On deformation sources in volcanic areas: modeling the Campi Flegrei (Italy) 1982-84 unrest

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Abstract

Deformation sources in volcanic areas are generally modeled in terms of pressurized tri-axial ellipsoids or other cavities with simple geometrical shapes embedded in homogeneous half-spaces. However, the assumption of a particular source mechanism and the neglect of medium heterogeneities bias significantly the estimate of source parameters. Leveling and EDM data, collected during the 1982-84 unrest episode at Campi Flegrei (Italy), are employed to retrieve source parameters according to a Bayesian inversion procedure, considering the heterogeneous elastic structure of the volcanic area. We describe a general deformation source in terms of a suitable moment tensor, through 3D finite element computations. Best fitting moment tensors are found to be incompatible with any pressurized ellipsoid. Taking into account the deflation of a deeper magma reservoir, which accompanies the inflation of the shallower moment source, data fit improves considerably but the retrieved moment tensor of the shallow source is found to be incompatible with pressurized ellipsoids, still. Looking for alternative physical

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models of the deformation source, we find that the best fit moment tensor can be best interpreted in terms of a mixed-mode (shear and tensile) dislocation at 5.5 km depth, striking EW and dipping by $\sim 30^\circ$ to the North. Gravity changes are found to be compatible with the intrusion of $\sim 60{\cdot}10^6$ m^3 of volatile rich magma with density ~ 2000 kg/m^3.

Keywords: caldera unrest, deformation, Campi Flegrei, numerical modeling

1. Introduction

Campi Flegrei (CF) is a nested caldera in Italy, close to the city of Naples. The area is characterized by high volcano hazard, due to the high density of inhabitants, and it is subject to intense geophysical and geochemical monitoring. A major unrest episode took place in 1982-84, when the town of Pozzuoli, located at the caldera center, was uplifted by 1.80 m. Since ground deformation is a reliable indicator of unrest, possibly resulting from the intrusion of fresh magma within the shallow rock layers, the deformation source is generally modeled as a pressurized cavity. The most popular of these models is the Mogi source (Mogi, 1958) which describes the deformation due to a spherical cavity with radius much smaller than its depth. The bell-shaped vertical pattern of leveling measurements at CF during the unrest is nicely fitted by a Mogi source located by many authors at about 3 km depth beneath the center of the caldera (e.g., Berrino et al., 1984; De Natale et al., 1991; Berrino, 1994; Fernandez et al., 2001). In recent years, the development of modern volcano geodesy and modeling techniques have clearly detected uplift episodes at CF in the 2000 and 2004-2006 amounting to few cm, renewing the interest to study of the 1982-1984 unrest episode, also leading to interpretations not in agreement with each other. Indeed, there was a controversy regarding the nature of the source (hydrothermal vs magmatic) and its overpressure. Battaglia et al. (2006) interpret the 1982-84 unrest in terms of a pressurized sill (among other pressurized sources such as Mogi and spheroid) in a homogeneous half-space, inferring from gravity data a very low “intrusion” density of 600±500 kg/m^3, compatible with supercritical water. Amoruso et al. (2008) support a much higher density for the sill-like source, compatible with trachybasaltic magma (2500±500 kg/m^3), by taking into account a horizontally layered medium which approximates the subsurface structure at CF. Both sources are localized at shallow depths of 2.5-3.5 km for Battaglia et al. (2006) and 3.0-3.5 km for Amoruso et al. (2008).

We must be aware that several common assumptions adopted for the CF caldera and in general for volcano geodetic modeling may bias the results.

1. Source geometry. Which geometry should be chosen for the deformation source (a sphere, an ellipsoid or a sill) clearly depends on the ability of the different models to reproduce the observed deformation. As illustrated by Dieterich and Decker (1975), the horizontal deformation pattern is particularly sensitive to the shape of the pressurized cavity, while the vertical deformation pattern is less constraining. It is not surprising that the choice of the source geometry, among the mentioned range of possibilities, may affect significantly the estimate of the depth, the position and (to a lesser extent) the volume of the source (Amoruso et al., 2007). Then, assigning an a priori shape of the source (within a very restricted “library” of available solutions) may bias considerably the inference of source parameters. Furthermore, as clearly shown by Trasatti and Bonafede (2008), the shape assumed for the overpressure source has great influence on the calculated

gravity changes, leading to very different inferred densities for the intrusion mass.

2. Medium complexity. Bonafede and Ferrari (2009) have shown that, as far as the medium is homogeneous, some source parameters (e.g. depth, location, incremental volume and intrusion density) depend only slightly from the assumed rheology (whether elastic or viscoelastic), while other parameters (notably the overpressure) are very sensitive to it. On the contrary, the deformation pattern depends strongly on the heterogeneity of the mechanical properties of the medium surrounding the source so that solutions computed in a homogeneous half-space may introduce a systematic bias in the interpretation of data collected in strongly heterogeneous regions. For instance, Trasatti et al. (2005) and Crescentini and Amoruso (2007) show that neglecting the elastic heterogeneities while inverting deformation data results in considerably inaccurate estimates of source depth. This is easily understood in terms of the low flexural rigidity of the soft shallow layers which conform easily to the deformation of the hard deeper layers.
3. Pressurized source assumption. An important limitation of pressurized cavities employed as deformation sources is that they do not provide any release of shear stress accompanying tensile opening due to magma overpressure. This assumption is appropriate if the cavity was filled with fluids even before the intrusion event so that any shear stress on the boundary of the cavity must vanish both before and after the intrusion. On the other hand, intrusion of fluid magma across pre-stressed solid rock provides the complete release of shear tractions which were present before magma emplacement over the source boundaries. This possibility is probably ignored because of the as-

sumption (plausible but unwarranted) that magma should open cracks in the direction of maximum tension, i.e. over a principal stress plane, where no shear stress can be present. But this implies to ignore the possibility that shear failure may precede magma emplacement (seismically induced intrusion), may accompany it (mixed mode-I and mode-II fracture) or that a pre-existent weakness plane is chosen by the ascending magma. In these cases the cavity boundary are required not to be a principal plane, and shear slip may take place in accordance with the observation that volcanic regions are strongly heterogeneous and seismically active. Furthermore, significant shear slip may take place on the boundary of a pressurized cavity if its shape is not symmetric or if strong heterogeneities are present; thus the assumption that the deformation source is a pressurized point-like cavity strongly constrains the variety of allowable moment tensors, as will be shown later.

