Earthquake Statistics in Models and Data

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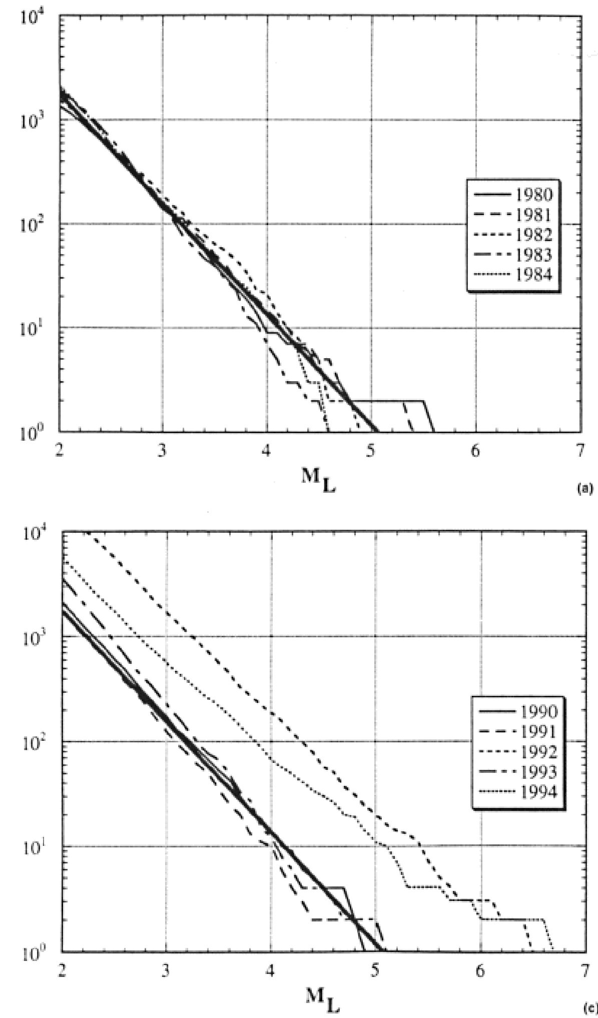
Motivation

- Most models of earthquake systems today focus, as a first order parameter, on reproducing the Gutenberg-Richter distribution.
- Most of these models then focus on reproducing various phenomena that we see in nature.
- As a result of both the growing complexity of these models and the expansion of the catalog date, integrating data analysis techniques with model validation today requires additional measures of evaluation.
- Here I introduce a statistical mechanics concept used to quantify model behavior, ergodicity, to investigate the behaviour of the natural system from seismicity data.



Cumulative number of earthquake, N, with magnitude greater than m for various years in southern California, a) 1980-1984 and b)1990-1994 (Turcotte 1997).

Ergodicity

- A system is determined to be ergodic if it visits every possible state in phase space over the course of time. It can be shown that, given enough sampling time, the temporal averages of a particular observable must equal the ensemble average.
- Recently, we employed a measure of ergodicity, the Thirumalai-Mountain (TM) metric (Thirumalai et al., 1989) to study both models and data from various tectonic locations (Tiampo et al., *PRL,* 2003; Tiampo et al., in press, *PRE*).

$$\Omega_e(t) = \frac{1}{N}\sum_{i=1}^{N}[\varepsilon_i(t) - \bar{\varepsilon}(t)]^2$$

$$\varepsilon_i(t) = \frac{1}{t}\int_0^t E_i(t')dt'$$

$$\bar{\varepsilon}(t) = \frac{1}{N}\sum_{i}^{N}\varepsilon_i(t)$$

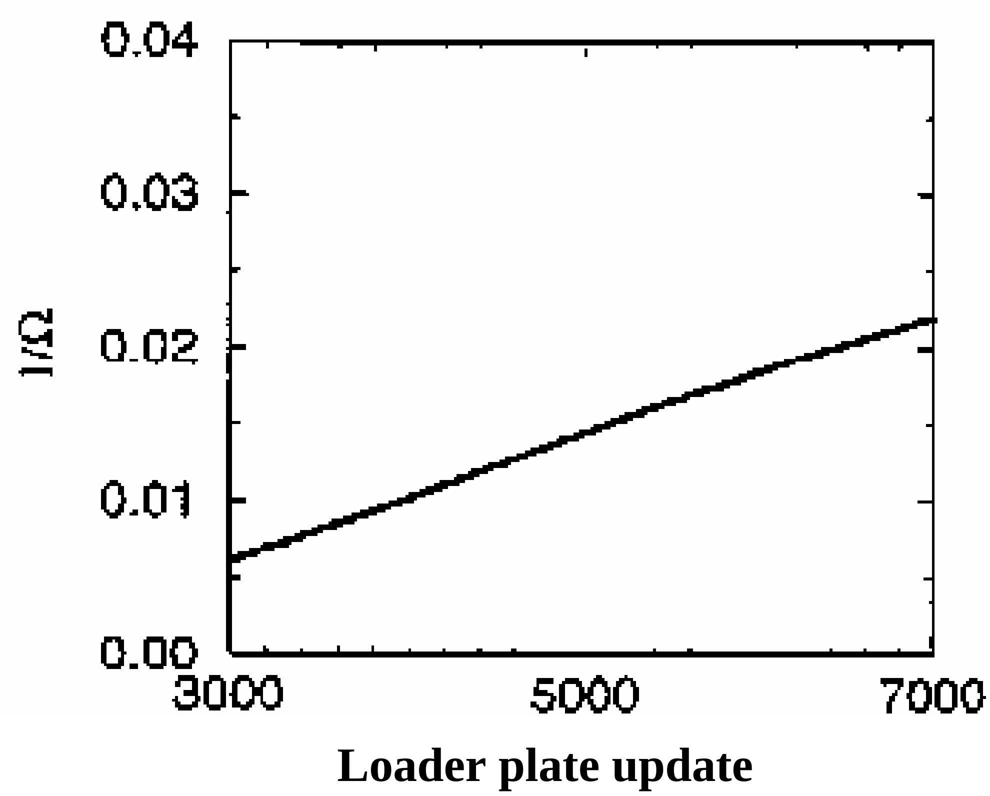
- Notice that the TM metric is the spatial variance of the temporal mean, and if those are equivalent, it not only goes to zero with time, it does so as 1/*t,* from the Central Limit Theorem.

Ergodicity in Fault Models

- Therefore, if the system is effectively ergodic at long times, the TM metric varies with time as

$$\frac{\Omega(0)}{\Omega_e(t)} = tD_e$$

- A number of earthquake system parameters can be formulated for analysis using the TM metric. At the right is shown a plot of the inverse TM metric for *energy* in a slider block model, from Ferguson et al., 1999.



Ergodicity

- An ergodic system is said to be in equilibrium, at least for some period of time, because it has no significant (large) excursions from the mean energy state for that same time interval.

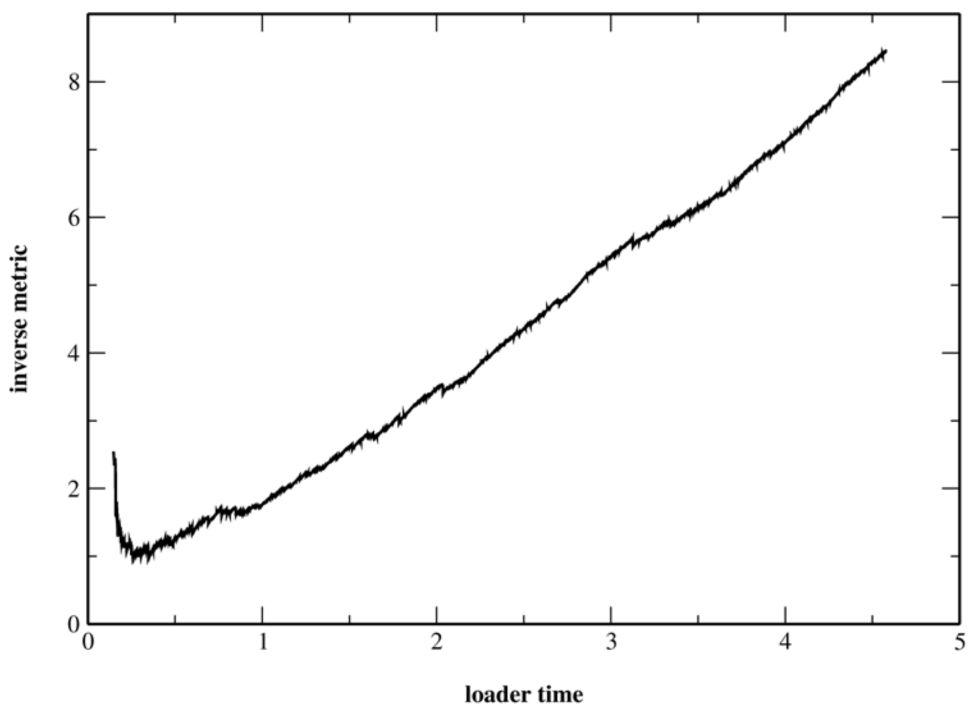
- In addition, a necessary condition for ergodicity is that the system be stationary. Stationarity implies that a system is well-behaved, and linear analyses such as Karhunen-Loeve (KL), or principal component, decomposition are applicable for those same spatial and temporal parameters.

- Note that the corollary of this is true – a linear KL analysis only contains a complete set of orthonormal basis functions if the system is ergodic (Holmes et al., 2006; Glösmann and Kreuzer, 2005).

- Finally, D_e is a diffusion constant, proportional to $1/t$, and related to the sampling rate of the system. Therefore, $1/D_e$ is the mixing time, or the time to reach effective ergodicity.

Ergodicity in Fault Models

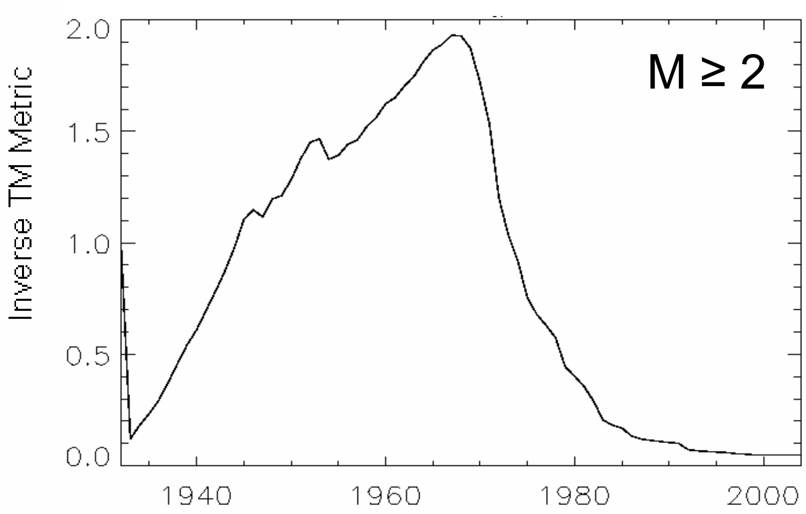
- Other parameters can be related to the energy of the earthquake fault system.

