Bayesian Event Tree for Eruption Forecasting (BET_EF) at Vesuvius, Italy: a retrospective forward application to the 1631 eruption.

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Abstract

Reliable forecasting of the next eruption at Vesuvius is the main scientific ingredient to define effective strategies to reduce volcanic risk in one of the most dangerous volcanic areas around the world. In this paper, we apply a recently developed probabilistic code for eruption forecasting to new and independent historical data related to the pre-eruptive phase of the 1631 eruption. The results obtained point out three main issues: 1) the importance of “cold” historical data (according to Guidoboni 2008) related to pre-eruptive phases for evaluating forecasting tools and possibly refining them; 2) the BET_EF code implemented for Vesuvius would have forecast the 1631 eruption satisfactorily, marking different stages of the pre-eruptive phase; 3) the code shows that pre-eruptive signals that significantly increase the probability of eruption were likely detected more than two months before the event.

Keywords: cold historical data, Vesuvius, 1631 eruption, BET_EF code, eruption forecasting

1. INTRODUCTION

Vesuvius is one of the highest risk volcanoes. Besides being located in a densely populated area, with circa one million of people living on its flanks and in the surrounding area, Vesuvius experienced several large explosive eruptions in the past that re-opened the conduit, ending so-called “close conduit” phases (e.g., Marzocchi and Zaccarelli 2006) such as the current phase of the volcano. Over the last decades, this large threat for society pushed scientists and the Italian Civil Protection to devote a significant effort in order to mitigate volcanic risk in this area. One of the most relevant strategies adopted is the development of an Emergency Plan that is periodically revised and that includes, prior to an eruption, a massive evacuation of the area that is likely to be affected by pyroclastic flows, lahars, and heavy ash falls (the so-called Red Zone). Thus, a key scientific ingredient for an evacuation to be effective and successful is a reliable forecast of the time evolution of the reactivation of Vesuvius.

Tracking quantitatively the evolution of the pre-eruptive phase of long-time dormant explosive volcanoes is the main purpose of a recent quantitative tool, named BET_EF (Bayesian Event Tree for Eruption Forecasting), developed by Marzocchi et al. (2004; 2008). The method makes probabilistic eruption forecasting, accounting for volcanological models, prior beliefs, past data, and monitoring measurements. Besides a general description of the probabilistic model that is potentially applicable to any explosive volcano, the papers also contain a set of monitoring parameters and relative thresholds, in order to apply the technique to pre-eruptive phases of Vesuvius.

Although the method is scientific, since it provides probabilities that can be used to test the model with independent data, in practice its verification is hampered by the lack of quantitative pre-eruptive data, for Vesuvius as well as for almost all volcanoes of this type. In this respect, we argue that historical documents can partially fill this void, providing useful information about pre-eruptive phenomena. A remarkable example is given by historical reports of the pre-eruptive phase of the 1631 eruption (Guidoboni 2008). These new data give us an unusual opportunity to perform a retrospective forward test of the BET_EF model implemented for Vesuvius. The “forward” nature of the test is guaranteed by the fact that we use the same parameters published in previous papers, keeping “frozen” all the quantitative rules/parameters/thresholds. The test consists of three basic steps: at first,

1. We translate, where possible, the historical information into reasonable values of some of the monitoring data required by the BET_EF model for eruption forecasting. In this step we keep separate, as clearly as possible, the “historical data” (in their so-called "cold" data form, i.e., not yet interpreted, see Guidoboni 2008) from our interpretation.
2. We run the BET_EF code to estimate the time evolution of the probability of eruption.

3. We perform stability checks of the results, acknowledging that our quantitative interpretation of historical reports is subjective and cannot be univocal. In practice, we evaluate the stability of the results when different interpretations of historical reports are applied.

The final goal of the paper is two-fold: beside testing the ability of BET_EF to forecast a large explosive eruption in a long-time quiescent volcano, we also highlight the importance of data coming from historical documents, as they can partly replace the lack of quantitative pre-eruptive data for large explosive eruptions.

2. THE 1631 VESUVIUS ERUPTION, AND A DETAILED HISTORICAL CHRONOLOGY OF THE PRE-ERUPTIVE PHASE

During its eruptive history, Vesuvius has produced several large explosive eruptions that devastated the surrounding area. It last erupted in 1944 with a VEI = 3 weakly explosive event. The most important previous eruptions were VEI = 4 events in 472 A.D. and 1631 (see, e.g., Scandone et al. 1993), and the well-known Pompeii eruption in 79 AD. Presumably, all of these large explosive eruptions brought to an end a closed conduit phase, intended, as in Marzocchi and Zaccarelli (2006), as a dormancy of the volcano with no activity, lasting at least few decades (e.g., Guidoboni and Boschi 2006) or centuries (e.g., Cioni et al. 2003) and allowing the closure of the volcanic conduit due to viscous relaxation and cooling of rocks within it (Quareni and Mulargia 1993).

After the reactivation of Vesuvius in 1631, volcanic activity switched to persistent activity typical of open conduit behavior, showing frequent eruptions of smaller size and intensity for decades to centuries. Persistent activity prevailed through the early 20th century until 1944. Since then, however, an unusually long repose (60 years and continuing) suggests this volcano may now be in a closed conduit condition (e.g., Marzocchi and Zaccarelli 2006).

The 1631 event had a particular importance because it was considered, in a previous version of the Emergency Plan (Presidenza del Consiglio dei Ministri 1995), as a reference scenario for the definition of the "Maximum Expected Event" (e.g., Barberi et al. 1995; Barberi et al. 1990; Esposti Ongaro et al. 2002; Cioni et al. 2003). The revised Emergency Plan has modified this view, recognizing that an eruption similar to the one occurred in 1631 is not actually the maximum expected event, and not even the most likely (Marzocchi et al. 2004), but it could represent a reasonable reference scenario that balances cost/benefit for practical actions. Nevertheless, the 1631 event is also remarkable because, being the most recent explosive eruption, it has also some noticeable historical reports about the pre-eruptive phase. While Rosi et al. (1993) let us learn what happened during the eruptive phases of 1631 event, recent works by Guidoboni (2004; 2008) made available a large amount of new information on what happened weeks and months before the eruption (see comments in Bertagnini et al. 2006).

2.1. The method and the materials studied: the role of "cold" historical data

One of the main problems in the use of historical data to learn about the phenomena of the past, like the activity of volcanoes, is the continuous and uncontrollable interpretative intervention effected by the volcanologists directly upon the sources. The descriptions of the Vesuvius activity (and of other volcanoes of the Italian area) in the treatises written by contemporaries are especially rich in detail, inserted within a lengthy chronological development. Most of the phenomena described in those treatises can thus be pinpointed in time and in geographical space and be elaborated as a fully-fledged sequence of “cold” data, that is, defined before the scientific interpretation. This way of elaborating the historical data is a novelty within the volcanological field, because it allows the relationship between source and interpretation to be made transparent, a relationship that hitherto in the literature had always been presented as a single phase and a single process. The separation of the two levels, i.e. historical and volcanological, which instead has been effected here, obviously does not come without its surprises as well as some problems, because it lays bare some unresolved aspects – and also at times ones that are hard to resolve – possible contradictions between the examined texts, or neglected elements that are not secondary. As compared with the knowledge already gained in the literature, it appears to be a more realistic set of data, but also in some ways more problematic, nonetheless meaningful, for anyone who has to interpret such data within the volcanological sphere.

In our opinion, no other paths can be followed for the critical use of a wealth of such particular historical data. The results presented here constitute a new way of using historical data in volcanology, which have been applied for the first time within the *Exploris* project (Guidoboni 2008).

2.2.The description of the 1631 eruption in five contemporaries treatises

On 16th December 1631 a violent eruption of Vesuvius started after a long period of silence that had led people to forget the danger of this volcano. In order of importance, that of 1631 is the third eruption occurring in the historical era, after those of 79 AD and 472 AD. This eruption in its most acute phase lasted several days, causing over a thousand deaths and substantial economic damage. The phenomenon ended completely only a few years later, but already in January 1632 a number of Neapolitan intellectuals were engaged in writing reports and treatises to recount, interpret, and explain that extraordinary reawakening of Vesuvius.

Between 1632 and 1634 numerous pamphlets, notices, letters and reports, along with some treatises written in Italian, Latin and Spanish, were published in Naples. These writings represent a heterogeneous set of materials, as a whole invaluable for becoming acquainted with that scenario. The four treatises (see also Appendix A) presented here have been analysed in different steps: the texts of Carafa (1632), Mascolo (1632) and Varrone (1634), in Latin, have been translated and analysed within the scope of the EXPLORIS project (Guidoboni 2004); the text of Braccini (1634) is in Italian, analysed within the Vulcan-3 project - RU 1-Guidoboni (2005-2007) agreement with the Dipartimento della Protezione

Civile and INGV. The text by Giovanni Domenico de Arminio (1632), which is a treatise in Latin up to now unknown to volcanological literature, has been translated and analysed for this work.

The list of chronologically ordered phenomena reported in the five treatises is given in table 1 (columns 1 to 5).

One of the most interesting aspects and so far scarcely used of such texts is their reconstruction of the chronology of the described phenomena. The treatises analysed are situated within production of the witnesses to the 1631 eruption. The authors, intellectuals and ecclesiastics of the day, were committed to analysing and explaining everything that they had previously observed before, during and after the eruption: obviously this was done within the cognitive frames of their times. The liveliness and immediacy of their descriptions accompany a literary scholarship, which was typical of the ecclesiastics and the men of law of that day and age.