4. Mass conservation. Finally, mass conservation requires that magma emplaced within a shallow reservoir must come from a (generally deeper) origin source. If the origin source is in the mantle, its deflation accompanying the inflation of the shallow source may be probably neglected when modeling surface deformation and gravity changes. However, in most volcanic regions, intermediate storage regions exist, whose deflation cannot be simply ignored: Okubo and Watanabe (1989) are among the few authors who account explicitly for both a shallow and a deep origin source while inverting deformation and gravity data.

From the previous considerations it appears that a reliable inference of source parameters in volcanically active areas should:

1. take into account a realistic description of the medium embedding the source;

2. avoid a priori assumptions regarding the geometrical shape of the deformation source;
3. include the possibility of shear stress release over the rock-magma interface;
4. account explicitly for mass conservation.

In previous papers Trasatti et al. (2008, 2009) perform data optimization at Mt Etna (Italy) without fixing a priori the source shape and including the heterogeneous elastic structure of the volcano. Models are based on Finite Element (FE) computation of the deformation field produced by a general moment tensor source: its interpretation in terms of a pressurized ellipsoid (Davis, 1986) is found to be plausible. In this paper we adopt the same methodology to study the 1982-84 unrest at CF by taking into account all the clues listed above. We perform a plausible physical interpretation of the retrieved moment tensor, extending the work by Bonafede and Ferrari (2009).

2. The Campi Flegrei 1982-84 unrest

The CF caldera is a complex resurgent caldera structure including submerged and continental parts at the western edge of the Bay of Naples. The last eruption took place in 1538 A.D. and since then secondary volcanism (intense degassing, seismic swarms and several episodes of ground uplift) is observed. The eruptive history and the structural setting of the area is reviewed, among others, by Rosi et al. (1983) and Orsi et al. (1996).

During the 1982-84 unrest episode, ground uplift was periodically monitored through leveling surveys, EDM surveys and 5 tide gauge placed in the harbor of Pozzuoli (close to the location of maximum uplift), along the coastline of the

Gulf of Pozzuoli and one in Naples (Fig. 1) (Berrino et al., 1984). The maximum uplift was 1.80 m in November 1984 (w.r.t. January 1982) recorded in the city of Pozzuoli, and the relative pattern of deformation remained practically unchanged during the unrest. The spatial pattern of uplift was nearly axi-symmetric (Fig. 1b), and this feature was generally considered as a strong indication that the source itself had to be isotropic or axi-symmetric (Berrino et al., 1984; Dvorak and Berrino, 1991; Battaglia et al., 2006; Amoruso et al., 2008). However, EDM data show significant asymmetry and a non-radial pattern of the horizontal displacements, the eastern sector of the caldera being characterized by larger displacements with respect to the western and northern sectors (e.g. Barbarella et al., 1984; Berrino et al., 1984; Bianchi et al., 1987). Seismic activity was mostly clustered in the northern sector (e.g. Dvorak and Berrino, 1991). EDM data were collected with several surveys during and after the unrest, however only in June 1980 and in June 1983 measurements were computed in a large number of benchmarks (Fig. 1c), allowing to map changes of horizontal distances during the unrest (Dvorak and Berrino, 1991). In this paper we employ a set of 36 EDM data from unpublished measurements, together with 66 leveling data collected in June 1980 and in June 1983. It must be mentioned that in Dvorak and Berrino (1991) EDM data are wrongly referred to September 1983 instead of June 1983.

Gravity data were also recorded regularly at a few benchmarks (Berrino et al., 1984; Berrino, 1994), but no control of the water table level was provided; at the Serapeo benchmark (a Roman market near the harbor of Pozzuoli) the water table is at sea level so that gravity data do not suffer from this problem. During the uplift phase, the gravity change at Serapeo, normalized to the uplift, was -215±6 μGal/m, in good agreement with the average of all the stations -213±6 μGal/m

(Fig. 1d) (Berrino, 1994).

The elastic structure of the shallow crust at CF is known from seismic tomography (Aster and Meyer, 1988; Zollo et al., 2003; Judenherc and Zollo, 2004; Chiarabba and Moretti, 2006; Zollo et al., 2008). The density structure is also constrained from deep wells and gravity inversions (Cassano and La Torre, 1987; Berrino et al., 2008; Zollo et al., 2008). Seismic tomography shows very soft shallow layers down to $\sim$ 0.6 km depth, where a large Poisson ratio ($\nu > 0.4$) is thought to be indicative of high porosity, liquid saturated yellow tuff. Below 0.6 km depth, the elastic parameters and the density progressively increase, with normal Poisson ratio $\nu \sim$ 0.28 up to values typical of a carbonatic basement below 3-5 km depth. The elastic structure varies also laterally: from active seismic experiments, Zollo et al. (2003) find evidence of the buried caldera rim off-shore, while Chiarabba and Moretti (2006) show a high v_p/v_s anomaly in the center of the caldera above 2 km depth, indicating the presence of liquid fluids. The vertical and lateral variations of the elastic structure below CF can be taken into account only by means of numerical tools.

3. FE inversion of the moment tensor

It is well known that any internal source of deformation can be described in terms of a moment tensor density distribution over a suitable source extent (e.g. Aki and Richards, 1980). If the source domain is small enough (e.g. it is much smaller than its depth) the point-source approximation is justified in the far-field and the surface deformation can be reproduced without considering the detailed moment density distribution. On the other hand, solutions were provided by Davis (1986) for a pressurized tri-axial ellipsoidal cavity under the point-source assumption. Following Davis (1986), the ellipsoid can be described by an equivalent system of double forces and double couples, i.e. a suitable moment tensor M_{ij}. Ellipsoid orientation is directly related to the orientation of the principal stress axes while the axes of the ellipsoid ($a > b > c$) are inversely related to the principal moments ($M_3 < M_2 < M_1$). We have to consider two main concerns regarding the ellipsoid and moment tensor relationship. Primarily, the relation is not biunivocal as already pointed out in Trasatti et al. (2009). If we plot M_2/M_1 vs M_3/M_1 ratios (Fig. 2) only the dark gray triangular area is permitted to obtain an ellipsoidal source. Furthermore, the analytical expressions provided by Davis (1986) allow us to compute the moment eigenvalues M_1, M_2, M_3 knowing a, b, c, but contain elliptic integrals that cannot be backward substituted. Therefore, the inversion for a moment tensor has the great advantage of describing a completely general point-source but its unambiguous interpretation in terms of a pressurized cavity is not always possible.

