Simulation of NMR relaxation time distribution for a fractal cluster of pores with surface relaxation, dephasing and diffusion

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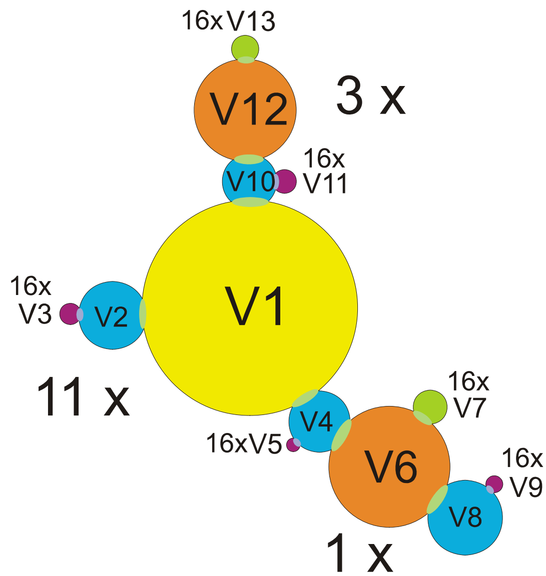
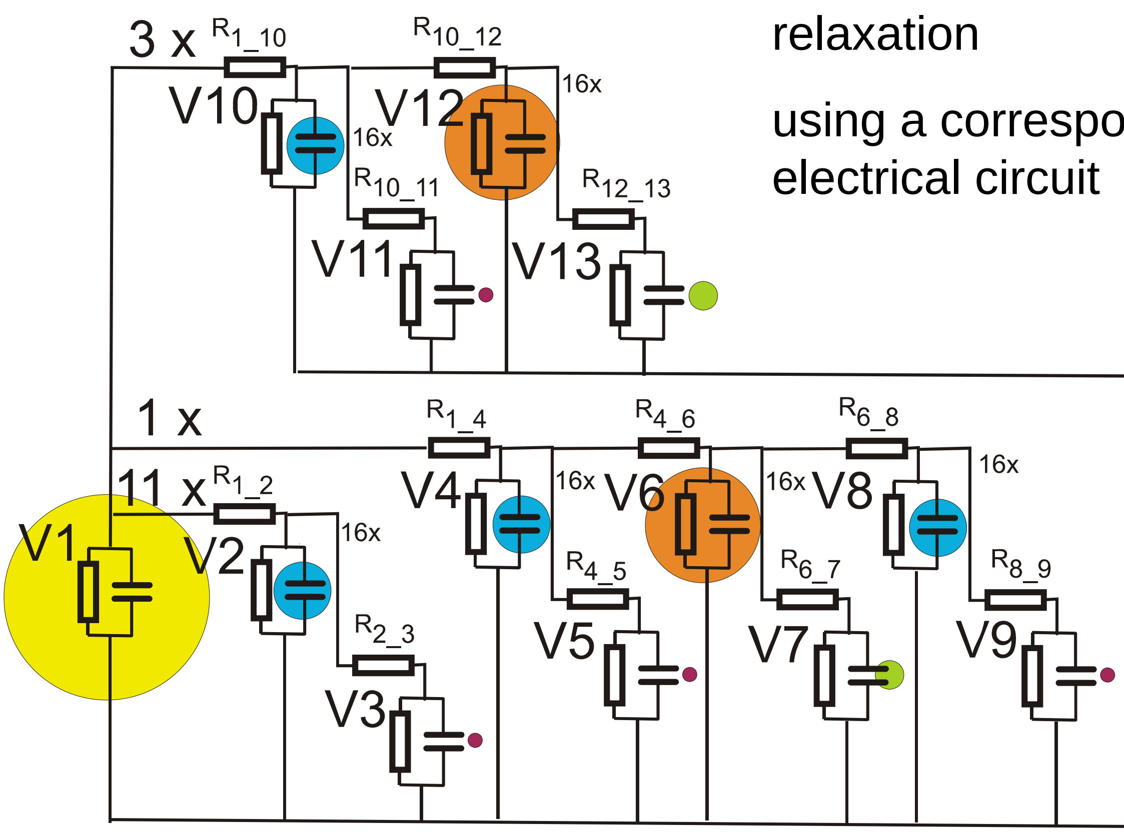
Simulation of NMR-relaxation

