amra analysis and monitoring of environmental risk

AGU FALL MEETING
San Francisco | 3–7 December 2012

ISTITUTO NAZIONALE DI GEOFISICA E VULCANOLOGIA

S43A-2470

ON THE UNCERTAINTIES OF SEISMIC PARAMETERS: A BAYESIAN FRAMEWORK FOR THEIR ESTIMATION USING BRUNE'S MODEL

Marco Caciagli [1,3] - (marco.caciagli@bo.ingv.it), **Alexander Garcia-Aristizabal** [2] - (alexander.garcia@amracenter.com), **Jacopo Selva** [3] - (jacopo.selva@bo.ingv.it)
1) Istituto Nazionale di Geofisica e Vulcanologia, Arezzo, Italy - 2) Center for the Analysis and Monitoring of Environmental Risk (AMRA), Napoli, Italy - 3) Istituto Nazionale di Geofisica e Vulcanologia, Bologna, Italy

ABSTRACT

The estimation of seismic parameters from ground-motion records is subject to many uncertainties, such as: (i) parameterization, modeling procedures and underlying hypotheses, (ii) approximated input parameters, (iii) instrumental errors on records and their impact in data post-processing, (iv) procedures to estimate model's parameters.

However these uncertainties are rarely treated and propagated to the final results. For example, on one side, density of rocks, velocity model, geometrical spreading, radiation pattern are just some of the common parameters needed to estimate the main seismic parameters of an earthquake and are generally used as average values.

On the other side, uncertainties derived from the acquisition system and processing of the data are often neglected.

Nevertheless, in many cases these uncertainties may be particularly important, as for example in the analysis of historical earthquakes, where both instrumental response and treatment of analog records intrinsically imply non negligible sources of uncertainty.

Here, we present a new Bayesian procedure to estimate seismic parameters that allows:

(i) to obtain a robust estimation of the Brune's model parameters (Brune 1970, 1971) and relative uncertainties,
(ii) to account for the uncertainty related to the Earth model parameters used, and
(iii) to propagate such uncertainties on the estimation of seismological parameters (seismic moment, moment magnitude, radius of the circular source zone and static stress drop).

It is important to highlight that this study does not intend to discuss the validity or the physical significance of the Brune's model, but it is focused on the details of how to fit it on a dataset in order to evaluate the seismological parameters, accounting and properly propagating a rather large range of uncertainties. These capabilities of the proposed procedure are finally demonstrated through an illustrative application analyzing seismic records from historical events.

