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Title: The calculation of ionospheric absorption with modern computers

Article Type: Earth magnetosphere and Upper Atmosphere

Keywords: Appleton-Hartree formula; more refined quasi-longitudinal approximation; usually employed non-deviative absorption; Booker's rule.

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Abstract: New outcomes are proposed for ionospheric absorption starting from the Appleton-Hartree formula, in its complete form. The range of applicability is discussed for the approximate formulae, which are usually employed in the calculation of non-deviative absorption coefficient. These results were achieved by performing a more refined approximation that is valid under quasi-longitudinal (QL) propagation conditions. The more refined QL approximation and the usually employed non-deviative absorption are compared with that derived from a complete formulation. Their expressions, nothing complicated, can usefully be implemented in a software program running on modern computers. Moreover, the importance of considering Booker's rule is highlighted. A radio link of ground range D = 1000 km was also simulated using ray tracing for a sample daytime ionosphere. Finally, some estimations of the integrated absorption for the radio link considered are provided for different frequencies.

Response to Reviewers:

Dear Advances in Space Research Co-Editor, Dr. Jan Laštovicka,
this cover letter is for re-submitting our manuscript (ASR-D-14-00164),

“The calculation of ionospheric absorption with modern computers",

by the Authors,

Carlo Scotto, Alessandro Settimi.

Please let me know of your decision at your earliest convenience.

Moreover, we would like thanking the Reviewers 1 and 2 for their useful suggestions. We re-wrote several sentences of the paper in order to avoid misunderstandings. We tried to change the paper in all the points that the Reviewers have suggested. We have provided a detailed explanation in the attached reply letters.
Finally, the Reviewers will notice that, in revised manuscript, the minor (or major) revisions occur in words and phrases highlighted by a green-yellow (or red-blue) colour.

With my best regards,
Sincerely yours,
Alessandro Settimi, PhD.

Reviewer #1: The authors present a new approximation of the Appleton-Hartree radio wave refractive index to estimate non-deviative absorption. They provide a good introduction into the classical A-H theory and its generalization by Sen and Wyller as well as by Budden.

The derived non-deviative approximation of the absorption coefficient is compared with the complete A-H formulation and its quasi-longitudinal approximation. The application of the non-deviative approximation in ray path calculations results in large differences compared to the other formulations. The manuscript in its present form does not explain explicitly under what conditions the proposed approximations will provide results comparable with the complete A-H formulation and its quasi-longitudinal approximation.

Authors: We agree with the Reviewer. This (major) comment was included throughout the manuscript by some lines highlighted in a yellow colour.

Page 13: “Only if $X << 1$ ($\omega << \omega_p$), when the ray wave is assumed in propagation conditions, away from the reflection, then $k_{ord} = k_{ord\text{-}long} = k_{ord\text{-}long[NoDev]}$, i.e. general formulation, a more refined QL approximation and the usually employed non-deviative absorption provide similar values for the local absorption coefficient”.

See also pages 3, 15: ...

Specific comments

P1, L 44/45: The sentence is misleading - if so correct no need would exist for this study since the complete formulation is easy to implement.

Authors: This (minor) comment was included throughout the manuscript by some lines highlighted in a green colour.

Page 1: “The more refined QL approximation and the usually employed non-deviative absorption are compared with that derived from a complete formulation. Their expressions, nothing complicated, can usefully be implemented in a software program running on modern computers”.

See also pages 4, 15: ...

P5 , L 43: More detailed information are missing how a potential user can obtain appropriate data of the effective collision frequency using the monoenergetic collision frequency as defined on P4, L43. The presentation of a formula would be desirable.

Authors: This (major) comment was included throughout the manuscript by some lines highlighted in a yellow colour.

Page 5: "Detailed information about data of the pressure can be obtained using the global climatology of atmospheric parameters from the Committee on Space Research (COSPAR) International Reference Atmosphere (CIRA-86) project. As recommended by the COSPAR, the CIRA-86 provides empirical models of atmospheric temperatures and densities. A global climatology of atmospheric temperature, zonal velocity and geopotential height was derived from a combination of satellite, radiosonde and ground-based measurements (Rees, 1988; Rees et al., 1990; Keating, 1996). The reference atmosphere extends from pole to pole and 0-120 km. CIRA-86 consists of tables of the monthly mean values of temperature and zonal wind with almost global coverage (80°N - 80°S). Two files were compiled by Fleming et al. (1988), one in pressure coordinates including also the geopotential heights, and one in height coordinates including also the pressure values".

Page 6: Equation (2) and following comments.

See also page 11: ...

P9/10, section 5: The description of the computation of radio wave absorption for a model ionosphere is insufficient: any information about the use of collision frequency is missing, the use of a flat earth propagation geometry is highly questionable. The propagation along the magnetic meridian is an optimum choice for the proposed approximation but the differences between the proposed approximation and the other two models are still large.

Authors: This (major) comment was included throughout the manuscript by some lines highlighted in a red colour.

Page 4: "Moreover, in this paper, an eikonal based ray tracing procedure was used to evaluate the ray path linking two sites 1000 km apart. Some limitations were imposed for simplifying the ray tracing computation. Azzarone et al. (2012) and Settimi et al. (2013, 2014) have already overcome these limitations, applying the more elaborate Haselgrove's (1955) ray theory and the Jones and Stephenson's (1975) method for ray tracing, which takes into account even the curvature of Earth's surface, and that the ionospheric medium can be characterized by large horizontal gradients.

Finally, in the paper, it is proved our ultimate purpose of underlining that, at any rate in some practical applications, the more refined QL approximation can be used, while the usually employed non-deviative absorption can lead to significant errors in the estimation of absorption. The expression of such QL approximation, nothing complicated, can usefully be implemented in a software program running on modern computers".

Pages 11-12: "... Some limitations were imposed for simplifying the ray tracing computation. Firstly, if the curvature of Earth's surface is ignored, then the flat earth geometry can be applied for wave propagation ...".

Page 13-14: "…The ray paths were assumed to be lying along the magnetic meridian. The wave propagation along the magnetic meridian is an optimum choice just for highlighting how much are even large the differences between the general formulation, the more refined QL approximation and the usually employed non-deviative absorption …".

See also pages 1, 2, 22: …

The quality of figures 1 to 4 is not sufficient for a publication.

Authors: All the new figures were attached as tiff files of 300 dpi resolution.

Figs. 3 and 4: The *x*-axis label is missing.

Authors: The $X=(\omega_p/\omega)^2$ symbol was inserted as the *x*-axis label.

Reviewer #2: The authors give the example of the calculation of the absorption using electron density profiles calculated from the global model of the ionosphere IRI for high solar activity conditions. Have such calculations for low solar activity? If so, is it possible to add data to the results in this article?

Authors: We agree with the Reviewer. This (major) comment was included throughout the manuscript by some lines highlighted in a blue colour.

The calculation of absorption was performed for both low and high solar activity conditions. An old bug has been fixed, so that now the new simulations are correctly computing old and new data results.

Page 11: "The June 15 at12.00 local time (LT) was taken as the input parameter for the IRI and CIRA-86 models, assuming either a low (R_{12} = 10) or a high (R_{12} = 100) solar activity level, where R_{12} is the monthly smoothed sunspot number".

Page 13: "The ray paths computed for a 1000 km radio link, at different frequencies, are plotted in Fig. 5", and following comments …

Page 14: "The corresponding absorption computed for the same 1000 km radio link, at different frequencies, is shown in Fig. 6", and following comments ...

See also pages 21-22: ...