The description of the eruption is preceded, in all five treatises, by scores of pages dedicated to philological, etymological and historical disquisitions, which are an example of how the culture of the day dealt with the great natural events.

The description of the events starts from the summer of 1631, that is, several months before the eruption of Vesuvius. Hence, the treatises give us the chance to observe, through the eyes of an intellectual of the day, a cinematic narrative of the events running up to the great eruption on 16th December 1631.

3. BET_EF APPLICATION TO THE 1631 ERUPTION

3.1 BET_EF rules for Vesuvius

The BET_EF software package implements the Bayesian Event Tree for Eruption Forecasting model published by Marzocchi et al. (2008). The model is based on an event tree in which individual branches are alternative steps from a general prior event evolving into increasingly specific subsequent events. By merging volcanological information, past data and monitoring measurements through a Bayesian inferential method, BET_EF computes the long- and short-term probability at each node of the event tree; particularly interesting from a volcanological point of view are the probabilities of volcanic unrest, magmatic unrest, eruption, vent location and eruption size. BET_EF also provides estimates of aleatory and epistemic uncertainties on such probabilities.

The upload of the available information regarding Vesuvius represents the main step in order to run BET_EF for this application. Here, we keep the same rules used by Marzocchi et al. (2008) during MESIMEX (Major Emergency SIMulation EXercise) experiment, in which the re-awakening of Vesuvius was simulated in order to test Civil Protection and Scientific Institution preparedness in case of such an event. On that occasion, BET_EF was run real-time, fed with the simulated monitoring data provided by a scientific pool of experts (Marzocchi et al. 2008).

In Appendix B, we describe these rules in detail again.

3.2 Strategy adopted for the volcanological interpretation of historical information

In order to verify the eruption forecasting ability of the BET_EF code on independent data, we apply it to the 1631 pre-eruptive phase, according to the historical accounts provided by Guidoboni (2004) and Guidoboni (2008).

We interpret the historical information in terms of some of the parameters routinely monitored at Vesuvius that are a direct input to the BET_EF code.

We stress that, in the process of translation of historical information into volcanological input to BET_EF, we have to interpret “anomalous” events reported in the chronicles with respect to a background activity of the volcano. In doing so, we necessarily refer to our volcanological experience gained from Vesuvius and to its present background state. Thus, the values suggested in the last column of Table 1 represent our subjective perception of the anomaly reported in the historical accounts with respect to the present-day background values. Actually, we do not know if the present- day state of the volcano is similar to the one before 1631 eruption. The results we obtain might be indeed biased by this assumption.

However, since the BET_EF code deals with monitoring measurements in a fuzzy approach (see Appendix B and Marzocchi et al. 2008), the results yielded by BET_EF do not change if we input a “wrong” value for a monitoring measure, provided that our value maintains a “degree of anomaly” similar to the one of the reality. For example, if the real occurred monthly largest magnitude is 2.1, but we give an interpretation of 3.0, the results will be exactly the same, since (see Table B2 in Appendix B) these values are both below the lowest threshold, thus their degree of anomaly is the same (in particular, they are indicative of NO anomaly). Similar considerations can be applied for any monitored variable that we consider.

Since historical chronicles refer to ground uplift and not to the corresponding strain, for the deformation parameters (cumulative strain ε and strain rate $d\varepsilon/dt$) we interpret the historical reports in terms of observed ground uplift (u), and translate the uplift in terms of ε (i.e., $\varepsilon=U/\Delta h$, where U is the cumulative total uplift since the beginning of the unrest, and Δh is the supposed thickness of the deformed layer) and $d\varepsilon/dt$ (i.e., $d\varepsilon/dt=u/(\Delta h\ \Delta t)$, where Δt is the time interval during which the uplift is observed, derived directly from the chronicles). Bertagnini et al. (2006) set the depth of the source of deformation (by a simple Mogi model) to 4Km, implying a deformed layer 4Km thick. Here we assume Δh=5Km, a bit more conservative choice (because, given a ground uplift, it implies lower cumulative strain and lower strain rate).

On purpose, we decide to translate into numerical values only the historical information undoubtedly linked to a specific monitoring parameter. For example, anomalous wild animal behavior (like fleeing from the Vesuvian area, as reported) might have been related to seismic activity, to rockfalls in the crater area, to anomalous gas emission, etc.; in few words, there is no specific and univocal phenomenon that caused the flee of wild animals from their usual territory. Therefore, we do not consider this information in BET_EF. Together with the fact that the historical information collected likely represents a subset of all the phenomena occurred in 1631 pre-eruptive phase, this choice implies that the results we will obtain are a lower limit on the eruption forecasting ability of BET_EF on independent data.

3.3 Volcanological interpretation of historical information

In Table 1 we provide our interpretation of the historical information in terms of some of the parameters routinely monitored at Vesuvius that are a direct input to the BET_EF code. In particular, in the two right-most columns of Table 1, we show our "verbal" interpretation of the information given on the rest of the table row (second last column), and its translation into the corresponding specific parameters and values of the monitoring data required as BET_EF input (last column).

In the following, we will give detailed explanation for the interpretations adopted, in order of appearance in time (see also Table 1). Since Bertagnini et al. (2006) and Rosi et al. (1993) already interpreted some of the historical chronicles used in the present study, we will explicitly compare our interpretation to theirs.

- "Smoke emissions" reported by C (see Table 1) on the beginning of September. We interpret this as a moderate increase (4 times the present average flux) in CO_2 flux. In doing so, we postulate that fumaroles composition remains unchanged (similar to the present-day one), but the flux becomes larger, and fumaroles become visible.

- "Elevation of the ground" reported by B between 16th November and 1st December. We interpret this as naked-eye detectable vertical ground uplift, i.e., around 10 cm, occurred during the period of observation (approx Δt=two weeks). Since BET_EF requires strain and strain rate, we transform this into $\varepsilon=2\times10^{-5}$ and $d\varepsilon/dt=1.3\times10^{-6}\ d^{-1}$ as illustrated in the previous section.

- "Moderate seismic activity in the coastal region of Vesuvius" reported by V between 19th and 20th November. Even if Bertagnini et al. (2006) suggest that this piece of Varrone's chronicle is a "convoluted sentence" and that its interpretation might be doubtful, this is not our opinion, because the Varrone's chronicle is not limited to the sentence reported by Bertagnini et al (2006). Indeed, the whole paragraph is undoubtably related to seismic activity felt by people on the coastal region of Vesuvius, thus we interpret this as an increase in the rate of earthquakes. At node 1, one of BET_EF's seismic input is the monthly number of earthquakes with magnitude larger or equal to 1.9 recorded at OVO station, located on the volcano. We imagine that, with the term "moderate seismicity", people might have felt a few earthquakes (let's say 5) of magnitude around 3; considering a power law between the energy of earthquakes and their frequency, i.e., the Gutenberg-Richter (GR) relationship, we have $Log(N_1)=a-bM_1$, where $N_1=5$ (earthquakes with magnitude larger or equal to 3), $M_1=3$ (magnitude), and $b=1$. In order to compute N_2, i.e., the number of earthquakes with magnitude larger or equal to 1.9, we consider again the GR law, i.e., $Log\ N_2=a-bM_2$, where $M_2=1.9$. By differentiating these two expressions of the GR law, we obtain $Log(N_2/N_1)=b(M_1-M_2)$ from which $N_2=N_1x10\char`^[b(M_1-M_2)]\approx50$. (Note that for this reason, from now on, every time the chronicles report "moderate seismic activity", we add other 50 earthquakes to the monthly count.) For the maximum magnitude of this earthquake burst, again on a monthly basis at OVO station, we set 3.0, below the lower threshold of anomaly. We keep it low because seismicity is felt in Vesuvian area only.

- “Emissions with effects on the herbaceous vegetation (wilting)” reported by V “just before December”. Similarly to Bertagnini et al. (2006), and to what observed during the last unrest episode at Long Valley Caldera (see e.g. Hill 1996), we interpret this as a further increase in CO_2 flux, up to 20 kg/m^2d^{-1}.
- “Major uplifting of the ground” occurring in 5 days and reported by V in the beginning of December. We interpret this as a ground uplift significantly larger than the one reported previously, thus we set a further uplift of 0.5 m in a time window $\Delta t=5$ days, and translate this into $\varepsilon=1.2\times10^{-4}$ and $d\varepsilon/dt=2x10^{-5}\ d^{-1}$. Due to cumulative strain, which becomes now larger than the upper anomaly threshold, the eruption probability takes a large jump.
- “Moderate seismic activity in the Vesuvius area” reported by V between 7th and 8th December. As mentioned above, we keep the same maximum magnitude but add other 50 earthquakes to the monthly count.
- “Small uplifting of the ground” reported by V between 9th and 15th December. We interpret this a further uplift of 0.1 m (yielding a total cumulated strain $\varepsilon=1.4\times10^{-4}$), occurring in a time window $\Delta t=7$ days, thus lowering the strain rate to $d\varepsilon/dt=2.9x10^{-6}\ d^{-1}$.
- “Emission of hot vapours (Vesuvian area)” reported by V in the same period. Similarly to Bertagnini et al. (2006), we interpret it as an increase of temperature of the fumaroles (T=110C, representing 15% of the present usual value).