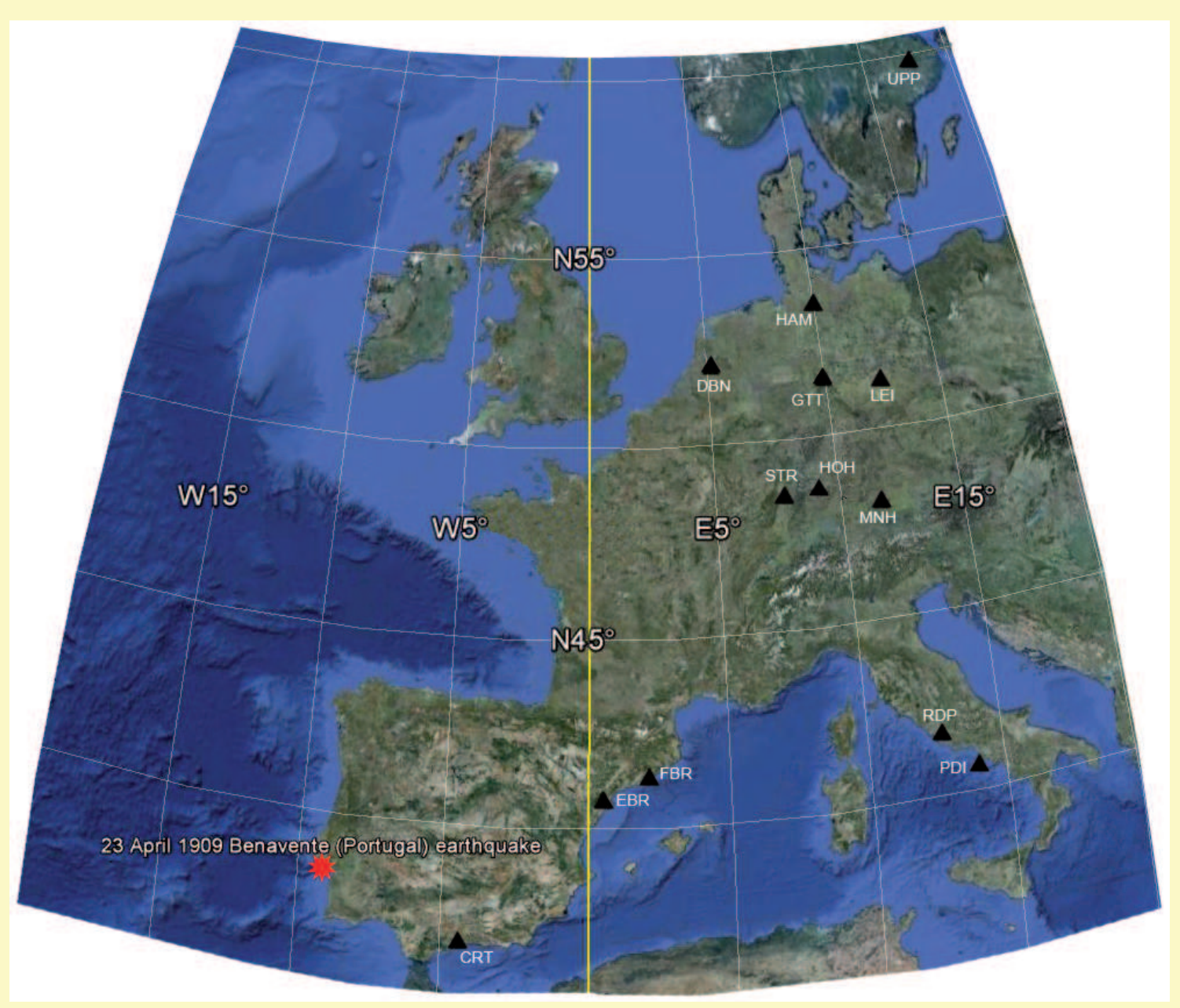


Figure 1. The epicentral location (red star) of 23rd april 1909 Benavente earthquake (from Dineva et al., 2000) and distribution of historical seismic stations (blacks triangles) from which it has been possible to recover seismograms that recorded the event (from Stich et al., 2005).

CASE STUDY and DATA: the 23rd april 1909 Benavente (Portugal) earthquake and its historical seismograms.

The re-assessment of the seismic parameters analyzing seismograms of an historic earthquake is a complex work.

The treatment of analogic records of seismic waves poses different technical challenges. The waveforms obtained are the result of different operations performed to overcome the numerous problems that may occur during the analog to digital conversion of a historical seismogram (Batllo et al, 2008). Furthermore, the instrumental characteristics of the seismometers, information necessary for proper correction of the waveform, are often approximative or lost (see table 1, also).

As case study to test our methodology, we analyzed the Benavente earthquake (Portugal) occurred in April 23, 1909 (Fig. 1). This earthquake has already been studied by various authors, and estimates of the seismic parameters of this event based on historical records are available in literature (Teves-Costa et al., 1999; Dineva et al., 2002; Stich et al., 2005; for details see Table 2). The number of components and seismic stations used for the computation changes according to the publications consulted. For example, Taves-Costa et al. (1999) analyzed three seismograms: two horizontal components from Uppsala (Sweden) sismic station (UPP) and the EW component of the Strasbourg seismic station (STR). Dineva et al. (2002) used six records (all horizontal components): two from Ebro (Spain), three from Cartuja (Spain) and one from De Bildt (Netherland) seismic stations. In this work, we used the same dataset used by Stich et al. (2005); the stations and components are listed in table 1.

MODEL PARAMETER ESTIMATION: A Bayesian approach

We start assuming that our data ***d* = (*d1, d2, ..., dn*)** and the model parameters are linked by a specific model (the "forward operator" often used in inverse theory, e.g. Menke 1989, Tarantola 2005). Our task is to infer the parameter values of a given model function ***g*(·)** that we sample in presence of noise:

$$d = g(m) + \epsilon$$

where ε represents the `error' component in the observed data. Let θ be the vector containing the model parameters, **f**(θ) the density of the prior distribution for θ, f(d|θ) the likelihood for θ. Then, Bayes' theorem states:

$$f(\theta|d) = \frac{f(\theta) f(d|\theta)}{\int_\theta f(\theta) f(d|\theta) d\theta}$$

Bayes' theorem provides a tool for converting a initial set of `beliefs' about θ, as represented by the prior distribution ***f***(θ), into a posterior distribution ***f***(θ|***d***), that includes the additional information provided by the data ***d***.

Integrating out in the data domain, the marginal probability density in the model parameter space provides the posterior distribution in the space of the model parameters.

