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Abstract.

We analyze binary choices in a random utility model assuming that the agent's preferences are affected by conformism (with respect to the behavior of the society) and coherence (with respect to his identity). We apply the analysis to sequential voting when voters like to win.

Keywords: identity, reputation, social interaction, random utility models, voting system.

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1 Introduction

Modern economic theory aims to investigate social and economic phenomena referring to agents' choices. To this end an assumption on their preferences is mandatory. The classical hypothesis is that it is possible to represent agents' preferences with a utility function defined on consumption goods - e.g. consumer theory, general equilibrium paradigm - or other items related to agent's behavior - e.g. effort, habit, etc..

According to this view, preferences are mostly exogenous and fixed. This approach is flexible enough to model different situations but recently a mounting evidence has shown its intrinsic limits in several fields (economics, sociology, anthropology, political science, psychology). We recall two main issues: agent's preferences are also defined on the behavior of the others; preferences are not fixed and exogenous, they evolve over time and are influenced by the agent's experience and the choices of the society as a whole, e.g. altruism, interdependent utilities, habit formation, network effects, externalities, see Bowles (1998), Sobel (2005), Akerlof and Kranton (2005). The behavior of other agents enters the utility function and influences tastes.

This evidence leads to go beyond self-interest as the unique motivation of the agents' behavior and the representation of preferences by a utility function of the quantity of the goods consumed by the agent. Preferences may include many different items and the maximization of the utility may produce results far away from the classical pure individualistic behavior. This is the case of choices that have social consequences: the choice has a private value for the agent and a social dimension with externalities on the behavior of the other agents. Analyzing social decisions, two features of agents' preferences turn out to be relevant: reputation and identity.

Reputation has been extensively analyzed in the literature. Going back to classical contributions, Veblen (1899) pointed out that economic behavior and preferences are influenced by the behavior of the others: agents care of their status, they desire prestige and esteem and therefore they are sensitive to social acceptance and reputation which induce them to conform to the behavior of the others. In the recent literature this intuition has been modeled in several different perspectives (learning, signaling, technology externalities), and in particular assuming a taste externality (individuals care directly about status, popularity, esteem, respect), see Akerlof (1980), Jones, (1984), Bernheim (1994), Akerlof (1997), Brock and Durlauf (2001). In this case, reputation is modeled through a utility function that positively depends on the fact that the agent's behavior agrees with that of the population. In this setting, social customs arise from the maximization of the utility function because deviation is punished by a loss of reputation and the social interaction outcome may be different from the pure individualistic aggregate behavior.

Pareto (1920) argued that utility is not only driven by tastes but also by identity and norms. The difference is subtle but important: tastes are not necessarily fixed, they may evolve over time but they are usually taken as fixed when the agent takes a decision or an action; norms and identity instead concern a mix of ingredients (social category, status, ideal) and introduce prescriptions on agent's behavior and on the behavior of the others (how they should behave) with a feedback effect on the agent's identity and on his utility. For some attempts to model identity see Stigler and Becker (1977), Akerlof (1980), Akerlof and Kranton (2000, 2005), Akerlof (2007), Akerlof and Kranton (2008, 2010), Cont and Löwe (2010). We observe an internalization of rules of behavior representing the identity or the ego of the agent with identity based payoff derived from the agent's action or from others' actions, see Akerlof and Kranton (2000).

In this paper we consider a binary choice model (random utility) in a dynamic setting taking into account both reputation and identity. Evaluating an action the agent considers its coherence with his identity and his reputation (or social acceptance) in the society taking the behavior of the whole population as a benchmark. Preferences are affected by coherence with respect to the identity of the agent and by conformism with respect to the behavior of the society. Starting from a random utility function *à la* Brock and Durlauf (2001), where conformism is already considered, we introduce identity and coherence in the utility. We model identity based payoffs derived from the action of the agent: as observed in Akerlof and Kranton (2000), an agent experiences anxiety (and a loss in the utility) when he doesn't obey to the rule prescribed by his identity. The analysis is completed by a law of motion of the identity. Identity is affected both by the agent behavior (action driven identity) and/or by the behavior of the society (society driven identity). In the first case the agent looks at his choice to discover his "ego", in the second case he is influenced by the behavior of economy. In the first case we have an individualistic identity formation in the second case we have that identity is influenced by the others. We also extend our analysis to an identity driven by the private utility.

The closest papers to our are Akerlof (1980), Akerlof and Kranton (2000, 2008). In a model on social customs and unemployment, Akerlof (1980) considers both a reputation loss from disobeying to the code of honor that depends on the fraction of people believing in the code and a the loss of utility from disobeying the code of honor by a believer in the code: the standard equilibrium always exists, an equilibrium with unemployment with labor-capital exchange at the code of honor exists if either the reputation effect or the loss from non coherence are large enough. Akerlof and Kranton (2000) provide a setting similar to our with an utility function that depends on the behavior of the society and on the identity with an identity function that depends on agents' actions, see Akerlof and Kranton (2008) for an application to contract theory. Our model can be interpreted as a dynamic version of their model.

We show that social interaction with conformism-reputation and coherence-identity may be different from the pure individualistic society (pure private utility with no interaction) and from a model based on conformism-reputation inspired by Brock and Durlauf (2001). In an individualistic society we have a unique equilibrium, in a model with conformism we may have one or three equilibria when reputation plays a relevant role or noise is limited. In the latter case the outcome is a polarized society with fads that looks very different from a society with a pure individualistic behavior. Coherence and an identity driven by the choice of the agent and not by those of the population contrast conformism: if coherence weight is large enough in the utility compared to that of reputation and coherence is affected mainly by the agent's behavior - and not by the behavior of the society - then social interaction leads to a unique (stable) equilibrium which coincides with the pure individualistic society.

We apply our methodology to sequential voting when voters like to win, see Callander (2007, 2008). In this setting voters observe a private signal on the value of the two candidates and vote in random order observing also the votes expressed by the others. They are influenced by the private signal, by ideology and by the vote of the society. We show that precise signals and a strong desire to vote for the winner lead the agents to vote independently of the value of the candidates with a large majority.

The paper is organized as follows. In Section 2 we introduce agent's preferences considering a random utility function with conformism and coherence. In Section 3 we model the identity formation. In Section 4 we analyze stationary equilibria of the model. In Section 5 we analyze model with identity driven by private utility. In Section 6 we apply our methodology to sequential voting.

2 Identity and Reputation in a binary choice model

We analyze how identity and reputation affect agent's choices in a random utility model *à la* Brock and Durlauf (2001). The economy is made up of I agents facing a discrete binary choice problem for all $t \geq 0$. There are only two possible choices labeled -1 and $+1$. We denote by $\omega_i(t) \in \{-1; 1\}$, where $i = 1, ..., I$ and $t \geq 0$, the choice of the $i - th$ agent at time t and by $\boldsymbol{\omega}(t) = (\omega_1(t), ..., \omega_I(t))$ the vector of the state variables, i.e., agents' choices at time t.

We develop our analysis in two steps. First, we introduce the agent's utility incorporating identity and reputation, then we analyze the dynamics of the entire population choices in a continuous time setting. Presenting the static problem we omit the time indicator t.