Following the approach by Trasatti et al. (2008, 2009), we perform inversions of the geodetic data at CF using the moment tensor source solutions generated by FE. We develop a FE model of the CF area including the elastic heterogeneities of the medium, while the surface is assumed to be flat (thus neglecting the mild topography). The model is made up of 150,000 8-nodes brick elements. The numerical domain is large enough (150 $\times$ 150 km horizontally and 60 km vertically) to avoid bias from the boundaries, where vanishing tractions at the surface or vanishing lateral and bottom boundaries displacements are assumed. The grid resolution is the highest near the center of the computational domain, and decreases toward the periphery. The central part of the domain is discretized into cubic cells with edge ℓ = 400 m, which are assumed as potential sources of deformation. We assign to each grid element independent elastic parameters and density, computed from the v_p and v_p/v_s anomalies from Chiarabba and Moretti (2006) for the caldera region. The tomography resolution is 1 km; parameters below 5 km depth are fixed to typical mid-crustal values, μ = 20 GPa and ν = 0.28. The commercial software MARC is employed to obtain solutions for the deformation field. We assign normal and shear stress components σ_{ij} on the opposite faces of each potential source and compute the surface deformation resulting from each distribution of force dipoles (normal stress) or each distribution of double couples (shear stress). The moment tensor source $M_{ij} = \ell^3 \sigma_{ij}$ is obtained through linear combination of the elementary solutions for a given cell (details can be found in Trasatti et al., 2008, 2009). The procedure is iterated for any of the 1000 cubic elements contained within a prescribed volume of 4 $\times$ 4 $\times$ 4 km centered in the caldera region. We build through FE computations a library of surface deformation fields, due to elementary moment sources located in any grid element of a prescribed volume beneath the caldera. The great deal of using this procedure is that data optimization can be performed taking into account the realistic elastic structure of the medium.

The inversion of the moment tensor source consists of a two steps approach: a direct search in the parameter space using the Neighbourhood Algorithm (Sambridge, 1999a), followed by a Bayesian inference (Sambridge, 1999b) to provide the posterior probability density distribution (PPD) of each parameter. Free parameters to be retrieved from the inversion are source coordinates x_S, y_S, z_S (East, North, up) and the moment tensor, given in terms of its eigenvalues M_1, M_2, M_3 (ordered according to their decreasing absolute value) and their respective eigenvectors $\hat{\mathbf{m}}_1, \hat{\mathbf{m}}_2, \hat{\mathbf{m}}_3$ described by the angles δ, ϕ, ψ (see supplementary material).

Angle δ is the dip of $\hat{\mathbf{m}}_3$ w.r.t. the horizontal plane, ϕ is the orientation of its surface projection measured anti-clockwise from x, ψ yields the rotation of $\hat{\mathbf{m}}_1$ from the vertical around $\hat{\mathbf{m}}_3$. Such an inversion provides the most probable source parameters and their uncertainties, the latter being estimated from the width of the PPD distribution.

The models considered are: HOM1 (HOMogeneous) assumes a moment source embedded in a homogeneous half-space, HET1 (HETerogeneous) accounts for a source in a heterogeneous medium. HOM2 and HET2 models include a deep deflating source and a shallow moment source inflating by the same volume, as discussed later on. After several trials performed with all the models described, the horizontal coordinates of the source were found to be always very close to the point of maximum recorded uplift, x_S = 426.2 km and y_S = 4518.8 km (UTM reference). This observation, together with the 400 m discretization of candidate source elements, led us to fix the horizontal coordinates, thus decreasing the number of free parameters from 9 to 7, with considerable benefit on the efficiency of the inversion procedure.

4. Single source models

4.1. Model HOM1

In order to elucidate the role of elastic heterogeneities, a preliminary inversion is performed assuming a homogeneous half-space. The best fit source parameters and their misfits are summarized in Table 1 for all the models considered in the paper. Probability distributions are shown as blue lines in Fig. 3 and the performance of the best fitting model can be inspected in Fig. 4 (blue circles): the overall misfit is 5.6 (average between the leveling misfit χ^2_{LEV1} = 3.5 and the

EDM misfit is χ^2_{EDM1} = 7.7). The HOM1 inversion yields a best fit source depth z_S = -3.9 km and sharply defined eigenvalues and eigenvectors. Very low PPD are associated to negative values of the eigen-moments. The eigenvectors orientation is approximately $\hat{\mathbf{m}}_1 \simeq$ up, $\hat{\mathbf{m}}_2 \simeq$ West, $\hat{\mathbf{m}}_3 \simeq$ South. We remind that the maximum eigen-moment M_1 (acting $\sim$ vertically along $\hat{\mathbf{m}}_1$) corresponds to the minimum axis for a pressurized ellipsoidal source. Therefore, the source seems to be characterized by horizontal dimensions much larger than the vertical. However, the mechanism provided by the moment tensor cannot be strictly associated with any pressurized point-like cavity, since the minimum ratio between moment eigenvalues is 1/3 for a flat crack (Fig. 2), while the ratio between best fit eigenvalues M_3/M_1 is close to zero (and even negative) and M_2/M_1 = 0.3. It may be mentioned that imposing an isotropic source mechanism (Mogi source) or a horizontal penny shaped crack (Battaglia et al., 2006) provides a shallower depth $\sim$ 3 km (Berrino et al., 1984; Trasatti et al., 2005), demonstrating that the a priori assumption of the source mechanism may provide biased estimates of source depth. In Fig. 4a the best fit model prediction (blue circles) are compared with leveling data displayed vs. radial distance from the surface projection of the source position. The different uplift computed for points at the same radial distance is a consequence of the asymmetry of the source mechanism; the fit to uplift data appears to be reasonably good, even if data are overestimated in the central region ($r <$ 1.5 km) and at the periphery ($r >$ 4 km). In Fig. 4b EDM distance changes between benchmarks are compared with model prediction (blue circles): the fit to EDM data is much worse, since the model underestimates systematically the data. Tests are performed successfully to check the accuracy of the FE model HOM1 compared with analytical solutions in a homogeneous half-space (Mindlin, 1936).

4.2. Model HET1

In model HET1 the heterogeneous elastic structure inferred from seismic tomography is accounted for and the PPD moment source parameters are shown in red in Fig. 3. As expected (Trasatti et al., 2005), the inferred source depth z_S = -5.2 km increases significantly w.r.t. model HOM1, due to the larger compliance of the shallower layers. The deeper source location requires significantly greater moment tensor eigenvalues in order to fit observed deformation, but strongly negative intermediate and minimum eigenvalues are inferred, with sharply defined PPD. The eigenvector orientation is approximately: $\hat{\mathbf{m}}_1 \simeq$ up, $\hat{\mathbf{m}}_2 \simeq$ North, $\hat{\mathbf{m}}_3 \simeq$ West. Despite of the order exchange between $\hat{\mathbf{m}}_2$ and $\hat{\mathbf{m}}_3$ w.r.t. model HOM1, the maximum eigen-vector $\hat{\mathbf{m}}_1$ remains oriented vertically confirming the larger horizontal extension of the source (due to the inverse relationship between moment eigenvectors and source extension). However, the negative values of M_2 andM_3 are even more difficult, not to say impossible, to interpret in terms of pressurized ellipsoids shown in Fig. 2.