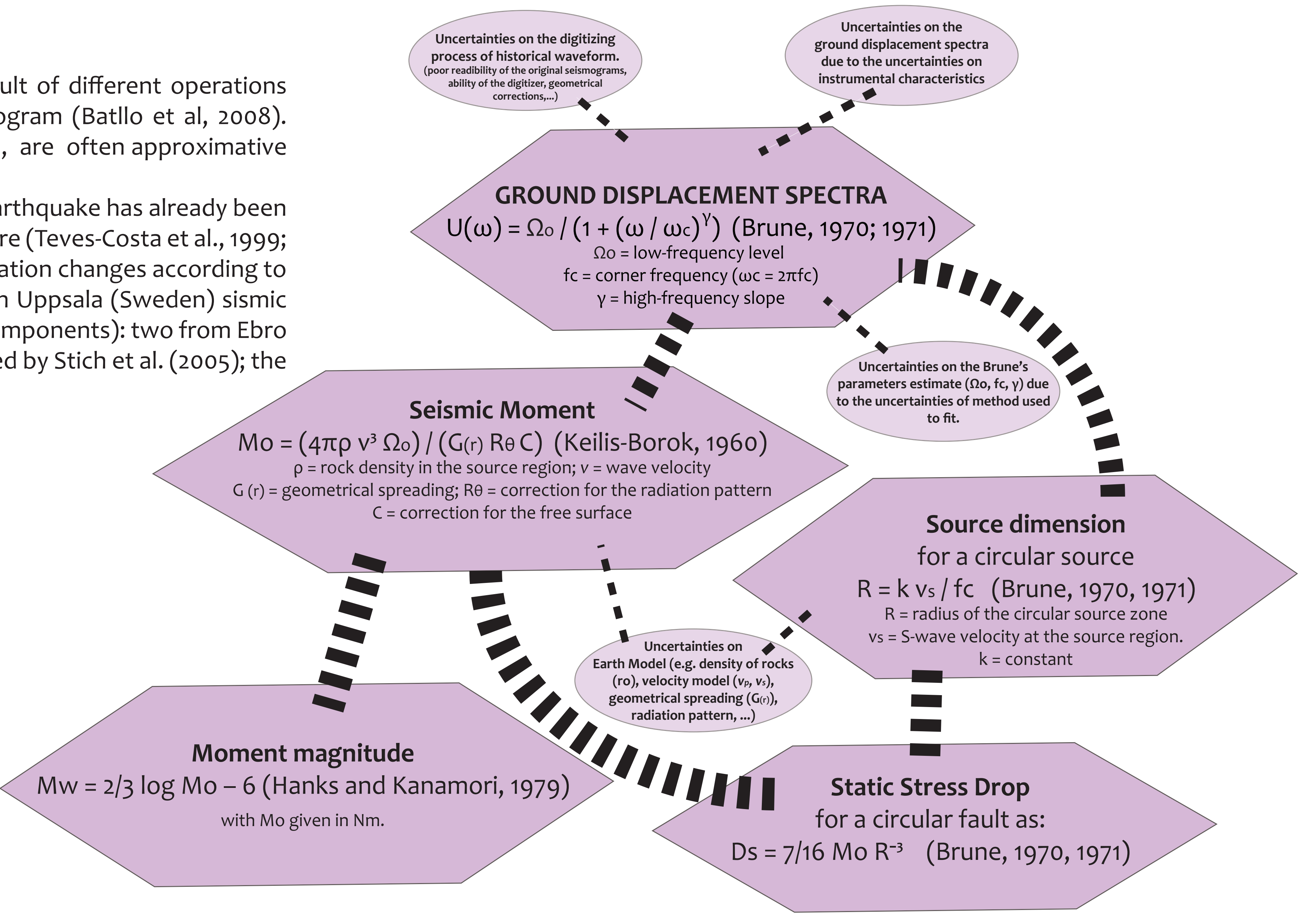
To obtain samples of the posterior distribution in the space of the model parameters we explore the model domain using a Markov-chain Monte Carlo (MCMC) approach based on the Metropolis algorithm (Metropolis and Ulam 1949; Metropolis et al. 1953; Hastings 1970). From the MCMC output (after eliminating a burn-in period and a thinning procedure), we get information about model parameters: an empirical probability distribution (CDF or PDF) for each model parameter, and (where possible) compute percentiles, moments, etc. to produce both best-guess values and associated uncertainties (e.g., see Fig 2-I).

Station	Instrument	T o [s]	α	Gain	Component
CRT (Cartuja)	Mod. Omori	14.00	0.4	33	NNW-SSE (N340-E)
	Wiechert	5.00	0.4	77	N-S
	Bifilar	6.03	0.45	80	E-W
DBN (De Bilt)	Bosch-Omori	18.00	0.4	20	(1 unknown component)
EBR (Ebre)	Grablovitz	13.00	0.4*	8	NE-SW; SE-NW
FBR (Fabra)	Cancani	4.0	0.4*	17.3	NE-SW; SE-NW
GTT (Göttingen)	Wiechert	11.7	0.35	147	N-S
		11.7	0.4	157	E-W
		5.7	0.55	159	Z
HAM (Hamburg)	Wiechert	10.5	0.48	190	N-S
	Hecker	9.9	0.48	195	[E-W]
		19.5	0.48	32	[N--S]
		20.0	0.45	32	[E-W]
HOH (Hohenheim)	Bosch-Omori	9.0	0.33	23	N-S; E-W
	Schmith Trifilar	1.5	.	400	[Z]
LEI (Leipzig)	Wiechert	8.5	0.34	227	N-S
		8.5	0.27	241	[E-W]
MNH (München)	Wiechert	12.5	0.4	240	N-S; E-W
PDI (Porto d'Ischia)	Grablovitz	(13.0)	(0.4)	(8)	NE-SW; NW-SE (attribution unknown)
RDP (Rocca di Papa)	Agamennone	4.2	0.4*	60	NE-SW; NW-SE
STR (Strasbourg)	Wiechert	8.3	0.46	200	E-W
UPP (Uppsala)	Wiechert	9.8	0.38	189	N-S
		9.4	0.38	191	[E-W]

Table 1. List of digitized waveforms for the 1909 Benavente earthquake, and instrumental parameters according to original bulletins and other historic sources (from Stich et al.,2005). Waveforms of the components in square brackets have not been used for the computation in this work. Damping values for instruments that were essentially undamped (except of dry friction) were set to a generic value for computational reasons and are marked with a star. Values in brackets are not documented for this specific instrument (PDI), and were adopted from similar instruments elsewhere.

