12-2-2020

What is Robotics: Why Do We Need It and How Can We Get It?

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Recommended Citation

Daniel E. Koditschek, "What is Robotics: Why Do We Need It and How Can We Get It?", *Annual Review of Control, Robotics, and Autonomous Systems* . December 2020.

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Disciplines

Electrical and Computer Engineering | Engineering | Systems Engineering

Preprint of an article to appear in
Annual Review of Control, Robotics, and Autonomous Systems

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December 2, 2020

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Robotics is an emerging synthetic science concerned with programming work. Robot technologies are quickly advancing beyond the insights of the existing science. More secure intellectual foundations will be required to achieve better, more reliable and safer capabilities as their penetration into society deepens. Presently missing foundations include the identification of fundamental physical limits, the development of new dynamical systems theory and the invention of physically grounded programming languages. The new discipline needs a departmental home in the universities which it can justify both intellectually and by its capacity to attract new diverse populations inspired by the age old human fascination with robots.

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 https://doi.org/10.1146/annurev-control-080320-011601

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1 INTRODUCTION

Robotics is a growing body of technology presently in the early stages of developing its disciplinary foundations. Emerging from decades of collaboration among biologists, computer scientists and engineers—its looming commercial presence, a mere harbinger of the enormous social impact to come—its reach has already begun to outstrip the grasp of its still meager foundations.

This article proposes that robotics is destined to become (eventually understood and explicitly advanced as) a new synthetic science concerned with programming work—the exchange of energy and information between a machine and its environment toward some specified set of goals. After exploring what these terms mean in Section 2, the argument turns to the value of their consideration for advancing the field in Section 3. The article proceeds in Section 4 to offer a speculative view of how this new discipline might come into being, and then concludes in Section 5 with a brief account of why it was written and who might benefit from reading it. [1]

2 WHAT IS ROBOTICS?

In contrast to the natural sciences—bodies of theory revealing the empirical realm as it exists—a synthetic science seeks theory that anticipates what could be brought into existence [1]. This requires that we go beyond Feynman's oft-quoted statement "What I cannot create I do not understand," [2] insisting not merely that we test our intuition and understanding by what we create, but that our understanding be codified in principles of design that predict its artifacts' empirical properties in advance of their creation. [3] Thus, a discipline of robotics must aspire to a formal body of theory that endows some language of goal specification with the blueprints of material construction for mechanisms that will exchange energy with their environment in a manner that provably achieves the goals. Such specifications must scope types of environments so as to express settings within which the goals must or cannot prevail. In particular, the theory must prevent the specification of goals that cannot prevail in any environment because their achievement would violate fundamental physical limits.

[1] The Supplemental Appendix offers a supporting (but necessarily abbreviated) sketch of ideas from dynamical systems theory (§A), some details on models of a robot's interface to its environment (§B) and further details (§C) supporting the speculation on how to advance the discipline of robotics (discussed in Section 4 of the main text); references led by these letters (often in combination with punctuated numbers) refer to the corresponding sections in the Supplemental Appendix. Footnotes using italicized lowercase letters rather than numerals similarly refer to footnotes in the Supplemental Appendix; these footnotes introduce an additional degree of technical detail that depends on notions that are introduced in (and, hence, must be relegated to) the supplement.

[2] Rendering the contrapositive of this quotation as "If I do understand then I can create," synthetic biologists have also begun to ponder the gap between that statement and its converse, "If I can create then I do understand" [2]—a central focus of this article.

[3] Here and throughout, such terms as synthesis, design principles, design methodologies, and so on are intended to connote a formal tool set whereby specification might map onto behavior. Addressing the profound and fascinating ascent from the science of tools toward the art of their application pondered over the centuries by thinkers far more capable than this author [3] lies well beyond the scope of this article.

 https://doi.org/10.1146/annurev-control-080320-011601

2.1 Lessons from Computer Science

Computer science, which is concerned with programming the exchange of information between a machine and its environment [4] [4] emerged over the course of the first half of the twentieth century as a body of theory and practice at the intersection of mathematical logic, linguistics, neuroscience, and electrical engineering [8]. It took a good part of the second half to win gradual recognition as a discipline [9]. Amidst the many controversies about what else it has become [10], the discipline arguably earned the distinction of synthetic science upon Landauer's [11]identification of the fundamental physical cost of information, following which a great deal more has been discovered about the fundamental limits of computing [12].

The lessons of computer science for robotics include the fruitful interplay between our ability to express design goals and the capacities of physical substrates to achieve them (§2.1.1); the vital role of modularity and its apotheosis in the form of a program (§2.1.2); and the central importance of the design triple—distinguishing between and relating tasks, architectures for accomplishing them, and the environments within which those accomplishments are sought (§2.1.3).

2.1.1 Computational substrates

Few would contest that relating formally specified architecture [13] to physical substrate [14, 15, 16] played a decisive role in the past half century's information technology revolution. The remarkable advances in theory that permit researchers to tie abstract features of logical operation to physical fabrication and design in light of such absolute barriers as Landauer's limit [17] span nearly 20 orders of magnitude—from nanoscale devices ($\sim 10^{-8}$ W) to regional data centers ($\sim 10^{9}$ W) [18]—a staggering testament to the success of computing as a physical synthetic science, underlying its inescapable social impact. Still more compelling from the envious view of a nascent-robotics scientist, is the computer engineer's crucial ability to define and operationalize design tradespaces whose dimensions freely mix and compare metrics of information (e.g. instruction word width, data vs. instruction memory address space, instruction loop length, etc.), against those of energy [19].

Ingenious technological work-arounds that continue to mitigate Landauer's thermodynamic threat to Moore's law [20] do not negate but rather underscore the value of identifying and designing explicitly with respect to such fundamental physical limits. In fact, a key difference between robotics and computer science is illuminated by Bennett's recourse to reversible computation [21]. Physically realized decades later [22], reversible computing might promise to free from Landauer's limit machines that merely exchange information with the environment. Regardless, however, the ingenuity of computer engineers has to date largely enabled users' abstractions to function as if this were the case. In contrast, computing might be cool, but robotics must be hot (§2.2.1.1).

2.1.2 From modules to programs

Key to this staggering information technology revolution has been the role of programming languages. Mead & Conway's [23] formal design rules that could scale with the computing substrates [15] fueled the development of register transfer languages for the specification of instruction set processors [24]. McCarthy [25] recognized that Church's λ-calculus could be used to represent programs , elevating them to first-class mathematical objects [26], eventually reconceived as affording the full expressive power of constructive mathematics itself [27].

Modular design via kits of interchangeable parts had been practiced for millennia [28, p. 26] before Simon's parable of the two watchmakers enshrined its merit for hierarchical composition [1]. Modularization promotes reuse, inviting grammars — general rules of recombination—to accommodate varying tasks. The Chomsky hierarchy calibrates a language, a subcollection of lexical strings, in terms of its computational model specified by the memory capacity and access of a discrete finite automaton that can recognize or generate it [8]. But robotics is initially concerned with a different class of dynamical systems (§B). Moreover, the imperatives of synthesis demand not merely a language, but a programming language [29]—a grammar of specification [30] — whose fundamental account follows the type-theoretic branch [5] [e] of the Church–Turing hypothesis first taken by McCarthy's turn toward AI [31].