REVIEW

Manuscript Number: ASR-D-14-00164

Title: The calculation of ionospheric absorption with modern computers

Article Type: EM

P. 4

Line 38 "ν" replaced by "*ν*" ("italic")

Line 43 "$s^{-1}\cdot N^{-1}$" replaced by "$s^{–1}\cdot N^{–1}$"

Line 56 "ν" replaced by "*ν*" ("italic")

P. 5

in formula (1) "ρ_N" replaced by "ρ_{N}"

Line 21 "ν" replaced by "*ν*" ("italic")

P. 6

in formula (3) "-" replaced by "–"

Line 37 "with" replaced by "where"

Line 40 "ω_p" replaced by "ω_{p}"

Line 43 "ε_0" replaced by "ε_{0}"

Line 48 "Y_{T}" and "Y_{L}" replaced by "Y_T" and "Y_L" (like in formula (5))

Line 56 "ν/ω" replaced by "*ν*/*ω*" ("italic")

Line 58 in n_{ord} = … and n_{ext} = … "-" replaced by "–"

P. 7

Line 9 "ν" replaced by "*ν*" ("italic")

Line 12 "ν" replaced by "*ν*" ("italic")

Line 12 "Y_{L} … Y" replaced by "Y_L … *Y*"

P.10

Line 9 requires a reference to IRI model

Line 9 "15 of June at12.00" replaced by "June 15 at 12.00"

Line 26 in expression for *B* "-" replaced by "–"

Line 26 "Tesla" replaced by "T"

Line 46 "Y_{T}" and "Y_{L}" replaced by "Y_T" and "Y_L"

Line 48 "ν = 105 s^{-1}" replaced by "ν = 105 $s^{–1}$"

P. 11

Line 19 "-" replaced by "–"

P. 12

Line 31 "χ" replaced by "χ" ("italic")

P. 16

In captions for Fig. 1 – 4 "s^{-1}" replaced by "$s^{–1}$"

In Figure 5 are required notations on the axes

Authors: All these (minor) comments were included throughout the manuscript by some signs/numbers/symbols highlighted in a green colour.

Highlights

- New outcomes for ionospheric absorption from the Appleton-Hartree formula.
- Their expressions can usefully be implemented in a software program.
- The importance of considering Booker's rule is highlighted.
- The integrated absorption for a radio link is provided for different frequencies.

The calculation of ionospheric absorption with modern computers

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Abstract

New outcomes are proposed for ionospheric absorption starting from the Appleton-Hartree formula, in its complete form. The range of applicability is discussed for the approximate formulae, which are usually employed in the calculation of non-deviative absorption coefficient. These results were achieved by performing a more refined approximation that is valid under quasi-longitudinal (QL) propagation conditions. The more refined QL approximation and the usually employed non-deviative absorption are compared with that derived from a complete formulation. Their expressions, nothing complicated, can usefully be implemented in a software program running on modern computers. Moreover, the importance of considering Booker's rule is highlighted. A radio link of ground range $D = 1000$ km was also simulated using ray tracing for a sample daytime ionosphere. Finally, some estimations of the integrated absorption for the radio link considered are provided for different frequencies.

Keywords:

Appleton-Hartree formula; more refined quasi-longitudinal approximation; usually employed non-deviative absorption; Booker's rule.

1. Introduction

When the ionospheric radio sounding technique was developed, the first recorded ionograms showed variations in amplitude of the received signal. It was immediately evident that ionospheric absorption occurred at lower altitudes, below those at which the electron density was sufficient to give rise to reflection (Pillet, 1960). Initially it was assumed that this absorption took place in the E region, and several studies were carried out, recording the amplitude of waves reflected from the F region, for both vertical and oblique incidence (Booker, 1935; White and Brown, 1936).

However, already in 1930, Appleton and Ratcliffe measured echo intensity after reflection from the E region, and concluded that the absorption occurs far below the level of reflection. In this way they discovered the existence of a distinct region, which they named the D region.

There was also significant progress in theoretical studies, including the contribution of Booker (1935). He demonstrated that a radio wave can be absorbed even at a level where the refractive index is slightly different from the unit. In practice, this region corresponded to the D region previously proposed by Appleton and Ratcliffe. Other experimental results confirmed the hypothesis of the existence of the D region, with the absorption properties mentioned above. For example, Farmer and Ratcliffe (1935) found a sharp increase in the reflection coefficient during the evening hours, which was attributed to the decreasing absorption coefficient in the D region at dusk.

Ever since the first formulation of the magneto-ionic theory, which is controversially attributed to Appleton and Ratcliffe (1930), or Lassen (1926), it was clear that collisions between electrons and neutral molecules influenced the local absorption coefficient of radio waves.

The magneto-ionic theory, in principle, allowed direct derivation of the local absorption coefficients for both the ordinary and the extraordinary, while also taking into account the presence of the magnetic field and collisions. These details can be studied by referring to the well known early publications of Ratcliffe (1959) and Budden (1961).

However, the formulae that can be derived are complicated and difficult to interpret. The focus of interest was therefore an approximate formula, which will be discussed in the following sections. This takes into account that, in most cases propagation takes place in QL approximation, and for non-deviative absorption $\mu \approx 1$ can be assumed, μ being the real part of the refractive index *n*. It was thus not considered necessary to substantially revise the theory of non-deviative absorption.

In high frequency (HF) radio propagation, the application of the approximate formula has also been proposed in recent studies, to assess for example the state of the D and E regions by establishing the local absorption coefficients of the ordinary and extraordinary components of radio waves, and making use of space-based facilities (Zuev and Nagorskiy, 2012). The effects of HF absorption in the ionosphere of Mars were also numerically simulated using the same approximate formula (Withers, 2011; Varun et al., 2012).

In the present paper, it is proposed that this mode of operation is no longer justified in all the applications, like for example riometry. A typical frequency used with this technique is 30 MHz, with which absorption changes of about 0.1 dB can be measured. Instead, it is preferable to use the exact formulation or even a more refined QL approximation for all the applications designed in the HF band, such that $\omega >> \omega_p$, ω being the angular frequency of the radio wave considered and ω_p the plasma frequency.

Moreover, in this paper, an eikonal based ray tracing procedure was used to evaluate the ray path linking two sites 1000 km apart. Some limitations were imposed for simplifying the ray tracing computation. Azzarone et al. (2012) and Settimi et al. (2013, 2014) have already overcome these limitations, applying the more elaborate Haselgrove's (1955) ray theory and the Jones and Stephenson's (1975) method for ray tracing, which takes into account even the curvature of Earth's surface, and that the ionospheric medium can be characterized by large horizontal gradients.

Finally, in the paper, it is proved our ultimate purpose of underlining that, at any rate in some practical applications, the more refined QL approximation can be used, while the usually employed non-deviative absorption can lead to significant errors in the estimation of absorption. The expression of such QL approximation, nothing complicated, can usefully be implemented in a software program running on modern computers.

2. The classical and generalized magneto-ionic theories

In the initial formulation of magneto-ionic theory, a frictional term is utilized that does not depend on the root-mean-square electron velocity and the electron velocity distribution. It represents a first approximation of the effective collision frequency due to the collisions between electrons and neutrals. Later, several studies were published that strived to improve this aspect of the theory.

Originally, Phelps and Pack (1959) measured the collision cross-section σ for electrons in the nitrogen N_2 — the most abundant atmospheric constituent up to 100 km — establishing that it is proportional to the root-mean-square electron velocity v_{rms}. Consequently, Sen and Wyller (1960) generalized the Appleton-Hartree magneto-ionic theory including a Maxwellian velocity distribution of the electrons (a), and extending the findings of Phelps and Pack (1959) to all constituents of air (b). However, Sen and Wyller (1960) made several key mistakes, later remedied by Manchester (1965). A valuable approximation of the generalized magneto-ionic theory exists in Flood (1980).

The momentum collision frequency ν of electrons with neutrals can be simply expressed by the product of pressure p times a constant α. Based on both laboratory and ionospheric data α can be estimated as $\alpha = 6.41 \cdot 10^5\ m^2 \cdot s^{-1} \cdot N^{-1}$ (Thrane and Piggott, 1966; Friedrich and Torkar, 1983; Singer et al., 2011).

Detailed information about data of the pressure can be obtained using the global climatology of atmospheric parameters from the Committee on Space Research (COSPAR) International Reference Atmosphere (CIRA-86) project. As recommended by the COSPAR, the CIRA-86 provides empirical models of atmospheric temperatures and densities. A global climatology of atmospheric temperature, zonal velocity and geopotential height was derived from a combination of satellite, radiosonde and ground-based measurements (Rees, 1988; Rees et al., 1990; Keating, 1996). The reference atmosphere extends from pole to pole and 0-120 km. CIRA-86 consists of tables of the monthly mean values of temperature and zonal wind with almost global coverage (80°N - 80°S). Two files were compiled by Fleming et al. (1988), one in pressure coordinates including also the geopotential heights, and one in height coordinates including also the pressure values.

