Effect of the intermediate principal stress on fault strike and dip - theoretical analysis and experimental verification

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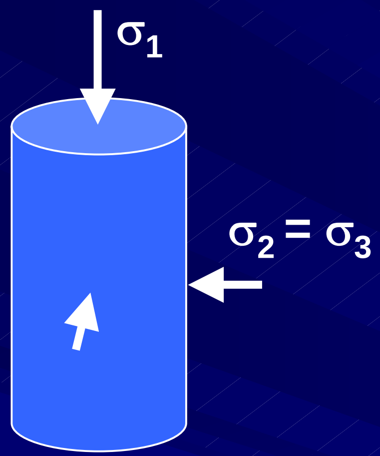
“We experimentalists are not like theorists: the originality of an idea is not for being presented in a paper but for being shown in implementation of an original experiment.”

Patrick M. S. Blackett, London, 1962

(from plaque outside lecture hall)

Conventional Triaxial Testing



Strength Criterion

$$\sigma_1 = f(\sigma_3)$$

$$q = g(p),\ q = \frac{1}{2}(\sigma_1 - \sigma_3), p = -\frac{1}{2}(\sigma_1 + \sigma_3)$$

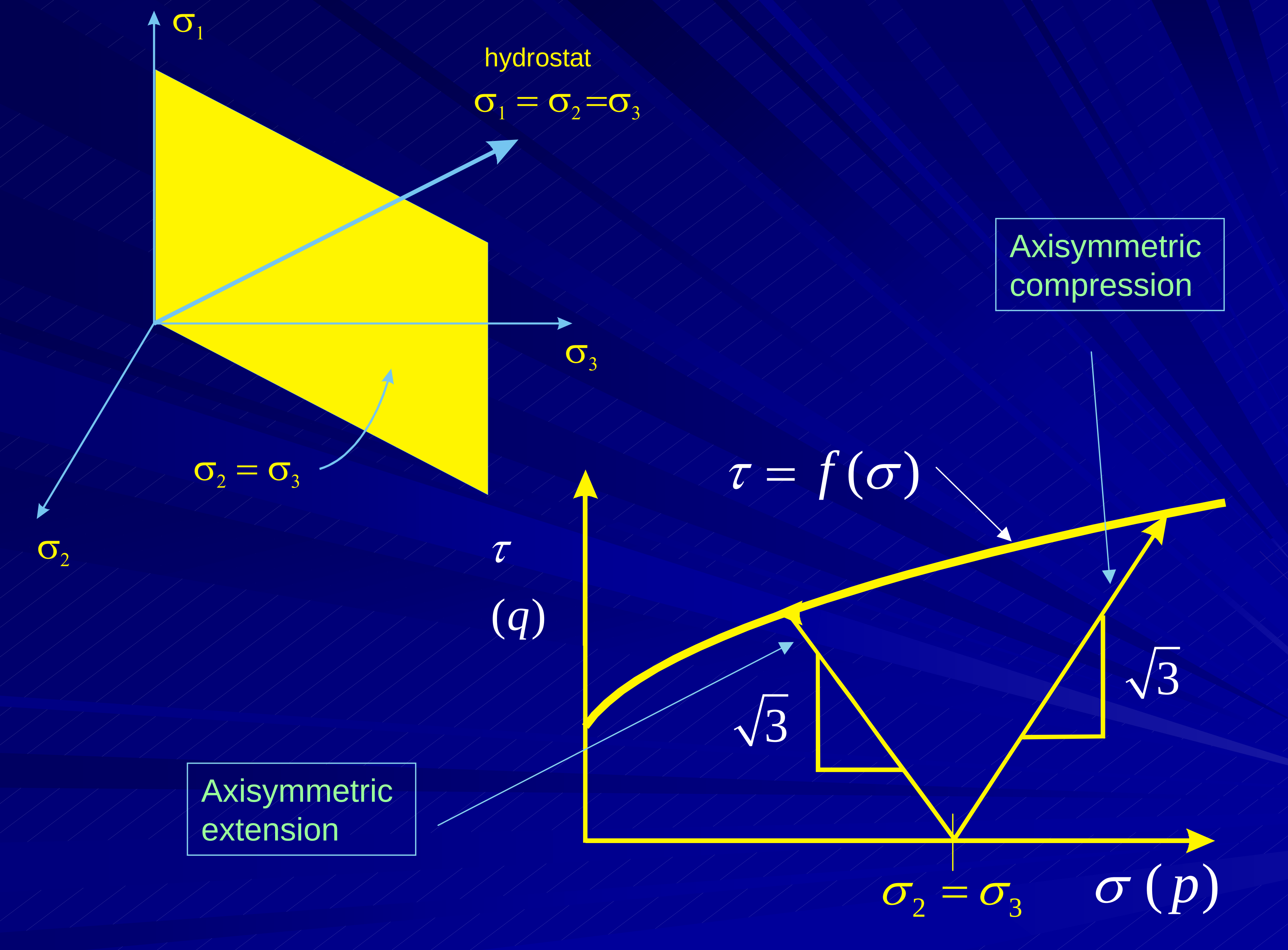
$$\tau = h(\sigma), \quad \sigma = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$\tau = \sqrt{s_{ij}s_{ij}/2}$$

$$= \sqrt{\frac{1}{6}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}}$$

Since, $\sigma_2 = \sigma_3$

$$\tau = \frac{2}{\sqrt{3}}q, \quad \sigma = p + \frac{1}{3}q$$



σ_1
hydrostat
$\sigma_1 = \sigma_2 = \sigma_3$
σ_3
$\sigma_2 = \sigma_3$
σ_2
Axisymmetric compression
$\tau = f(\sigma)$
τ
(q)
$\sqrt{3}$
$\sqrt{3}$
Axisymmetric extension
$\sigma_2 = \sigma_3$
$\sigma\,(p)$

What if $\sigma_2 \neq \sigma_3$?

Ans.: Make assumptions, e.g.

Mohr Coulomb:

$$\frac{1}{2}(\sigma_1 - \sigma_3) + \mu_c \frac{1}{2}(\sigma_1 + \sigma_3) = c$$

No dependence on σ_2 !