[4] The technological prescience of this late twentieth-century paradigm shift to concurrent models of computing—specification of the computer–world information interface [5, chap. 41]—is by now amply attested not only by the preponderance of communications applications [6] but the unremitting rack- and chip-level interconnect bottleneck that has begun to encroach upon even algorithmic complexity theory [7].

 https://doi.org/10.1146/annurev-control-080320-011601

2.1.3 The design triple

Throughout his book *The Sciences of the Artificial*, Simon [1] calls our attention to a designer's problem triple: the goal sought, by an artifact, in its environment. Yet his unforgettable image of the ant steered by its surroundings seems to have been quickly forgotten within the AI field he helped found. Notwithstanding important exceptions [32], AI has been preoccupied with its architectures and their efficacy in achieving its goals, reserving very little systematic attention for even empirical study of their situated behavior, much less an attempt to theorize about which classes of environments will abet or impede them. [5]

The broader reaches of the discipline came to accept and embrace the centrality to computer science of understanding and formally specifiying the interaction of an agent with its environment [4]. Yet even the most scholarly, comprehensive and contemporaneously valuable text in the field of AI [33], while emphasizing this interaction does not offer a unified formalism for its specification and analysis — a conundrum inherited by robotics (§B.2).

2.2 Problems Awaiting the Discipline of Robotics

The essential job of a robot [6] is to perform work on its environment specified by its user. Thus, the discipline of robotics must address three main problems:

Problem 1 (the robot): Build a physical body equipped with material resources that can move energy from a (typically chemical) reservoir to the environment directed along the right degrees of freedom at the right time.

Problem 2 (the robot program): Accept a specification of a goal over the state space of a robot conjoined to its environment and then either declare failure in consequence of some insufficient resource or execute a solution to Problem 1 that achieves the goal.

Problem 3 (the robot design): Accept a specification of a world model (a class of environments) and a task domain (a class of goals), then return a specification of resources along with a solution to Problem 1 and Problem 2 that achieve tasks in those environments given those resources.

Section 2.2.1 sketches what is known about the challenges of Problem 1—realizing the working substrate. Section 2.2.2 addresses Problem 2, introducing models of the robot–environment interface and examining the conceptual gaps impeding the programming and execution of abstract goals. Problem 3 is taken up in Section 4 through the more speculative discussion required to do justice to terms such as task domain and class of environments. [7]

2.2.1 Physical resources: substrates of work

Solutions to Problem 1—limbs and body, and their endowment with actuation, perception, and manipulation capabilities—have exhibited substantial advances since the first modern designs appeared roughly half a century ago [38]. But, possibly apart from the benefits of better chips and sensors, there is no hint of

[5] It would require a digression too long for this article to discuss the appropriate role of benchmarking—empirical performance in curated environments — in robotics. By itself, a preoccupation with benchmarks cannot be construed as proper empirical science. Surely, testing against benchmarks cannot be mistaken for theorizing (forming precisely stated hypotheses to be tested in new environments of a postulated type) about what features of what types of environments will elicit what sorts of behaviors from an artifact. Alternatively (veering a bit dangerously toward the art of science practice banned in Footnote 3 from consideration in this article), it is hard to imagine any novel empirical discovery ever arising from a collection of examples chosen to summarize what is already known to be hard about a design problem. Benchmarking can play a vital role in helping assess the likely performance of an architecture in the presence of environmental conditions or task specifications which go beyond the scope of its (typically disappointingly conservative) provably sufficient conditions for success along the lines discussed in Section 3.1.

[6] To bound its scope, this article focuses on the specific case of mechanical robots (whose energetic exchange is described by classical mechanics) at the human scale (lengths of roughly $10^{-2} - -10^{2}$ m). Extending or improving on these ideas toward the promise of power- and information-autonomous robots down below the millimeter scale [34] and of machines that work by chemistry [35] will become the urgent business of the emerging future discipline.

[7] Again, constraints of space and time preclude consideration in this article of agency—the capacity of a system to develop (and perhaps execute) its own goals. Inevitably, gradations of autonomy required to achieve users' goals will begin to encompass increasingly broad decision-making capabilities and motivational dynamics [36] as specifications become more abstract and the environment departs from their designers' anticipated type. The embodied situation of robotics presents an ideal setting for empirically grounded advancement of the science of autonomy [37], deserving of its own dedicated discussion.

 https://doi.org/10.1146/annurev-control-080320-011601

accelerating progress comparable to that already evident within the first two decades of the information technology revolution [13, 14]. Researchers have only begun to explore the intuitively compelling questions of how and why mechanical circuits—interconnected systems whose assigned tasks require exchange of Joules as well as bits—are fundamentally more difficult to modularize, design and scale than very large scale integrated (VLSI) circuits [39][8]

This section proposes that insufficient attention to fundamental physical resources and the way they must interact to drive progress remains a key obstacle to accounting for—and, hence, accelerating— its slow pace. The physical setting of robotics is distinguished by three specific resources (energy, information, and bonds); their rates; and, most particularly, the complicated interplay between them that is required to harness and effectively deploy any of them. The sidebar titled The Challenge of Problem 1 summarizes the discussion in the rest of this section that addresses these fundamental physical resources of robotics.

2.2.1.1 Energy and information rates Because there is a premium on getting work done quickly, power—the rate at which actuators can move joules—is a first scarce resource. Prior to Huygens's invention of a gunpowder-forced linear actuator in the 1670s, animal muscle was the only source of taskable mechanical work [43]. Different arrangements of muscle fibers and skeletal attachment work as motors, brakes, springs, and struts with huge variations in actuation capacity, achieving, for example up to 10^3 W/kg at 10 Hz or 10^2 W/kg at 10^2 Hz [44], over mass scales spanning seven orders of magnitude, 10^{-4}–10^3 kg [45]. Meanwhile, the twentieth century's handful of physical principles and mechanisms available for synthetic actuator design [46] has blossomed into a hothouse of active materials energized by a diverse range of physical effects [43]. These abundant variations reveal substantially contrasting performance along such dimensions of merit as power density, bandwidth, stress, strain, efficiency and linearity, many exceeding biological performance over a delimited subset [47]. However, no single synthetic approach seems close to matching the pluripotency of animal muscle tissue, much less its capacities of self-assembly, regeneration and intrinsic adaptation [43].

The Challenge of Problem 1

Addressing Problem 1 requires materials affording scalable mechanical power and information processing (§2.2.1.1) as well as grip (§2.2.1.2). Design methodologies that can specify their distribution across the interior volumes and interface surfaces of robot bodies as a function of task–environment pairings require models of the robot–environment work and information interface (§2.2.2.1) that incorporate explicit representations of these material resources.

