

1 **Bayesian Event Tree for Eruption Forecasting (BET_EF) at**
2 **Vesuvius, Italy: a retrospective forward application to the 1631**
3 **eruption.**

4
5 Laura Sandri¹, Emanuela Guidoboni¹, Warner Marzocchi¹, Jacopo Selva¹

6
7 1- Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Bologna, Italy

8
9
10 Contacting author:

11 Sandri Laura

12 Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Bologna

13 Via Donato Creti, 12

14 40128 Bologna

15 Italy

16 Tel: +390514151422

17 Fax: +390514151498

18 e-mail: sandri@bo.ingv.it

19
20
21
22 Accepted for publication on *Bull. Volcanol.*, Oct 8, 2008

1 **Abstract**

2 Reliable forecasting of the next eruption at Vesuvius is the main scientific
3 ingredient to define effective strategies to reduce volcanic risk in one of the
4 most dangerous volcanic areas around the world. In this paper, we apply a
5 recently developed probabilistic code for eruption forecasting to new and
6 independent historical data related to the pre-eruptive phase of the 1631
7 eruption. The results obtained point out three main issues: 1) the
8 importance of “cold” historical data (according to Guidoboni 2008) related
9 to pre-eruptive phases for evaluating forecasting tools and possibly
10 refining them; 2) the BET_EF code implemented for Vesuvius would have
11 forecast the 1631 eruption satisfactorily, marking different stages of the
12 pre-eruptive phase; 3) the code shows that pre-eruptive signals that
13 significantly increase the probability of eruption were likely detected more
14 than two months before the event.

15

16 **Keywords:** cold historical data, Vesuvius, 1631 eruption, BET_EF code,
17 eruption forecasting

18

19

20

21

22

23

24

1. INTRODUCTION

Vesuvius is one of the highest risk volcanoes. Besides being located in a densely populated area, with circa one million of people living on its flanks and in the surrounding area, Vesuvius experienced several large explosive eruptions in the past that re-opened the conduit, ending so-called “close conduit” phases (e.g., Marzocchi and Zaccarelli 2006) such as the current phase of the volcano. Over the last decades, this large threat for society pushed scientists and the Italian Civil Protection to devote a significant effort in order to mitigate volcanic risk in this area. One of the most relevant strategies adopted is the development of an Emergency Plan that is periodically revised and that includes, prior to an eruption, a massive evacuation of the area that is likely to be affected by pyroclastic flows, lahars, and heavy ash falls (the so-called Red Zone). Thus, a key scientific ingredient for an evacuation to be effective and successful is a reliable forecast of the time evolution of the reactivation of Vesuvius.

Tracking quantitatively the evolution of the pre-eruptive phase of long-time dormant explosive volcanoes is the main purpose of a recent quantitative tool, named BET_EF (Bayesian Event Tree for Eruption Forecasting), developed by Marzocchi et al. (2004; 2008). The method makes probabilistic eruption forecasting, accounting for volcanological models, prior beliefs, past data, and monitoring measurements. Besides a general description of the probabilistic model that is potentially applicable to any explosive volcano, the papers also contain a set of monitoring parameters

1 and relative thresholds, in order to apply the technique to pre-eruptive
2 phases of Vesuvius.

3 Although the method is scientific, since it provides probabilities that can be
4 used to test the model with independent data, in practice its verification is
5 hampered by the lack of quantitative pre-eruptive data, for Vesuvius as
6 well as for almost all volcanoes of this type. In this respect, we argue that
7 historical documents can partially fill this void, providing useful information
8 about pre-eruptive phenomena. A remarkable example is given by
9 historical reports of the pre-eruptive phase of the 1631 eruption
10 (Guidoboni 2008). These new data give us an unusual opportunity to
11 perform a retrospective forward test of the BET_EF model implemented for
12 Vesuvius. The “forward” nature of the test is guaranteed by the fact that
13 we use the same parameters published in previous papers, keeping
14 “frozen” all the quantitative rules/parameters/thresholds. The test consists
15 of three basic steps: at first,

16 1. We translate, where possible, the historical information into
17 reasonable values of some of the monitoring data required by the
18 BET_EF model for eruption forecasting. In this step we keep
19 separate, as clearly as possible, the “historical data” (in their so-
20 called "cold" data form, i.e., not yet interpreted, see Guidoboni
21 2008) from our interpretation.

22 2. We run the BET_EF code to estimate the time evolution of the
23 probability of eruption.

1 3. We perform stability checks of the results, acknowledging that our
2 quantitative interpretation of historical reports is subjective and
3 cannot be univocal. In practice, we evaluate the stability of the
4 results when different interpretations of historical reports are
5 applied.

6
7 The final goal of the paper is two-fold: beside testing the ability of BET_EF
8 to forecast a large explosive eruption in a long-time quiescent volcano, we
9 also highlight the importance of data coming from historical documents,
10 as they can partly replace the lack of quantitative pre-eruptive data for
11 large explosive eruptions.

12 13 **2. THE 1631 VESUVIUS ERUPTION, AND A DETAILED HISTORICAL** 14 **CHRONOLOGY OF THE PRE-ERUPTIVE PHASE**

15 During its eruptive history, Vesuvius has produced several large explosive
16 eruptions that devastated the surrounding area. It last erupted in 1944 with
17 a VEI = 3 weakly explosive event. The most important previous eruptions
18 were VEI = 4 events in 472 A.D. and 1631 (see, e.g., Scandone et al.
19 1993), and the well-known Pompeii eruption in 79 AD. Presumably, all of
20 these large explosive eruptions brought to an end a closed conduit phase,
21 intended, as in Marzocchi and Zaccarelli (2006), as a dormancy of the
22 volcano with no activity, lasting at least few decades (e.g., Guidoboni and
23 Boschi 2006) or centuries (e.g., Cioni et al. 2003) and allowing the closure

1 of the volcanic conduit due to viscous relaxation and cooling of rocks
2 within it (Quarenì and Mulargia 1993).

3 After the reactivation of Vesuvius in 1631, volcanic activity switched to
4 persistent activity typical of open conduit behavior, showing frequent
5 eruptions of smaller size and intensity for decades to centuries. Persistent
6 activity prevailed through the early 20th century until 1944. Since then,
7 however, an unusually long repose (60 years and continuing) suggests
8 this volcano may now be in a closed conduit condition (e.g., Marzocchi
9 and Zaccarelli 2006).

10 The 1631 event had a particular importance because it was considered, in
11 a previous version of the Emergency Plan (Presidenza del Consiglio dei
12 Ministri 1995), as a reference scenario for the definition of the “Maximum
13 Expected Event” (e.g., Barberi et al. 1995; Barberi et al. 1990; Esposti
14 Ongaro et al. 2002; Cioni et al. 2003). The revised Emergency Plan has
15 modified this view, recognizing that an eruption similar to the one occurred
16 in 1631 is not actually the maximum expected event, and not even the
17 most likely (Marzocchi et al. 2004), but it could represent a reasonable
18 reference scenario that balances cost/benefit for practical actions.
19 Nevertheless, the 1631 event is also remarkable because, being the most
20 recent explosive eruption, it has also some noticeable historical reports
21 about the pre-eruptive phase. While Rosi et al. (1993) let us learn what
22 happened during the eruptive phases of 1631 event, recent works by
23 Guidoboni (2004; 2008) made available a large amount of new information
24 on what happened weeks and months before the eruption (see comments
25 in Bertagnini et al. 2006).

26 27 **2.1. The method and the materials studied: the role of “cold”** 28 **historical data**

29 One of the main problems in the use of historical data to learn about the
30 phenomena of the past, like the activity of volcanoes, is the continuous

1 and uncontrollable interpretative intervention effected by the
2 volcanologists directly upon the sources. The descriptions of the Vesuvius
3 activity (and of other volcanoes of the Italian area) in the treatises written
4 by contemporaries are especially rich in detail, inserted within a lengthy
5 chronological development. Most of the phenomena described in those
6 treatises can thus be pinpointed in time and in geographical space and be
7 elaborated as a fully-fledged sequence of “cold” data, that is, defined
8 before the scientific interpretation. This way of elaborating the historical
9 data is a novelty within the volcanological field, because it allows the
10 relationship between source and interpretation to be made transparent, a
11 relationship that hitherto in the literature had always been presented as a
12 single phase and a single process. The separation of the two levels, i.e.
13 historical and volcanological, which instead has been effected here,
14 obviously does not come without its surprises as well as some problems,
15 because it lays bare some unresolved aspects – and also at times ones
16 that are hard to resolve – possible contradictions between the examined
17 texts, or neglected elements that are not secondary. As compared with
18 the knowledge already gained in the literature, it appears to be a more
19 realistic set of data, but also in some ways more problematic, nonetheless
20 meaningful, for anyone who has to interpret such data within the
21 volcanological sphere.

22 In our opinion, no other paths can be followed for the critical use of a
23 wealth of such particular historical data. The results presented here

1 constitute a new way of using historical data in volcanology, which have
2 been applied for the first time within the *Exploris* project (Guidoboni 2008).

3 4 **2.2.The description of the 1631 eruption in five contemporaries** 5 **treatises**

6 On 16th December 1631 a violent eruption of Vesuvius started after a long
7 period of silence that had led people to forget the danger of this volcano.
8 In order of importance, that of 1631 is the third eruption occurring in the
9 historical era, after those of 79 AD and 472 AD. This eruption in its most
10 acute phase lasted several days, causing over a thousand deaths and
11 substantial economic damage. The phenomenon ended completely only a
12 few years later, but already in January 1632 a number of Neapolitan
13 intellectuals were engaged in writing reports and treatises to recount,
14 interpret, and explain that extraordinary reawakening of Vesuvius.

15 Between 1632 and 1634 numerous pamphlets, notices, letters and reports,
16 along with some treatises written in Italian, Latin and Spanish, were
17 published in Naples. These writings represent a heterogeneous set of
18 materials, as a whole invaluable for becoming acquainted with that
19 scenario. The four treatises (see also Appendix A) presented here have
20 been analysed in different steps: the texts of Carafa (1632), Mascolo
21 (1632) and Varrone (1634), in Latin, have been translated and analysed
22 within the scope of the EXPLORIS project (Guidoboni 2004); the text of
23 Braccini (1634) is in Italian, analysed within the Vulcan-3 project - RU 1-
24 Guidoboni (2005-2007) agreement with the Dipartimento della Protezione

1 Civile and INGV. The text by Giovanni Domenico de Arminio (1632), which
2 is a treatise in Latin up to now unknown to volcanological literature, has
3 been translated and analysed for this work.

4 The list of chronologically ordered phenomena reported in the five treatises
5 is given in table 1 (columns 1 to 5).

6 One of the most interesting aspects and so far scarcely used of such texts
7 is their reconstruction of the chronology of the described phenomena. The
8 treatises analysed are situated within production of the witnesses to the
9 1631 eruption. The authors, intellectuals and ecclesiastics of the day, were
10 committed to analysing and explaining everything that they had previously
11 observed before, during and after the eruption: obviously this was done
12 within the cognitive frames of their times. The liveliness and immediacy of
13 their descriptions accompany a literary scholarship, which was typical of
14 the ecclesiastics and the men of law of that day and age.

15 The description of the eruption is preceded, in all five treatises, by scores
16 of pages dedicated to philological, etymological and historical
17 disquisitions, which are an example of how the culture of the day dealt with
18 the great natural events.