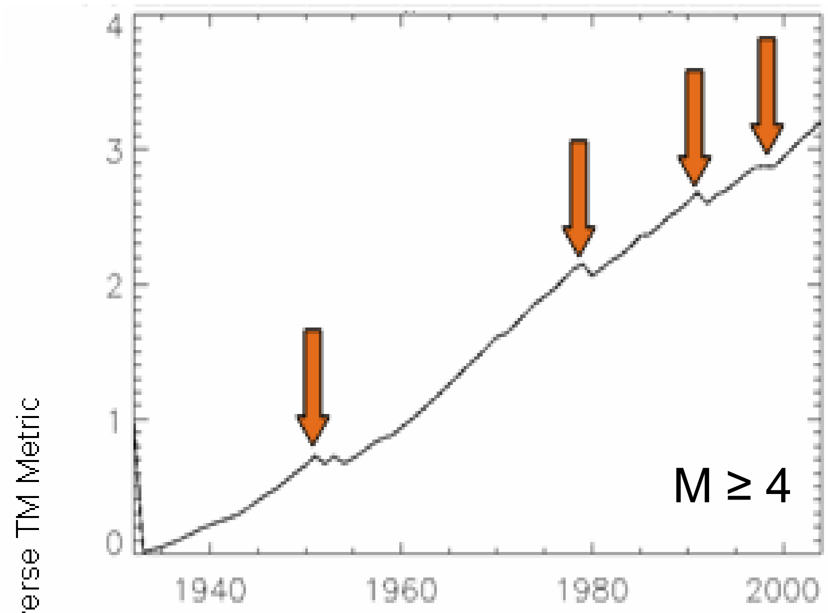


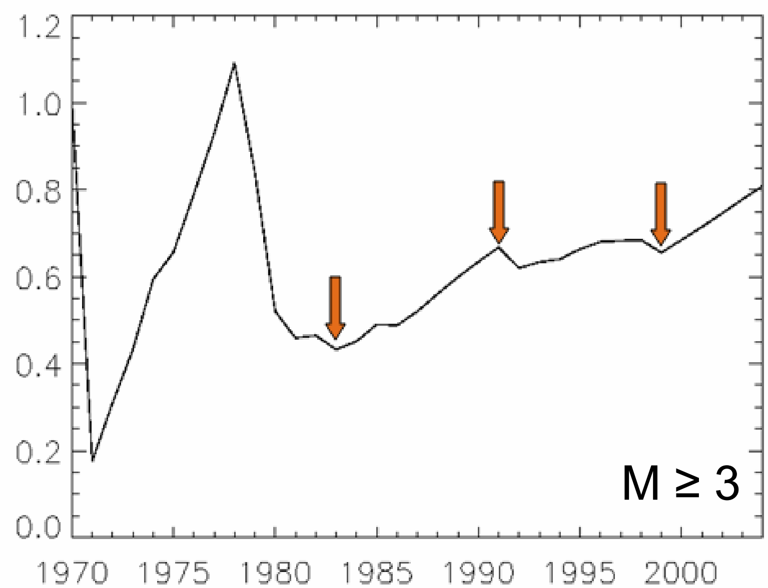
- At the left is shown the inverse TM metric for *numbers of events*, in a slider block model with precursory slip (Tiampo et al., 2003).
- Note that, while the linear regions here indicate ergodicity, or punctuated ergodicity, there are also certain ranges of parameters, not shown for which these models are *not* ergodic (Tiampo et al., in press, 2007).

Ergodicity in Natural Catalogs

- Here, we bin two very different tectonic regions, California and eastern Canada, into a set of locations, and use the number of events as our parameter of interest, in order to investigate under what conditions the system is, or is not, ergodic (Tiampo et al., in press, *PRE*).







California

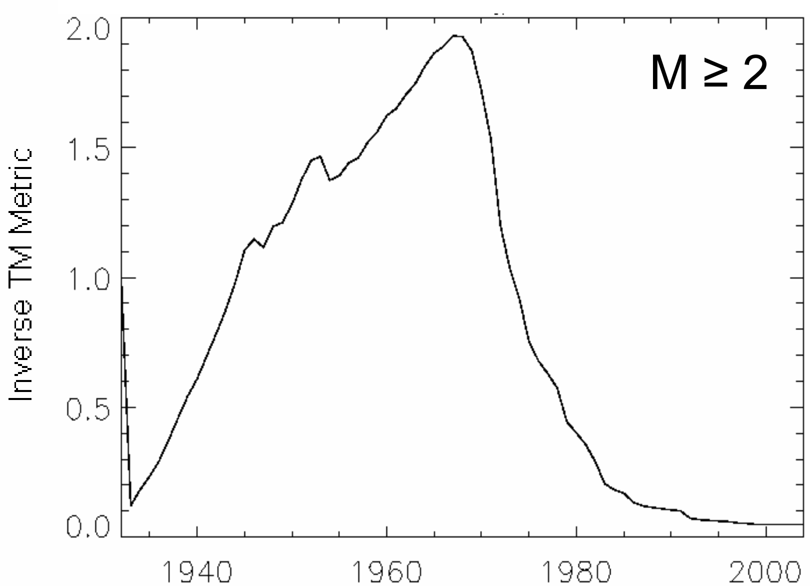
Seismicity Data

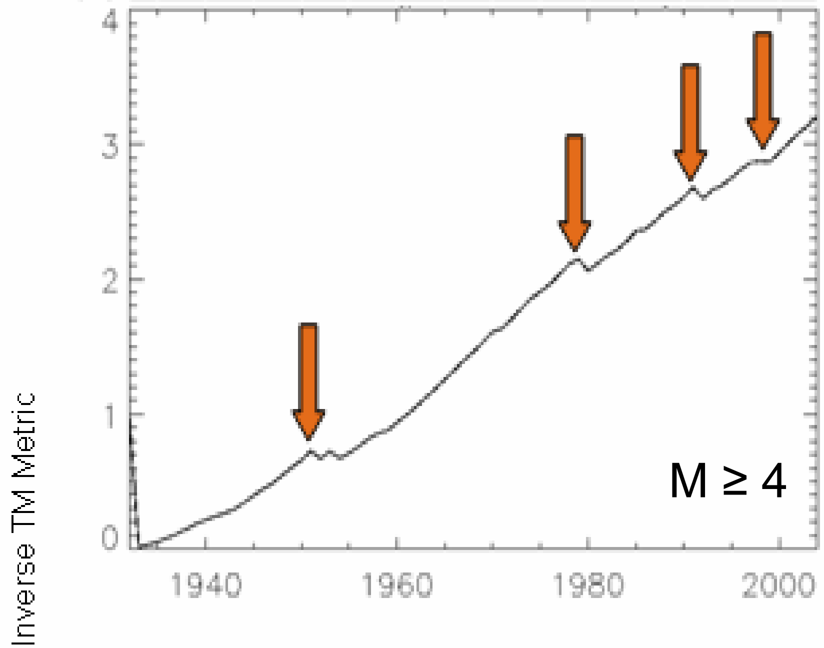
- Southern California Earthquake Center (SCEC) and Northern California Seismic Network (NCSN) earthquake catalogs, or the ANSS catalog, for the period 1932-2004.
- Events are binned into areas 0.1° to a side (approximately 11 kms).
- Analysis is performed for an area ranging from 32° to 39° latitude, -123° to -115° longitude, or some subset thereof. No declustering is performed, except for a particular magnitude cutoff.
- A matrix is created consisting of the seismicity time series (n time steps) for each location (p locations).

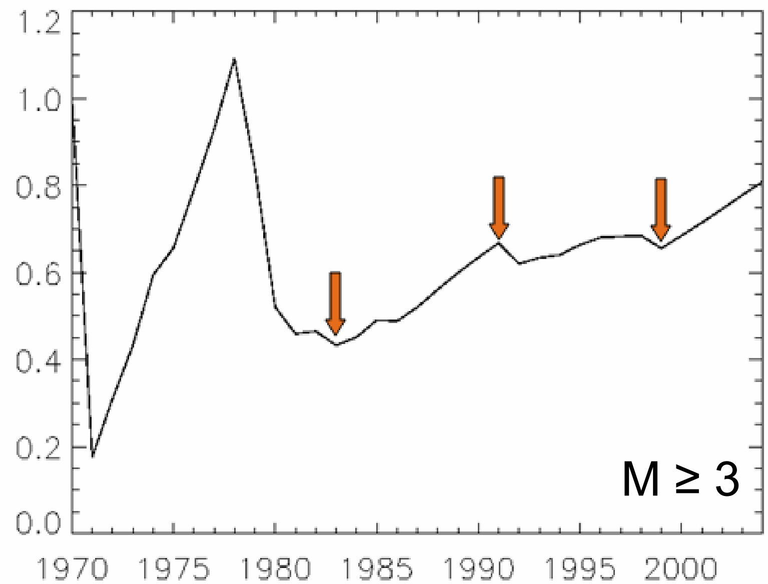
$$T = [\bar{y}_1, \bar{y}_2, \ldots \bar{y}_p] = \begin{bmatrix} y_1^1 & y_1^2 & \cdots & y_1^p \\ y_2^1 & y_2^2 & \cdots & y_2^p \\ \vdots & \vdots & \ddots & \vdots \\ y_n^1 & y_n^2 & \cdots & y_n^p \end{bmatrix}$$

Ergodicity in Natural Catalogs

- Again, we bin the California region into a set of locations, and use the numbers of events as our parameter of interest







California

Karhunen-Loeve Analysis

- A Karhunen-Loeve (KL) expansion analysis is a method for decomposing large data sets into their orthonormal eigenvectors and associated time series based upon the correlations that exist in the data.

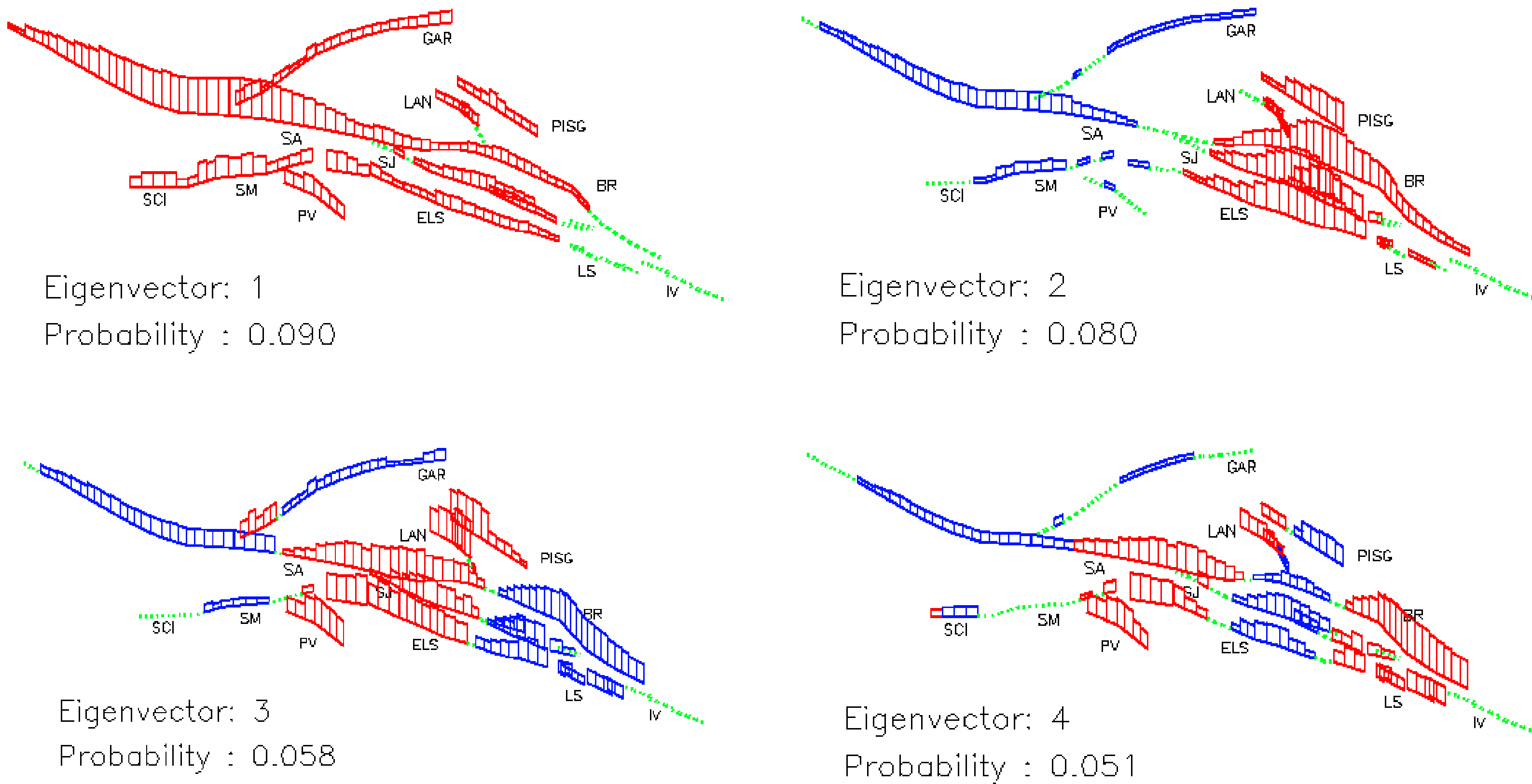
- The vector space is spanned by the eigenvectors, or eigenpatterns, of an N-dimensional correlation matrix, $C(x_i,x_j)$. The elements of C are obtained by cross-correlating this set of location time series, T.

