Natural Hazards manuscript No.
(will be inserted by the editor)

Long-term multi-risk assessment: statistical treatment of interaction among risks

Jacopo Selva

Received: date / Accepted: date

Abstract Multi-risk approaches have been recently proposed to assess and compare different risks in the same target area. The key points of multi-risk assessment are the development of homogeneous risk definitions and the treatment of risk interaction. The lack of treatment of interaction may lead to significant biases and thus to erroneous risk hierarchization, which is one of primary output of risk assessments for decision makers. In this paper, a formal statistical model is developed to treat interaction between two different hazardous phenomena in long-term multi-risk assessments, accounting for possible effects of interaction at hazard, vulnerability and exposure levels. The applicability of the methodology is demonstrated through two illustrative examples, dealing with the influence of (i) volcanic ash in seismic risk and (ii) local earthquakes in tsunami risk. In these applications, the bias in single-risk estimation induced by the assumption of independence among risks is explicitly assessed. An extensive application of this methodology at regional and sub-regional scale would allow to identify when and where a given interaction has significant effects in long-term risk assessments, and thus it should be considered in multi-risk analyses and risks hierarchization.

Keywords multi-risk · multi-hazard

1 Introduction

In most of the areas in the world, more than one hazard may act in the same time frame, leading to different risks. Until recent years, risks were often assessed with different definitions/approaches/assumptions, making them substantially not comparable (e.g., [24]). Recently, different analyses and case studies have been proposed in order to make comparable assessments, with the goal of comparing and

J. Selva
Istituto Nazionale di Geofisica e Vulcanologia, sezione di Bologna, Via Donato Creti 12 - 40128 Bologna, Italy
Tel.: +39-051-4151457
Fax: +39-051-4151499
E-mail: jacopo.selva@bo.ingv.it

ranking the different risks (e.g., [15,17,13]). Very recently, the analysis of cascade (or domino) effects highlighted the importance of the interaction among different risks, demonstrating that in multi-risk approaches the different risks should not only be compared, but also made interact [6,24,23].

Classical risk assessments are based on the independence of risks, and thus may be substantially biased due to the fact that this assumption is not always and/or everywhere true. For example, the assessment of expected damages due to a given hazard is commonly made through vulnerability assessments based on fragility models of the target assets. The structural analyses usually adopted to develop fragility curves are based on the assumption of not perturbed structures, that is, only one specific hazard acts on the structure at the same time (e.g., [29]). In this case, it is evident that the possible simultaneous action of two hazards is not considered at all. Such interactions are of course in many case statistically irrelevant. However, on one hand, many times the eventuality of two hazards acting at the same time is not unlikely at all, like for example when one hazard increases the probability of occurrence of a second hazard (e.g., earthquakes during volcanic eruptions) or the two hazards may share a common source (e.g., seismic and tsunami hazards), or when the action of one hazard covers quite large time windows (e.g., snow on roofs of buildings). On the other hand, the consequence of simultaneous hazards may be so catastrophic that their impact on risk assessments may be significant, even in case of a rare simultaneous events.

Focusing on one specific hazard, the mechanism leading to losses in case of single or simultaneous events is the same. For example, focusing on seismic risk, inter-story drift may be used as leading criterium for damages in case of ground shaking (e.g., [32]), both in presence or in absence of volcanic ash on roofs. Therefore, each specific mitigation action (e.g., retrofit, land-use plans, etc) decreases the total risk, that is, both single- and multi- risk components. Thus, a complete and coherent risk comparison is meaningful only when such interactive effects are accounted for. In other words, the bias of single-risk assessments (without interaction) may lead to erroneous assessments of risks hierarchy and actions' priority.

A quantitative analysis of this possible bias in long-term risk assessments is still lacking. Indeed, the treatment of interaction among risks is still a quite open field, in which several case studies of cascade effects have been developed [22,42, 24], while a complete formalization of the problem is not available. Recently, such interaction at hazard level (multi-hazard) have been treated by [23], where the formal distinction between single non-interacting hazard and complete multi-hazard assessment is proposed. However, the interaction of risks may act also at levels other than hazard, fact that deserves a specific treatment. Indeed, in several case studies, it has been shown how the contemporaneous action of different hazards may significantly either change the response of assets to the hazards (vulnerability interaction, e.g., [22,42]), or induce changes to the distribution of 'goods', as for example when people moves from their original 'standard' position (exposure interaction, e.g., [20,38]). Even restricting to natural hazards only, such interaction is conceivable in many realistic cases, such as for earthquakes striking areas in which are present volcanic ash, snow, or even floods (different fragility, as in application 1); tsunami striking shortly after an earthquake (different exposure, as in application 2); generic events striking pre-damaged (and not repaired) structures by, for example, earthquakes (different fragility and exposure). In other words, significant interaction is possible at both vulnerability and exposure level.

In this paper, a formal procedure is developed to account for interaction in risk assessments at all levels (hazard, vulnerability and exposure), in the case of two interacting hazardous phenomena. In this framework, the effect of interaction on single loss/risk assessment can be quantitatively evaluated, and the assumption of complete independence of risks verified. Two illustrative applications are then presented, demonstrating the practical applicability of the methodology in real case studies.

2 Interaction in multi-risk assessment

The risk curve due to a generic event $E1$ for a given asset in a given exposure time ΔT represents the probability that a given loss value l is overcome in a target area and in the exposure time ΔT. By use of the total probability rule, it can be written

$$\mathcal{R}c^{(E1)}(l) = \int_d \int_x \mathcal{E}(l|d) \cdot d\mathcal{F}(d|x) \cdot d\mathcal{H}(x) \tag{1}$$

where l is a loss measure in a specific metrics, d a given damage measure, x a given hazard intensity measure, and

- $\mathcal{H}(x)$ represents the cumulative hazard assessment (survivor function), in terms of its intensity x
- $\mathcal{F}(d|x)$ represents the fragility of the target asset, that is, the probability that the damage level d is overcome due to an intensity x [13]
- $\mathcal{E}(l|d)$ is the probability that a given loss level l is reached or overcome, given the damage level d. Since $\mathcal{E}(l|d)$ accounts for the consequence of damage d with its specific metrics (economic loss, casualties, dead), hereinafter it is referred to as the 'exposure' term.

The formulation in Eq. 1 represents a generalization for a generic ΔT of the Pacific Earthquake Engineering Research (PEER) formula [10, 12], it may be used as a general formulation for any kind of natural risks [23], and formally it holds for small probabilities for the hazardous phenomenon [34]. Given the large number of symbols used throughout the paper, in Tab. 1 a complete list of symbols is reported.

In case of non-systemic risk assessments (e.g., [7]), losses due to different assets in the target area can be assessed independently and then summed up over all the present assets (e.g., damages to buildings in seismic risk [40]). Commonly, damages d, for each single asset, are expressed through a discrete number of damage states d_i (e.g., [29]). In addition, also the hazard assessment is commonly approximated for discrete intervals of intensity x_j (e.g., [4]). With these simplifications and using the notations in Tab. 1, Eq. 1 becomes

$$\mathcal{R}c^{(E1)}(l) \approx \sum_i \left\{ \mathcal{E}^{(E1)}(l|d_i) \cdot \left[\sum_j \delta\mathcal{F}^{(E1)}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1)}(x_j) \right] \right\} \tag{2}$$

where the symbol δ, instead of d, is used, to highlight the discretization. The superscript $(E1)$ indicates that all the quantities refer to the specific hazardous phenomenon $E1$ (e.g., ground shaking). The term into square brackets is often referred to as *physical vulnerability* (e.g., [14]), and it represents the probability that a damage state d_i is observed for the asset in the exposure time. In its cumulative form reads

$$\mathcal{P}V^{(E1)}(d_i) = \sum_j \mathcal{F}^{(E1)}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1)}(x_j) \tag{3}$$

A quite special case, but very common for most of natural hazards, occurs when the hazard term $\mathcal{H}^{(E1)}(x_j)$ is assumed Possonian, with annual rate $\lambda^{(E1)}_{\geq x_j}$. In this case, also the physical vulnerability is Poissonian (from [9]), with annual rates

$$\lambda^{(E1)}_{\geq d_i} = \sum_j \mathcal{F}^{(E1)}(d_i|x_j) \cdot \delta\lambda^{(E1)}_{\geq x_j} \tag{4}$$

which is numerically equivalent to Eq. 3 for small $\lambda^{(E1)}_{x_j}$ [12]. This formulation allows to automatically account for the repeatability of the hazardous phenomenon (many earthquakes in the exposure time), problem that should be specifically addressed in Eq. 3 in case of significant probability of multiple events in the exposure time [34].