The agent's choice enters the agent's utility through three different channels. As in the classical setting, we have the monetary value/utility associated with the specific choice (private utility). Preferences are also characterized by two further features: introspection and conformism. Introspection is modeled assuming that the agent compares his behavior with his identity and gets a reward from the coherence of his behavior and what is established by the identity. The mechanism is as follows: the agent internalizes rules of behavior that are captured by the identity which prescribes a certain behavior and the agent is rewarded by adopting a coherent behavior. The second feature concerns the fact that the agent compares his behavior with the behavior of the economy as a whole and he gets a benefit from behaving (or not) as the majority of the population does. In this case preferences are affected by conformism or an antisocial attitude (anticonformism) with respect to the behavior of the majority of the population.

In this setting, agent's preferences are endogenous and in particular the agent compares his choice with what the others do and with his norm-identity. We have to build a benchmark for the behavior of the society and for the identity. As far as the behavior of the society is concerned, we assume that the agent takes the expectation of the average of the choices of the economy as a proxy of the behavior of the society, i.e., agent i considers $\bar{m}_i^e = \frac{1}{I-1} E[\sum_{j \neq i} \omega_j]$ as a sufficient statistics on the choices of the economy as a whole. As far as the agent's identity is concerned, we consider the binary variable $J_i \in \{-1; 1\}$. At this stage of our analysis we consider the agent's identity as an exogenous constant, the next Section is devoted to its law of motion and to its evolution. We assume that the identity J_i is strictly related to the choice ω_i and, without loss of generality, we assume that if the agent's identity is $J_i = 1$ (-1) then he should take the decision $\omega_i = 1$ (-1). In these cases we say that the agent acts coherently: identity $J_i = 1$ prescribes $\omega_i = 1$, identity $J_i = -1$ prescribes $\omega_i = -1$.

In a random additive utility setting, these features are fully described by the following utility function:

$$u_i(\omega_i) = v(\omega_i) + \gamma\omega_i J_i + \alpha^{\omega}\omega_i \bar{m}_i^e + \epsilon(\omega_i). \tag{1}$$

$v(\omega_i)$ is the private utility function associated with the binary choice. Private utility only depends upon the choice made by the agent and can be interpreted as the monetary value associated with that particular choice. To simplify the analysis we assume that agents have the same private utility, we can allow for heterogeneity in the private utility at the cost of computational problems.

$\gamma\omega_i J_i$ captures the utility that the agent gets by comparing his behavior with his identity. Let $\gamma > 0$. If $sign(\omega_i) = sign(J_i)$ then the agent is coherent, his behavior agrees with the prescriptions of his identity and therefore he gets a reward in terms of utility. The magnitudo of this component is provided by γ. This way to introduce an interaction between identity and behavior is similar to Akerlof (1980). The main difference is that he only considers a loss for an agent who believes in a code of behavior but disobeys to his prescriptions, no loss occurs in the case of an agent who obeys the code but doesn't believe in its social value, in our setting instead the loss is symmetric, i.e., when J_i and ω_i have different sign.

$\alpha^{\omega}\omega_i \bar{m}_i^e$ captures the utility that the agent gets comparing his behavior with that of the economy as a whole. Conformism is introduced assuming directly an externality on preferences, see Bernheim (1994) for a motivation. Assuming $\alpha^{\omega} > 0$ we have conformism. if $sign(\omega_i) = sign(\bar{m}_i^e)$ then the agent gets a benefit from the fact that his behavior conforms with that of the majority of the population. Assuming $\alpha^{\omega} < 0$ we have anticonformism or an antisocial attitude. The magnitudo of the externality is captured by $|\alpha^{\omega}|$. Note that this feature of preferences is already captured by the classical random utility function model, see Brock and Durlauf (2001). A positive α^{ω} leads to strategic complementarities as in Cooper and John, (1988): the marginal utility associated with an action increases with the average action taken by the population. This formulation captures a social component which is negatively affected by the distance between the agent's behavior and the average behavior of the population, as a matter of fact the metric $-\frac{J}{2}(\omega_i - \bar{m}_i^e)^2$ can be written as $J\omega_i \bar{m}_i^e - \frac{J_i}{2}(1 + (\bar{m}_i^e)^2)$ which differs from (1) only in the level while utility dependency on the agent's choice coincides. This way to introduce conformism in the utility function is similar to the "conformist model" proposed by Akerlof (1997), to the reputation function considered in Akerlof (1980), i.e., the loss from disobeying to the code of behavior that depends on the fraction of people believing in the code.

$\epsilon(\omega_i)$ is a random term whose distribution is extreme value, i.e.,

$$P(\epsilon(-1) - \epsilon(1) \leq x) = \frac{1}{1 + e^{-\beta^\omega x}}. \tag{2}$$

$\beta^\omega > 0$ is a measure of the impact of the random component in the decision process. A large β^ω means that the deterministic part plays a relevant role in the maximization of the utility; instead when β^ω tends to zero the error term dominates and the choice between $\omega = 1$ or $\omega = -1$ becomes a coin tossing. The error component can be interpreted as a bounded rationality component on the behavior of the agents: agents are nearly rational in the sense that their behavior can differ from the optimal one by a noise term.

Let $h = \frac{1}{2}(v(1) - v(-1))$. When $h > 0$ the choice $\omega = 1$ leads to a higher private value and therefore $\omega = 1$ is risk-dominant in the sense that the agent is more likely to choose $\omega = 1$ instead of $\omega = -1$.

At time t agent i observes his specific noise realization $(\epsilon(1),\ \epsilon(-1))$ and takes the decision (-1 or 1) comparing $u_i(1)$ to $u_i(-1)$ on the basis of his expectation on the behavior of the population ($\bar{m}_i^e$): if $u_i(1) > u_i(-1)$, then $\omega_i = 1$ and $\omega_i = -1$ otherwise. As the environment is stochastic the choice is non deterministic and satisfies a probability law. Thanks to (2) it can be shown that the agent's choice obeys the probability

$$P(\omega_i | \, \bar{m}_i^e) = \frac{e^{\beta^\omega \omega_i \left(h + \gamma J_i + \alpha^\omega \bar{m}_i^e\right)}}{e^{-\beta^\omega \omega_i \left(h + \gamma J_i + \alpha^\omega \bar{m}_i^e\right)} + e^{\beta^\omega \omega_i \left(h + \gamma J_i + \alpha^\omega \bar{m}_i^e\right)}}. \tag{3}$$

The analysis of the model calls for an assumption on agents' expectations $\bar{m}_i^e$. We can address this issue in a static setting or introducing a dynamics. In the first case the natural candidate is the rational expectations hypothesis: agents possess homogeneous expectations on the average behavior of the population and this hypothesis is consistent with the outcome of the model: the expectation on the average choice is confirmed by the realization of the economy. Utility maximization and rational expectations allow us to identify equilibria for the average behavior of the population thorough a simple fixed point argument: with homogeneous and fixed J, Brock and Durlauf (2001) show that the model admits one or three equilibria. The same paper also addresses expectation formation in a dynamic setting: in a discrete time model the expectation of the behavior of the population at time t is given by the realized behavior at time $t-1$, i.e., agents' expectations are backward looking and myopic. Equilibria of the system of difference equations coincide with those of the static problem with rational expectations. In our analysis instead we consider a continuous time dynamics with a continuous time Markov chain governing both agents' choices and their identity.