References	Mo (Nm)	Mw	R(m)	Stress drop (Pa)
Teves-Costa P. et alii (1999)	$1.03\ 10^{18}$	6	$1.15\ 10^{4}$	$3.05\ 10^{7}$
Dineva S. et alii (2002)	$2.30\ (\pm0.93)\ 10^{18}$	6.08 (±0.21)	$4.30\ (\pm1.6)\ 10^{3}$	$1.00\ 10^{7}$
			$2.30\ (\pm0.9)\ 10^{3}$	$7.90\ 10^{7}$
Stich D. et alii (2005)	$1.08\ 10^{18}$	6.00 (±0.1)	-	-

Table 2. Seismic parameters of 23rd april 1909 Benavente earthquake obtained from analysis of historical seismograms by previous Authors. Note that uncertainty associated to results (where present) is the sigma error of the average (Dineva et al 2002) or of the best solution chosen (Stich et al., 2005) with respect to all results computed. In Dineva et al (2002), fault plane radius, R, and static stress drop, have been calculated using the Brune (1970) and Madariaga (1976) models, listed respectively in the first and in the second row.



Uncertainties on the digitizing process of historical waveform.
(poor readibility of the original seismograms, ability of the digitizer, geometrical corrections,...)

Uncertainties on the ground displacement spectra due to the uncertainties on instrumental characteristics

GROUND DISPLACEMENT SPECTRA
$U(\omega) = \Omega_0 / (1 + (\omega / \omega_c)^{\gamma})$ (Brune, 1970; 1971)
Ω_0 = low-frequency level
fc = corner frequency ($\omega_c = 2\pi f_c$)
γ = high-frequency slope

Uncertainties on the Brune's parameters estimate (Ω_0, fc, γ) due to the uncertainties of method used to fit.

Seismic Moment
$M_0 = (4\pi\rho v^3 \Omega_0) / (G(r) R_\theta C)$ (Keilis-Borok, 1960)
ρ = rock density in the source region; *v* = wave velocity
G (r) = geometrical spreading; Rθ = correction for the radiation pattern
C = correction for the free surface

Source dimension
for a circular source
$R = k v_s / f_c$ (Brune, 1970, 1971)
R = radius of the circular source zone
vs = S-wave velocity at the source region.
k = constant

Uncertainties on Earth Model (e.g. density of rocks (ro), velocity model (v_p, v_s), geometrical spreading ($G_{(r)}$), radiation pattern, ...)

Moment magnitude
Mw = 2/3 log Mo – 6 (Hanks and Kanamori, 1979)
with Mo given in Nm.

Static Stress Drop
for a circular fault as:
$Ds = 7/16\ M_0 R^{-3}$ (Brune, 1970, 1971)

