Hydro-thermal coupling in a rough fracture

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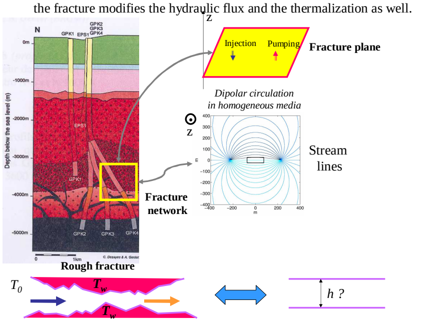
ABSTRACT

Heat exchange during laminar flow is studied at the fracture scale on the basis of the Stokes equation. The aperture is obtained from a self-affine geometrical model shown to be a realistic description of a natural fracture. We study the influence of the fracture roughness on the heat flux through the fracture sides when a cold fluid is injected and we estimate at which distance the thermal equilibration between the fluid and the rock temperature is reached. We show that at a coarse grained scale, the basic equation for heat flux is identical to the one for parallel plates, but with a different characteristic thermal length. Statistical computations and comparisons with flat parallel plates are made for the hydraulic and thermal results. The hydraulic aperture of rough fractures can be higher or lower than the one of parallel plates having the same mean mechanical aperture (or geometrical aperture) : the aspect ratio of the fracture appears to be an important parameter. Our model also shows that the fracture roughness induces channeling effects in hydraulic and thermal flows. Although fracture roughness is shown to induce a large variability of behaviors, the thermal equilibration is often reached with a higher characteristic thermal length than for a system with parallel plates having the same hydraulic aperture. A boundary element model describing how the fracture elastically closes is used to introduce the study of the hydro-thermo mechanical coupling of a rough fracture.

EXAMPLE : GEOTHERMAL EXPLOITATION

The context of study is an area where stream lines and isobars would be flat if the fracture were plane. For example, it could take place in the framed area between injection and pumping wells. Then we will see how the roughness of the fracture modifies the hydraulic flux and the thermalization as well.



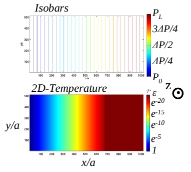
Impact of roughness on the fluid thermalization ?

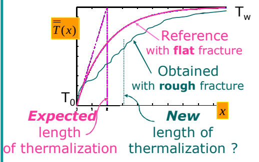
Equivalence at coarsened scale ?

EQUIVALENT PROBLEM AT COARSENED SCALE

As reference case, the fracture is modeled by two parallel plates which are separated by a distance h with a pressure P_0 at the inlet and P_L at the outlet.

We aim at estimating the characteristic length of thermalization l_{ref} which depends on the fracture morphology





$$\overline{T}_{ref} - T_w = (T_0 - T_w)\exp\left(-\frac{x}{l_{ref}}\right)$$

$$l_{ref} = \frac{h^2 u}{2\,Nu\,\chi} = \frac{h\,Pe}{Nu}$$

Peclet number $Pe = \frac{h \cdot u}{2\chi}$

u : Velocity averaged on the aperture height

METHOD

HYDRAULIC FLOW

Properties of fluid :

Density [kg/m^3] : ρ	Specific heat capacity [J/kg/K] : c
Conductivity [W/m/K] : λ	Diffusivity [m^2/s] : χ = λ / (ρc)

- Permanent
- Laminarity

} Stokes : $\vec{\nabla}P = \eta\Delta\vec{v}$

- Lubrication => locally smooth surface
  * Velocity $\vec{v} = \begin{pmatrix} v_x(x,y,z) \\ v_y(x,y,z) \\ 0 \end{pmatrix}$ Local parabolic law
  * Hydraulic flow $\vec{q} = \int_h \vec{v}\,dz = -\frac{h^3}{12\eta}\vec{\nabla}P$ Local cubic law
- Incompressibility : $\vec{\nabla}.\vec{u} = 0$

➢ ***Equation to be solved (2D) :***

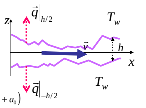
$$\boxed{\vec{\nabla}.(h(x,y)^3\vec{\nabla}P) = 0}$$

ENERGY CONSERVATION

- Conduction $\vec{\varphi} = -\lambda\vec{\nabla}T$
- Convection $\rho c\vec{v}T$

} $\Rightarrow \vec{v}.\vec{\nabla}T = \chi\Delta T$

- Assumptions :
  * T_w constant and invariant
  * $\vec{v} = \begin{pmatrix} v_x(x,y,z) \\ v_y(x,y,z) \\ 0 \end{pmatrix}$ => In plane convection
  * Normal to plane conduction
  * Lubrication



- Local temperature law
  $\Rightarrow T - T_w = \frac{f(x,y)}{h^2(x,y)}\left(a_4 z^4 + a_2 z^2 + a_0\right)$

Averaging over thickness

2D-TEMPERATURE LAW

- Energy conservation $\overline{T}(x,y) = \frac{\int_h v(x,y,z)T(x,y,z)dz}{\int_h v(x,y,z)dz}$
- Boundary flow $\varphi_w = -\lambda\frac{\partial T}{\partial z}\Big|_{h/2}$

