Andreev tunnelling in quantum dots: A slave-boson approach

P. Schwab[a], R. Raimondi[b]

[a] Istituto di Fisica della Materia e Dipartimento di Fisica, Universitá degli Studi di Roma "La Sapienza", Piazzale Aldo Moro 2, I-00185 Roma, Italy

[b] Istituto di Fisica della Materia e Dipartimento di Fisica E. Amaldi, Terza Università degli Studi di Roma, Via della Vasca Navale 84, I-00146 Roma, Italy

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We study a strongly interacting quantum dot connected to a normal and to a superconducting lead. By means of the slave-boson technique we investigate the low temperature regime and discuss electrical transport through the dot. We find that the zero bias anomaly in the current-voltage characteristics which is associated to the occurance of the Kondo resonance in the quantum dot, is enhanced in the presence of superconductivity, due to resonant Andreev scattering.

With the advent of nanotechnology, recent years have witnessed an impressive experimental activity, studying various properties of small mesoscopic structures. In particular, the transport properties of hybrid superconducting structures and the associated Andreev scattering mechanism have been investigated intensively after observing several new phenomena [1]. Many of these phenomena have been successfully explained in terms of a one-particle picture essentially based on the BCS theory either via the Bogolubov-de Gennes equations or via quasiclassical Green's function methods, as documented by various review articles [1].

Besides this, electrical transport through small confined regions, where electron-electron interactions are strong, has also attracted a lot of interest. Such experimental setups, *e.g.* quantum dots (QDs), allows one to investigate in a controlled way the interplay of the electron-electron interaction and disorder. In particular it has been pointed out that a QD attached to two metallic leads resembles an impurity level in a metal. As a consequence, even when the dot level is far from the Fermi energy of the leads, transport will occur due to the Kondo effect [2]. This is due to the formation of a spin singlet between the impurity level and the conduction electrons, which gives rise to a quasiparticle peak at the Fermi energy in the dot spectral function. This suggestion has been explored theoretically by several groups [3–7] and lead to the prediction that one should observe a zero-bias anomaly in the current voltage characteristics. Such an anomaly has been indeed observed in different QD systems [8,9].

In a recent letter [10], it has been suggested that, if the QD is coupled to two different types of leads, *i.e.*, a normal and a superconducting lead, resonant Andreev tunnelling yields a stronger zero-bias anomaly with a broader temperature region where the effect occurs. In the analysis of [10] an approach based on the equation-of-motion method which is mainly valid at high temperature was used. In this paper, we extend the analysis of [10] to the extreme low temperature regime. To this end we use slave-boson mean field theory. This approach has been successfully applied to the low temperature properties of a Kondo impurity in presence of normal [11,12] as well as for superconducting conduction electrons [13]. Despite its simplicity, this method captures the main physical aspects of the Fermi liquid regime at low temperatures, *i.e.*, the formation of a many-body resonance at the Fermi energy. For this reason it presents a convenient framework in which to study the interplay between Andreev scattering and Coulomb interactions.

We model the N-QD-S system with the Hamiltonian

$$H = H_{\rm N} + H_{\rm S} + H_{\rm D} + H_{\rm T,N} + H_{\rm T,S} \qquad (1)$$

where $H_{\rm N} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{{\rm N},\mathbf{k}\sigma} c_{{\rm N},\mathbf{k}\sigma}$, $H_{\rm S} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c^\dagger_{{\rm S},\mathbf{k}\sigma} c_{{\rm S},\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\Delta c^\dagger_{{\rm S},\mathbf{k}\uparrow} c^\dagger_{{\rm S},-\mathbf{k}\downarrow} + c.c.)$ and $H_{\rm D} = \epsilon_d d^\dagger_\sigma d_\sigma + U n_{d\uparrow} n_{d\downarrow}$ are the Hamiltonians of the normal lead, the superconducting lead (Δ is the superconducting gap) and the dot respectively. The single particle energy $\epsilon_{\rm d}$ is double degenerate in the spin index σ and the interaction is included through the on-site repulsion U. The position of the dot level can be modulated by an external gate voltage. Tunnelling between the leads and the dot is described by $H_{{\rm T},\alpha} = \sum_{\mathbf{k}\sigma} (V_\alpha c^\dagger_{\alpha,\mathbf{k}\sigma} d_\sigma + c.c.)$ where $\alpha = {\rm N}, {\rm S}$ and V_α is the tunnelling amplitude. For simplicity we assumed V_α independent from $\mathbf{k}$ and σ.