The atmosphere in the E and D layers consists mainly of nitrogen N_2 (about 78%), with atomic and molecular oxygen O_2 as the next most important constituents. The relatively large cross section for N_2 makes it likely, as a first-order approximation, that the height variation of collision frequency ν is proportional to the partial pressure of the N_2. Experiments show that the cross section for O_2 also varies by the square root of T so that the two contributions can be combined (Davies, 1990).

When there is complete mixing of the atmospheric gases the following relationship holds:

$$\frac{dp}{p} = \frac{d\rho_{\mathrm{N}}}{\rho_{\mathrm{N}}} + \frac{dT}{T} = -\frac{dH}{H}, \tag{1}$$

where p is the total pressure, ρ_{N} the number density, T the absolute temperature of molecules, and $H = k_{\mathrm{B}}T/mg$ the atmospheric scale height, with g the gravity acceleration and m the mean molecular mass. For this reason, the collision frequency ν varies by the height h above ground as (Thrane and Piggott, 1966):

$$\nu(h) = \nu_0 \frac{p(h)}{p_0}. \tag{2}$$

Theoretically, a decreasing exponential law holds in an atmosphere which is constant in composition (Budden, 1961): $\nu(h) = \nu_0\exp[-(h-h_0)/H]$, where ν_0 is a constant, i.e. $\nu_0 = \nu(h_0)$, and h_0 is the height corresponding to the maximum electron density N_0, i.e. $N_0 = N(h_0)$. On equal terms, this maximum occurs for a null solar zenith angle χ, i.e. $\chi = 0$. In practice, H takes different values at different levels, and the law can only be expected to hold over ranges of h so small that H may be treated as constant. Experimentally, in the thermosphere (above about 100 km) CIRA-86 is identical with the Mass-Spectrometer-Incoherent-Scatter (MSIS-86) model (Hedin, 1987). In the lower part of thermosphere (at 120 km altitude) CIRA-86 was merged with MSIS-86.

According to Budden (1965), while the generalized theory (Sen and Wyller, 1960) is important in the detailed quantitative interpretation of certain experiments, for most practical radio propagation problems the classical theory (Appleton and Chapman, 1932) is adequate, especially when appropriate values are used for the effective collision frequency.

3. Absorption theory in general formulation

It is known that, in general, the integral absorption of a radio wave through the ionosphere can be described in differential form by the exponential decrease in the field amplitude *E*, which can be expressed using a relationship of the type:

$$E(s) = E_0 \cdot \exp(-k \cdot s), \tag{3}$$

s being the curvilinear abscissa along the ray path, and *k* the local absorption coefficient. This can be expressed by the following relation:

$$k = \omega \cdot \chi / c, \tag{4}$$

where χ is the imaginary part of complex refractive index $n = \mu - i \cdot \chi$ and *c* is the velocity of light. Both μ and χ can be derived from the Appleton-Hartree equation:

$$n^2_{\mathrm{ord,\,ext}} = 1 - \frac{X}{1 - iZ - \frac{1}{2}\frac{Y_{\mathrm{T}}^2}{(1 - X - iZ)} \pm \sqrt{\frac{Y_{\mathrm{T}}^4}{4(1 - X - iZ)^2} + Y_{\mathrm{L}}^2}}, \tag{5}$$

where:

$X = \omega_{\mathrm{p}}^2/\omega^2$ (where ω is the angular frequency of the radio wave, $\omega_{\mathrm{p}} = \sqrt{Ne^2/m\varepsilon_0}$ the plasma frequency, *N* the profile of electron density, *m* the electron mass, *e* the electron charge, and ε_0 the constant permittivity of vacuum);

$Y_T = Y \cdot \sin(\theta)$, $Y_L = Y \cdot \cos(\theta)$ (where θ is the angle between the wave vector and the Earth's magnetic field), and $Y = \omega_B/\omega$ ($\omega_B = Be/m$ being the gyro-frequency, and B the amplitude of the Earth's magnetic field);

$Z = \nu/\omega$ (where ν is the collision frequency).

This equation gives two indices of refraction $n_{ord} = \mu_{ord} - i \cdot \chi_{ord}$ and $n_{ext} = \mu_{ext} - i \cdot \chi_{ext}$ for the known birefringence of ionospheric plasma. The two refractive indices are obtained from Eq. (5) through the choice of positive or negative signs, which must be decided applying the so-called Booker's rule. Once the critical frequency is defined $\omega_c = (\omega_B/2) \cdot \sin^2(\theta)/\cos(\theta)$, this rule states that, to achieve continuity of μ_{ord} (μ_{ext}) and χ_{ord} (χ_{ext}), if $\omega_c/\nu > 1$, the positive (negative) sign in Eq. (5) must be adopted both for $X < 1$ and for $X > 1$; while, if $\omega_c/\nu < 1$, the positive (negative) sign for $X < 1$ and negative (positive) for $X > 1$ must be adopted.

It is clearly not a simple task to analytically derive μ_{ord} (μ_{ext}) and χ_{ord} (χ_{ext}) from Eq. (5). However, this is facilitated by some commercial commercial mathematical software tool packages able to perform symbolic computation. Using those tools, it is easy to obtain analytical expressions for μ_{ord} (μ_{ext}) and χ_{ord} (χ_{ext}), which are extremely complicated, difficult to interpret, and not worth reporting, but nevertheless providing relationships that can be effectively and easily introduced into calculation algorithms.

Moreover, from χ_{ord} and χ_{ext}, applying Eq. (4), gives k_{ord} and k_{ext}, with obvious symbol meanings.

4. The theory of non-deviative absorption

If the QL propagation approximation is assumed to be valid, it holds that:

$$\frac{Y_{\mathrm{T}}^{4}}{4\cdot\left[(1-X)^{2}+Z^{2}\right]} << Y_{\mathrm{L}}^{2}. \tag{6}$$

From this relationship, considering that:

$$Z << 1, \tag{7}$$

then $\theta << 1 \rightarrow Y_{\mathrm{L}} \cong \mathrm{Y}$ and Eq. (5) can be reduced to the simplified form:

$$n_{\text{ord-long,ext-long}}^{2} = (\mu_{\text{ord-long,ext-long}} - i\chi_{\text{ord-long,ext-long}})^{2} = 1 - \frac{X}{1 - iZ \pm Y}. \tag{8}$$

Once some mathematical steps have been performed, Eq. (8) is split into two equations, one for the real part,

$$\mu_{\text{ord-long,ext-long}}^{2} - \chi_{\text{ord-long,ext-long}}^{2} \cong \mu_{\text{ord-long,ext-long}}^{2} = 1 - \frac{X(1 \pm Y)}{(1 \pm Y)^{2} + Z^{2}}, \tag{9}$$

and one for the imaginary part,

$$2\mu_{\text{ord-long,ext-long}} \cdot \chi_{\text{ord-long,ext-long}} = \frac{X \cdot Z}{(1 \pm Y)^{2} + Z^{2}}. \tag{10}$$

Under the simplifying condition $\mu << \chi$, once the real part μ of the refractive index is calculated from Eq. (9), the imaginary part χ of the refractive index can be derived from Eq. (10), by a simple passage, obtaining:

$$\chi_{\text{ord-long,ext-long}} = \frac{1}{2\mu_{\text{ord-long,ext-long}}} \frac{X \cdot Z}{(1 \pm Y)^2 + Z^2} . \tag{11}$$

This relation, by introducing Eq. (4), gives:

$$k_{\text{ord-long,ext-long}} = \frac{1}{2\mu_{\text{ord-long,ext-long}} c} \frac{Ne^2}{m\varepsilon_0} \frac{\nu}{(\omega \pm \omega_{\text{B}})^2 + \nu^2} . \tag{12}$$

It is obvious that this formula is used in practice only assuming (in non-deviative absorption approximation): $\mu_{\text{ord-long}} \approx 1$ ($\mu_{\text{ext-long}} \approx 1$). The local absorption coefficient, which is obtained from Eq. (12) by replacing $\mu_{\text{ord-long}} \approx 1$ ($\mu_{\text{ext-long}} \approx 1$), will be indicated as $k_{\text{ord-long[NoDev]}}$, ($k_{\text{ext-long[NoDev]}}$). The positive sign has to be applied to the ordinary and the negative to the extraordinary. Note that Eq. (12) is valid in QL conditions. In this case, similarly to what happens for longitudinal propagation, Booker's rule should not be considered. If not performing the approximation $\mu_{\text{ord-long}} \approx 1$ ($\mu_{\text{ext-long}} \approx 1$), from Eq. (8) it is possible to derive relationships for $\mu_{\text{ord-long}}$ ($\mu_{\text{ext-long}}$) and $\chi_{\text{ord-long}}$ ($\chi_{\text{ext-long}}$). In this case complicated expressions are obtained, difficult to interpret and not worth reporting. Besides, applying $\chi_{\text{ord-long}}$ ($\chi_{\text{ext-long}}$) it is possible to compute $k_{\text{ord-long}}$ ($k_{\text{ext-long}}$) through Eq. (4).