Whereas computation need not intrinsically entail energetic cost [21], robots must expend energy, not merely when working explicitly on their environment [48] but also to achieve any task formulated in terms of dynamical attractor basins as advocated below (Section 4.1.2.2). [9] This motivates the question, What are the fundamental limits of specific power? It seems inevitable that dynamical versions of Landauer's limit tying together not simply energy and information but also power and bit rate (§C.1.1) will play an essential role in any formulation of fundamental physical constraints on robotics. Understanding these relationships has in the last decade become an active area of theoretical inquiry [55] and increasingly practical device design [56] at the nanoscale. This article argues (§4.1.1.1) that exploring the implications of such fundamental constraints at the mesoscale of conventional robotics will play a key role in establishing the discipline. Equally important, of course, is the question of how to effectively use whatever specific power a robot has been endowed with (§4.1.3.1).

2.2.1.2 Making and breaking bonds Whether fixed at design time or, crucially, designed to be made and broken through the course of a task along the degrees of freedom relevant to the robot's goals, bonds must be strong enough to withstand the forces they communicate. Apparently, grip—materials properties conferring friction and adhesion, their removal, and their facility for higher rates of alternation—is also scarce.

[8] One anecdotal but revealing measure of the challenge is presented by the effort of one of VLSI's co-founders [23] to foment a similar revolution for analog circuit design [40]—an exciting and influential [there are now thousands of papers working within this promising framework [41]] but surely not (at least yet) comparably paradigm-shifting development. Section B.2.2 offers some discussion of the crucial role that might be played in robotics by analog computation and its recent advances, e.g. [42].

[9] Unlike classical Hamiltonian systems, hybrid compositions of piecewise lossless holonomic systems can sustain stable, partially asymptotically stable attractors [49, 50, 51], a phenomenon that has been exploited for inspirationally efficient legged locomotion [52, 53]. Basins of ambient volume as advocated in Section 4.1.2.2 are not possible however, so the resulting steady-state behaviors are neutrally stable and can be pushed around—perhaps to useful effect by the controller [54], but just as easily by the environment to the detriment of the prescribed goal.

 https://doi.org/10.1146/annurev-control-080320-011601

Joining of materials takes its place among the very oldest human technologies [57] and in this article the term framing cost refers to the added mass incurred in achieving sufficiently strong and robust permanent mechanical bonds between disimilar materials, as is typically required to integrate components such as actuators and sensors into a targeted morphology. Rapid advances in materials science and engineering anticipate an eventual future wherein fundamental principles of condensed matter physics drive 3D printing of working devices and complex structures from homogeneous ingredients in analogy to the 2D inkjet printing of arbitrary images from a few colored ink reservoirs [58]. Already, algorithmic thinking about the design of pluripotent materials from cutting and folding of homogeneous sheets [59] is achieving tunably compliant and shaped robot limbs [60], and programmable spatiotemporally complex self assembly of similarly active materials seems to be on the way as well [61]. A major challenge for robotics is the development of a systematic framework for specifying the distribution of materials properties across the interior volumes and interface surfaces of the permanently (or, perhaps, developing – or even evolving) integrated body (§4.1.1.2).

A contrasting and still greater challenge is the development of materials to interface with the environment that are capable of making and breaking strong bonds, rapidly on command. Difficult though it may be to join with high mass-specific adhesion (normal) and friction (shear) forces, it is truly daunting to arrange for rapid, effortless detachment as well. Multicellular life is enabled by adhesion on many scales; in particular, animals use quickly reversibly forcible grippers to eat, crawl, climb, and capture prey in every terrestrial environment imaginable [62]. In turn, impressive feats of manipulation abound in the animal world [63]. Unsurprisingly then, the architecture of grippers has appropriately preoccupied robotics for decades [64]. But the physical basis of specific animals' grip has only recently begun to be understood [65] with the first revelation [66] only following millennia of puzzlement. Section 4.1.1.1 examines the prospects for identifying the fundamental limits of grip, touching as well upon the question of how to identify and use whatever grip the environment affords.

2.2.2 Programming: architectures for tasks undertaken in environments

Robots are quite different from computers as physical machines; hence, their Lagrangian internal models [67, 68] are different from the discrete finite automaton models of computing machines [8]. But a program is a mathematical object as first viewed by McCarthy's [26] λ-calculus representation, and more generally as a function defined with respect to a theory of types—the specification of available domains and codomains [27]. Types, the "central organizing principle of the theory of programming languages" [5, p. xvii], impose constraints on syntax ensuring that its evaluation always yields a valid function—one whose "behavior" is defined by the resulting domain and codomain provided in the theory. In this view, the meaning of the mathematical objects specified by the syntax is given by the operational semantics of any evaluation step [29].

There are two essential challenges to achieving the physically grounded type theory required by Problem 2. First, prescribing the behavior of a system in its environment in any formal terms presupposes a description of the interface between them relative to which goals can then be specified. Section 2.2.2.1 assesses the availability of interface descriptions that clearly manifest the underlying physical resources so that alternative architectures can be considered relative to their requirements and operation in different environments. Second, since complex systems entail a hierarchy of interfaces [1] a behaviorally interesting robot will need a deep specification, requiring many layers of programming languages with clearly related operational semantics by which the meaning of a specification at any given layer is clearly expressed in terms of the behaviors of the child and sibling components it calls out [30]. Section 2.2.2.2 discusses the challenges of identifying useful internal layers and appropriate abstraction barriers to separate them. The sidebar titled The Challenge of Problem 2 summarizes the discussion in the rest of this section that addresses the problems of robot programming.

2.2.2.1 Describing the robot–environment interface A robot's working interface is specified by the properties of (a) its actuarium (a representation of its energy ports modeled by the first equation of Section B.1.1), [10]

whose capacity is fundamentally characterized by specific power in W/kg; (b) its sensorium [a representation of its information channels [69]], which is characterized by bit rate in B/s; and (c) its tenacium (a representation of the speed and strength with which it can grip) characterized by rate at magnitude of

[10] See Section B.1.2 for a brief discussion about the prospects for a more general behavioral representation of this interface.

 https://doi.org/10.1146/annurev-control-080320-011601

reversal force ratio (N/s). [11] The brazenly Latinate terms aim to underscore the increasingly rich variety of smart materials [71] that offer new opportunities for distributing these capacities [e.g. $\mathrm{W}/(\mathrm{kg}\cdot\mathrm{m}^3)$ or $\mathrm{B}/(\mathrm{s}\cdot\mathrm{m}^2)$] without suffering the framing and other scaling costs incurred by traditional actuators [46] or by integrating local, scalable computation in networked communications channels [72].