3 Choice and Identity Dynamics

Building on Blume and Durlauf (2003) and Barucci and Tolotti (2009), we propose a dynamic decision process based on the maximization of the utility (1). Agents update their decisions at random Poissonian times: when the Poissonian clock of the $i-th$ agent rings, he takes a decision according to his utility function and to his expectation of the others' behavior. Under this specification, the system evolves as a continuous time Markov chain on the state space $\{-1;1\}^I$. We can study the system dynamics and its invariant distributions that characterize the steady states and hence the equilibria of the system. Indeed, we transpose the static probability (3) into a dynamic (instantaneous) probability defined by

$$\lambda_i^{\omega}(t) = \lim_{\tau \to 0} \frac{1}{\tau} P(\omega_i(t+\tau) \neq \omega_i(t) | \,\underline{\omega}(t)\,) = e^{-\beta^{\omega}\omega_i(t)(h+\gamma J_i(t)+\alpha^{\omega} s_I(t))}, \quad i = 1, \ldots, I, \qquad (4)$$

where now all the state variables are indexed with time and the expectation by agent i of the behavior of the others ($\bar{m}_i^e$) has been substituted by the empirical mean

$$s_I(t) := \frac{1}{I} \sum_{i=1}^{I} \omega_i(t). \qquad (5)$$

$\lambda_i(t)$ represents the local rate of probability that agent i changes his choice between time t and t^+, given the state of the system $s_I(t)$ at time t. This model requires all agents to share the same information at any time, in particular they know the statistical mean of the choices of the entire population and therefore they observe the behavior of the others instantaneously. We can also use $\sum_{j \neq i} \omega_j(t)$ in place of $s_I(t)$, i.e., excluding ω_i from the mean, this only requires to multiply all λ_i's by a constant term with no qualitative effects on the dynamics of the model.

We can formally prove that (4) represents the continuous dynamic counterpart of (3). According to (4) agents change their opinion at random times $\{\tau_n^i\}_{n \in \mathbb{N}}$ such that $\tau_n^i - \tau_{n-1}^i$ are exponentially distributed with mean $1/\lambda_i$. At time τ_n^i, agent i revises his choice according to a process driven by (3). For a more detailed discussion on this point, see Barucci and Tolotti (2009).

We can rewrite (4) in a compact way omitting time dependency as

$$\omega_i \mapsto -\omega_i \qquad \text{with intensity} \qquad \lambda_i^\omega = e^{-\beta^\omega \omega_i (h+\gamma J_i + \alpha^\omega s_I)}, \tag{6}$$

where $\beta^\omega > 0$ and $J_i \in \{-1, +1\}$. Let $\alpha^\omega > 0$ and $\gamma > 0$, the interpretation is as follows: a positive and large value of s_I or $J_i = 1$ imply a high probability for agent i to confirm $\omega_i = 1$ or to switch from $\omega_i = -1$ to $\omega_i = 1$; if $s_I < 0$ and $J_i = -1$, then there is a high probability of choosing $\omega_i = -1$. In these cases, coherence and conformism go in the same directions. Instead, if $s_I > 0$ and $J_i = -1$ or $s_I < 0$ and $J_i = +1$, then coherence and conformism go in different directions. A non conformism attitude ($\alpha^\omega < 0$) induces a different effect. Obviously an $h > (<)\ 0$ implies a high probability for agent i to choose $\omega_i = 1\ (-1)$.

Identity of agent i is represented by the variable $J_i = \{-1, 1\}$. Identity is not a constant, it varies over time and is influenced both by the agent's behavior (ω_i) and by the behavior of the society (s_I). We investigate identity formation directly in a dynamic setting. Agents reconsider their identity at random time according to a dynamics similar to (6), We assume a continuous time Markovian evolution for J_i, $i = 1, ..., I$. In particular, we assume the following dynamics:

$$J_i \mapsto -J_i \qquad \text{with intensity} \qquad \lambda_i^J = e^{-\beta^J J_i ((1-\alpha^J)\omega_i + \alpha^J s_I)} \tag{7}$$

where $\alpha^J \in [0, 1]$. Let $\beta^J > 0$ (assuming a negative coefficient the interpretation is reversed), according to (7) the probability that the agent changes his identity from -1 to $+1$ or that he confirms his identity $+1$ is the exponential (and therefore an increasing function) of a convex linear combination of $J_i\omega_i$ and $J_i s_I$. The first component says that agent's identity is driven by his behavior, not only identity affects the agent behavior as shown in (1), also the agent behavior affects the identity, i.e., if $\omega_i = 1$ then there is a high probability that $J_i = 1$ persists or that there is a switch from $J_i = -1$ to $J_i = 1$. In this way we model an *action driven identity* (pure individualistic identity formation). The second effect says that agent's identity is driven by what the others do, not only the behavior of the others affects the agent's utility and his behavior as shown in (1), the social behavior also affects the identity formation, i.e., if $s_I = 1(-1)$ then there is a high probability that $J_i = 1(-1)$. In this way we model a *society driven identity*. This second effect is similar to the hypothesis of Akerlof (1980), where it is assumed that the fraction of the population believing in the code increases in the fraction of the population that obeys the code.

Specifications as (6)-(7) make the state space variables evolve as a continuous time Markov chain on $\{-1,1\}^{2N}$ with the following infinitesimal generator

$$\mathcal{G}_I f(\boldsymbol{\omega},\boldsymbol{J}) = \sum_{i=1}^{I} \lambda_i^{\omega}\left(f(\boldsymbol{\omega}^i,\boldsymbol{J}) - f(\boldsymbol{\omega},\boldsymbol{J})\right) + \sum_{i=1}^{I} \lambda_i^{J}\left(f(\boldsymbol{\omega},\boldsymbol{J}^i) - f(\boldsymbol{\omega},\boldsymbol{J})\right) \tag{8}$$

where $\boldsymbol{\omega}^i = (\omega_1,\ldots,\omega_{i-1},-\omega_i,\omega_{i+1},\ldots,\omega_I)$ (resp. $\boldsymbol{J}^i$) denotes the vector with a switched $i-$th component.

In order to study the dynamics of the system induced by (8), we can rely on techniques developed in Barucci and Tolotti (2009). Compared to Barucci and Tolotti (2009), despite the common theoretical bases, it turns out that the derivation of the equations and the corresponding equilibria is more complex.