- The eigenvalues and eigenvectors of C are computed, such that they represent the correlations in the seismicity data in space and time.

- These eigenvectors are ordered from largest to smallest percent of the correlation. *This also results in an ordering from largest to smallest scales*.

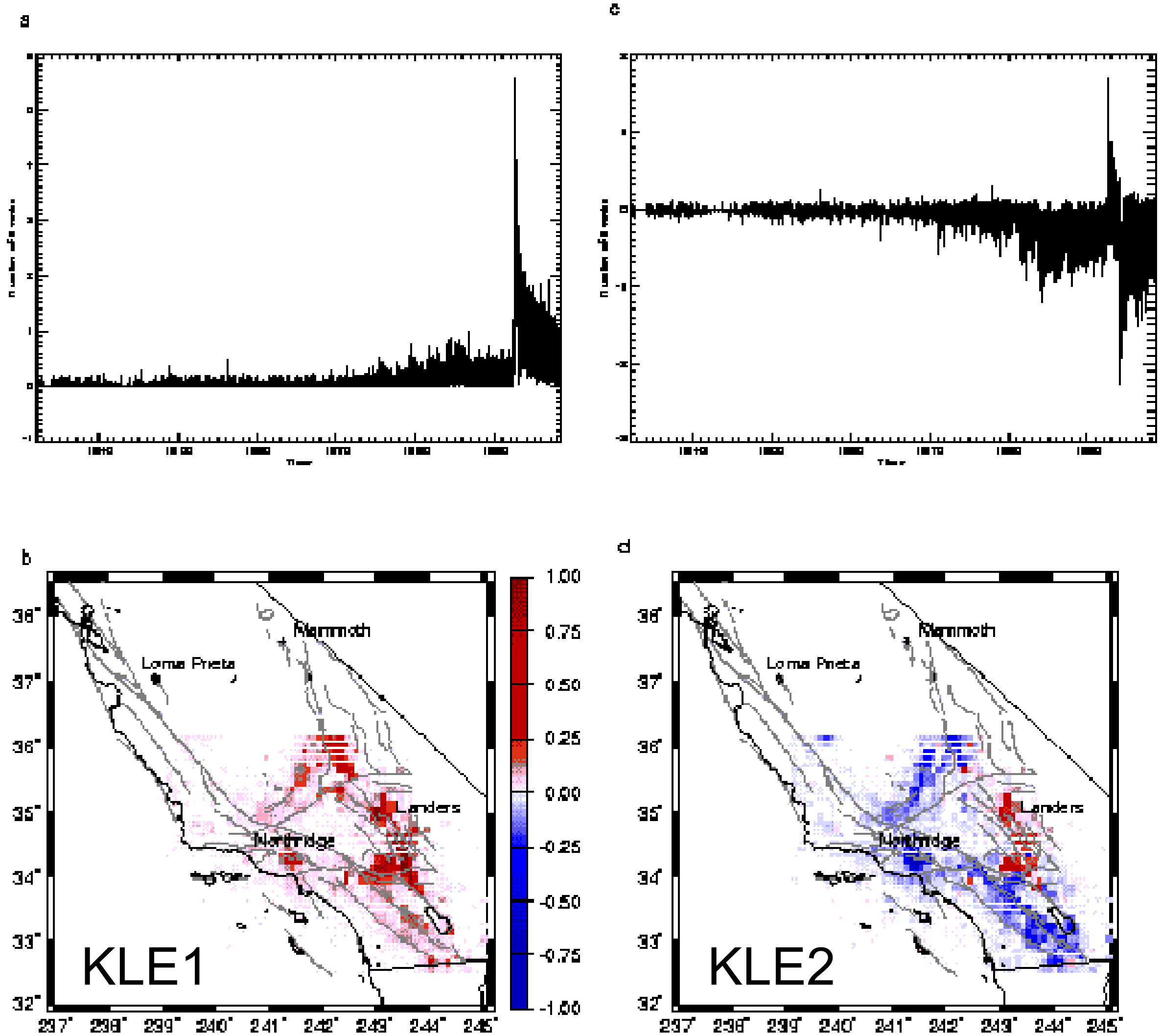
- This method can be used to study those modes most responsible for these correlations and their sources (Savage, 1988), to remove the uninteresting modes from the system (Preisendorfer, 1988), or project their trajectories forward in time (Penland and others).

Computer Simulations, Virtual California Eigenpatterns 1 – 4



Note that there are no small events (M ≤ 6) in this model.

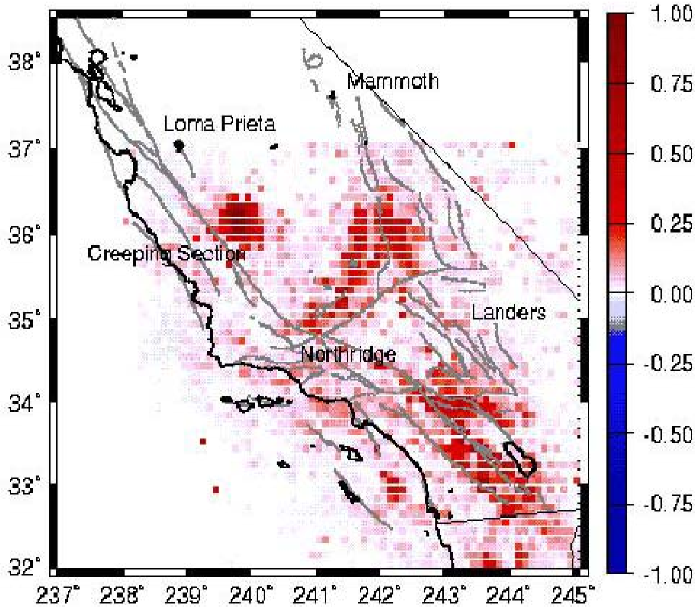
Correlated Patterns in Historic Seismicity Data



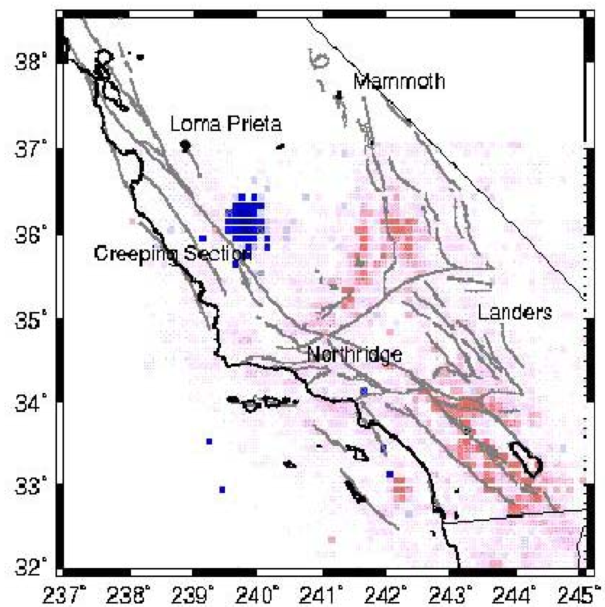
Southern California seismicity, all events, 1932 through 1999