One potentially confounding aspect of these specifications is that all resources play a role in both interfaces: There is information (which degrees of freedom get what rates) in the work exchanged through ports, information channels (regarding both computation and communications) inevitably have an energy cost, and grip plays a central role in working on and receiving information about the environment. Notably, proprioceptive devices that participate in both actuation and sensing have long played an important role in robotics [73] and complicate the characterization of capacities as they enrich the behavioral suite of robots.

The Challenge of Problem 2

Addressing Problem 2 requires reasoning about the complete synthesis triple (§2.1.3) including the robot's interface to its environment (§2.2.2.1) and its architecture in relation to its targeted tasks along with the environment's affordances toward achieving them (§2.2.2.2). Developing models of a robot's information interface to match the consensus model of its work interface remains a key challenge (§B).

The consensus view within robotics of the work interface to the environment leads to an internal model over a conjoined robot–environment state, $x_v \in \mathcal{X}_v$, indexed by the mode of contact, $v \in V$ (§B.1). Although this consensus may well be undercut by the introduction of advanced materials in view of their promise of distributed interfaces [71] and ubiquitous compliance [74], it remains a useful point of departure in considering the specification of robot architectures and their deployment. That caveat in place, it is convenient for purposes of exposition to posit the standard model for the work interface as having ports taking the form of the first equation in Section B.1.1.

Given this work interface model, power resources are manifest by the properties of the input signal, τ, in that equation—most simply its dimension relative to that of the state vector, x (the degree of underactuation), or, more accurately, by a range of increasingly detailed internal dynamical models, e.g., [75, 76, 77] [and eventually requiring specific robot-environment modeling [78]], as discussed in Section B.1. By contrast, the immaturity of the discipline is such that it does not yet seem possible to propose a standard model for a robot's information interface (§B.2). A likely general candidate for such models may be found in the process algebra literature [69]; hence, it is convenient, even if merely as a conceptual placeholder, [12] to use the terminology channels (§B.2.1) when discussing requirements of and prospects for representations of a robot's information resources.

2.2.2.2 Specification: internal layers and models Notwithstanding the central importance of robot perception [81, 82], the absence of a consensus model of channels (§B.2.1) corresponding to the work interface of ports (§B.1.1) compounds the challenge of designing a robot's deep layers. This is an intrinsically fraught enterprise because interior processing interfaces—information processing modules and their inter-relations—are confusingly underconstrained. Viewed at the work interface modeled the first equation of Section B.1.1 it is conceptually straightforward (albeit often technically challenging) to develop sufficient conditions for the success of an architecture in achieving a specified task relative to a specified environment since Newtonian mechanics is physically ineluctable. Furthermore, at the resolution of local behavior (§B.2.2), careful control-theoretic reasoning can yield necessary conditions with generality adequate even to constrain animal architecture in the light of clever experiments [83, 84, 85]. A central motivation for precision in describing a task domain as discussed in Section 4.1.2.2 is the possibility of at least demonstrating sufficiency. Deeper conceptual progress in the form of necessary conditions on the internal architecture will require carefully articulated reasoning about generative models [86, 37] to establish how fundamental resource limits constrain it. [13]

These last observations underscore the obviously crucial but still inscrutable role and structure of memory—

[11] This very speculative suggestion for appropriate units of grip in terms of time rate and magnitudes of load-to-preload and reversal ratio [70] is contextualizd in Section 4.1.1.1.

[12] It is daunting to contemplate the challenge here since the eventual conceptual apparatus will need to encompass a vast scope of intricately entangled phenomena ranging from "transparency" of drivetrains (§4.1.1.2) to the notorious correspondence problem [79] and statistical active perception [80] (§B.2.1) through representation and use of analog computation (§B.2.2).

[13] Animal architecture offers a tantalizing source of necessary conditions for any performance model of animals' agency. However, it remains to be explored whether such constraints have any purchase over the competence models of behavior that would be adequate (and likely preferred) for prescribing those properties of a robot's internal architecture required to achieve a specified task in a given environment.

 https://doi.org/10.1146/annurev-control-080320-011601

prior information about the robot and environment and the history of their encounters. The necessary critique of AI's obsession with representation in the designed architecture has unfortunately been clouded by charismatic calls "to use the world as its own model" [87, p. 140]. [14] The true disciplinary question concerns representation of architecture: interior interface specifications that elucidate the design triple (§2.1.3). This fundamental problem seems to have been traditionally avoided by both the internal-model and the reactive-behavior camps of AI. Both traditions shy away from any study of the robot–environment pairing beyond the most cursory level of empirical anecdote [93]. But robotics cannot afford this luxury: The key challenge is developing tools for reasoning about the degree to which some internal model of a particular environmental affordance (and the conjoined robot–environment state in relation to it) is necessary or sufficient to achieve a particular goal [94].

There are two tightly related but distinct dimensions of depth in the interfacing layers to be accounted for. The first, manifesting the needs of task specification, arises from the human predilection for abstraction as a means of taming behavioral complexity and is particularly challenged by the signal–symbol divide (§4.1.2.2). The second, introduced by the physical resources of information and grip arises from the ever more spatiotemporally distal aspects of the environment that must be mechanically engaged or perceptually experienced as behavioral complexity increases. Here the conceptual bottleneck lies in appropriately abstract models of the environment, both endowed by the designer (§B.2.3.1) as well as learned from experience (§B.2.3.2). [15] This article seeks to advance the perspective (to be first articulated in Section 4.1.2.2 and pursued with more technical detail in Section B.2 and Section C.2) that at whatever level of specification, the daunting challenge of sensorimotor coherence — keeping symbols arising from learned models and their sensed referents relevant to programmed task expressions—can be overcome by grounding them all in the sublevel sets of the energy landscape. [16]

3 WHY DO WE NEED IT?

3.1 Foundations of Intellectual Progress

Are robots getting better? Certainly their computers and sensors are. If we set up competitions, we can pretty well discern when there is a winner, measure how much progress has been made between iterations [98], and surely recognize technological inadequacies from the post hoc scorn of the lay public [99]. But why? Continually relearning that the technology project is very hard [100, 101, 102, 103] does not seem to diminish its hundreds of billions of capital inflow [104, 105, 106]. But neither does merely throwing more money and replacing one test with another test seem to bring technological progress beyond that all too readily ascribable to improved component hardware or advancements in algorithms imported from distinct fields. [17]

Important conceptual progress in robotics has surely accrued and can be roughly charted by the appearance of landmark monographs. The algorithmic foundations of motion planning [109] were greatly enriched and made practicable by the adaptation of Bayesian filtering to navigation and mapping [110], and it can be expected that the huge impact of learning in this domain will in time generate a comparably high-impact next summary account. Insight into the mechanics of manipulation has grown dramatically [111, 112]. The

[14] Sophomore control engineers who have contemplated stabilizing a force-controlled mass–spring–damper system with only position feedback understand the necessity of augmenting sensory cues with internal representations of certain environments to achieve certain tasks [88, example 6.2.1]. More general formal reasoning reveals that a complete internal model of the relevant environmental disturbance is necessary for any control architecture capable of stably and robustly rejecting it [89]. The reinvention of decades-prior hierarchically arranged inner and outer (or minor) loops [90] reveal such "new approaches" to robotics [87] as uninformed by and uncommitted to science. Section B.2.3 briefly re-examines such questions by considering a range of design settings. At the limit of this range in task–environment pairings lies the extreme of "kicking the sensing habit" [91] entirely via open-loop procedures that require no measurement of the world state at all—provably guaranteed and empirically demonstrated to succeed in such structured settings [92].