Theorem 3.1. *Consider the Markov process $(\boldsymbol{\omega}(t),\boldsymbol{J}(t))_{t\geq 0}$ with generator (8) and assume that at time $t=0$ the random variables $(\omega_i(0),J_i(0))$, $i=1,\ldots,I$, are independent and identically distributed. Consider the following* empirical means

$$s_I^{\omega}(t) = \frac{1}{I}\sum_i \omega_i(t), \quad s_I^{J} := \frac{1}{I}\sum_i J_i(t), \quad s_I^{\omega J} := \frac{1}{I}\sum_i \omega_i(t)J_i(t).$$

Then for $I\to\infty$ the triplet $(s_I^{\omega}(t), s_I^{J}(t), s_I^{\omega J}(t))$ converges (in the sense of weak convergence of stochastic processes) to a triplet $\boldsymbol{m}(t) = (m^{\omega}(t), m^{J}(t), m^{\omega J}(t))$ such that

$$\dot{\boldsymbol{m}}(t) = A(m^{\omega}(t))\cdot\boldsymbol{m}(t) - b(m^{\omega}(t)), \tag{9}$$

where $A(m^{\omega}(t))$ is a 3×3 matrix and $b(m^{\omega}(t))\in\mathbb{R}^3$, both depending on $m^{\omega}(t)$ defined as

$$A(m^{\omega}) = 2\cdot\begin{pmatrix} -C(\mathfrak{a})C(\mathfrak{b}) & C(\mathfrak{a})S(\mathfrak{b}) & -S(\mathfrak{a})S(\mathfrak{b}) \\ C(\mathfrak{c})S(\mathfrak{d}) & -C(\mathfrak{c})C(\mathfrak{d}) & -S(\mathfrak{c})S(\mathfrak{d}) \\ -S(\mathfrak{a})S(\mathfrak{b})+S(\mathfrak{c})C(\mathfrak{d}) & S(\mathfrak{a})C(\mathfrak{b})-S(\mathfrak{c})S(\mathfrak{d}) & -C(\mathfrak{a})C(\mathfrak{b})-C(\mathfrak{c})C(\mathfrak{d}) \end{pmatrix} \tag{10}$$

and the three dimensional vector

$$b(m^{\omega}) = 2\cdot\begin{pmatrix} -S(\mathfrak{a})C(\mathfrak{b}) \\ -S(\mathfrak{c})C(\mathfrak{d}) \\ -C(\mathfrak{a})S(\mathfrak{b})-C(\mathfrak{c})S(\mathfrak{d}) \end{pmatrix},$$

where $S(x) = \sinh(x)$, $C(x) = \cosh(x)$, $T(x) = \tanh(x)$ *and*

$$\begin{cases} \mathfrak{a} = \beta^\omega h + \beta^\omega \alpha^\omega m^\omega \\ \mathfrak{b} = \beta^\omega \gamma \\ \mathfrak{c} = \beta^J \alpha^J m^\omega \\ \mathfrak{d} = \beta^J (1 - \alpha^J)\,. \end{cases}$$

Proof. See Appendix A. ∎

Notice that equation (9) simply says that the differential system leading the dynamics of the triplet $\boldsymbol{m}(t)$ can be written as a *generalized* linear system in $\boldsymbol{m}(t)$ where the coefficients explicitly depend on $m^\omega(t)$.

4 Stationary equilibria and comparative statics

Having established the dynamics of the infinite dimensional system, we can now look at the steady states of the system.

We define $A^{(k)}(m^\omega), k = 1, 2, 3$ as the matrix where we substitute the $k - th$ row of $A(m^\omega)$ with $b(m^\omega)$. Now we are ready to state the Proposition that characterizes the steady states of the system.

Proposition 4.1. *Consider the dynamical system* (9)*. Then the steady states* $(\bar{m}^\omega, \bar{m}^J, \bar{m}^{\omega J})$ *of the system are characterized by a fixed point argument:*

$$\bar{m}^\omega = F_1(\bar{m}^\omega), \tag{11}$$

where

$$F_1(m^\omega) = \frac{|A^{(1)}(m^\omega)|}{|A(m^\omega)|}$$

and $\bar{m}^J = F_2(\bar{m}^\omega)$*,* $\bar{m}^{\omega J} = F_3(\bar{m}^\omega)$*, with* $F_k(m^\omega) = \frac{|A^{(k)}(m^\omega)|}{|A(m^\omega)|}$ *for* $k = 2, 3$.

Proof. See Appendix A. ∎

Proposition 4.1 suggests that the analysis of the stationary equilibria of the system can be reduced to the study of the equation in one variable reported in (11). This is highly nonlinear and therefore its solutions can only be obtained numerically.

We now provide an analysis of stationary equilibria, their stability and some comparative statics. In order to simplify the analysis, we restrict our attention to the case

$\alpha^\omega \in [0,1]$ and $\gamma = 1 - \alpha^\omega$. This assumption puts an antisymmetric relation between the identity driven component of the utility (represented by γ) and the conformism effect (represented by α^ω). Thus, we are left with five parameters, see Table 1.

Table 1: The parameters and the aggregate variables of the model

Parameter	Range	Economic interpretation
h	$\mathbb{R}$	preferability of $\omega = +1$ over $\omega = -1$
β^ω	$(0,\infty)$	The noise level related to the choice of the agent
β^J	$(0,\infty)$	The noise level related to the identity of the agent
α^ω	$[0,1]$	The impact of conformism on the choice of the agents
α^J	$[0,1]$	The impact of conformism on the identity of the agent
Variable		
m^ω	$[-1,1]$	The share of agents choosing $\omega = +1$ ($m^\omega = 0$ means half/half)
m^J	$[-1,1]$	The share of agents with $J = +1$ ($m^J = 0$ means half/half)
$m^{\omega J}$	$[-1,1]$	The share of agents for which $\omega = J$ ($m^{\omega J} = 0$ means half/half)

As far as stationary equilibria is concerned, equation (11) admits one or three solutions depending on the values of the parameters $(h, \alpha^\omega, \beta^\omega, \alpha^J, \beta^J)$. When the solution is unique, it has the same sign of h. When there are three solutions, the two extreme ones have opposite sign; in particular for $h = 0$, the two extreme equilibria are symmetric. Note that $m^{\omega J} > 0$ means that a fraction of the population larger than 50% behaves coherently with his identity, $m^{\omega J} < 0$ means that less than 50% of the population behaves coherently with his identity.

When the stationary solution is unique, it is stable. When there are three equilibria, the two extreme ones are stable and the intermediate one is always unstable. So with multiple stationary equilibria there is a tendency towards a polarized society.

Analyzing the relationship between stationary equilibria and the parameters, we set $h = 0$ which renders symmetric stationary equilibria. $h > 0$ ($h < 0$) simply amounts to a de-symmetrization of the model towards $\omega = 1$ ($\omega = -1$). Existence of multiple equilibria is driven by two key features: randomness measured by β^ω and β^J, the relevance of the behavior of the aggregate economy on agent's choice (measured by α^ω) and on his identity (measured by α^J).

Small values of β^ω and/or β^J denote more randomness in agent's choice and/or in the identity formation process that will eventually drive the dynamics of the system to a (unique) stationary equilibrium in which (approximately) half of the agents chose $\omega = 1$ and/or half of the population is characterized by the identity $J = 1$. The limiting case $\beta^\omega = 0$ (resp. $\beta^J = 0$) represents the case where exactly half of the agents choose $\omega = 1$ (resp. are in the state $J = 1$). On the opposite, as in Brock and Durlauf (2001), a large β^J or a large β^ω make the presence of multiple equilibria more likely.

α^ω and α^J measure the dependence among action, identity and the behavior of the economy as a whole, i.e., conformism in the agent's choice and in his identity. As α^ω and/or α^J increase, the system is characterized by three stationary equilibria. The resulting equilibrium is the positive or negative one, depending on the initial conditions. This effect is not surprising, it confirms the result obtained in Brock and Durlauf (2001) on the relation between conformism and agent's choices and it extends the argument to the relation between the behavior of the economy as a whole and agent's identity. On the contrary small values of α^ω and α^J induce a strict relation between action and identity through the utility function and a role of the agent's choice in forming his identity.