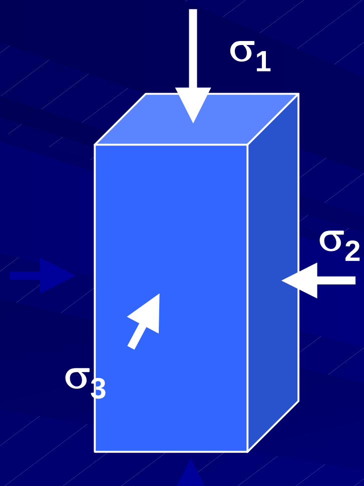
Drucker-Prager (Rudnicki-Rice):

$$\tau = h(\sigma)$$

No dependence on 3rd stress invariant!

$$J_3 = \det(s_{ij}),\ s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$$

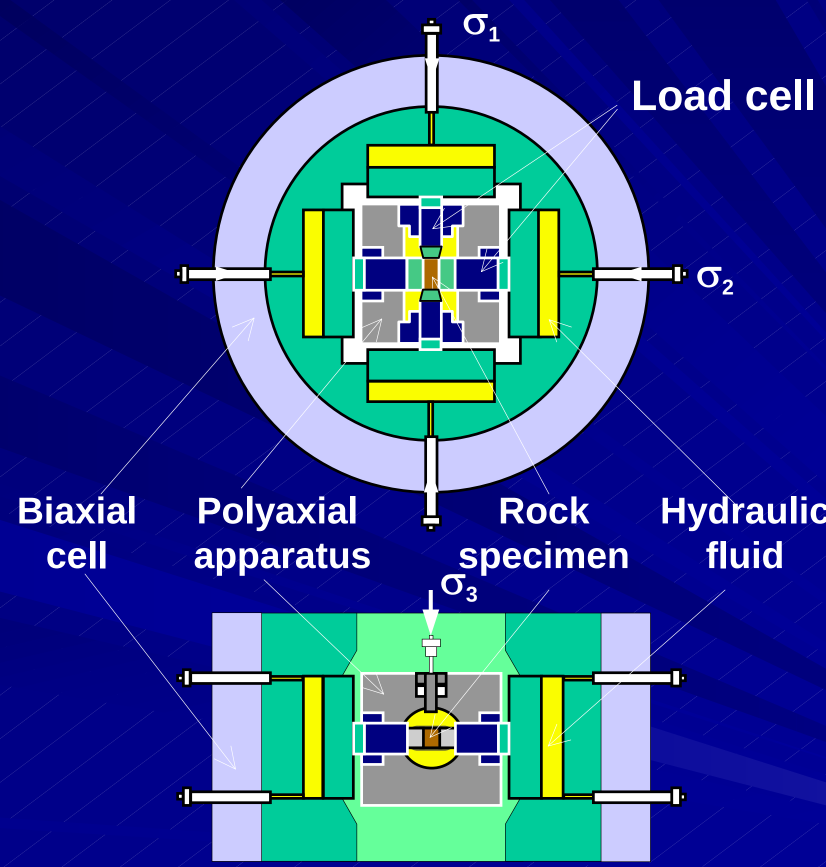
True- Triaxial (polyaxial) testing



Strength criterion:

$\sigma_1 = f(\sigma_2, \sigma_3)$

(Mises, Nadai, Mogi)