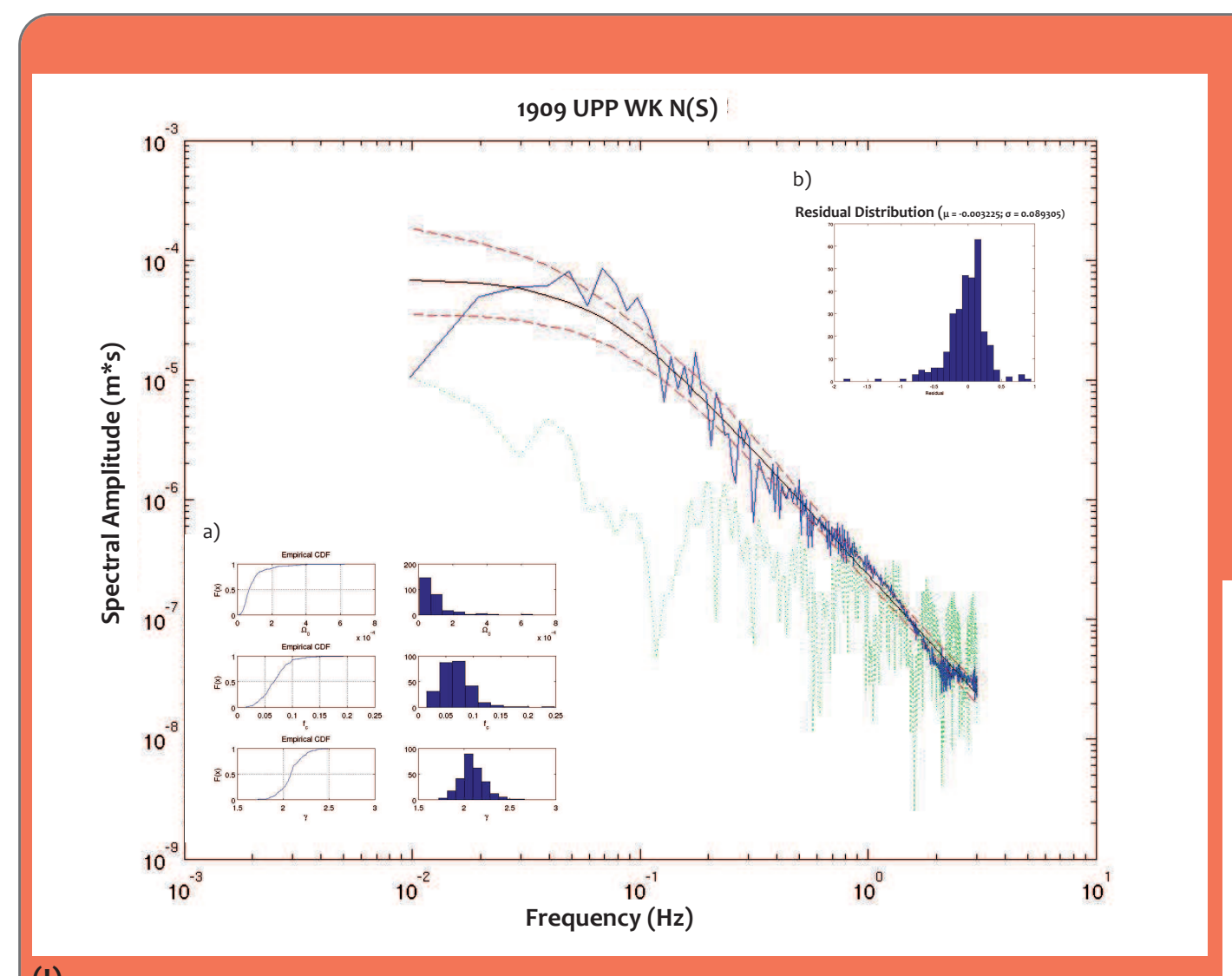
Sismic parameters (exagonal boxes) estimated using our approach and uncertainties sources (ellipsoidal boxes) that influence the different steps of computation.The formulas inside the boxes are those used for computations. The increase of the thickness of the dashed lines has the purpose of graphically display the variability of the uncertainties associated with each step of the process and that are propagated in the final results of the analysis.

POINTS TO TAKE HOME:

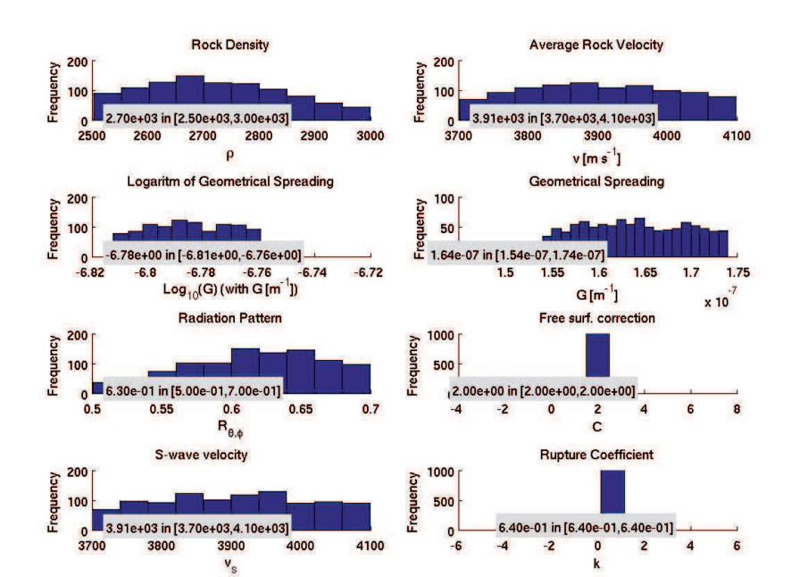
We have presented a methodological approach to perform inference on seismic parameters. To illustrate it, we have used for reference Brune's model applied to a dataset form historical seismograms of the 1909 Benavente (Portugal) earthquake.

This analysis allows:

- *Fitting Brune's model to spectra:*
  - *> homogeneous treatment of different order of magnitude in spectrum amplitudes (log treatment)*
  - *> eventual inclusion of operator experience (in prior distributions)*
- *Estimating seismic parameters:*
  - *> objective and robust assessessment from single & multiple seimograms*
  - *> full propagation of all uncertainties (all spectra & Earth model), quantifying*
    1. *best guess values*
    2. *distributions & confidence intervals*