19 The description of the events starts from the summer of 1631, that is,
20 several months before the eruption of Vesuvius. Hence, the treatises give
21 us the chance to observe, through the eyes of an intellectual of the day, a
22 cinematic narrative of the events running up to the great eruption on 16th
23 December 1631.

24

3. BET_EF APPLICATION TO THE 1631 ERUPTION

3.1 BET_EF rules for Vesuvius

The BET_EF software package implements the Bayesian Event Tree for Eruption Forecasting model published by Marzocchi et al. (2008). The model is based on an event tree in which individual branches are alternative steps from a general prior event evolving into increasingly specific subsequent events. By merging volcanological information, past data and monitoring measurements through a Bayesian inferential method, BET_EF computes the long- and short-term probability at each node of the event tree; particularly interesting from a volcanological point of view are the probabilities of volcanic unrest, magmatic unrest, eruption, vent location and eruption size. BET_EF also provides estimates of aleatory and epistemic uncertainties on such probabilities.

The upload of the available information regarding Vesuvius represents the main step in order to run BET_EF for this application. Here, we keep the same rules used by Marzocchi et al. (2008) during MESIMEX (Major Emergency SIMulation EXercise) experiment, in which the re-awakening of Vesuvius was simulated in order to test Civil Protection and Scientific Institution preparedness in case of such an event. On that occasion, BET_EF was run real-time, fed with the simulated monitoring data provided by a scientific pool of experts (Marzocchi et al. 2008).

In Appendix B, we describe these rules in detail again.

3.2 Strategy adopted for the volcanological interpretation of historical information

In order to verify the eruption forecasting ability of the BET_EF code on independent data, we apply it to the 1631 pre-eruptive phase, according to the historical accounts provided by Guidoboni (2004) and Guidoboni (2008).

1 We interpret the historical information in terms of some of the parameters
2 routinely monitored at Vesuvius that are a direct input to the BET_EF
3 code.

4 We stress that, in the process of translation of historical information into
5 volcanological input to BET_EF, we have to interpret “anomalous” events
6 reported in the chronicles with respect to a background activity of the
7 volcano. In doing so, we necessarily refer to our volcanological experience
8 gained from Vesuvius and to its present background state. Thus, the
9 values suggested in the last column of Table 1 represent our subjective
10 perception of the anomaly reported in the historical accounts with respect
11 to the present-day background values. Actually, we do not know if the
12 present- day state of the volcano is similar to the one before 1631
13 eruption. The results we obtain might be indeed biased by this
14 assumption.

15 However, since the BET_EF code deals with monitoring measurements in
16 a fuzzy approach (see Appendix B and Marzocchi et al. 2008), the results
17 yielded by BET_EF do not change if we input a “wrong” value for a
18 monitoring measure, provided that our value maintains a “degree of
19 anomaly” similar to the one of the reality. For example, if the real occurred
20 monthly largest magnitude is 2.1, but we give an interpretation of 3.0, the
21 results will be exactly the same, since (see Table B2 in Appendix B) these
22 values are both below the lowest threshold, thus their degree of anomaly
23 is the same (in particular, they are indicative of NO anomaly). Similar
24 considerations can be applied for any monitored variable that we consider.

1 Since historical chronicles refer to ground uplift and not to the
2 corresponding strain, for the deformation parameters (cumulative strain
3 ε and strain rate $d\varepsilon/dt$) we interpret the historical reports in terms of
4 observed ground uplift (u), and translate the uplift in terms of
5 ε (i.e., $\varepsilon=U/\Delta h$, where U is the cumulative total uplift since the beginning of
6 the unrest, and Δh is the supposed thickness of the deformed layer) and
7 $d\varepsilon/dt$ (i.e., $d\varepsilon/dt =u/(\Delta h \Delta t)$, where Δt is the time interval during which the
8 uplift is observed, derived directly from the chronicles). Bertagnini et al.
9 (2006) set the depth of the source of deformation (by a simple Mogi
10 model) to 4Km, implying a deformed layer 4Km thick. Here we assume
11 $\Delta h=5Km$, a bit more conservative choice (because, given a ground uplift, it
12 implies lower cumulative strain and lower strain rate).

13 On purpose, we decide to translate into numerical values only the
14 historical information undoubtedly linked to a specific monitoring
15 parameter. For example, anomalous wild animal behavior (like fleeing from
16 the Vesuvian area, as reported) might have been related to seismic
17 activity, to rockfalls in the crater area, to anomalous gas emission, etc.; in
18 few words, there is no specific and univocal phenomenon that caused the
19 flee of wild animals from their usual territory. Therefore, we do not
20 consider this information in BET_EF. Together with the fact that the
21 historical information collected likely represents a subset of all the
22 phenomena occurred in 1631 pre-eruptive phase, this choice implies that
23 the results we will obtain are a lower limit on the eruption forecasting ability
24 of BET_EF on independent data.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23

3.3 Volcanological interpretation of historical information

In Table 1 we provide our interpretation of the historical information in terms of some of the parameters routinely monitored at Vesuvius that are a direct input to the BET_EF code. In particular, in the two right-most columns of Table 1, we show our “verbal” interpretation of the information given on the rest of the table row (second last column), and its translation into the corresponding specific parameters and values of the monitoring data required as BET_EF input (last column).

In the following, we will give detailed explanation for the interpretations adopted, in order of appearance in time (see also Table 1). Since Bertagnini et al. (2006) and Rosi et al. (1993) already interpreted some of the historical chronicles used in the present study, we will explicitly compare our interpretation to theirs.

- “Smoke emissions” reported by C (see Table 1) on the beginning of September. We interpret this as a moderate increase (4 times the present average flux) in CO₂ flux. In doing so, we postulate that fumaroles composition remains unchanged (similar to the present-day one), but the flux becomes larger, and fumaroles become visible.
- “Elevation of the ground” reported by B between 16th November and 1st December. We interpret this as naked-eye detectable vertical ground uplift, i.e., around 10 cm, occurred during the period of observation (approx Δt =two weeks). Since BET_EF requires strain and strain rate, we transform

1 this into $\varepsilon=2 \times 10^{-5}$ and $d\varepsilon/dt=1.3 \times 10^{-6} \text{ d}^{-1}$ as illustrated in the previous
2 section.

3 - “Moderate seismic activity in the coastal region of Vesuvius” reported by V
4 between 19th and 20th November. Even if Bertagnini et al. (2006) suggest
5 that this piece of Varrone’s chronicle is a “convoluted sentence” and that its
6 interpretation might be doubtful, this is not our opinion, because the
7 Varrone’s chronicle is not limited to the sentence reported by Bertagnini et
8 al (2006). Indeed, the whole paragraph is undoubtedly related to seismic
9 activity felt by people on the coastal region of Vesuvius, thus we interpret
10 this as an increase in the rate of earthquakes. At node 1, one of BET_EF’s
11 seismic input is the monthly number of earthquakes with magnitude larger
12 or equal to 1.9 recorded at OVO station, located on the volcano. We
13 imagine that, with the term “moderate seismicity”, people might have felt a
14 few earthquakes (let’s say 5) of magnitude around 3; considering a power
15 law between the energy of earthquakes and their frequency, i.e., the
16 Gutenberg-Richter (GR) relationship, we have $\text{Log}(N_1)=a-bM_1$, where $N_1=5$
17 (earthquakes with magnitude larger or equal to 3), $M_1=3$ (magnitude), and
18 $b=1$. In order to compute N_2 , i.e., the number of earthquakes with
19 magnitude larger or equal to 1.9, we consider again the GR law, i.e., Log
20 $N_2=a-bM_2$, where $M_2=1.9$. By differentiating these two expressions of the
21 GR law, we obtain $\text{Log}(N_2/N_1)=b(M_1-M_2)$ from which $N_2=N_1 \times 10^{[b(M_1-$
22 $M_2)]} \approx 50$. (Note that for this reason, from now on, every time the chronicles
23 report “moderate seismic activity”, we add other 50 earthquakes to the
24 monthly count.) For the maximum magnitude of this earthquake burst, again

- 1 on a monthly basis at OVO station, we set 3.0, below the lower threshold of
2 anomaly. We keep it low because seismicity is felt in Vesuvian area only.
- 3 - “Emissions with effects on the herbaceous vegetation (wilting)” reported by
4 V “just before December”. Similarly to Bertagnini et al. (2006), and to what
5 observed during the last unrest episode at Long Valley Caldera (see e.g.
6 Hill 1996), we interpret this as a further increase in CO₂ flux, up to 20
7 kg/m²d⁻¹.
- 8 - “Major uplifting of the ground” occurring in 5 days and reported by V in the
9 beginning of December. We interpret this as a ground uplift significantly
10 larger than the one reported previously, thus we set a further uplift of 0.5 m
11 in a time window $\Delta t=5$ days, and translate this into $\epsilon=1.2 \times 10^{-4}$ and
12 $d\epsilon/dt=2 \times 10^{-5} \text{ d}^{-1}$. Due to cumulative strain, which becomes now larger than
13 the upper anomaly threshold, the eruption probability takes a large jump.
- 14 - “Moderate seismic activity in the Vesuvius area” reported by V between 7th
15 and 8th December. As mentioned above, we keep the same maximum
16 magnitude but add other 50 earthquakes to the monthly count.
- 17 - “Small uplifting of the ground” reported by V between 9th and 15th
18 December. We interpret this a further uplift of 0.1 m (yielding a total
19 cumulated strain $\epsilon=1.4 \times 10^{-4}$), occurring in a time window $\Delta t=7$ days, thus
20 lowering the strain rate to $d\epsilon/dt=2.9 \times 10^{-6} \text{ d}^{-1}$.
- 21 - “Emission of hot vapours (Vesuvian area)” reported by V in the same
22 period. Similarly to Bertagnini et al. (2006), we interpret it as an increase of
23 temperature of the fumaroles (T=110C, representing 15% of the present
24 usual value).

- 1 - "Moderate seismic activity (Vesuvian area)" reported by B on 10th
2 December. As above, we keep the same maximum magnitude, but we add
3 other 50 earthquakes to the monthly count.
- 4 - "Repeated seismic activity felt at Naples and in the Vesuvius area, for a
5 range of about 7.5Km" reported by C, V and M about 8 hours before the
6 onset of the eruption. We interpret this as a major jump in the monthly count
7 of earthquakes (further 150 events), and, more important, an acceleration of
8 the seismic energy released ($d^2E/dt^2=1$), acknowledged also by Bertagnini
9 et al. (2006); we also set a larger maximum magnitude (up to 4.0) motivated
10 by the larger area in which earthquakes are felt.
- 11 - "Seismic activity, opening of faults" reported by V about 1 hour before
12 eruption onset. We interpret this as a further jump in the monthly count of
13 earthquakes (further 200 events), and persisting acceleration of the seismic
14 energy released ($d^2E/dt^2=1$). Furthermore, we also interpret the opening of
15 faults as strain acceleration ($d^2\varepsilon/dt^2=1$). This is a very important parameter
16 because, having a double weight, it implies a large jump in eruption
17 probability.
- 18 - "Opening of the mount in the Atria" reported by B about 1 hour before
19 eruption onset. We interpret this as a major ground deformation (which
20 possibly could be the same fact reported by V and just discussed above), in
21 which cumulative strain exceeds rock strength; we set a further ground
22 displacement of 1 m in a very short time window ($\Delta t=6$ hours), giving
23 $\varepsilon=3.4 \times 10^{-4}$ and $d\varepsilon/dt=8 \times 10^{-4} \text{ d}^{-1}$

- 1 - “Intense smoke emission” reported by V about 1 hour before eruption onset.
2 We interpret this as a further strong increase in CO₂ flux ($\Pi_{\text{CO}_2}=100 \text{ m}^{-2} \text{ d}^{-1}$).
3 - “Rock expulsion (from central crater)” reported by V about 1 hour before
4 eruption onset. We interpret this as occurrence of phreatic explosions
5 (PE=1). This parameter also concurs in causing a large jump in eruption
6 probability.