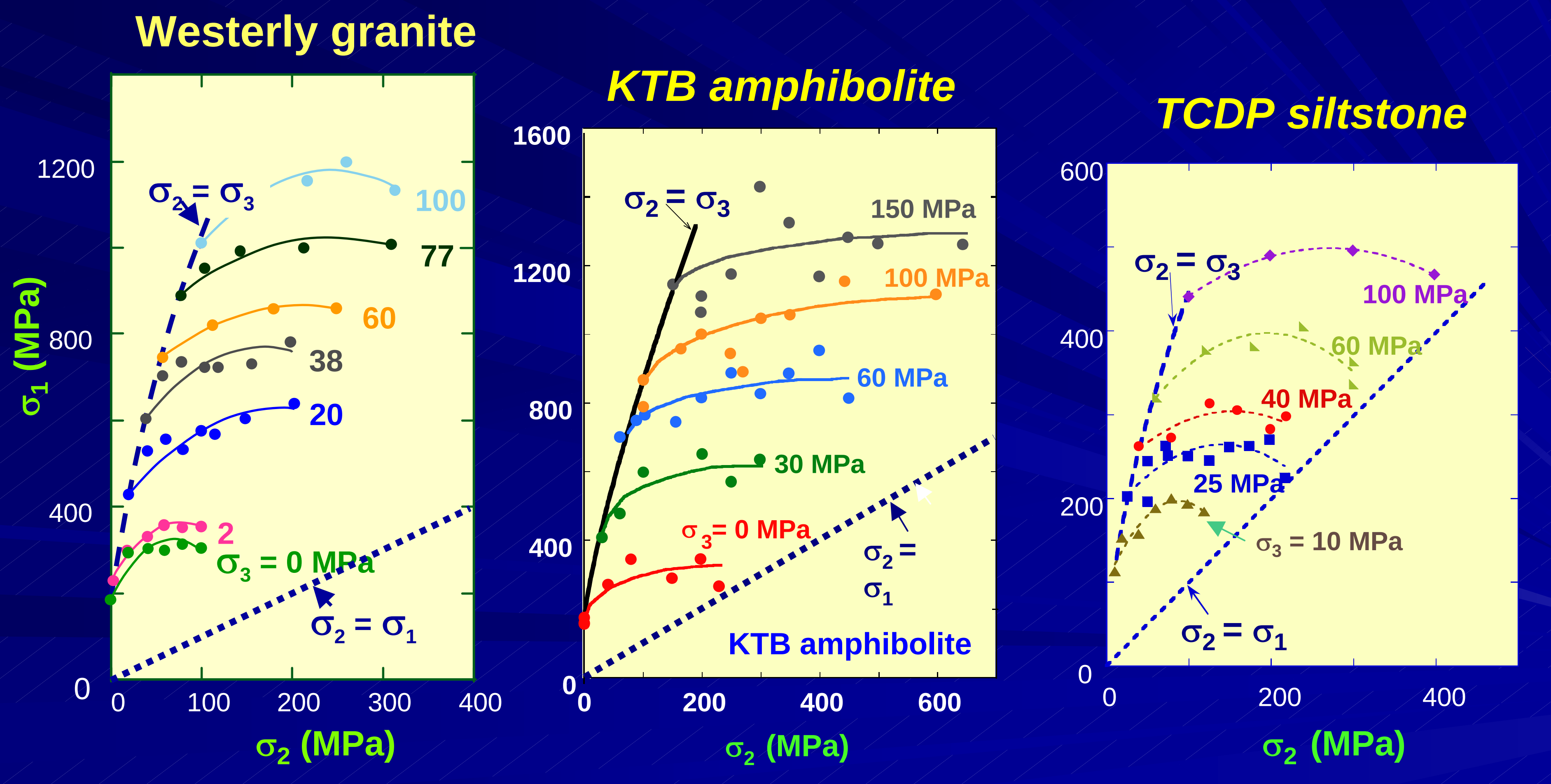
True-triaxial testing apparatus

Three rocks tested

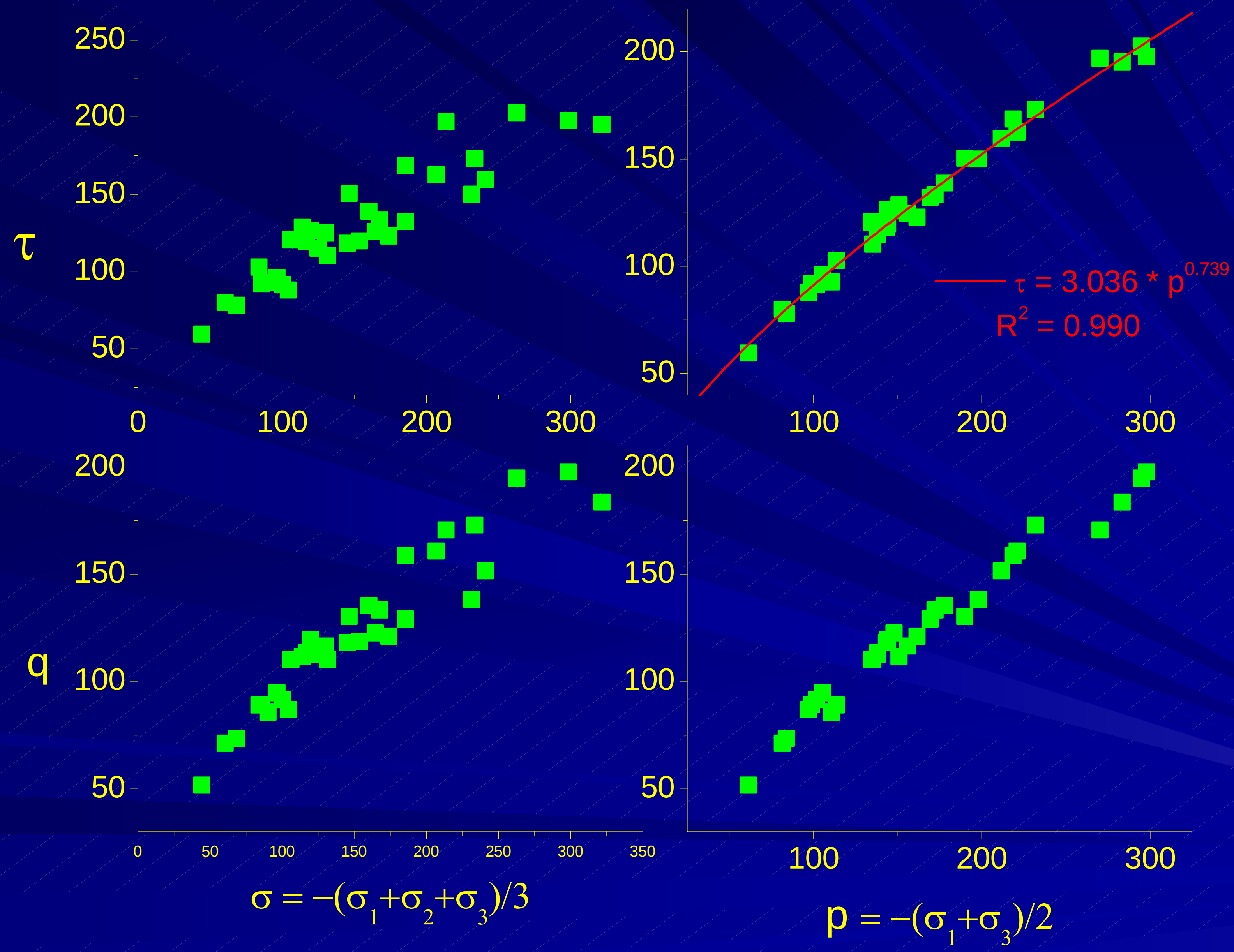
Mineral	Volume percentage (%)		
	Westerly granite (igneous)	KTB amphibolite (metamorphic)	TCDP siltstone (sedimentary)
Feldspar	66	25	10
Quartz	28		68
Clay			20
Mica	3	2	3
Amphibole		58	

Property	Westerly granite	KTB amphibolite	TCDP siltstone
Density, kg/m^3	2630	2920	2594
Porosity, %	0.9	0.7	6.9
UCS, MPa	201	164	80
Elastic Modulus, GPa	59	95	14