In many areas, two hazardous events ($E1$ and $E2$) can act on the same structure in the exposure time ΔT (e.g., ground shaking and snow). Interaction among the consequent risks occurs when $E1$ acts in a temporal windows in which $E2$, or its consequence, is still acting (e.g., ground shaking occurs when snow is present on roofs). If the effects of $E2$ may influence the expected losses due to $E1$, either through the exposure $\mathcal{E}^{(E1)}$ and/or the vulnerability $\mathcal{F}^{(E1)}$, their consequences are completely neglected whenever $E1$ risk is assessed through Eq. 2. In other words, to account for the interaction between $E1$ and $E2$ in losses/risk assessment of $E1$, it must be considered that for limited time windows (e.g., snow is present on roofs) the expected losses due to the event $E1$ are modified by $E2$, potentially influencing the overall long-term assessment. In other words, following the reported example, in assessing the probability of damages due to ground shaking in the exposure time ΔT, it must be considered that (i) the presence of snow on roofs alters the fragility $\mathcal{F}^{(E1)}$ [22], and (ii) the snow is present only in limited time windows.

To account for this interaction, the different contributions to damages due to $E1$ in presence or not of $E2$ should be factorized. The probability of $E1$ in ΔT in presence of the effects of $E2$ can be defined as

$$\begin{aligned}\mathcal{H}^{(E1,E2)}(x_j) &= pr(\geq x_j \text{ at time } t \text{ in } \Delta T \;\&\; t - t_{E2} < \Delta T_p) \\ &= pr(\geq x_j; \Delta T_p | E2) \cdot pr(E2, \Delta T) = \\ &= \mathcal{H}^{(E1|E2)}(x_j; \Delta T_p) \cdot pr(E2, \Delta T)\end{aligned} \tag{5}$$

which represents the probability that, during the exposure time ΔT, $E1$ is preceded by E_2 within a time window ΔT_p. In the second row, this probability is factorized in conditional probabilities, where $\mathcal{H}^{(E1|E2)}$ represents the probability of $x \geq x_j$ in a time window ΔT_p, given that $E2$ has occurred. Note that this does not necessary imply a cause-effect relationship between $E1$ and $E2$, but possibly just a temporal coincidence. The term $pr(E2, \Delta T)$ represents the probability of $E2$ in the exposure time ΔT. For sake of simplicity, just at this stage, the secondary event $E2$ is assumed Boolean (yes/no), but this restriction will be overcome in paragraph 2.2. The time window ΔT_p, herein referred to as *persistence time window*, represents for how long the effect of $E2$ will be active after the occurrence of the event $E2$, so potentially influencing $E1$ vulnerability or exposure terms. The length of ΔT_p strongly varies for different $E2$. To better understand the meaning of all terms in 5, we can follow the example reported above ($E1$ is 'ground shaking', $E2$ is 'snow on roofs'): $pr(E2, \Delta T)$ is the probability of significant snow in ΔT, ΔT_p is the time window in which snow melts, and $\mathcal{H}^{(E1,E2)}$ represents the probability of significant ground shaking in presence of snow.

Since the events $x \geq x_j$ within ΔT_p after $E2$ represent a subset of the events $x \geq x_j$ in ΔT, the hazard $\mathcal{H}^{(E1,E2)}$ is only a part of the total $E1$ hazard $\mathcal{H}^{(E1)}$; thus

$$\mathcal{H}^{(E1)}(\geq x_j) = \mathcal{H}^{(E1,E2)}(\geq x_j) + \mathcal{H}^{(E1,\overline{E2})}(\geq x_j) \tag{6}$$

where $\mathcal{H}^{(E1,E2)}$ is the same of Eq. 5, and $\mathcal{H}^{(E1,\overline{E2})}$ represents the probability of $x \geq x_j$ in ΔT not preceded by an event $E2$ in ΔT_p. Following the reported example, $\mathcal{H}^{(E1,\overline{E2})}$ represents the probability of ground shaking in ΔT when snow is not present on roofs. Hereinafter, $\mathcal{H}^{(E1,E2)}$ and $\mathcal{H}^{(E1,\overline{E2})}$ will be referred to as co-active and isolated-hazard factors, respectively. Note that this factorization is similar to the one proposed by [23] for multi-hazard assessments. The fundamental difference is that this factorization specifies a temporal limit ΔT_p for interaction. Without this specification, none of the following developments would be possible.

The probability of damages $\geq d_i$ due to any x_j is evaluated by assessing the physical vulnerability through Eq. 3. However, the two complementary (to the total hazard) hazard factors now must be kept separated, that is:

$$\begin{aligned}\mathcal{P}V^{(E1)}(\geq d_i) = \textstyle\sum_j \mathcal{F}^{(E1,E2)}(\geq d_i|x_j) \cdot \delta\mathcal{H}^{(E1,E2)}(x_j) \\ + \textstyle\sum_j \mathcal{F}^{(E1,\overline{E2})}(\geq d_i|x_j) \cdot \delta\mathcal{H}^{(E1,\overline{E2})}(x_j)\end{aligned} \tag{7}$$

where different symbols for the fragility terms are reported, to highlight that they should be evaluated in condition of occurrence and non-occurrence of $E2$, respectively. Following the reported example, $\mathcal{F}^{(E1,E2)}$ represents the fragility to ground shaking assuming snow on roofs, while $\mathcal{F}^{(E1,\overline{E2})}$ assumes no snow on roofs.These two fragilities, as discussed above, may significantly differ in these two different conditions.

To complete the risk analysis, the consequences of damages should be considered through the exposure term, as in Eq. 2. The expression for the $\mathcal{P}V^{(E1)}$ can be substituted in Eqs. 3 and 2, obtaining

$$\mathcal{R}c^{(E1)}(\geq l) = \mathcal{R}c^{(E1,E2)}(\geq l) + \mathcal{R}c^{(E1,\overline{E2})}(\geq l) \tag{8}$$

where

$$\begin{cases} \mathcal{R}c^{(E1,E2)}(\geq l) = \sum_i \mathcal{E}^{(E2)}(l|d_i) \sum_j \delta\mathcal{F}^{(E1,E2)}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1,E2)}(x_j) \\ \mathcal{R}c^{(E1,\overline{E2})}(\geq l) = \sum_i \mathcal{E}^{(\overline{E2})}(l|d_i) \sum_j \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1,\overline{E2})}(x_j) \end{cases} \tag{9}$$

and, as for fragilities above, different symbols for the exposure terms are reported, in condition of occurrence or non-occurrence of $E2$.

In Eqs. 8 and 9, the contributions to the total risk of the co-active and the isolated-risk factors result completely separated. With this formulation, the effects of interaction on damaging are accounted for whenever risk is assessed. Since only in this case a complete risk assessment in a multi-risk perspective is made, $\mathcal{R}c^{(E1)}$ will be referred to as *multi-risk* for $E1$.

The co-active risk factor $\mathcal{R}c^{(E1,E2)}$ represents the risk posed by the event $E1$ in the time lapses ΔT_p, in which it is active the hazard $E2$. The effects of $E2$ in both fragility and exposure are accounted for in this term. Considering Eq. 5, the co-active risk factor reads

$$\begin{cases} \mathcal{R}c^{(E1,E2)}(\geq l) = \sum_i \mathcal{E}^{(E2)}(l|d_i) \cdot \delta\mathcal{P}V^{(E1|E2)}(d_i) \cdot pr(E2;\Delta T) \\ \mathcal{P}V^{(E1|E2)}(d_i) = \sum_j \delta\mathcal{F}^{(E1,E2)}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1|E2)}(x_j) \end{cases} \tag{10}$$

where the physical vulnerability $\delta\mathcal{P}V^{(E1|E2)}$ is highlighted. This vulnerability is exactly as a canonical physical vulnerability, but it is conditioned to the occurrence of the event $E2$ and it is referred to a time window ΔT_p. Following the reported example, this risk term refers to losses occurring when ground shaking strike structures covered by snow.

The isolated risk factor $\mathcal{R}c^{(E1,\overline{E2})}$ represents the residual risk posed by $E1$, when this is not influenced by the occurrence of $E2$. To better understand its meaning, it can be rewritten in light of Eq. 6, so that

$$\begin{aligned} \mathcal{R}c^{(E1,\overline{E2})}(\geq l) &= \sum_i \mathcal{E}^{(\overline{E2})}(l|d_i) \sum_j \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1,\overline{E2})}(x_j) = \\ &= \mathcal{R}c^{(E1,s)}(\geq l) - \mathcal{R}c^{(E1,v)}(\geq l) \end{aligned} \tag{11}$$

where the two terms identified have a clear physical meaning. Indeed, the first term

$$\begin{cases} \mathcal{R}c^{(E1,s)}(\geq l) = \sum_i \mathcal{E}^{(\overline{E2})}(l|d_i) \cdot \delta\mathcal{P}V^{(E1,s)}(d_i) \\ \delta\mathcal{P}V^{(E1,s)}(d_i) = \sum_j \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1)}(x_j) \end{cases} \tag{12}$$

represents the risk evaluated considering the isolated fragility $\delta\mathcal{F}^{(E1,\overline{E2})}$ and exposure $\mathcal{E}^{(\overline{E2})}$ factors, and the total hazard $\delta\mathcal{H}^{(E1)}(\geq x_j)$. This is what it is usually done in the literature, whenever the hazard is assessed from undifferentiated catalogs [23]. For this reason, $\mathcal{R}c^{(E1,s)}$ will be referred to as *single-risk* for $E1$. Following the reported example, this risk term considers fragility and exposure evaluated assuming no snow on roofs, while hazard is assessed independently from the fact that snow is present on roofs.