In equilibrium we may have coincidence or not between choice and identity. identity of the population goes in the same direction as the aggregate behavior if $m^\omega = m^J = 0$ or m^ω and m^J have the same sign. Discrepancy between the agent's choice and the identity at the aggregate level is measured by $|m^\omega - m^J|$, instead an index of discrepancy at the agent level - the agent doesn't behaves as his identity predicts - is provided by $m^{\omega J}$.

To provide the flavor of the comparative static results we analyze stationary equilibria varying the parameters of the model. The values of the parameters in the different cases are reported in Tables 2 and 3.

Case 1. Low values of betas and high values of alphas: large noise both in agent's choices and in their identity formation; identity doesn't affect agent's choices and agent's choice doesn't affect his identity; only the behavior of the society as a whole affects choices and identity. There is a unique stationary equilibrium with a completely randomized society: the noise component of the utility dominates the conformism effect and agents are perfectly dispersed both at the aggregate level and at the micro level, i.e., $|m^\omega - m^J| = |0 - 0| = m^{\omega J} = 0$, i.e., half of the population chooses $\omega = 1$ and the identity of half of the population is $J = 1$ but only half of the population acts coherently (i.e. $J = \omega$).

Case 2. Low values of betas, small but positive $1 - \alpha^\omega$ and $1 - \alpha^J$: large noise both in agent's choices and in their identity formation; identity affects agent's choices and agent's choice affects identity, but the behavior of the society as a whole plays a more significant

Table 2: Comparative statics (1)

	Case 1	Case 2	Case 3	Case 4	Case 5
β^{ω}	1	1	2.5	2.5	2.5
β^{J}	1	1	2.5	2.5	2.5
α^{ω}	1	0.6	0	0.2	1
α^{J}	1	0.6	0	0.2	1
N.Eq.	1	1	1	3	3
m^{ω}	0	0	0	$\pm$ 0.13 / 0	$\pm$ 0.98 / 0
m^{J}	0	0	0	$\pm$ 0.13 / 0	$\pm$ 0.98 / 0
$m^{\omega J}$	0	0.28	0.98	0.96	0.97 / 0

effect both on choices and identity. There is a unique stationary equilibrium. A significant fraction of agents follow their identity and synchronizes ω and J.

Case 3: High values of betas and $\alpha^{\omega} = \alpha^{J} = 0$: little noise, irrelevance of the behavior of the economy as a whole on agent's choices/identity. There is a unique stationary equilibrium. Here $m^{\omega J}$ is next to the maximum value as no conformism/social interaction implies dominance of identity on agent's choice.

Case 4: High values of betas and positive (but small) $\alpha^{\omega}, \alpha^{J}$: little noise, moderate relevance of the behavior of the economy as a whole on agent's choices/identity. There are three stationary equilibria. There is still high concordance between ω and J in all the stationary equilibria ($m^{\omega J}$ next to one) but the society is "partially" polarized (positive and negative equilibrium with small absolute value).

Case 5: High values of betas and alphas: small noise both in agent's choices and in their identity formation; identity doesn't affect agent's choices and agent's choice doesn't affect his identity; only the behavior of the society as a whole affects choices and identity. Strong conformism leads to "extreme" stationary equilibria. We have a strong concordance between ω and J at the extreme equilibria ($m^{\omega J}$ near to one). A society with small noise in the utility function and strong conformism is characterized by a polarized outcome and coherence.

Case 6-9: different degree of noise and of conformism. When noise and conformism are high in identity formation or agent's choice (case 7 and 8) there is a unique equilibrium; in these cases there is coherence between choice and identity; the polarized effect of little

Table 3: Comparative statics (2)

	Case 6	Case 7	Case 8	Case 9	Case 10	Case 11
β^ω	2.5	2.5	0.1	0.1	1	2.5
β^J	0.1	0.1	2.5	2.5	2.5	1
α^ω	0.6	0	0.6	0	0.8	0.8
α^J	0	0.6	0	0.6	0.8	0.8
N.Eq.	3	1	1	1	3	3
m^ω	$\pm$ 0.14 / 0	0	0	0	$\pm$ 0.5 / 0	$\pm$ 0.97 / 0
m^J	$\pm$ 0.01 / 0	0	0	0	$\pm$ 0.81 / 0	$\pm$ 0.75 / 0
$m^{\omega J}$	0.5	0.85	0.85	0.5	0.52 / 0.33	0.75 / 0.33

noise and conformism is stronger on the choice than on the identity: in case 6 we have three equilibria, in case 9 a unique equilibrium, uniqueness of the equilibrium is also confirmed with $\alpha^\omega = 1$.

Case 10-11: strong conformism, little noise in the choice or in the identity formation process. We have a polarized society with a coherent behavior. Little noise in the choice renders a more polarized/coherent society.

The analysis provides us with the following main results.

Result 1. As noise in agent's choices and/or identity formation decreases or conformism in agent's choices and/or identity formation increases we have multiple stationary equilibria (compare case 1 and 5 and case 3 and 5).

Result 2. With a lot of noise or little conformism in agent's choices and identity formation we have a unique stationary equilibrium with the individualistic outcome; the difference is that noise leads to decoupling between choice and identity (case 1 and 2), instead little conformism leads to a behavior coherent with the agent's identity (case 3).

Result 3. With multiple equilibria, in equilibrium $|m^\omega|$ is increasing in α^ω and β^ω, $|m^J|$ is increasing in α^J and β^J (case 4 and 5).

Result 4. In equilibrium $m^{\omega J}$ increases when α^ω and/or α^J decrease (case 1 and 2, case 3 and 5), and when β^ω and/or β^J increase.

Result 5. In equilibrium $|m^\omega - m^J|$ is high when $|\beta^\omega - \beta^J|$ is high.

Result 6. Coherence between agent's behavior and identity is observed with little noise when there is no conformism (mixed-individualistic outcome), see case 3 and 4, and when there is strong conformism (polarized outcome), see case 5, 10 and 11.

Result 7. In a polarized society (multiple stationary equilibria), identity and choice as a whole are coherent ($sign(m^\omega) = sign(m^J)$) but $|m^\omega| > |m^J|$ if $\beta^\omega > \beta^J$.

Result 8. Difference between choice and identity is driven by the precision of noise int he choices (β^ω) or by by the identity conviction (β^J) rather than by the difference of conformism of the two processes.

5 Utility driven identity

We consider the setting of Section 3 assuming that identity is driven by two components: the behavior of the society (s_I) and the private utility represented by h. Agents reconsider their identity at random time according to a dynamics similar to (6). In particular, we assume the following dynamics for the J component:

$$J_i \mapsto -J_i \qquad \text{with intensity} \qquad \mu_i^J = e^{-\beta^J J_i((1-\alpha^J)h+\alpha^J s_I)} \tag{12}$$

The following result holds.

