamma}u^\gamma,$$

and substituting $w + n\frac{r}{\gamma}u^\gamma$ for the income and $p(nu)^{\frac{1}{1-\sigma}}$ for the price index, we obtain:

$$(67) \quad \pi = \frac{\mu}{\sigma n} + \frac{\mu r}{\sigma \gamma}u^\gamma - wF - \frac{r}{\gamma}u^\gamma,$$

finally. Solving equation (11) for n and using the expression for Y as above, leads to:

$$(68) \quad n = \left(\frac{\mu\gamma}{\sigma\gamma-\mu}\right)\frac{1}{ru^\gamma}.$$

Equation (68) describes the dependency between n and u. If we set $\frac{\mu}{\sigma\gamma-\mu}$ for r, and substitute (68) into the (67), totally differentiating the profit function yields the expression (29).

In section 3, the total differential is:

$$(69) \quad d\pi = \left[\frac{1}{\sigma}\right]dx + \left[\underbrace{\left(\frac{\alpha}{(1-\sigma)(1-\alpha)}\right)}_{<0}\underbrace{\left(\frac{\mu - (\gamma\sigma - \alpha(\gamma\sigma - 1))}{\gamma\sigma + \gamma\alpha(1-\sigma) - \mu}\right)}_{<0}FP\right]\frac{du}{u}.$$

As in the model without linkages, profits and firm number respond positively on demand and quality. For a further detailed analysis of the disaggregated model, see Kranich (2006).

Table 1: Simulation Results for Selected Basic European Industries

NACE	Industrial Sector	μ	n	nR in Mio. €	npx in Mio. €	R/px in Mio. €	σ	γ	r	u	∂R/∂u	r_{max}
DA	Food products, beverages and tobacco	0.0874	282,876	2,195	871,000	0.0025	3.40	116.7091	220.18	1.0739	0.8433	0.0000
DG24	Chemicals and chemical products (without pharmaceuticals)	0.0427	26,604	7,550	425,988	0.0177	5.28	10.6860	757.75	2.1732	1.3954	0.0000
DG244	Pharmaceuticals	0.0177	4,111	15,647	176,013	0.0889	9.53	1.1804	1,571.67	847.2409	0.0053	0.0015
DH25	Rubber and plastic products	0.0229	61,430	2,181	228,358	0.0096	4.82	21.7227	218.78	1.4564	0.5295	0.0000
DJ27	Basic metals	0.0241	15,000	1,122	240,000	0.0047	3.53	60.5960	112.54	1.1912	3.8050	0.0000
DJ28	Fabricated metal products, except machinery and equipment	0.0372	370,000	1,271	371,055	0.0034	4.85	60.1936	127.48	1.1307	0.1829	0.0000
DK29	Machinery and equipment n.e.c	0.0501	157,244	9,669	499,212	0.0194	6.98	7.3969	970.62	2.2965	0.1981	0.0000
DL30	Office machinery and computers	0.0067	9,482	4,296	66,500	0.0646	11.02	1.4047	431.02	180.3889	0.0035	0.0004
DL33	Medical, precision and optical instruments, watches and clocks	0.0120	90,000	6,081	120,000	0.0507	6.72	2.9365	610.22	7.1689	0.0277	0.0004
DM34	Motor vehicles, trailers and semi-trailers	0.0741	16,921	20,364	739,000	0.0276	7.11	5.1040	2,046.44	4.8006	1.2795	0.0008

References

Brakman, S. and Heijdra, B. (eds): 2004, *The Monopolistic Competition Revolution in Retrospect*, Cambridge University Press, Cambridge UK.

Dixit, A. K. and Stiglitz, J.: 1977, Monopolistic competition and optimum product diversity, *American Economic Review* **67**, 297–308.

Ethier, W.: 1982, National and international returns to scale in the modern theory of international trade, *American Economic Review* **72**, 389–405.

Forslid, R.: 1999, Agglomeration with human and physical capital: an analytically solvable case, *CEPR Discussion Paper 2102*, Centre for Economic Policy Research.

Fujita, M., Krugman, P. and Venables, A.: 1999, *The Spatial Economy*, The MIT Press, Cambridge, London.

Hummels, D.: 1999, Toward a geography of trade costs, *GTAP Working Paper 17*.

Kranich, J.: 2006, The strength of vertical linkages, *Working Paper Series in Economics 20*, University of Lueneburg, Lueneburg.

Krugman, P.: 1991, Increasing returns and economic geography, *Journal of Political Economy* **99**, 483–499.

Krugman, P. and Venables, A. J.: 1995, Globalization and the inequality of nations, *The Quarterly Journal of Economics* **110**, 857–880.

Neary, P. J.: 2001, Of hype and hyperbolas: Introducing the new economic geography, *Journal of Economic Literature* **39**, 536–561.

Ottaviano, G.: 1996, Monopolistic competition, trade, and endogenuous spatial fluctuations, *CEPR Discussion Paper 1327*, Centre for Economic Policy Research.

Schmalensee, R.: 1992, Sunk costs and market structure. a review article, *Journal of Industrial Economics* **40**(2), 125–134.

Sutton, J.: 1991, *Sunk Costs and Market Structure*, The MIT Press, Cambridge.

Sutton, J.: 1998, *Technology and Market Structure*, The MIT Press, Cambridge/MA, London.