True triaxial strengths (peak σ_1) of three tested rocks

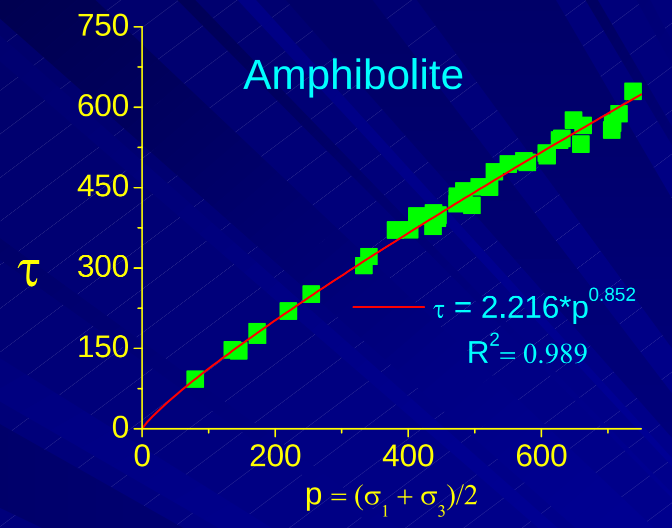


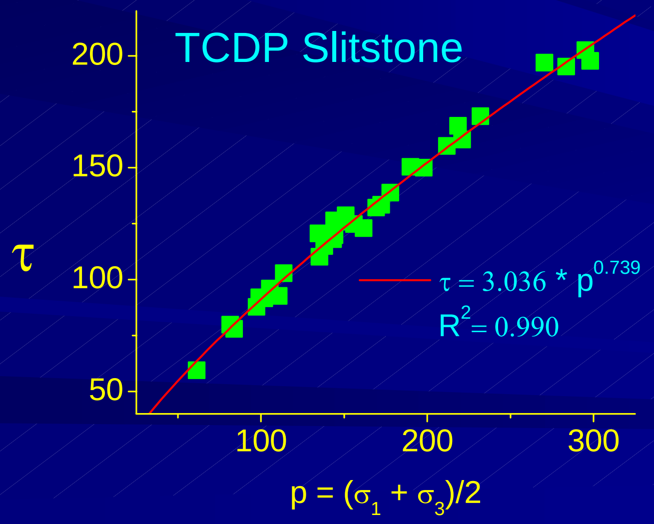
TCDP Siltstone

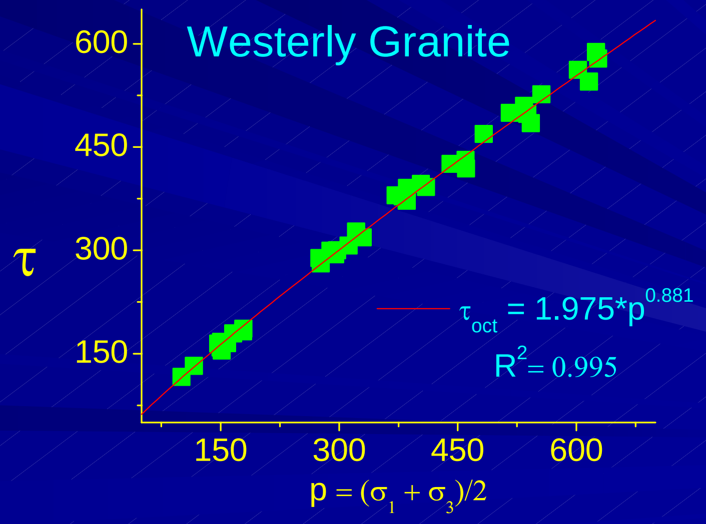


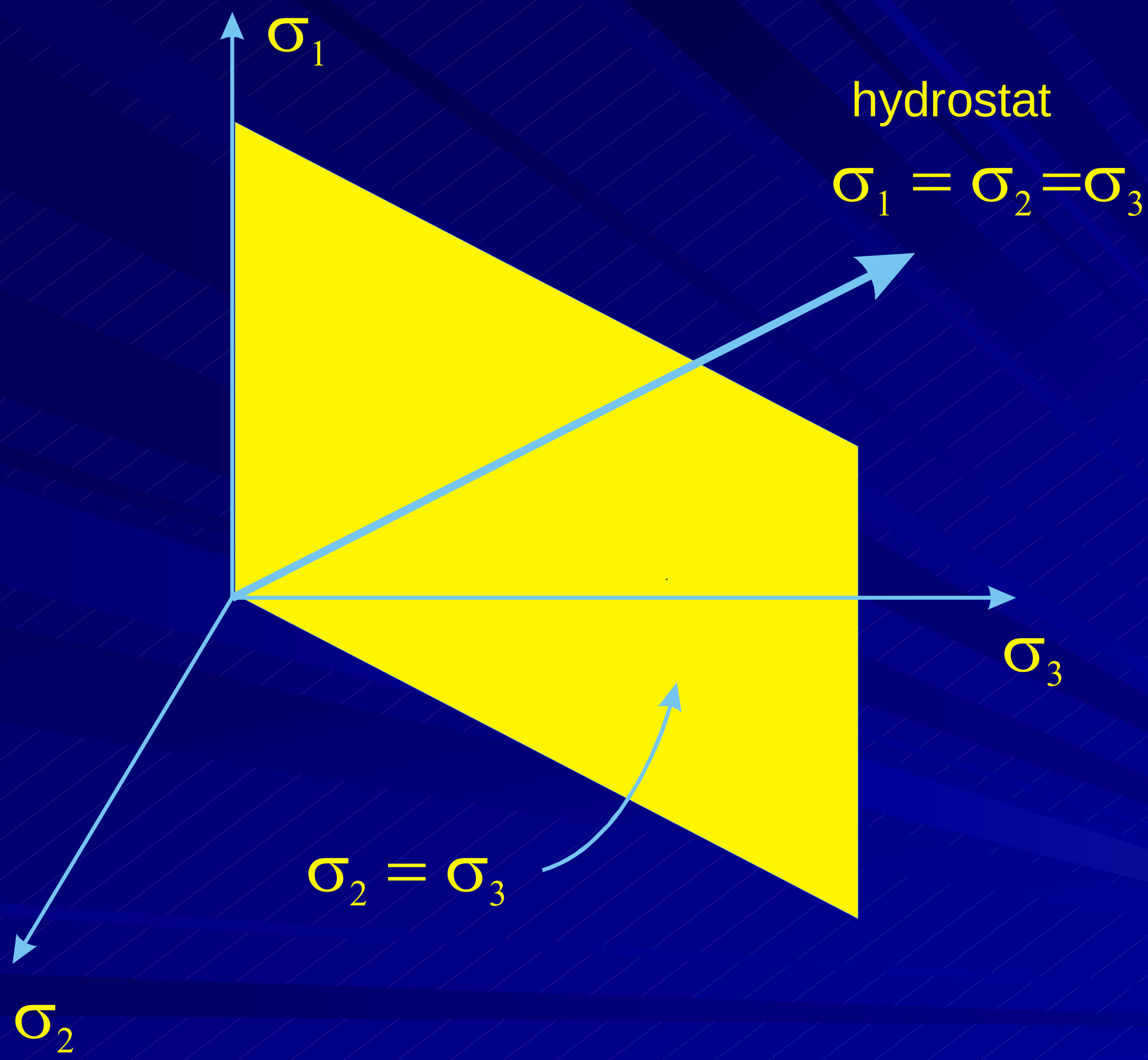
True triaxial strength criteria

(All stresses MPa)