The second term in Eq. 11 reads

$$\begin{cases} \mathcal{R}c^{(E1,v)}(\geq l) = \sum_i \mathcal{E}^{(\overline{E2})}(l|d_i) \cdot \delta\mathcal{P}V^{(E1|E2,v)}(d_i) \cdot pr(E2;\Delta T) \\ \delta\mathcal{P}V^{(E1|E2,v)}(d_i) = \sum_j \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j) \cdot \delta\mathcal{H}^{(E1,E2)}(x_j) \end{cases} \tag{13}$$

and it represents the risk that would have been forecast in case of occurrence of $E2$, if no changes to fragility and exposure were expected. Since the physical vulnerability term $\delta\mathcal{P}V^{(E1|E2,v)}$ has the same meaning of $\delta\mathcal{P}V^{(E1|E2)}$ in Eq. 10, it is used the same symbol with the addition of v, to highlight that the fragility term here is the isolated one, instead of the coactive one. $\mathcal{R}c^{(E1,v)}$ will be referred to as *virtual* risk factor, and it is fundamental to compensate the coactive-risk factor in Eq. 10. Indeed, if both fragility and exposure are not affected by the occurrence of $E2$ (fragility and exposure to ground shaking are equal, with or without snow), $\mathcal{R}c^{(E1,E2)}$ equals $\mathcal{R}c^{(E1,v)}$, for all l, and thus, from Eq. 8,

$$\mathcal{R}c^{(E1)}(\geq l) = \mathcal{R}c^{(E1,s)}(\geq l) \tag{14}$$

meaning that the multi-risk assessment $\mathcal{R}c^{(E1)}$, in this case, is exactly equal to the single-risk assessment $\mathcal{R}c^{(E1,s)}$, for all l.

In practice, the multi-risk analysis can be performed through Eqs. 8 and 9. The coactive-risk factor $\mathcal{R}c^{(E1,E2)}$ is assessed though Eq. 10. The assessment of the isolated risk factor $\mathcal{R}c^{(E1,\overline{E2})}$ is based either on a direct assessment through Eq. 11, first row, or through the evaluation of the two further risk factors, that is, the single risk factor $\mathcal{R}c^{(E1,s)}$ (Eq. 12) and the virtual risk factor $\mathcal{R}c^{(E1,v)}$ (Eq. 13).

All physical vulnerability terms share the same functional form, that is:

$$\mathcal{P}V^{(*)} = \sum_j \mathcal{F}^{(*)}(\geq d_i|x_j) \cdot \delta\mathcal{H}^{(*)}(x_j) \tag{15}$$

which is identical to the classical assessment, as reported in Eq. 3. In case of Poissonian hazards, they can also be assessed through 4, as commonly reported in the literature (e.g., [14]). However, it is worth noting that $\mathcal{P}V^{(E1,s)}$ (single-risk factor, Eq. 12) is referred to the hazard of $E1$ in the exposure time ΔT. On the contrary, both $\mathcal{P}V^{(E1|E2)}$ (co-active risk factor, Eq. 10) and $\mathcal{P}V^{(E1|E2,v)}$ (virtual-risk factor, Eq. 13) are referred to time windows ΔT_p just after the occurrence of $E2$ in ΔT. During these periods, all terms (including the hazard term $\mathcal{H}^{(E1,E2)}$) may be largely influenced [23].

2.1 Single-risk assessment and its bias

In the literature, sometimes only one of the two hazard factors in Eq. 6 is assessed, but more commonly they are assessed jointly like, for example, when hazard is assessed from undifferentiated catalogs [23]. On the contrary, just one fragility term is usually assessed (the isolated one), that is, the one in absence of external influences (e.g., [29]). The same is valid for the exposure term, for which either the variability due to external factors (day/night, summer/winter) or a averaged value is assumed, but in any case not considering the effects of $E2$. Since, in case of occurrence of $E2$, one or more of these terms may be potentially significantly influenced, single-risk assessment may result significantly biased.

To evaluate the effective strength of the interaction among risks, in the followings single and multi-risk assessments are compared, as assessed through Eqs. 8 and 12, respectively. In the single-risk formulation, the assessment is based on the isolated fragility $\mathcal{F}^{(E1,\overline{E2})}$ and exposure $\mathcal{E}^{(\overline{E2})}$ terms, and the total hazard $\mathcal{H}^{(E1)}(\geq x_j)$. This is the case in most of applications, even though sometimes $\mathcal{H}^{(E1,\overline{E2})}$ is assessed instead. In these cases, the single-risk assessment is equal to the isolated risk factor $\mathcal{R}c^{(E1,\overline{E2})}$ in Eq. 8.

In order to allow a simpler comparison among the same risk in different areas, as well as, different risks in the same area, a single risk index is often considered, instead of using the whole risk curve (e.g., [24]). As risk index, the average (mean) of losses in the target area in the exposure time is considered, which reads:

$$\begin{aligned}\mathcal{R}^{(*)} &= \int_l l \cdot d\mathcal{R}c^{(*)}(\geq l) = \int_l \int_d \int_x l \cdot d\mathcal{E}(l|d) \cdot d\mathcal{F}(d|x) \cdot d\mathcal{H}(x) = \\ &\approx \sum_i l_{ave}^{(*)}(d_i) \cdot \delta\mathcal{P}V^{(*)}(\geq d_i)\end{aligned} \tag{16}$$

where $l_{ave}^{(*)}(d_i)$ is the average loss caused by the damage state d_i for the generic $(*)$ risk factor (e.g., [5]). The risk index $\mathcal{R}^{(*)}$ is often expressed for $\Delta T = 1$ year and, in the case of earthquakes, referred to as Average Annual Earthquake Losses (AEL in [28]). The average in Eq. 16 applies to both the single-risk assessment, and to all the factors of the multi-risk assessment, so that

$$\begin{cases} \mathcal{R}^{(E1,s)} &= \sum_i l_{ave}^{(\overline{E2})}(d_i) \cdot \delta\mathcal{P}V^{(E1,s)}(\geq d_i) \\ \mathcal{R}^{(E1)} &\equiv \mathcal{R}^{(E1,E2)} + \mathcal{R}^{(E1,s)} - \mathcal{R}^{(E1,v)} = \\ &= \sum_i l_{ave}^{(E2)}(d_i) \cdot \delta\mathcal{P}V^{(E1|E2)}(\geq d_i) \cdot pr(E2;\Delta T) \\ &\quad + \sum_i l_{ave}^{(\overline{E2})}(d_i) \cdot \delta\mathcal{P}V^{(E1,s)}(\geq d_i) \\ &\quad - \sum_i l_{ave}^{(\overline{E2})}(d_i) \cdot \delta\mathcal{P}V^{(E1|E2)}(\geq d_i) \cdot pr(E2;\Delta T) \end{cases} \tag{17}$$

The bias that a single-risk assessment introduces, since it does not account for the effects of $E2$, reads:

$$\begin{aligned} \delta\mathcal{R}^{(E1)} &= \mathcal{R}^{(E1)} - \mathcal{R}^{(E1,s)} = \mathcal{R}^{(E1,E2)} - \mathcal{R}^{(E1,v)} = \\ &= \textstyle\sum_{ij} \left[l_{ave}^{(E1,E2)}(d_i) \delta\mathcal{F}^{(E1,E2)}(d_i|x_j) - l_{ave}^{(\overline{E2})}(d_i) \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j) \right] \delta\mathcal{H}^{(E1,E2)}(x_j) = \\ &= \textstyle\sum_j \left[\mathcal{L}^{(E1,E2)}(x_j) - \mathcal{L}^{(E1,\overline{E2})}(x_j) \right] \delta\mathcal{H}^{(E1,E2)}(x_j) \end{aligned} \tag{18}$$

and, normalized by the single-risk:

$$\delta\mathcal{R}^{(E1)}/\mathcal{R}^{(E1,s)} = \frac{\sum_j \left[\mathcal{L}^{(E1,E2)}(x_j) - \mathcal{L}^{(E1,\overline{E2})}(x_j) \right] \delta\mathcal{H}^{(E1,E2)}(x_j)}{\sum_j \mathcal{L}^{(E1,\overline{E2})}(x_j) \delta\mathcal{H}^{(E1)}(x_j)} \tag{19}$$

This means that the bias depends, for each level of intensity x_j, on two factors: (i) how strong the interaction at damaging level is (the term in brackets), and (ii) how probable it is the occurrence of the event $E1$, within the persistence time window ΔT_p, just after $E2$ in ΔT.

In many cases, the bias of single-risk assessments will be negligible. In particular, this is certainly true whenever (i) $pr(E2) \approx 0$ (approximately no $E2$ hazard), that is, $\delta\mathcal{H}^{(E1,E2)} \approx 0$, or (ii) the damaging term is not influenced by $E2$, that is, $\mathcal{L}^{(E1,E2)}(x_j) = \mathcal{L}^{(E1,\overline{E2})}(x_j)$, for all x_j.