Corollary 5.1. *Consider the dynamical system induced by* (9) *where now* λ_i^J *in* (8) *is replaced by* μ_i^J *defined in* (12)*. Then Proposition 4.1 still holds where the matrix* A *and the vector* b *are now defined as*

$$A(m^\omega) = \begin{pmatrix} -C(\mathfrak{a})C(\mathfrak{b}) & C(\mathfrak{a})S(\mathfrak{b}) & -S(\mathfrak{a})S(\mathfrak{b}) \\ -C(\mathfrak{c}+h\mathfrak{d}) & 0 & 0 \\ -S(\mathfrak{a})S(\mathfrak{b})+S(\mathfrak{c}+h\mathfrak{d}) & S(\mathfrak{a})C(\mathfrak{b}) & -C(\mathfrak{a})C(\mathfrak{b})-C(\mathfrak{c}+\mathfrak{d}) \end{pmatrix} \tag{13}$$

$$b(m^\omega) = \begin{pmatrix} -S(\mathfrak{a})C(\mathfrak{b}) \\ -S(\mathfrak{c}+h\mathfrak{d}) \\ C(\mathfrak{a})S(\mathfrak{b}) \end{pmatrix},$$

where $S(x) = \sinh(x)$, $C(x) = \cosh(x)$ *and*

$$\begin{cases} \mathfrak{a} = \beta^\omega h + \beta^\omega \alpha^\omega m^\omega \\ \mathfrak{b} = \beta^\omega \gamma \\ \mathfrak{c} + h\mathfrak{d} = \beta^J \alpha^J m^\omega + h \cdot \beta^J (1-\alpha^J)\,. \end{cases}$$

Proof. The Corollary can be proved arguing in the same way as in Proposition 4.1 with slightly modifications in the calculation due to the change in the jump intensity of the J component. It only remains to ensure that the determinant of A is non zero. A straightforward computation shows that

$$\det(A) = C(\mathfrak{c} + h\mathfrak{d}) \cdot \left[C(\mathfrak{a})^2 C(\mathfrak{b}) S(\mathfrak{b}) (T(\mathfrak{a})^2 - 1) - C(\mathfrak{a}) S(\mathfrak{b}) C(\mathfrak{c} + h\mathfrak{d}) \right].$$

Being $C(x) > 0$, $T(x)^2 < 1$ for all x and $S(\mathfrak{b}) > 0$, the determinant is strictly negative. ∎

Table 4: Comparative statics - Utility driven identity

	Case 1	Case 2	Case 3	Case 4	Case 5
$\boldsymbol{h}$	**0**	**0**	**0**	**0.3**	**0.3**
β^ω	2.5	2.5	2.5	2.5	2.5
β^J	2.5	2.5	1	1	1
α^ω	0.2	0.2	0.8	0.8	**1**
α^J	0.2	**0.4**	0.8	0.8	**1**
N.Eq.	1	3	3	1	3
m^ω	0	± 0.52 / 0	± 0.97 / 0	0.99	-0.91 / -0.54 / 0.99
m^J	0	± 0.47 / 0	± 0.65 / 0	0.77	-0.54 / -0.24 / 0.86
$m^{\omega J}$	0.76	0.90 / 0.76	0.65 / 0.24	0.77	0.50 / 0.13 / 0.86

Comparative static results are similar to those obtained in the previous section. In Table 4 we discuss the cases analyzed in the previous Section considering $h = 0$ and $h \neq 0$.

The main difference with respect to the model analyzed in the previous Section is that an agent whose identity is driven by his private utility (h) is more likely to behave in an individualistic way. To have multiple stationary equilibria we need a higher degree of conformism in choice/identity compared to the case analyzed in the previous section. The effect is made clear by case 1 and 2 in Table 4 and case 4 in 2: with the same parameters the economy with identity driven by ideology is characterized by multiple equilibria and those stationary do not correspond to agents' private preferences. The rationale is that in this setting we have a more relevant role of fundamental value associated with the choice.

Considering case 3 and 4 in Table 4 we observe that with a zero private utility differential we have multiple stationary equilibria, assuming $h \neq 0$ we have a unique stationary equilibrium. To observe multiple equilibria, we need to increase α^ω and α^J, see case 5. Notice that when $h \neq 0$, equilibria are no longer symmetric.

6 Voting when voters like to win

We apply our analysis to sequential voting. Agents choose between two candidates/parties ($+1$ and -1). We consider a common value setting a' la Austen-Smith and Banks (1996) with incomplete information, agents vote sequentially observing a private signal about the value of the two candidates. Voters vote sequentially in a random order. As agent i is selected to vote, he first observes a private signal about who is the better candidate ($\epsilon_i(+1) - \epsilon_i(-1)$): if $v(+1) + \epsilon_i(+1) - v(-1) - \epsilon_i(-1) > 0$ he votes for candidate $+1$. The voting system is peculiar, voters may change their vote and the competition ends when an equilibrium is reached.

The sequential structure of voting systems has been investigated in the literature considering the tradeoff between efficiency (aggregation of information) and momentum (voting bandwagon) with departure from the individualistic behavior; in the first case the better candidate wins the election, in the second case the winner may not be the better candidate. The equilibrium is efficient (informative) if $sign(m^{\omega}) = sign(h)$. We follow Callander (2007, 2008) assuming that voters possess a desire to conform with the majority. Their analysis shows that as long as the desire to vote for the winner is non-zero a bandwagon will ultimately begin, see Hung and Plott (2001), Dasgupta et al. (2007) for experimental results. The bandwagon threshold that induces the voter to follow the vote of the majority independently of their signal is decreasing in the degree of voter to be in the winning majority.

We enrich the setting assuming that each voter is also endowed with an ideology (J_i). We consider the interplay between ideology and desire to vote with the winning majority (conformism), measured by α^{ω}. In this setting β^J provides an index of the ideological conviction. The agent gets a benefit by voting in agreement with his signal, his ideology and the majority. Our analysis developed in the previous sections sheds some interesting results with respect to previous studies. We summarize the results as follows:

1. Bandwagons/departure from the true value of the candidates (informative equilibrium) are more likely when private signals on the value of the candidates are precise.

2. Strong ideological conviction and imprecise private information signals lead to the informative equilibrium.

3. When signals on the value of the candidates are precise and conformism high (either on choices or on ideology), the informative equilibrium (the one that coincides with the individualistic outcome) is not stable and we observe a polarized vote with a

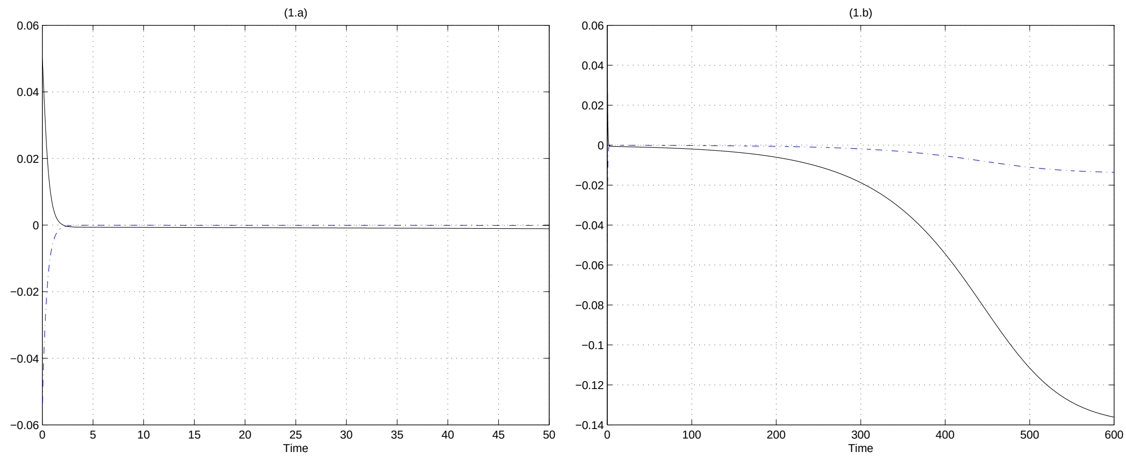

Figure 1: Dynamics of $m^\omega(t)$ (solid) and $m^J(t)$ (dash-dot), with parameters as in Case 6 of Table 3. The time horizon is short ($T = 50$ days) on the left panel and long ($T = 600$ days) on the right.

large majority (bandwagon).