[15] Given the recent triumphal emergence of computational learning [95] it is particularly unfortunate that limitations of space and time preclude anything close to a consideration of their import for robotics. Their huge potential for control has been understood for decades [96]. Used with precision in architectural design [97] they hold at least comparable value for robotics; see Footnote 16 just below and Section B.2.3.2 for brief speculative remarks bearing on the matter.

[16] These terms and concepts related to sensorimotor coherence are given brief technical descriptions in Footnote m, Footnote n, and the text that calls them out in the Supplemental Appendix. Footnote k provides a speculative but succinct general statement of this idea in that more technical context.

[17] Economists have understood for decades that demand-side pressure is inadequate to generate new technology absent appropriate scientific foundations [107, chap. 14]. There is at least some empirical evidence that radical invention in the sphere of mechanical devices may be particularly driven by new advances in fundamental knowledge [108].

 https://doi.org/10.1146/annurev-control-080320-011601

empirical foundations of dynamical locomotion [113] were greatly strengthened by the introduction of more formal ideas from nonlinear feedback control [114]. But it might not be apparent to a student—or even an accomplished practitioner—how these books relate to each other. Indeed, it does not yet seem clear how to build machines that benefit simultaneously from all three traditions of insight. In particular, the physical resources whose scarcity most dramatically obstructs performance seem different in each: information flows for navigation, gripping affordance for manipulation, and power budgets for locomotion (although all three make an appearance in each). How do these different sorts of robotic capabilities fit together? What are the fundamental limits to performance for any or all of them?

It is the job of a scientific discipline to pose carefully and answer such questions. Formal synthesis—a precisely stated hypothesis of what properties must inhere from a design in advance of its construction—is a profound enabler of better technology even if it is construed as merely a debugging tool. If a correct theorem states that a particular architecture must be capable of achieving a specified task in a type of environment, and empirical evidence contradicts that conclusion then we know that there is some discrepancy between the assumptions in the hypothesis and the prevailing conditions in the physical world. Either the architecture fails to achieve some specific capability listed as necessary or the environment fails to conform to the properties of the assumed type.

This last possibility underscores the driving intellectual importance of clearly posited assumptions and the proofs they enable. Without them we have no methodical way of drilling down into the details of what makes a synthesis problem hard. By definition, the environmental model is an abstraction that will miss details of the physical setting. Explicitly stating what environmental properties must be assumed in order for the architecture to be appropriate crystallizes the role of that affordance in enabling the task. It can clarify the appropriate target of benchmarking (e.g., facilitate the curation of key out-of-scope settings along the lines discussed in Footnote 5). Thus, such abstraction plays a central role in teasing out what details are essential to take into account and what specific design challenges arise from what specific adverse conditions.

3.2 Foundations of Research and Teaching

A logistical reason to establish the discipline of robotics is that contemporary civilization enshrines disciplines in universities that commit substantial resources to departments for their propagation and advancement in the broadest interests of human knowledge. This may change—many prophesy, and some already find evidence of great disruption. But for present purposes, it is convenient to envision the prospects for a discipline of robotics through the lens of its departmental manifestation in a research university. Here, pedagogical imperatives confer the greatest intellectual benefit. Delivery to a novice provides the best motivation for and evidence of a deep understanding. The conceptual barriers between the sorts of benchmark monographs just discussed underscore the huge advantage their impressive authors and indebted readers would all accrue from the obligation to explain to a sophomore robotics major how they fit together.

The next most important role of the robotics department is to hire its replacements. While the key criteria for wise faculty appointments remain creative talent and intellectual ambition, the accompanying arguments about what direction to push in and why play a crucial role in the maturing of a discipline. The droning on about how the department needs not just one but actually three more scholars in the area of one's five most recent publications ultimately confers significant intellectual value in the aggregate, however near unendurable in the moment. For the vision of what should come ahead must be contested not merely in dropfuls by the grant but in bushels — or, with extreme luck, in tons—by the career. As robotic technology's impacts deepen, these vital arguments about where the fundamental questions lie are increasingly camouflaged or distorted when cramped into the mold of neighboring disciplines. Correspondingly, their potential benefit is lost to the field whose present coherence and future invention depends upon them.

If the history of computer science holds the lessons for robotics urged in this article, then not the least important role of the discipline's creation will be to referee the tussle over theory and practice. In one convincing reading [115] the discipline of computing emerged from specialized corporate training programs (in the 1950s) to educate a practice of software engineering promulgated by universities (in the 1960s), shortly encapsulated within an academically focused canon of theory (in the 1970s and 1980s), the escape beyond which was engineered by a creed of disciplinary problem solving that persists to the present day. The cycles of tension, expansion of purview, and re-emergence of consensus regarding curricula and foundational agendas that characterize departmental incarnations of disciplines seem to provide essential ballast for any technology that boasts the accelerating social impact to which the argument now turns.

 https://doi.org/10.1146/annurev-control-080320-011601

3.3 Imperatives of Social Impact

Ready or not, robots seem finally to be on the way. They have already transformed factories. Bold announcements and acquisitions by large corporate actors herald their appearance with greater autonomy and in less structured settings throughout the commercial sphere. But roboticists understand that such pronouncements are the mark of irrational exuberance [116] and dangerously misleading product advertising [117]. Following nearly a decade of promised disruption, automated vehicles at level 4 of the SAE J3016 classification [118] seem unlikely to operate securely in the face of general highway hazard scenarios for another decade while full autonomous operation at level 5 is many decades away [119]. Of course, the very notion of levels is suspect given the huge importance of the local cityscape, an environmental context whose vital characterization is in its very infant stages [120]. Meanwhile, robots in still less structured settings do not deploy with much of any repeatable pattern: Successful applications result from elaborate, one-off, multihuman team exertion and still do not function in any predictable manner, failing regularly—or, worse, succeeding unexpectedly—from setting to setting [121].

Notwithstanding the accumulating multiple fatal accidents [117], physically embodied agents endowed with poorly understood, sloppily conceived, demonstrably dangerous partial autonomy are already being released into the human and natural environment by both commercial and state actors. While there is no dearth of similarly sloppy practice in the software industry, growing evidence suggests that the increasing power and practicability of formal methods are beginning to play an important role in at least life-critical applications [122]. Robots, to the extent that they are useful at all, must be presumed to fall into this same category of life-critical applications. The demand-side pressure for such technology is rapidly growing, and many would-be customers will not want to delay the benefits of—much less impose a moratorium on—apparently useful machines no matter how imperfectly characterized. But there is presently no available formal methodology of correct robotics even to offer in case industry seeks it—or society comes to demand it.