Start with a pore radius distribution and a model of diffusion coupled pores

End with a relaxation time distribution



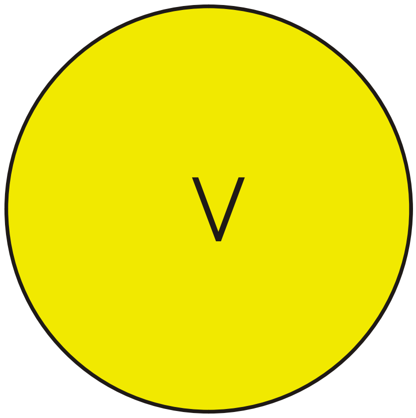
cluster of pores

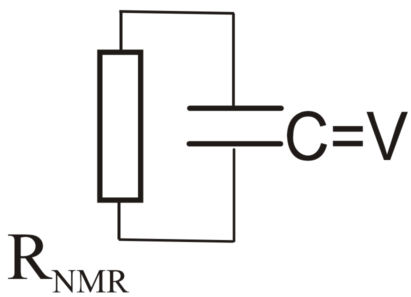


simulation of NMR-relaxation

using a corresponding electrical circuit

Simulation of surface relaxation for one pore class and longitudinal magnetization





corresponding electrical circuit

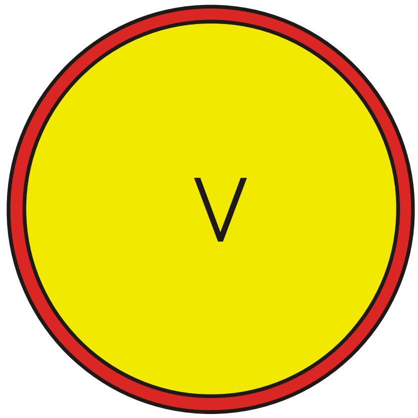
$$R_{NMR} = \rho_1^{-1}/S$$

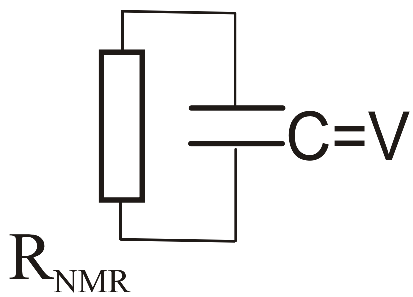
$$T = R\,C = (\rho_1^{-1}/S)V$$

$$= (\rho_1^{-1} r/3)$$

relaxation time

Simulation of surface relaxation for one pore class and transverse magnetization





corresponding electrical circuit

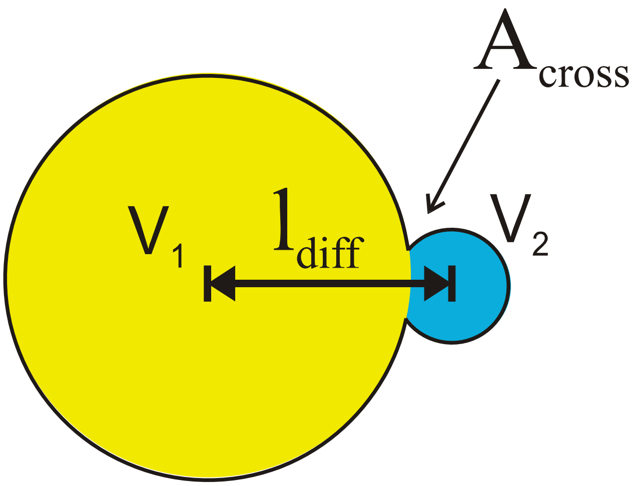
$$R_{NMR} = \rho_2^{-1}/S$$

$$T = R\,C = (\rho_2^{-1}/S)V$$
$$= (\rho_2^{-1} r/3)$$

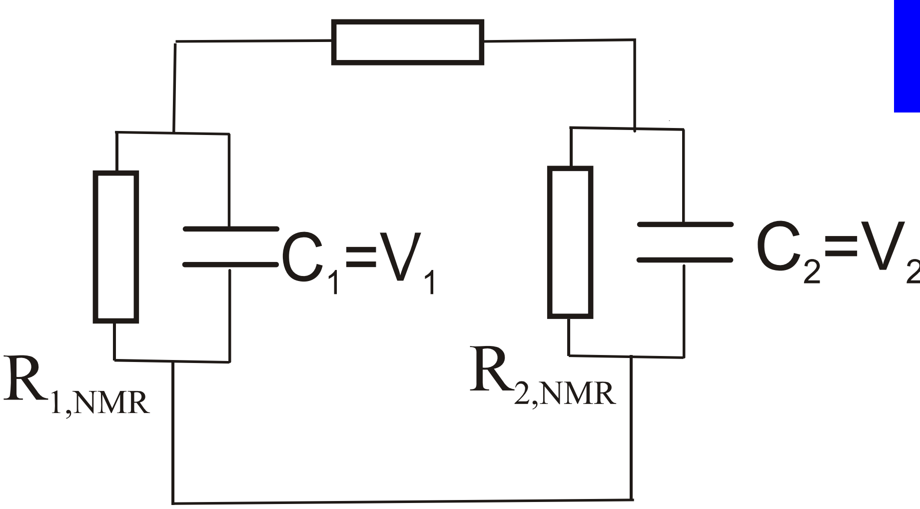
relaxation time

$$\rho_2 = 5.85\rho_1(9.53/r)^{0.43}$$

Simulation of surface relaxation and diffusion for two connected pores