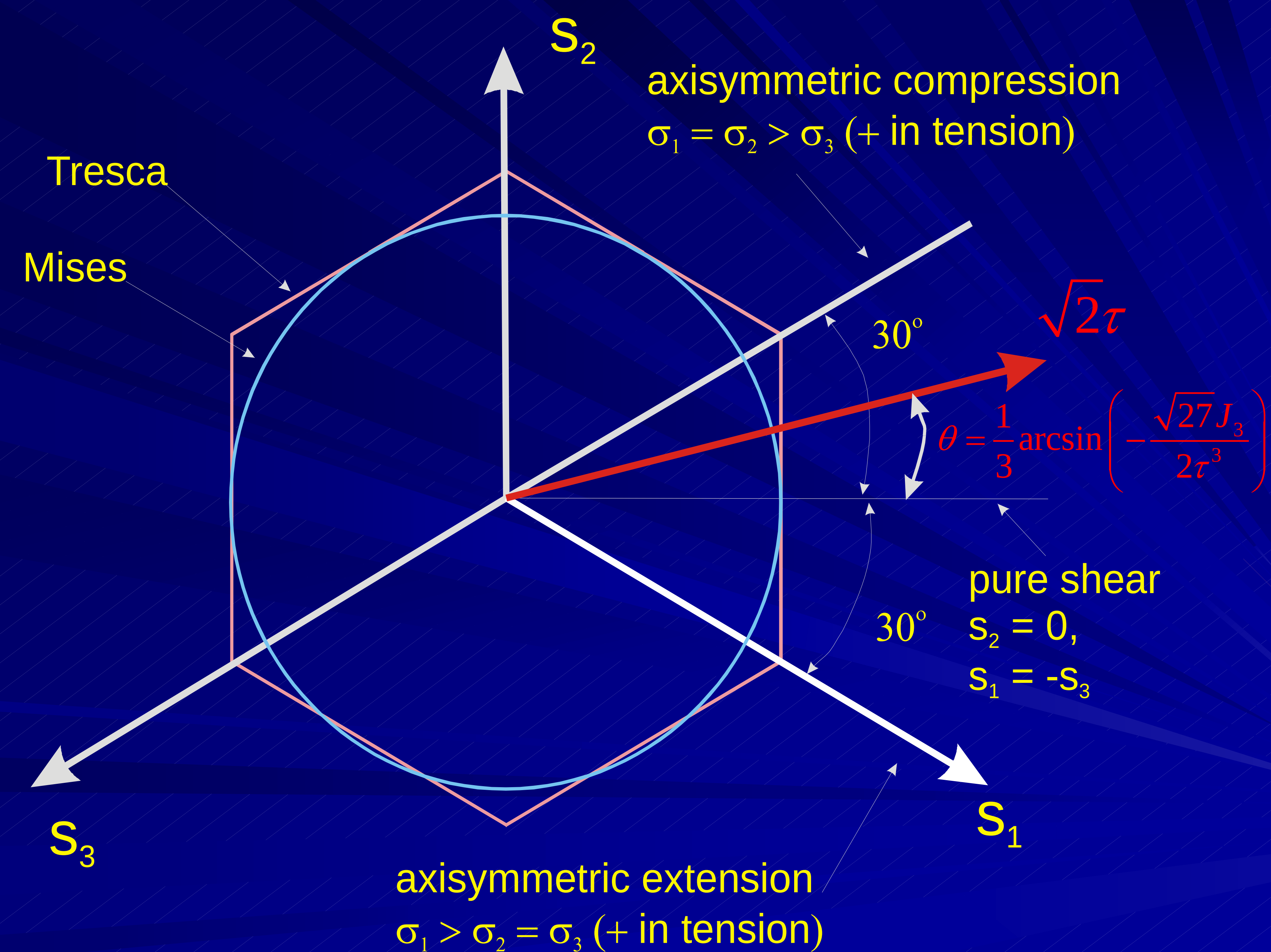






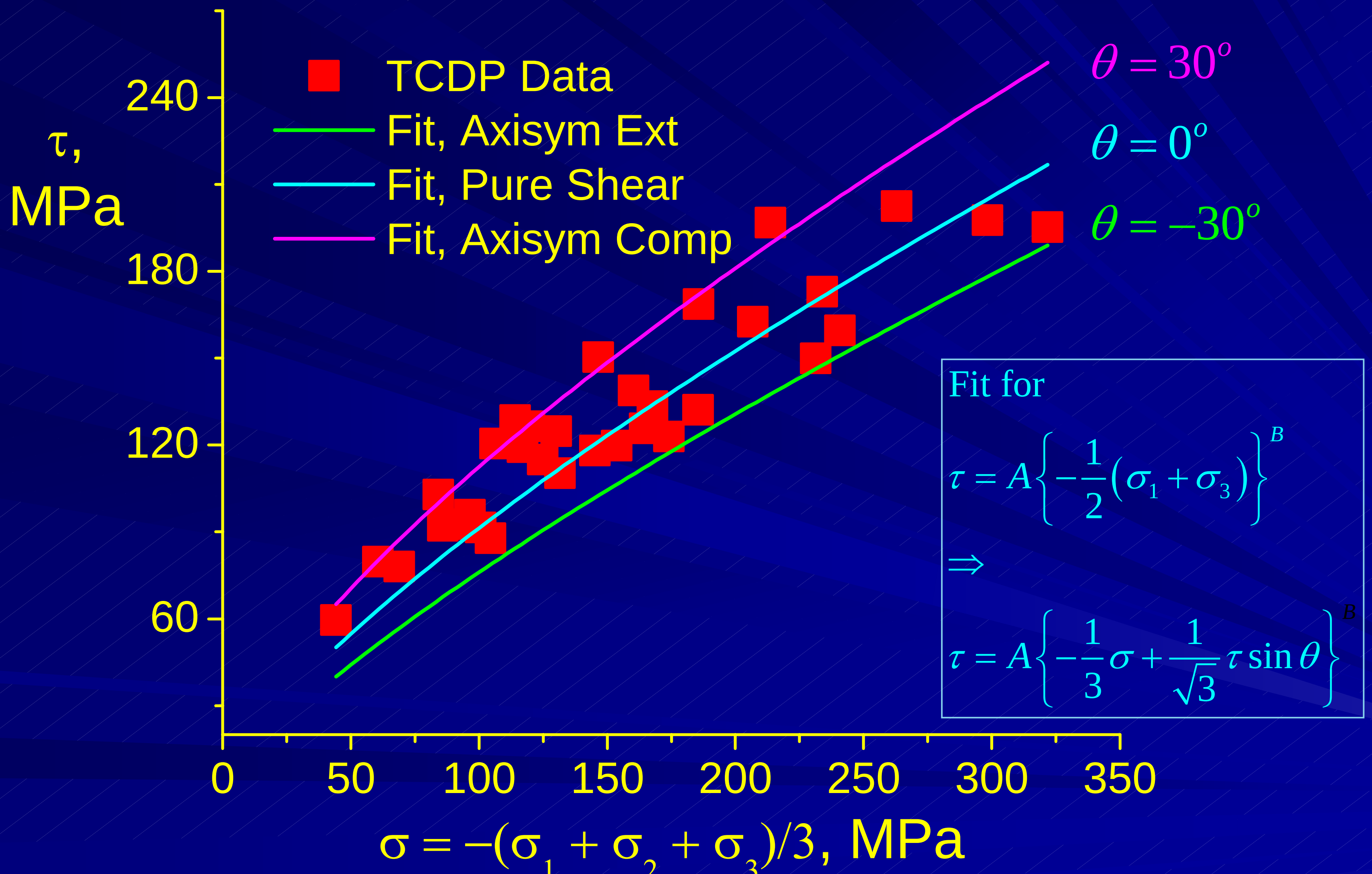


σ_1
hydrostat
$\sigma_1 = \sigma_2 = \sigma_3$
σ_3
$\sigma_2 = \sigma_3$
σ_2

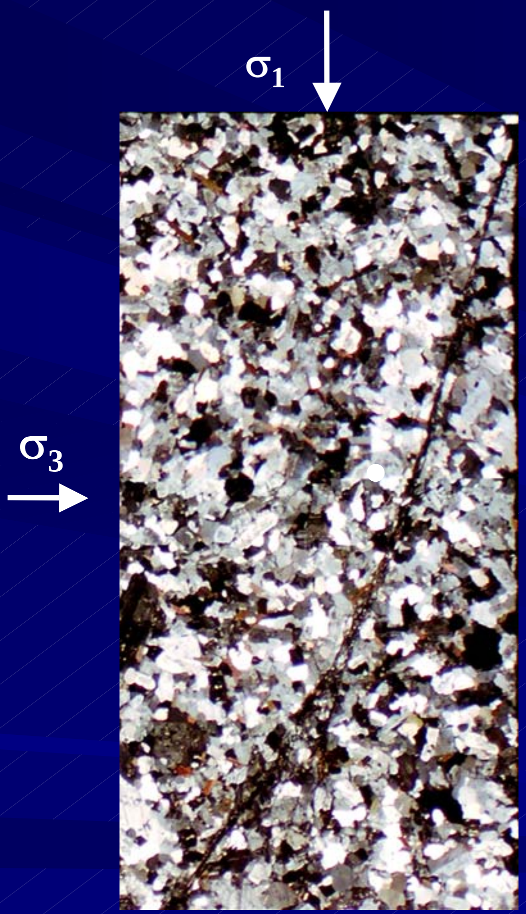


s_2
axisymmetric compression
$\sigma_1 = \sigma_2 > \sigma_3$ (+ in tension)
Tresca
Mises
$\sqrt{2}\tau$
30^o
$\theta = \frac{1}{3}\arcsin\left(-\frac{\sqrt{27}J_3}{2\tau^3}\right)$
pure shear
$s_2 = 0$,
$s_1 = -s_3$
30^o
s_3
s_1
axisymmetric extension
$\sigma_1 > \sigma_2 = \sigma_3$ (+ in tension)