4 HOW CAN WE GET IT?

The synthetic sciences are so young that adapting the right model for robotics will require both deliberate introspection and historical insight. Kinematics, the discipline focused on design of mechanical motion, was the first aspirant to synthetic science [123]. It plays a role in robotics nearly as critical as computing, but seems less instructive because it concerns behaviors whose architectures do not require internal state, and seems never to have been concerned with physical limits. Cybernetics burst into mid-twentieth-century science with the proposal that emerging theories of information flow and its regulation could unify the study of animal nervous systems, computing machines, economies, electromechanical circuits, languages, psychopathologies, and social organizations [124]. Under the weight of these breathtaking burdens — unsupportable even by the dazzling brilliance (and weakly armored emotions) of its proponent [125]—the field shattered into new engineering disciplines whose narrowed foci reflected different aspects of its origins in dynamical control theory [126]. Embracing mathematical synthesis, these offspring—modern-day control, communications, and signal processing—fled the domain of synthetic science by rejecting the commitment to any specific physical setting. Synthetic biology is a fascinating younger cousin of robotics that is seemingly even less settled in its foundations. There remains the example of computer science.

From that perspective, the problems that robots face (and that humans must help them overcome) can be formulated in terms of a space of agent–environment states within which agent-initiated actions instigate transitions to new states toward some task-designated goal subspace [127]. Uncertainty is rife: Models are wrong by intent of abstraction, and information is incomplete and noisy. Thus, inevitably, solutions—policies for progressing to goals (§4.1.2.2)—must be applied iteratively, requiring the agent to check its progress and re-plan according to the mismatch between anticipated and actual accomplishment, notwithstanding its noisy perception (§4.1.2.1). This iterated re-planning view defines a closed loop that locks problem solving into the setting of dynamical systems theory (§A). Section 4.1 brings this dynamical systems point of view to bear on the problems of robot body and program design as summarized in the sidebar title The Challenge of Problem 3. Section 4.2 takes an analogous view of the problems facing the birth of a new discipline.

4.1 Building and Testing Robots, Theories and Programs

 https://doi.org/10.1146/annurev-control-080320-011601

Speculating about approaches to Problem 3 necessitates some account of its antecedents. Section 4.1.1 addresses the problem of bodies (Problem 1), Section 4.1.2 proposes a theoretical framework that might yield programming languages of work (Problem 2), and Section 4.1.3 imagines what it might be like to actually use them in putting robots to task (Problem 3).

> ## The Challenge of Problem 3
>
> Addressing Problem 3 is bound up in solving its antecedents: Progress in building and programming robots invites a speculative view of how to reason about their design.
>
> ### Problem 1
>
> Progress is being made using task–environment templates to reason about resource requirements (§4.1.1.1) and the transparency–dexterity trade-offs in deploying them (§4.1.1.2). Outstanding challenges include seeking fundamental limits to power, bit rate and grip (§4.1.1.1) and formalizing principles of codesign for distributing the available resources across the body's interior and surface interface to the environment (§4.1.1.2).
>
> ### Problem 2
>
> Progress is being made by advances in the qualitative theory of robot hybrid dynamical systems (§4.1.2.1) arising from the consensus work-interface model (§B.1) and the consequent prospects for grounded symbols (§4.1.2.2). Outstanding challenges include advancing and deepening the categorical account of template compositions (§4.1.3.1) and reworking them to ensure the empirical utility of the associated functional programming languages (§4.1.3.2).
>
> ### Problem 3
>
> A well-defined notion of task domain might be conceived as arising from the application of available grounded compositional operators (§4.1.3.1) to the available lexicon of grounded symbols (§4.1.2.2) via expressions allowed in the resulting programming language (§4.1.3.2). The prospects for defining and reasoning about types of environments seem to rest on further progress modeling a robot's information interface (§B.2.3).

4.1.1 Robots

As commercially available robot technologies improve their capabilities to operate in more diverse environments, the disciplinary project of robot design and building comes into finer resolution. Researchers [18] must articulate what fundamental resources (e.g., power, information rate, or grip)—or perhaps some more carefully refined or newly identified fundamental limit not considered in Section 2.2.1—are better recruited or coordinated in their new designs, or else their contributions are more suited to evaluation by markets than by peer review. [19]

In that context, Problem 1 might be more carefully articulated as follows. Suppose a designer is given some material budget affording a sensorimotor and grip endowment with known and reliable scaling and distribution properties. Now, how do required tasks situated in discovered environments dictate a morphology and the distribution of power, bit rate and gripping resources assigned it across space and time? Section 4.1.1.1 assesses the prospects for developing well-characterized appropriate robotic material resources and Section 4.1.1.2 reviews what is known about how to distribute them. [20]

4.1.1.1 Resources: power, information, grip and their trade-offs For the foreseeable future, roboticists must closely study the requirements of specific problem triples (§2.1.3) in order to design their robot's actuarium. This article makes a case in Section 4.1.3.1 for using templates (§C.2.3.1) as the modeling framework for so doing. Recent work on projectile launch against gravity [131] presents an archetypal example of how to pursue such challenging analysis for task domains—here, single-shot leaps or hammering. Modeling the complex interplay between power constraints (speed–torque curve), compliance non-idealities (spring inertia), and grip limitations (latch geometry) to achieve launch energy over a range of environments (load mass) yields fundamental insight into what power train may be necessary or sufficient for what regions of this task domain [131]. Analogously, the first vertical dynamical climbing robot [132] was achieved by insights from a bioinspired template [133] revealing the necessity for parallel springs to assist the available motor-specific power in supporting the machine's three-orders-of-magnitude increase in load (albeit with the simplification of assuming perfect grip) [134].

[18] Once again, time and space constraints restrict the scope of this article to the consideration of general-purpose robot architectures for general-purpose environments. For example, that restrictive scope entirely ignores such crucial applications areas as design for physical [128] or psychological [129] human–robot interaction.

[19] By the same token, robot companies that care about advancing the discipline underlying their technologies must be willing to expose to the broad research community (with suitable nondisclosure protections) hardware interfaces that permit testing of new theory.