In the following, we consider the on-site repulsion U is infinite, so processes where the dot level is doubly occupied are excluded. This condition allows to apply the slave boson technique [11]. The dot level is represented as $d^\dagger_\sigma = f^\dagger_\sigma b$, where the fermion f_σ and the boson b describe the singly occupied and empty dot states. Since the dot is either empty or singly occupied, the constraint $b^\dagger b + \sum_\sigma f^\dagger_\sigma f_\sigma = 1$ has to be fulfilled.

In the mean field approximation, the operator b is replaced by a c-number b_0, and the constraint is fulfilled only on average. This is achieved by introducing a chemical potential λ_0 for the pseudo particles. Notice that one ends up with a non interacting-like problem with renormalized parameters, *i.e.*, an energy shift for the dot level $\epsilon_{\rm d} \to \epsilon_{\rm d} + \lambda_0 = \tilde{\epsilon}_{\rm d}$ and a multiplicatively renormalized tunnelling $V_\alpha \to b_0 V_\alpha$.

We discuss the mean field equations and its solution first in equilibrium and then generalize to non-equilibrium. We start from the impurity part of the free energy, which in presence of both normal and superconducting leads is given by

$$F = -T \sum_{\epsilon_n} \mathrm{Tr} \ln[\mathrm{i}\epsilon_n \hat{\sigma}^0 - \tilde{\epsilon}_{\rm d}\hat{\sigma}^z - b_0^2 \hat{\Gamma}(\mathrm{i}\epsilon_n)] + \lambda_0 b_0^2 + \epsilon_{\rm d} - \mu, \tag{2}$$

where ϵ_n is a fermionic Matsubara frequency, $\hat{\sigma}^i$ are the Pauli matrices, and

$$\hat{\Gamma}(\mathrm{i}\epsilon_n) = \sum_{\mathbf{k},\alpha} |V_\alpha|^2 \hat{\sigma}^z \hat{g}_{\mathbf{k},\alpha}(\mathrm{i}\epsilon_n)\hat{\sigma}^z \tag{3}$$

with $\hat{g}_{\mathbf{k},\alpha}$ being the Green's function of the lead α.

By minimizing the free energy with respect to λ_0 and b_0 we find the equations

$$b_0^2 + T \sum_{\epsilon_n} \mathrm{Tr}\left[\hat{\mathcal{G}}(\mathrm{i}\epsilon_n)\hat{\sigma}^z\right] = 0, \tag{4}$$

$$b_0\lambda_0 + b_0 T \sum_{\epsilon_n} \mathrm{Tr}\left[\hat{\mathcal{G}}(\mathrm{i}\epsilon_n)\hat{\Gamma}(\mathrm{i}\epsilon_n)\right] = 0, \tag{5}$$

which have to be solved self-consistently. $\hat{\mathcal{G}}(\mathrm{i}\epsilon_n)$ is the pseudo fermion Green function given by $\hat{\mathcal{G}}(\mathrm{i}\epsilon_n) = [\mathrm{i}\epsilon_n\hat{\sigma}^0 - \tilde{\epsilon}_{\rm d}\hat{\sigma}^z - b_0^2\hat{\Gamma}(\mathrm{i}\epsilon_n)]^{-1}$.

Before presenting a numerical solution of the above equations, it is useful to get some insight from an approximate analytical treatment. The first equation, eq.(4), is the constraint. Since the pseudo fermion level is at maximum singly occupied, the renormalized level is above the Fermi energy. In the Kondo limit, where the occupancy is nearly one, it is found $0 < \tilde{\epsilon}_{\rm d} < b_0^2(\Gamma_{\rm N} + \Gamma_{\rm S})$, *i.e.* $\lambda_0 \approx |\epsilon_{\rm d}|$ and $\tilde{\epsilon}_{\rm d} \approx 0$. The renormalization of the tunnelling amplitude is determined from eq.(5). A trivial solution $b_0 = 0$ always exists. The solutions which minimize the free energy, however, are those with $b_0 \neq 0$. By introducing a flat density of states in the leads and the tunnelling rates $\Gamma_\alpha = \pi N_{0\alpha}|V_\alpha|^2$, the elements of the matrix $\hat{\Gamma}(\mathrm{i}\epsilon_n)$ are $\Gamma_{11} = \Gamma_{22} = -\mathrm{i}\gamma_1$ and $\Gamma_{12} = \Gamma_{21}^* = \gamma_2$, where

$$\gamma_1 = \mathrm{sign}(\epsilon_n)\Gamma_{\rm N} + \Gamma_{\rm S}\frac{\epsilon_n}{\sqrt{\epsilon_n^2 + |\Delta|^2}},\ \gamma_2 = \Gamma_{\rm S}\frac{\Delta}{\sqrt{\epsilon_n^2 + |\Delta|^2}}. \tag{6}$$