As is explained clearly in Ratcliffe's well known early publication (1959), in a very wide range of ω, ν, and θ, propagation occurs in QL conditions. In practice, QL conditions are always verified, except for $X \sim 1$. Eq. (12), considering $\mu \approx 1$, is therefore often used to calculate the non-deviative absorption coefficients of the ordinary and extraordinary rays except when $X \sim 1$, for example, for frequencies $\omega >> \omega_p$ ($X << 1$).

A better approximation for k, also limited to the case of QL conditions, can be derived using Eq. (8), and deducing from this $\chi_{\text{ord-long}}$ (and $\mu_{\text{ord-long}}$), from which k can be derived using Eq. (4). In this case, complicated expressions are obtained, difficult to interpret, and not worth reporting, but that can usefully be incorporated inside commercial mathematical software tool packages.

5. The computation of absorption in a modelled ionosphere

It is interesting to make further comparisons of full equations with approximations using well-known literature models of electron density and collision frequency. For practical applications, radio wave absorption can be expressed in decibels (dB). As an example, a numerical simulation calculates the output of absorption, having as inputs: an electron density N obtained from the International Reference Ionosphere (IRI) model (Bilitza, 1990; Bilitza and Reinisch, 2008), and a collision frequency ν proportional to the pressure data obtained from the CIRA-86 model. The June 15 at12.00 local time (LT) was taken as the input parameter for the IRI and CIRA-86 models, assuming either a low (R_{12} = 10) or a high (R_{12} = 100) solar activity level, where R_{12} is the monthly smoothed sunspot number. Basing on these N and ν models, an eikonal based ray tracing procedure was used to evaluate the ray path linking two sites 1000 km apart. Some limitations were imposed for simplifying the ray tracing computation. Firstly, if the curvature of Earth's surface is ignored, then the flat earth geometry can be applied for wave propagation. Secondly, if the ionospheric medium is characterized by small horizontal gradients, then the azimuth angle of transmission can be assumed to be a constant along the great circle path (Davies, 1990). All the more, considering a flat layering ionosphere, so without any horizontal gradient, the profiles of electron density *N(h)* and collision frequency *ν(h)* are assumed to be functions only of the height. At the limit, a single profile for both *N(h)* and *ν(h)* recurs throughout the latitude and longitude grid of points involved in the ray tracing computation.

6. Results and discussion

In Fig. 1(a)-(d), μ_{ord} e $\mu_{ord\text{-}long}$ (μ_{ext} e $\mu_{ext\text{-}long}$) are reported for different values of the θ angle, having considered a radio wave with $Y = (Y_T^2 + Y_L^2)^{1/2} = 0.5$, frequency $f = 4$ MHz, and a minimal collision frequency $\nu = 10^5$ s^{-1}, typical of the high D region around an altitude of 90 km, which maximises the absorption variances among the general formulation, QL, and non-deviative approximations. The curves are shown with different colours, as indicated in the figure legend. In essence, it demonstrates the possibility of approximating μ_{ord} with $\mu_{ord\text{-}long}$ and μ_{ext} with $\mu_{ext\text{-}long}$, as long as conditions do not require changing sign for $X = 1$, as specified in Booker's rule. This fact is reflected in the similar curves χ_{ord} and $\chi_{ord\text{-}long}$ (χ_{ext} and $\chi_{ext\text{-}long}$), which are shown in Fig. 2 (a)-(d). In fact, when Eq. (5) is approximated to Eq. (8) an assumption more limiting than QL conditions is made, considering the propagation as perfectly longitudinal. Now, to study the propagation, it is particularly important to investigate the conditions for which $\mu = 0$, when ionospheric reflection takes place. In this regard, it is known that, in the absence of collisions, even a small value of the θ angle is sufficient to ensure that the ordinary ray has critical frequency of reflection for $X = 1$ and the extraordinary for $X = 1 \pm Y$. Only for $\theta = 0$ is the ordinary ray reflected in $X = 1 + Y$ and the extraordinary in $X = 1 - Y$. In other words, in the absence of collisions, it is only for $\theta = 0$ that propagation can be considered, with good reason, to be perfectly longitudinal. Effectively, in the presence of collisions, if the condition $X \sim 1$ is not verified, propagation occurs in QL conditions.

However, the same reflection conditions of perfectly longitudinal propagation occur only if: ω_c/v < 1. This can be verified by observing the graphs of μ_{ord} and $\mu_{ord\text{-}long}$ (μ_{ext} and $\mu_{ext\text{-}long}$) [Fig. 1 (a) - (d)]. The same behaviour is observed in the graphs of χ_{ord} and $\chi_{ord\text{-}long}$ (χ_{ext} and $\chi_{ext\text{-}long}$) [Fig. 2 (a) - (d)]. Therefore, even if the range of QL conditions is very wide, the possibility of considering propagation to be perfectly longitudinal, and approximating Eq. (5) with Eq. (8), is limited by the condition $\omega_c/v < 1$. This is evident in Figs. 3 (d) and 4 (d), where $\omega_c/v > 1$. These figures show for example that for $X \sim 0.5$, k_{ord} and $k_{ord\text{-}long}$ deviate appreciably from $k_{ord\text{-}long[NoDev]}$. Only if $X << 1$ ($\omega << \omega_p$), when the ray wave is assumed in propagation conditions, away from the reflection, then $k_{ord} = k_{ord\text{-}long} = k_{ord\text{-}long[NoDev]}$, i.e. general formulation, a more refined QL approximation and the usually employed non-deviative absorption provide similar values for the local absorption coefficient.

Generally, the eikonal based ray tracing has to assume the absence of geomagnetic field. Conversely, the presence of the geomagnetic field has to be considered when computing absorption. The geomagnetic field was assumed as $B \approx 4.5 \cdot 10^{-5}$ T. Absorption was computed for the ordinary ray along the ray paths. The ray paths were assumed to be lying along the magnetic meridian. The wave propagation along the magnetic meridian is an optimum choice just for highlighting how much are even large the differences between the general formulation, the more refined QL approximation and the usually employed non-deviative absorption. The ray paths computed for a 1000 km radio link, at different frequencies, are plotted in Fig. 5. The low ray paths occur in a narrower HF band (3-11 MHz) assuming low solar activity, and in a wider HF band (2-14 MHz) assuming high solar activity. The apogee of low ray paths reaches a similar altitude ($h \approx 100$ km) for both solar activity levels. Indeed, that altitude corresponds to the bottom of E-layer (reflecting the low ray paths as a mirror). Instead, the high ray paths occur in the HF band (9-14 MHz), similarly for both the solar activity levels. The apogee of high ray paths reaches a higher altitude (h

> 200 km, throughout the HF band 9-14 MHz) assuming low solar activity, and a lower altitude (100 < h < 200 km, especially for lowest HFs 9-11 MHz) assuming high solar activity. Indeed, the F1-layer, which appears for both the solar activity levels, is characterized by a higher critical plasma frequency assuming high solar activity. The corresponding absorption computed for the same 1000 km radio link, at different frequencies, is shown in Fig. 6. Firstly, the refined QL approximation and the usually employed non-deviative approximation provide comparable values for low ray paths, throughout the whole HF band (2-14 MHz). Indeed, aside from the reflection due to the bottom of E-layer, the low ray propagation is just non-deviative, such that the real part of refractive index can be assumed as unitary, i.e. $\mu = 1$. Secondly, the refined QL approximation provides more accurate values than the usually employed non-deviative approximation for high ray paths, especially for lowest HFs (9-11 MHz). Indeed, the high ray propagation in the F1-F2 layers is even deviative, when that the real part of refractive index must be assumed less than unit, i.e. $\mu < 1$. Thirdly, the refined QL approximation and the usually employed non-deviative approximation provide underestimates compared to the general formulation, assuming either a low or a high solar activity, and both the approximations tend to the general formulation, especially for highest HFs (12-14 MHz). Indeed, the general formulation takes into account the deviative and non-deviative propagations occurring across the E and F1-F2 layers, where the real part of refractive index is generally assumed as $\mu \leq 1$. However, Figs. 5 and 6 prove our ultimate purpose of underlining that, at any rate in some practical applications, the more refined QL approximation can be used, while the usually employed non-deviative absorption can lead to significant errors in the estimation of absorption.