When comparing model HET1 with data (Fig. 4 red circles), we may appreciate a significantly better fit, even though the number of free parameters is the same as in model HOM1: compared with model HOM1, the misfit between best model HET1 and leveling data decreases by $\sim$ 22% to χ^2_{LEV1} = 2.8 and the misfit with EDM data decreases by $\sim$ 57% to χ^2_{EDM1} = 4.3, with an overall average misfit decrease by $\sim$ 43%. It must be stressed that the improvement of fit w.r.t. model HOM1 is obtained employing independent evidence regarding the elastic structure of the medium: no adjustable parameters are added to the inversion scheme. However, the fit of EDM data remains unsatisfactory.

The lesson learned from these models is that data fit improves appreciably when the realistic elastic heterogeneities of the medium are accounted for, but the fit of EDM data remains unsatisfactory and the physical interpretation of the source is not devoid of difficulties.

5. Introduction of a deep deflating source

As mentioned in the introduction, a constraint generally ignored when modeling deformation in volcanic areas is mass conservation: if the intrusion of a magmatic mass is responsible for the inflation of a cavity, the same mass must disappear from somewhere else. Since the deformation due to internal sources typically decreases as r^{-3} away from the source, this constraint may be not accounted only if the magma origin is much deeper than the inflating cavity. At CF, high resolution seismic reflection surveys suggest the presence of a large magma reservoir at 7.5 km depth (Zollo et al., 2008). Since the shallow inflating source was previously inferred at $\sim$ 5 km depth, it appears that the role of the deep origin source cannot be neglected. In order to avoid the proliferation of new free parameters we constrain the deep source to be vertically below the shallow source. The deep source is assumed to be a horizontal sill at 7.5 km depth, endowed with opposite moment trace w.r.t. the shallow source, i.e. deflating by the same volume which goes to inflate the shallow source (in this way we assume also that the density of transferred magma remains constant). We considered also the deep source as a deflating sphere, but results remained practically unchanged.

5.1. Model HOM2

We perform a preliminary inversion assuming a deep deflating horizontal sill at 7.5 km depth and a shallow inflating moment source above it, both embedded in a homogeneous half-space. PPD distributions of source parameters are shown in blue in Fig. 5 and predictions from the best fit model HOM2 are compared with data in Fig. 6.

The depth of the inflating source is ill-determined, with 3 PPD maxima at $\sim$ 4.5, 5.4 and 5.7 km, systematically deeper than inferred employing only one source ($\sim$ 3.9 km). The eigenvalues of the shallow source moment tensor are much greater than inferred assuming one source; they are all positive but ill constrained, even though the eigenvectors are sharply defined. The eigenvectors are oriented as $\hat{\mathbf{m}}_1 \sim$ up, $\hat{\mathbf{m}}_2 \sim$ West, $\hat{\mathbf{m}}_3 \sim$ South. The best fit mechanism is still out of the region allowed for ellipsoidal cavities (Fig. 2) since M_2/M_1 = 0.48 and M_3/M_1 = 0.33.

A comparison between models HOM1 and HOM2 shows an interesting decrease of misfit for EDM data from χ^2_{EDM1} = 7.7 to χ^2_{EDM2} = 3.5, while the misfit of leveling data remains practically unchanged (χ^2_{LEV1} = 3.5, χ^2_{LEV2} = 3.6); slightly negative uplift values are predicted for $r > 6$ km, that are not observed. EDM data are fitted significantly better by HOM2 model than HOM1 but they still appear systematically underestimated. The global misfit provided by HOM2 model is lower than HET1: considering the simultaneous role of a deflating and an inflating source in a homogeneous medium provides better results than considering only one source in a realistically layered medium. It must be stressed that no additional free parameters are introduced in the inversion.

5.2. Model HET2

Our most complete model is HET2, in which the elastic heterogeneities of the medium are accounted for, and both the deep deflating and the shallow inflating sources are included. PPD distributions are shown in red in Fig. 5. The depth of the shallow source is inferred at z_S = -5.5 km similar to model HOM2 but much better constrained.

Moment eigenvalues are all positive and have a sharply defined PPD maximum, but a secondary maximum is present, close to an isotropic source (nearly equal eigenvalues). The eigenvectors are also sharply defined: the largest moment $\hat{\mathbf{m}}_1$ is nearly vertical, while the smallest $\hat{\mathbf{m}}_3$ points nearly South (276° from East) and the intermediate $\hat{\mathbf{m}}_2$ to West. The improvement of fit can be visually appreciated in Fig. 6 (red circles). The misfit between data and predictions decreases further: for the leveling data we get χ^2_{LEV2} = 2.7 and for EDM data χ^2_{EDM2} = 1.4, the lowest values obtained so far. EDM data are fitted within experimental errors even though some systematic underestimate seems to persist. The best fit HET2 moment cannot be interpreted strictly in terms of a tri-axial pressurized cavity since M_2/M_1 = 0.3 and M_3/M_1 = 0.1 (see Fig. 2).

The source volume change (the volume of magma transferred from the deep to the shallow source) can be estimated by an accurate numerical integration of the normal displacement over the cell boundary for model HET2:

$$\Delta V_0 = \oint_{\partial V_0} \mathbf{u} \cdot \mathbf{n} \, dS = 20.9 \cdot 10^6 \, \mathrm{m}^3 \tag{1}$$

which coincides with the value $\Delta V_0 = \frac{M_{kk}}{3(\lambda+2\mu)}$ provided by three dipoles with moments M_{11}, M_{22} and M_{33}, applied in the center of the cell, with λ and μ values pertinent to the source depth (5.5 km). From the previous estimate, the typical source dimension is suggested to be $\Delta V_0^{1/3} \simeq$ 275 m, supporting the point-source assumption. Another indication in favor of the point-source approximation is the observation that the uplift increased uniformly during the 1982-84 unrest, without changing its shape. However, the possibility that the inflating source may be very thin in one direction, so that its length may be much larger than the previous estimate (e.g. Amoruso et al., 2008) cannot be ruled out.