7

8 **3.4 BET_EF run (the “reference case”)**

9 At first, we divide the period of time considered into six different sub-
10 periods, corresponding to (interpreted) significant changes in the
11 monitoring parameters. For each sub-period, we run a BET_EF simulation
12 in order to compute the various probability distributions of interest (valid in
13 the sub-period of the simulation). The results are displayed in Table 2. For
14 the sake of conciseness, for each simulation we show the average
15 absolute probability of unrest and of eruption, the relative 10-, 50- and 90-
16 th percentiles of their distributions, and the same quantities for the
17 conditional probabilities of magmatic unrest given unrest and of eruption
18 given magmatic unrest. We define this set of simulations the “reference
19 case”.

20 The most striking features of BET_EF results for the reference case are:

- 21 ✓ immediate jump in the probability of unrest already from sub-period 1
22 (the long-term average for Vesuvius is in the order of 10^{-3} (see Marzocchi
23 et al. 2008) while here in sub-period 1 the average jumps to about 10%),
24 i.e., months before the eruption; however, the conditional probability of

1 magma given unrest and of eruption given magmatic unrest are still
2 around 50% because unrest is not yet completely clear;

3 ✓ absolute probability of eruption has a constantly increasing trend; in
4 particular this probability is low (around 10%) with a large uncertainty in
5 sub-period 1, while it becomes quite high (more than 30%) about 7-10
6 days before the eruption (i.e., from sub-period 4); note that these
7 probability values could justify the call for an evacuation on the basis of a
8 rationale cost/benefit analysis (Marzocchi and Woo 2007);

9 ✓ aleatory uncertainty on the probability of eruption is drastically reduced
10 only few hours before the eruption (sub-period 6). This is a quite
11 common feeling among present-day volcanologists who have dealt with
12 large explosive eruptions (for example, the eruption of Mt Pinatubo in
13 1991, see e.g. Cornelius and Voight 1996).

14

15 **3.5 Control experiments**

16 Since the interpretation of historical information is not univocal, we run
17 some control experiments. The main idea is to assume that the monitoring
18 parameters identified in Table 1 for each sub-period from the historical
19 accounts are exact, but their specific values might change inside
20 reasonable ranges, because our interpretation is intrinsically subjective. To
21 account for this, we random vary the values of the monitoring parameters
22 inside these ranges for 1000 times, by random sampling (1000 times) from
23 uniform distributions. Then, we run 1000 control simulations for each sub-
24 period. In doing so, we obviously must respect the temporal trend of the

1 parameters through the pre-eruptive phase (for example, it is clear from
2 the historical accounts that the monthly number of seismic events had
3 been continuously increasing throughout the whole period). Furthermore,
4 we do not vary the values of the yes/no parameters, i.e., the parameters
5 related to the presence of phreatic explosions and to accelerations in
6 seismic energy release and deformation, since they appear to be certain
7 from the historical chronicles.

8 The monitoring parameters whose values are varied in the control
9 experiments are n_e , M_d , Φ_{CO_2} , $d\varepsilon/dt$, T and ε . For all of them, we start from
10 the present-day background value from actual monitoring of Vesuvius
11 (listed in Table 3), and then we set specific ranges in which the increase in
12 their values can be uniformly random sampled, according to what reported
13 in Table 1 from the historical chronicles. Except for T , whose value
14 changes only once during the pre-eruptive crises, we define a range for
15 the increase in the parameter for each time it is reported to have changed.
16 In particular (see also Table 3):

- 17 - T varies only in sub-period 4; we identify a reasonable increase in T in
18 the order of 5 to 25 degrees Celsius with respect to the present
19 temperature of the fumaroles, corresponding to a 5 to 30% increase.
- 20 - M_d varies twice, i.e. once in sub-periods 2 and once in 5. We identify a
21 reasonable increase in M_d in sub-period 2 in the order of 0 to 1.5 units of
22 magnitudes with respect to the present monthly maximum magnitude of
23 earthquakes. For sub-period 5, since chronicles report “seismic activity felt

1 in Naples”, we define a further increase in the range of 0 to 1.5 units of
2 magnitude.

3 - n_e varies in sub-periods 2 (once), 4 (twice), 5 (once) and 6 (once). We
4 identify a reasonable increase in n_e in sub-period 2 in the order of 0 to 100
5 events, corresponding to a 0 to 10 times increase of the present rate.
6 Equal ranges are assumed for the two further increases in sub-period 4.
7 For sub-periods 5 and 6 we assume a more important increase (from 50 to
8 300 events per month more) due to the specification of “repeated seismic
9 activity” in the accounts.

10 - Φ_{CO_2} varies (once) in sub-periods 1, 3 and 6. For sub-periods 1 we define
11 a reasonable range of increase between 0 and $30 \text{ kg m}^{-2} \text{ d}^{-1}$, and an equal
12 range for the further increase in sub-period 3 is assumed. For the last sub-
13 period we identify a larger increase (20 to $300 \text{ kg m}^{-2} \text{ d}^{-1}$, representing
14 about 10 and 100 times the present background emission rate) due to the
15 specification of “strong gas emission” in the historical report.

16 - For the deformation parameters (ε and $d\varepsilon/dt$), we define ranges for the
17 increase in the observed ground uplift, and translate the uplift in terms of ε
18 and $d\varepsilon/dt$ as in section 4.1. In this view, ground uplift varies (once) in sub-
19 periods 2, 3, 4 and 6. In sub-period 2, the account simply reports
20 “elevation of the ground” in 15 days. We assume that this might
21 correspond to an uplift of 5 cm (about the minimum observable without
22 instruments) to 50 cm. In terms of $d\varepsilon/dt$ this range corresponds to 6.7×10^{-7}
23 d^{-1} to $6.7 \times 10^{-6} \text{ d}^{-1}$, while for ε this corresponds to 10^{-5} to 10^{-4} . In sub-period
24 3, we define a larger further uplift (“major ground uplifting” in 5 days)

1 between 20 cm and 1 m. In terms of $d\varepsilon/dt$ this range corresponds to 8×10^{-6}
2 d^{-1} to $4 \times 10^{-5} d^{-1}$, while for ε this corresponds to a further increase of 5×10^{-5}
3 to 3×10^{-4} . In sub-period 4, we define a small uplift ("small uplifting of the
4 ground" in 7 days) between 5 and 20 cm. In terms of $d\varepsilon/dt$ this range
5 corresponds to $1.4 \times 10^{-6} d^{-1}$ to $5.7 \times 10^{-6} d^{-1}$, while for ε this corresponds to a
6 further increase of 6×10^{-5} to 3.4×10^{-4} . In sub-period 6, the deformation
7 must be large and rapid ("opening of mount" in few hours, here we take
8 $\Delta t = 6$ hours), between 50 cm and 3 m. In terms of $d\varepsilon/dt$ this range
9 corresponds to $4 \times 10^{-4} d^{-1}$ to $2.4 \times 10^{-3} d^{-1}$, while for ε this corresponds to
10 1.6×10^{-4} to 9.4×10^{-4} .

11 In this way, we perform 1000 control experiments; in practice, we build up
12 1000 different pre-eruptive phases, with the same anomalous parameters,
13 but assuming different values for them. This is a way of controlling the
14 dependence of BET_EF forecasting results on the thresholds fixed for
15 Vesuvius and on the errors in input monitoring measurements. In
16 particular, we want to check the stability of our best guess result (the
17 average of the probability distribution) with respect to reasonable errors on
18 our subjective interpretation. Since the best guess value is largely related
19 to aleatory uncertainty, i.e., a measure of the intrinsic unpredictability of
20 the system, with the control experiment we want to check how stable is the
21 system predictability in relation to errors in our interpretation. Because of
22 this, we will concentrate on the statistics of the control experiments'
23 average, rather than on the statistics of the dispersion.

1 In figure 1 we show a plot of the dispersion in the estimate of the average
2 probability of unrest versus time approaching the eruption among the 1000
3 control experiments. For comparison, we also show the results from the
4 reference case. It is important to note that the increases of the monitoring
5 parameters in the reference case are all contained in the ranges identified
6 above and given in Table 3.

7 From figure 1, we see that the BET_EF model immediately takes a jump in
8 the probability of unrest some months before the eruption, and it
9 recognizes undoubtedly an unrest phase about a month before the
10 beginning of the eruption.

11

12 In figures 2, 3 and 4 we show the same as in figure 1, except that we show
13 respectively the average absolute probability of eruption, the average
14 conditional probability of magma given unrest, and the average conditional
15 probability of eruption given magmatic unrest. These figures show that, as
16 time approaches the eruption, an escalating trend towards an eruptive
17 characterization of the crisis is evident (figure 2). The conditional
18 probability of magma given unrest (figure 3) is very high (around 80%)
19 already about 10 days before the eruption onset, while the conditional
20 probability of eruption given magma (figure 4) is stable around 30-50%,
21 except for a large step up to 90% in the few hours preceding the onset,
22 implying a substantial reduction of the aleatory uncertainty on the eruption
23 absolute probability (up to 70-80%). This is actually a common experience

1 of present-day volcanologists who have witnessed a large explosive
2 eruption.

3

4 **4. DISCUSSION AND CONCLUSIONS**

5 We have applied the BET_EF code (Marzocchi et al. 2004; Marzocchi et
6 al. 2008) to characterize the time evolution of the 1631 pre-eruptive phase
7 at Vesuvius, by using new and independent (i.e., they were not used to set
8 up the model) “cold” historical data. This application highlights four major
9 points.

10 - Historical researches to study and to model pre-eruptive phases, overall
11 for explosive volcanoes that do not have recent and monitored volcanic
12 eruption, as for Vesuvius, are of prominent importance, as suggested also
13 by Bertagnini et al. (2006).

14 - BET_EF code (Marzocchi et al. 2004; Marzocchi et al. 2008) applied to
15 the 1631 pre-eruptive phase is able to track the time evolution leading to
16 the eruption, marking steps in probability of eruption as time approaches
17 the eruption onset.

18 - The 1631 pre-eruptive phase shows signals that were able to increase
19 the absolute probability of eruption up to 10% about a month before the
20 beginning of the eruption; this probability reaches more than 30% 7-10
21 days before the onset of the event. In this respect, we do not agree with
22 Bertagnini et al. (2006) when they conclude that “the anomalous
23 phenomena reported before the end of November appear to be of doubtful
24 significance and reliability”. Rather, we think that even if the chronicles

1 report a filtered subset of occurred medium-term precursors, they are
2 sufficient for estimating a 10% probability of eruption, a month in advance.
3 We argue that also figures like these are worth being considered. In fact,
4 Marzocchi and Woo (2007) showed that the call for an evacuation based
5 on cost/benefit analysis is usually much lower than the higher probabilities
6 usually adopted by volcanologists, for instance during the MESIMEX
7 experiment.