In other cases, a more specific analysis of both terms is necessary. A quite common situation is when $E1$ and $E2$ are two independent and rare events, that is

$$\delta\mathcal{H}^{(E1,E2)} = pr(E1;\Delta T_p) \cdot pr(E2;\Delta T) \tag{20}$$

due to their independence, and

$$\delta\mathcal{H}^{(E1,E2)} = pr(E1;\Delta T_p) \cdot pr(E2;\Delta T) \ll pr(E1;\Delta T) = \delta\mathcal{H}^{(E1)} \tag{21}$$

due to the fact that $pr(E2;\Delta T) \ll 1$ and $pr(E1;\Delta T_p) < pr(E1;\Delta T)$, for all intensity levels x. This means, in practice, that the possibility of concomitant events $E1$ and $E2$ is quite rare. However, this can be sufficient to neglect the bias only assuming that this difference in probability is sufficient to compensate the increase in vulnerability due to $E2$. This is valid only if $\mathcal{L}^{(E1,\overline{E2})}$ is smaller, but approximately of the same order of magnitude, of $\mathcal{L}^{(E1,E2)}$, at all intensity levels x. In general, this can be the case, but a careful assessment of both isolated and coactive-vulnerability factors should be preferable.

Whenever the events $E1$ and $E2$ effectively interact, that is, the occurrence of $E2$ increases the probability of $E1$, it is surely necessary a careful evaluation of both terms. Indeed, $\delta\mathcal{H}^{(E1|E2)}$ may be close to 1 and then $\delta\mathcal{H}^{(E1,E2)} \sim pr(E2;\Delta T)$, that is, all the hazard terms have the same order of magnitude. In this case, $\mathcal{R}^{(E1)}$ and $\mathcal{R}^{(E1,s)}$ may actually significantly differ.

2.2 From Boolean to discrete intensity values

In most of cases, the secondary event $E2$ cannot be considered as Boolean, since different intensities for $E2$ may lead to different levels of interaction with the $E1$ risk terms.

The influence of the different levels of intensity of $E2$ must be considered in both the co-active (all terms) and the virtual-risk (hazard term) factors. Starting from the co-active risk factor $\mathcal{R}^{(E1,E2)}$ in Eq. 10, it is noted that the event $E2$ can lead to different levels of intensity y_k, with probability $pr(y_k|E2)$. By definition, the y_k, for all k and given the occurrence of $E2$, represent a complete and mutually exclusive set of events ($\sum_k pr(y_k|E2) = 1$), and thus:

$$\begin{aligned}
\mathcal{R}c^{(E1,E2)}(l) &\equiv \sum_i \mathcal{E}^{(E2)}(l|d_i) \sum_j \delta\mathcal{F}^{(E1,E2)}(d_i|x_j)\delta\mathcal{H}^{(E1,E2)}(x_j) \equiv \\
&\equiv pr(\geq l|E2)pr(E2;\Delta T) = \\
&= \left[\sum_k pr(\geq l|E2)pr(y_k|E2)\right] pr(E2;\Delta T) = \\
&= \sum_{i,k}[\mathcal{E}^{(E2,y_k)}(l|d_i) \sum_j \delta\mathcal{F}^{(E1,y_k)}(d_i|x_j,y_k)\delta\mathcal{H}^{(E1|y_k)}(x_j,y_k)]\delta\mathcal{H}^{(E2)}(y_k) = \\
&= \sum_{i,k} \mathcal{E}^{(E2,y_k)}(l|d_i)\delta\mathcal{P}V^{(E1|y_k)}(d_i|y_k)\delta\mathcal{H}^{(E2)}(y_k)
\end{aligned} \tag{22}$$

where

- each y_k represents a specific interval of values for the intensity of $E2$, and the index k covers all possible values for the intensity, so that $\sum_k pr(y_k|E2) = 1$
- in the second row, all probability terms are highlighted using the symbol $pr(e)$, which generically indicates the probability of the event e
- for each of the original terms (first row), symbols are modified to highlight their potential dependence on the value y_k. Hazard, fragility and exposure may all theoretically depend on y_k. In practical applications, exposure is often not dependent on y_k and can exit from the sum in k.
- the term $\delta\mathcal{H}^{(E1|y_k)}(x_j)$ represents the (discrete) non-cumulative hazard curve for the primary event $E1$ in the persistence time window ΔT_p after an event $E2$ with intensity y_k in ΔT. This conditional hazard term, combined to the fragility, allows the assessment of the conditional physical vulnerability $\delta\mathcal{P}V^{(E1|y_k)}(d_i|y_k)$, as identified in the last row. This physical vulnerability is equivalent to the one in Eq. 3, in its cumulative version, and it can be assessed as the ordinary one, but it is conditioned to the occurrence of $E2$ with intensity y_k (one separate assessment for each k), and relative to the time window ΔT_p.
- the term $pr(y_k|E2)pr(E2)$ is identified as the (discrete) non-cumulative hazard curve for the secondary event $E2$, in the exposure time ΔT, i.e., $\delta\mathcal{H}^{(E2)}(y_k)$.

The formulation in Eq. 22 allows accounting for the effects on hazard, fragility and exposure of the different intensities of $E2$. Of course, all these additional terms need specific assessments, strongly increasing the effort necessary for a complete multi-risk assessment. However, on one side, not necessarily both vulnerability and exposure depend on y_k, on the other side, the width of $E2$ intensity intervals (the number of y_k) can be rather large (i.e., few ks), depending on the sensitivity on y of the $E1$ risk terms.

The same development is necessary for the virtual-risk factor, in Eq. 13. The result is exactly the same, but in this case neither fragility nor exposure terms do depend on $E2$, and consequently on y_k, that is

$$\begin{aligned}\mathcal{R}c^{(E1,v)}(l) &= \sum_{i,k}[\mathcal{E}^{(\overline{E2})}(l|d_i)\sum_j \delta\mathcal{F}^{(E1,\overline{E2})}(d_i|x_j)\delta\mathcal{H}^{(E1|y_k)}(x_j,y_k)]\delta\mathcal{H}^{(E2)}(y_k) = \\ &= \sum_{i,k}\mathcal{E}^{(\overline{E2})}(l|d_i)\delta\mathcal{PV}^{(E1|y_k,v)}(d_i|y_k)\delta\mathcal{H}^{(E2)}(y_k)\end{aligned} \quad (23)$$

where in each physical vulnerability term (one for each k), there is a dependence on y_k only at the hazard level, if any. In any case, as for $\mathcal{PV}^{(E1|y_k)}(d_i|y_k)$ in Eq. 22, each one of these terms is equivalent to the one in Eq. 3. Thus, as above, it can be assessed as the ordinary physical vulnerability, but it is conditioned to the occurrence of $E2$ with intensity y_k (one separate assessment for each k), and relative to the time window ΔT_p.

Equations 22 and 23 contain the hazard terms in both x_j and y_k. The total (discrete) non-cumulative hazard curve for the primary event $E1$, in case of occurrence of $E2$, can be derived by marginalizing with respect to y_k, that is:

$$\delta\mathcal{H}^{(E1,E2)}(x_j) = \sum_k \delta\mathcal{H}^{(E1|y_k)}(x_j)\delta\mathcal{H}^{(E2)}(y_k) \quad (24)$$

Note that, if there is not a specific dependence between the intensity of primary and secondary events, as in most of real cases, this expression again collapses to the formulation in Eq. 5, since $\delta\mathcal{H}^{(E1|y_k)}$ does not depend on y_k and can exit from the sum, and thus the sum simply becomes the probability of the event $E2$ in ΔT.

3 Applications

3.1 Case study 1: fragility dependence in ground-motion/ash-fall interaction

This application focuses in assessing the long-term seismic risk ($E1 \equiv$ ground shaking) related to economic loss, in presence ($E2$) and in absence ($\overline{E2}$) of previous volcanic eruptions with significant ash fall (loading > 3 kPa) [42]. In particular, it is discussed here the effect of a possible increase of the vulnerability to earthquakes when buildings' roofs are loaded by significant ash. This increase is discussed in [42] and it is reported here as illustrative example of interaction at the fragility level.

This illustrative analysis is indicatively located in the area of Naples (Italy). This area is subject to seismic hazard due to both tectonic earthquakes located the Apennines chain, and to seismic events located in the nearby volcanic areas (e.g., [8]). In addition to this, this area is subject to significant volcanic hazard due to the three active volcanoes in the Neapolitan area, Mt. Vesuvius, Campi Flegrei and Ischia (e.g., [3]). For this application, an exposure time ΔT of 50 years is selected, in agreement with the official seismic hazard for Italy [18]. For simplicity, it is considered as target just one specific building, of a specific class.

The seismic fragility curves adopted are the ones in [42] for a class B_s building, that is, good masonry with iron beam floor. Such fragility curves change for various levels of ash load on the roof. The fragility in presence of significant ash loading ($\mathcal{F}^{(E1,y_k)}$) can be deduced from the one in absence of loading ($\mathcal{F}^{(E1,\overline{E2})}$), with the relationship proposed in Tab. 8 of [42] with loading variable from 3 to 20 kPa, at intervals of 1 kPa. No significant changes to fragilities are expected for $y < 3$ kPa. The seismic intensity adopted for seismic hazard (x_j) is macro-seismic intensity EMS92. Such fragility curves consider 5 damage states, ranging from minor to complete damages.