- “Moderate seismic activity (Vesuvian area)” reported by B on 10th December. As above, we keep the same maximum magnitude, but we add other 50 earthquakes to the monthly count.
- “Repeated seismic activity felt at Naples and in the Vesuvius area, for a range of about 7.5Km” reported by C, V and M about 8 hours before the onset of the eruption. We interpret this as a major jump in the monthly count of earthquakes (further 150 events), and, more important, an acceleration of the seismic energy released ($d^2E/dt^2=1$), acknowledged also by Bertagnini et al. (2006); we also set a larger maximum magnitude (up to 4.0) motivated by the larger area in which earthquakes are felt.
- “Seismic activity, opening of faults” reported by V about 1 hour before eruption onset. We interpret this as a further jump in the monthly count of earthquakes (further 200 events), and persisting acceleration of the seismic energy released ($d^2E/dt^2=1$). Furthermore, we also interpret the opening of faults as strain acceleration ($d^2\varepsilon/dt^2=1$). This is a very important parameter because, having a double weight, it implies a large jump in eruption probability.
- “Opening of the mount in the Atria” reported by B about 1 hour before eruption onset. We interpret this as a major ground deformation (which possibly could be the same fact reported by V and just discussed above), in which cumulative strain exceeds rock strength; we set a further ground displacement of 1 m in a very short time window ($\Delta t=6$ hours), giving $\varepsilon=3.4\times10^{-4}$ and $d\varepsilon/dt=8\times10^{-4}\ d^{-1}$

- “Intense smoke emission” reported by V about 1 hour before eruption onset. We interpret this as a further strong increase in CO_2 flux (Π_{CO2}=100 m^{-2} d^{-1}).
- “Rock expulsion (from central crater)” reported by V about 1 hour before eruption onset. We interpret this as occurrence of phreatic explosions (PE=1). This parameter also concurs in causing a large jump in eruption probability.

3.4 BET_EF run (the “reference case”)

At first, we divide the period of time considered into six different sub-periods, corresponding to (interpreted) significant changes in the monitoring parameters. For each sub-period, we run a BET_EF simulation in order to compute the various probability distributions of interest (valid in the sub-period of the simulation). The results are displayed in Table 2. For the sake of conciseness, for each simulation we show the average absolute probability of unrest and of eruption, the relative 10-, 50- and 90-th percentiles of their distributions, and the same quantities for the conditional probabilities of magmatic unrest given unrest and of eruption given magmatic unrest. We define this set of simulations the “reference case”.

The most striking features of BET_EF results for the reference case are:

- ✓ immediate jump in the probability of unrest already from sub-period 1 (the long-term average for Vesuvius is in the order of 10^{-3} (see Marzocchi et al. 2008) while here in sub-period 1 the average jumps to about 10%), i.e., months before the eruption; however, the conditional probability of magma given unrest and of eruption given magmatic unrest are still around 50% because unrest is not yet completely clear;

- ✓ absolute probability of eruption has a constantly increasing trend; in particular this probability is low (around 10%) with a large uncertainty in sub-period 1, while it becomes quite high (more than 30%) about 7-10 days before the eruption (i.e., from sub-period 4); note that these probability values could justify the call for an evacuation on the basis of a rationale cost/benefit analysis (Marzocchi and Woo 2007);
- ✓ aleatory uncertainty on the probability of eruption is drastically reduced only few hours before the eruption (sub-period 6). This is a quite common feeling among present-day volcanologists who have dealt with large explosive eruptions (for example, the eruption of Mt Pinatubo in 1991, see e.g. Cornelius and Voight 1996).

3.5 Control experiments

Since the interpretation of historical information is not univocal, we run some control experiments. The main idea is to assume that the monitoring parameters identified in Table 1 for each sub-period from the historical accounts are exact, but their specific values might change inside reasonable ranges, because our interpretation is intrinsically subjective. To account for this, we random vary the values of the monitoring parameters inside these ranges for 1000 times, by random sampling (1000 times) from uniform distributions. Then, we run 1000 control simulations for each sub-period. In doing so, we obviously must respect the temporal trend of the parameters through the pre-eruptive phase (for example, it is clear from the historical accounts that the monthly number of seismic events had been continuously increasing throughout the whole period). Furthermore, we do not vary the values of the yes/no parameters, i.e., the parameters related to the presence of phreatic explosions and to accelerations in seismic energy release and deformation, since they appear to be certain from the historical chronicles.

The monitoring parameters whose values are varied in the control experiments are n_e, M_d, Φ_{CO2}, $d\varepsilon/dt$, T and ε. For all of them, we start from the present-day background value from actual monitoring of Vesuvius (listed in Table 3), and then we set specific ranges in which the increase in their values can be uniformly random sampled, according to what reported in Table 1 from the historical chronicles. Except for T, whose value changes only once during the pre-eruptive crises, we define a range for the increase in the parameter for each time it is reported to have changed. In particular (see also Table 3):

- T varies only in sub-period 4; we identify a reasonable increase in T in the order of 5 to 25 degrees Celsius with respect to the present temperature of the fumaroles, corresponding to a 5 to 30% increase.
- M_d varies twice, i.e. once in sub-periods 2 and once in 5. We identify a reasonable increase in M_d in sub-period 2 in the order of 0 to 1.5 units of magnitudes with respect to the present monthly maximum magnitude of earthquakes. For sub-period 5, since chronicles report "seismic activity felt

in Naples", we define a further increase in the range of 0 to 1.5 units of magnitude.

- n_e varies in sub-periods 2 (once), 4 (twice), 5 (once) and 6 (once). We identify a reasonable increase in n_e in sub-period 2 in the order of 0 to 100 events, corresponding to a 0 to 10 times increase of the present rate. Equal ranges are assumed for the two further increases in sub-period 4. For sub-periods 5 and 6 we assume a more important increase (from 50 to 300 events per month more) due to the specification of "repeated seismic activity" in the accounts.
- Φ_{CO2} varies (once) in sub-periods 1, 3 and 6. For sub-periods 1 we define a reasonable range of increase between 0 and 30 kg m^{-2} d^{-1}, and an equal range for the further increase in sub-period 3 is assumed. For the last sub-period we identify a larger increase (20 to 300 kg m^{-2} d^{-1}, representing about 10 and 100 times the present background emission rate) due to the specification of "strong gas emission" in the historical report.
- For the deformation parameters (ε and $d\varepsilon/dt$), we define ranges for the increase in the observed ground uplift, and translate the uplift in terms of ε and $d\varepsilon/dt$ as in section 4.1. In this view, ground uplift varies (once) in sub-periods 2, 3, 4 and 6. In sub-period 2, the account simply reports "elevation of the ground" in 15 days. We assume that this might correspond to an uplift of 5 cm (about the minimum observable without instruments) to 50 cm. In terms of $d\varepsilon/dt$ this range corresponds to $6.7\text{x}10^{-7}$ d^{-1} to $6.7\text{x}10^{-6}$ d^{-1}, while for ε this corresponds to 10^{-5} to 10^{-4}. In sub-period 3, we define a larger further uplift ("major ground uplifting" in 5 days)

between 20 cm and 1 m. In terms of dε/dt this range corresponds to 8x10^{-6} d^{-1} to 4x10^{-5} d^{-1}, while for ε this corresponds to a further increase of 5x10^{-5} to 3x10^{-4}. In sub-period 4, we define a small uplift ("small uplifting of the ground" in 7 days) between 5 and 20 cm. In terms of dε/dt this range corresponds to 1.4x10^{-6} d^{-1} to 5.7x10^{-6} d^{-1}, while for ε this corresponds to a further increase of 6x10^{-5} to 3.4x10^{-4}. In sub-period 6, the deformation must be large and rapid ("opening of mount" in few hours, here we take Δt=6 hours), between 50 cm and 3 m. In terms of dε/dt this range corresponds to 4x10^{-4} d^{-1} to 2.4x10^{-3} d^{-1}, while for ε this corresponds to 1.6x10^{-4} to 9.4x10^{-4}.

In this way, we perform 1000 control experiments; in practice, we build up 1000 different pre-eruptive phases, with the same anomalous parameters, but assuming different values for them. This is a way of controlling the dependence of BET_EF forecasting results on the thresholds fixed for Vesuvius and on the errors in input monitoring measurements. In particular, we want to check the stability of our best guess result (the average of the probability distribution) with respect to reasonable errors on our subjective interpretation. Since the best guess value is largely related to aleatory uncertainty, i.e., a measure of the intrinsic unpredictability of the system, with the control experiment we want to check how stable is the system predictability in relation to errors in our interpretation. Because of this, we will concentrate on the statistics of the control experiments' average, rather than on the statistics of the dispersion.

In figure 1 we show a plot of the dispersion in the estimate of the average probability of unrest versus time approaching the eruption among the 1000 control experiments. For comparison, we also show the results from the reference case. It is important to note that the increases of the monitoring parameters in the reference case are all contained in the ranges identified above and given in Table 3.

From figure 1, we see that the BET_EF model immediately takes a jump in the probability of unrest some months before the eruption, and it recognizes undoubtedly an unrest phase about a month before the beginning of the eruption.

In figures 2, 3 and 4 we show the same as in figure 1, except that we show respectively the average absolute probability of eruption, the average conditional probability of magma given unrest, and the average conditional probability of eruption given magmatic unrest. These figures show that, as time approaches the eruption, an escalating trend towards an eruptive characterization of the crisis is evident (figure 2). The conditional probability of magma given unrest (figure 3) is very high (around 80%) already about 10 days before the eruption onset, while the conditional probability of eruption given magma (figure 4) is stable around 30-50%, except for a large step up to 90% in the few hours preceding the onset, implying a substantial reduction of the aleatory uncertainty on the eruption absolute probability (up to 70-80%). This is actually a common experience of present-day volcanologists who have witnessed a large explosive eruption.

4. DISCUSSION AND CONCLUSIONS

We have applied the BET_EF code (Marzocchi et al. 2004; Marzocchi et al. 2008) to characterize the time evolution of the 1631 pre-eruptive phase at Vesuvius, by using new and independent (i.e., they were not used to set up the model) “cold” historical data. This application highlights four major points.