8 - The BET_EF aleatory uncertainty on eruption probability for 1631 event
9 resembles the experience of present-day volcanologists who have
10 witnessed large explosive eruption, i.e., the aleatory uncertainty on the
11 eruption occurrence is low only hours before its onset.

12

13 Finally, we want to remark that the translation of historical information into
14 monitoring parameters always involves “subjective” choices. Here, we
15 have deeply explored the stability of the results (“control experiments”) as
16 a function of the assumptions made, and, in any case, we have usually
17 chosen the most conservative options in order to not optimize the results.

18 It is also worth remarking that these results could represent a lower bound
19 of the BET_EF forecasting capability, because we use only a subset of the
20 real pre-eruptive signals, i.e., we use monitoring parameters that produced
21 signals felt by human beings and not only by instruments. We think that
22 the inclusion of the latter type of signals can improve the results reported
23 here.

24

1 **References**

2 Arminio, (de) Giovanni Domenico (1632), *De terremotibus et incendiis eorumque*
3 *causis et signis naturalibus et supranaturalibus Item de flagratione Vesuvii*
4 *eiusque mirabilibus eventis et auspiciis, Xenodochii Incurabilium Neap. urbis a*
5 *secretis*, Naples.

6

7 Braccini, Giulio Cesare (1632) *Dell'incendio fattosi nel Vesuvio a XVI. di*
8 *Dicembre M.DC.XXXI. e delle sue cause, ed effetti.* | *Con la narrazione di quanto*
9 *è seguito in esso per tutto Marzo 1632. e con la Storia di tutti gli altri Incendij nel*
10 *medesimo Monte avvenuti. Discorrendosi in fine delle Acque, le quali in questa*
11 *occasione hanno danneggiato le campagne, e di molte altre cose curiose,*
12 *dell'Abbate Giulio Cesare Braccini da Gioviano di Lucca Dottor di Leggi, e*
13 *Protonotario Appostolico. Napoli.*

14

15 Barberi F, Macedonio G, Pareschi MT, Santacroce R (1990) Mapping the
16 tephra fallout risk: An example from Vesuvius, Italy. *Nature* 344:142–144

17

18 Barberi F, Principe C, Rosi M, Santacroce R (1995) Scenario eruttivo al Vesuvio
19 nel caso di riattivazione a medio-breve termine. Aggiornamento al 20 gennaio
20 1995. Rapporto GNV-Protezione Civile, pp 14 (in Italian)

21

22 Bertagnini A, Cioni R, Guidoboni E, Rosi M, Neri A, Boschi E (2006) Eruption
23 early warning at Vesuvius: The A.D. 1631 lesson. *Geophys Res Lett*
24 33:L18317. DOI:10.1029/2006GL027297.

25

1 Carafa, Gregorio (1632) In opusculum de novissima Vesuvij conflagratione,
2 epistola isagogica, 2nd ed Naples
3

4 Cioni R, Longo A, Macedonio G, Santacroce R, Sbrana R, Sulpizio R,
5 Andronico D (2003) Assessing pyroclastic fall hazard through field data and
6 numerical simulations: Example from Vesuvius. J Geophys Res 108:B2:2063.
7 DOI:10.1029/2001JB000642
8

9 Cornelius RR, Voight B (1996) Real-time Seismic Amplitude Measurement
10 (RSAM) and Seismic Spectral Amplitude Measurement (SSAM) Analyses with
11 the Materials Failure Forecast Method (FFM), June 1991 Explosive Eruption at
12 Mount Pinatubo. In: Newhall CG, Punongbayan RS (Eds) Fire and Mud:
13 Eruptions and Lahars of Mount Pinatubo, Phillipines. University of Washington
14 Press, Seattle, WA, pp 249-268
15

16 Esposti Ongaro T, Neri A, Todesco M, Macedonio G (2002) Pyroclastic flow
17 hazard assessment at Vesuvius (Italy) by using numerical modeling II. Analysis
18 of flow variables. Bull Volcanol 64:178–191. DOI:10.1007/s00445-0010190-1
19

20 Guidoboni E (2004) Analysis of three coeval Treatises on the 1631 Vesuvius
21 eruption, finalised to the evidence of the precursors: Gregorio Carafa, Giovan
22 Battista Mascolo, Salvatore Varonius. Internal Report D2.4 of the EU Project:
23 Explosive eruption risk and decision support for EU population threatened by

1 volcanoes, EXPLORIS, Contract no. CT-EVR1- 2002-40026, Coordinator Neri
2 A, INGV
3
4 Guidoboni (2005-2007). Vulcan-3 project - RU 1, Historical Research on the
5 Volcanoes of the Campania Region: Campi Flegrei, Vesuvius, Ischia,
6 agreement with the Dipartimento della Protezione Civile and INGV
7
8 Guidoboni E, Boschi E (2006) Vesuvius before the 1631 Eruption. EOS,
9 TRANSACTIONS AMERICAN GEOPHYSICAL UNION 87:40:417
10
11 Guidoboni E (2008) Vesuvius: A Historical Approach To The 1631 Eruption -
12 “Cold Data” From The Analysis Of Three Contemporary Treatises. J Volcanol
13 Geotherm Res. DOI:10.1016/j.jvolgeores.2008.09.020
14
15 Hill DP (1996) Earthquakes and carbon dioxide beneath Mammoth Mountain,
16 California. Seismol Res Lett 67:8-15
17
18 Marzocchi W, Sandri L, Gasparini P, Newhall CG, Boschi E (2004) Quantifying
19 probabilities of volcanic events: the example of volcanic hazard at Mount
20 Vesuvius. J Geophys Res 109:B11201. DOI:10.1029/2004JB003155
21
22 Marzocchi W, Zaccarelli L (2006) A quantitative model for the time-size
23 distribution of eruptions. J Geophys Res 111:B04204.
24 DOI:10.1029/2005JB003709

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

Marzocchi W, Woo G (2007) Probabilistic eruption forecasting and the call for an evacuation. *Geophys Res Lett* 34:L22310. DOI:10.1029/2007GL031922

Marzocchi W, Sandri L, Selva J (2008) BET_EF: a probabilistic tool for long- and short-term eruption forecasting. *Bull Volcanol* 70:623–632. DOI:10.1007/s00445-007-0157-y

Mascolo, Giovan Battista (1634) *De incendio Vesuvii excitato xvij. Kal. Ianuar. anno trigesimo primo sæculi Decimiseptimi libri X. Cum Chronologia superiorum incendiorum; & Ephemeride ultimi.* Napoli

Presidenza del Consiglio dei Ministri, Dipartimento della Protezione Civile, Pianificazione Nazionale d'emergenza dell'area vesuviana, 1995

Quarenì F, Mulargia F (1993) Modeling of closure of volcanic conduits with an application to Mount Vesuvius. *J Geophys Res* 98:4221–4229

Scandone R, Giacomelli L, Gasparini P (1993) Mount Vesuvius: 2000 years of volcanological observations. *J Volcanol Geotherm Res* 58:5–25

Varrone (Varonius), Salvatore (1634), *Vesuviani incendii historiae libri tres,* Naples

Zollo A, Marzocchi W, Capuano P, Lomax A, Iannaccone G (2002) Space and

- 1 Time Behavior of Seismic Activity at Mt. Vesuvius Volcano, Southern Italy. Bull
- 2 Seismol Soc Am 92:625–640
- 3

TABLE 1. Pre-eruptive phase for the 1631 eruption: from the summer to December 16th 4-5am, GMT

Note that the thick black lines represent different time sub-periods related to different applications of the BET_EF code (see section 4). The question mark “?” means “missing or not defined data”, while “ca” means “circa”. Legend:

Leftmost column indicates the time sub-period

Date/chronological range = month day (and, when defined, hour in italics) of the beginning of the phenomenon described, and, when available, its end. Time is given in GMT

Duration= duration of the phenomenon (in the units specified)

Time to eruption = time in months, days or hours (as specified) elapsing between the start of the phenomenon described and the onset of the eruption

Source: A=de Arminio (1632); B=Braccini (1634); C=Carafa (1634); M=Mascolo (1634);

V=Varrone (1632)

	<i>Date/chronological range</i>	<i>Duration</i>	<i>Time to eruption</i>	<i>Source</i>	<i>Summary of the description</i>	<i>Volcanological interpretation</i>	<i>BET_EF parameters (for the meaning of the symbols, see Section 3)</i>
1	Summer/August?	1month	ca 120days	V	Fires visible at night at Herculaneum (<i>Resina</i> in the texts) eastern slopes of the volcano		
	Aug	?	ca 120days	A	Smoke emissions from the crater	Increase in CO ₂ flux (about 4 times the average)	$\Pi_{CO_2}=10 \text{ kg m}^{-2} \text{ d}^{-1}$
	Sep 1 - Sep 10	11days	107days	C	Lowering of the ground; landslides; ground faulting; smoke emissions; fiery emissions (north-eastern slope)		
	Sep 1 - Dec 16 05am	3.5months	ca 107days	V	Considerable anomalous restlessness among the domestic animals; flight of wild animals from the Vesuvius area		
	Oct 1 - Nov 30	2months	77days	V	Underground noises (from Portici)		
2	ca Nov 16 - Dec 1	0.5month	30days	B	Elevation of the ground	Detection of positive strain (about 0.1m of vertical uplift over $\Delta h=5\text{km}$, i.e., the supposed thickness of the deformed layer); positive strain rate (0.1m of uplift in 0.5mo)	$\epsilon=2 \times 10^{-5}$ $d\epsilon/dt=1.3 \times 10^{-6} \text{ d}^{-1}$
	Nov 19 10pm - Nov 20 07am	9hours	27days	V	Rough seas; moderate seismic activity in the coastal region of Vesuvius	About 50 earthquakes M1.9+ recorded at OVO; maximum magnitude about 3.0	$n_e=50$ $M_d=3.0$
3	"Just before December"	?	ca 18days	V	Emissions, with effects on the herbaceous vegetation [wilting] (Vesuvius area)	Stronger increase in CO ₂ flux	$\Pi_{CO_2}=20 \text{ kg m}^{-2} \text{ d}^{-1}$
	Beginning of December	5days	ca 16days	AV	Loud noises in the Vesuvius area		
	Nov 28 - Dec 02	5days	14days	V	Darkening and variation in the water chemistry [salinity] (Vesuvius area); collapse of a great overhanging rock (eastern slope); major uplifting of the ground - (slopes, particularly western)	Increase in cumulative strain (further 0.5m of vertical uplift); higher strain rate, about 0.5m in 5d	$\epsilon=1.2 \times 10^{-4}$ $d\epsilon/dt=2 \times 10^{-5} \text{ d}^{-1}$