The exposure $\mathcal{E}(l|d_i)$ is assumed not dependent on the presence or absence of ash loading. For simplicity, the uncertainty on the losses due to each d_i is not considered (e.g., [39,5]), in which case the distribution $\mathcal{E}(l|d_i)$ can be modelled as a step function θ centered on an average value $l_{ave}(d_i)$. The average loss l_{ave} is set as $RC \cdot CDF_i$, that is, the 'replacement cost' RC of the structure multiplied by the 'cost damage factor' CDF_k, which represents the fraction of RC to repair the k-th damage state [39], that is

$$\begin{cases} l_{ave}^{(E2,y_k)}(d_i) = l_{ave}^{(\overline{E2})}(d_i) = RC \cdot CDF_i \\ \mathcal{E}^{(E2,y_k)}(l|d_i) = \mathcal{E}^{(\overline{E2})}(l|d_i) = \theta(RC \cdot CDF_i - l) \end{cases} \tag{25}$$

In order to provide numerical results, RC is set to 10 million Euro (MEuro), and CDF_i is set as in Tab. 2. These values are reliable and not critical, since in common for both single and multi-risk assessments.

3.1.1 Single-risk assessment

The single-risk assessment is performed implementing Eq. 12. As total seismic hazard, the annual rates for the city of Naples (LAT 40.8322, LON 14.2832, ID: 33201) provided by the official Italian hazard map [18,41] are taken, where PGA values are transformed to macroseismic EMS92 intensity values as in [27]. For simplicity, epistemic uncertainties are neglected, and only the best guess curve is considered. This hazard assessment formally includes also volcanic sources and it is produced starting from an undifferentiated seismic catalogue. This hazard is in the Poissonian form, and reads

$$\mathcal{H}^{(E1)}(x_j) = 1 - exp\{-\lambda_{x_j}^{(E1)} \cdot \Delta T\} \tag{26}$$

where the annual rates $\lambda_{x_j}^{(E1)}$ are available online [21]. The obtained seismic hazard curve for $\Delta T = 50$ years is reported in Figure 1.

Since $\mathcal{H}^{(E1)}$ is Poissonian, the physical vulnerability $\mathcal{P}V^{(S1;s)}(\geq d_i)$ can be assessed through Eq. 4, that is

$$\begin{cases} \mathcal{P}V^{(S1;s)}(\geq d_i) = 1 - exp\{-\lambda_{\geq d_i}^{(E1,s)} \Delta T\} \\ \lambda_{\geq d_i}^{(E1,s)} = \sum_j \mathcal{F}^{(E1,\overline{E2})}(\geq d_i|x_j) \times \delta\lambda_{x_j}^{(E1)} \end{cases} \tag{27}$$

where the fragility models for B_s building are available in [42]. The single-risk curve is then evaluated by substituting in Eq. 12, that is

$$\mathcal{R}c_{\geq l}^{(E1,s)} = \sum_i \theta(RC \cdot CDF_i - l) \cdot \delta\mathcal{P}V^{(S1;s)}(\geq d_i) \tag{28}$$

and it is reported in Figure 2 (black line).

3.1.2 Multi-risk assessment

The multi-risk assessment is performed through Eqs. 8 and 9, as modified in Eqs. 22 and 23 for non Boolean $E2$.

The co-active risk factor is evaluated through Eq. 10. To assess the volcanic hazard $\mathcal{H}^{(E2)}(y_k)$, ash loadings ≥ 3 kPa, up to 10 kPa, with intervals of 1 kPa, are considered. Under the assumption of independence of time intervals, this hazard can be set as

$$\mathcal{H}^{(E2)}(\geq y_k) = 1 - \left[1 - pr_{(ER)} \cdot pr(\geq y_k|ER)\right]^{12\cdot\Delta T} \qquad \text{k=1,2,...,10} \tag{29}$$

where $pr_{(ER)}$ is the probability of eruption per month, whose a reliable order of magnitude for the volcanoes in the Napolitan area is $1 \cdot 10^{-3}$ per month [25, 26, 35], and $pr(\geq y_k|ER)$ represents the probability to have ash loading $\geq y_k$ given the occurrence of an eruption. To set this probability, a target point 4 km eastward of eruptive vent is selected and the hazard is modelled as in COMBO1 analysis in [36] (in which the Bayesian Event Tree method is set for the Campi Flegrei caldera). The COMBO1 analysis considers 4 possible eruption's sizes for 1 specific eruptive vent, typical configuration of most of central volcanoes, and it is used here to derive the entire hazard curve for all the y_k values. For simplicity, also in this case, epistemic uncertainties are neglected and only the best guess curve is considered. The obtained hazard curve is reported in Figure 3. The distance of 4 km is reasonable in the Napolitan area, since it represents approximately the distance between Naples (central area) and the most likely vent position for the Campi Flegrei caldera [37]. Of course, the results will strongly depend on this distance and here only one representative value is selected. Indeed, the quantification in all the urbanized areas around the volcano is important, but it is beyond the goals of this illustrative application.

The seismic hazard, conditioned to the occurrence of $E2$, is assumed to not depend on y_k, meaning that it is assumed to be equal for all eruptions. To set reliable values for $\mathcal{H}_{E1|E2}(x_j)$, it is noted that (i) the persistence time is quite long and, for this application, ΔT_p is set to 3 months [8], (ii) it is quite unlikely concomitant earthquakes and volcanic eruptions, except for the case of earthquakes occurring during the eruptive dynamics. As first approximation, the syn-eruptive seismicity is assumed Poissonian, that is

$$\mathcal{H}^{(E1|E2)}(x_j) = 1 - exp\{-\lambda_{x_j}^{(E1|E2)} \cdot \Delta T_p\} \tag{30}$$

The annual rates are calculated from the 1982-84 seismicity at Campi Flegrei [11]. Macro-seismic intensities are estimated from M_s and attenuated for distances of 4 km as in [42]. The resulting non cumulated monthly rates, $d\lambda_{x_j}^{(E1|E2)}$ are reported in Figure 4, with maximum expected intensities at site equal to 7 (epicentral intensity

8). Note that, a rather spread seismicity is expectable, so that the attenuation selected might underestimate peak intensities at the target site. On the other hand, it is noted that the 1982-84 did not lead to eruptions. On one side, higher intensities are expected during crises leading to eruptions (e.g., the maximum macroseismic intensity registered during the last eruption, Monte Nuovo 1582 AD, is 10 [19]). On the other side, most of those seismic events are expected before the actual eruption. However, the eruption sizes that contribute more to the hazard are quite large (classes 3 and 4 in [31]), for which a complex eruptive dynamics is expected, with several eruptive phases [30]. For comparison with the total hazard reported above ($\mathcal{H}^{(E1)}$, as used in the single-risk assessment), the combined seismic hazard curve, as computed applying Eq. 24, is reported in Figure 1 (grey dots).

The physical vulnerability of the co-active risk factor is then assessed through Eq. 4, so that

$$\begin{cases} \mathcal{P}V^{(E1|y_k)}(\geq d_i|y_k) = 1 - exp\{-\lambda^{(E1|y_k)}_{\geq d_i} \cdot \Delta T_p\} \\ \lambda^{(E1|y_k)}_{\geq d_i} = \sum_j \mathcal{F}^{(E1,y_k)}(\geq d_i|x_j, y_k) \cdot \delta\lambda^{(E1|E2)}_{x_j} \end{cases} \quad (31)$$

in which fragility functions are set as in Table 8 of [42], and the hazard is set as in Eq. 30. The co-active risk factor can be evaluated by substituting in Eqs. 22 and 10, that is

$$\mathcal{R}c^{(E1,E2)}(\geq l) = \sum_i \theta(RC \cdot CDF_i - l) \sum_k \delta\mathcal{P}V^{(E1|y_k)}(d_i|y_k)\delta\mathcal{H}^{(E2)}(y_k) \quad (32)$$

where the volcanic hazard term is assessed as in Eq. 29. The obtained co-active risk factor is reported in Figure 2 (dashed line).

The isolated risk factor is assessed through Eq. 11, from the single-risk and virtual-risk factors. The single-risk term is exactly the one assessed above, for the single-risk assessment, as reported in Figure 2 (black line). The virtual-risk factor can be assessed directly from Eq. 13, since there is not dependence of the $E1$ hazard on y_k. The physical vulnerability, as above, is assessed through Eq. 4, that is

$$\begin{cases} \mathcal{P}V^{(E1|E2,v)}(\geq d_i) = 1 - exp\{-\lambda^{(E1|E2,v)}_{\geq d_i} \cdot \Delta T_p\} \\ \lambda^{(E1|E2,v)}_{\geq d_i} = \sum_j \mathcal{F}^{(E1,\overline{E2})}(\geq d_i|x_j) \cdot \delta\lambda^{(E1|E2)}_{x_j} \end{cases} \quad (33)$$

where the fragilities are exactly the same adopted above for the single-risk assessment (Eq. 27), while the annual rates are the ones adopted to assess the co-active risk factor (Eq. 31). The virtual-risk factor is then evaluated by substituting in Eq. 13 (or, equivalently, Eq. 23), that is

$$\mathcal{R}c^{(E1,v)}(\geq l) = \sum_i \theta(RC \cdot CDF_i - l)\delta\mathcal{P}V^{(E1|E2,v)}(d_i)\mathcal{H}^{(E2)}(\geq y_1) \quad (34)$$

and it is reported in Figure 2 (dotted line).