- Historical researches to study and to model pre-eruptive phases, overall for explosive volcanoes that do not have recent and monitored volcanic eruption, as for Vesuvius, are of prominent importance, as suggested also by Bertagnini et al. (2006).
- BET_EF code (Marzocchi et al. 2004; Marzocchi et al. 2008) applied to the 1631 pre-eruptive phase is able to track the time evolution leading to the eruption, marking steps in probability of eruption as time approaches the eruption onset.
- The 1631 pre-eruptive phase shows signals that were able to increase the absolute probability of eruption up to 10% about a month before the beginning of the eruption; this probability reaches more than 30% 7-10 days before the onset of the event. In this respect, we do not agree with Bertagnini et al. (2006) when they conclude that “the anomalous phenomena reported before the end of November appear to be of doubtful significance and reliability”. Rather, we think that even if the chronicles

report a filtered subset of occurred medium-term precursors, they are sufficient for estimating a 10% probability of eruption, a month in advance. We argue that also figures like these are worth being considered. In fact, Marzocchi and Woo (2007) showed that the call for an evacuation based on cost/benefit analysis is usually much lower than the higher probabilities usually adopted by volcanologists, for instance during the MESIMEX experiment.

- The BET_EF aleatory uncertainty on eruption probability for 1631 event resembles the experience of present-day volcanologists who have witnessed large explosive eruption, i.e., the aleatory uncertainty on the eruption occurrence is low only hours before its onset.

Finally, we want to remark that the translation of historical information into monitoring parameters always involves "subjective" choices. Here, we have deeply explored the stability of the results ("control experiments") as a function of the assumptions made, and, in any case, we have usually chosen the most conservative options in order to not optimize the results. It is also worth remarking that these results could represent a lower bound of the BET_EF forecasting capability, because we use only a subset of the real pre-eruptive signals, i.e., we use monitoring parameters that produced signals felt by human beings and not only by instruments. We think that the inclusion of the latter type of signals can improve the results reported here.

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TABLE 1. Pre-eruptive phase for the 1631 eruption: from the summer to December 16th 4-5am, GMT

Note that the thick black lines represent different time sub-periods related to different applications of the BET_EF code (see section 4). The question mark “?” means “missing or not defined data”, while “ca” means “circa”. Legend:
Leftmost column indicates the time sub-period
Date/chronological range = month day (and, when defined, hour in italics) of the beginning of the phenomenon described, and, when available, its end. Time is given in GMT
Duration= duration of the phenomenon (in the units specified)
Time to eruption = time in months, days or hours (as specified) elapsing between the start of the phenomenon described and the onset of the eruption
Source: A=de Arminio (1632); B=Braccini (1634); C=Carafa (1634); M=Mascolo (1634); V=Varrone (1632)

	Date/chronological range	*Duration*	*Time to eruption*	*Source*	*Summary of the description*	*Volcanological interpretation*	*BET_EF parameters (for the meaning of the symbols, see Section 3)*
1	Summer/August?	1month	ca 120days	V	Fires visible at night at Herculaneum (*Resina* in the texts) eastern slopes of the volcano		
	Aug	?	ca 120days	A	Smoke emissions from the crater	Increase in CO_2 flux (about 4 times the average)	Π_{CO2}=10 kg m^{-2} d^{-1}
	Sep 1 - Sep 10	11days	107days	C	Lowering of the ground; landslides; ground faulting; smoke emissions; fiery emissions (north-eastern slope)		
	Sep 1 – Dec 16 *05am*	3.5months	ca 107days	V	Considerable anomalous restlessness among the domestic animals; flight of wild animals from the Vesuvius area		
	Oct 1 - Nov 30	2months	77days	V	Underground noises (from Portici)		
2	ca Nov 16 – Dec 1	0.5month	30days	B	Elevation of the ground	Detection of positive strain (about 0.1m of vertical uplift over Δh=5km, i.e., the supposed thickness of the deformed layer); positive strain rate (0.1m of uplift in 0.5mo)	ε=2x10^{-5} dε/dt=1.3x10^{-6} d^{-1}
	Nov 19 *10pm* - Nov 20 *07am*	9hours	27days	V	Rough seas; moderate seismic activity in the coastal region of Vesuvius	About 50 earthquakes M1.9+ recorded at OVO; maximum magnitude about 3.0	n_e=50 M_d=3.0
3	“Just before December”	?	ca 18days	V	Emissions, with effects on the herbaceous vegetation [wilting] (Vesuvius area)	Stronger increase in CO_2 flux	Π_{CO2}=20 kg m^{-2} d^{-1}
	Beginning of December	5days	ca 16days	AV	Loud noises in the Vesuvius area		
	Nov 28 - Dec 02	5days	14days	V	Darkening and variation in the water chemistry [salinity] (Vesuvius area); collapse of a great overhanging rock (eastern slope); major uplifting of the ground - (slopes, particularly western)	Increase in cumulative strain (further 0.5m of vertical uplift); higher strain rate, about 0.5m in 5d	ε=1.2x10^{-4} dε/dt=2x10^{-5} d^{-1}

4	Dec 07 *05pm* - Dec 08 *05pm*	2days	10days	VCM	Moderate seismic activity in the Vesuvius area V; underground noises: in the Vesuvius area M V, and at Herculaneum for some days C	Other 50 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude	n_e=100 M_d=3.0
	Dec 08 *05pm* - Dec 09 *05pm*	1days	9days	V	Underground noises in the western and south-western slopes of Vesuvius		
	Dec 08 *05pm* - Dec 15	8days	9days	VCM	Underground noises Vesuvius area; underground noises (Herculaneum and Vesuvius area)		
	8 days before	?	8days	V	Underground noises and thunder in the Vesuvius area		
	Dec 09 - Dec 15	7days	8days	V	Restlessness among the domestic animals in the Vesuvius area; Small uplifting of the ground; emissions of hot vapours (Vesuvius area)	Increase in cumulative strain (further 0.1m of vertical uplift); lower strain rate, about 0.1m in 7d; high temperature of the fumaroles	$\varepsilon=1.4x10^{-4}$ $d\varepsilon/dt=2.9x10^{-6}\ d^{-1}$ T=110 C
	Dec 10	?	6days	B	Underground noises and thunder in the Vesuvius area		
	Dec 10	?	6days	B	Moderate seismic activity (Vesuvius area)	Other 50 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude	n_e=150 M_d=3.0
	Dec 10	?	6 days	B	Darkening of the waters of the wells (Vesuvius area)		
	Dec 10	6days	7days	V	Tranquillity of the air (Vesuvius area); calm sea (sea facing the Vesuvius area)		
	Dec 11 *05pm* - Dec 13 *05pm*	3days	6days	V	Moderate seismic activity (Vesuvius area)	Same considerations as above, probably related to the same period of time	
	Dec 13 – Dec 16	?	3days	C	Darkening of the waters of the wells (Vesuvius area)		
	Dec 13 – Dec 15 *08pm*	2days	5days	C	Tranquillity of the air in the area of Vesuvius and in Naples		
	Dec 14 *05pm* - Dec 15 *09pm*	28hours	35hours	V	Tranquillity of the air and restlessness of the animals in the Vesuvius area, calm sea in the coastal area of Vesuvius		
	Dec 15 morning	?	1day	B	flashing arcs upon Vesuvius		
5	Dec 15 *08pm* - Dec 16 *01am*	5hours	8hours	V	Calm sea (coastal area)		
	Dec 15 *08pm* - Dec 16 *04am*	8hours	8hours	CVM	Repeated seismic activity felt at Naples and in the Vesuvius area, for a range of ca. 7,5 km	Other 150 earthquakes M1.9+ recorded at OVO; maximum magnitude increases because the earthquakes are felt in Naples; acceleration in seismic energy release	n_e=300 M_d=4.0 $d^2E/dt^2=1$
	Dec 15 *09pm* - Dec 16 *04am*	7 hours	7hours	B	Repeated seismic activity felt at Naples and in the Vesuvius area	Same considerations as above, related to the same period of time	
	Dec 15 *08pm* - Dec 16 *05am*	8hours	8hours	V	Restlessness among the domestic animals (Vesuvius area, from Portici); moderate and repeated seismic activity (30 shocks); underground noises – Vesuvius area	Same considerations as above, related to the same period of time	