Restricting ourselves to zero temperature, we replace the Matsubara sum in eq.(5) by an integral and obtain

$$|\epsilon_{\rm d}| = 4\int_0^W \frac{\mathrm{d}\epsilon}{2\pi}\frac{\gamma_1(\epsilon + b_0^2\gamma_1) + b_0^2|\gamma_2|^2}{(\epsilon + b_0^2\gamma_1)^2 + b_0^4|\gamma_2|^2}, \tag{7}$$

where W is a cut-off of order of the band width. We simplify the integral by approximating γ_1 and γ_2 as

$$\gamma_1 = \begin{cases} \Gamma_{\rm N} & \text{for } \epsilon < \Delta \\ \Gamma_{\rm N} + \Gamma_{\rm S} & \text{for } \epsilon > \Delta \end{cases},\ \gamma_2 = \begin{cases} \Gamma_{\rm S} & \text{for } \epsilon < \Delta \\ 0 & \text{for } \epsilon > \Delta \end{cases} \tag{8}$$

The result is

$$|\epsilon_{\rm d}| = \frac{\Gamma_{\rm N}}{\pi}\ln\frac{(\Delta + b_0^2\Gamma_{\rm N})^2 + b_0^4\Gamma_{\rm S}^2}{b_0^4(\Gamma_{\rm N}^2 + \Gamma_{\rm S}^2)} + \frac{\Gamma_{\rm N} + \Gamma_{\rm S}}{\pi}\ln\frac{W^2}{(\Delta + b_0^2\Gamma_{\rm S} + b_0^2\Gamma_{\rm N})^2}, \tag{9}$$

where we neglect a term proportional to $\Gamma_{\rm S}$, but without any logarithmic factor. For small superconducting gap, *i.e.* Δ much smaller than the Kondo temperature which is given by $T_{\rm K} = b_0^2\Gamma_{\rm N} + b_0^2\Gamma_{\rm S}$, Δ is negligible. One can then easily solve eq.(9) for b_0^2 and obtains the result for two normal leads with total tunnelling rate $\Gamma_{\rm N} + \Gamma_{\rm S}$:

$$b_0^2(\Gamma_{\rm N} + \Gamma_{\rm S}) = W\exp\left(-\frac{\pi}{2}\frac{|\epsilon_{\rm d}|}{\Gamma_{\rm N} + \Gamma_{\rm S}}\right). \tag{10}$$

In the opposite limit, where Δ is much larger than the Kondo temperature, we find

$$b_0^2\sqrt{\Gamma_{\rm N}^2 + \Gamma_{\rm S}^2} = W\exp\left(-\frac{\pi}{2}\frac{|\epsilon_{\rm d}| - (2\Gamma_{\rm S}/\pi)\ln(W/\Delta)}{\Gamma_{\rm N}}\right). \tag{11}$$

The results agree qualitatively with what we expect from scaling arguments for the Anderson model [14]. In the perturbative regime, a logarithmic correction to $\epsilon_{\rm d}$ has been found. In the case of a large gap scaling due to the superconducting electrons stops at energies of the order Δ, giving rise to a finite renormalization of $\epsilon_{\rm d}$, as seen in eq.(11). A similar shift in the dot level has also been reported within the equation-of-motion approach of ref. [10]. In case of a small gap, the superconducting lead contributes to scaling down to low energies, where one enters the strong coupling regime. Presumably, the fixed point is still reached for energies of the order of $T_{\rm K}$, much greater than Δ, so that the Kondo temperature does not depend on Δ, compare eq.(10).

Notice that in presence of normal electrons, we always find a non-trivial solution of the mean field equations. This is to be contrasted with the case of superconducting electrons only, $\Gamma_{\rm N} = 0$, where for large gap only the solution $b_0 = 0$ exists, and there is no Kondo effect [13].