$$R_{1\_2} = D^{-1}\, l_{diff}/A_{cross}$$



corresponding electrical circuit

NMR relaxation

$$P(t) = \sum_{i=1}^{n} P_i$$

relaxation curve of pore cluster

normed to $P_{t=0} = 1$

for isolated pores

$$P_i = V_i \exp\left(-\frac{t}{T_i}\right)$$

relative volumes V_i with

$$\sum_{i=1}^{n} V_i = 1$$

T_i is a function of pore radius r_i

for connected pores

$$P_i = \sum_{j=1}^{n} b_{ij} \exp\left(-\frac{t}{T_j}\right)$$

$$\sum_{j=1}^{n} b_{ij} = V_i$$

The relaxation times T_j are common for all connected pores, however the coefficients b_{ij} are different.

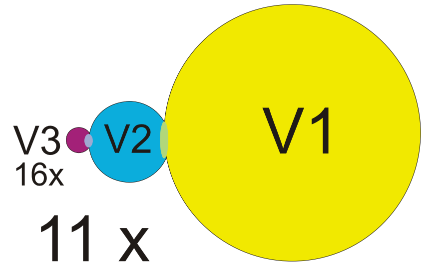
Some b_{ij} may be negative.

relaxation curve of pore cluster
from coefficients of connected pores

$$P(t) = \sum_{j=1}^{n} \left(\sum_{i=1}^{n} b_{ij}\right) \exp\left(-\frac{t}{T_j}\right)$$

analytical calculation of NMR relaxation by solving a system of linear differential equations

example for a cluster of three pore classes



$$Y' = AY$$

$$Y' = \begin{matrix} P_1' \\ P_2' \\ P_3' \end{matrix} \qquad Y = P_1 \quad P_2 \quad P_3$$