6. Interpretation of the moment tensor source

The best fitting moment tensor of model HET2 falls outside the domain of pressurized cavities as shown in Fig. 2, and the same considerations may apply to all models retrieved in Table 1. It appears that a "complex" inflation mechanism is needed to interpret the inferred moment tensor. A pressurized cavity can explain only a fraction of the released moment: we may separate M_{ij} into an isotropic component $\frac{1}{3}M_{kk}\delta_{ij}$ and a deviatoric component $M'_{ij} = M_{ij} - \frac{1}{3}M_{kk}\delta_{ij}$. A spherical Mogi-like pressurized cavity may be associated to the isotropic component, while the deviatoric component may be ascribed to one or more shear dislocations (e.g., obliquely dipping shear faults, as already envisaged by De Natale et al., 1997; Troise et al., 2003). For the best fit HET2 model M_{kk} = 38.2·10^{17} Nm and, in the reference system provided by best fit moment eigenvectors ($\hat{\mathbf{m}}_1 \simeq$ vertical, $\hat{\mathbf{m}}_2 \simeq$ West, $\hat{\mathbf{m}}_3 \simeq$ South), we have

$$M'_{ij} = \begin{vmatrix} 14.4 & 0 & 0 \\ 0 & -4.6 & 0 \\ 0 & 0 & -9.8 \end{vmatrix} \cdot 10^{17} \quad [Nm]$$

which may be decomposed, for instance, in an EW striking reverse fault with eigenvalues (9.8, 0, -9.8)·10^{17} Nm and a NS striking reverse fault with eigenvalues (4.6, -4.6, 0)·10^{17} Nm. The shear deformation may be localized over ring faults as suggested by De Natale et al. (1997) or may be distributed as plastic deformation around the inflating source, as envisaged by Trasatti et al. (2005). Of course, such a decomposition is largely non-unique. Moreover, there is the usual ambiguity between a shear fault plane and its conjugate "auxiliary" plane.

At the opposite extreme, we may consider a flat pressurized cavity (tensile dislocation or penny shaped crack), to which all the isotropic component and a fraction of the deviatoric component may be ascribed, since the eigen-moments of a tensile dislocation M_1^n, M_2^n, M_3^n are proportional to $(\lambda + 2\mu), \lambda, \lambda$, respectively. For ν = 0.28 we have:

$$M_{ij}^n = \begin{vmatrix} 21.5 & 0 & 0 \\ 0 & 8.4 & 0 \\ 0 & 0 & 8.4 \end{vmatrix} \cdot 10^{17} \quad [Nm]$$

and the remaining deviatoric component to be explained by shear dislocations is

$$M_{ij}'' = \begin{vmatrix} 5.7 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -5.5 \end{vmatrix} \cdot 10^{17} \quad [Nm]$$

which may be interpreted as EW striking reverse faulting, with a negligible contribution from NS striking faulting. The two extreme decompositions illustrated above are largely non unique, since an infinite variety of tri-axial pressurized cavities may be proposed according to Davis (1986), even though the tensile crack (degenerate ellipsoid with vanishing minor axis) is preferable since it requires less (residual) deviatoric moment and much less overpressure to accommodate the same magma volume.

The previous interpretation of the moment tensor in terms of a pressurized cavity and a residual deviatoric moment associated with shear failure on nearby faults is a possibility, but a few inversions performed assuming three sources (a deep deflating sill, a shallow inflating isotropic source and a deviatoric source at different depth), provided very ill constrained source parameters even though data fit improved significantly. Accordingly, the assumption of a shear source differently located than the shallow inflating source was shelved. Furthermore, a problem with the double mechanism source model is that the global seismic moment released by earthquakes at CF (maximum magnitude 4.6) was a negligible fraction of the retrieved moment tensors, so that the shear dislocations should be practically aseismic, in spite of the large strain and their very fast evolution. Trasatti et al. (2005) interpret the large deviatoric strain release in terms of a plastic rheology at shallow depth within the inner caldera, showing that in this case the source depth can be deeper than 5 km even for a spherical overpressure source.

In the following sections we introduce two new source mechanisms to interpret the retrieved moment tensors, with particular attention to the HET2 source mechanism (our preferred and most complete model) and its predicted gravity change.

6.1. Parallelepipedal cavity

Bonafede and Ferrari (2009) illustrate the equivalence between moment sources and pressurized cavities assuming an isotropic cubic source, but the same scheme can be easily generalized to a parallelepiped with edges d_1, d_2, d_3 along the coordinate axes x_1, x_2, x_3 (Fig. 7a). Over each face, a rectangular pressurized crack is considered with surface area $A_i^{\pm}$ (where $\pm$ denote the orientation of opposite faces normal to x_i): of course $A_1^{\pm} = d_2 d_3, A_2^{\pm} = d_1 d_3, A_3^{\pm} = d_1 d_2$. According to Kirchoff uniqueness theorem (e.g. Fung, 1965), the deformation field outside a pressurized parallelepiped is identical to that provided by these 6 pressurized rectangular cracks over its faces. According to the boundary element method, these cracks may be approximated in the far field as 6 dislocations if their Burgers vectors $b_i^{\pm}$ are computed from Okada solutions (Okada, 1992) in order that they provide the same overpressure ΔP (and vanishing shear tractions) at the center of each face (Fig. 7b). Furthermore, in the point-source approximation, these 6 dislocations are equivalent to 3 orthogonal tensile dislocations, located in the center of the cavity, with surface areas $A_i = A_i^{\pm}$ and Burgers vectors $b_i = b_i^{+} - b_i^{-}$ (Fig. 7c).

The moment tensor describing these three orthogonal tensile dislocations is simply obtained (employing the axes x_1, x_2, x_3 as basis vectors) from the theorem of body force equivalents (Burridge and Knopoff, 1964):

$$M_{ij} = A_1 b_1 \begin{vmatrix} \lambda + 2\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} + A_2 b_2 \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda + 2\mu & 0 \\ 0 & 0 & \lambda \end{vmatrix} + A_3 b_3 \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda + 2\mu \end{vmatrix} \quad (2)$$

The relationship between parallelepiped edges and moment tensor eigenvalues is provided in the supplementary material, assuming that $d_1 \leq d_2 \leq d_3$. Solutions depend on the product of the overpressure ΔP times the cavity volume $V_0 = d_1 d_2 d_3$, which is reported in the last column. A direct comparison with tri-axial ellipsoidal cavities (Table 1 in Davis, 1986) is not possible, due to the different source geometry, but some similarities and differences may be noted: as in Davis (1986) $M_1 > M_2 > M_3$ if the parallelepiped edges (the ellipsoid axes) are in the reverse order $d_1 < d_2 < d_3$; the cubic source ($d_1 = d_2 = d_3$) and the flat square source ($d_1 \simeq 0, d_2 = d_3$) yield the same results as a spherical source and the flat circular crack, respectively. In both cases, moment ratios M_2/M_1 and M_3/M_1 must be positive and ratios lower than $1/3$ (Poisson approximation) cannot be obtained. However, the domain of possible moment ratios is significantly wider (Fig. 8).