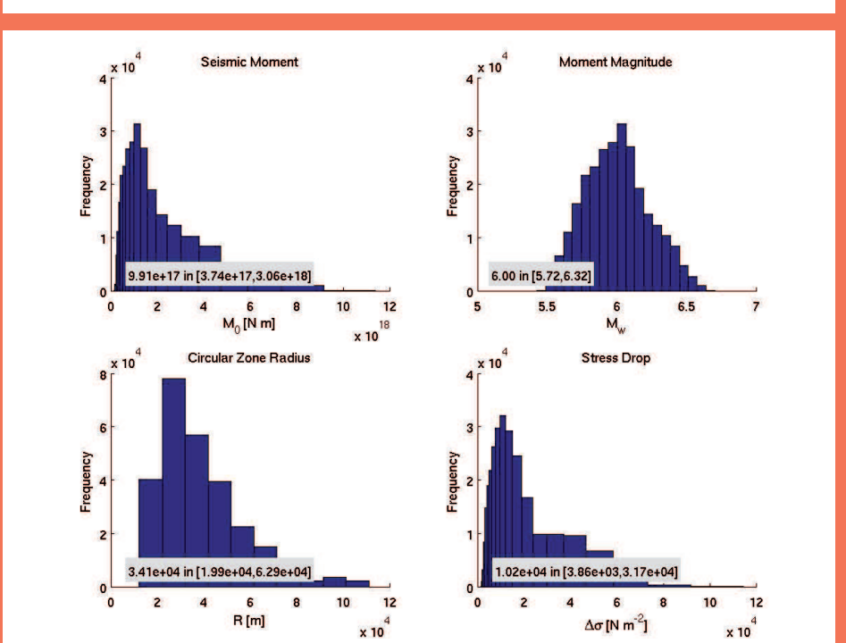
(I)
For each wave train (P, S) of each component, Bruene's model is fitted with a Bayesian approach (see Model Parameter Estimation) and all the possible Brune's parameters are estimated. (a): In blue is plotted the Wiechert north component S-wave displacement spectra (recorded in Uppsala station); in green, the spectra of a sample of noise recorded previous to the arrival of the earthquake signal.
The black dashed line represents the best guess fit of Brune's model (calculated using the median values of the empirical distributions representing the model parameters solutions). The red dashed lines are the 10th and 90th percentiles representing the uncertainties due to variability of hte solutions. (b): Plot of residuals resepct to the best guess solution.



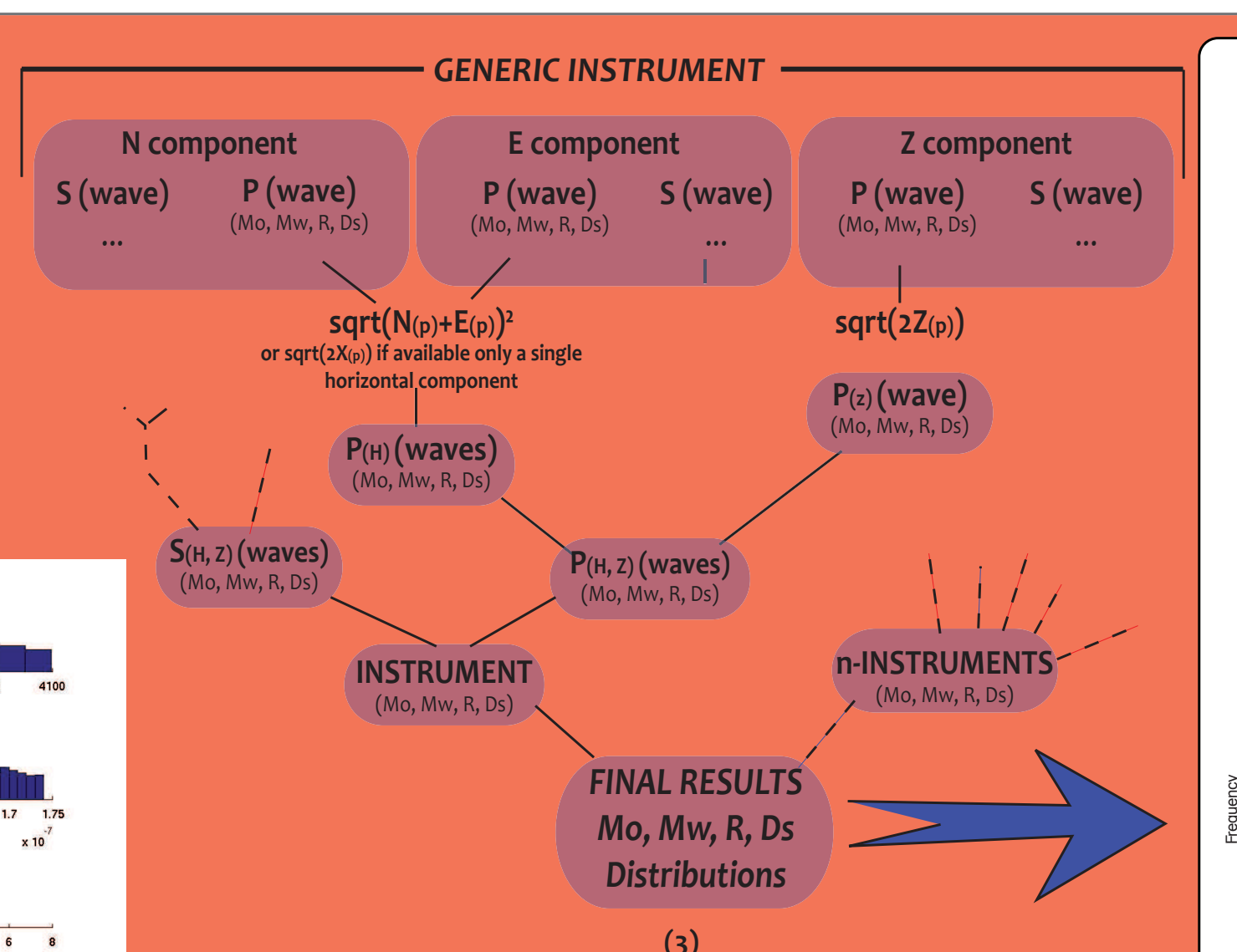
Monte Carlo sampling of truncated gaussian distribution of Earth model parameters