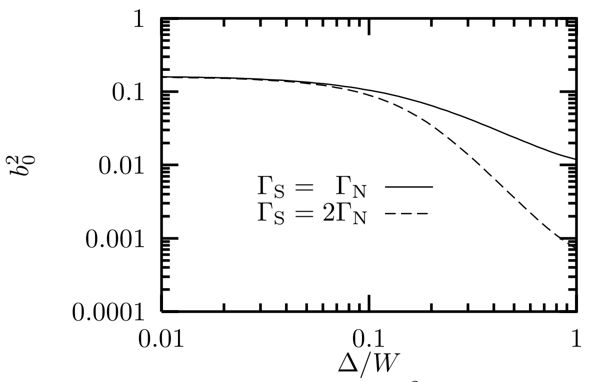

FIG. 1. The mean field parameter b_0^2, which is a measure for the Kondo temperature, as a function of the superconducting gap. For both curves the energy of the dot level is $\epsilon_d = -W/3$ and the total tunnelling rate $\Gamma_S + \Gamma_N = 0.15W$. The Kondo temperature estimated from these parameters is $T_K \approx 0.03W$.

In Fig.1 we report numerical results for b_0^2 as a function of Δ. Instead of a sharp cut-off in the density of states, we used an exponential, $\exp(-\epsilon^2/W^2)$, in our numerics. The bare d-level is $\epsilon_d = -W/3$. For $\Gamma_N + \Gamma_S = 0.15W$ the Kondo temperature is approximately given by $T_K \approx 0.03W$. As long as the gap remains below this energy scale, b_0^2 is almost constant. It drops quickly for $T_K < \Delta < W$, with a region, where $\log b_0^2$ decreases linearly in $\log \Delta$, in agreement with eq.(11).

In a non-equilibrium situation, when a voltage is applied between the two leads, the mean field parameters cannot be obtained by minimizing the free energy. However one can derive self-consistency equations following the arguments given in ref. [16]. These impose the vanishing of so called tadpole diagrams order by order in perturbation theory. The equations read

$$b_0^2 - \mathrm{i}\int \frac{\mathrm{d}\epsilon}{2\pi}\mathrm{Tr}\left[\hat{\mathcal{G}}^<(\epsilon)\hat{\sigma}^z\right] = 0 \quad (12)$$

$$\lambda_0 b_0 - \mathrm{i}b_0\int \frac{\mathrm{d}\epsilon}{2\pi}\mathrm{Tr}\left[\hat{\mathcal{G}}^R(\epsilon)\hat{\Gamma}^<(\epsilon) + \hat{\mathcal{G}}^<(\epsilon)\hat{\Gamma}^A(\epsilon)\right] = 0, \quad (13)$$

where the lesser Green's function $\hat{\mathcal{G}}^<(t,t') = \mathrm{i}\langle\phi^\dagger(t')\phi(t)\rangle$ has been introduced, with $\phi = (f_\uparrow, f_\downarrow^\dagger)$. The lesser and advanced matrix $\hat{\Gamma}$ is defined in analogy to its equilibrium version in eq.(3). To obtain $\hat{\mathcal{G}}^<$, we use the general relation $\hat{\mathcal{G}}^< = \hat{\mathcal{G}}^R\hat{\Sigma}^<\hat{\mathcal{G}}^A$, where at mean-field level $\hat{\Sigma}^< = b_0^2\hat{\Gamma}^<$ and

$$\hat{\Gamma}^<(\epsilon) = -\sum_{\alpha,\mathbf{k}}|V_\alpha|^2\hat{\sigma}^z\left[\hat{g}^R_{\alpha,\mathbf{k}}(\epsilon)\hat{f}_\alpha(\epsilon) - \hat{f}_\alpha(\epsilon)\hat{g}^A_{\alpha,\mathbf{k}}(\epsilon)\right]\hat{\sigma}^z. \quad (14)$$

If the chemical potential of the normal electrode differs by eV from that of the superconductor, the matrices $\hat{f}_\alpha$ have elements $f_{\alpha,11} = f(\epsilon + eV_\alpha^x)$ and $f_{\alpha,22} = 1 - f(-\epsilon + eV_\alpha^x)$, with $V_N^x = V$, $V_S^x = 0$ and $f(\epsilon)$ being the Fermi function. Note that the superconducting lead does not contribute to $\Sigma^<(\epsilon)$ for $\epsilon < \Delta$.