Southern California Seismicity, 1932 through 1991

KLE1

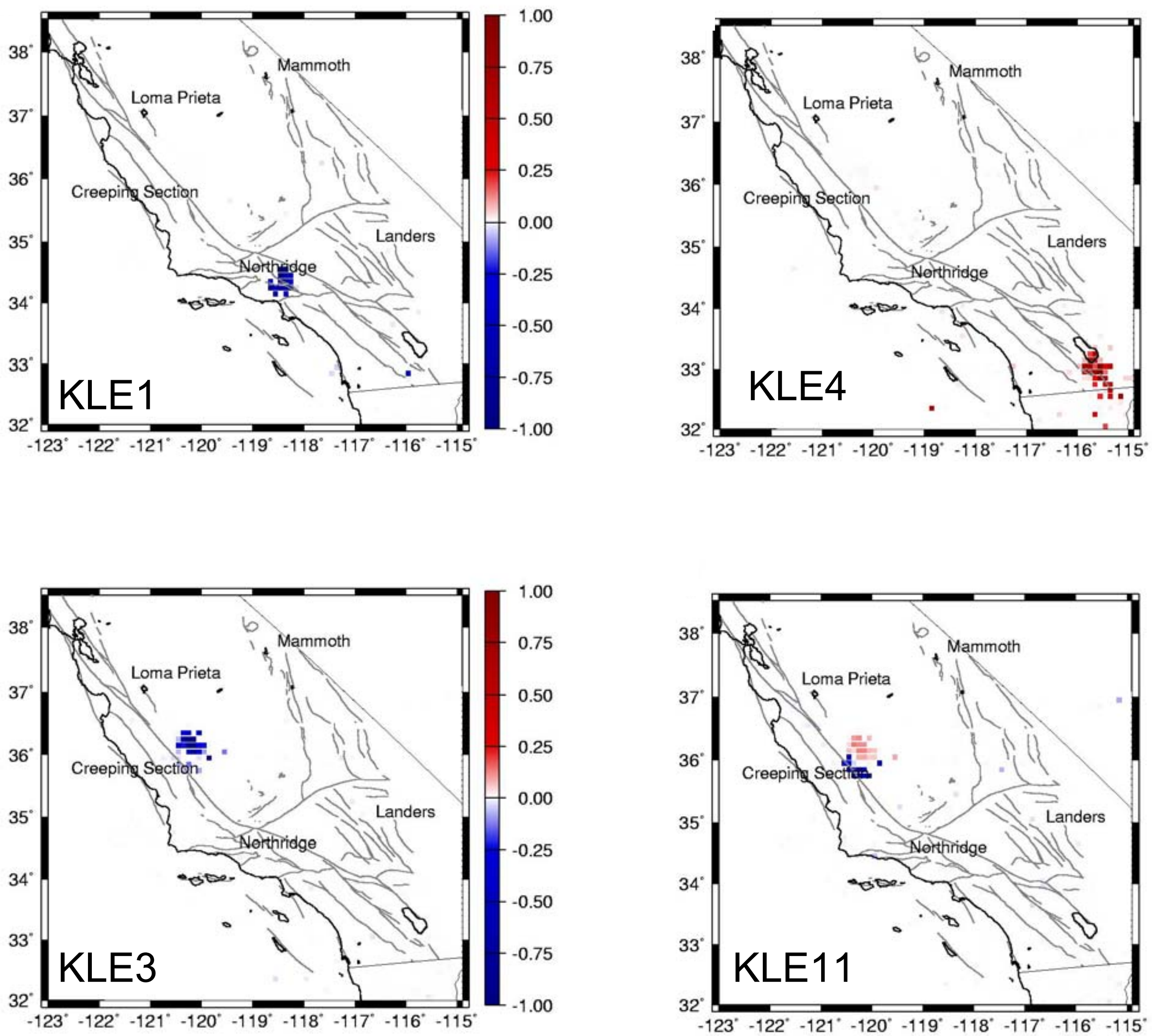


KLE2



Again, all events M ≥ 0

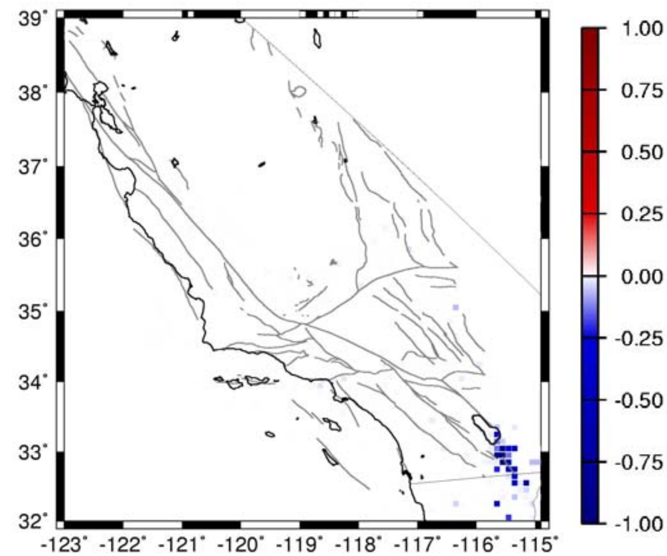
Spatial Eigenvectors, 1932 through 1991



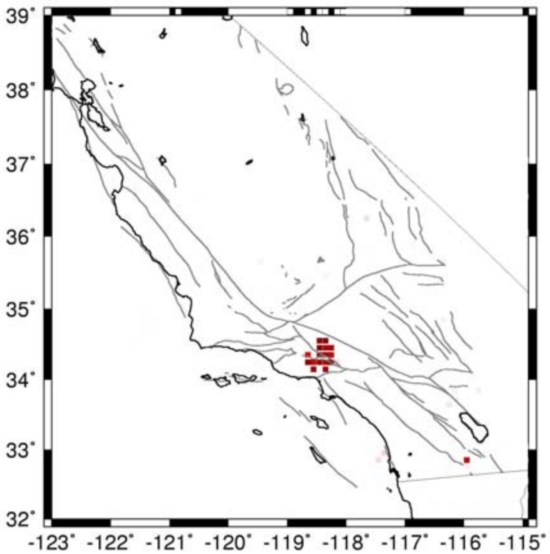
Southern California seismicity, M ≥ 3

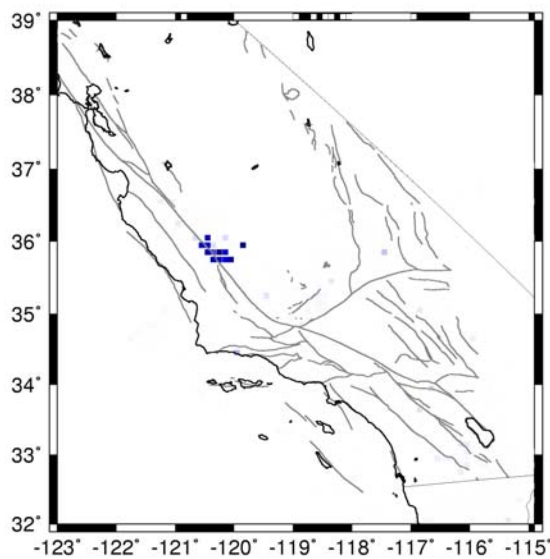
1932 through 1982

KLE3



KLE1

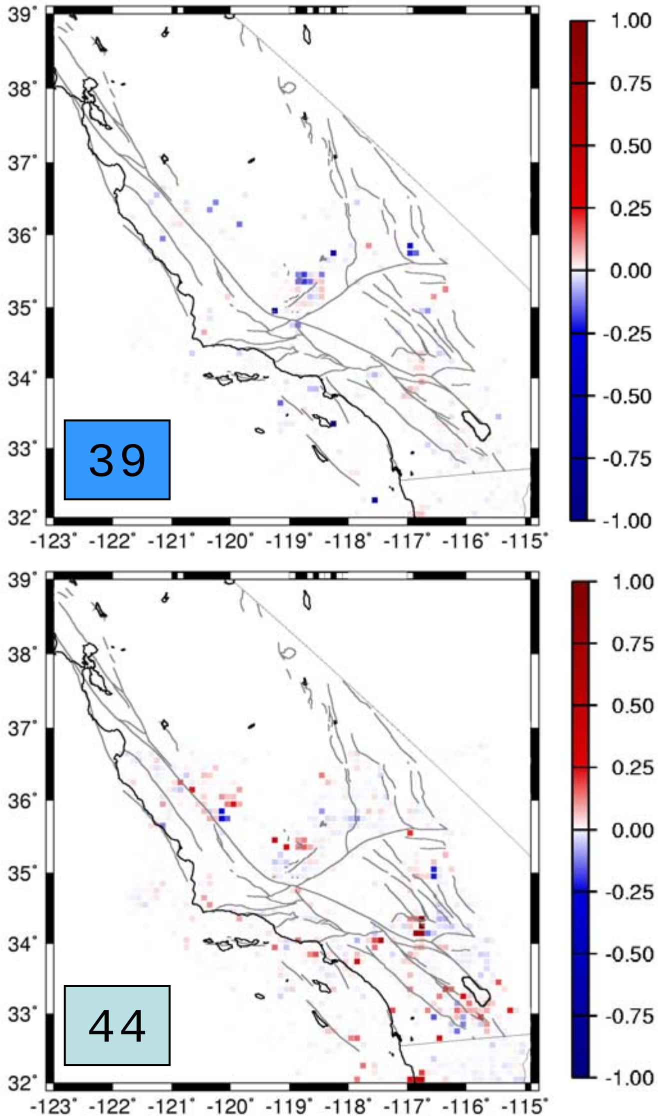


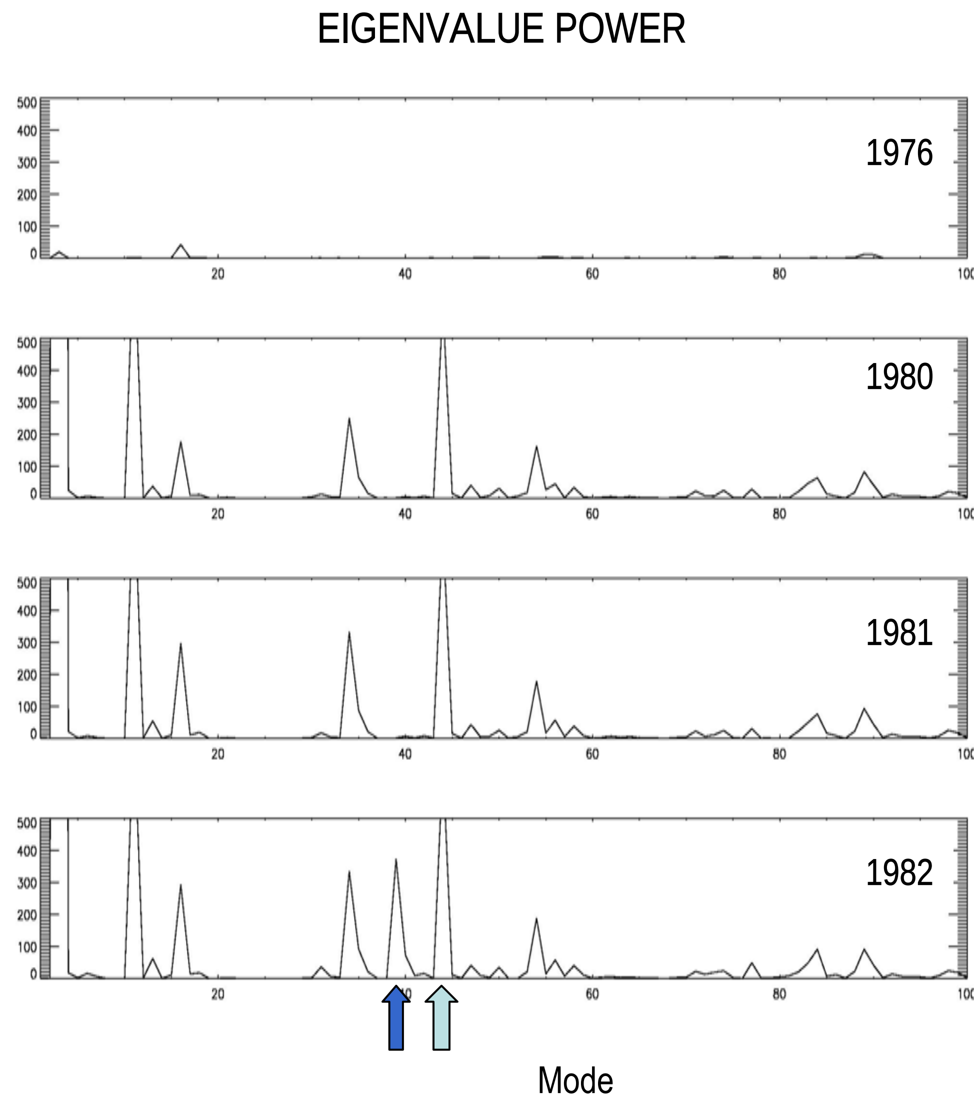


KLE7

M ≥ 3

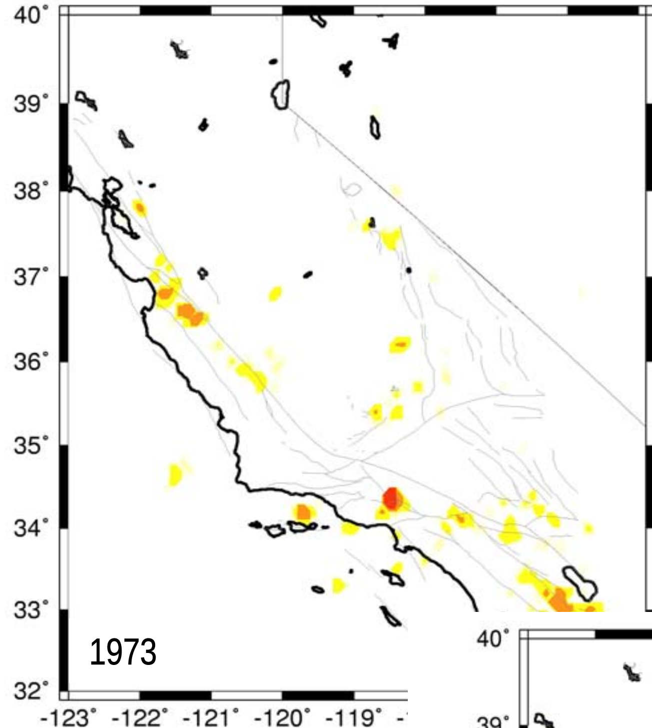
Decomposition of Annual Seismicity - 1982