(2)
The variability of Earth model parameters (density of rocks, velocity model, geometrical spreading, radiation pattern) is arbitrarily defined and described using truncated Gaussian distributions.
Monte Carlo sampling is used to draw parameter values from all the distributions of the parameters used to calculate seismic parameters (Mo, Mw, R, Ds).

The final results are expressed as distributions representing the variability of the calculated parameters due to all the uncertainties considered during the process.



Final results obtained from S-wave (north component) spectral analysis of UPP



GENERIC INSTRUMENT

N component: S (wave) ... ; P (wave) (Mo, Mw, R, Ds)
E component: P (wave) (Mo, Mw, R, Ds) ; S (wave) ...
Z component: P (wave) (Mo, Mw, R, Ds) ; S (wave) ...

$\mathrm{sqrt}(N_{(p)}+E_{(p)})^2$
or $\mathrm{sqrt}(2X_{(p)})$ if available only a single horizontal component

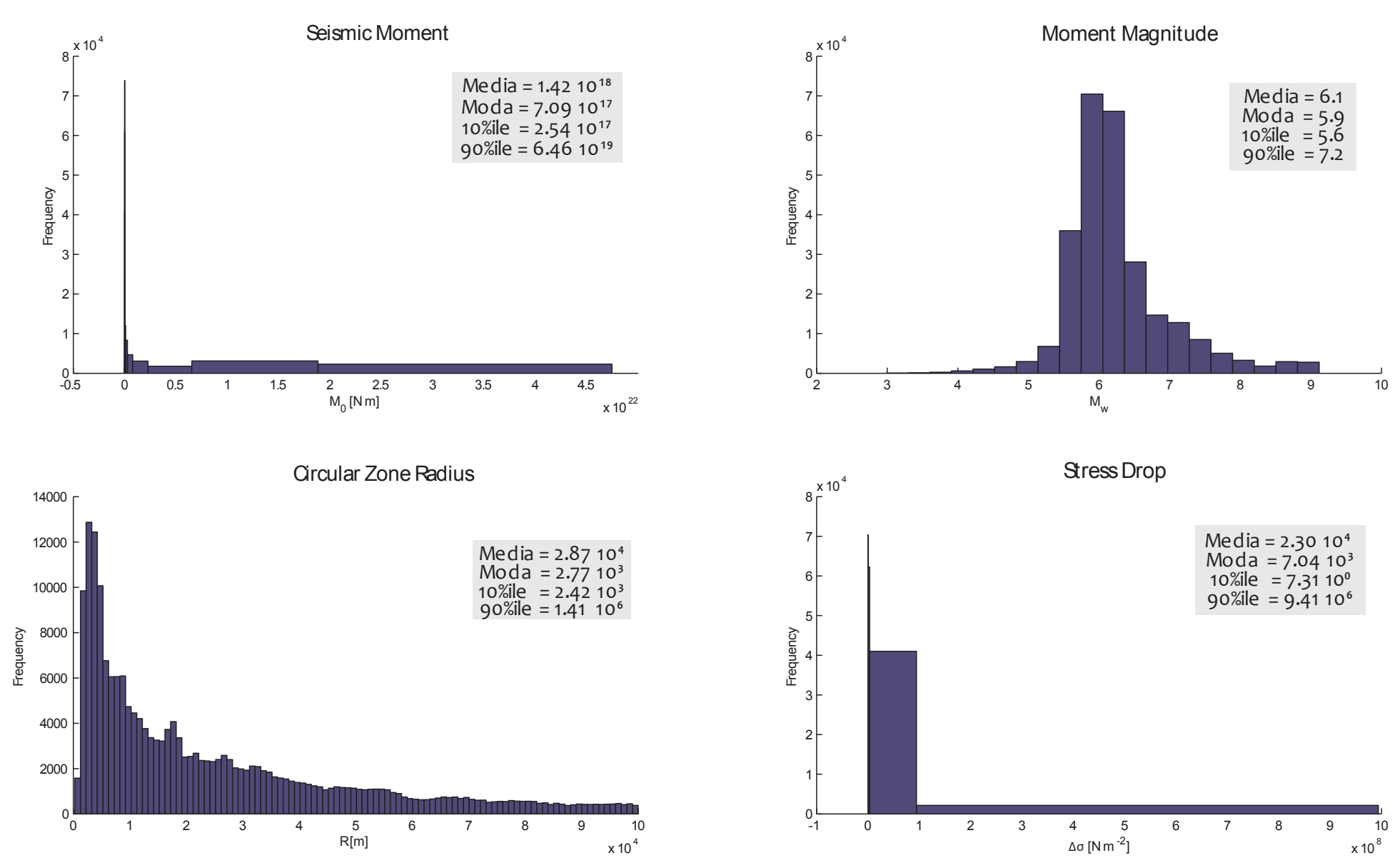
$\mathrm{sqrt}(2Z_{(p)})$

$P_{(H)}$ (waves) (Mo, Mw, R, Ds)
$P_{(z)}$ (wave) (Mo, Mw, R, Ds)
$S_{(H, Z)}$ (waves) (Mo, Mw, R, Ds)
$P_{(H, Z)}$ (waves) (Mo, Mw, R, Ds)
INSTRUMENT (Mo, Mw, R, Ds)
n-INSTRUMENTS (Mo, Mw, R, Ds)

FINAL RESULTS
Mo, Mw, R, Ds
Distributions

(3)
After the two previous steps, merging procedures are performed.
i) For each single phase:
the horizontal components are merged, trough MC sampling of the results obtained in step 2, and the samples obtained are used in the relation $\mathrm{sqrt}(N_{(phase)}+E_{(phase)})^2$. If it is present only one horizontal component (X) the relation used is $\mathrm{sqrt}(2X_{(phase)})$. Latter formula, is used also for the vertical component. The obtained solutions (horizontal and vertical) are then merged, always with a MC sampling, to reach a solution relating to each single phase.
ii) For each single instrument *(if more than one phase is availble):*
the resulting solutions are merged with the aim to define seismic parameters solutions for each instrument.
iii) The last merging step:
Finally, a merging of all the solutions for all the instruments is performed.
The final result, for any seimic parameter estimated, is a sample of possible solutions representing the variability due to the uncertainties considered in the process.
The final results obtained in this application for the 23rd april 1909 Benavente (Portugal) earthquake are:

	Mo	Mw	R(m)	Ds(N/m²)
Median	$1.42\ 10^{18}$	6.1	$2.87\ 10^{4}$	$2.30\ 10^{4}$
Mode	$7.09\ 10^{17}$	5.9	$2.77\ 10^{3}$	$7.04\ 10^{3}$