	Dec 15 *08pm* - Dec 16 *07am*	2days	8hours	M	Intense and repeated seismic activity in the area of Vesuvius and Naples	Same considerations as above, related to the same period of time	
	Dec 15 night - Dec 16	ca 14hours	?	A	Flames and lights from the crater visible at night		
	Dec 15 *09pm*	7hours	7hours	V	Underground noises and seismic activity – Vesuvius area		
6	Dec 16 *04am*	?	ca 1hour	C	Seismic activity; opening of faults and landslides; landslides between the Atria and the summit of the Veolo, just above the path that goes from Atria and circles the Veolo, in the middle of the mountain's slopes, closer to the one facing Atria	Other 200 earthquakes M1.9+ recorded at OVO; no significant differences in magnitude; acceleration in seismic energy release; acceleration in strain	$n_e=500$ $M_d=4.5$ $d^2E/dt^2=1$ $d^2\varepsilon/dt^2=1$
	Dec 16 *after 04am*	?	ca 1hour	B	Opening of the mount in the Atria	Cumulative strain exceeds rock strength; strain rate must be very high (at least 1m of ground displacement in less than 1d)	$\varepsilon=3.4\times10^{-4}$ $d\varepsilon/dt=8\times10^{-4}\ d^{-1}$
	Dec 16 *04am* - Dec 16 *05am*	ca 1hour	ca 1hour	V	Very loud underground noises in the Vesuvius area, ebbing of the sea in the Gulf of Naples, between Pozzuoli and the coast facing the mouth of the River Sarno; earthquakes, landslides, ground faulting; loud underground; noises in the Atria; intense smoke emissions; fiery emissions; rock expulsion (from the central crater, on the eastern slope, seawards)	Same considerations as above, related to the same period of time. Furthermore: stronger increase in CO_2 flux; phreatic explosions	$\Pi_{CO2}=100$ kg m^{-2} d^{-1} PE=1
	Dec 16 *after 04am*	ca 1hour	ca 1hour	B	Opening secondary bocca	Same considerations as above, related to the same period of time	
	Dec 16 *04am* - Dec 16 *05am*	ca 1hour	ca 1hour	VCM B	Fiery emissions; thunder, underground noises and lightening, rock expulsion, ash and soot, emission of a cloud (Vesuvius area), violent seismic activity	Same considerations as above, related to the same period of time	
	Dec 16 *05am*	**ERUPTION**		VB	Smoke emissions; loud underground noises; ground faulting; opening of a large chasm; cloud emission reaching about 37 km in height; haze (from the fracture appearing in the Atria)		

TABLE 2. Results of the simulations over the 6 different time sub-periods for the reference case

Time Sub-period and relative "anomalous monitoring"	Absolute Probability of Unrest				Absolute Probability of Eruption				Conditional Probability of Magma (given Unrest)				Conditional Probability of Eruption (given Magmatic Unrest)			
	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc	Average	10 perc	50 perc	90 perc
1 (Aug to Nov 15) Π_{CO2}=10 kg m^{-2} d^{-1}	0.10	0.10	0.10	0.10	0.02	0.002	0.02	0.05	0.48	0.12	0.47	0.84	0.48	0.12	0.47	0.84
2 (Nov 16 to Nov 27) Π_{CO2}=10 kg m^{-2} d^{-1} ε=2x10^{-5} dε/dt=1.3x10^{-6} d^{-1} n_e=50 M_d=3.0	1	1	1	1	0.08	0	0.002	0.29	0.28	0	0.12	0.86	0.28	0	0.09	0.86
3 (Nov 28 to Dec 02) Π_{CO2}=20 kg m^{-2} d^{-1} ε=1.2x10^{-4} dε/dt=2x10^{-5} d^{-1} n_e=50 M_d=3.0	1	1	1	1	0.13	0	0.02	0.50	0.43	0.004	0.37	0,96	0.34	0	0.22	0,92
4 (Dec 03 to Dec 15 *morning*) Π_{CO2}=20 kg m^{-2} d^{-1} T=110 C ε=1.4x10^{-4} dε/dt=2.9x10^{-6} d^{-1} n_e=150 M_d=3.0	1	1	1	1	0.28	0	0.13	0,83	0.82	0.21	1	1	0.35	0	0.24	0,92
5 (Dec 15 *08pm* to Dec 16 *night*) Π_{CO2}=20 kg m^{-2} d^{-1} T=110 C ε=1.4x10^{-4} dε/dt=0 d^{-1} n_e=300 M_d=4.0 d^{2}E/dt^2=1	1	1	1	1	0.27	0	0.10	0,85	0.82	0.22	1	1	0.36	0	0.24	0,93
6 (Dec 16 *04 am to* Eruption Onset) Π_{CO2}=100 kg m^{-2} d^{-1} T=110 C ε=3.4x10^{-4} dε/dt=8x10^{-4} d^{-1} n_e=500 M_d=4.0 d^{2}E/dt^2=1 d$^2\varepsilon$/dt^2=1 PE=1	1	1	1	1	0.77	0.13	0.98	1	0.87	0.38	1	1	0.87	0.41	1	1

TABLE 3. Control experiments: parameters that are varied in the experiments, present-day background value and relative ranges for the increments in each sub-period. For the deformation parameters ($d\varepsilon/dt$ and ε), we vary the supposed uplift and translate it into the desired parameters.

Parameter	Present-day background	Range of increase
T	95 °C	Sub-period 4: 5 – 25 °C
M_d	About 2 – 2.5/month	Sub-period 2: 0 – 1.5 /month Sub-period 5: 0 – 1.5 /month
n_e	About 10 events/month	Sub-period 2: 0 – 100 events/month Sub-period 4 (1): 0 – 100 events/month Sub-period 4 (2): 0 – 100 events/month Sub-period 5: 50 – 300 events/month Sub-period 6: 50 – 300 events/month
Φ_{CO2}	2.5 kg m^{-2} d^{-1}	Sub-period 1: 0 – 30 kg m^{-2} d^{-1} Sub-period 3: 0 – 30 kg m^{-2} d^{-1} Sub-period 6: 20 – 300 kg m^{-2} d^{-1}
Uplift	0 m	Sub-period 2: 5 – 50cm/0.5month ($d\varepsilon/dt$: $6.7\text{x}10^{-7}$ – $6.7\text{x}10^{-6}$ d^{-1}; ε: 10^{-5} – 10^{-4}) Sub-period 3: 20 – 100 cm/5 days ($d\varepsilon/dt$: $8\text{x}10^{-6}$ – $4\text{x}10^{-5}$ d^{-1}; ε: $5\text{x}10^{-5}$ - $3\text{x}10^{-4}$) Sub-period 4: 5 - 20 cm/7days ($d\varepsilon/dt$: $1.4\text{x}10^{-6}$ – $5.7\text{x}10^{-6}$ d^{-1}; ε: $6\text{x}10^{-5}$ – $3.4\text{x}10^{-4}$) Sub-period 6: 50 – 300 cm/6hours ($d\varepsilon/dt$: $4\text{x}10^{-4}$ - $2.4\text{x}10^{-3}$ d^{-1}; ε: $1.6\text{x}10^{-4}$- $9.4\text{x}10^{-4}$)

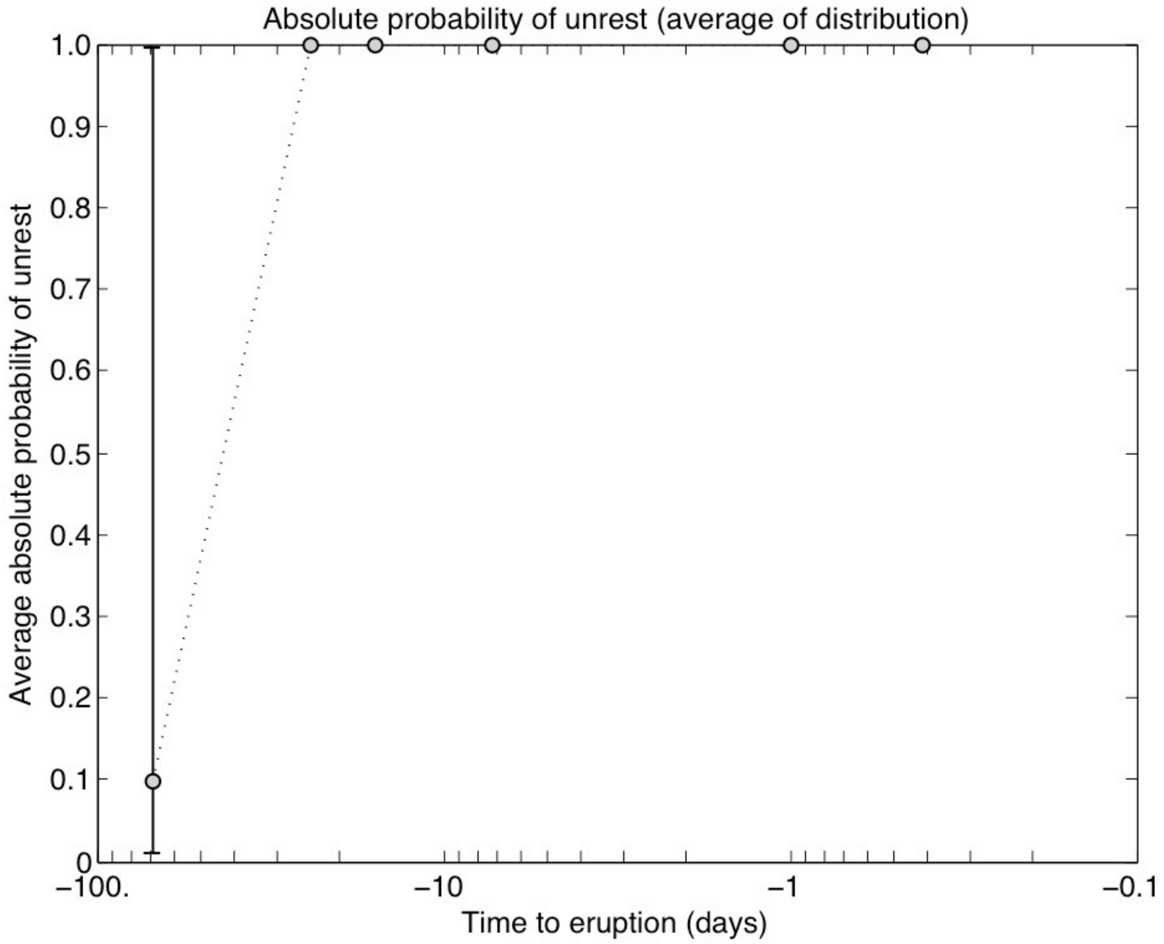

Figure 1. Average absolute probability of unrest in the control experiments for different sub-periods versus time approaching the eruption. The intervals represent the 10-90 percentiles of the average probabilities obtained in the 1000 simulations. The circles represents the value obtained for the reference case.