The final multi-risk curve, as obtained through Eqs. 8 and 11, is reported in Figure 2 (grey line).

3.1.3 Single vs Multi-risk assessments

The bias between single (black) and multi-risk (grey) curves is evident in Figure 2. A more quantitative evaluation of this bias is given by average losses (risk index). By substituting in Eq. 17, the single-risk assessment reads:

$$\mathcal{R}^{(E1,s)} = RC \cdot \sum_i CDF_i \cdot \delta\mathcal{P}V^{(S1;s)}(\geq d_i) = 1.2003 \text{ MEuro} \qquad (35)$$

and for the multi-risk assessment:

$$\begin{cases} \mathcal{R}^{(E1)} &= \mathcal{R}^{(E1,E2)} + \mathcal{R}^{(E1,s)} - \mathcal{R}^{(E1,v)} = \\ &= RC \cdot \sum_{i,k} CDF_i \cdot \delta\mathcal{P}V^{(E1|y_k)}(d_i|y_k)\delta\mathcal{H}^{(E2)}(y_k) \\ &\quad +RC \cdot \sum_i CDF_i \cdot \delta\mathcal{P}V^{(S1;s)}(\geq d_i) \\ &\quad -RC \cdot \sum_{i,k} CDF_i \cdot \delta\mathcal{P}V^{(E1|y_k,v)}(d_i|y_k)\delta\mathcal{H}^{(E2)}(y_k) = \\ &= 1.3389 \text{ MEuro} \end{cases} \qquad (36)$$

that is, the bias of single-risk assessment results:

$$\begin{cases} \delta\mathcal{R}^{(E1)} = 0.13859 \text{ MEuro} \\ \delta\mathcal{R}^{(E1)}/\mathcal{R}^{(E1,s)} = 0.115 \approx 10\% \end{cases} \qquad (37)$$

These results are based on very first-order, but reasonable values, and they show that the single-risk assessment underestimates the actual risk by about 10 percent. It is important to note that this results strongly depend on the selected position, since the long-term volcanic hazard decays quite quickly with distance (e.g., [36]). Therefore, this bias could strongly vary also at urban scale, affecting in a non-uniform manner the total multi-risk assessment. This bias, together with the intrinsic epistemic uncertainties associated to any risk assessment [1], could have large effects on the resulting risk hierarchization presented to decision makers, at least in specific areas.

3.2 Case study 2: exposure dependence in tsunami/earthquake interaction

This application focuses in assessing the tsunami risk ($E1 \equiv$ tsunami) related to human life losses, in presence and in absence of a previous seismic event ($E2$ is a significant earthquake) that can influence the exposure in coastal areas. In particular, it is investigated here the effect on tsunami risk of changes in the exposure to tsunamis due to local strong earthquakes striking the target area. Indeed, such local earthquakes may significantly modify the exposure to tsunamis, either increasing it (e.g., concentration in seaside areas of people escaping from damaged buildings), or decreasing it (spontaneous evacuation of seaside areas of adequately informed population). This effect is here reported as illustrative example of interaction at the exposure level. For significant earthquakes, events strong enough to generate significant damages in the target area are considered, so that such damages could induce changes to the tsunami exposure. It is set, as risk metrics, the number of deaths and, for simplicity, only direct effects of tsunami are considered. An exposure time of 1 year ($\Delta T = 1$ yr) is considered.

With these choices, only one damage state contribute to the loss assessment ($d_1 \equiv d$ means 'death'). The fragility (mortality) of persons exposed to tsunami waves is independent from the occurrence/non-occurrence of significant earthquakes in the area. In this application, the formulation in [33] is assumed, where the rates of deaths (and injuries) as functions of water depth are assessed by using information from both the survey and prior events for the 17 July 2006 Java tsunami. This fragility function reads:

$$\mathcal{F}^{(E1,\overline{E2})}(d=1|x_j) = \mathcal{F}^{(E1,E2)}(d=1|x_j) = \mathcal{F}(d=1|x_j) \approx \frac{0.4}{10} x_j \qquad (38)$$

where x_j is expressed in meter. In this formulation, differences (e.g., age, sex, etc.) among the people exposed to the tsunami waves are not considered.

As regards the exposure, each person counts as one in the risk assessment. The total risk should consider the total number of people exposed to tsunami in the target area. While an explicit formulation for the total risk curve is more complicated, under the assumption of identical and independent individuals and identical capability in movements at all times (day/night, summer/winter), the risk index can be written as

$$\mathcal{R}^{(*)} = \int_d \int_x < N^{(*)} >_t \cdot d\mathcal{F}(d|x) \cdot d\mathcal{H}(x) \qquad (39)$$

where $< N^{(*)} >_t$ is the average number (in time) of exposed people to tsunamis of intensity x, and it plays the role of $l_{ave}^{(*)}$ in Eq. 16. For simplicity, this application is then limited to the assessment of risk indexes, in order to quantify the bias between single- and multi-risk assessments.

In general, in case of non occurrence of $E2$, the number of exposed people strongly varies through time, in particular for day/night as well as summer/winter changes, and its average reads

$$l_{ave}^{(\overline{E2})} = \frac{1}{\Delta T} \int_0^{\Delta T} N(t) dt \equiv N_{ave} \qquad (40)$$

In case of occurrence of $E2$, the number of exposed persons may drastically vary. Indeed, it may be extremely high if the population is not correctly informed about tsunamis, since many people can move to the seaside in order to escape from falling buildings (e.g., Lisbon 1755, Messina 1908). If, on the opposite, the population has been adequately prepared to the risk of tsunamis, the exposure to tsunamis may be very low (provided that they have sufficient time to evacuate), since the informed population runs toward less exposed areas (e.g., Padang 2009). In either case, in first approximation, it does not depend on time, so that

$$l_{ave}^{(E2)} = N_{EQ} \qquad (41)$$

which can be, for limited time windows, very different from N_{ave}.

To assess the single-risk index, the total hazard curve for a given location is considered. In Fig. 5, for example, it is reported the tsunami hazard curve $\delta\mathcal{H}^{E1}(x_j)$ for a representative location offshore Seaside, Oregon, as obtained by [16]. Note that this hazard curve is not obtained for inundation, and also it is relative to a completely different environment with respect to fragilities. Thus, the risk assessment obtained by combining this hazard curve with the fragility reported above is only really indicative, with the solely goal of illustrating a complete analysis. Applying Eq. 17, first row, single-risk indexes can be assessed as

$$\mathcal{R}^{(E1,s)} = \sum_i N_{ave} \cdot \delta\mathcal{F}(d=1|x_j) \cdot \delta\mathcal{H}^{(E1)}(x_j) = N_{ave} \times 1.4 \cdot 10^{-3} \tag{42}$$

where $\delta\mathcal{H}^{(E1)}(x_j)$ is the non-cumulative hazard curve relative to the target coastline area.

To assess the multi-risk index, the hazard factor $\mathcal{H}^{(E1,E2)}(\geq x_j)$ must be defined, that is, the probability in ΔT that a significant seismic event $E2$ occurs, followed within ΔT_p by a tsunami with intensity $\geq x_j$. The persistence time ΔT_p can be approximately set to few hours, that is the time after which people start moving back to cities, and/or emergency actions start taking place. The probability to have a tsunami within few hours after an earthquake that causes large damages in the target coastline is surely very low, unless the tsunami is caused by the earthquake itself or by close in time aftershocks. Thus, the term $\mathcal{H}^{(E1,E2)}(x_j)$ essentially refers to the tsunami caused by earthquakes close enough to the target area to cause significant damages to structures. This allows a more quantitative definition of significant earthquakes, since such events must be strong enough to generate significant tsunami (e.g., $M > 7$), and close enough to generate significant damages due to seismic waves (e.g., at distances < 100 km). Note that all the other possible tsunamis, due to either distant earthquakes or non-seismic sources, for which no significant seismic damages are experienced at the target site, will contribute through the isolated hazard term $\mathcal{H}^{(E1,\overline{E2})}$.

In Figure 5, the contribution to the hazard curve of near seismic sources (green line), and the one of the other (only seismic, in this case) sources (red lines) are individuated [16], allowing the identification of $\delta\mathcal{H}^{(E1,E2)}(x_j)$. This allows an explicit assessment of the multi-risk factors, since the co-active risk factor reads

$$\begin{aligned} \mathcal{R}^{(E1,E2)} &= N_{EQ} \textstyle\sum_j \delta\mathcal{F}(d=1|x_j) \cdot \delta\mathcal{H}^{(E1,E2)}(x_j) = \\ &= N_{EQ} \times 1.6 \cdot 10^{-4} \end{aligned} \tag{43}$$

where $\delta\mathcal{H}^{(E1,E2)}(x_j)$ is the hazard related to the near seismic sources. The virtual-risk factor reads

$$\begin{aligned} \mathcal{R}^{(E1,v)} &= N_{ave} \textstyle\sum_j \delta\mathcal{F}(d=1|x_j) \cdot \delta\mathcal{H}^{(E1,E2)}(x_j) = \\ &= N_{ave} \times 1.6 \cdot 10^{-4} \end{aligned} \tag{44}$$

that is, obviously, equal to co-active risk factor, a part for the number of exposed persons.