The best fit mechanism of HOM2, outside the region allowed for ellipsoidal cavities, is close to pressurized parallelepipeds, since M_2/M_1 = 0.48 and M_3/M_1 = 0.33. Therefore the closest cavity is a thin horizontal crack. The HET1 moment tensor, composed of intermediate and minimum negative principal values, may be interpreted in terms of three orthogonal tensile dislocations, with surface areas A_1, A_2, A_3 and Burgers vectors b_1, b_2, b_3, respectively, without imposing constraints on b_i. Solving separately for the incremental volumes V_1, V_2, V_3 in terms of values inferred from HET1 model for M_1, M_2, M_3, we obtain strongly negative values for both V_2 and V_3, indicating that a vertical expansion of the source should be accompanied by significant horizontal contractions ($b_2 < 0$ and $b_3 < 0$). This is physically possible avoiding matter compenetration only if a pre-existent cavity expands vertically and contracts laterally.

6.2. Mixed mode (tensile & shear) crack

In the previous sections, we have shown that pressurized cavities are by no means the most general internal sources. They assume that shear tractions vanish both before and after the inflation and accordingly they are suited to describe magma addition to pre-existent fluid-filled reservoirs. However, if the intrusion of magma takes place across pre-stressed solid rock, shear tractions must be released over the boundaries of the intrusion. The best fit moment tensor of model HET2 (and of the other models, too), although falling outside the region of pressurized cavities, is closer to flat pressurized cavities than to thick 3D cavities (see Fig. 8). We should be ready to accept that some release of shear stress may have taken place over the inflating source itself.

Let us consider a flat pressurized cavity over which the intrusion of fluid magma provides the release of overpressure and of any pre-existent shear tractions. In order to describe the full moment tensor, let us consider a reference frame with axes along $\hat{\mathbf{n}}$ (normal to the dislocation surface A), $\hat{\mathbf{s}}$ (in the shear slip direction), $\hat{\mathbf{t}} = \hat{\mathbf{n}} \times \hat{\mathbf{s}}$, perpendicular to $\hat{\mathbf{n}}$ and $\hat{\mathbf{s}}$ according to the right-hand convention. The Burgers vector is $\mathbf{b} = (b^n, b^s, 0)$ where b^n is the normal component and

b^s the shear component. Let $b^n = b\cos\theta$ and $b^s = b\sin\theta$ (θ is the angle between $\mathbf{b}$ and $\hat{\mathbf{n}}$). The moment tensor of the mixed mode dislocation is:

$$M_{ij}^{md} = M_{ij}^{n} + M_{ij}^{s} = Ab^n \begin{vmatrix} \lambda+2\mu & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} + Ab^s \begin{vmatrix} 0 & \mu & 0 \\ \mu & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = \mu Ab \begin{vmatrix} (k+2)\cos\theta & \sin\theta & 0 \\ \sin\theta & k\cos\theta & 0 \\ 0 & 0 & k\cos\theta \end{vmatrix}$$

where $k = \lambda/\mu$ is employed in the last equality. The eigenvalues are found to be simply

$$\begin{cases} M_1 = \mu Ab[(k+1)\cos\theta + 1] \\ M_2 = \mu Abk\cos\theta \\ M_3 = \mu Ab[(k+1)\cos\theta - 1] \end{cases} \tag{3}$$

These values of M_2/M_1 and M_3/M_1 are shown in Fig. 9 as functions of θ if $k =$ 1 (i.e. $\lambda = \mu$). It may be easily shown from Eq. (3) that M_3/M_1 vs. M_2/M_1 is a straight line joining the points $(\frac{\lambda}{\lambda+2\mu}, \frac{\lambda}{\lambda+2\mu})$ and $(0, -1)$: as far as $\theta < 15°$, the mixed mode dislocation is hardly distinguishable from a pure tensile crack (the moment ratios are close to each other), but moment ratios may be much smaller than $\frac{\lambda}{\lambda+2\mu}$: M_2/M_1 may vanish and M_3/M_1 may be even negative when $\theta >$ 60°). In the reference frame $\hat{\mathbf{n}}, \hat{\mathbf{s}}, \hat{\mathbf{t}}$ the intermediate eigenvector $\hat{\mathbf{m}}_2$ of the moment tensor is along $\hat{\mathbf{t}}$, while the maximum and minimum eigenvectors are (for any values of λ and μ):

$$\hat{\mathbf{m}}_1 = \frac{(1+\cos\theta, \sin\theta, 0)}{\sqrt{2}(1+\cos\theta)^{1/2}} \quad \text{and} \quad \hat{\mathbf{m}}_3 = \frac{(-\sin\theta, 1+\cos\theta, 0)}{\sqrt{2}(1+\cos\theta)^{1/2}}$$

The eigenvectors $\hat{\mathbf{m}}_1$ and $\hat{\mathbf{m}}_3$ are found to be simply rotated anti-clockwise by $\alpha = \theta/2$ around $\hat{\mathbf{t}}$ with respect to $\hat{\mathbf{n}}$ and $\hat{\mathbf{s}}$, since $\cos\alpha = \cos(\theta/2) = \hat{\mathbf{n}} \cdot \hat{\mathbf{m}}_1$.

In Fig. 8 a summary is provided of all the moment ratios admissible for pressurized parallelepipeds (red triangle), mixed mode cracks and CLVD sources (with vanishing moment trace). It appears that the HET2 moment is very close to a mixed mode dislocation with $\theta \sim 58^\circ$. Since $\hat{\mathbf{m}}_1$ is nearly vertical for model HET2, then the dislocation plane is inferred to dip approximately by $\alpha \sim 29^\circ$ with respect to the horizontal, with the northern block overriding the southern block. It is interesting to note that there is no ambiguity with an “auxiliary fault plane”, due to the constraint that the failure surface is the same for both the shear and the tensile dislocations. The same may apply to HOM1 source being very close to a mixed mode dislocation with $\theta \sim 64^\circ$, dipping $\alpha \sim 32^\circ$ from the horizontal.

