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Fractal time statistics of AE-index burst waiting times: evidence of metastability

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Abstract. Recent observations and analyses evidenced that the magnetotail, as well as the magnetospheric dynamics are characterised by a scale-free behaviour and intermittence. These results, along with numerical simulations on cellular automata, suggest that the observed scale-invariance may be due to *forced and/or self-organised criticality* (FSOC), meaning that the magnetotail operates near a marginally stable state (Chang, 1999). On the other hand, it was underlined that a complex magnetic field topology in the geotail regions may play a relevant role in the impulsive energy relaxation (Consolini and Chang, 2001).

1 Introduction

The conditions of solar wind and the interplanetary magnetic field strongly affect the state of the Earth's magnetosphere, which responds to the external driving in a highly organised and complex way (Klimas et al., 1996). This complex behaviour is due mainly to a nonlinear dynamics related to the energy storage, transport and release in the geomagnetic tail regions. Moreover, as a consequence of the continuous solar wind driving, the coupled magnetosphere-ionosphere system is believed to be in an out-of-equilibrium configuration.

In the past, significant progress in the knowledge of the general features of the magnetospheric dynamics and response to solar wind changes was achieved on the basis of the magnetohydrodynamic (MHD) concepts. These advances have led to the realisation that a deeper understanding of the magnetospheric phenomena, and, in particular, of the highly dynamic character of the geotail dynamics might benefit from new concepts and approaches based on the physics of complex systems (Consolini and Chang, 2001, 2002). In detail, it has been realised that topological structures and connectivity, associated with nonlocal and global features, play a relevant role in the magnetospheric dynamics.

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Recently, modern techniques, based on nonlinear dynamics, have been applied to study the magnetospheric activity as revealed by geomagnetic indices. These new techniques have led to different views of the magnetospheric system. At the beginning of the 1990s, some analyses suggested the magnetospheric dynamics to be characterised by low-dimensional chaos (Baker et al., 1990; Roberts et al., 1991; Vassiliadis et al., 1990). At the present time, autonomous attractor dynamics does not seem to be relevant; the Earth's magnetosphere is, indeed, better described as an input-output nonlinear dynamical system (Klimas et al., 1996).

Looking at the magnetosphere as a nonlinear stochastic system, Chang (Chang, 1992a, 1992b, 1999) proposed a new point of view. He showed that a nonlinear stochastic system, driven far from equilibrium near criticality, can exhibit low-dimensional behaviour, and that the relevant number of dimensions could change continuously due to the evolution of the system itself from one critical point to another. He suggested that the magnetosphere might be an open, dissipative dynamical system near a forced and/or self-organised critical (FSOC) state showing anomalous dimensions. In such a state the temporal output of the magnetosphere should be intermittent and characterised by power-law power spectra, exhibiting scale-invariant and self-similar spatial structure. Evidences of this intermittent dynamics were found by Consolini et al. (1996), who investigated the scaling properties of the auroral electrojet index. The non-Gaussian shape of the distribution function of the AE index fluctuations (Consolini and De Michelis, 1998) further confirmed this intermittence. In the same period, Consolini (1997), analysing the AE-index bursty behaviour, revealed the existence of a power-law distribution in the energy released during substorm events. Further experimental evidences of this scale-invariance in the energy releases have been found in the duration of the "bursty bulk flows" (BBF) (Angelopoulos et al., 1999) and in the auroral displays (Lui et al., 2000). All of these observations of free-scale phenomena have been considered to be an indication of a near-criticality dynamics, supporting the previous suggestion by Chang (Chang, 1992a, 1992b). However, this interpretation is still quite controversial; some authors do not, indeed, rule out the possibility that the observed scale invariant features may be a simple manifestation of solar wind properties and that the Earth's magnetosphere responds passively to solar wind changes (Freeman et al., 2000; Takalo et al., 2000; Price and Newman, 2001).