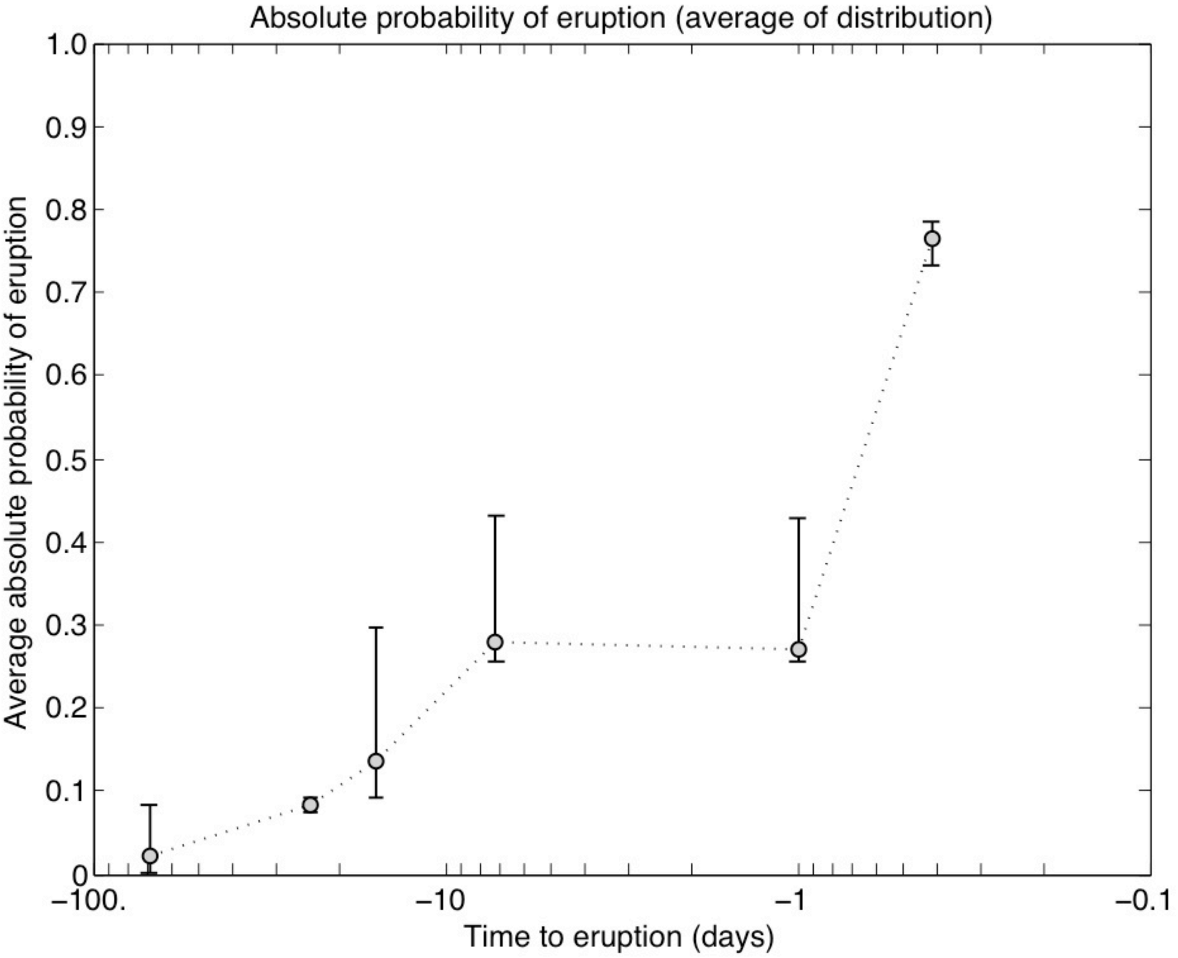

Figure 2. Average absolute probability of eruption in the control experiments for different sub-periods versus time approaching the eruption. The intervals represent the 10-90 percentiles of the average probabilities obtained in the 1000 simulations. The circles represents the value obtained for the reference case.

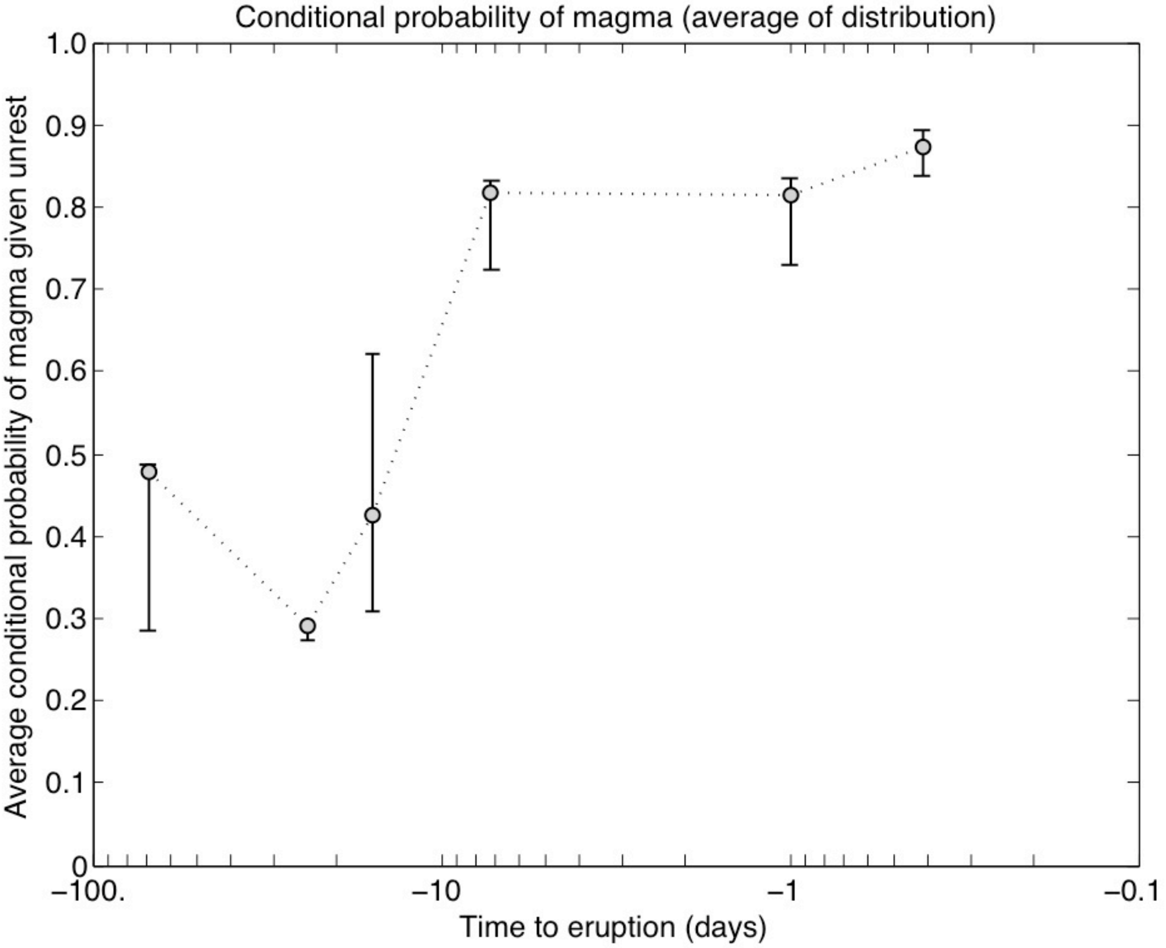

Figure 3. Average conditional probability of magma given unrest in the control experiments for different sub-periods versus time approaching the eruption. The intervals represent the 10-90 percentiles of the average probabilities obtained in the 1000 simulations. The circles represents the value obtained for the reference case.