The multi-risk index can finally be computed:

$$\begin{aligned} \mathcal{R}^{(E1)} &= \mathcal{R}^{(E1,E2)} + \mathcal{R}^{(E1,s)} - \mathcal{R}^{(E1,v)} = \\ &= [0.16 \cdot N_{EQ} + 1.24 \cdot N_{ave}] \cdot 10^{-3} \end{aligned} \tag{45}$$

and the bias between single and multi-risk assessed:

$$\begin{cases} \delta\mathcal{R}^{(E1)} = 0.16 \cdot (N_{EQ} - N_{ave}) \cdot 10^{-3} \\ \delta\mathcal{R}^{(E1)}/\mathcal{R}^{(E1,s)} = 0.11 \cdot (\frac{N_{EQ}}{N_{ave}} - 1) \end{cases} \tag{46}$$

Note that, in this illustrative application, a significant bias (> 0.05, that is, 5%) is obtained already for an increase of 50% of the exposure in presence of $E2$

($N_{EQ} = 1.5 \cdot N_{ave}$), which is a rather small increase in case of significant local earthquakes. Note also that $\delta\mathcal{R}$ also quantifies the long-term benefit (decrease of risk) in case of correct education of people and/or management plan about the possibility of tsunamis just after a large local earthquake, in which case $N_{EQ} \approx 0$. In this case, indeed, the earthquake works as an efficient precursor for local tsunamis, reducing the long-term tsunami risk by $\approx 10\%$.

4 Final Remarks

The presented method allows a full assessment of one specific long-term risk, considering the interaction that its terms may have with other hazards and/or external events. Beside considering interaction at the hazard level (e.g., [23]), the method focuses to the possibility that one secondary hazard triggers changes to the vulnerability and exposure terms relative to the primary hazard (e.g., [42]). To do that, the method (i) makes use of interacting (sometimes called time-dependent) vulnerability and exposure terms, in which the effect on the target assets of combined hazards is accounted for, and (ii) it introduces the concept of persistence time window for the hazard that triggers the interaction. Combining such terms with the hazard assessments, the presented method allows an explicit quantification (eqs. 8 and followings) of the long-term risk associated to the primary hazard in a multi-risk perspective, that is, considering risk interactions at all levels. This quantification finally allows an explicit estimate (eqs. 18 and 19) of the bias that it is induced by neglecting risk interactions (as in single-risk analyses), and thus an explicit assessment of the statistical significance in long-term risk assessments of any conceivable interaction among two risks.

In assessing risk, the method makes use of fully aggregated hazard assessments. This characteristic makes it applicable in systematic analyses of the strength of interactions in extended target areas, since it does not imply large computational efforts. This is of primary importance, since only such systematic analyses (i) allow identifying if and where a specific interaction is significant and thus when it is important to consider strictly multi-hazard/risk procedures, and (ii) may help implementing effective multi-risk mitigation actions, focusing to specific interactions and selected areas.

This method is limited to applications with only two interacting risks. In several case studies, however, more than two hazards may potentially interact in the same area. In theory, the developed method could be recursively extended to more than two hazards, but this further development would lead to an explosion in the number of the terms necessary to the analysis (e.g., all terms could eventually depend on three or more intensity measures). On the other side, the presented method may be applied to each couple of hazards, and it may be used to filter out interactions that lead to statistically non significant effects in long-term risk assessments. In alternative, other methods considering combinations of single scenarios (cascade events) may be applied (e.g., [2]). However, the large number of scenarios to be considered in long-term risk assessments may limit their effective applicability.

Acknowledgements

The work described in this paper has been carried out in the framework of the project *Quantificazione del Multi-Rischio con approccio Bayesiano: un caso studio per i rischi naturali della città di Napoli* (2010-2013), funded by the Italian Ministry of Education, Universities and Research (Ministero dell'Istruzione, dell'Università e della Ricerca) [1].

Fig. 1 Complete (black) and co-active (grey) Probabilistic Seismic Hazard Assessment in the area of Naples (Italy), with an exposure time $\Delta T = 50$ years. The co-active hazard factor considers only seismic event occurring in presence of significant ash loading (> 3 kPa) on roofs, within a persistence time window $\Delta T_p = 3$ months.

Fig. 2 Seismic risk curves in the area of Naples (Italy), with an exposure time $\Delta T = 50$ years. Single (black) and multi-risk (grey) assessments are reported, together with multi-risk factors, i.e., the co-active (dashed grey line) and the virtual (dotted grey line) ones.

Fig. 3 Probabilistic Volcanic Hazard Assessment for ash fall in the area of Naples (Italy), with an exposure time $\Delta T = 50$ years. This hazard curve is assessed assuming a target area 4 km eastward of a possible eruptive vent, with 4 possible eruption sizes typical of the Campi Flegrei caldera, Italy (see text for more details).

Fig. 4 Annual rates of macroseismic intensity at site (attenuated at 4 km) during the 1982-1984 unrest episode in Campi Flegrei, Italy. See text for more details.

Fig. 5 Probabilistic Tsunami Hazard Assessment offshore Seaside, Oregon, as obtained by [16]. With different colours, the distinct contributions to the hazard of near (green) and other (red) sources are highlighted.

Table 1 List of the symbols used in the paper.

Symbol	Description
x	Intensity measure for the hazard of the primary event ($E1$)
y	Intensity measure for the hazard of the secondary event ($E2$)
d	Damage state
l	Loss
ΔT	Exposure time window
ΔT_p	Persistence time window
$\mathcal{H}^{(*)}(x)$	Cumulative hazard, i.e., $pr(\geq x; \Delta T)$
$\mathcal{F}^{(*)}(d\|x)$	Fragility curve, i.e., $pr(\geq d\|x)$
$\mathcal{PV}^{(*)}(d)$	Physical vulnerability, i.e., $pr(\geq d; \Delta T)$
$\mathcal{E}^{(*)}(l)$	Cumulative 'exposure' term, i.e., $pr(\geq l\|d)$
$l_{ave}^{(*)}(d)$	Mean losses caused by damages d
$\mathcal{L}^{(*)}(x)$	Mean losses caused by intensity x
$\mathcal{R}c^{(*)}(\geq l)$	Cumulative risk curve, i.e., $pr(\geq l; \Delta T)$
$\mathcal{R}^{(*)}$	Risk index, i.e., herein mean loss in ΔT
for the generic cumulative F	
dF	non-cumulative function
δF	non-cumulative function, for discrete intervals

NOTES:
$*$ may stand for: '$E1, E2$' (co-active factor); '$E1, \overline{E2}$' (isolated factor); '$E1, s$' (single factor, only for risk); '$E1, v$' (virtual factor, only for risk)

Table 2 Cost damage factors (CDFs) for each damage state in Application 1. CDF_k represents the fraction of the replacement cost (RC) to repair the k-th damage state.