A possible way to reconcile the apparent controversy between the organised low-dimensional nonlinear behaviour and the near-criticality (SOC-like) behaviour seems to be to look at magnetic substorms in terms of noise induced nonequilibrium and/or topological phase transitions (Chang, 2001a,; Sharma et al., 2001; Sitnov et al., 2000, 2001; Consolini and Chang, 2002). As a matter of fact, the theory of topological phase transitions in out-of-equilibrium systems provides a natural framework for the understanding of the solar wind-magnetosphere coupling mainly because of its input-output nature. As is well known, one of the main features of extended out-of-equilibrium systems is their ability to organise themselves in metastable states. The dissipation events will then be the consequence of first and/or second order phase transition among these metastable configurations.

Here, looking at the statistical features of the waiting times among successive activity bursts in AE-index, we will investigate the possible occurrence of metastability in the magnetospheric dynamics. In detail, we will show how fractal time statistics of AE-index burst waiting statistics is in agreement with a random walk in a complex free-energy landscape.

2 Data description and analysis

Data used in this work refer to the auroral electrojet (AE) index and come from both the National Geophysical Data Centre (NGDC, Boulder, Colorado) and the World Data Centre I (Kyoto, Japan). In detail, we have considered a continuous time series of the AE-index covering the period from 1 January 1978 to 30 June 1988. Data time resolution is 1 min for a total amount of points of the order of 6 M-points. We selected such a long period without any preliminary assumption in order to have good statistics for our analysis. Moreover, the choice of the AE-index as an indicator of the global magnetospheric activity has been made because of the common point of view that AE-indices are able, in some sense, to sample the state space of the magnetospheric system.

As previously shown (Consolini and De Michelis, 1998), one of the main features of the AE-index is its intermittent character, which is evidence of a punctuated dynamics of the magnetospheric system in response to solar wind changes. In detail, the AE-index is characterised by periods of relative stasis punctuated by crises of different sizes. The existence of two dynamical phases has also been confirmed by the bimodal feature of the AE-index probability distribution function (Pdf). In Fig. 1, we report the probability density function of the AE-index as evaluated on the basis of the data set considered here.

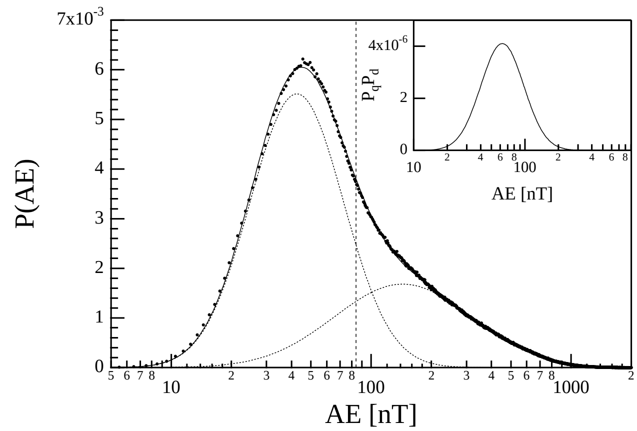

Fig. 1. The probability density function $P(AE)$ of the auroral electrojet (AE) index data set, considered in this work. Solid line refers to a nonlinear best fit using the superposition of two *Log-normal* distribution function (see text Eq. 1). The two dotted lines show the two components (P_q and P_d) relative to quiet and active periods, respectively. The vertical dashed line is the threshold evaluated on the basis of expression (2). The inset shows the product of the two components $P_q P_d$.

Due to the avalanche-like and multifractal nature of the AE-index, the Pdf can be fitted using a superposition of two *Log-normal* distributions:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{x} \exp\left[-\frac{\ln^2(x/x_0)}{2\sigma^2}\right] \Leftrightarrow 0^+ < x < \infty \quad (1)$$

associated with quiet ($AE < 100$ nT) and active ($AE > 100$ nT) periods, respectively. In order to discriminate between quiet and active times, taking into account the bimodal character of the AE-index Pdf, we have introduced a threshold value ϕ_{thr}, defined in a different way with respect to our previous analysis (Consolini and De Michelis, 1998) and evaluated as follows:

$$\phi_{thr} = \frac{\int_{0^+}^{\infty} \phi P_q(\phi) P_d(\phi) d\phi}{\int_{0^+}^{\infty} P_q(\phi) P_d(\phi) d\phi}, \quad (2)$$

where $P_q(\phi)$ and $P_d(\phi)$ are the two components of the Pdf associated with quiet and active periods, respectively (see the inset in Fig. 1). The numerical value of the threshold is $\phi_{thr} = 80$ nT, which is well in agreement with the usual definition of the quiet condition of the AE-index ($AE < 100$ nT). In Fig. 2, we show a two-day interval of the AE-index. The horizontal dashed line refers to the threshold, which allowed us to discriminate the quiet periods from the active ones.