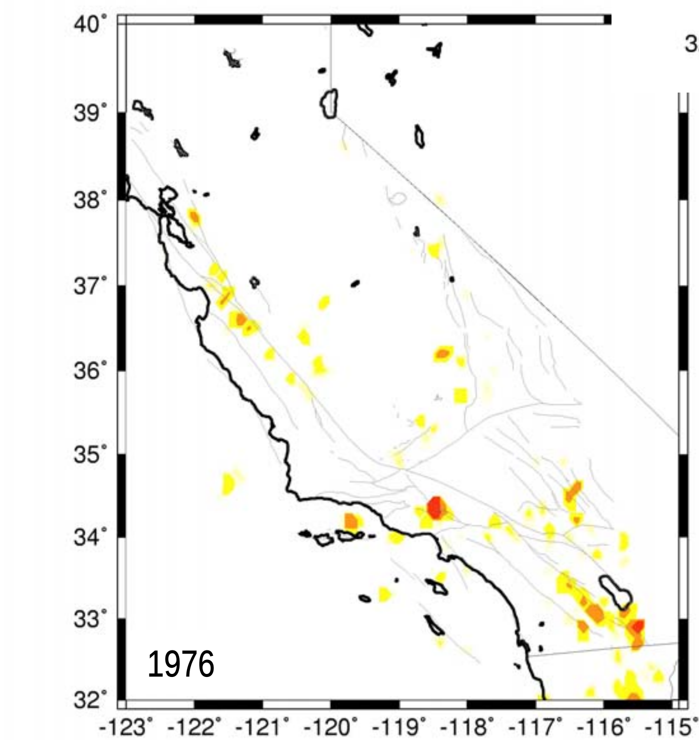


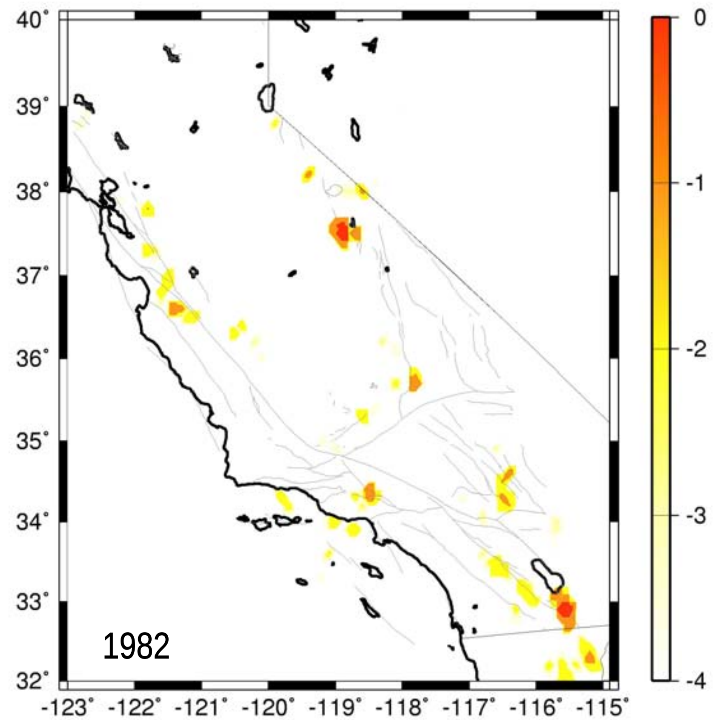


The PI Index:

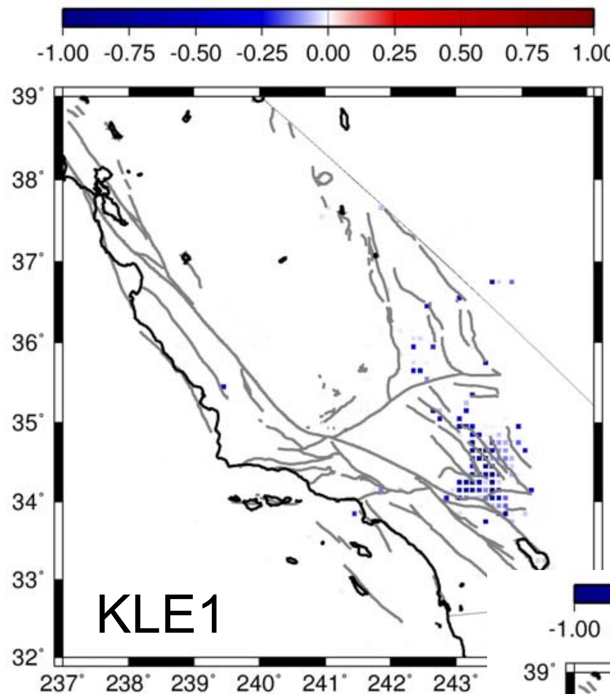
As a measure of seismicity, is a linear combination of linear eigenvectors