4	Dec 07 05pm - Dec 08 05pm	2days	10days	VCM	Moderate seismic activity in the Vesuvius area V; underground noises: in the Vesuvius area M V, and at Herculaneum for some days C	Other 50 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude	$n_e=100$ $M_d=3.0$
	Dec 08 05pm - Dec 09 05pm	1days	9days	V	Underground noises in the western and south-western slopes of Vesuvius		
	Dec 08 05pm - Dec 15	8days	9days	VCM	Underground noises Vesuvius area; underground noises (Herculaneum and Vesuvius area)		
	8 days before	?	8days	V	Underground noises and thunder in the Vesuvius area		
	Dec 09 - Dec 15	7days	8days	V	Restlessness among the domestic animals in the Vesuvius area; Small uplifting of the ground; emissions of hot vapours (Vesuvius area)	Increase in cumulative strain (further 0.1m of vertical uplift); lower strain rate, about 0.1m in 7d; high temperature of the fumaroles	$\epsilon=1.4 \times 10^{-4}$ $d\epsilon/dt=2.9 \times 10^{-6} \text{ d}^{-1}$ $T=110 \text{ C}$
	Dec 10	?	6days	B	Underground noises and thunder in the Vesuvius area		
	Dec 10	?	6days	B	Moderate seismic activity (Vesuvius area)	Other 50 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude	$n_e=150$ $M_d=3.0$
	Dec 10	?	6 days	B	Darkening of the waters of the wells (Vesuvius area)		
	Dec 10	6days	7days	V	Tranquility of the air (Vesuvius area); calm sea (sea facing the Vesuvius area)		
	Dec 11 05pm - Dec 13 05pm	3days	6days	V	Moderate seismic activity (Vesuvius area)	Same considerations as above, probably related to the same period of time	
	Dec 13 – Dec 16	?	3days	C	Darkening of the waters of the wells (Vesuvius area)		
	Dec 13 – Dec 15 08pm	2days	5days	C	Tranquility of the air in the area of Vesuvius and in Naples		
	Dec 14 05pm - Dec 15 09pm	28hours	35hours	V	Tranquility of the air and restlessness of the animals in the Vesuvius area, calm sea in the coastal area of Vesuvius		
	Dec 15 morning	?	1day	B	flashing arcs upon Vesuvius		
5	Dec 15 08pm - Dec 16 01am	5hours	8hours	V	Calm sea (coastal area)		
	Dec 15 08pm - Dec 16 04am	8hours	8hours	CVM	Repeated seismic activity felt at Naples and in the Vesuvius area, for a range of ca. 7,5 km	Other 150 earthquakes M1.9+ recorded at OVO; maximum magnitude increases because the earthquakes are felt in Naples; acceleration in seismic energy release	$n_e=300$ $M_d=4.0$ $d^2E/dt^2=1$
	Dec 15 09pm - Dec 16 04am	7 hours	7hours	B	Repeated seismic activity felt at Naples and in the Vesuvius area	Same considerations as above, related to the same period of time	
	Dec 15 08pm - Dec 16 05am	8hours	8hours	V	Restlessness among the domestic animals (Vesuvius area, from Portici); moderate and repeated seismic activity (30 shocks); underground noises – Vesuvius area	Same considerations as above, related to the same period of time	

	Dec 15 08pm - Dec 16 07am	2days	8hours	M	Intense and repeated seismic activity in the area of Vesuvius and Naples	Same considerations as above, related to the same period of time	
	Dec 15 night - Dec 16	ca 14hours	?	A	Flames and lights from the crater visible at night		
	Dec 15 09pm	7hours	7hours	V	Underground noises and seismic activity – Vesuvius area		
6	Dec 16 04am	?	ca 1hour	C	Seismic activity; opening of faults and landslides; landslides between the Atria and the summit of the Veolo, just above the path that goes from Atria and circles the Veolo, in the middle of the mountain's slopes, closer to the one facing Atria	Other 200 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude; acceleration in seismic energy release; acceleration in strain	$n_e=500$ $M_d=4.5$ $d^2E/dt^2=1$ $d^2\varepsilon/dt^2=1$
	Dec 16 after 04am	?	ca 1hour	B	Opening of the mount in the Atria	Cumulative strain exceeds rock strength; strain rate must be very high (at least 1m of ground displacement in less than 1d)	$\varepsilon=3.4 \times 10^{-4}$ $d\varepsilon/dt=8 \times 10^{-4} \text{ d}^{-1}$
	Dec 16 04am - Dec 16 05am	ca 1hour	ca 1hour	V	Very loud underground noises in the Vesuvius area, ebbing of the sea in the Gulf of Naples, between Pozzuoli and the coast facing the mouth of the River Sarno; earthquakes, landslides, ground faulting; loud underground; noises in the Atria; intense smoke emissions; fiery emissions; rock expulsion (from the central crater, on the eastern slope, seawards)	Same considerations as above, related to the same period of time. Furthermore: stronger increase in CO ₂ flux; phreatic explosions	$\Pi_{CO_2}=100 \text{ kg m}^{-2} \text{ d}^{-1}$ $PE=1$
	Dec 16 after 04am	ca 1hour	ca 1hour	B	Opening secondary bocca	Same considerations as above, related to the same period of time	
	Dec 16 04am - Dec 16 05am	ca 1hour	ca 1hour	VCM B	Fiery emissions; thunder, underground noises and lightening, rock expulsion, ash and soot, emission of a cloud (Vesuvius area), violent seismic activity	Same considerations as above, related to the same period of time	
	Dec 16 05am	ERUPTION		VB	Smoke emissions; loud underground noises; ground faulting; opening of a large chasm; cloud emission reaching about 37 km in height; haze (from the fracture appearing in the Atria)		

1
2

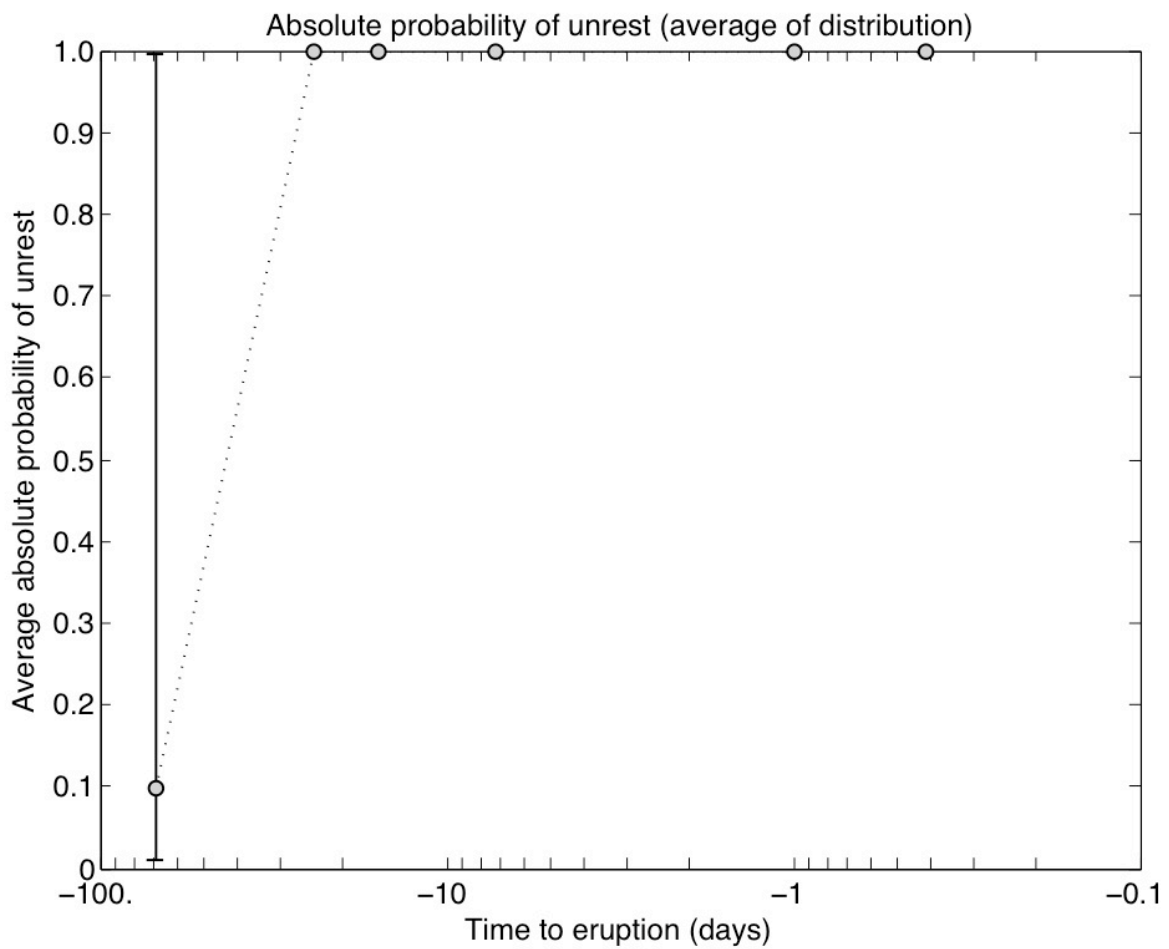
1 **TABLE 2. Results of the simulations over the 6 different time sub-periods for**
 2 **the reference case**
 3

Time Sub-period and relative "anomalous monitoring"	Absolute Probability of Unrest				Absolute Probability of Eruption				Conditional Probability of Magma (given Unrest)				Conditional Probability of Eruption (given Magmatic Unrest)			
	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc
1 (Aug to Nov 15) $\Pi_{CO_2}=10 \text{ kg m}^{-2} \text{ d}^{-1}$	0.10	0.10	0.10	0.10	0.02	0.002	0.02	0.05	0.48	0.12	0.47	0.84	0.48	0.12	0.47	0.84
2 (Nov 16 to Nov 27) $\Pi_{CO_2}=10 \text{ kg m}^{-2} \text{ d}^{-1}$ $\epsilon=2 \times 10^{-5}$ $d\epsilon/dt=1.3 \times 10^{-6} \text{ d}^{-1}$ $n_e=50$ $M_d=3.0$	1	1	1	1	0.08	0	0.002	0.29	0.28	0	0.12	0.86	0.28	0	0.09	0.86
3 (Nov 28 to Dec 02) $\Pi_{CO_2}=20 \text{ kg m}^{-2} \text{ d}^{-1}$ $\epsilon=1.2 \times 10^{-4}$ $d\epsilon/dt=2 \times 10^{-5} \text{ d}^{-1}$ $n_e=50$ $M_d=3.0$	1	1	1	1	0.13	0	0.02	0.50	0.43	0.004	0.37	0.96	0.34	0	0.22	0.92
4 (Dec 03 to Dec 15 morning) $\Pi_{CO_2}=20 \text{ kg m}^{-2} \text{ d}^{-1}$ $T=110 \text{ C}$ $\epsilon=1.4 \times 10^{-4}$ $d\epsilon/dt=2.9 \times 10^{-6} \text{ d}^{-1}$ $n_e=150$ $M_d=3.0$	1	1	1	1	0.28	0	0.13	0.83	0.82	0.21	1	1	0.35	0	0.24	0.92
5 (Dec 15 08pm to Dec 16 night) $\Pi_{CO_2}=20 \text{ kg m}^{-2} \text{ d}^{-1}$ $T=110 \text{ C}$ $\epsilon=1.4 \times 10^{-4}$ $d\epsilon/dt=0 \text{ d}^{-1}$ $n_e=300$ $M_d=4.0$ $d^2E/dt^2=1$	1	1	1	1	0.27	0	0.10	0.85	0.82	0.22	1	1	0.36	0	0.24	0.93
6 (Dec 16 04 am to Eruption Onset) $\Pi_{CO_2}=100 \text{ kg m}^{-2} \text{ d}^{-1}$ $T=110 \text{ C}$ $\epsilon=3.4 \times 10^{-4}$ $d\epsilon/dt=8 \times 10^{-4} \text{ d}^{-1}$ $n_e=500$ $M_d=4.0$ $d^2E/dt^2=1$ $d^2\epsilon/dt^2=1$ $PE=1$	1	1	1	1	0.77	0.13	0.98	1	0.87	0.38	1	1	0.87	0.41	1	1