By means of the threshold, we have evaluated the duration of the quiet periods as the time interval between two successive (downward-upward) crossings of the threshold (see Fig. 2). In the following, these time intervals will be called "waiting times" (τ). Successively, the probability distribution function $\psi(\tau)$ of the waiting times has been computed.

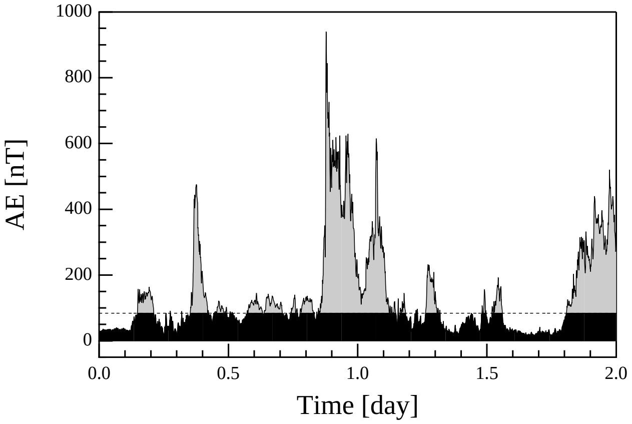

Fig. 2. A sample of a 2-day interval of AE-index records. The horizontal dashed line refers to the threshold used to discriminate quiet and active periods (grey in the figure).

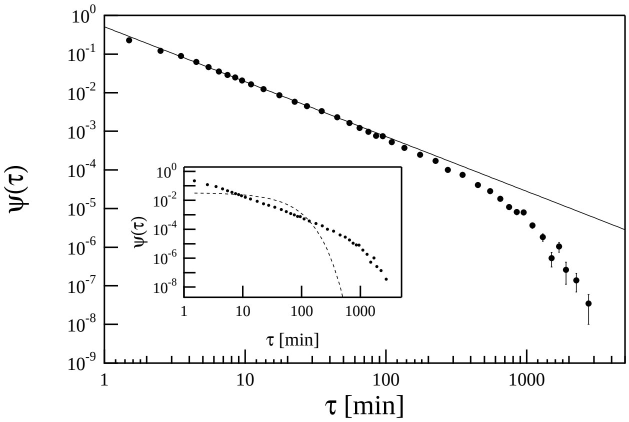

Fig. 3. The waiting times distribution function $\psi(\tau)$. The solid line refers to a power-law nonlinear best fit. The inset shows a comparison between the observed distribution and the expected Poissonian distribution characterized by the same average waiting time.

Figure 3 shows the waiting times Pdf $\psi(\tau)$ in a double logarithmic plot. The results are robust with respect to the peculiar choice of the threshold in the range $60 \div 120$ nT.

From a preliminary analysis of the trend of the probability distribution function $\psi(\tau)$, we find that it follows a power law $\psi(\tau) \propto \tau^{-\eta}$ over more than 2 orders of magnitude with a scaling exponent equal to $\eta = [1.42 \pm 0.01]$. If the $\psi(\tau)$ would be a simple power law, then, in the case of $\eta < 2$, the average waiting time $\langle\tau\rangle$ would be infinite. This result of an approximate power law in a limited range of scales may be seen as evidence of a scale-invariance of the Pdf of at least two orders of magnitude that we will term as "fractal statistics" of waiting times. Moreover, scale-invariance of the waiting times' statistics could be an indication of a time correlation among the bursts, i.e. of a sort of aging effect (Sornette, 2000).