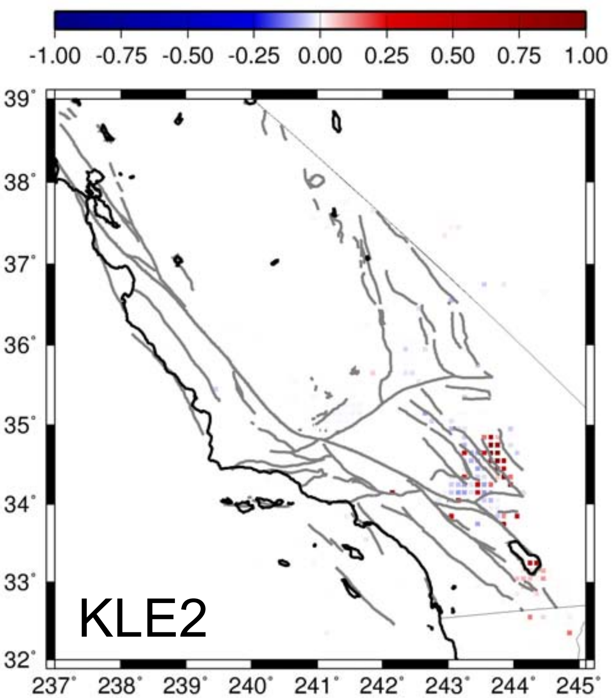


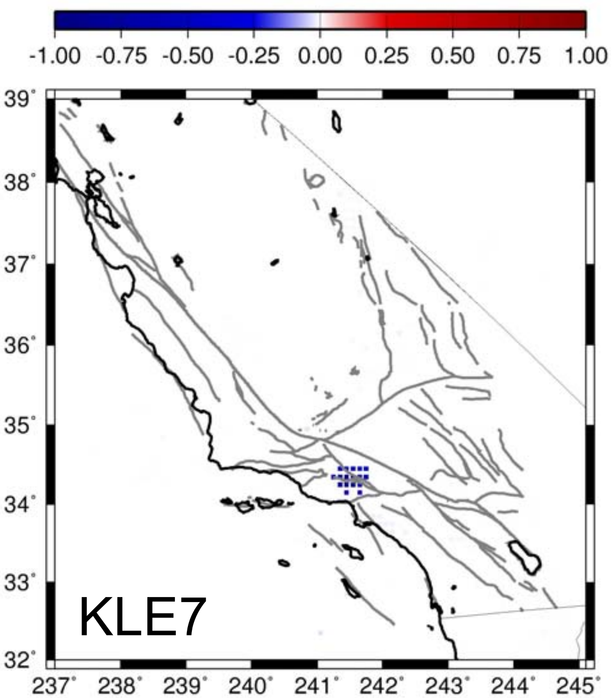




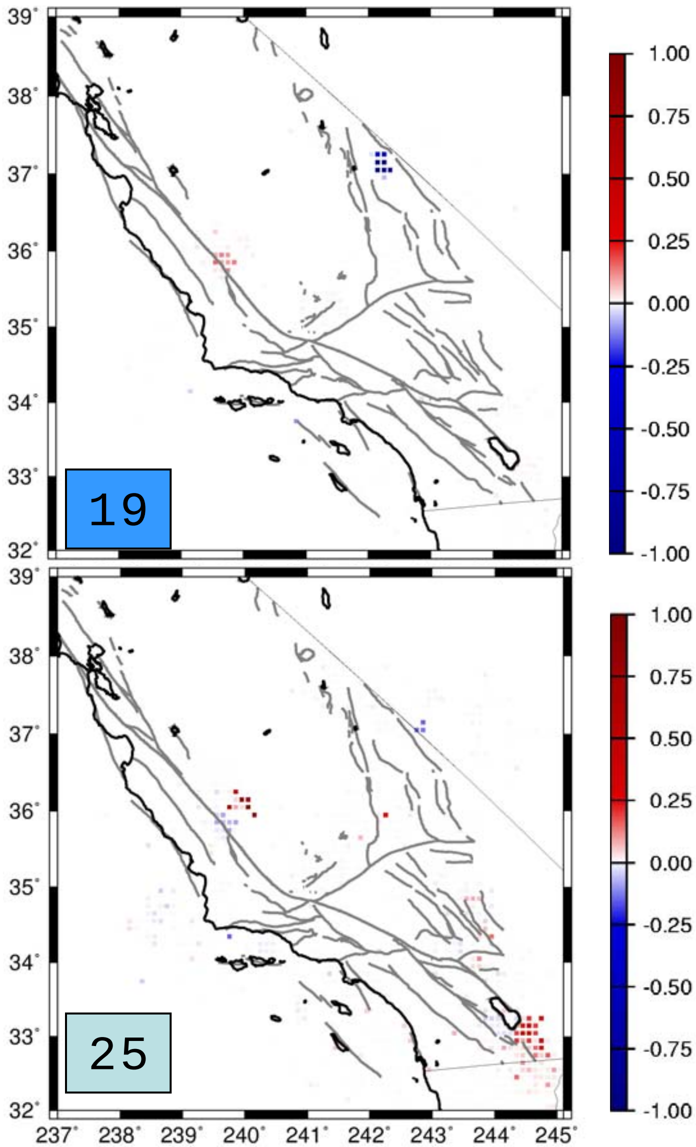
1932 through 2003

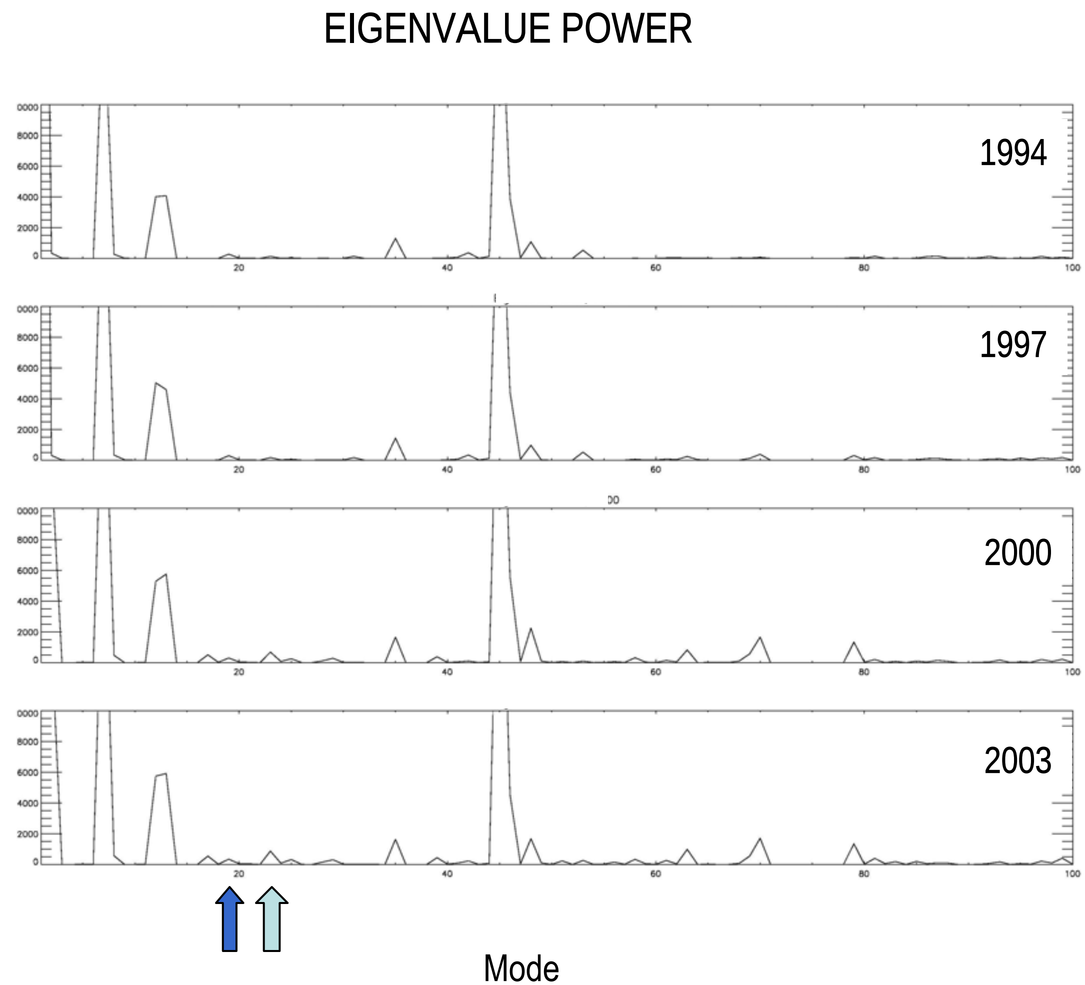




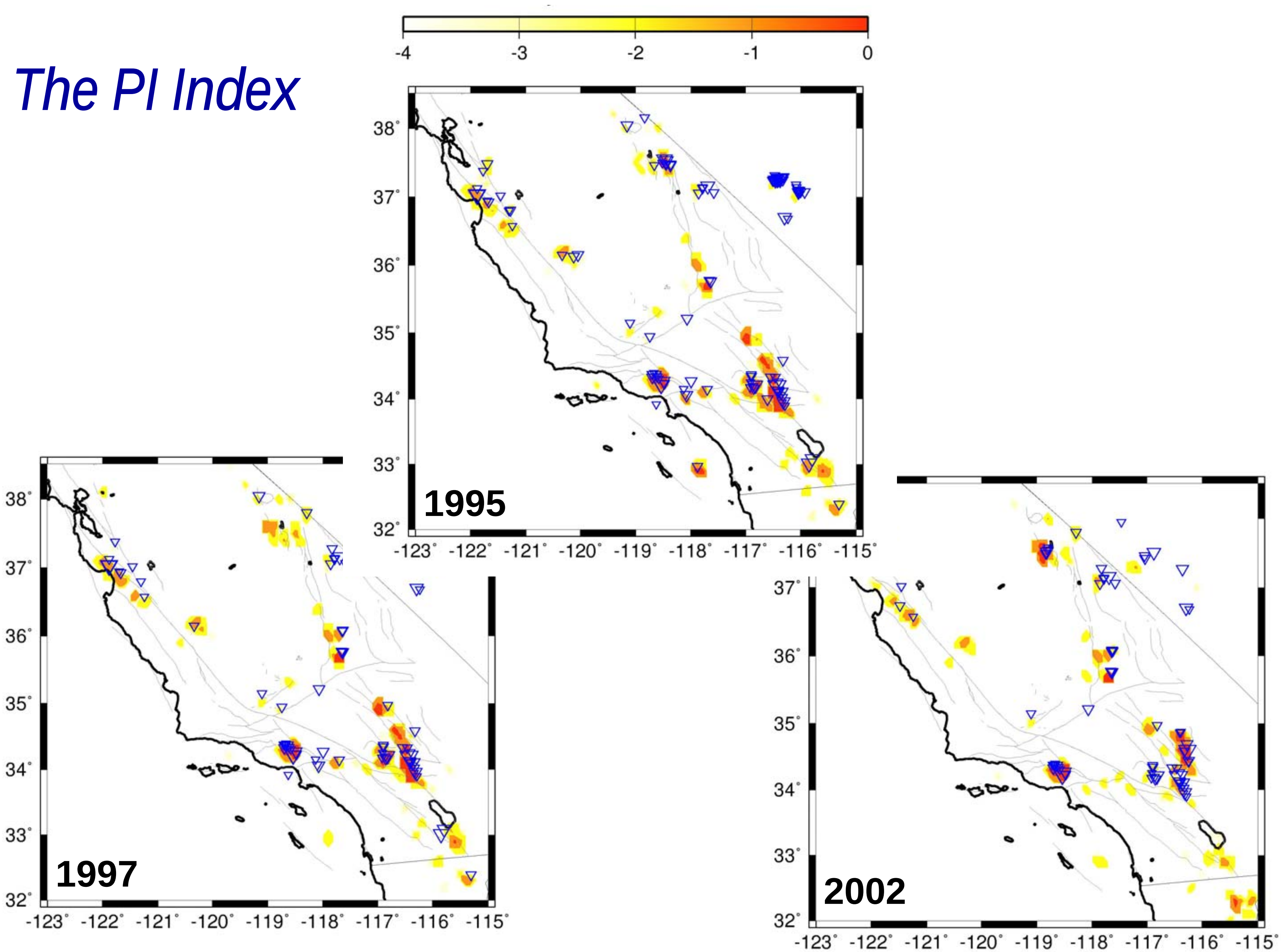


Decomposition of Annual Seismicity - 2003



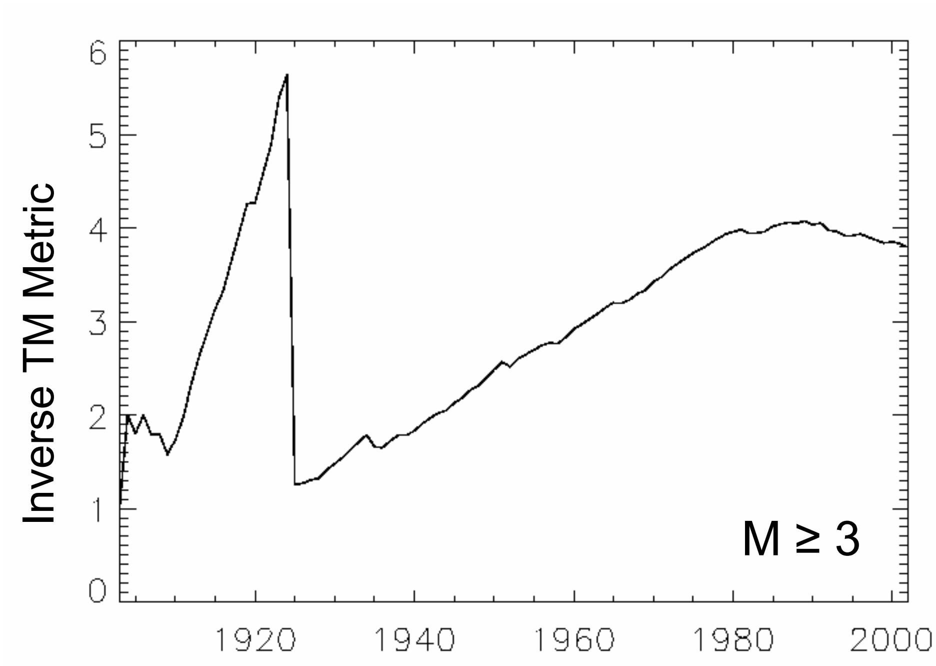


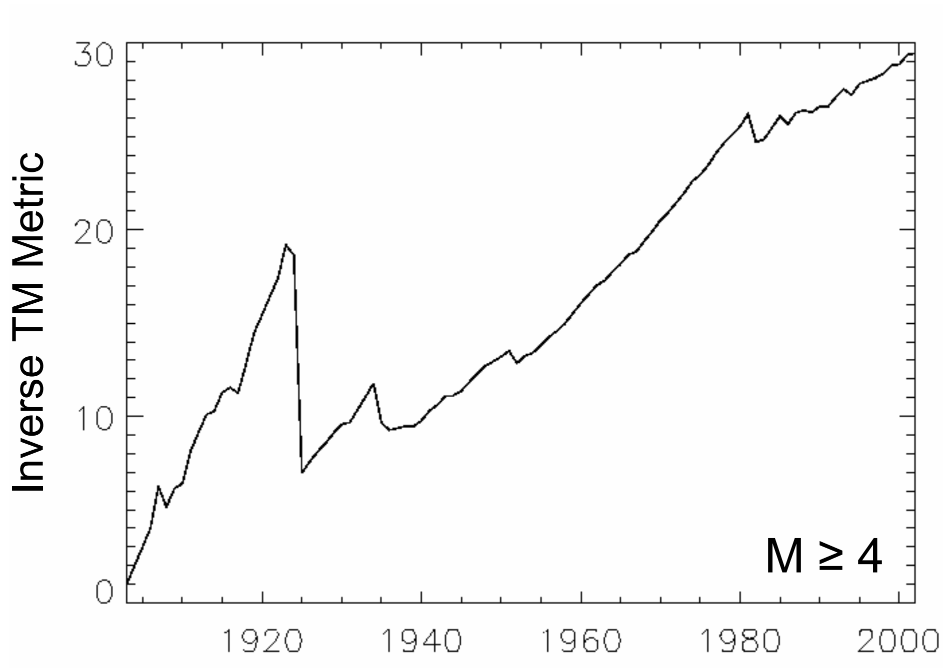
The PI Index



-4
-3
-2
-1
0
1995
1997
2002
38˚
37˚
36˚
35˚
34˚
33˚
32˚
-123˚ -122˚ -121˚ -120˚ -119˚ -118˚ -117˚ -116˚ -115˚