d_i	Description	CDF_i
0	No damages	0
1		0.1
2		0.2
3		0.6
4		0.8
5	Complete	1.0

References

1. ByMuR project: Quantificazione del Multi-Rischio con approccio Bayesiano: un caso studio per i rischi naturali della città di Napoli (Bayesian Multi-Risk assessment: a case study for natural risks in the city of Naples) (2010-2013). URL http://bymur.bo.ingv.it
2. MATRIX - New Multi-Hazard and Multi-Risk Assessment Methods for Europe (2010-2013). URL http://matrix.gpi.kit.edu
3. Alberico, I., Petrosino, P., Lirer, L.: Volcanic hazard and risk assessment in a multi-source volcanic area: the example of Napoli city. Natural Hazards - Earth System Science (11), 1057–1970 (2011). DOI 10.5194/nhess-11-1057-2011
4. Bazzurro, P., Cornell, C.: Disaggregation of seismic hazard. Bulletin of the Seismological Society of America **89**(2), 501–520 (1999)
5. Bommer, J., Spence, R., Erdik, M., Tabuchi, S., Aydinoglu, N., Booth, E., del Re, D., Peterken, O.: Development of an earthquake loss model for Turkish catastrophe insurance. Journal of Seismology **6**, 431–446 (2002). URL http://dx.doi.org/10.1023/A:1020095711419. 10.1023/A:1020095711419
6. Bovolo, A., Abele, S., Bathurst, J., Caballero, D., Ciglan, M., Eftichidi, G., Simo, B.: A distributed framework for multi-risk assessment of natural hazards used to model the effects of forest fire on hydrology and sediment yield. Computers and Geosciences **35**, 924–945 (2009). DOI 10.1016/j.cageo.2007.10.010
7. Cavalieri, F., Franchin, P., Gehl, P., Khazai, B.: Quantitative assessment of social losses based on physical damage and interaction with infrastructural systems. Earthquake Engineering and Structural Dynamics (2012). DOI 10.1002/eqe.2220. URL http://dx.doi.org/10.1002/eqe.2220
8. Convertito, V., Zollo, A.: Assessment of pre-crisis and syn-crisis seismic hazard at Campi Flegrei and Mt. Vesuvius volcanoes, Campania, southern Italy. Bulletin of Volcanology **73**, 767–783 (2011). DOI 10.1007/s00445-011-0455-2
9. Cornell, C.: Engineering seismic risk analysis. Bulletin of the Seismological Society of America **58**, 1583–1606 (1968)
10. Cornell, C., Krawinkle, H.: Progress and challanges in seismic performance assessment. PEER Center News **3**(2) (2000). URL http://peer.berkeley.edu/news/2000spring/index.html
11. De Natale, G., Zollo, A.: Statistical analysis and clustering features of the Phlegraean Fields earthquake sequence (May 1983 - May 1984). Bulletin of the Seismological Society of America **76**(3), 801–814 (1986)
12. Der Kiureghian, A.: Non-ergodicity and PEER's framework formula. Earthquake Engineering and Structural Dynamics **34**, 1643–1652 (2005). DOI 10.1002/eqe.504
13. Douglas, J.: Physical vulnerability modelling in natural hazard risk assessment. Natural Hazards - Earth System Science **7**, 283–288 (2007)
14. Elefante, L., Jalayer, F., Iervolino, I., Manfredi, G.: Disaggregation-based response weighting scheme for seismic risk assessment of structures. Soil Dynamics and Earthquake Engineering **30**, 1513–1527 (2010)
15. FEMA: Multi hazard identification and risk assessment:a cornerstone of the national mitigation strategy. Tech. rep., FEMA (1997)
16. Gonzalez, F., Geist, E., Jaffe, B., Kanoglu, U., Mofjeld, H., Synolakis, C., Titov, V., Arcas, D., Bellomo, D., Carlton, D., Horning, T., Johnson, J., Newman, J., Parsons, T., Peters, R., Peterson, C., Priest, G., Venturato, A., Weber, J., Wong, F., Yalciner, A.: Probabilistic tsunami hazard assessment at Seaside, Oregon, for near- and far-field seismic sources. Journal of Geophysical Research **114**(C11023) (2009). DOI 10.1029/2008JC005132
17. Grunthal, G., Thieken, A., Schwarz, J., Radtke, K., Smolka, A., Merz, B.: Comparative Risk Assessments for the City of Cologne – Storms, Floods, Earthquakes. Natural Hazards **38**(1-2), 21–44 (2006). DOI 10.1007/s11069-005-8598-0
18. Gruppo di Lavoro: Redazione della mappa di pericolosita' sismica prevista dall'Ordinanza PCM 3274 del 20 marzo 2003. Rapporto Conclusivo per il Dipartimento della Protezione Civile. Tech. rep., Istituto Nazionale di Geofisica e Vulcanologia, Milano-Roma (2004)
19. Guidoboni, E., Ciucciarelli, C.: The Campi Flegrei caldera: historical revision and new data on seismic crises, bradyseisms, the Monte Nuovo eruption and ensuing earthquakes (twelfth century 1582 AD). Bulletin of Volcanology **73**, 655–677 (2011). DOI 10.1007/s00445-010-0430-3

20. Guidoboni, E., Mariotti, D.: Il terremoto e il maremoto del 1908: effetti e parametri sismici. In: G. Bertolaso, E. Boschi, E. Guidoboni, G. Valensise (eds.) Il terremoto e il maremoto del 28 dicembre 1908: analisi sismologica, impatto, prospettive, p. 814pp. SGA edizioni (2008)
21. INGV-DPC: Interactive Seismic Hazard Maps (2004). URL http://esse1-gis.mi.ingv.it
22. Lee, K., Rosowsky, D.: Fragility analysis of woodframe buildings considering combined snow and earthquake loading. Structural Safety **28**(3), 289–303 (2006)
23. Marzocchi, W., Garcia-Aristizabal, A., Gasparini, P., Mastellone, M., Di Ruocco, A.: Basic principles of multi-risk assessment: a case study in Italy. Natural Hazards pp. 1–23 (2012). URL http://dx.doi.org/10.1007/s11069-012-0092-x. 10.1007/s11069-012-0092-x
24. Marzocchi, W., Mastellone, M.L., Di Ruocco, A., Novelli, P., Romeo, E., Gasparini, P.: Principles of multi-risk assessment 2009. Principles of multi-risk assessment: interactions amongst natural and man-induced risks, Project Report - FP6 NARAS project (contract No. 511264). Tech. Rep. EUR23615, European Commision - Directorate-General Research – Environment (2009)
25. Marzocchi, W., Sandri, L., Gasparini, P., Newhall, C., Boschi, E.: Quantifying probabilities of volcanic events: the example of volcanic hazard at Mt. Vesuvius. Journal of Geophysical Research **109**(B11201) (2004). DOI 10.1029/2004JB003155
26. Marzocchi, W., Sandri, L., Selva, J.: BET_EF: a probabilistic tool for long- and short-term eruption forecasting. Bulletin of Volcanology **70**, 623 632 (2008)
27. Masi, A.: Seismic Vulnerability Assessment of Gravity Load Designed R/C Frames. Bulletin of Earthquake Engineering **1**, 371–395 (2003)
28. NIBS: HAZUS99 Estimated Annualized Earthquake Losses for the United States. Tech. Rep. 366, Federal Emergency Management Agency, Washington, D.C. (2000)
29. NIBS: HAZUS, Technical manuals, vol. Earthquake loss estimation methodology, federal emergency management agency edn. Washington, D.C. (2004)
30. Orsi, G., Civetta, L., Del Gaudio, C., de Vita, S., di Vito, M.A., Isaia, R., Petrazzuoli, S., Ricciardi, G., Ricco, C.: Short-term ground deformations and seismicity in the nested Campi Flegrei caldera (Italy): an example of active block-resurgence in a densely populated area. Journal of Volcanology and Geothermal Research **91**(2-4), 415–451 (1999)
31. Orsi, G., Di Vito, M.A., Selva, J., Marzocchi, W.: Long-term forecast of eruption style and size at Campi Flegrei caldera (Italy). Earth and Planetary Science Letters **287**, 265–276 (2009)
32. Pinto, P.E. (ed.): Probabilistic Methods for Seismic Assessment of Existing Structures, vol. LESSLOSS Report No. 2007/06. IUSS Press, Pavia, Italy (2007)
33. Reese, S., Cousins, W., Power, W., Palmer, N., Tejakusuma, I., Nugrahadi, S.: Tsunami vulnerability of buildings and people in South Java - field observations after the July 2006 Java tsunami. Natural Hazards - Earth System Science **7**(5), 573–589 (2007)
34. Ryden, J., Rychlik, I.: Probability and Risk Analysis, An Introduction for Engineers, springer edn. Springer Berlin / Heidelberg, Berlin NY (2006)
35. Sandri, L., Guidoboni, E., Marzocchi, W., Selva, J.: Bayesian event tree for eruption forecasting (BET_EF) at Vesuvius, Italy: a retrospective forward application to the 1631 eruption. Bulletin of Volcanology **71**(7), 729–745 (2009)
36. Selva, J., Costa, A., Marzocchi, W., Sandri, L.: BET_VH: Exploring the influence of natural uncertainties on long-term hazard from tephra fallout at Campi Flegrei (Italy). Bulletin of Volcanology **72**(6), 717–733 (2010). DOI 10.1007/s00445-010-0358-7
37. Selva, J., Orsi, G., Di Vito, M., Marzocchi, W., Sandri, L.: Probability hazard map for future vent opening at the Campi Flegrei caldera, Italy. Bulletin of Volcanology **74**, 497–510 (2012). DOI 10.1007/s00445-011-0528-2. 10.1007/s00445-011-0528-2
38. Spahn, H., Hoppe, M., Vidiarina, H., Usdianto, B.: Experience from three years of local capacity development for tsunami early warning in Indonesia: challenges, lessons and the way ahead. Natural Hazards - Earth System Science **10**, 1411–1429 (2010). DOI 10.5194/nhess-10-1411-2010
39. Stergiou, E., Kiremidjian, A.: Treatment of Uncertainties in Seismic Risk Analysis of Transportation Systems. Tech. Rep. 154, John A. Blume Earthquake Engineering Center, Civil Engineering Department, Stanford University, Stanford, CA (2006)
40. Strasser, F., Bommer, J., Sesetyan, K., Erdik, M., Cagnan, Z., Irizarry, J., Goula, X., Lucatoni, A., Sabetta, F., Bal, I., Crowley, H., Lindholm, C.: A Comparative Study of European Earthquake Loss Estimation Tools for a Scenario in Istanbul . Journal of Earthquake Engineering **12**(S2), 246–256 (2008). DOI 10.1080/13632460802014188

41. Stucchi, M., Meletti, C., Montaldo, V., Crowley, H., Calvi, G.M., Boschi, E.: Seismic Hazard Assessment (2003-2009) for the Italian Building Code. Bulletin of the Seismological Society of America **101**, 1885–1911 (2011)
42. Zuccaro, G., Cacace, F., Spence, R., Baxter, P.: Impact of explosive eruption scenarios at Vesuvius. Journal of Volcanology and Geothermal Research **178**, 416–453 (2008)