6.3. Gravity change and intrusion density

Several studies have shown the importance of hydrothermal contributions to the deformation field in volcanic areas (e.g. Rinaldi et al., 2010, and references therein). However, it is difficult to accommodate in this way more than $\sim$ 10 cm uplift and, in any case, a big instability of the hydrothermal system necessarily requires a big energy input from magmatic fluids. Gravity measurements can discriminate between magma and volatiles.

The observed gravity change may be decomposed in a sum of different contributions: Δg_L, due to displacement of density layers including the free surface, Δg_V, due to density variations of the compressible material surrounding the source, the free air correction Δg_{FA} due to benchmark uplift, and the mass shift $\Delta M = \rho_m \Delta V_0$ from the deep source (at 7.5 km) to the shallow source (at 5.5 km) in the specific case of model HET2.

Following the approach described in Trasatti et al. (2009) in which gravity variations were computed in FE models of pressurized cavities in elastic heterogeneous media, we compute the gravity changes due to a general moment tensor, as described in section 3. According to this algorithm, we may finally compute the deformation (displacement and strain fields) everywhere in the medium surrounding the source from the moment density distribution of our best fitting HET2 model. From this, the gravity changes Δg_L and Δg_V may be computed by numerical integration over the FEM grid. Since ΔV_0 may be also computed from Eq. (1), ρ_m can be finally inferred.

At CF the gravity/uplift ratio was measured as $\Delta g/\Delta h$ = -215±6 μGal/m during 1982-84 unrest (Fig. 1) and the measured free-air gravity gradient is -290±5 μ Gal/m (Berrino, 1994) so that the residual (free-air corrected) $\Delta g/\Delta h$ amounts to +75± 8 μGal/m. From numerical integration of density changes due to HET2 model, the difference between observed and computed $[\Delta g_{FA} + \Delta g_L + \Delta g_V]/\Delta h$ amounts to 7.0 μGal/m only (ignoring the experimental uncertainty) and must be attributed to the intrusion mass shifted from the deep source at depth z_d = -7.5 km to the shallow source in z_s = -5.5 km according to the formula (for a benchmark vertically above the source):

$$\Delta g_S = G\rho_m \Delta V_0 \left[\frac{1}{z_s^2} - \frac{1}{z_d^2} \right]$$

A source volume change ΔV_0 = 20.9·10^6 m^3 is computed from model HET2 and the inferred intrusion density value is ρ_m = 2043 kg/m^3 even though it is poorly constrained due to the experimental uncertainty. Similar densities are compatible with volatile rich basaltic magma, rather than hydrothermal fluids. From the previous computations result that most of the residual gravity change is due to the deformation of the medium, and only a minor (if any) release of the mass emplaced into the shallow source is needed to explain the gravity change during the deflation phase starting in November 1984, which amounts to -224± 24 μGal/m (very similar to the uplift phase). The deflation phase following the unrest after 1984 may be probably interpreted as the result of the release of exsolved volatiles (water and $\mathrm{CO_2}$) by magma depressurization. If an isotropic (Mogi-like) source is assumed, the residual gravity change should be entirely attributed to the emplaced mass, since $\Delta g_L + \Delta g_V + \Delta g_{FA}$ vanish identically for an isotropic source (Walsh and Rice, 1979). The gravity change observed during the post-1984 deflation phase would then require that the mass entering a Mogi source from remote distance during inflation should disappear to remote distance during deflation, which does not seem plausible. We remark that this result is a by-product of our inverse modeling of surface deformation data, since no model optimization was performed to fit gravity data.

7. Discussion and conclusions

Pressurized cavities are generally employed as source models of deformation in volcanic areas. The geometrical shape assumed for the cavity has important effects on the inferred source parameters, but no general inversion scheme is available to retrieve the source shape from the observations: thus, inversions are generally performed assuming (at most) a tri-axial pressurized point-like ellipsoid. On the other side, any internal deformation source, including pressurized ellipsoids, can be described in terms of a moment tensor under the point-source assumption.

If a moderate component of deviatoric moment tensor is inferred from data, the source may be interpreted as a simple pressurized ellipsoidal cavity (see Fig. 2), going from a Mogi-like sphere (eigenvalues in ratios $1 : 1 : 1$), along the sub-domain of oblate axi-symmetric ellipsoids ($1 : a : a$), where $1/3 < a < 1$, down to the circular penny shaped crack ($1 : 1/3 : 1/3$, in the Poisson approximation $\lambda = \mu$), or along the sub-domain of prolate axi-symmetric ellipsoids ($1 : 1 : b$), where $2/3 < b < 1$, down to the thin “cigar-like” ellipsoid ($1 : 1 : 2/3$).

If pressurized parallelepipeds are considered, the domain of admissible moment tensors increases somewhat, from moment ratios $1:1:1$ of an isotropic cubic cavity, to $1:1/3:1/3$ of the flat square cavity, and to $1:1:1/2$ of the thin finger-like conduit.

Even in the presence of a moderate deviatoric component, and allowing for a non vanishing component of shear dislocations, the interpretation of the source geometry is not unique: for instance, a pressurized penny-shaped crack is equivalent to an isotropic source plus a shear dislocation source; it is noteworthy that the inferred incremental volume of magma does not change, since it is proportional to the moment trace M_{kk}. In any case, no ellipsoidal or parallelepipedal pressurized cavity (in the point-like approximation) can provide a larger deviatoric component of moment tensor than a flat tensile crack.

The source responsible for the 1982-84 uplift at CF caldera is found to be significantly out of the domain of pressurized cavities if the inversion of geodetic data is performed assuming a homogeneous half-space (model HOM1). If the realistic heterogeneous structure of the medium, as inferred from seismic tomography, is accounted for (model HET1), the misfit between data and model decreases by 43% but the best fitting moment source is even more difficult to reconcile with a pressurized cavity (see Fig. 8). In such a model, the moment source can be interpreted in terms of a tensile crack plus reverse-slip shear faults.