10th perc.	$2.54\ 10^{17}$	5.6	$2.42\ 10^{3}$	$7.31\ 10^{0}$
90th perc.	$6.46\ 10^{19}$	7.2	$1.41\ 10^{6}$	$9.41\ 10^{6}$



ACKNOWLEDGEMENTS

We are extremely greatful to Josep Batlló, Ramon Macia, Jose Morales, Daniel Stich and Paula Teves-Costa for the great availability demonstrated in sharing with us their dataset on the earthquake studied. Thank you very much!

REFERENCES

Batlló, J., D. Stich, R. Macià (2008). *Quatitative Analysis of Early Seismograph Recordings.* Historical Seismology: Interdisciplinary Studies of Past and Recent Earthquakes. J. M. Fréchet, M.; Stucchi, M. (eds.). Berlin, Springer Verlag. XVIII: 379-396.

Brune, J. N. (1970). Tectonic Stress and Spectra of Seismic Shear Waves from Earthquakes. Journal of Geophysical Research 75(26): 4997-5009.

Brune, J. N. (1971). Correction. Journal of Geophysical Research 76(20): 5002.

Dineva, S., J. Batlló, D. Mihaylov, T. van Eck, (2002). *Source parameters of four strong earthquakes in Bulgaria and Portugal at the beginning of the 20th century.* Journal of Seismology 6(1): 99-123.

Hanks, T. C. and H. Kanamori (1979). *Moment Magnitude Scale.* Journal of Geophysical Research 84(Nb5): 2348-2350.

Hastings, W. K., 1970. Monte Carlo sampling methods using Markov chains and their applications, Biometrika, 57(1), 97-109.

Keilis-Borok, V. I. (1960). Investigation of the mechanism of earthquakes. Sov. Res. Geophys. (English trans.) 4, 29.

Menke, W. (1989). *Geophysical Data Analysis: Discrete Inverse Theory.* Revised Edition, vol. 45 of International Geophysics Series, Academic Press, Inc., New York.

Metropolis, N. and Ulam, S. (1949). *The Monte Carlo method.* J. Amer. Stat. Assoc., 44, 335-341.

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., and Teller, E. (1953). *Equation of state calculations by fast computing machines.* J. Chem. Phys., 21, 1081-1092.

Stich, D., J. Batlló, R. Macia, P. Teves-Costa, J. Morales (2005). *Moment tensor inversion with single-component historical seismograms: The 1909 Benavente (Portugal) and Lambesc (France) earthquakes.* Geophysical Journal International 162(3): 850-858.

Tarantola, A. (2005). Inverse problem theory and methods for model parameter estimation. SIAM (Society for industrial and applied mathematics), Philadelphia.

Teves-Costa, P., J. F. Borges, I. Rio, R. Ribeiro, C. Marreiros, (1999). *Source Parameters of Old Earthquakes: Semi-Automatic Digitization of Analog Records and Seismic Moment Assessment.* Natural Hazards 19(2): 205-220.