$$A =$$

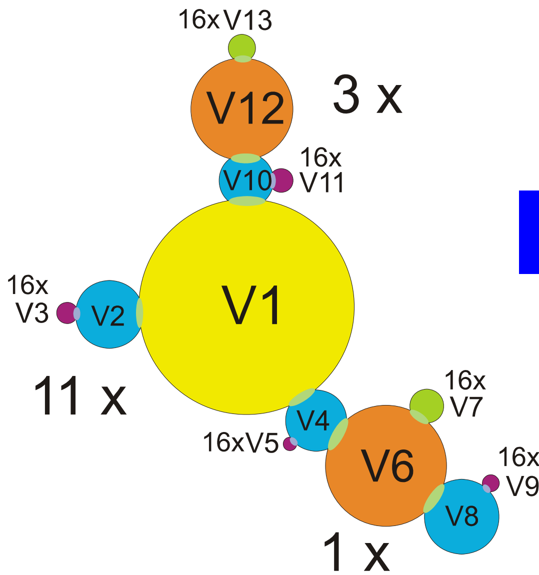
$$\begin{matrix}
\frac{\left(\left(R_{NMR,1}\right)^{-1} + 11\left(R_{diff,1_2}\right)^{-1}\right)}{C_1} & \frac{-11\left(R_{diff,1_2}\right)^{-1}}{C_1} & 0 \\
\frac{-\left(R_{diff,1_2}\right)^{-1}}{C_2} & \frac{\left(\left(R_{NMR,2}\right)^{-1} + \left(R_{diff,1_2}\right)^{-1} + 16\left(R_{diff,2_3}\right)^{-1}\right)}{C_2} & \frac{-16\left(R_{diff,2_3}\right)^{-1}}{C_2} \\
0 & \frac{-\left(R_{diff,2_3}\right)^{-1}}{C_3} & \frac{\left(\left(R_{NMR,3}\right)^{-1} + \left(R_{diff,2_3}\right)^{-1}\right)}{C_3}
\end{matrix}$$

$$R_{NMR,i} = \left(\rho S_i\right)^{-1}$$

$$R_{diff,1\_2} = D^{-1} l_{1\_2} A_{cross,1\_2}$$

$$R_{diff,2\_3} = D^{-1} l_{2\_3} A_{cross,2\_3}$$

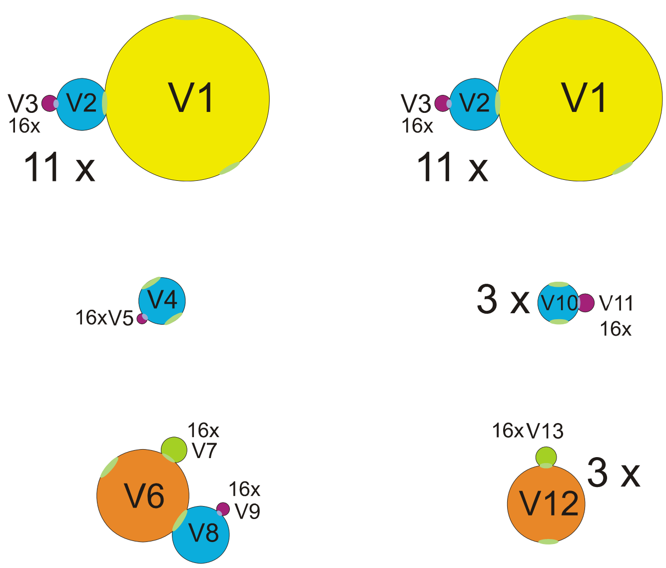
relaxation time distribution for sub-clusters