7. Summary

The main results can be summarized as follows:

1) Commercial mathematical software tool packages make it easy to obtain exact values of k_{ord} (k_{ext}), which are obtained from χ_{ord} (χ_{ext}) [Eq. (4)] and applying the Appleton-Hartree formula [Eq. (5)].

2) The local absorption coefficient $k_{\mathrm{ord\text{-}long[NoDev]}}$ ($k_{\mathrm{ext\text{-}long[NoDev]}}$), calculated by Eq. (12) setting $\mu_{\mathrm{ord\text{-}long[NoDev]}} \approx 1$ ($\mu_{\mathrm{ext\text{-}long[NoDev]}} \approx 1$), is an acceptable approximation only for $X << 1$ ($\omega << \omega_p$).

3) A better approximation for $k_{\mathrm{ord\text{-}long}}$ ($k_{\mathrm{ext\text{-}long}}$) can be obtained from $\chi_{\mathrm{ord\text{-}long}}$ ($\chi_{\mathrm{ext\text{-}long}}$), which are calculated from Eq. (8) without setting $\mu_{\mathrm{ord\text{-}long[NoDev]}} \approx 1$ ($\mu_{\mathrm{ext\text{-}long[NoDev]}} \approx 1$). The expression of such QL approximation, nothing complicated, can usefully be implemented in a software program running on modern computers.

4) It is important to consider the application of Booker's rule, applied equally to the calculation of μ and χ, which is required when calculating ionospheric absorption.

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Figure captions

Figure 1. The graphs μ_{ord}, $\mu_{ord\text{-}long}$, μ_{ext} and $\mu_{ext\text{-}long}$ for different values of the θ angle, considering a radio wave with Y = 0.5, frequency f = 4 MHz, a collision frequency $\nu = 10^5$ s^{-1}. The curves are shown in different colours, as reported in the legend.

Figure 2. The graphs χ_{ord}, $\chi_{ord\text{-}long}$, χ_{ext} and $\chi_{ext\text{-}long}$ for different values of the θ angle, considering a radio wave with Y = 0.5, frequency f = 4 MHz, a collision frequency $\nu = 10^5$ s^{-1}. The curves are shown in different colours, as reported in the legend.

Figure 3. The graphs k_{ord}, $k_{ord\text{-}long}$ and $k_{ord\text{-}long[NoDev]}$ for different values of the θ angle, considering a radio wave with Y = 0.5, frequency f = 4 MHz, a collision frequency $\nu = 10^5$ s^{-1}. The curves are shown in different colours, as reported in the legend.

Figure 4. The graphs k_{ext}, $k_{ext\text{-}long}$ and $k_{ext\text{-}long[NoDev]}$ for different values of the θ angle, considering a radio wave with Y = 0.5, frequency f = 4 MHz, a collision frequency $\nu = 10^5$ s^{-1}. The curves are shown in different colours, as reported in the legend.

Figure 5. The ray paths computed for a 1000 km radio link, at different frequencies. The simulations are based on the eikonal equation, using an IRI derived ionosphere for June 15 at 12.00 LT, and assuming either a low (R_{12} = 10) or a high (R_{12} = 100) solar activity level. Both low and high ray paths can be distinguished.

Figure 6. With reference to Fig. 5, the corresponding absorption computed for the same 1000 km radio link, at different frequencies, assuming either a low (R_{12} = 10) or a high (R_{12} = 100) solar activity level, for both low and high ray paths. The simulations are based on general formulation, a more refined QL approximation and the usually employed non-deviative absorption.