➢***Equation to be solved (2D)***

$$\boxed{\vec{u}.\vec{\nabla}\overline{T} + \frac{2\chi}{h^2(x,y)}Nu(\overline{T} - T_w) = 0}$$

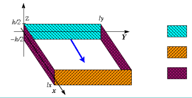
with $h, \vec{u}, \overline{T}$ depend on *(x,y)*; $\vec{u}(x,y) = \frac{\int_{h(x,y)} \vec{v}(x,y,z)\,dz}{h(x,y)}$

Convection — Conduction

➢Nusselt number

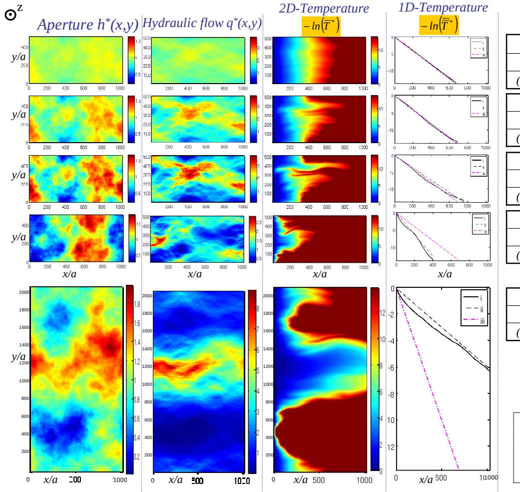
$$Nu = \frac{-\varphi_w}{\varphi_{ref}} = \frac{70}{17} \approx 4.12 \quad \text{where} \quad \varphi_{ref} = \lambda\frac{T_w - \overline{T}}{h}$$

BOUNDARY CONDITIONS



- ▨ $P = P_0$ and $\bar{T} = T_0$
- ▨ $P = P_L$ and $\bar{T} = T_w$
- ▨ $\nabla P = 0$ and $\bar{T} = T_w$

RESULTS ILLUSTRATION



Characteristics
RMS : Root Mean Square of the aperture
<h> : mean aperture

Same aperture features

(RMS , <h>)	*(0.05 , 1)*
$<h_h^*>$	*1.00*
$(l_{ref}^*, <h_{th}^*>)$	*(0.97 , 0.99)*

(RMS , <h>)	*(0.15 , 1)*
$<h_h^*>$	*0.98*
$(l_{ref}^*, <h_{th}^*>)$	*(0.98 , 1.00)*

(RMS , <h>)	*(0.25 , 1)*
$<h_h^*>$	*0.94*
$(l_{ref}^*, <h_{th}^*>)$	*(1.09 , 1.02)*

(RMS , <h>)	*(0.35 , 1)*
$<h_h^*>$	*0.82*
$(l_{ref}^*, <h_{th}^*>)$	*(0.61 , 0.89)*

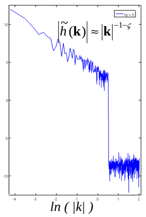
(RMS , <h>)	*(0.35 , 1)*
$<h_h^*>$	*1.02*
$(l_{ref}^*, <h_{th}^*>)$	*(3.36 , 1.35)*

Other aspect ratio ; case requiring a pressure and temperature boundary conditions further away along *x*

- i : $-\ln(\overline{\overline{T}}^*)$ *for rough fracture*
- ii : *Model line of* i
- iii : *Model line for parallel plates separated by* $<h_h^*>=1$

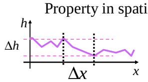
ROUGH FRACTURE : SELF AFFINE APERTURE *(e.g. Ref. [5])*

Aperture : $h(x,y)$
2D Fourier transform : $\tilde{h}(\mathbf{k})$



ζ : Roughness exponent
ζ ≈ 0.8

Property in spatial domain



$$\lambda^\zeta g\left(\lambda^\zeta \Delta h, [\lambda\Delta x, \lambda\Delta y]\right) = g\left(\Delta h, [\Delta x, \Delta y]\right)$$

Where *g* gives the probability to get a height difference *Δh* between two coordinate points separated by [*Δx*,*Δy*].

1D-TEMPERATURE :

$$\overline{\overline{T}}(x) = \frac{\int_{l_y} u_x(x,y)\overline{T}(x,y)dy}{\int_{l_y} u_x(x,y)dy}$$

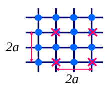
Using the parallel plates reference solutions,

- l_{ref} *for rough apertures is determined by computing the slope of* $-\ln(\overline{\overline{T}})$ *vs x.*
- *Then it is possible to define a thermal aperture* h_{th} *equivalent at coarse grained scale with :*

$$l_{ref} = \frac{(h_{th})^4}{24Nu \cdot \eta \cdot \chi} \cdot \frac{\Delta P}{L_x}$$