clusters

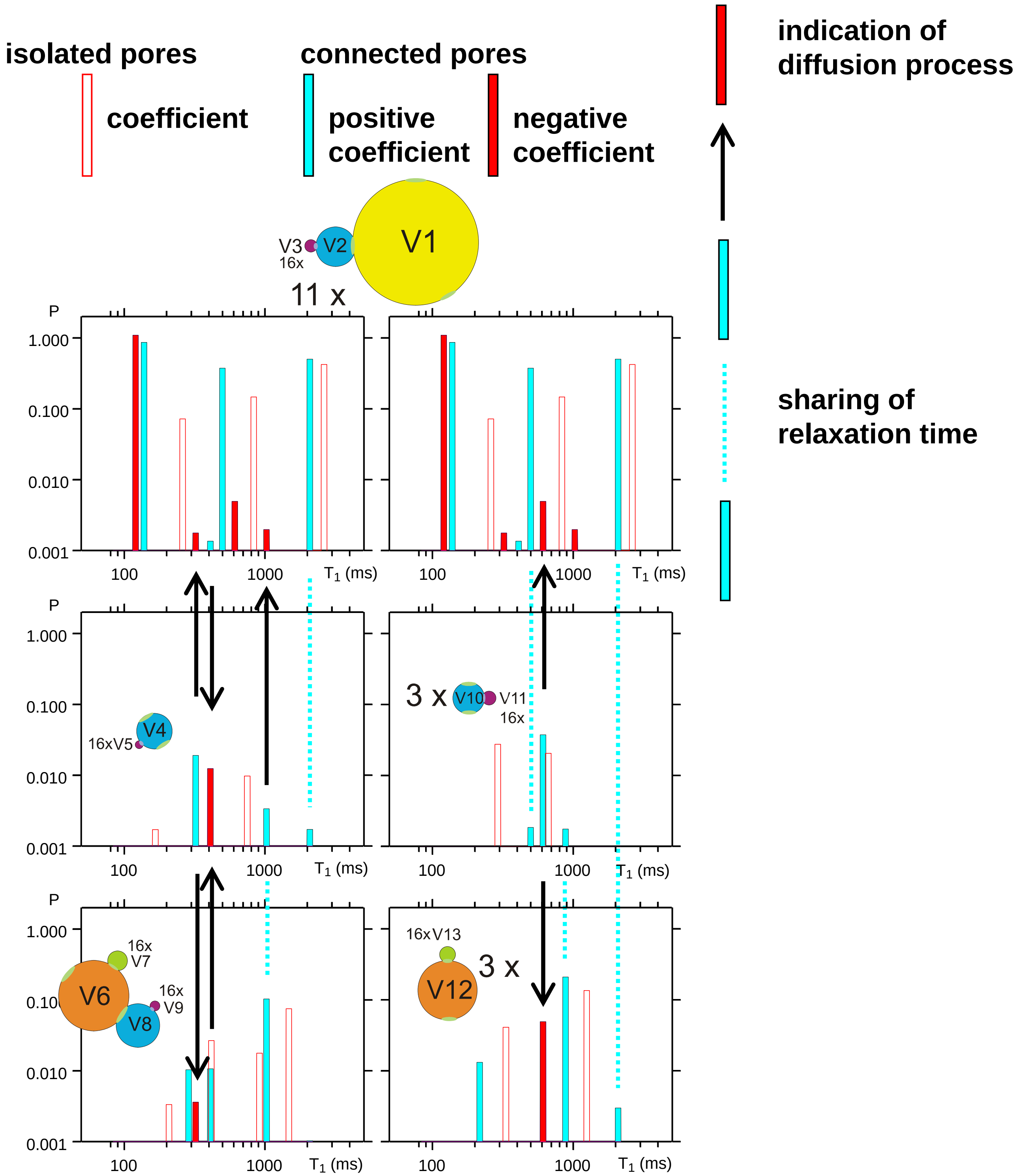
$$P_{k,l} = \sum_{i=k}^{l} P_i = \sum_{j=1}^{n} \left(\sum_{i=k}^{l} b_{ij} \right) \exp\left(-\frac{t}{T_j} \right)$$