4. We may have a large majority in a society with a weak ideological conviction when private signals are precise and conformism is strong.

5. There is coherence between ideology and vote with little noise both at the informative equilibrium (weak conformism) or in a bandwagon with a large majority (strong conformism).

Relying on our model is it also possible to analyze the dynamics of the system. The dynamics of sequential vote may be peculiar with a tight competition for a long time before convergence to one (stable) equilibrium. To illustrate this point we provide a numerical example. We look at the model with parameters as described in case 6 of Table 4. As already discussed, in this case we have a society whose citizens are at any stable equilibrium almost equally divided between the two major parties $m^J \approx 0$ from an ideological viewpoint, but where a significant part of the voters chooses one of the two parties because of a bandwagon effect. In the numerical example we have chosen the negative equilibrium $m^\omega = -0.14$.

In Figure 1 we analyze a trajectory of the pair $(m^\omega(t), m^J(t))$ where we have chosen as initial values $m^\omega(0) = 0.05$ and $m^J(0) = -0.055$. In the two panels we compare the short horizon behavior and the long run behavior of the society. In particular we see that the trajectory is initially driven towards the unstable equilibrium ($m^\omega = 0$, $m^J = 0$), see Figure 1.a where we have taken $T = 50$ as final horizon (assume t is the number of days under account). The stable equilibrium is reached only after a considerable transition time, see Figure 1.b where we consider $T = 600$, i.e. almost two years.

7 Conclusions

Social interaction is a complex issue. In this paper we have provided a model to analyze social interaction when agents are affected both by the behavior of the society as a whole and by their identity. We may have a polarized society with a deep departure from the outcome with individualistic behavior. Conformism induces this phenomenon, identity drives towards the private utility outcome provided that identity is affected by the agent's choices or private utility.

A polarized outcome is observed mainly when there is little noise in choices, i.e., private signals are precise, or in identity formation, i.e., agents have a strong conviction in their identity. A polarized outcome is usually accompanied by coherence between choice and identity, i.e., choice and identity go in the same direction.

A Proofs

Proof of Theorem 3.1.

Notice that this theorem is twofold. On one hand it ensures convergence of the I dimensional empirical process to an infinite dimensional one. On the other hand, it characterizes the infinite dimensional process by means of the *deterministic* three dimensional system given by (9).

The proof of the convergence part of the theorem follows directly from the proof of Theorem 3.1 and Theorem A.1 in Barucci and Tolotti (2009) with minor modifications in the functions involved. We thus refer the reader to that theorem.

The convergence part of the theorem, ensures the existence of a limiting process $\boldsymbol{m}(t) = (m^\omega(t), m^J(t), m^{\omega j}(t))$. We still have to show that $\boldsymbol{m}(t)$ solves (9).

One of the consequences of Theorem 3.1 in Barucci and Tolotti (2009) is that we can rewrite $\boldsymbol{m}(t) = (m^\omega(t), m^J(t), m^{\omega j}(t))$ as follows:

$$m^\omega(t) = \sum_{\omega,J=\pm 1} \omega q_t(\omega, J), \quad m^J(t) = \sum_{\omega,J=\pm 1} J q_t(\omega, J), \quad m^{\omega J}(t) = \sum_{\omega,J=\pm 1} \omega J q_t(\omega, J); \quad (14)$$

where $(q_t)_{\{t\in[0,T]\}}$ is family of measures such that for all $t \in [0, T]$

$$\dot{q}_t = \nabla^\omega \left[e^{-\beta^\omega \omega(h+\gamma J+\alpha^\omega m^\omega(t))} \cdot q_t(\omega, J)\right] + \nabla^J \left[e^{\beta^J J\,((1-\alpha^J)\omega+\alpha^J m^\omega(t))} \cdot q_t(\omega, J)\right] \tag{15}$$

with $(\omega, J) \in \{-1; 1\}^2$ and where $\nabla^x f(x, y) = f(-x, y) - f(x, y)$.

To simplify the notations, in what follows we shall write m_t in place of $m(t)$. Relying on (14) and (15), it is possible to prove that

$$\left\{\begin{array}{lcl} \dot{m}_t^\omega & = & -2C(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)C(\beta^\omega \gamma)\, m_t^\omega + 2C(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)S(\beta^\omega \gamma)\, m_t^J \\ & & \quad -2S(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)S(\beta^\omega \gamma)\, m_t^{\omega J} + 2S(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)C(\beta^\omega \gamma) \\ \dot{m}_t^J & = & +2S(\beta^J(1-\alpha^J))C(\beta^J \alpha^J m_t^\omega)\, m_t^\omega - 2C(\beta^J(1-\alpha^J))C(\beta^J \alpha^J m_t^\omega)\, m_t^J \\ & & \quad -2S(\beta^J(1-\alpha^J))S(\beta^J \alpha^J m_t^\omega)\, m_t^{\omega J} + 2C(\beta^J(1-\alpha^J))S(\beta^J \alpha^J m_t^\omega) \\ \dot{m}_t^{\omega J} & = & -2[S(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)S(\beta^\omega \gamma) - C(\beta^J(1-\alpha^J))S(\beta^J \alpha^J m_t^\omega)]\, m_t^\omega \\ & & \quad +2[S(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)C(\beta^\omega \gamma) - S(\beta^J(1-\alpha^J))S(\beta^J \alpha^J m_t^\omega)]\, m_t^J \\ & & \quad -2[C(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)C(\beta^\omega \gamma) + C(\beta^J(1-\alpha^J))C(\beta^J \alpha^J m_t^\omega)]\, m_t^{\omega J} \\ & & \quad +2[C(\beta^\omega h + \beta^\omega \alpha^\omega m_t^\omega)S(\beta^\omega \gamma) + S(\beta^J(1-\alpha^J))C(\beta^J \alpha^J m_t^\omega)]. \end{array}\right. \tag{16}$$

where $C(x) = \cosh(x)$ and $S(x) = \sinh(x)$.

The definitions of $A(m^\omega(t))$ and $b(m^\omega(t))$, given in (10), immediately follow from (16).