TCDP Siltstone



Typical fault planes under true triaxial stress

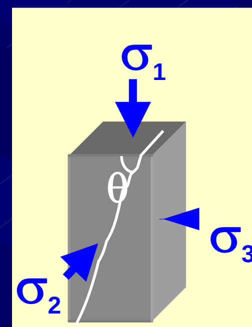


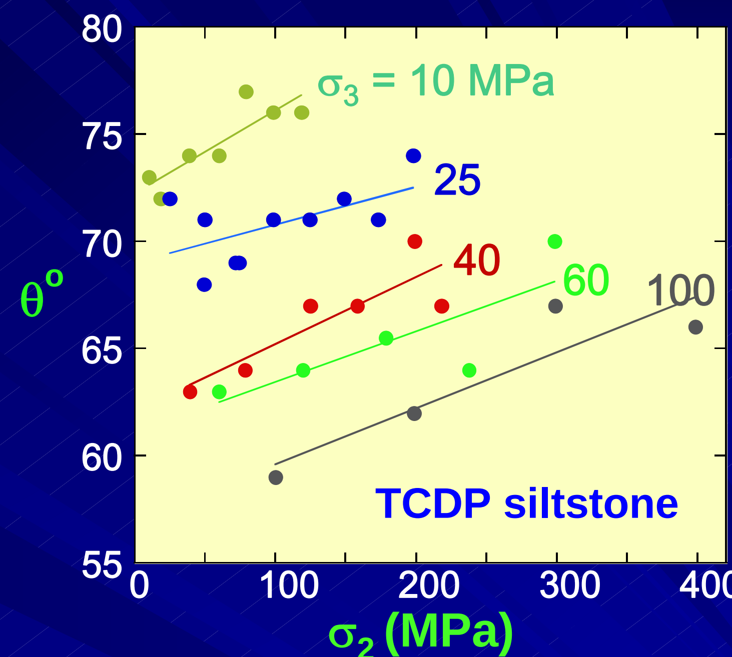
Westerly granite

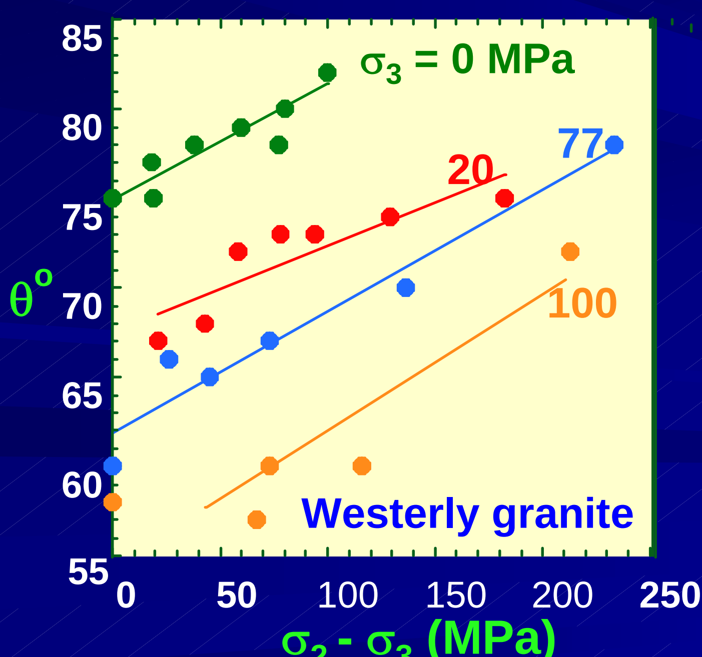
KTB amphibolite

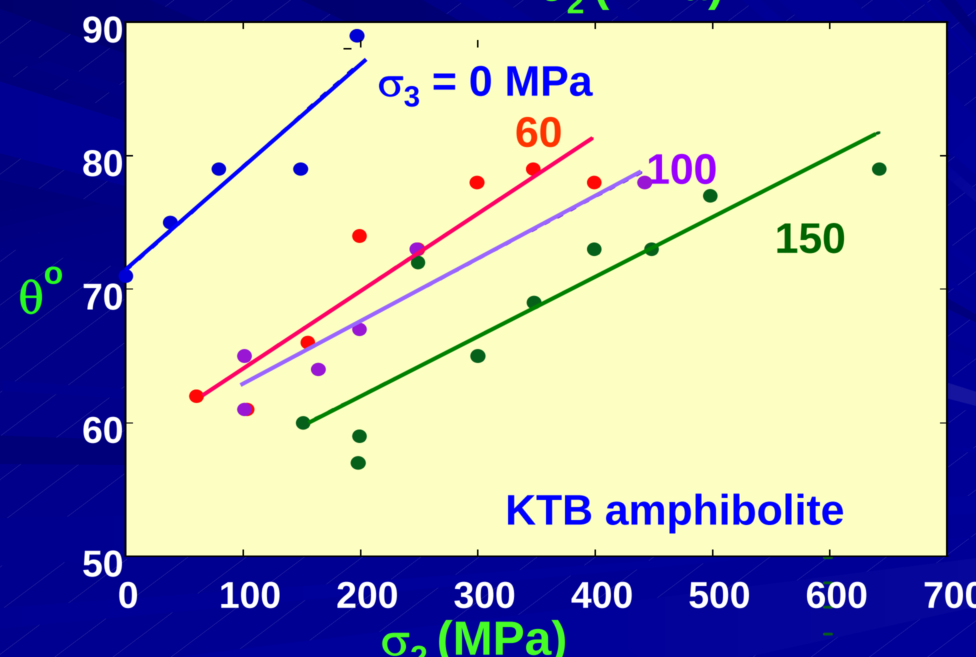
TCDP siltstone

Fracture dip angle increases with σ_2 (for given σ_3)