relaxation function of sub-cluster



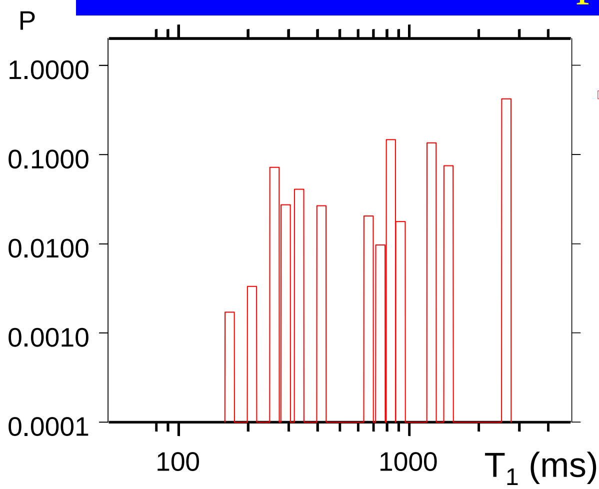
sub-clusters

relaxation time distribution for longitudinal magnetization

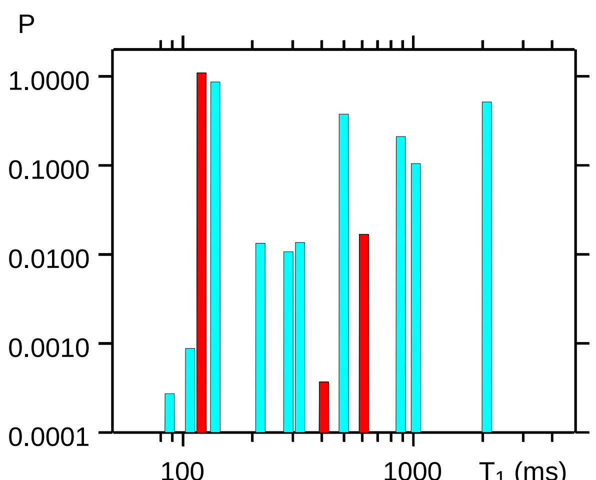


relaxation time distribution
for longitudinal magnetization

for the complete cluster

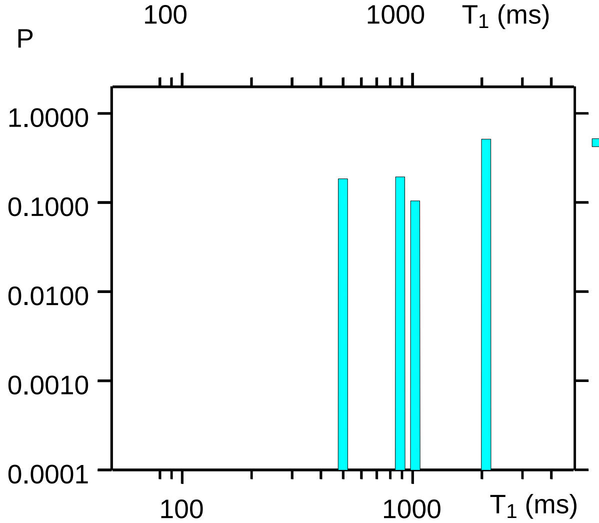


coefficients of isolated pores



positive / negative { coefficients of cluster

lines are shifted to smaller relaxation times