[20] Another important, fascinating topic that lies beyond the scope of this article is the development of new materials for sensing. For example, given the huge role played by olfaction in the evolution of animal cognition [130], it is remarkable that robotics has not yet found a way to widely integrate some corresponding technology.

 https://doi.org/10.1146/annurev-control-080320-011601

The need for grip seems even more complicated to characterize and trade off in the context of such task–environment pairings. [21] Considered as a metamaterial property, grip seems most carefully studied within the two-decade-old literature on synthetic dry adhesives, arguably initiated by discovery of the van der Waals force mechanism underlying Gecko toe attachment and detachment [66]. Principal figures of merit entail strength (typically measured in terms of surface energies) and reversibility (relevant quantities entailing time rate and magnitudes of load-to-preload and reversal ratios) [70]. But numerous additional criteria such as durability, propensity for fouling vs. self-cleaning, and the difficulty of performing many of the relevant measurements greatly complicate its physical characterization [70]. Engineers' growing insight stemming from carefully informed bioinspiration [136, 137, 138] and improving materials and fabrication methods have spurred notable advances in rapidly reversible high-strength bonds through ingenious arrangement of hierarchical mechanisms [139] exploiting anisotropic compliance [140]. Correspondingly, the environment's affordance [141] of grip has commanded at least a comparable degree of attention regarding its native composition [142, 143] or design [144] as well as assessment whether by remote anticipation [145] or direct proprioception [146, 147]. It seems plausible that the urgently needed characterization of grip may be emerging with advances departing from these two opposite poles of the task–environment axis.

By contrast, equally central, yet almost entirely ignored within robotics is the question of whether the trade-offs among grip, power, and information rate are fundamental or merely artifacts of presently available (or perhaps even poorly deployed) technology. Apparently all three resources are bound up together and simultaneously coordinated as well as co-limited in a robot's exploitation of any one. Greater power implies an ability to more rapidly and securely grasp and release using whatever adhesion a given object's surface affords. Reciprocally, substrates with higher friction coefficient afford broader ground reaction force cones that increase the stance travel distance along which a given actuation power budget can add kinetic energy to a running body. Meanwhile, more timely contact information is required to increase the impulse that can be lent the body by the same power train and traction condition. Or, again reciprocally, the more secure the grip, the faster the proprioceptively gleaned information about an object's mass distribution. An urgent agenda for the discipline of robotics is to uncover the relationships among these resources and their ultimate physical co-limitations (§C.1.1).

Computer engineers' ability to trade energy against information is well established [19]. Preliminary exploration of the nature and implications of analogous information–energy rate limits relevant to robotics are beginning to appear in the literature. Empirical observation suggests that mass-specific force (bonding strength) rather than power fundamentally limits actuator work rates [148]. However the constrained interaction between mechanical power and rapidly received, computed and transmitted information has begun to be established at the mesoscale as affecting both output ports [76] and input channels [149].

4.1.1.2 Distribution: compliance, modularity and codesign Compliance ideally is characterized by a memoryless force-extension function or, equivalently, a scalar-valued potential energy function. The term memoryless means that there is no internal state; hence, in the absence of any intrinsic time constants, ideal compliant elements incur no power limits and can support arbitrarily fast energy flows with no losses. Springs are good [150]: Given the inevitable limits on actuation power, a designer is strongly motivated to introduce compliant elements in the body that can intermediate between the actuators' slow extraction of joules from the energy supply and the loads' fast time constants associated with the kinetic energy shifts required by the target mass states. However, the introduction of compliance typically incurs lowered information rates on the proprioceptive interfaces: diminished transparency (in their channels) and dexterity (through their ports). [22]

Channels suffer since, once coupled to an inertial load, the compliant elements dramatically alter the overall system time constants. The desire for transparency (the ability of an actuator to quickly and accurately read the loads' states) motivates the introduction of direct-drive (neither gearing nor compliance) robot technology [73], dramatically increasing the specific power, albeit at the cost of tricky trade-offs in specific force for locomotory systems [151]. At the ports, series compliance has long been understood as offering enhanced output force accuracy [152] at the cost of severe bandwidth loss [153]. Alternatively, parallel compliance can

[21] Discussion of grip is a particularly illuminating setting for understanding that resource requirements can only be characterized with respect to a pairing of task and environment. For example, locomotion constrains animals' use of their remarkable grippers while, simultaneously, animal gaits are known to vary dramatically depending upon the friction and adhesive properties of the substrate [135].

[22] Once again, limitations of space preclude a proper treatment in this article of so-called soft robotics—a popularizing term for the systematic introduction of tunable compliance afforded by recent advances in materials science and engineering.

 https://doi.org/10.1146/annurev-control-080320-011601

be used to amplify force magnitude but only at specifically designated phases in the work loop [132], limiting the generality of tasks that can be performed.

The loss of transparency (accuracy–bandwidth trade-offs in proprioceptive channels) and dexterity (accuracy–bandwidth or magnitude–timing trade-offs through proprioceptive ports), motivates the consideration of sensorimotor specialization whereby, for example, compliance can be associated with high-power actuation at the body core (i.e., proximal to the mass center) [154] where neither dexterity nor state information is crucial whereas highly dexterous, sensitive, lowered power actuation can be placed at the periphery (i.e., distal to the mass center) where the body meets the environment [155]. In contrast, the appeal of composable resources has motivated proposals for programmability [156] and reconfigurability [157] of modules, some originating in the familiar traditions of mechatronics [158, 159, 160] and others from microelectromechanical systems [161] or materials science [71]. The hierarchical, multiscale nature of biological morphology confers decisive advantage on engineering designs clever enough to achieve it [138], heightening the challenge of finding simple recipes for composition of either form or function.

The general question of how to rationally distribute a robot's physical resources to perform a set of tasks in a class of environments has come to be called the problem of codesign (§C.1.2). A more constrained version of this problem inspired by the common recourse to preflexes [162, 163] (§B.2.1) observed in animals seeks to design what has come to be called morphological computation (§C.1.2). Accelerating progress in posing and solving such design problems gives the promise of advancing the field's insight into how to pose Problem 3.

4.1.2 Theories

The mechanics of work expressed by the first equation of Section B.1.1 imbues any reasoning about robotics with the study of dynamics. Embracing the iterated version of Newell & Simon's [127] general-problem solving formulation further implies closed-loop dynamics that result from feedback. [23] Absent a general information interface model (§B.2.1), it is now convenient to assume that this feedback takes the form of assigning to actuators some function of the entire history of sensor readings.

These assumptions bring to bear the theory of dynamical systems (§A), and, thereby, the tools of topology that hold a relationship to robotics roughly analogous to that exhibited by logic relative to computer science. Originating in Poincaré's investigation of celestial mechanics [164], topology engages robotics through dynamics (§B.1) to present an intrinsic, robust account of uncertainty and cost (§4.1.2.1) [24] and offer a formal characterization of grounded symbols (§4.1.2.2).