Beside the significant better fit of model HET1 w.r.t. HOM1, EDM data are still poorly fitted. Furthermore, the assumption of an inflating source, without considering a deflating source somewhere, violates mass conservation. Exploiting the recent finding of a very large magma reservoir at 7.5 km depth below CF, models HOM2 and HET2, accounting for a deep origin source at 7.5 km depth (deflating by the same volume which inflates the shallow source) are considered. Although no additional free parameters are introduced (the deep source is assumed vertically below the shallow source), data fit improves significantly and EDM data are satisfactorily well reproduced by model HET2. The shallow HET2 source still requires a deviatoric component larger than the amount attributable to a pressurized cavity. Additional reverse faulting, mostly on EW striking faults, is a possibility, as already discussed for model HET1, but the moment tensor may be probably best interpreted in terms of one mixed mode (tensile and shear) dislocation. In this case, one inflating source is assumed (apart from the deflating "origin" source), over which shear slip accompanies the opening due to a fluid intrusion. The HET2 moment tensor is found very close to that provided by a dislocation plane dipping by 29° Northward, with Burgers vector pointing 29° South from the vertical. In this model there is no ambiguity with an auxiliary fault plane, since the same dislocation plane accommodates both the slip and opening components. This mechanism of magma emplacement is similar to that modeled for dike arrest by Dahm (2000) in presence of stress heterogeneities and by Maccaferri et al. (2010) in proximity of elastic discontinuities. If this model is accepted, we also get the important additional hint that magma was emplaced across solid rock, releasing the shear traction over the dislocation plane and, as a consequence, the incremental magma volume inferred from the trace of the moment tensor is the total amount of magma present in the shallow source location.

All interpretative models discussed above require a large component of reverse slip, mostly over EW striking sources, in addition to an inflation component. The northward dipping dislocation plane (whether it is interpreted as shear slip on the same or on a different plane, or else as diffuse anelastic deformation) is compatible with the presence of ancient eruptive vents only in the Northern sector of the caldera and with the presence of uplifted marine terraces striking EW, close to coastline (e.g. “La Starza” terrace). Seismic activity was also strongly clustered in the northern sector, close to the coast (e.g. Dvorak and Berrino, 1991), with hypocenter depths typically above 4.5 km (i.e. just above the inferred source depth).

At the end of the uplift phase, in November 1984, the maximum uplift was $\sim$ 1.80 m and the uplift pattern was very similar to that shown in Fig. 1b, multiplied by a factor of ~ 3. If the same source mechanism is assumed for the entire inflation 1982-84, as seems plausible because of the constant shape of the inflation and the constant $\Delta g/\Delta h$ ratio (Fig. 1d), the moment eigenvalues should be multiplied by a factor of 3, due to the linearity of the equations. Thus, the inferred volume ΔV_0 of magma transferred from the deep to the shallow source according to model HET2 may be estimated as $\Delta V_0^{tot} \simeq 60 \cdot 10^6$ m^3 at the end of the inflation period. The magma volume is much greater for HET2 than HET1.

Finally, in this paper we have always adopted the point-source assumption. Instead, an important role may be played by the finite dimensions of the source. Amoruso et al. (2008) have shown that a circular horizontal penny-shaped crack at shallow depth ($\sim$ 3 km), with 2.7 km radius may reproduce the observed deformation and gravity change better than a point-like pressurized crack at 5 km depth, inferred by them as the best fitting point-source. However, the assumption of a flat, circular and horizontal intrusion may bias the solution even more than the point-source assumption. In particular, the presence of seismicity down to 4.5 km depth and the relatively cold temperatures $\sim$ 420 °C met by deep drillings down to 2.7 km depth at CF, seems difficult to reconcile with the presence of magma at 3 km depth only. Moreover, no evidence of a large magma reservoir at depths shallower than 7 km is found from tomographic studies (Aster and Meyer, 1988; Chiarabba and Moretti, 2006; Zollo et al., 2008). Of course, the presence of a reservoir smaller than the resolving power of tomographic data ($\sim$ 1 km) cannot be excluded and the problem of a finite source with one dimension shorter than this requires a deeper evaluation. In any case, no convenient inversion scheme is presently available for finite sources of arbitrary shape.

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Figure 1: Sketch of Campi Flegrei (CF) caldera data set. (a) Geodetic and gravity benchmarks surveyed during the 1982-84 crisis: leveling (blue circles), EDM (red triangles) and gravity stations (yellow squares). (b) Spatial pattern of uplift measured in June 1983 (black) and in June 1984 (red) w.r.t. January 1982; the approximate axial symmetry is shown from the dotted lines (Pozzuoli-Quarto): the maximum uplift was always found at benchmark no. 15 close to the center of Pozzuoli. (c) EDM distance changes between June 1980 and June 1983 (referred to benchmark no. 15). (d) Gravity change Δg vs. uplift Δh at Serapeo benchmark no. 19 ($\sim$ 1 km NW of no. 15).

Figure 2: Moment ratios $M_3/M_1, M_2/M_1$ admissible for pressurized ellipsoids (dark gray subset) in the Poisson approximation. By assumption, $M_1 \geq M_2 \geq M_3$ (light gray area). Best fit moment tensors are shown as solid diamonds for three models (out of four) discussed in the text (model HET1 is off-scale).

Figure 3: PPD distributions of source parameters for model HOM1 (blue) and model HET1 (red).

Figure 4: Best fit model prediction compared with leveling (a) and EDM (b) data (black bars) for model HOM1 (blue circles) and model HET1 (red circles).

Figure 5: PPD distributions of source parameters for model HOM2 (blue) and model HET2 (red).

Figure 6: Best fit model prediction compared with (a) leveling data and (b) EDM elongations for model HOM2 (blue circles) and model HET2 (red circles).

Figure 7: In the point-source approximation, the deformation field outside a pressurized parallelepiped (a) is the same as provided by 6 tensile dislocations (b) with Burgers vectors computed in order to provide normal stress $\sigma_n = \Delta P$ at the center of each face. This system, in turn, is equivalent to three orthogonal tensile dislocations placed at the center of the cavity. In (b) and (c) the edge d_3 and the surfaces $A_3^{\pm}$ are not drawn for clarity.

Figure 8: Domains of possible moment ratios for pressurized parallelepipeds embedded in an elastic medium with ν = 0.28, (red triangle), mixed mode dislocations (red line) and CLVD sources (black line). The moment tensor inferred from HET1 model is incompatible with any plausible point-source and requires a significant release of deviatoric moment on shear dislocations. Model HET2 is compatible with a mixed mode dislocation with $\theta \sim 58^\circ$, dipping by $\alpha \sim 29^\circ$ Northward.

Figure 9: Values of M_2/M_1 and M_3/M_1 in a mixed mode dislocation as functions of θ (in the Poisson approximation $\lambda = \mu$).

Table 1: Results of the Bayesian Neighbourhood Algorithm inversion and misfits associated to the different models considered in the paper. The total misfit is the average between those computed for the leveling dataset and for the EDM dataset separately. The source position is fixed at x_S = 426.2 km and y_S = 4518.8 km (UTM reference); z_S is the inferred depth (negative below sea level). M_1, M_2 and M_3 are the principal moments computed from the inverted stress tensor $M_{ij} = \ell^3 \sigma_{ij}$. The last 3 columns are the angles of the principal moments described in the text and in the supplementary material.