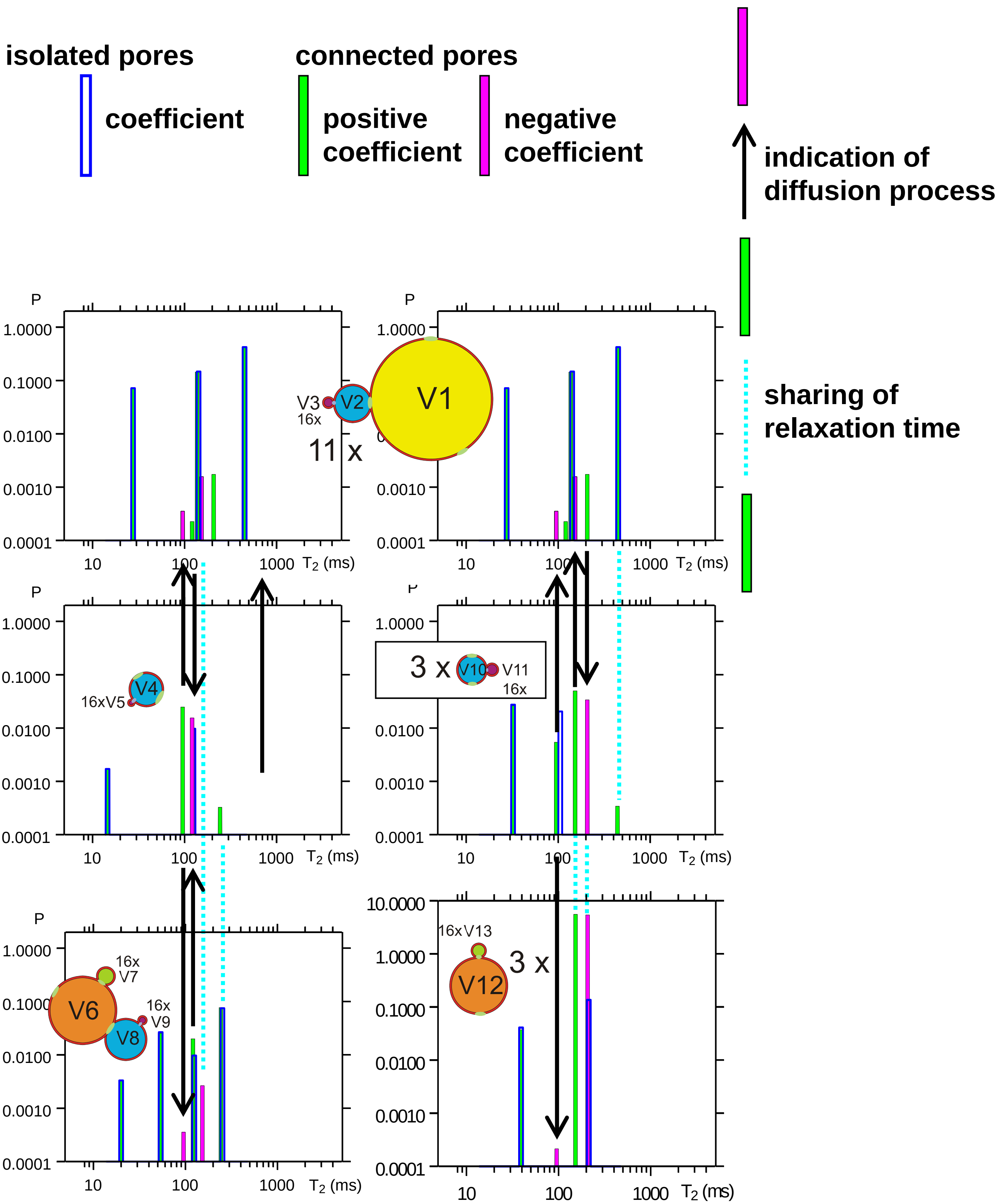


approximate coefficients of cluster

the resulting spectrum is compressed

similar results by Arns et al.,2005
on the base of digitized core images
and NMR response simulation by random walkers

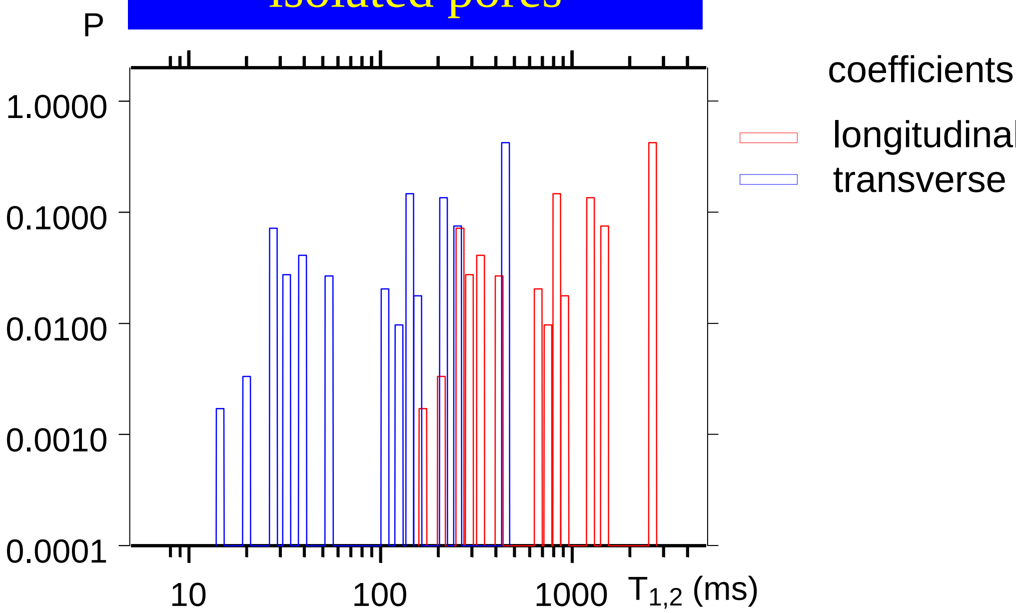
relaxation time distribution for transverse magnetization



relaxation time distribution

transverse magnetization compared to longitudinal magnetization

isolated pores



pores coupled by diffusion