1 **TABLE 3. Control experiments: parameters that are varied in the experiments,**
 2 **present-day background value and relative ranges for the increments in each**
 3 **sub-period. For the deformation parameters ($d\varepsilon/dt$ and ε), we vary the**
 4 **supposed uplift and translate it into the desired parameters.**
 5

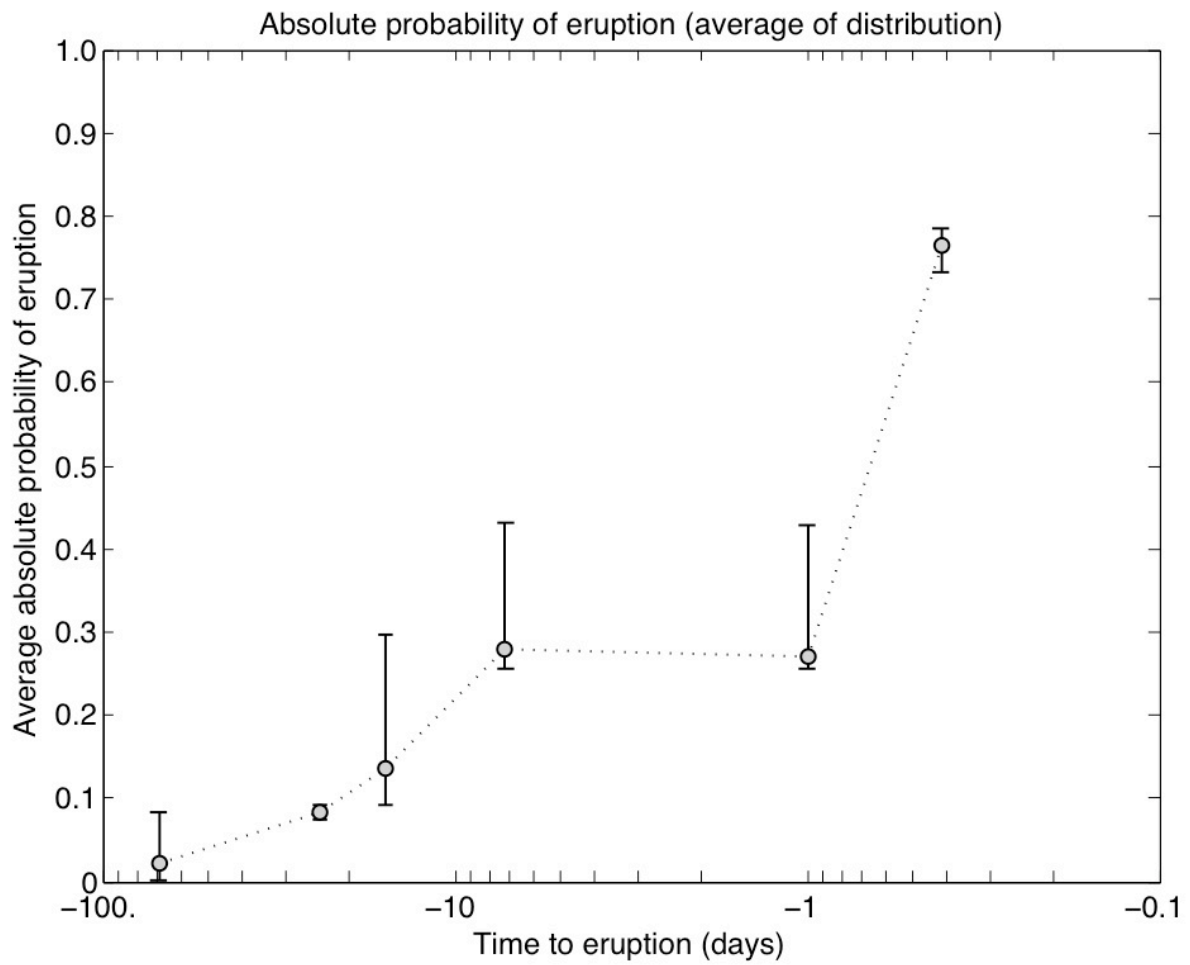
Parameter	Present-day background	Range of increase
T	95 °C	Sub-period 4: 5 – 25 °C
M_d	About 2 – 2.5/month	Sub-period 2: 0 – 1.5 /month Sub-period 5: 0 – 1.5 /month
n_e	About 10 events/month	Sub-period 2: 0 – 100 events/month Sub-period 4 (1): 0 – 100 events/month Sub-period 4 (2): 0 – 100 events/month Sub-period 5: 50 – 300 events/month Sub-period 6: 50 – 300 events/month
Φ_{CO_2}	2.5 kg m ⁻² d ⁻¹	Sub-period 1: 0 – 30 kg m ⁻² d ⁻¹ Sub-period 3: 0 – 30 kg m ⁻² d ⁻¹ Sub-period 6: 20 – 300 kg m ⁻² d ⁻¹
Uplift	0 m	Sub-period 2: 5 – 50cm/0.5month ($d\varepsilon/dt$: 6.7×10^{-7} – 6.7×10^{-6} d ⁻¹ ; ε : 10^{-5} – 10^{-4}) Sub-period 3: 20 – 100 cm/5 days ($d\varepsilon/dt$: 8×10^{-6} – 4×10^{-5} d ⁻¹ ; ε : 5×10^{-5} – 3×10^{-4}) Sub-period 4: 5 – 20 cm/7days ($d\varepsilon/dt$: 1.4×10^{-6} – 5.7×10^{-6} d ⁻¹ ; ε : 6×10^{-5} – 3.4×10^{-4}) Sub-period 6: 50 – 300 cm/6hours ($d\varepsilon/dt$: 4×10^{-4} – 2.4×10^{-3} d ⁻¹ ; ε : 1.6×10^{-4} – 9.4×10^{-4})

6
7
8
9
10



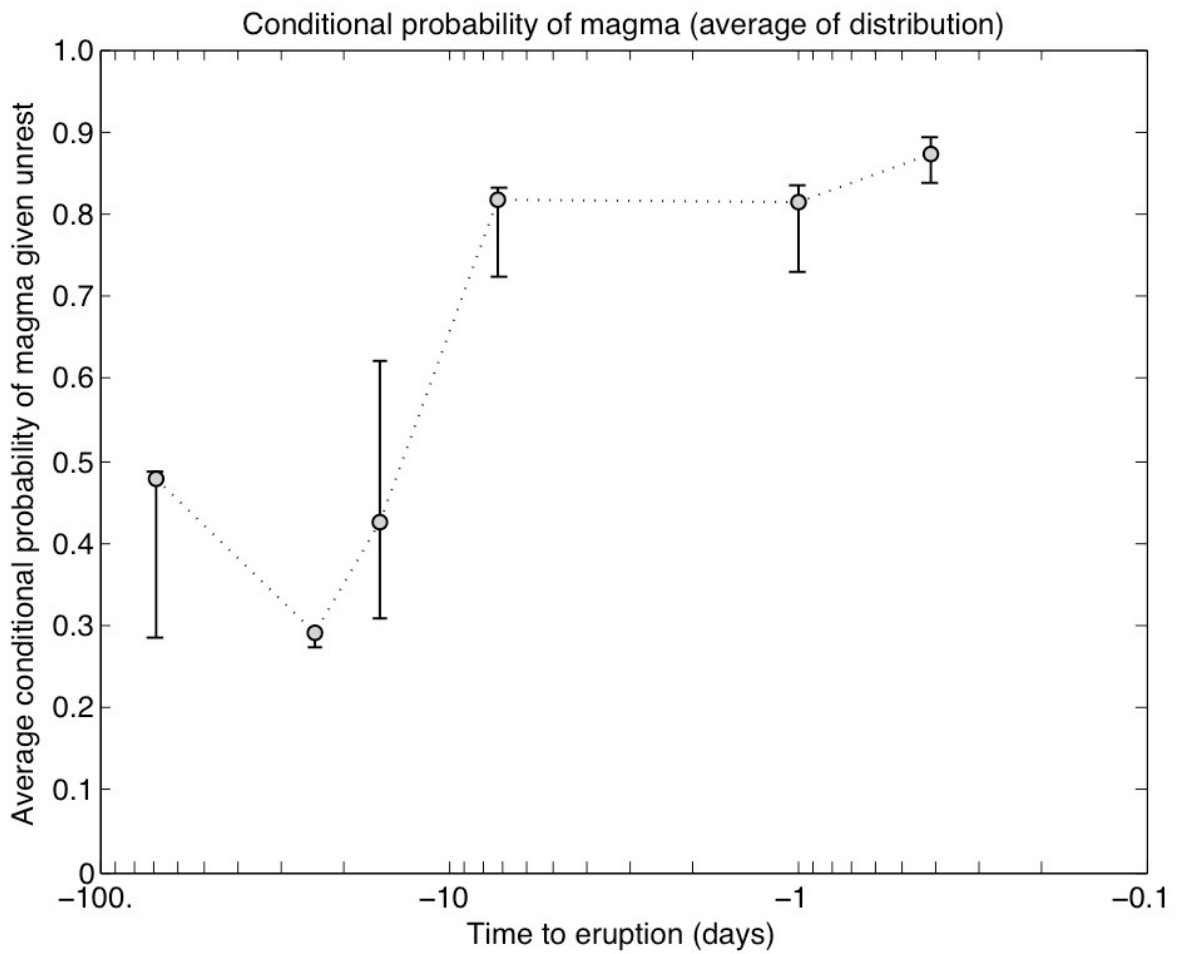
1
2

3 **Figure 1.** Average absolute probability of unrest in the control experiments for
 4 different sub-periods versus time approaching the eruption. The intervals represent
 5 the 10-90 percentiles of the average probabilities obtained in the 1000 simulations.
 6 The circles represents the value obtained for the reference case.