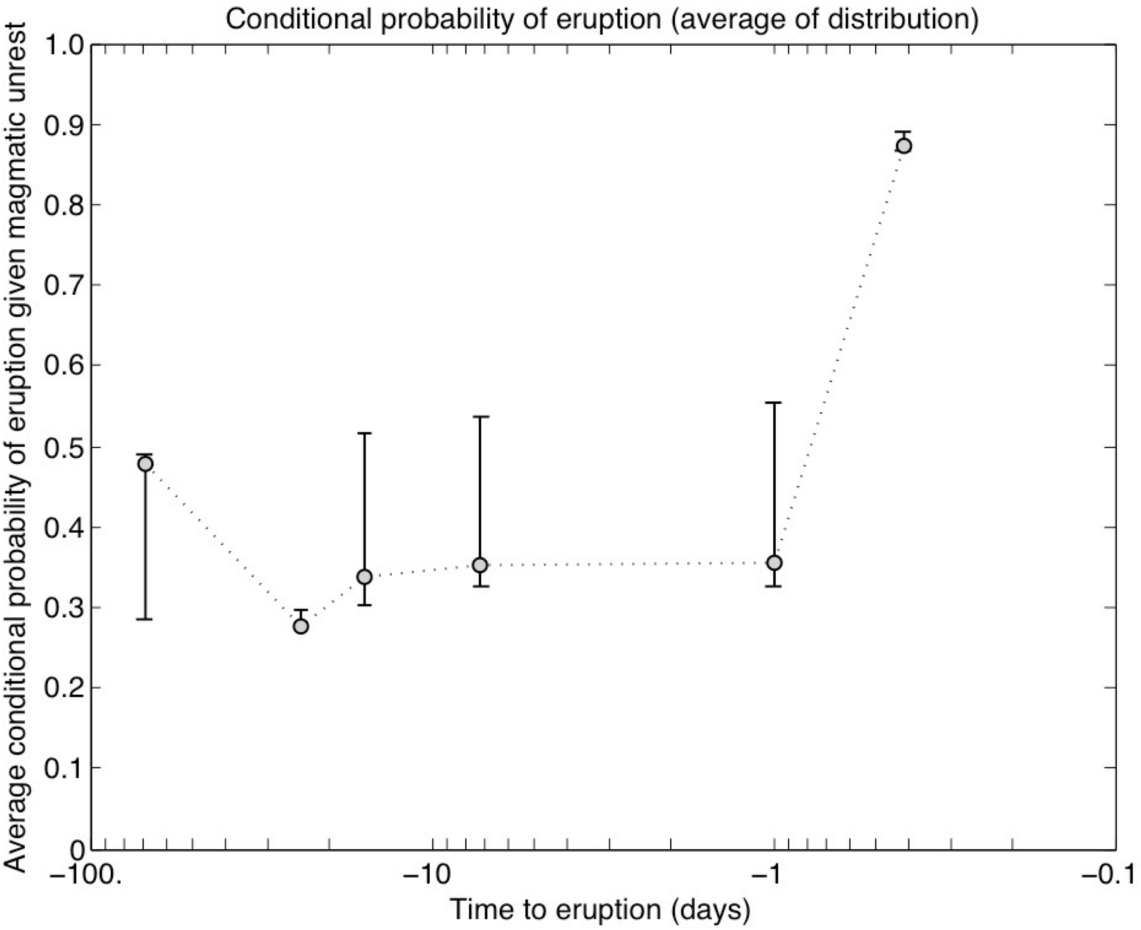

Figure 4. Average conditional probability of eruption given magmatic unrest in the control experiments for different sub-periods versus time approaching the eruption. The intervals represent the 10-90 percentiles of the average probabilities obtained in the 1000 simulations. The circles represents the value obtained for the reference case.

Appendix A: The authors of the five treatises examined

The authors of these four texts were Neapolitan intellectuals, men of the cloth or ecclesiastic, all present at the eruption of Vesuvius in 1631. Here is their concise biography, that is useful in order to frame the cultural ambient those authors belonged to.

Giovanni Domenico de Arminio. Hardly anything is known about this author, apart from the fact that he was a doctor at the Ospedale degli Incurabili of Naples, as he himself declares in the presentation of the Treatise. So he was a direct witness to what had happened, above all of what was seen in Naples. His treatise contains many theoretical elements on the causes of the eruption, mainly oriented to an almost anthropomorphic interpretation of the volcanic activity.

Gregorio Carafa (1588–1675). Carlo Marcello (later Gregorio) Carafa was born in Naples. He was a philosopher and theologian; he joined the Teatini fathers of Naples, where he held the chair of Philosophy and Theology. It was in those years that he acquired great fame as a preacher and a cultured intellectual. The fame he earned among his contemporaries and his noble origins allowed him to obtain the highest positions in his order. he was also the Bishop of Cassano (Calabria) and Archbishop of Salerno. He held some diplomatic posts: he was the special diplomatic representative of the Emperor Philip IV of Spain with the Pope Innocence X.

Giovanni Battista Mascolo (1583–1648*).* Born in Naples, he became a Jesuit at a very young age, in 1598. He taught Theology and Philosophy at the College of his order, of which he was the rector for some time; later he held a school of rhetoric at his home. He was famed for being a good Latin scholar. His treatise *Vesuviani Incendii Historiae libri tres* (Naples, 1634), his most complex work, was written in his full maturity, probably using some of his previous unpublished writings.

***Giulio Cesare Braccini* (1570-1632)** He was a man of law, later an ecclesiast and abbot, born at Gioviano di Lucca. He was appointed apostolic proto-notary by Urban VIII and between 1629 and 1632 he was in Naples, in touch with many leading political characters of the day. As regards Vesuvius, abbot Braccini published two texts: one was a letter to cardinal Girolamo Colonna, disseminated in the first few days after the eruption, to the extent that already between the end of December 1631 and the beginning of 1632 three editions had been published (*Relazione dell'incendio del Vesuvio alli 16 dicembre 1631 in una lettera diretta all'Em.mo e Re. mo Signore Card. Colonna*, printed in Naples at S. Roncagliolo); and the treatise examined here, printed in Naples in September 1632.

Salvatore Varrone (1593–1656.) We have very little information about him. He became a Jesuit in 1612; he taught grammar, humanities and rhetoric, and for six years scholastic theology and morals. He was famous for being a very learned intellectual. In the period of the eruption of Vesuvius, Varrone was staying at Portici, a village on the slopes of Vesuvius, and therefore was an eyewitness to the whole eruption.

Appendix B: Summarizing tables of the BET_EF rules for Vesuvius

Here we show the rules uploaded in BET_EF in this application. They are exactly the same used in Marzocchi et al (2008).

A summary of all the rules is provided in Table B1.

At each node k of the Bayesian Event Tree we compute two different probability distributions, one by using only monitoring data and the other one by using only other kinds of data, e.g. models, past occurrence, expert opinions. These two probability distributions are indicated respectively by [$\theta_k^{(M)}$] and [$\theta_k^{(NM)}$], where the index k stands for the k-th node and the square brackets denote a probability distribution. In order to compute the actual probability distribution at node k, we linearly combine [$\theta_k^{(M)}$] and [$\theta_k^{(NM)}$] with a relative weight that is function of the state of unrest. Both [$\theta_k^{(M)}$] and [$\theta_k^{(NM)}$] are computed through the Bayes theorem, i.e., by starting from a prior distribution (based on models and beliefs for [$\theta_k^{(NM)}$], and on present monitoring for [$\theta_k^{(M)}$]). The prior distribution is characterized by a mean ($\Theta_k^{(NM)}$ and $\Theta_k^{(M)}$ in the two cases), representing our best prior guess on the probability at node k, and by a measure of the variance that we call "equivalent number of data" ($\Lambda_k^{(NM)}$ and $\Lambda_k^{(M)}$ in the two cases) because intuitively it translates the confidence we have in our prior guess in terms of number of data. The minimum possible "equivalent number of data" is 1 and it represents the maximum variance allowed, implying a very low confidence on our prior guess, while there is no upper limit to its maximum value. The prior distribution is then transformed into the posterior distribution through Bayes theorem, i.e., by multiplying it by the likelihood function, based on past frequencies of occurrence for [$\theta_k^{(NM)}$] and on past monitoring (if any) for [$\theta_k^{(M)}$].

An important aspect of BET is the way it deals with monitoring measurements. Through a fuzzy approach, the measured values are translated into degrees of anomaly, from whom the mean of the monitoring probability distribution ($\Theta_k^{(M)}$) is derived (Marzocchi et al, 2008). In practice, for each monitoring parameter, we define a lower and an upper threshold, and an order relationship, used to infer the degree of anomaly for every specific measured value.

In the following, we give a detailed account of the choices on all BET parameters for Mt. Vesuvius (see also Marzocchi et al, 2004; 2008), as frozen before this retrospective application.

See Marzocchi et al (2008) for a deeper discussion on BET structure (the nodes), general rules and related concepts.

B.1 First node: Unrest

B.1.1 Non-monitoring part: [$\theta_1^{(NM)}$]

There is no theoretical model or expert belief to be used for assessing the probability of a volcano entering an unrest phase. Thus here we set up a maximum ignorance prior distribution with mean $\Theta_1^{(NM)}$=0.5 and $\Lambda_1^{(NM)}$=1.

Regarding past data, we know that OVO seismic station (see Zollo et al., 2002) has been monitoring Mt. Vesuvius continuously since 1972. We assume that there has been no episode of unrest ever since. Thus, we have 0 unrest episodes out of 420 months (i.e., 35 years).

B.1.2 Monitoring part: [$\theta_1^{(M)}$]

From now on, for the monitoring part at each node we give the list of selected monitoring variables used in Marzocchi et al (2004). In particular, for node 1, as summarized in Table B2, they are (from now on, for monitoring parameters we denote in braces the order relationship and the upper and lower thresholds used to define the degree of anomaly of the measured parameter):

- n_e {>;23;150} number of seismic events per month with $M_d \geq 1.9$ recorded at OVO station; the thresholds have been chosen on the basis of the monthly distribution of the number of seismic events observed at OVO, and they represent respectively the 55-th and the 95-th percentiles
- M_d {>;3.4;4.3} largest duration magnitude of the earthquakes recorded during last month at OVO station; the thresholds have been chosen on the basis of the monthly distribution of the magnitude of seismic events observed at OVO, and they represent respectively the 55-th and the 95-th percentiles
- n_{LF} {>;1;3} number of low-frequency (LF) events deeper than 1 Km per month; since only sporadic and temporally isolated LF events have been observed so far, even a small swarm is indicative of unrest
- Π_{SO2} {=;1;1} significant presence SO_2 (0, no; 1, yes)
- Φ_{CO2} {>;5;30 kg m^{-2} d^{-1}} daily CO_2 emission rate; the thresholds represent respectively about twice and 10 times the background value

- $d\varepsilon/dt$ {>;0;0 d^{-1}} strain rate (inflation), assuming that any detected uplift implies unrest
- T {>;98;105 °C} temperature of the fumaroles inside the crater; the thresholds represent respectively about 3% and 10% higher than the value observed since 1990 (95°C).

No past monitoring is available. Thus, the posterior for [$\theta_1^{(M)}$] is equal to the prior.