Band Angle Predictions

Mohr Coulomb: $\theta_{MC} = \dfrac{\pi}{4} + \dfrac{1}{2}\arctan \mu_{MC}$

Rudnicki-Rice: $\theta_{RR} = \dfrac{\pi}{4} + \dfrac{1}{2}\arcsin \alpha$

dilatancy factor

where

friction coefficient

$$\alpha = \frac{(2/3)(\beta + \mu) - N(1-2\nu)}{\sqrt{4-3N^2}}$$

Poisson's ratio

$$N = \frac{s_2}{\tau} = \frac{2}{\sqrt{3}}\sin(\theta)$$

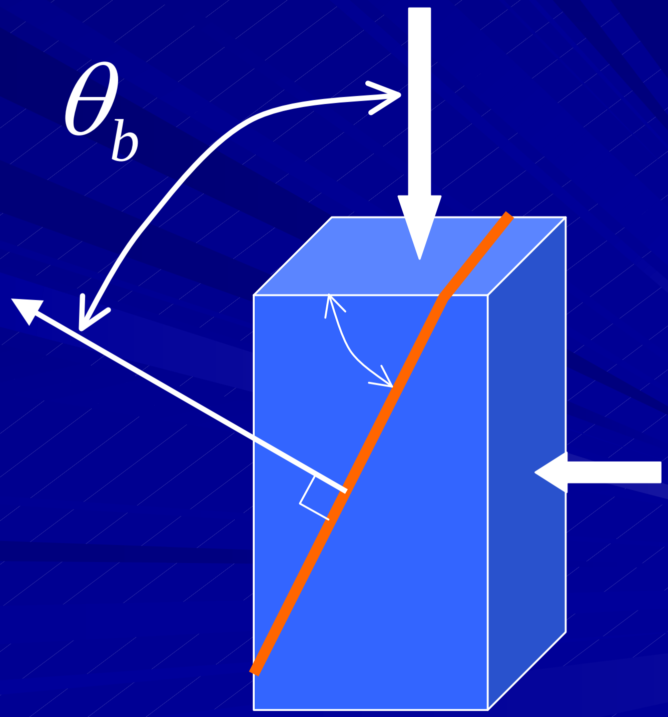
Generalize, for yield condition $f(\tau,\sigma,\theta) = 0$

and plastic potential $g(\tau,\sigma,\theta) = 0$

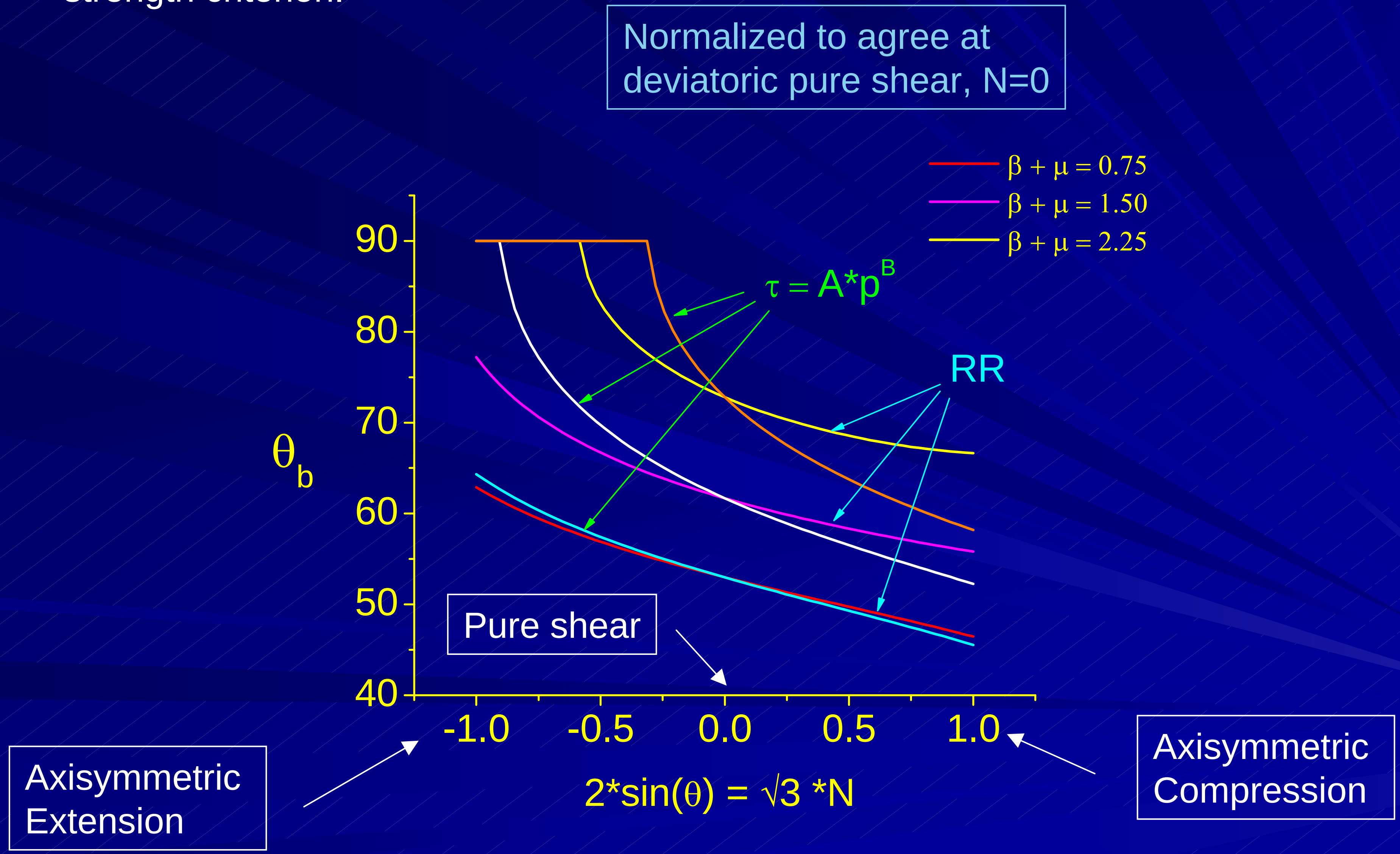
with $g_\tau = f_\tau,\ g_\theta = f_\theta$ but $g_\sigma \neq f_\sigma$

$$N \rightarrow \frac{2}{\sqrt{3}}\sin(\theta + \phi), \quad \tan\phi = \frac{g_\theta / \tau}{g_\tau}$$

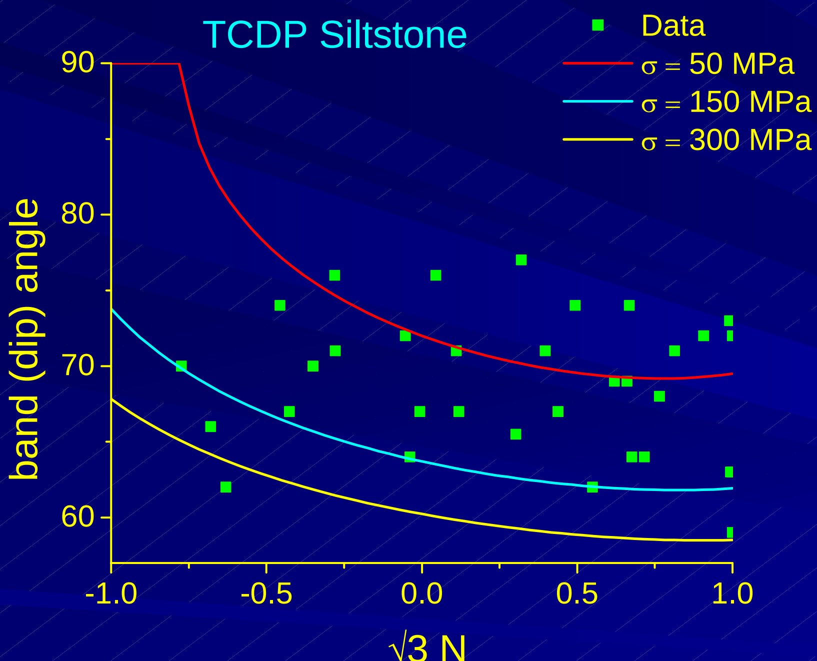
$$\frac{1}{3}(\beta + \mu) \rightarrow \frac{(g_\sigma + f)}{g_\tau}\cos\phi$$