As noted by Boffetta et al. (1999), fractal waiting time statistics seems to be in contrast with the occurrence of a classical SOC dynamics (Bak et al., 1987). As a matter of fact, in the case of a SOC system, we would expect a near Poisson distribution function of the waiting times: i.e. $\psi(\tau) = \langle\tau\rangle^{-1} \exp(-\tau/\langle\tau\rangle)$. The inset of Fig. 3 shows a comparison between the expected Poisson distribution and the observed one. However, the presence of fractal time statistics of the waiting times could be interpreted in terms of more general complex dynamics associated with the existence of metastable states in an out-of-equilibrium system (Conoslini and Chang, 2001, 2002). Recently, Chang (Chang, 2001a, 2001b) has introduced a new point of view where the configuration of the magnetospheric plasma and field topology might be close to a colloidal phase. In particular, it has been proposed that the dynamics of coherent structures could be similar to that of a stirred colloidal suspension, showing topological phase transitions in the evolution from one critical state to another. Applying such a view to the magnetotail plasma sheet regions, the magnetotail system itself might naturally evolve toward a region of the configuration space characterised by many metastable states. Therefore, the evolution of the global system could be equivalent to a random walk in this configuration fractal space.

3 Topological randomness and waiting time statistics

In the last years, a great interest has been focused on the relevance of topological disorder in physical systems. Converse to the traditional approach that treats disorder in terms of a perturbation, it was realised that randomness and disorder may introduce new and unexpected behaviours in physical systems. One of the most relevant characteristics of disordered systems is the occurrence of metastability as a consequence of the intrinsic space-time randomness. For example, quenched disorder in spin systems involves the existence of many competing minima in the energy landscape that prevents the ergodicity. In such disordered systems, the dynamics may result in a wandering in a separate multi-valley energy landscape; i.e. a sequence of jumps between the many competing minima. The ergodicity breaking and the existence of a complex topology of the energy landscape will strongly affect the statistics of residence times, i.e. of the waiting time τ (which can be, indeed, associated with the time between two successive jumps from a local minimum to another).

In this framework, let us consider a complex energy landscape characterised by a stretched exponential distribution of the local minima:

$$f(E) \propto \exp\left[-\left(\frac{E}{E_0}\right)^{\alpha}\right]. \qquad (3)$$

Assuming that the jumping mechanism among the local minima may be described within the framework of classical Kramer's reaction rate theory (Hänggi et al., 1990), the typ-

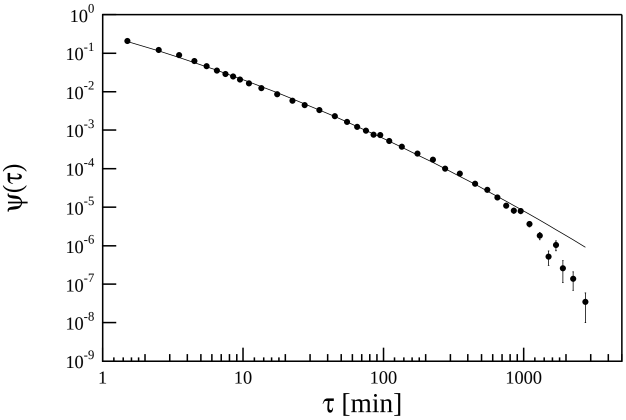

Fig. 4. The comparison between the observed waiting times distribution function $\psi(\tau)$ and the expression of Eq. (5). The solid line refers to a nonlinear best fit.

ical residence (waiting) time will follow the well-known Arrhenius activation law:

$$\tau \propto \tau_0 \exp\left[\beta E\right], \tag{4}$$

where β plays the role of the inverse of an equivalent temperature. From here, the resulting distribution function of the waiting times should be as follows:

$$\psi(\tau) = f(E)\frac{dE}{dt} \propto \frac{1}{\tau} \exp\left[-A \ln^{\alpha}(\tau)\right]. \tag{5}$$

We may note that in the case of a Poisson statistics ($\alpha = 1$) of the energy local minima, the waiting time distribution function will be $\psi(\tau) \sim \tau^{-1-\mu}$, with $\mu = 1/\beta E_0$ (Klafter et al., 1997; Sornette, 2000).

We report in Fig. 4 the nonlinear best fit of the AE-index waiting time statistics using the expression of Eq. (5). This fit agrees with data over 3 orders of magnitude, showing a better confidence than the simple power-law behaviour of Fig. 3. The observed value of α is $\alpha = [2.0 \pm 0.1]$. This result suggests that the magnetospheric dynamics may be treated in terms of a random walk in a complex free energy landscape with Gaussian statistics for the local minima. In the following section, we will discuss this result in connection with metastability and topological complexity of magnetic field in the tail regions.