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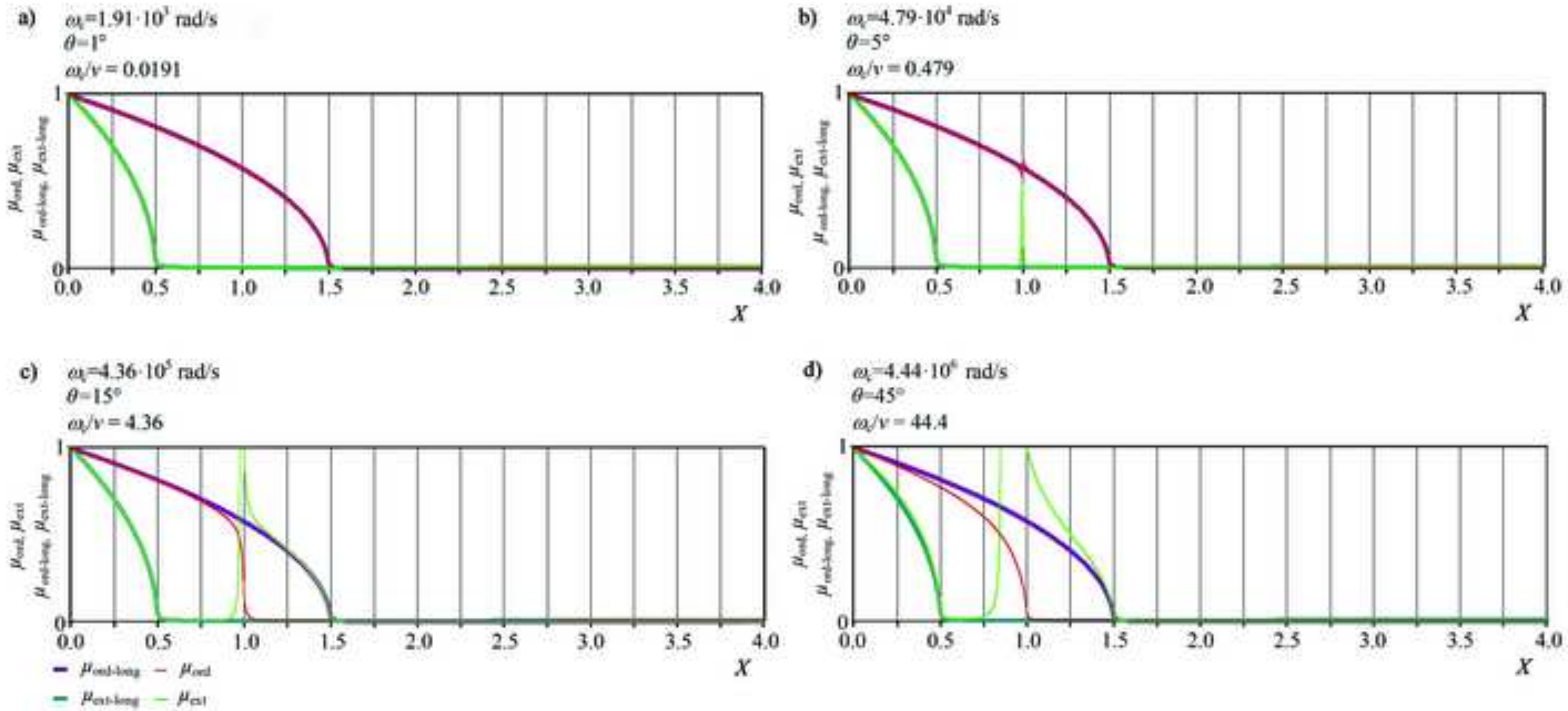
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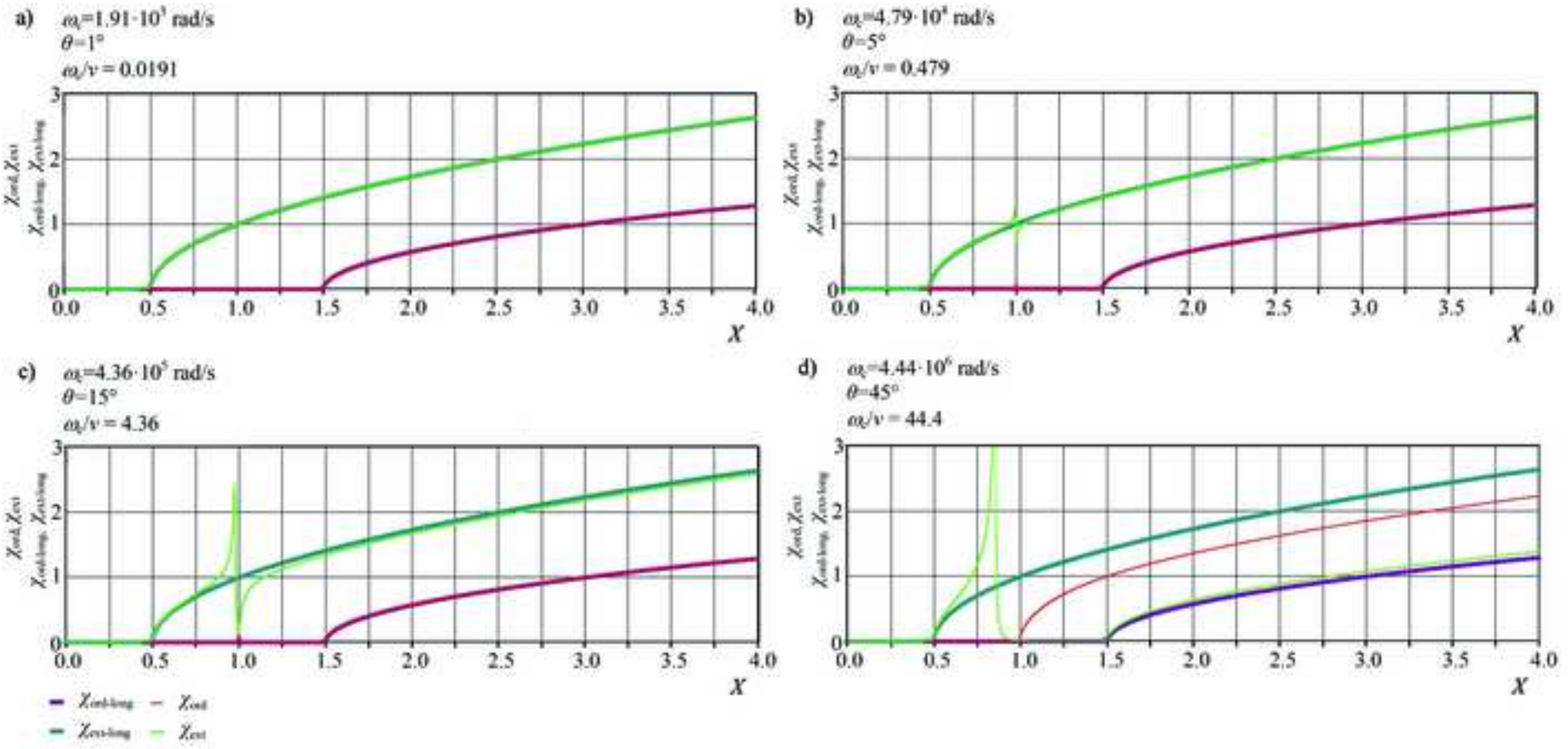
b) ω_c=4.79·10[4] rad/s
θ=5°
ω_c/v = 0.479
c) ω_c=4.36·10[5] rad/s
θ=15°
ω_c/v = 4.36
d) ω_c=4.44·10[6] rad/s
θ=45°
ω_c/v = 44.4
X
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
μ_{ord}, μ_{ext}
$\mu_{ord\text{-}long}$, $\mu_{ext\text{-}long}$
$\mu_{ord\text{-}long}$
μ_{ord}
$\mu_{ext\text{-}long}$
μ_{ext}

NEW_Fig(2)



a)
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θ=1°
ω_c/ν = 0.0191
b)
ω_c=4.79·10[4] rad/s
θ=5°
ω_c/ν = 0.479
c)
ω_c=4.36·10[5] rad/s
θ=15°
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d)
ω_c=4.44·10[6] rad/s
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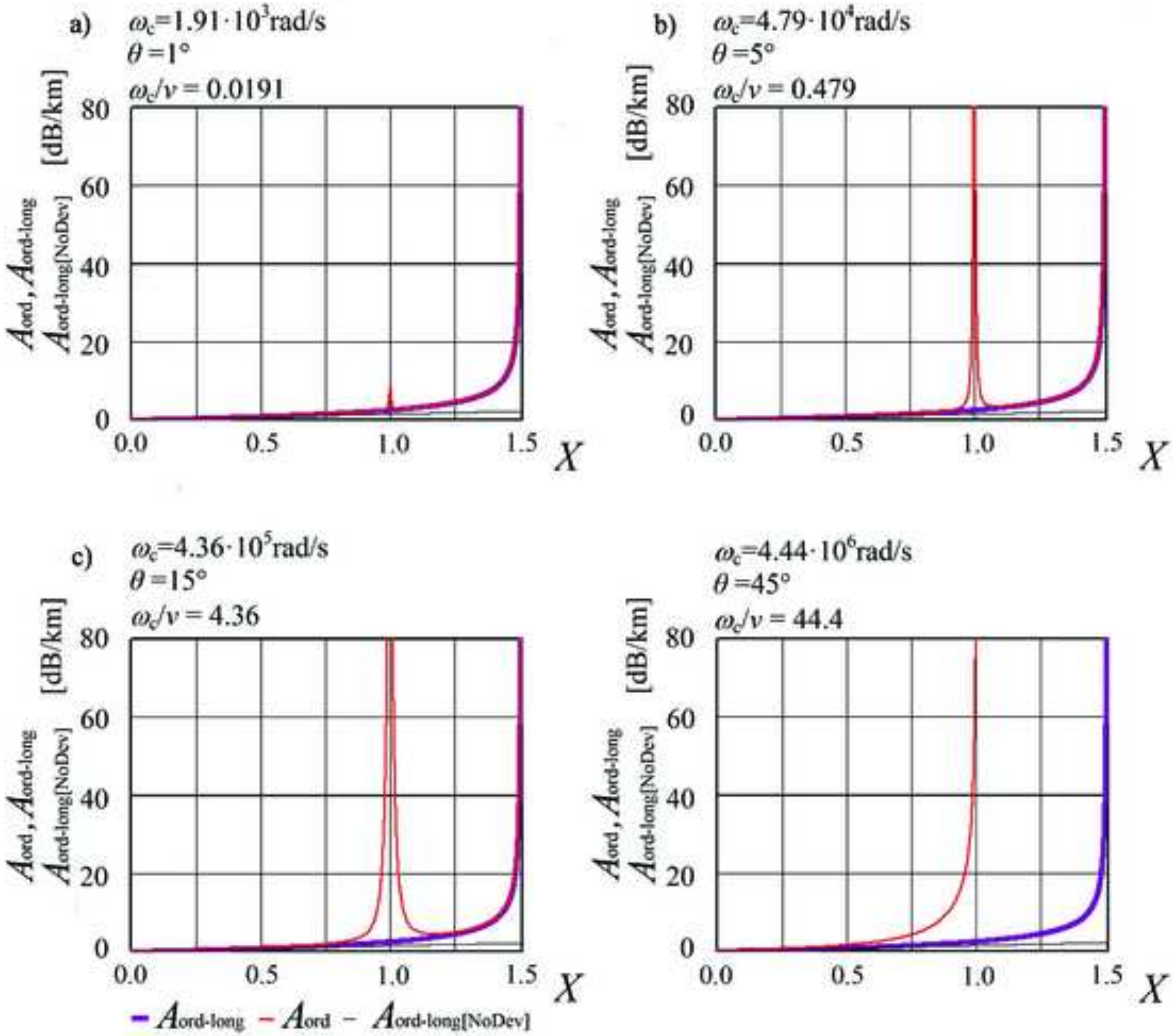
ω_c/ν = 44.4
X