Ergodicity in Natural Catalogs

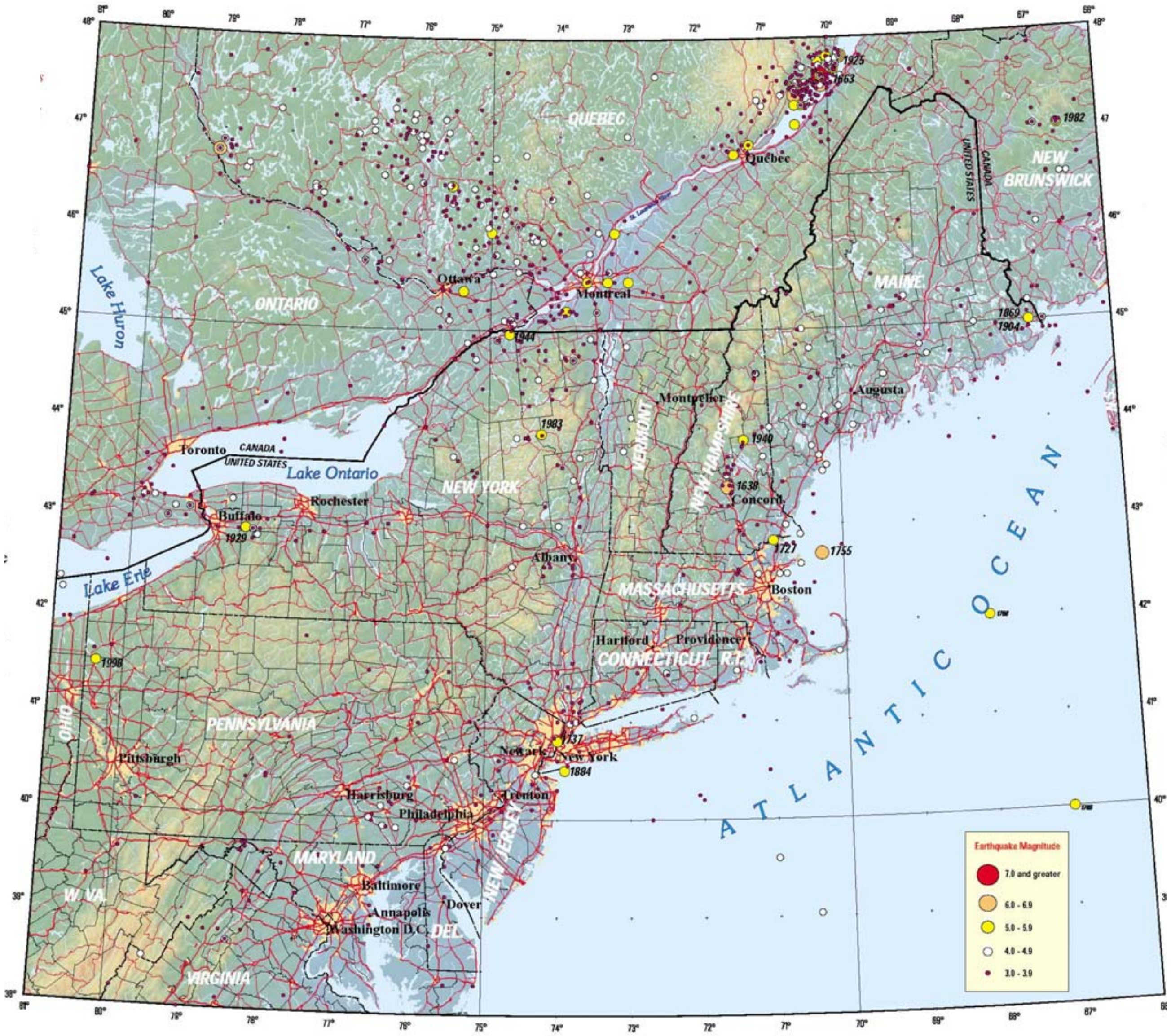




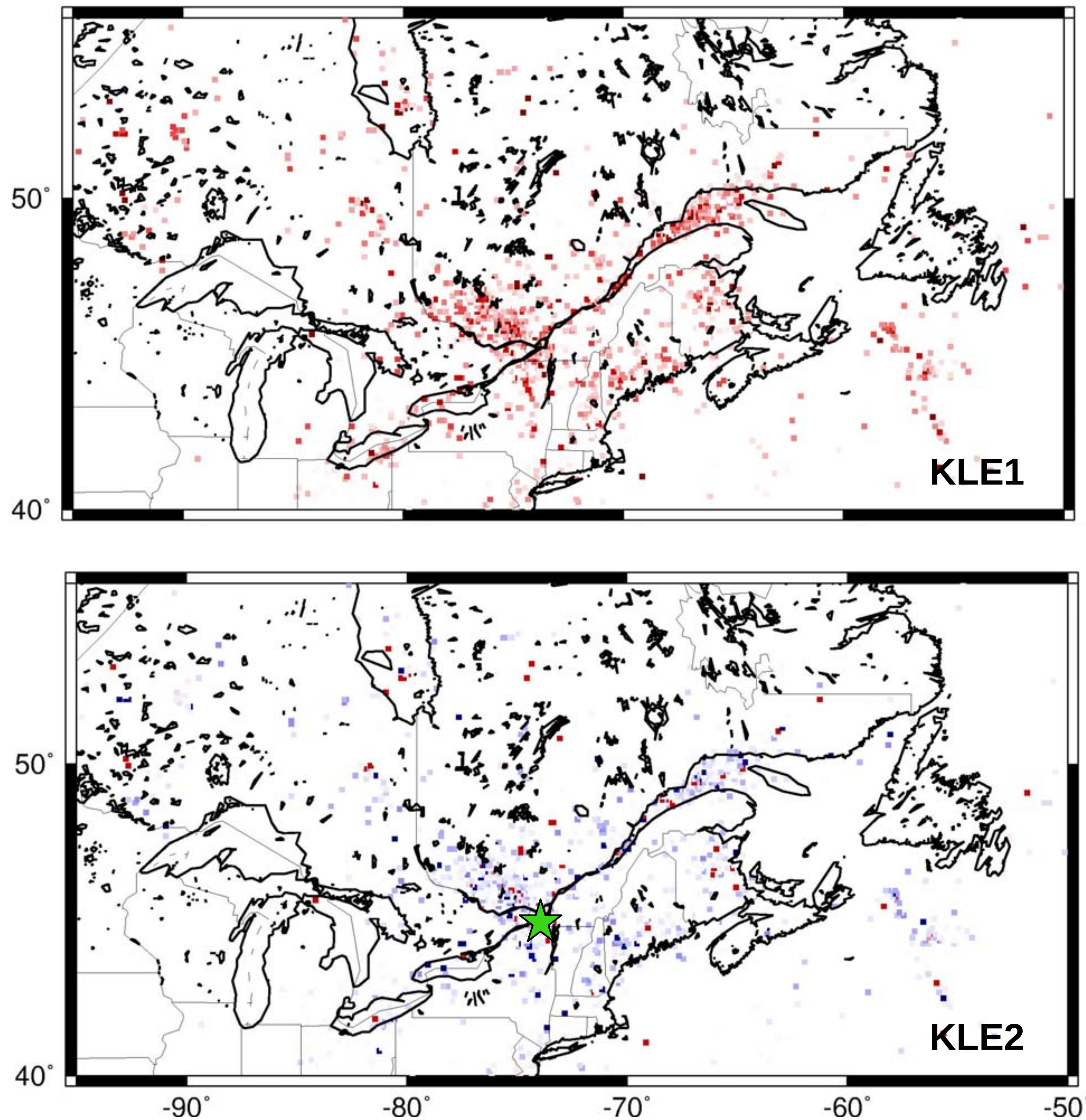
Eastern Canada: 1900 – present.
Catalog courtesy of J. Adams, GSC