The solution of the mean-field equations, in presence of an external voltage, can then be obtained along the lines of the equilibrium case. As long as $|eV| < T_K$ the solution is almost independent of the voltage. For large voltage, $|eV| \gg T_K$, we found the Kondo peak pinned to the chemical potential at the normal side, *i.e.* $\tilde{\epsilon}_d \to \tilde{\epsilon}_d - eV$, with a decreased width.

Following ref. [10], the Andreev current through an interacting quantum dot is

$$I = 2\mathrm{i}e\int \frac{\mathrm{d}\epsilon}{2\pi}\Gamma_N \mathrm{Tr}\left\{\hat{\sigma}^z\hat{G}^R(\epsilon)\left[\hat{\Sigma}^R(\epsilon), \hat{f}_N(\epsilon)\right]\hat{G}^A(\epsilon)\right\}, \quad (15)$$

where $\hat{G}$ and $\hat{\Sigma}$ are the Green's function and self-energy of the dot electrons. Within the present mean-field approach, $\hat{G} = b_0^2\hat{\mathcal{G}}$. Explicitly, the Andreev current is given by

$$I(V) = \frac{4e^2}{h}\int_{-\infty}^{\infty}\mathrm{d}\epsilon\frac{f(\epsilon - eV) - f(\epsilon + eV)}{2e}G_{NS}(\epsilon) \quad (16)$$

with

$$G_{NS}(\epsilon) = \frac{4(\tilde{\Gamma}_N\tilde{\Gamma}_S)^2}{(\tilde{\epsilon}^2 - \tilde{\epsilon}_d^2 - \tilde{\Gamma}_N^2 - \tilde{\Gamma}_S^2)^2 + 4\tilde{\Gamma}_N^2\tilde{\epsilon}^2} \quad (17)$$

Here the tunnelling rates $\tilde{\Gamma}_{S,N} = b_0^2\Gamma_{S,N}$, and $\tilde{\epsilon} = \epsilon(1 + b_0^2\Gamma_S/\Delta)$. Then one recovers the current formula for a non-interacting quantum dot, with renormalized parameters which are voltage dependent. We see that on resonance, when $\tilde{\epsilon}_d \approx 0$ and $\epsilon = 0$, the small renormalization factor b_0 drops out and the conductance has the same value as in the non-interacting case [18]. The peak strength becomes maximal when $\tilde{\Gamma}_N = \tilde{\Gamma}_S$ with $G_{NS,\max} = 1$, twice the maximum for a N-QD-N system.

Finally, we want to comment on the reliability of our results. The success of slave-boson mean field theory stems from the fact that it captures the Fermi-liquid regime at low temperature. If the N-QD-S system scales to a Fermi liquid at low temperature, G_{NS} as given in eq.(17) is exact in the low temperature, low voltage limit, with the parameters to be determined. Within slave-boson mean field theory Γ_N and Γ_S renormalize equally, although this may no longer be the case when considering higher order corrections. For illustration, we estimate the effect of residual quasiparticle interaction in the limit $\Delta \ll T_K$. By assuming an effective quasiparticle interaction of the form $H_{\mathrm{int}} = \tilde{U}n_\uparrow n_\downarrow$, we find to first order in $\tilde{U}$ no corrections to $\tilde{\Gamma}_N$, while, as one could have expected, repulsive quasiparticle interaction suppresses $\tilde{\Gamma}_S = b_0^2\Gamma_S[1 - (\tilde{U}/\pi T_K)(\Delta/T_K)\ln T_K/\Delta]$.

We studied Andreev tunnelling through a strongly interacting quantum dot, focussing on the extreme low temperature limit. In agreement with a previous study, we found an enhanced Andreev current at low bias voltage, due to the Kondo effect. The zero-bias conductance is maximum with the universal value $G_{\rm NS} = 1$ when the renormalized tunnelling rates $\tilde{\Gamma}_{\rm N}$ and $\tilde{\Gamma}_{\rm S}$ are equal. We identified the ratio $\Delta/T_{\rm K}$ as an important parameter. In the case $\Delta \ll T_{\rm K}$, the Kondo resonance forms as for two normal leads. The condition $\tilde{\Gamma}_{\rm N} = \tilde{\Gamma}_{\rm S}$ coincides with equal bare tunnelling rates $\Gamma_{\rm N} = \Gamma_{\rm S}$. In case of large gap, quasiparticle interaction suppresses $\tilde{\Gamma}_{\rm S}$, nevertheless the conductance maximum condition may be achieved with large bare tunnelling rate $\Gamma_{\rm S} > \Gamma_{\rm N}$.

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