NEW_Fig(3)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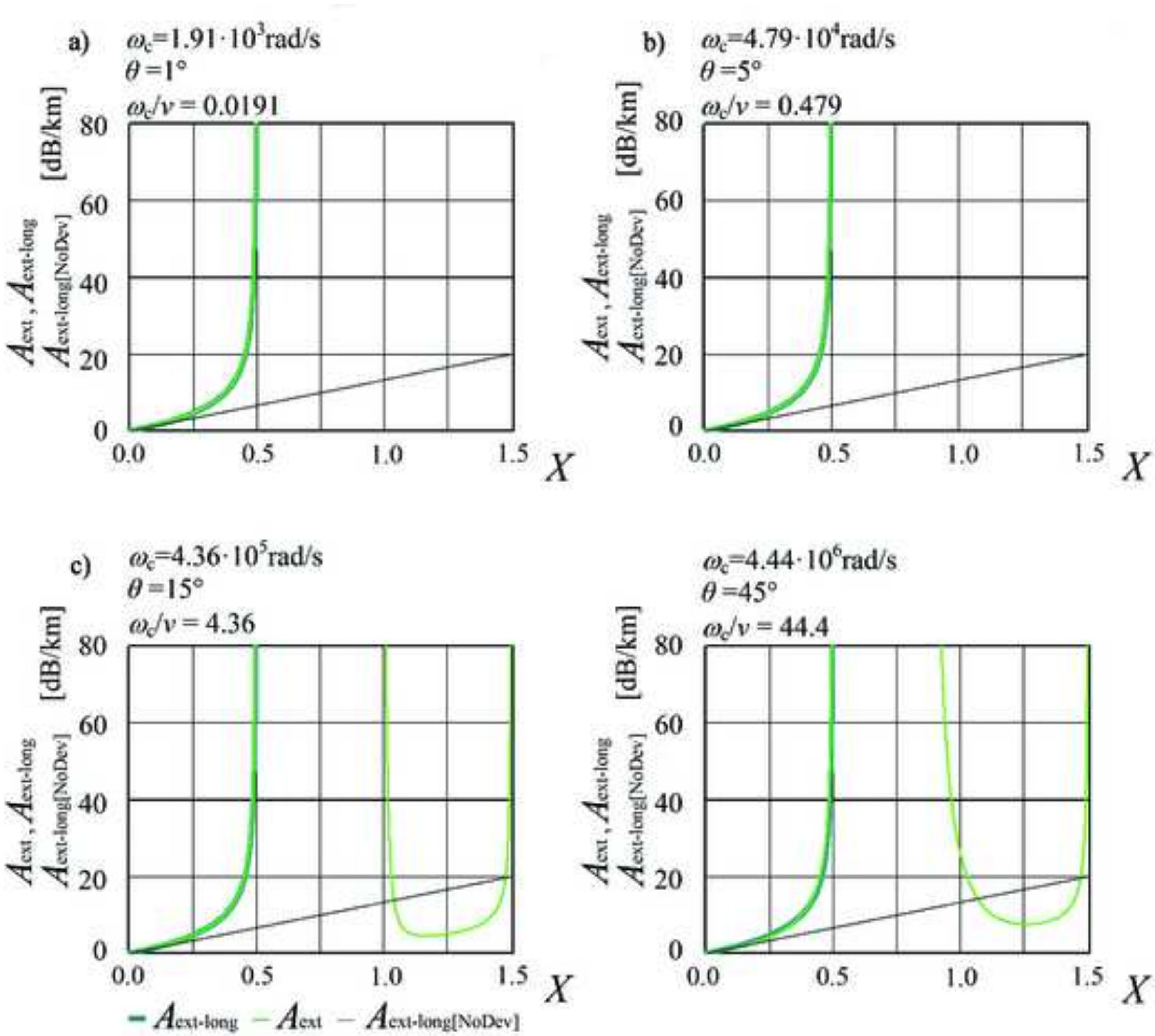
ω_c/v = 0.479
c)
ω_c=4.36·10^{5}rad/s
θ=15°
ω_c/v = 4.36
ω_c=4.44·10^{6}rad/s
θ=45°
ω_c/v = 44.4
A_{ord}, $A_{ord\text{-}long}$ $A_{ord\text{-}long[NoDev]}$ [dB/km]
0
20
40
60
80
0.0
0.5
1.0
1.5
X
$A_{ord\text{-}long}$
A_{ord}
$A_{ord\text{-}long[NoDev]}$