4 Conclusions

The study of out-of-equilibrium, heterogeneous and disordered systems has improved the development of new concepts and of a new branch of statistical mechanics, called the physics of complex systems. One of the main features of complex systems is the role that the topological disorder plays in such systems. As a matter of fact, it has been noted that disorder generally introduces new and surprising effects not expected from the simple microscopic evolution rules. Complex systems, indeed, self-organise their internal structure and their dynamics showing novel and surprising macroscopic properties. For example, a complex system may display metastability, non-ergodicity, and coherent large-scale collective behaviours that are the consequence of the repeated nonlinear interactions among its elementary parts.

In this framework, our findings on the waiting times distribution function seem to support the hypothesis that the Earth's magnetotail might work as a complex system. As a matter of fact, if we figure out the magnetic topological complexity emerging from Chang's model (Chang, 1999), in terms of topological disorder, we may immediately realise that the associated dynamics should be characterised by metastability. If we consider a physical system with a given disorder magnetic field structure, the minimum free energy is a function of the topological complexity of the magnetic field. In such a system, any non-ideal process that modifies the overall topology may be associated with a sort of a dynamical transition between two different local minima in the configuration space during which a certain amount of free energy is relaxed. The emerging dynamical framework is that of a random walk in a complex free energy landscape. If the system evolves near criticality, this random walk in the free energy space will be characterised by a time correlation in the jumps and non-ergodicity.

In this framework, magnetic substorms are better described in terms of noise-induced topological transitions in an extended out-of-equilibrium system. In other words, the magnetic substorm is the set of phenomena during which a reduction in topological complexity in the tail regions takes place (see also Chang, 2001a, 2001b, Consolini and Chang, 2001, 2002). The role of the solar-wind driver would be to enhance the internal noise (i.e. the internal fluctuations) that could induce a topological transition among metastable complex topologies. In such a case, the evolution of the magnetospheric system (and in detail of the magnetotail regions) will be the result of the combined effects of local couplings of the magnetic and plasma structures, and of the noise intensity through the nonlinearities of the system. This point of view also supports the recent results of Sitnov et al. (2001) that the substorm activity resembles the nonequilibrium (first and/or second order) phase transitions.

At the moment, we believe that this framework is of a more general character than the classical SOC phenomena (Bak et al., 1987) and may involve space-time coupling among the transition events, as revealed by our analysis on the waiting time statistics. Clearly, much more work will be necessary to better address this new picture of magnetospheric substorms in terms of a noise-induced topological transition.

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References

Angelopoulos, V., Mukai, T., and Kokubun, S.: Evidence for intermittency in earth's plasma sheel and implications for self-organized criticality, Phys. Plasmas, 6, 4161–4165, 1999.

Baker, D. N., Klimas, A. J., McPherron, R. I., and Büchner, J.: The evolution from weak to strong geomagnetic activity: and interpretation in terms of deterministic chaos, Geophys. Res. Lett., 17, 41–44, 1990.

Bak, P., Tang, C., and Wiesenfeld, K.: Self-organized criticality: an explanation of $1/f$ noise, Phys. Rev. Lett., 59, 381–383, 1987.

Boffetta, G., Carbone, V., Giuliani, P., Veltri, P., and Vulpiani, A.: Power laws in solar flares: self-organized criticality or turbulence? Phys. Rev. Lett., 83, 4662–4665, 1999.

Chang, T.: Low-dimensional behavior and symmetry breaking of stochastic system near criticality: can these effects be observed in space and in laboratory? IEEE Trans on Plasma Sci., 20, 691–694, 1992a.

Chang, T.: Path integral approach to stochastic systems near self-organized criticality. in: Research Trends in Physics: Nonlinear Space Plasma Physics, (Eds) Alfvén, H., et al., A.I.P., 165–171, 1992b.

Chang, T.: Self-organized criticality, multi-fractal spectra, sporadic localized reconnections and intermittent turbulence in magnetotail, Phys. Plasmas, 6, 4137, 1999.