To show the validity of (16), consider $m^\omega(t) = \sum_{\omega,J} \omega q_t(\omega, J)$, hence $\dot{m}_t^\omega = \sum_{\omega,J} \omega \dot{q}_t(\omega, J)$. Relying on (15) we have

$$\begin{aligned} \dot{m}_t^\omega = \sum_{\omega,J} \omega \left(\nabla^\omega \left[e^{-\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q(\omega, J)\right] + \nabla^J \left[e^{-\beta^J J((1-\alpha^J)\omega+\alpha^J m_t^\omega)} q(\omega, J)\right]\right) = \\ = \sum_{\omega,J} \omega \left(e^{\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q_t(-\omega, J) - e^{-\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q_t(\omega, J)\right) + \\ + \sum_{\omega,J} \omega \left(e^{\beta^J J((1-\alpha^J)\omega+\alpha^J m_t^\omega)} q_t(\omega, -J) - e^{-\beta^J J((1-\alpha^J)\omega+\alpha^J m_t^\omega)} q_t(\omega, J)\right). \end{aligned}$$

We now use the following facts

$$\sum_{\omega,J} \omega e^{\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q_t(-\omega, J) = -\sum_{\omega,J} \omega e^{-\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q_t(\omega, J),$$

$$\sum_{\omega,J} \omega e^{\beta^J J((1-\alpha^J)\omega+\alpha^J m_t^\omega)} q_t(\omega, -J) = \sum_{\omega,J} \omega e^{-\beta^J J((1-\alpha^J)\omega+\alpha^J m_t^\omega)} q_t(\omega, J).$$

So that

$$\dot{m}_t^\omega = -2 \sum_{\omega,J} \omega e^{-\beta^\omega \omega(h+\gamma J+\alpha^\omega m_t^\omega)} q_t(\omega, J).$$

Moreover, it is easy to check that for $\omega, J \in \{-1;+1\}$, it holds

$$e^{-\beta^\omega \omega(h+\alpha^\omega m_t^\omega)} = -\omega \frac{e^{\beta^\omega(h+\alpha^\omega m_t^\omega)} - e^{-\beta^\omega(h+\alpha^\omega m_t^\omega)}}{2} + \frac{e^{\beta^\omega(h+\alpha^\omega m_t^\omega)} + e^{-\beta^\omega(h+\alpha^\omega m_t^\omega)}}{2},$$

$$e^{-\beta^\omega \omega\gamma J} = -\omega J \frac{e^{\beta^\omega\gamma} - e^{-\beta^\omega\gamma}}{2} + \frac{e^{\beta^\omega\gamma} + e^{-\beta^\omega\gamma}}{2}.$$

Using the definitions $S(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$ and $C(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$, we have

$$\dot{m}_t^\omega = -2\sum_{\omega,J} \omega[-\omega S(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega) + C(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega)][-\omega J S(\beta^\omega\gamma) + C(\beta^\omega\gamma)]q_t(\omega, J).$$

Now, using the fact that $\omega^2 = 1$ and (14), it follows that

$$\dot{m}_t^\omega = -2C(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega)C(\beta^\omega\gamma)\, m_t^\omega + 2C(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega)S(\beta^\omega\gamma)\, m_t^J$$
$$-2S(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega)S(\beta^\omega\gamma)\, m_t^{\omega J} + 2S(\beta^\omega h + \beta^\omega\alpha^\omega m_t^\omega)C(\beta^\omega\gamma).$$

A similar argument is used to compute $\dot{m}_t^J$ and $\dot{m}_t^{\omega J}$. ■

Proof of Proposition 4.1.

Being $(\bar{m}^\omega, \bar{m}^J, \bar{m}^{\omega J})$ the steady states of (9), they solve $A(m^\omega)\boldsymbol{m} = b(m^\omega)$.

The proposition then follows from the classical Cramer's Theorem on linear systems, once we ensure that the determinant of $A(m^\omega)$ is non zero. We distinguish two cases:

a) $T(\mathfrak{b}) \geq T(\mathfrak{d})$.

In this case we operate a manipulation (we omit the details) on the rows of the matrix obtaining:

$$\tilde{A} = \begin{pmatrix} -1 & T(\mathfrak{b}) & -T(\mathfrak{a})T(\mathfrak{b}) \\ 0 & T(\mathfrak{b})T(\mathfrak{d}) - 1 & -T(\mathfrak{d})(T(\mathfrak{c}) + T(\mathfrak{a})T(\mathfrak{b})) \\ 0 & a_{23} & a_{33} \end{pmatrix}$$

where

$$\begin{cases} a_{23} = S(\mathfrak{a})C(\mathfrak{b}) - S(\mathfrak{c})S(\mathfrak{d}) - T(\mathfrak{b})S(\mathfrak{a})S(\mathfrak{b}) + T(\mathfrak{b})S(\mathfrak{c})C(\mathfrak{d}); \\ a_{33} = -C(\mathfrak{a})C(\mathfrak{b}) - C(\mathfrak{c})C(\mathfrak{d}) - T(\mathfrak{a})T(\mathfrak{b})(-S(\mathfrak{a})S(\mathfrak{b}) + S(\mathfrak{c})C(\mathfrak{d})). \end{cases}$$

All the operations used in order to transform A into $\tilde{A}$ are preserving the rank. Then $\det(A)$ is non zero as soon as $\det(\tilde{A})$ is non zero. We now have

$$\det(\tilde{A}) = -\left\{a_{33} \cdot [T(\mathfrak{b})T(\mathfrak{d}) - 1] + a_{23} \cdot [T(\mathfrak{c})T(\mathfrak{d}) + T(\mathfrak{a})T(\mathfrak{b})T(\mathfrak{d})]\right\}.$$

Consider now m^ω and h non negative. In this case, all the hyperbolic functions are non negative. Moreover, $T(\mathfrak{b})T(\mathfrak{d}) < 1$. A tedious calculation shows that $a_{23} \geq 0$ and $a_{33} < 0$. Thus $\det(\tilde{A}) < 0$.

b) $T(\mathfrak{b}) < T(\mathfrak{d})$.

As before we transform A obtaining:

$$\bar{A} = \begin{pmatrix} T(\mathfrak{d}) & -1 & -T(\mathfrak{c})T(\mathfrak{d}) \\ 0 & T(\mathfrak{b})T(\mathfrak{d}) - 1 & -T(\mathfrak{d})(T(\mathfrak{c}) + T(\mathfrak{a})T(\mathfrak{b})) \\ 0 & a_{23} & a_{33} \end{pmatrix}$$

where

$$\begin{cases} a_{23} = T(\mathfrak{d})S(\mathfrak{a})C(\mathfrak{b}) - T(\mathfrak{d})S(\mathfrak{c})S(\mathfrak{d}) - S(\mathfrak{a})S(\mathfrak{b}) + S(\mathfrak{c})C(\mathfrak{d}); \\ a_{33} = -T(\mathfrak{d})C(\mathfrak{a})C(\mathfrak{b}) - T(\mathfrak{d})C(\mathfrak{c})C(\mathfrak{d}) + T(\mathfrak{c})T(\mathfrak{d})(-S(\mathfrak{a})S(\mathfrak{b}) + S(\mathfrak{c})C(\mathfrak{d})) . \end{cases}$$

We now have

$$\det(\tilde{A}) = T(\mathfrak{d}) \cdot \{a_{33} \cdot [T(\mathfrak{b})T(\mathfrak{d}) - 1] + a_{23} \cdot [T(\mathfrak{c})T(\mathfrak{d}) + T(\mathfrak{a})T(\mathfrak{b})T(\mathfrak{d})]\} .$$

Consider now m^ω and h non negative. In this case, all the hyperbolic functions are non negative. Moreover, $T(\mathfrak{b})T(\mathfrak{d}) < 1$. A tedious calculation shows that $a_{23} > 0$ and $a_{33} < 0$. Thus $\det(\bar{A}) > 0$. Similar arguments are used for different values of m^ω and h. ∎

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