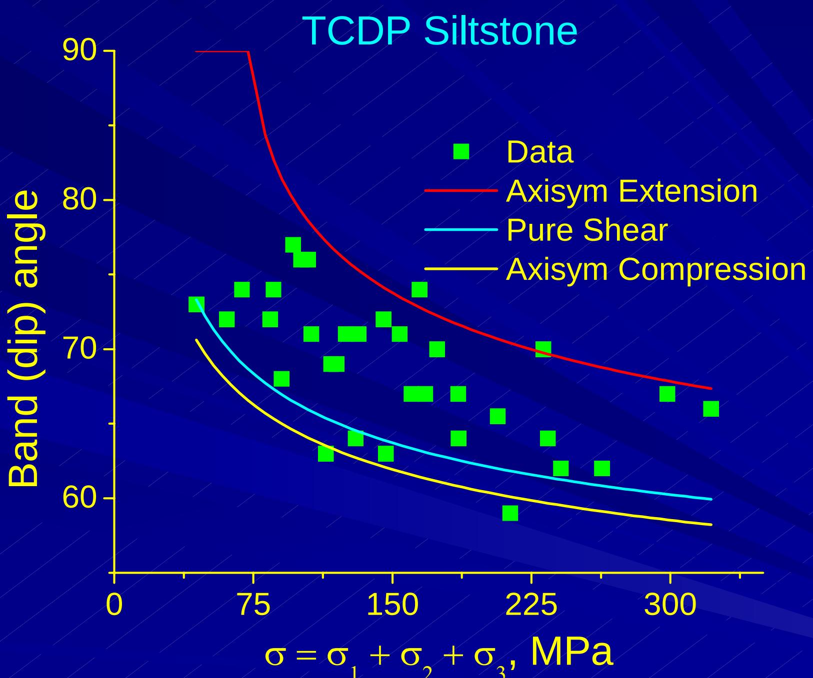
Comparison of band angle predictions vs. deviatoric stress state for Rudnicki-Rice (Drucker Prager) with constitutive relation derived from Haimson strength criterion.



Band angle data against deviatoric stress state with predictions for fixed mean normal stress.



Band angle data against mean normal Stress with predictions for axisym ext, axisym comp and pure shear.

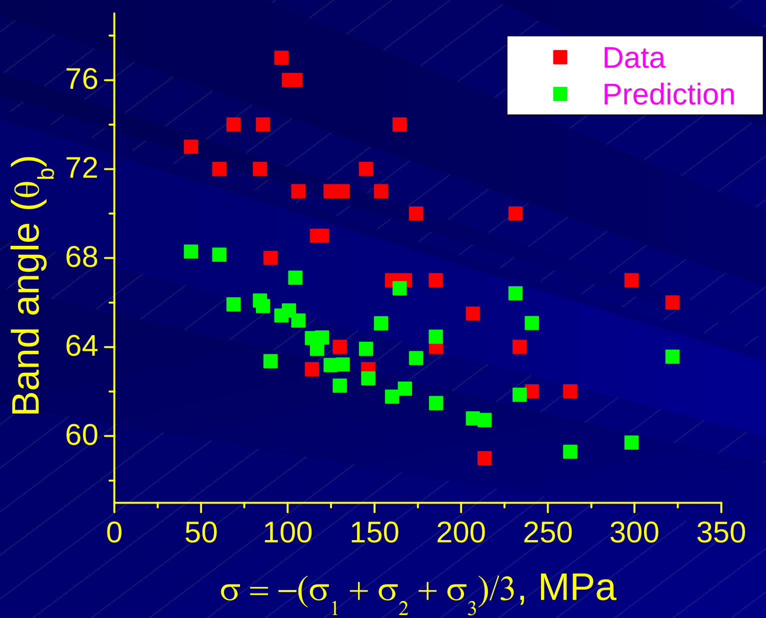


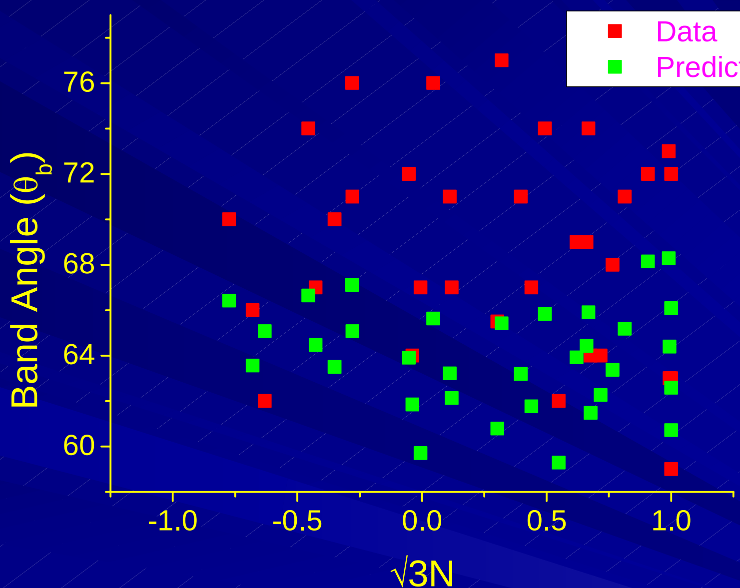
Predictions from

$$\tau = 3.036\left\{-\frac{1}{2}\left(\sigma_1 + \sigma_3\right)\right\}^{0.739}, \nu = 0.35$$

Band angle vs. mean normal stress (for different deviatoric stress states, i. e. N)

Band angle vs. deviatoric stress state for different mean normal stresses.





Predictions from

$$\tau = 3.036\left\{-\frac{1}{2}\left(\sigma_1 + \sigma_3\right)\right\}^{0.739}$$

$$\nu = 0.35$$

Conclusions

- The intermediate principal stress σ_2 affects all aspects of mechanical behavior of rock under compressive stresses.
- The strength is well-described by a relation $\tau = Ap^B$ (neither Mohr-Coulomb nor Drucker Prager (RR)).
- Fault dip angle increases steadily as σ_2 is raised for a given σ_3 (prediction based on $\tau = Ap^B$ models trends with mean stress and deviatoric stress state adequately but, in general, angles are less than observed).
- True triaxial testing is essential for constraining constitutive relations for applications and numerical calculations.
- True triaxial testing provides the opportunity to interrogate the role of constitutive behavior in predicting failure strength and fault orientation.