Chang, T.: Colloid-like behavior and topological phase transitions in space plasmas: intermittent low frequency turbulence in the auroral zone, Phys. Scripta, T89, 80–83, 2001a.

Chang, T.: An example of resonances, coherent structures and topological phase transition - the origin of the low frequency broadband spectrum in the auroral zone, Nonlinear Proc. Geophys., 8, 175–180, 2001b.

Consolini, G., Marcucci, M. F., and Candidi, M.: Multifractal structure of auroral electrojet index data, Phys. Rev. Lett., 76, 4082–4085, 1996.

Consolini, G.: Sandpile cellular automata and magnetospheric dynamics, in: Cosmic Physics in the year 2000, (Eds) Aiello et al., 58, pp. 123–126, SIF, Bologna, Italy, 1997.

Consolini, G. and De Michelis, P.: Non-gaussian distribution function of AE-index fluctuations: evidence for time intermittency, Geophys. Res. Lett., 25, 4087–4090, 1998.

Consolini, G. and Chang, T.: Magnetic field topology and criticality in geotail dynamics: relevance to substorm phenomena, Space Sci. Rev., 95, 309–321, 2001.

Consolini, G. and Chang, T.: Complexity, magnetic field topology, criticality, and metastability in magnetotail dynamics. J. Atmos. Sol. Terr. Phys., in press, 2002.

Freeman, M. P., Watkins, N. W., and Riley, T.: Evidence for a solar origin of the power law burst lifetime distribution of the AE indices, Geophys. Res. Lett., 27, 1087–1090, 2000.

Hänggi, P., Talkner, P. and Borkovec, M.: Reaction-rate theory – 50 years after Kramers, Rev. Mod. Phys., 62, 251–350, 1990.

Klafter, J., Zumofen, G., and Shlesinger, M. F.: Long-tailed distributions and non-Brownian transport in complex systems, in: The Physics of Complex Systems, Mallamace, F. and Stanley, H. E., IOS Press, Amsterdam (The Netherlands), 85–99, 1997.

Klimas, A. J., Vassiliadis, D., Baker, D. N., and Roberts, D. A.: The organised nonlinear dynamics of the magnetosphere, J. Geophys. Res., 101, 13 089–13 113, 1996.

Lui, A. T. Y., Chapman, S. C., Liou, K., Newell, P. T., Meng, C. I., Britthacher, M., and Parks, G. D.: Is the dynamic magnetosphere an avalanching system? Geophys. Res. Lett., 27, 911–914, 2000.

Price, C. P. and Newman, D. E.: Using the R/S statistic to analyze AE data, J. Atmos. Sol. Terr. Phys., 63, 1387–1397, 2001.

Roberts, D. A., Baker, D. N., Klimas, A. J., and Brgatze, L. F.: Indications of low dimensionality in magnetospheric dynamics, Geophys. Res. Lett., 18, 151–154, 1991.

Sharma, A. S., Sitnov, M. I., and Papadopoulos, K.: Substorms as nonequilibrium transitions of the magnetosphere, J. Atmos. Sol. Terr. Phys., 63, 1399–1406, 2001.

Sitnov, M. I., Sharma, A. S., Papadopoulos, K., Vassiliadis, D., Valdivia, J. A., Klimas, A. J., and Baker, D. N.: Phase transition-like behavior of the magnetosphere during substorms, J. Geophys. Res., 105, 12 955–12 974, 2000.

Sitnov, M. I., Sharma, A. S., Papadopoulos, K., and Vassiliadis, D.: Modeling substorm dynamics of the magnetosphere: from self-organization and self-organized criticality to nonequilibrium phase transitions, Phs. Rev. E., 65, 016116, 2001.

Sornette, D.: Critical Phenomena in Natural Sciences, Springer-Verlag, Berlin Heidelberg, 2000.

Takalo, J., Mursula, K., and Timonen, J.: Role of the driver in the dynamics of a coupled-map model of the magnetotail: does the magnetosphere act as a low-pass filter? J. Geophys. Res., 105, 27 665–27 672, 2000.

Vassiliadis, D. V., Sharma, A. S., Eastman, T. E., and Papadopoulos, K.: Low-dimensional chaos in magnetospheric activity from AE time series, Geophys. Res. Lett., 17, 1841–1844, 1990.