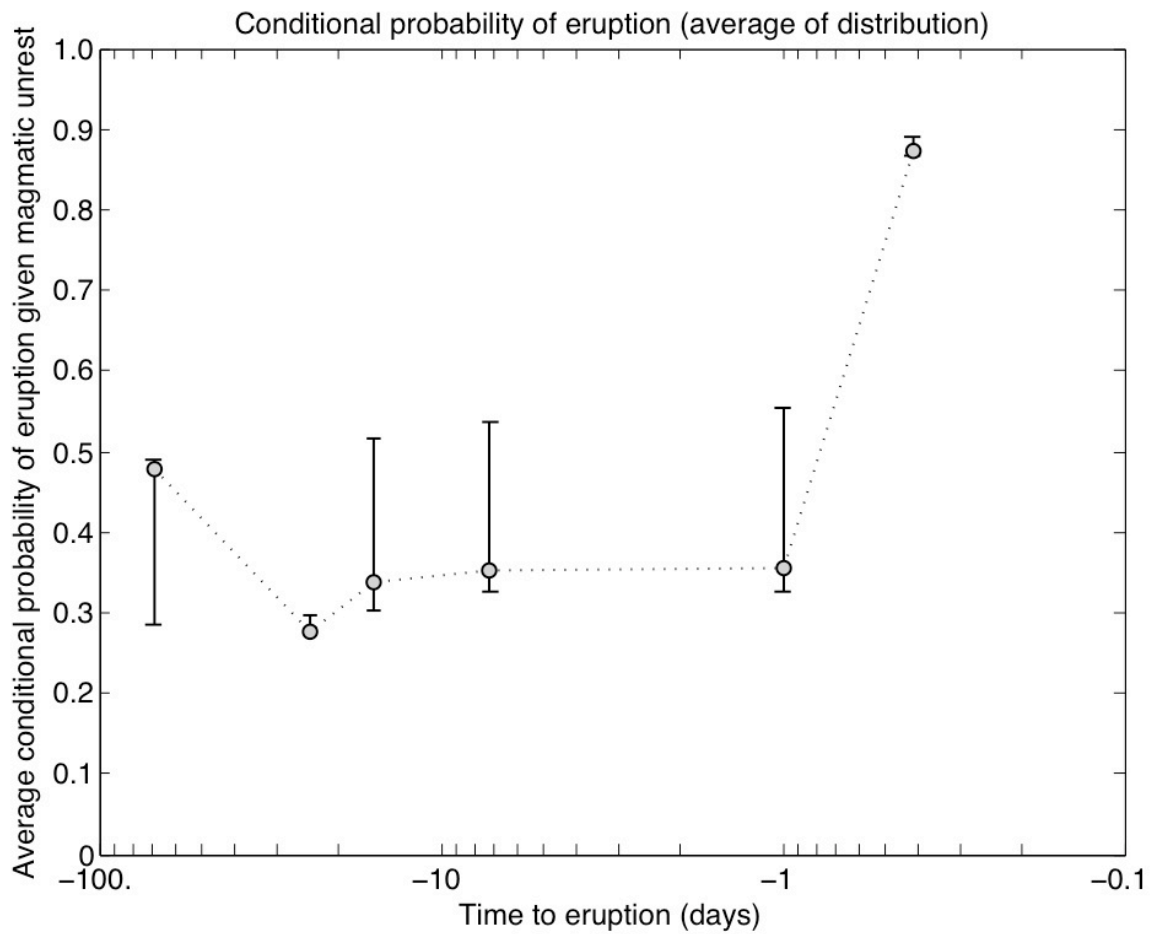


1
2

3 **Figure 2.** Average absolute probability of eruption in the control experiments for
 4 different sub-periods versus time approaching the eruption. The intervals represent
 5 the 10-90 percentiles of the average probabilities obtained in the 1000 simulations.
 6 The circles represents the value obtained for the reference case.



1
2
3 **Figure 3.** Average conditional probability of magma given unrest in the
4 control experiments for different sub-periods versus time approaching the
5 eruption. The intervals represent the 10-90 percentiles of the average
6 probabilities obtained in the 1000 simulations. The circles represents the
7 value obtained for the reference case.



1
2

3 **Figure 4.** Average conditional probability of eruption given magmatic unrest
 4 in the control experiments for different sub-periods versus time approaching
 5 the eruption. The intervals represent the 10-90 percentiles of the average
 6 probabilities obtained in the 1000 simulations. The circles represents the
 7 value obtained for the reference case.

1 **Appendix A: The authors of the five treatises examined**

2 The authors of these four texts were Neapolitan intellectuals, men of the cloth or
3 ecclesiastic, all present at the eruption of Vesuvius in 1631. Here is their
4 concise biography, that is useful in order to frame the cultural ambient those
5 authors belonged to.

6 ***Giovanni Domenico de Arminio***. Hardly anything is known about this author,
7 apart from the fact that he was a doctor at the Ospedale degli Incurabili of
8 Naples, as he himself declares in the presentation of the Treatise. So he was a
9 direct witness to what had happened, above all of what was seen in Naples. His
10 treatise contains many theoretical elements on the causes of the eruption,
11 mainly oriented to an almost anthropomorphic interpretation of the volcanic
12 activity.

13 ***Gregorio Carafa (1588–1675)***. Carlo Marcello (later Gregorio) Carafa was born
14 in Naples. He was a philosopher and theologian; he joined the Teatini fathers of
15 Naples, where he held the chair of Philosophy and Theology. It was in those
16 years that he acquired great fame as a preacher and a cultured intellectual. The
17 fame he earned among his contemporaries and his noble origins allowed him to
18 obtain the highest positions in his order. he was also the Bishop of Cassano
19 (Calabria) and Archbishop of Salerno. He held some diplomatic posts: he was
20 the special diplomatic representative of the Emperor Philip IV of Spain with the
21 Pope Innocence X.

22 ***Giovanni Battista Mascolo (1583–1648)***. Born in Naples, he became a Jesuit
23 at a very young age, in 1598. He taught Theology and Philosophy at the
24 College of his order, of which he was the rector for some time; later he held a

1 school of rhetoric at his home. He was famed for being a good Latin scholar.
2 His treatise *Vesuviani Incendii Historiae libri tres* (Naples, 1634), his most
3 complex work, was written in his full maturity, probably using some of his
4 previous unpublished writings.

5 **Giulio Cesare Braccini (1570-1632)** He was a man of law, later an ecclesiast
6 and abbot, born at Gioviano di Lucca. He was appointed apostolic proto-notary
7 by Urban VIII and between 1629 and 1632 he was in Naples, in touch with
8 many leading political characters of the day. As regards Vesuvius, abbot
9 Braccini published two texts: one was a letter to cardinal Girolamo Colonna,
10 disseminated in the first few days after the eruption, to the extent that already
11 between the end of December 1631 and the beginning of 1632 three editions
12 had been published (*Relazione dell'incendio del Vesuvio alli 16 dicembre 1631*
13 *in una lettera diretta all'Em.mo e Re. mo Signore Card. Colonna*, printed in
14 Naples at S. Roncagliolo); and the treatise examined here, printed in Naples in
15 September 1632.

16 **Salvatore Varrone (1593–1656.)** We have very little information about him. He
17 became a Jesuit in 1612; he taught grammar, humanities and rhetoric, and for
18 six years scholastic theology and morals. He was famous for being a very
19 learned intellectual. In the period of the eruption of Vesuvius, Varrone was
20 staying at Portici, a village on the slopes of Vesuvius, and therefore was an
21 eyewitness to the whole eruption.

1 **Appendix B: Summarizing tables of the BET_EF rules for Vesuvius**

2 Here we show the rules uploaded in BET_EF in this application. They are
3 exactly the same used in Marzocchi et al (2008).

4 A summary of all the rules is provided in Table B1.

5 At each node k of the Bayesian Event Tree we compute two different
6 probability distributions, one by using only monitoring data and the other
7 one by using only other kinds of data, e.g. models, past occurrence, expert
8 opinions. These two probability distributions are indicated respectively by
9 $[\theta_k^{(M)}]$ and $[\theta_k^{(NM)}]$, where the index k stands for the k -th node and the
10 square brackets denote a probability distribution. In order to compute the
11 actual probability distribution at node k , we linearly combine $[\theta_k^{(M)}]$ and
12 $[\theta_k^{(NM)}]$ with a relative weight that is function of the state of unrest. Both
13 $[\theta_k^{(M)}]$ and $[\theta_k^{(NM)}]$ are computed through the Bayes theorem, i.e., by
14 starting from a prior distribution (based on models and beliefs for $[\theta_k^{(NM)}]$,
15 and on present monitoring for $[\theta_k^{(M)}]$). The prior distribution is characterized
16 by a mean ($\Theta_k^{(NM)}$ and $\Theta_k^{(M)}$ in the two cases), representing our best prior
17 guess on the probability at node k , and by a measure of the variance that
18 we call “equivalent number of data” ($\Lambda_k^{(NM)}$ and $\Lambda_k^{(M)}$ in the two cases)
19 because intuitively it translates the confidence we have in our prior guess
20 in terms of number of data. The minimum possible “equivalent number of
21 data” is 1 and it represents the maximum variance allowed, implying a very
22 low confidence on our prior guess, while there is no upper limit to its
23 maximum value. The prior distribution is then transformed into the
24 posterior distribution through Bayes theorem, i.e., by multiplying it by the

1 likelihood function, based on past frequencies of occurrence for $[\theta_k^{(NM)}]$
2 and on past monitoring (if any) for $[\theta_k^{(M)}]$.

3 An important aspect of BET is the way it deals with monitoring
4 measurements. Through a fuzzy approach, the measured values are
5 translated into degrees of anomaly, from whom the mean of the monitoring
6 probability distribution ($\Theta_k^{(M)}$) is derived (Marzocchi et al, 2008). In
7 practice, for each monitoring parameter, we define a lower and an upper
8 threshold, and an order relationship, used to infer the degree of anomaly
9 for every specific measured value.

10 In the following, we give a detailed account of the choices on all BET
11 parameters for Mt. Vesuvius (see also Marzocchi et al, 2004; 2008), as
12 frozen before this retrospective application.

13 See Marzocchi et al (2008) for a deeper discussion on BET structure (the
14 nodes), general rules and related concepts.

15

16 B.1 First node: Unrest

17 B.1.1 Non-monitoring part: $[\theta_1^{(NM)}]$

18 There is no theoretical model or expert belief to be used for assessing the
19 probability of a volcano entering an unrest phase. Thus here we set up a
20 maximum ignorance prior distribution with mean $\Theta_1^{(NM)}=0.5$ and $\Lambda_1^{(NM)}=1$.

21 Regarding past data, we know that OVO seismic station (see Zollo et al.,
22 2002) has been monitoring Mt. Vesuvius continuously since 1972. We
23 assume that there has been no episode of unrest ever since. Thus, we
24 have 0 unrest episodes out of 420 months (i.e., 35 years).

1 B.1.2 Monitoring part: $[\theta_1^{(M)}]$

2 From now on, for the monitoring part at each node we give the list of
3 selected monitoring variables used in Marzocchi et al (2004). In particular,
4 for node 1, as summarized in Table B2, they are (from now on, for
5 monitoring parameters we denote in braces the order relationship and the
6 upper and lower thresholds used to define the degree of anomaly of the
7 measured parameter):

- 8 - $n_e \{>;23;150\}$ number of seismic events per month with $M_d \geq 1.9$ recorded
9 at OVO station; the thresholds have been chosen on the basis of the
10 monthly distribution of the number of seismic events observed at OVO,
11 and they represent respectively the 55-th and the 95-th percentiles
- 12 - $M_d \{>;3.4;4.3\}$ largest duration magnitude of the earthquakes recorded
13 during last month at OVO station; the thresholds have been chosen on the
14 basis of the monthly distribution of the magnitude of seismic events
15 observed at OVO, and they represent respectively the 55-th and the 95-th
16 percentiles
- 17 - $n_{LF} \{>;1;3\}$ number of low-frequency (LF) events deeper than 1 Km per
18 month; since only sporadic and temporally isolated LF events have been
19 observed so far, even a small swarm is indicative of unrest
- 20 - $\Pi_{SO_2} \{=;1;1\}$ significant presence SO_2 (0, no; 1, yes)
- 21 - $\Phi_{CO_2} \{>;5;30 \text{ kg m}^{-2} \text{ d}^{-1}\}$ daily CO_2 emission rate; the thresholds represent
22 respectively about twice and 10 times the background value

1 - $d\varepsilon/dt \{>0;0 d^{-1}\}$ strain rate (inflation), assuming that any detected uplift
2 implies unrest

3 - $T \{>98;105 \text{ }^\circ\text{C}\}$ temperature of the fumaroles inside the crater; the
4 thresholds represent respectively about 3% and 10% higher than the value
5 observed since 1990 (95°C).