NEW_Fig(4)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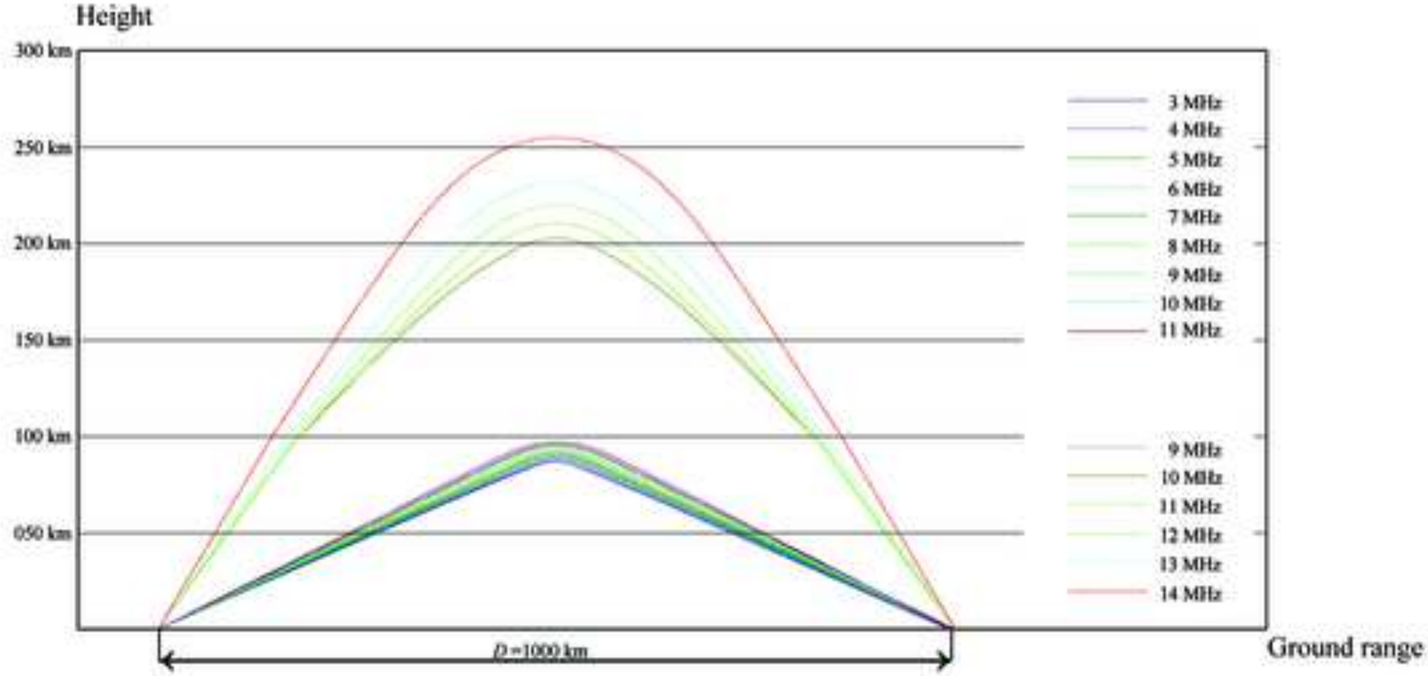
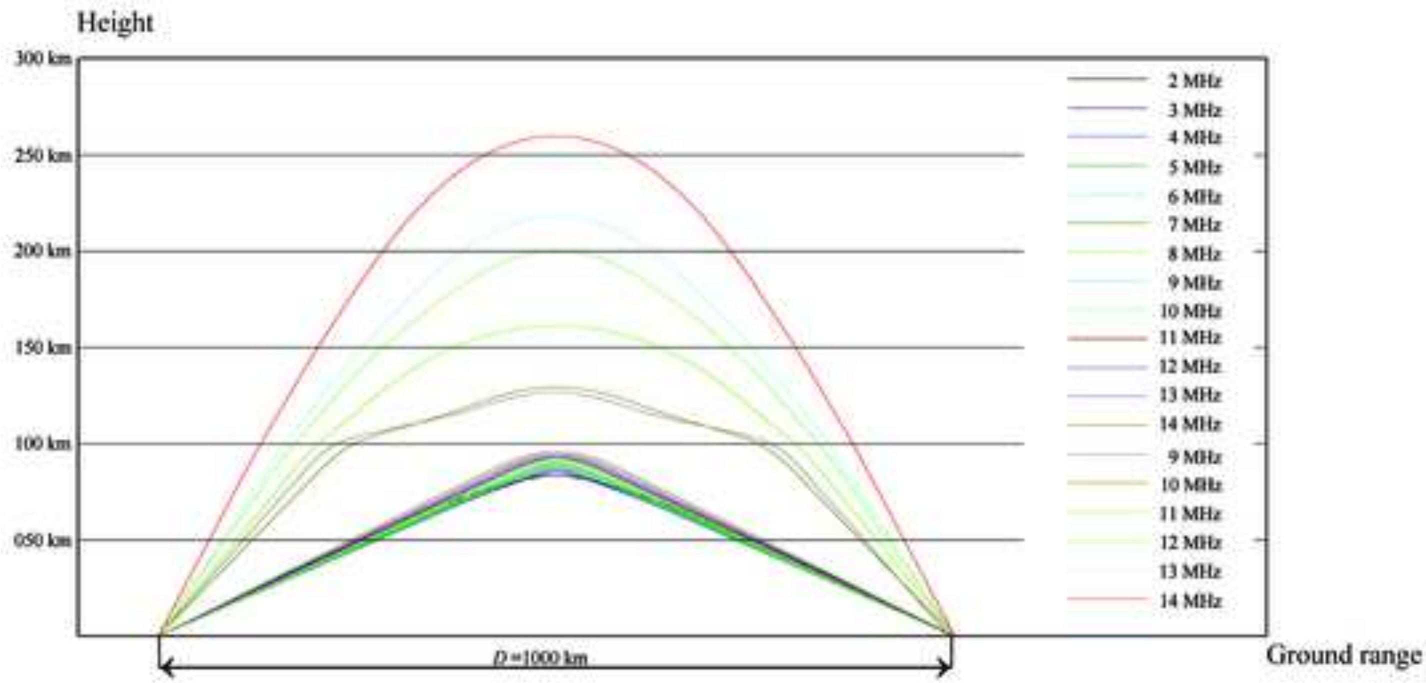
c)
ω_c=4.36·10^{5}rad/s
θ=15°
ω_c/v = 4.36
ω_c=4.44·10^{6}rad/s
θ=45°
ω_c/v = 44.4
A_{ext}, $A_{ext\text{-}long}$
$A_{ext\text{-}long[NoDev]}$ [dB/km]
0
20
40
60
80
0.0
0.5
1.0
1.5
X
$A_{ext\text{-}long}$
A_{ext}
$A_{ext\text{-}long[NoDev]}$

NEW_Fig(5)

LOW SOLAR ACTIVITY (*R12* = 10)



HIGH SOLAR ACTIVITY (*R12* = 100)



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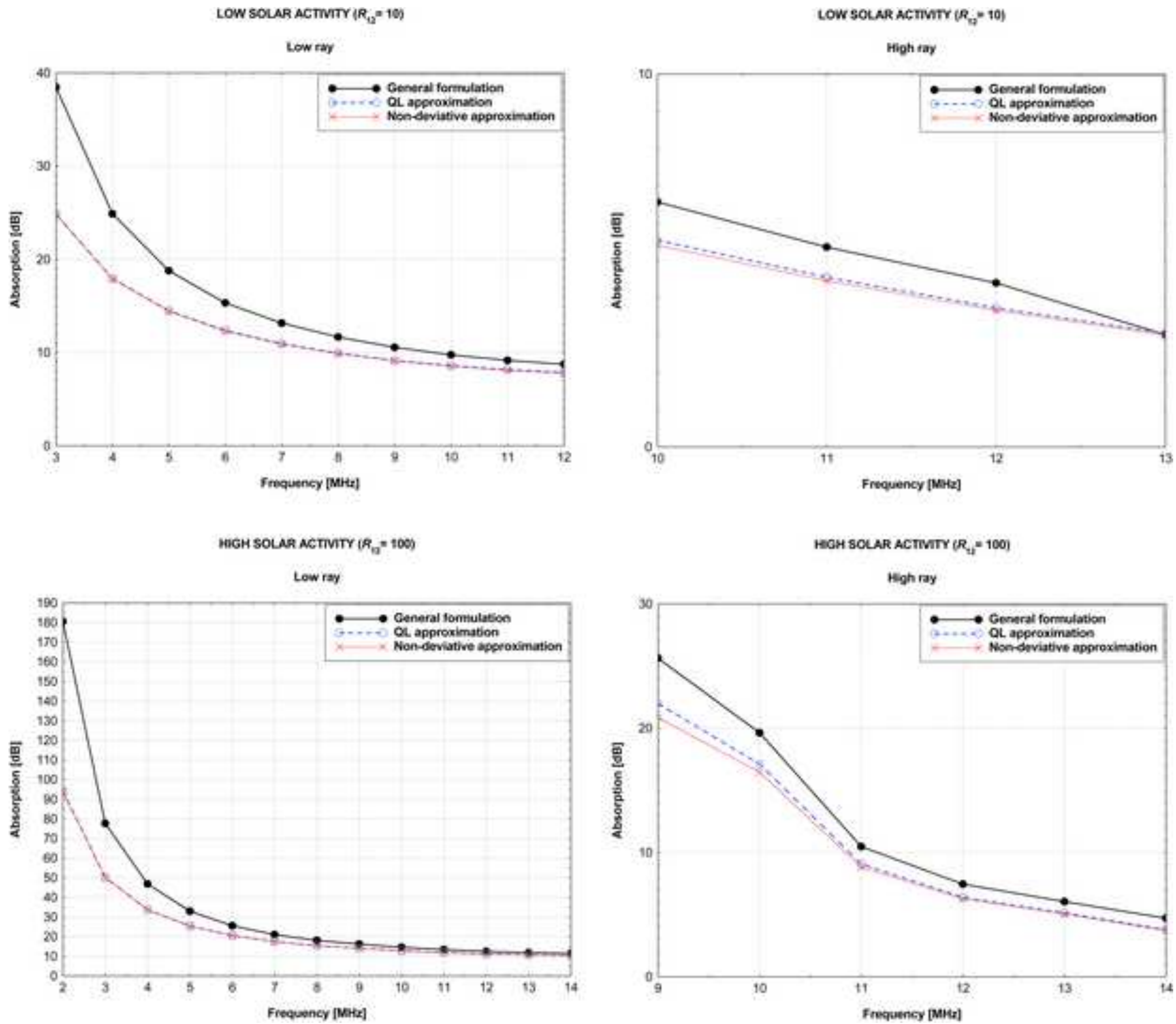
LOW SOLAR ACTIVITY (R_{12}= 10)
Low ray
General formulation
QL approximation
Non-deviative approximation
Absorption [dB]
Frequency [MHz]
LOW SOLAR ACTIVITY (R_{12}= 10)
High ray
General formulation
QL approximation
Non-deviative approximation
Absorption [dB]
Frequency [MHz]
HIGH SOLAR ACTIVITY (R_{12}= 100)
Low ray
General formulation
QL approximation
Non-deviative approximation
Absorption [dB]
Frequency [MHz]
HIGH SOLAR ACTIVITY (R_{12}= 100)
High ray
General formulation
QL approximation
Non-deviative approximation
Absorption [dB]
Frequency [MHz]