FINITE DIFFERENCES NUMERICAL SCHEME

- Discretization



- ● Apertures
- × Temperatures, Hydraulic flux

a : Aperture mesh size

$$\boxed{\vec{\nabla}.(h(x,y)^3\vec{\nabla}P) = 0}$$

$$h^3_{i+0.5,j}(P_{i+1,j} - P_{i,j}) + h^3_{i-0.5,j}(P_{i-1,j} - P_{i,j}) + h^3_{i,j+0.5}(P_{i,j+1} - P_{i,j}) + h^3_{i,j-0.5}(P_{i,j-1} - P_{i,j}) = 0$$

$$\boxed{\vec{u}.\vec{\nabla}\overline{T} + \frac{2\chi}{h^2}Nu(\overline{T} - T_w) = 0}$$

$$u^x_{i,j}(\overline{T}_{i+1,j} - \overline{T}_{i-1,j}) + u^y_{i,j}(\overline{T}_{i,j+1} - \overline{T}_{i,j-1}) + \frac{8a\,\chi\,Nu}{h^2_{i,j}}(\overline{T}_{i,j} - \overline{T}_w) = 0$$

- Systems solved with the biconjugate gradient method
- Use of dimensionless variables : h^*, q^*, T^*, …

RANGE OF VALIDITY

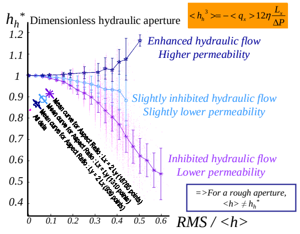
- ρ constant
  Low thermal dilatation coefficient
- *z*-Conduction >> *(x,y)*-Conduction
  Re >> 1.2
- *x*-Conduction << *x*-Convection
  Re >> 0.43
- Lubrication
  Re << 10²
- No heat source because of viscosity
  $Ec = \frac{u^2}{c(T_w - T_0)} << 4.1\,10^{-3}$
  Re << 4.10⁴ for ΔT=100 C

STATISTICAL RESULTS WITH

- Various realizations
- Various RMS (Root Mean Square of the aperture)
- Same mean aperture *<h>*
- Various aspect ratios

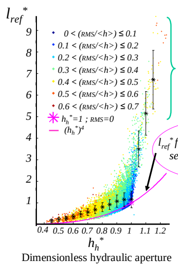
HYDRAULIC APERTURE

$<h_h^3> = -<q_x> 12\eta\frac{L_x}{\Delta P}$



=>*For a rough aperture,* $<h> \neq h_h^*$

THERMAL LENGTH



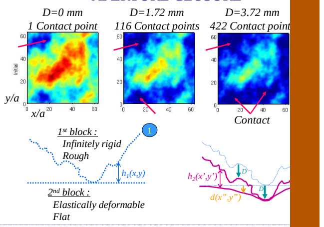
On average, l_{ref}^* *for rough apertures is superior to* l_{ref}^* *for parallel plates with the same hydraulic aperture :*
The "thermalization" is inhibited

l_{ref}^* *for parallel plates separated by* h_h^* *:*
$l_{ref}^* = (h_h^*)^4$

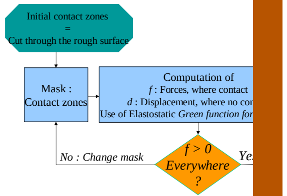
=> *For a rough aperture,* $h_h^* \neq h_{th}^*$

ELASTIC CLOSURE OF THE FRAC
HYDRO-MECHANICAL COUPLIN

APERTURE CLOSURE



MECHANICS : METHOD (Ref. [1])



Initial contact zones
=
Cut through the rough surface

Mask :
Contact zones

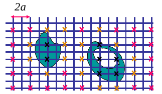
Computation of
f : Forces, where contact
d : Displacement, where no con
Use of Elastostatic *Green function for*

f > 0
Everywhere
?

No : Change mask

Ye

HYDRO-COUPLING (ongoing)



- ▦ *Aperture grid*
- × *No connected*
- × *Boundary pres*
- × *Other pressure*

CONCLUSION

- Implications of fracture roughness on ***hydraulic*** properties
  * Channeling
  * Aspect ratio influence on the hydraulic aperture:
    ○ On average :
      ❖ $L_y > L_x$: Enhanced hydraulic flow
      ❖ $L_y = L_x$: Slightly inhibited hydraulic flow
      ❖ $L_y < L_x$: Inhibited hydraulic flow
    ○ With high variability
  * Possible explanation :
    ○ Parallel channels, perpendicular to the flow ($L_x \geq L_y$)
    ○ Parallel Channels, parallel to the flow ($L_x < L_y$)
- Implications of fracture roughness on ***thermal*** properties
  * Channeling
  * On average inhibited heating of the fluid
  * Definition of a thermal aperture for a coarse grained equivalence

PROSPECTS

- Improvement of boundary conditions
  * Mechanics : variable contact zone (fracture closu
  * Variations of the host rock temperature
  * Fracture geometry : Anisotropy of the aperture (g anisotropic features)
- Application to real apertures
- Which Fourier modes are dominating the flows ?
- Using *ρ(T)* dependence allowing natural convection
- Lattice Boltzmann implementation
  * Full Navier Stokes equation
  * 3D-aspect
  * Time dependence

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