6 No past monitoring is available. Thus, the posterior for $[\theta_1^{(M)}]$ is equal to
7 the prior.

8

9 B.2 Second node: Magma given Unrest

10 B.2.1 Non-monitoring part: $[\theta_2^{(NM)}]$

11 There is no model to be used for assessing the probability of a volcano
12 unrest being due to magma. Thus here we set up a maximum ignorance
13 distribution with mean $\Theta_2^{(NM)}=0.5$ and $\Lambda_2^{(NM)}=1$.

14 Regarding past data, we have no unbiased past data to infer the origin of
15 past unrest at Vesuvius, since we know much better the magmatic unrest
16 episodes (at least those that yielded an eruption), compared to the
17 hydrothermal ones. Thus, the posterior for $[\theta_2^{(NM)}]$ remains equal to the
18 prior.

19 B.2.2 Monitoring part: $[\theta_2^{(M)}]$

20 As summarized in Table B3, from Marzocchi et al (2004), the selected
21 monitored parameters at node 2 are:

22 - $\Pi_{\text{SO}_2} \{=;1;1\}$ significant presence SO_2 (0, no; 1, yes)

- 1 - $d\varepsilon/dt \{>5 \times 10^{-6}; 5 \times 10^{-5} \text{ d}^{-1}\}$ strain rate (inflation), considering that a rapid
2 and localized positive strain of the volcanic edifice is usually indicative of
3 rising magma
- 4 - $\nu \{<2.5; 3.5 \text{ Hz}\}$ the dominant spectral frequency of earthquakes; if it is
5 around the frequency specified by the thresholds, there are probably LF
6 events or tremor, that might be indicative of magma motion
- 7 - $\xi_e \{<0.3; 0.4\}$ ratio between average and dispersion of the depth of the
8 earthquakes during unrest. The present value of ξ_e for Mt. Vesuvius from
9 seismic events recorded in the last 30 years is about 0.4. The thresholds
10 are chosen in order to fuzzify this value, assuming that earthquakes
11 coming closer to the surface and/or occurring in a larger range of depths
12 may indicate an upward migration of magma or a coalescence of small
13 fractures that may facilitate the magma uprising. At the same time, we do
14 not consider shallow earthquakes concentrated in a very small range of
15 depths to be indicative of magma, as they could also be due to
16 hydrothermal activity
- 17 - $T \{>98; 105 \text{ }^\circ\text{C}\}$ the temperature of the fumaroles inside the crater; the
18 thresholds are chosen as for node 1.
- 19 Remarkably, we assume that the parameters Π_{SO_2} and T have a weight
20 twice as much as the other parameters in determining a magmatic origin
21 for the unrest.
- 22 Neither here past monitoring is available. Thus, the posterior for $[\theta_2^{(M)}]$ is
23 equal to the prior.
- 24

1 B.3 Third node: Eruption given Magmatic Unrest

2 B.3.1 Non-monitoring part: $[\theta_3^{(NM)}]$

3 There is no model to be used for assessing the probability of a volcano
4 erupting given that there is an unrest of magmatic origin. Thus here we set
5 up a maximum ignorance distribution with mean $\Theta_3^{(NM)}=0.5$ and $\Lambda_3^{(NM)}=1$.

6 Regarding past data, we have no unbiased past data to infer the
7 frequency of eruptions following past magmatic unrest at Vesuvius, since
8 we know much better the magmatic unrest episodes that yielded an
9 eruption with respect to those that died out without eruption. Thus, the
10 posterior for $[\theta_3^{(NM)}]$ remains equal to the prior.

11 B.3.2 Monitoring part: $[\theta_3^{(M)}]$

12 As summarized in Table B4, from Marzocchi et al (2004), the selected
13 monitored parameters at node 2 are:

- 14 - PE $\{=;1;1\}$ presence of phreatic explosion (0, no; 1, yes)
- 15 - $d\nu/dt \{<;0;0 \text{ Hz d}^{-1}\}$ rate of change of the average spectral frequency
16 content of earthquakes
- 17 - $\xi_e \{<;0.3;0.4\}$ ratio between average and dispersion of the depth of the
18 earthquakes during unrest; the thresholds are chosen as for node 2
- 19 - $d^2E/dt^2 \{=;1;1\}$ acceleration of seismic energy release (0, no; 1, yes)
- 20 - $d^2\varepsilon/dt^2 \{=;1;1\}$ acceleration of inflation (0, no; 1, yes)
- 21 - $\varepsilon \{>; 5 \times 10^{-5}; 5 \times 10^{-4}\}$ cumulative strain since the beginning of the unrest;
22 the thresholds are just a little bit less than a reasonable value for the
23 maximum prefracture strain

- 1 - $d\rho/dt \{>;0;0\}$ change of the ratios HCl/SO₂ and/or HF/SO₂
2 - REV $\{=;1;1\}$ sudden reversal of at least one of the above parameters (0,
3 no; 1, yes)

4 For most of these parameters the thresholds are based on a “yes/no”
5 choice. This is because we are mainly interested in the time trend of the
6 parameters.

7 We assume that the parameters $d^2\varepsilon/dt^2$, PE and REV have a weight twice
8 as much as the other parameters in determining the occurrence of the
9 eruption. Neither here past monitoring is available. Thus, the posterior for
10 $[\theta_3^{(M)}]$ is equal to the prior.

11

12 B.4 Forth node: Location of vent given that there is an Eruption

13 B.4.1 Non-monitoring part: $[\theta_4^{(NM)}]$

14 Since Mt. Vesuvius is a central volcano, and its activity has been mainly
15 concentrated in the crater, we assume that there is a very high probability
16 of vent opening in the crater area, and a very small one outside it. With
17 this idea in mind, we divide the volcanic edifice into 5 areas: the crater
18 area, and 4 outer, equal sized areas. We assume that next eruption will
19 take place in one and only one of them. Because of these assumptions,
20 we set up a Dirichlet distribution (see Marzocchi et al, 2008 for further
21 details on such distribution) with mean $\Theta_4^{(NM)(1)}=0.99$ in the crater area
22 (area 1) and $\Theta_4^{(NM)(i)}=0.0025$ ($i=2,\dots,5$) for the surrounding areas (i.e.,
23 Areas 2, 3, 4 and 5). Since we are very confident on this assumption, we
24 set $\Lambda_4^{(NM)}=50$, which is an (arbitrarily) high equivalent number of data.

1 B.4.2 Monitoring part: $[\theta_4^{(M)}]$

2 BET allows to localize some or all of the monitored measurements used at
3 previous nodes to assess the probability of vent opening in different areas,
4 according to monitoring (see Marzocchi et al, 2008 for further details on
5 this issue).

6

7 B.5 Fifth node: Size of the Eruption given that there is an Eruption

8 We parametrize the possible sizes with VEI. Since Mt. Vesuvius has been
9 dormant for over 60 years now, it is presumably in a closed conduit
10 regime. Therefore, we assume that next eruption will be at least of VEI=3
11 in order to have sufficient energy to re-open the system. For practical
12 purposes, we define three classes of possible sizes, according to this idea:
13 VEI=3, VEI=4 and VEI \geq 5.

14 B.5.1 Non-monitoring part: $[\theta_5^{(NM)}]$

15 We use the observation that the worldwide log(frequency)-size relationship
16 for volcanoes is a straight line, implying that the most frequent, and
17 therefore likely, eruptions are the smaller, with a power law relationship.
18 We translate this information by setting up a Dirichlet distribution for the
19 three size classes. The mean of each class is set equal to the probability
20 for that class given by the above mentioned worldwide relationship. In this
21 way we have the following means: $\theta_5^{(NM)(1)}=0.83$ for VEI=3, $\theta_5^{(NM)(2)}=0.14$
22 for VEI=4 and $\theta_5^{(NM)(3)}=0.03$ for larger eruptions. We also assume the
23 maximum variance on this model, thus $\Lambda_5^{(NM)}=1$.

1 For past data, we use the catalog of Mt. Vesuvius eruptions with a repose
 2 time larger or equal to 60 years, so that we are sure to consider only past
 3 eruptions occurred in closed conduit regime, as it is now the case. We
 4 have 7 such eruptions in the catalog, in particular 4 eruptions of VEI=3, 2
 5 of VEI=4 and 1 larger.

6 B.5.2 Monitoring part: $[\theta_5^{(M)}]$

7 Up to present, there is no reliable precursor heralding the size of
 8 eruptions. Therefore, we do not use monitoring data at this node.

9

TABLE B1: BET_EF settings

NODE	prior model	past data	monitoring parameters	past monitored episodes
1	NO	0 of 384	7 (see Table B2)	0
2	NO	NO	5 (see Table B3)	0
3	NO	NO	8 (see Table B4)	0
4	Means of the 5 areas: 0.99, 0.0025, 0.0025, 0.0025, 0.0025 with $\Lambda_4=50$	13, 0, 0, 0, 0		
5	0.83, 0.14, 0.03 with $\Lambda_5 = 1$	4, 2, 1		

10

TABLE B2: Monitoring parameters at NODE 1

symbol	description	thresholds	units
n_e	monthly number of events with $M_d \geq 1.9$ at OVO station	>23;150	month ⁻¹
M_d	monthly largest magnitude at OVO station	>3.4;4.3	month ⁻¹
n_{LF}	monthly number of LF events deeper than 1 km	>1;3	month ⁻¹
Π_{SO_2}	presence of significant SO ₂	=1	
Φ_{CO_2}	daily CO ₂ emission rate	>5;30	kg m ⁻² day ⁻¹
$d\varepsilon/dt$	strain rate (inflation)	>0;0	day ⁻¹
T	temperature of fumaroles in the crater	>98;105	°C

TABLE B3: Monitoring parameters at NODE 2

symbol	description	thresholds	units	weight
Π_{SO_2}	presence of significant SO ₂	=1		2
$d\varepsilon/dt$	strain rate	>5x10 ⁻⁶ ;5x10 ⁻⁵	day ⁻¹	1
$\langle v \rangle$	average spectral frequency	<2.5;3.5	Hz	1
ξ_e	ratio between average and dispersion of earthquake depths	<0.3;0.4		1
T	as in NODE 1	>98;105	°C	2

TABLE B4: Monitoring parameters at NODE 3

symbol	description	thresholds	units	weight
PE	Presence of phreatic explosions	=1		2
$d\langle v \rangle/dt$	rate of change of $\langle v \rangle$	<0;0	Hz	1
ξ_e	as NODE 2	<0.3;0.4		1
d^2E/dt^2	acceleration of seismic energy release	>0;0	J day ⁻²	1
$d^2\varepsilon/dt^2$	acceleration of strain (inflation)	>0;0	day ⁻²	2
ε	cumulative strain (inflation)	>5x10 ⁻⁵ ;5x10 ⁻⁴		1
r	change of the ratios HCl/SO ₂ and/or HF/SO ₂	=1		1
REV	sudden reversal of at least one of the parameters above	=1		2