B.2 Second node: Magma given Unrest

B.2.1 Non-monitoring part: [$\theta_2^{(NM)}$]

There is no model to be used for assessing the probability of a volcano unrest being due to magma. Thus here we set up a maximum ignorance distribution with mean $\Theta_2^{(NM)}=0.5$ and $\Lambda_2^{(NM)}=1$.

Regarding past data, we have no unbiased past data to infer the origin of past unrest at Vesuvius, since we know much better the magmatic unrest episodes (at least those that yielded an eruption), compared to the hydrothermal ones. Thus, the posterior for [$\theta_2^{(NM)}$] remains equal to the prior.

B.2.2 Monitoring part: [$\theta_2^{(M)}$]

As summarized in Table B3, from Marzocchi et al (2004), the selected monitored parameters at node 2 are:

- Π_{SO2} {=;1;1} significant presence SO_2 (0, no; 1, yes)

- $d\varepsilon/dt$ {>;5x10^{-6}; 5x10^{-5} d^{-1}} strain rate (inflation), considering that a rapid and localized positive strain of the volcanic edifice is usually indicative of rising magma
- ν {<;2.5;3.5 Hz} the dominant spectral frequency of earthquakes; if it is around the frequency specified by the thresholds, there are probably LF events or tremor, that might be indicative of magma motion
- ξ_e {<;0.3;0.4] ratio between average and dispersion of the depth of the earthquakes during unrest. The present value of ξ_e for Mt. Vesuvius from seismic events recorded in the last 30 years is about 0.4. The thresholds are chosen in order to fuzzify this value, assuming that earthquakes coming closer to the surface and/or occurring in a larger range of depths may indicate an upward migration of magma or a coalescence of small fractures that may facilitate the magma uprising. At the same time, we do not consider shallow earthquakes concentrated in a very small range of depths to be indicative of magma, as they could also be due to hydrothermal activity
- T {>;98;105 °C} the temperature of the fumaroles inside the crater; the thresholds are chosen as for node 1.

Remarkably, we assume that the parameters Π_{SO2} and T have a weight twice as much as the other parameters in determining a magmatic origin for the unrest.

Neither here past monitoring is available. Thus, the posterior for [$\theta_2^{(M)}$] is equal to the prior.

B.3 Third node: Eruption given Magmatic Unrest

B.3.1 Non-monitoring part: $[\theta_3^{(NM)}]$

There is no model to be used for assessing the probability of a volcano erupting given that there is an unrest of magmatic origin. Thus here we set up a maximum ignorance distribution with mean $\Theta_3^{(NM)}=0.5$ and $\Lambda_3^{(NM)}=1$.

Regarding past data, we have no unbiased past data to infer the frequency of eruptions following past magmatic unrest at Vesuvius, since we know much better the magmatic unrest episodes that yielded an eruption with respect to those that died out without eruption. Thus, the posterior for $[\theta_3^{(NM)}]$ remains equal to the prior.

B.3.2 Monitoring part: $[\theta_3^{(M)}]$

As summarized in Table B4, from Marzocchi et al (2004), the selected monitored parameters at node 2 are:

- PE {=;1;1} presence of phreatic explosion (0, no; 1, yes)
- $d\nu/dt$ {<;0;0 Hz d^{-1}} rate of change of the average spectral frequency content of earthquakes
- ξ_e {<;0.3;0.4} ratio between average and dispersion of the depth of the earthquakes during unrest; the thresholds are chosen as for node 2
- d^2E/dt^2 {=;1;1} acceleration of seismic energy release (0, no; 1, yes)
- $d^2\varepsilon/dt^2$ {=;1;1} acceleration of inflation (0, no; 1, yes)
- ε {>; $5x10^{-5}$; $5x10^{-4}$} cumulative strain since the beginning of the unrest; the thresholds are just a little bit less than a reasonable value for the maximum prefracture strain

- $d\rho/dt$ {>;0;0} change of the ratios HCl/SO_2 and/or HF/SO_2
- REV {=;1;1} sudden reversal of at least one of the above parameters (0, no; 1, yes)

For most of these parameters the thresholds are based on a "yes/no" choice. This is because we are mainly interested in the time trend of the parameters.

We assume that the parameters $d^2\varepsilon/dt^2$, PE and REV have a weight twice as much as the other parameters in determining the occurrence of the eruption. Neither here past monitoring is available. Thus, the posterior for $[\theta_3^{(M)}]$ is equal to the prior.

B.4 Forth node: Location of vent given that there is an Eruption

B.4.1 Non-monitoring part: $[\theta_4^{(NM)}]$

Since Mt. Vesuvius is a central volcano, and its activity has been mainly concentrated in the crater, we assume that there is a very high probability of vent opening in the crater area, and a very small one outside it. With this idea in mind, we divide the volcanic edifice into 5 areas: the crater area, and 4 outer, equal sized areas. We assume that next eruption will take place in one and only one of them. Because of these assumptions, we set up a Dirichlet distribution (see Marzocchi et al, 2008 for further details on such distribution) with mean $\Theta_4^{(NM)(1)}=0.99$ in the crater area (area 1) and $\Theta_4^{(NM)(i)}=0.0025$ (i=2,...5) for the surrounding areas (i.e., Areas 2, 3, 4 and 5). Since we are very confident on this assumption, we set $\Lambda_4^{(NM)}=50$, which is an (arbitrarily) high equivalent number of data.

B.4.2 Monitoring part: [$\theta_4^{(M)}$]

BET allows to localize some or all of the monitored measurements used at previous nodes to assess the probability of vent opening in different areas, according to monitoring (see Marzocchi et al, 2008 for further details on this issue).

B.5 Fifth node: Size of the Eruption given that there is an Eruption

We parametrize the possible sizes with VEI. Since Mt. Vesuvius has been dormant for over 60 years now, it is presumably in a closed conduit regime. Therefore, we assume that next eruption will be at least of VEI=3 in order to have sufficient energy to re-open the system. For practical purposes, we define three classes of possible sizes, according to this idea: VEI=3, VEI=4 and VEI≥5.

B.5.1 Non-monitoring part: [$\theta_5^{(NM)}$]

We use the observation that the worldwide log(frequency)-size relationship for volcanoes is a straight line, implying that the most frequent, and therefore likely, eruptions are the smaller, with a power law relationship. We translate this information by setting up a Dirichlet distribution for the three size classes. The mean of each class is set equal to the probability for that class given by the above mentioned worldwide relationship. In this way we have the following means: $\Theta_5^{(NM)(1)}=0.83$ for VEI=3, $\Theta_5^{(NM)(2)}=0.14$ for VEI=4 and $\Theta_5^{(NM)(3)}=0.03$ for larger eruptions. We also assume the maximum variance on this model, thus $\Lambda_5^{(NM)}=1$.

For past data, we use the catalog of Mt. Vesuvius eruptions with a repose time larger or equal to 60 years, so that we are sure to consider only past eruptions occurred in closed conduit regime, as it is now the case. We have 7 such eruptions in the catalog, in particular 4 eruptions of VEI=3, 2 of VEI=4 and 1 larger.

B.5.2 Monitoring part: [$\theta_5^{(M)}$]

Up to present, there is no reliable precursor heralding the size of eruptions. Therefore, we do not use monitoring data at this node.

TABLE B1: BET_EF settings

NODE	prior model	past data	monitoring parameters	past monitored episodes
1	NO	0 of 384	7 (see Table B2)	0
2	NO	NO	5 (see Table B3)	0
3	NO	NO	8 (see Table B4)	0
4	Means of the 5 areas: 0.99, 0.0025, 0.0025, 0.0025, 0.0025 with Λ_4=50	13, 0, 0, 0, 0		
5	0.83, 0.14, 0.03 with Λ_5 = 1	4, 2, 1		

TABLE B2: Monitoring parameters at NODE 1

symbol	description	thresholds	units
n_e	monthly number of events with Md ≥ 1.9 at OVO station	>23;150	month^{-1}
M_d	monthly largest magnitude at OVO station	>3.4;4.3	month^{-1}
n_{LF}	monthly number of LF events deeper than 1 km	>1;3	month^{-1}
Π_{SO2}	presence of significant SO$_2$	=1	
Φ_{CO2}	daily CO2 emission rate	>5;30	kg m^{-2} day^{-1}
$d\varepsilon/dt$	strain rate (inflation)	>0;0	day^{-1}
T	temperature of fumaroles in the crater	>98;105	°C

TABLE B3: Monitoring parameters at NODE 2

symbol	description	thresholds	units	weight
Π_{SO2}	presence of significant SO$_2$	=1		2
$d\varepsilon/dt$	strain rate	>5x10^{-6};5x10^{-5}	day^{-1}	1
$<\nu>$	average spectral frequency	<2.5;3.5	Hz	1
ξ_e	ratio between average and dispertion of earthquake depths	<0.3;0.4		1
T	as in NODE 1	>98;105	°C	2

TABLE B4: Monitoring parameters at NODE 3

symbol	description	thresholds	units	weight
PE	Presence of phreatic explosions	=1		2
$d<\nu>/dt$	rate of change of $<\nu>$	<0;0	Hz	1
ξ_e	as NODE 2	<0.3;0.4		1
d^2E/dt^2	acceleration of seismic energy release	>0;0	J day^{-2}	1
$d^2\varepsilon/dt^2$	acceleration of strain (inflation)	>0;0	day^{-2}	2
ε	cumulative strain (inflation)	>5x10^{-5};5x10^{-4}		1
r	change of the ratios HCl/SO$_2$ and/or HF/SO$_2$	=1		1
REV	sudden reversal of at least one of the parameters above	=1		2