Tiampo et al., *PRE,* in press

Eastern North American Seismicity, USGS

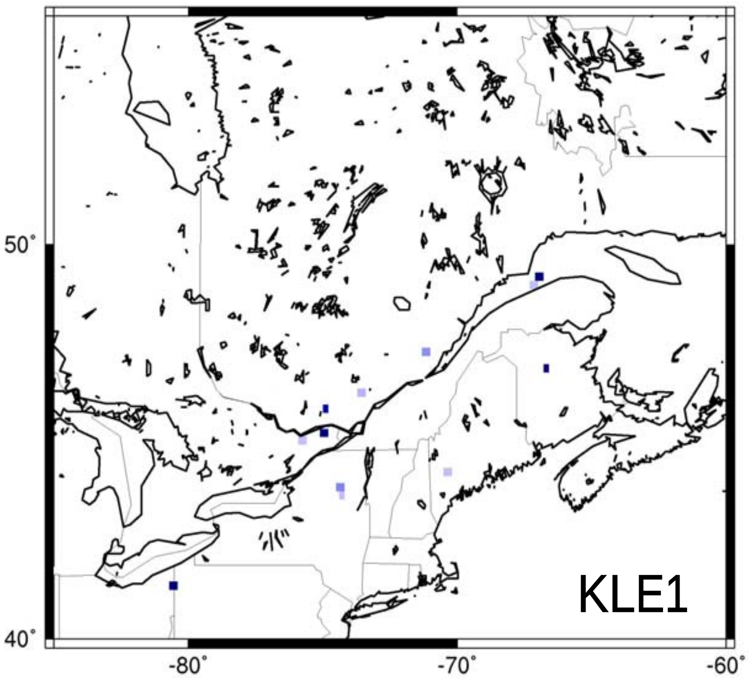


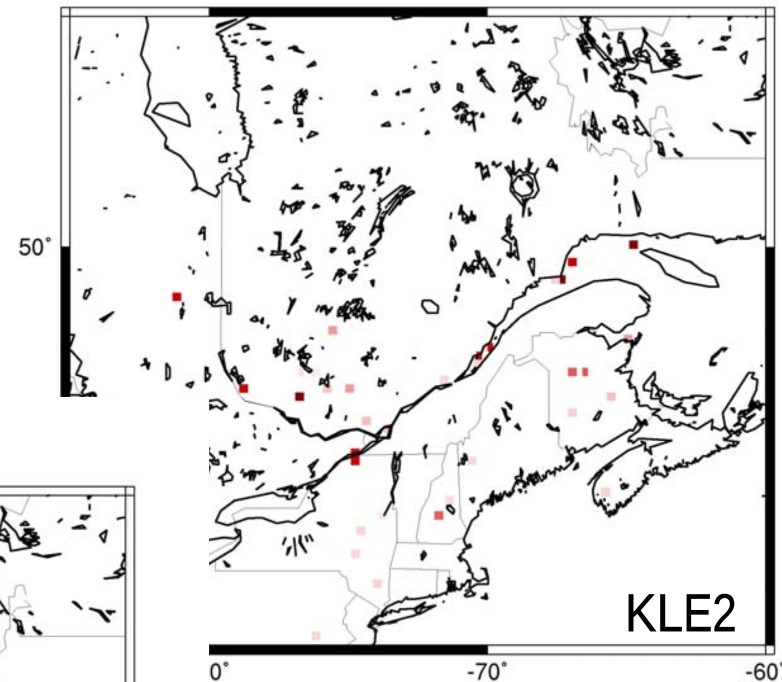
KL Decomposition, eastern Canada

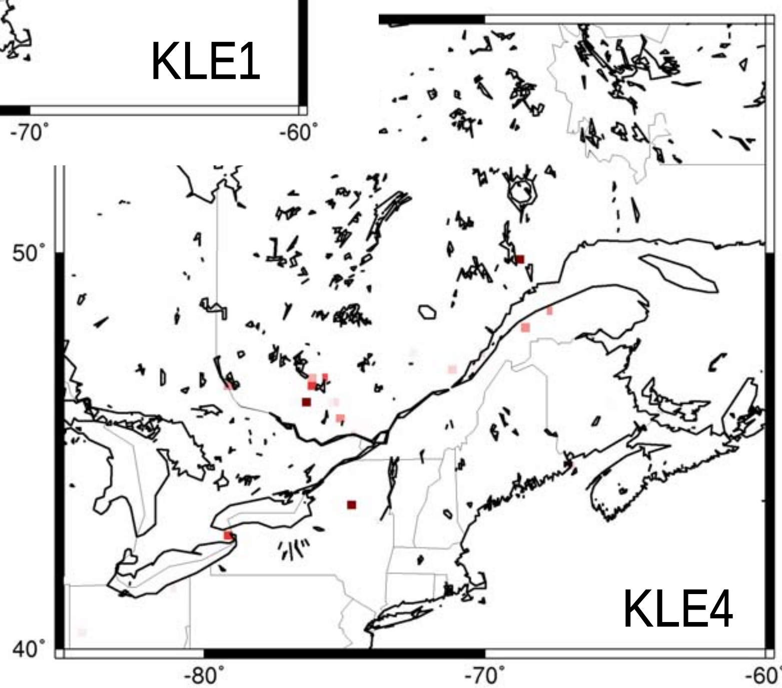


1900 through
2001, M ≥ 0

KL Decomposition, eastern Canada

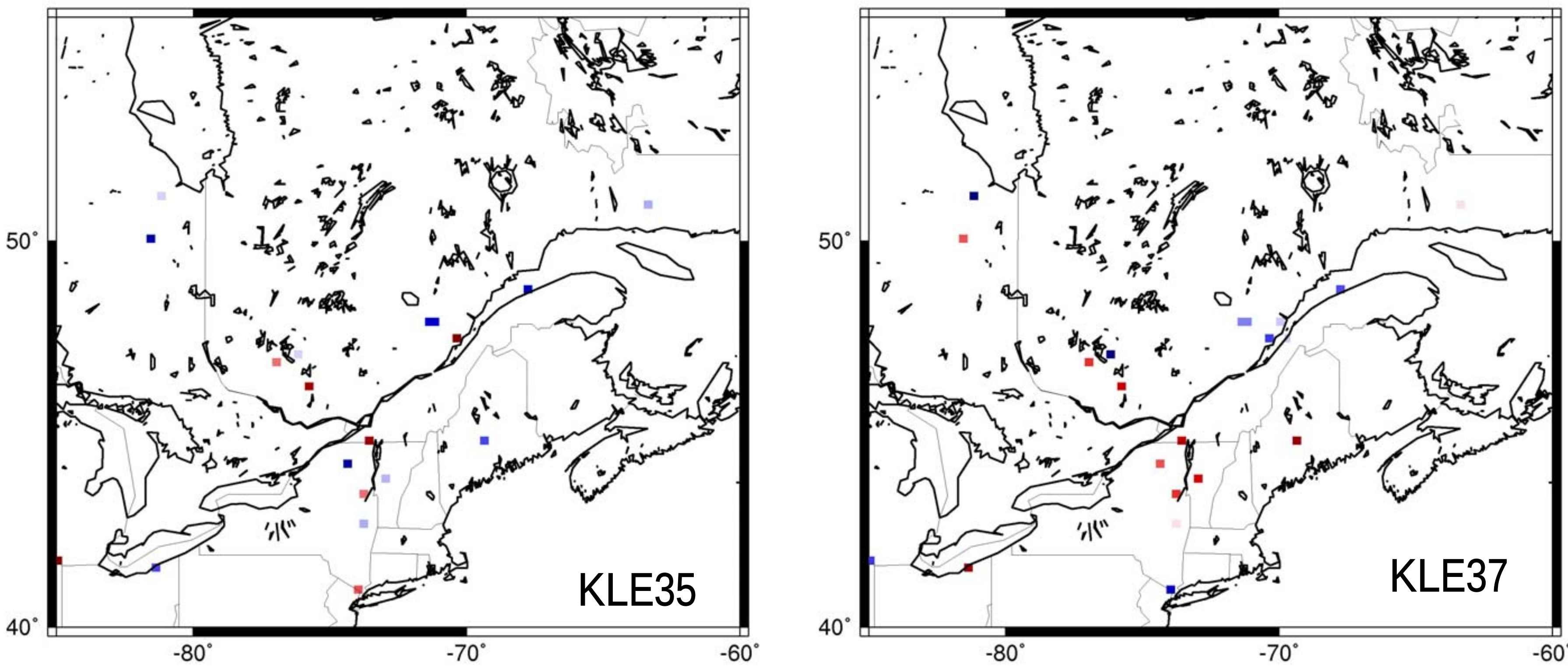






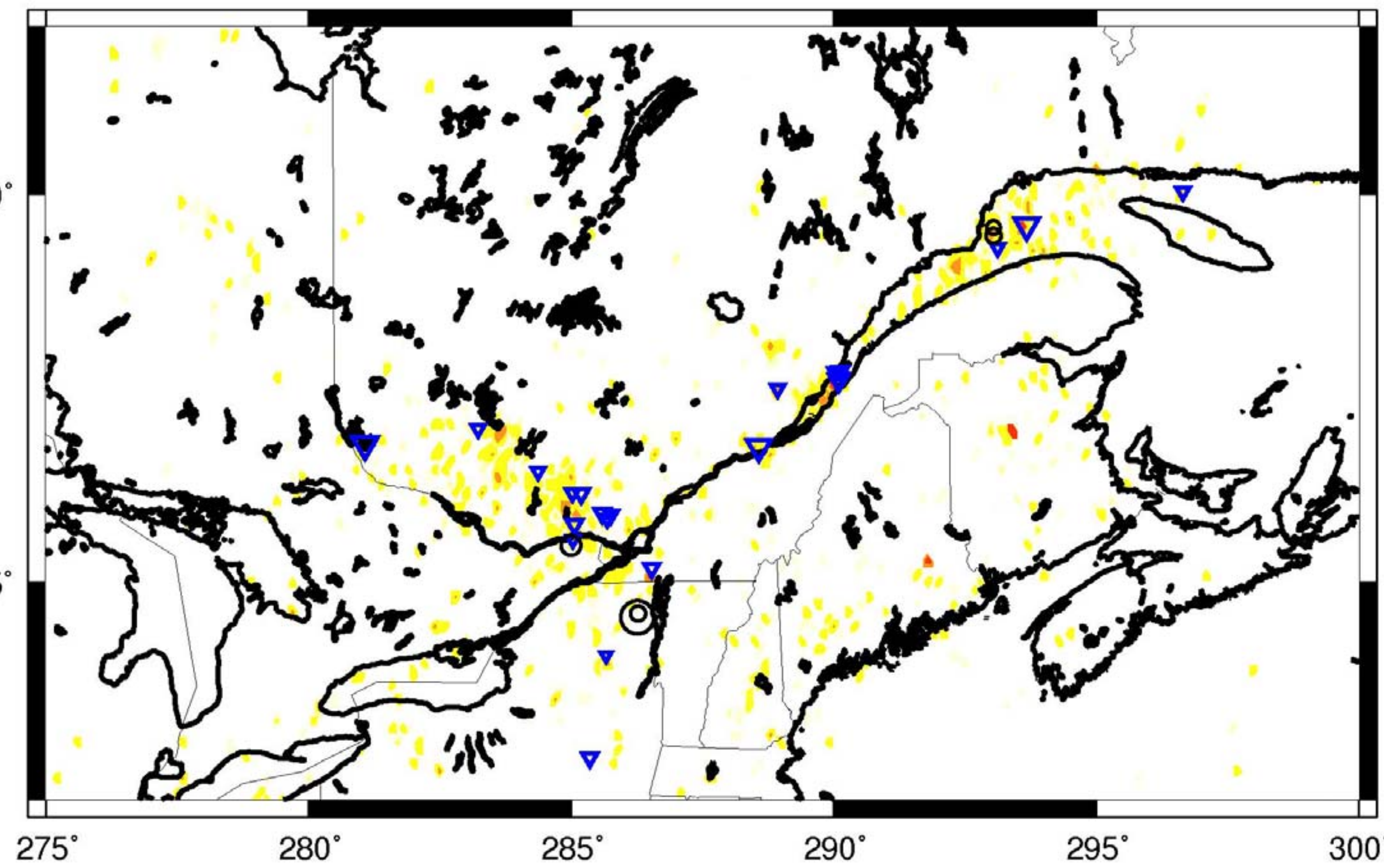
1900 through
2001, M ≥ 4

KL Decomposition, eastern Canada



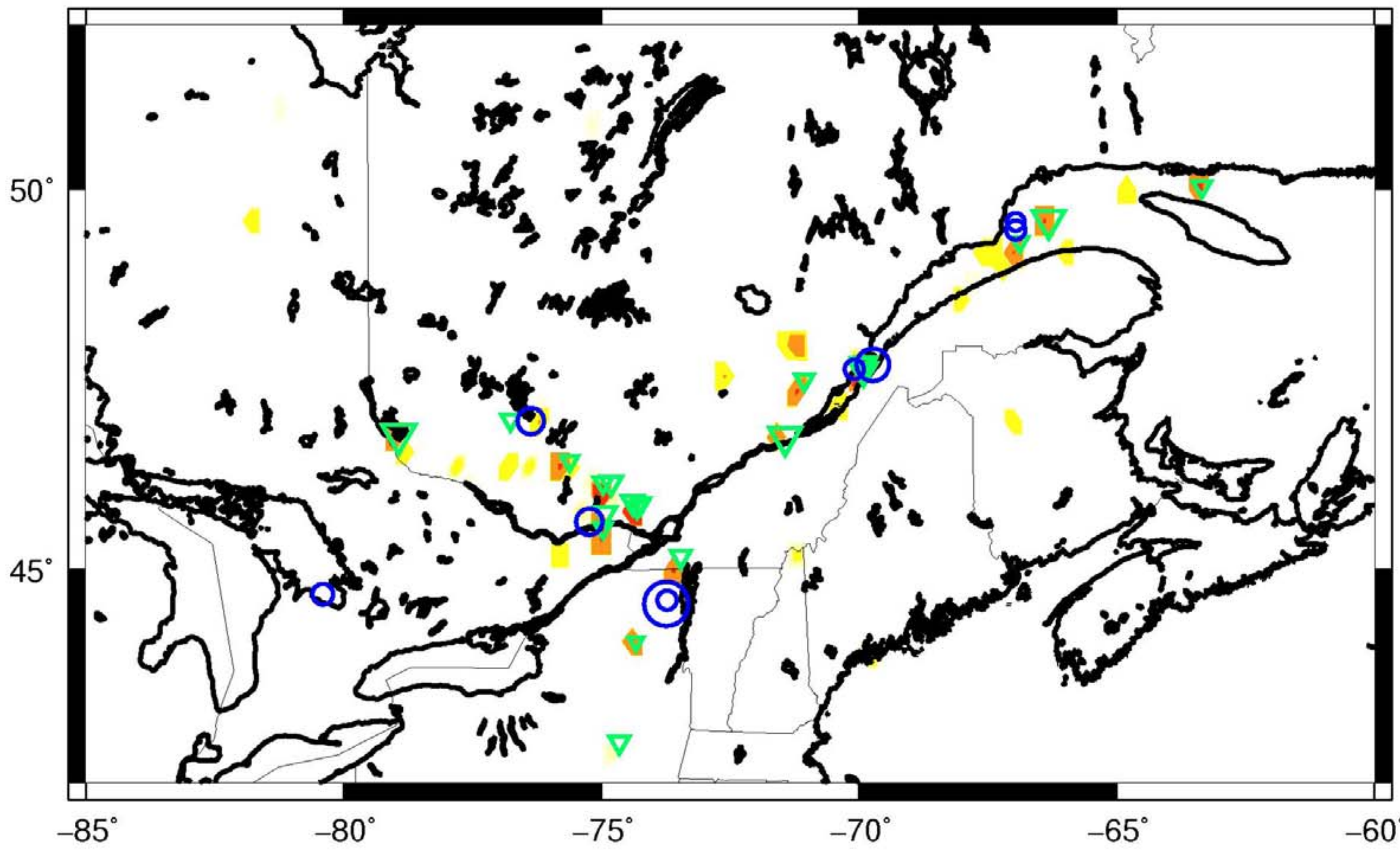
1900 through 2001, M ≥ 4

PI Index, eastern Canada



PI forecast for eastern Canada, 2002-2012, M ≥ 3 (no data is used after 2001).

PI Index, eastern Canada



PI forecast for eastern Canada, 2002-2012. Scale as shown earlier.

Conclusions

- The TM metric can be used to investigate those regions of parameter space (magnitude, region size, spatial discretization) for which the natural fault system, and those models that are used to study it, are ergodic.
- Stationarity is a necessary, although not a sufficient, condition for ergodicity, so that an ergodic condition implies both that the system is stationary and in equilibrium.
- Preliminary results suggest that earthquake fault systems are ergodic for various spatial and temporal regimes, in metastable equilibrium for some period of time.
- D_e is of similar order for both tectonic regions, eastern Canada and California, but better models are needed to determine if this is universal or related to the parameter space in some way.
- For those periods of time, linear operators can be used to analyze the fault system.
- Using this, and other statistical tests derived from and tested on appropriate models, can be used to test both the applicability of those models and our assumptions about the underlying physical properties.