4.1.2.1 Intrinsic models of uncertainty and cost Uncertainty in the models and measurements underlying a robot's interface to the environment motivates the consideration of chains [166]—solutions of dynamical systems with arbitrary, small, but arbitrarily persistent errors (§A.1). From the foundational view, this completely unstructured model of uncertainty is very attractive: It is intrinsic in the sense that there is no requirement for further models of information (§B.2.1). Happily, the formal notion of a chain can be extended to the hybrid setting [167], and with it a version of steady-state behavior [168] for at least a large subclass of physically practicable but well-behaved versions of the work interface model (§B.1) derived from Reference [169]. However, usefully more structured representations of uncertainty such as parameterized families of models lead to differential inclusions that can also be incorporated in useful hybrid systems models [170]. It seems urgent to establish which empirically effective robotics models do or do not admit what version of chains with accompanying guarantees of well-behaved steady state. [25]

Those guarantees include the existence of an intrinsic scalar-valued Lyapunov function down which flows must decrease along the way to their steady-state attracting sets (§A.2). While there is no canonical

[23] For example, the distinction between deliberative and reactive planning so firmly entrenched in the idioms of contemporary robotics blurs when one considers that reactive policies must be constructed in advance of their execution while deliberative policies will inevitably be iterated in some fashion. At best, one might expect that these categories describe a relationship wherein policies seem deliberative to other policies at a finer spatiotemporal scale that they call out and reactive to spatiotemporally broader policies that call them in turn.

[24] Limitations of space and insight restrict this article's focus to the work exchanged between a robot and its environment as specified in Section B.1. This comes at the expense of more elaborated uncertainty and cost models whose careful representation awaits the development of an information interface specification (§B.2). Meanwhile, of course, methods of both stochastic filtering and control [110] and optimization [165] are, justifiably, deeply ingrained in robotics practice. Their formal integration will unquestionably be essential to an eventual discipline. Brief mention of the optimality point of view is made in Section 4.1.2.1.

[25] Here and throughout the article, steady state is a convenient but potentially misleading term for the robust, long-term behaviors exhibited by dynamical systems. Section A.1 offers a brief sketch of the powerful theory establishing the emergence and persistence of such attracting sets whose complicated spatiotemporal structure lies far beyond the tidy equilibrium or oscillatory behaviors that the colloquial use of the term might intuitively connote.

 https://doi.org/10.1146/annurev-control-080320-011601

choice of such functions in general, when carried over to the setting of hybrid robot dynamics (§B.1), it is appropriate to assume that they will be closely related to the physical total energy [171]. [c] This article takes the speculative step of simply presuming that such a natural energy-like function is available to play the role of whatever version of a Lyapunov function the particular dynamics will afford (§A.2). Then, in the smooth case (§A.3), its derivative along the system's motions will yield an expression of mechanical power (instantaneous energy expenditure). Henceforth, it is convenient to simply refer to the various versions of these scalar-valued functions as energy landscapes and their scalar-valued descent rate functions as power landscapes .

4.1.2.2 From goal primitives to tasks via grounded symbols Models of mechanical behavior (§B.1.1) describe the interaction of spatiotemporally continuous, and hence uncountable, quantities (energy and information flows); but languages are defined over countable alphabets. With the tools and concepts of the previous discussion (§4.1.2.1) in place it becomes possible to address the essential barrier to the agenda of Problem 3: the gulf between signals and symbols (§C.2.1). One concludes that the valleys of the energy landscape—the basins of attracting invariant manifolds perceptually marked by their energy sublevel sets [n]—are viable candidates for physically grounded symbols. Happily, these symbolic goal primitives also come ready-equipped with actual task specifications, as follows.

Historically, robotic tasks were confined to motion planning in presumed known workspaces—initially Euclidean spaces, necessitating only robot kinematics models [172], and subsequently rigid placements in spaces punctured by fixed obstacles [173]. Thus, path planning was the first algorithmically posed problem of robotics and remains a central focus of the field [174]. The computational complexity of a deterministic solution must grow exponentially in the degrees of freedom [175], motivating a shift toward sampling-based formulations [176] that can only be probabilistically complete. Yet finding a free path between a given initial–final pair of configurations is essentially a topological problem whose complexity can be alternatively quantified via the cardinality of a lexicon over the space of pairs each of whose symbols is a continuous path planning function [177]. The unavoidable need for repeated replanning now enforces a reformulation in dynamical terms: iterated maps or vector field flows that bring (almost) any initial choice to a desired final configuration [178].

This reformulation was first expressed in terms of artificial potential energy [179] and subsequently shown in principle to afford almost global dynamical solutions to any motion planning problem [180]. [26] The sequential composition of artificial energy basins [184] (§C.2.3.1) was anticipated by the notion of pre-image back-chaining [185] inherited from AI [186]. The proposal to use artificial potential functions as a specification of the effective impedance that a robot should present to the environment [187] represents an important parallel starting point in the agenda to encode goal primitives via reference dynamics rather than reference motions (§C.2.2). [27] These ideas can be extended to algorithmically generate artificial potential functions for a large variety of robot dynamics models of the sort modeled by the first equation of Section B.1.1 [189].

Because the combinatorial complexity of symbol manipulation can be dramatically reduced by appeal to the algebra of basin adjacency revealed by a vector field planner [190], [g] this approach to task planning seems worth embracing even from the perspective of computational efficiency alone. In this view, it seems urgent to work out the topological perplexity [182] (see Footnote 26) of task domains along lines parallel to the ongoing progress in characterizing topological complexity [191]. More broadly, the adjacency or disconnection of basins reveals intrinsic spelling rules for transitional tasks arising from specific environmental affordances [192, 193, 94] or task relationships [194] in a manner touched upon in Section B.2.2.

The question now arises of how to ground such planning vector fields in the dynamics of a working robot given by the first equation of Section B.1.1. Conceived literally in the manner of their origins as artificial potential fields [179], the goals and anti-goals [m] of the associated quasi-static gradient field can be immediately rendered as dynamically grounded symbols by recourse to Lord Kelvin's insight that dissipative second-order systems asymptotically reach the minima of their potential energy [171, 195]. However, the motion planning

[26] Here, the term "almost global" means that the defect (i.e., the complement of the basin of the attracting final destination) has empty interior in the configuration space. In typical application settings (indeed, barring pathologically wild cases; M.D. Kvalheim, manuscript in preparation), basins have the same homotopy type as their attracting sets [181]. Hence, single-point goals typically have contractible basins (topological disks) that cannot cover the entirety of non-contractible configuration spaces. This motivates the problem of how few basins must be required do so, as exemplified by Reference [182], coined as topological perplexity in Reference [183].

[27] This idea has grown to be influential enough in animal motor science to have achieved clinical application in human rehabilitation therapies [188]. See Footnote 30 for references to the animal motor science literature that further extend the concept into composition of motor primitives in a manner analogous to that traced for robotics in Section 4.1.3.1.

 https://doi.org/10.1146/annurev-control-080320-011601

literature has appropriately emphasized the importance of kinodynamic plans [196] entailing trajectories with specifically tailored transient properties. Analogously tailored properties can be imposed upon planning fields and those desirable transient properties closely approximated by the appropriately compensated working robot dynamics [197, 198, 199] as illustrated in Section C.2.2.3.

The Supplemental Appendix also briefly addresses the alternative of generating reference dynamics indirectly from a cost function [200] using methods of optimal control (§C.2.2.2). A preference for task specification via direct reference dynamics emerges from a number of considerations, [o] but the driving motivation stems from the conclusion that reference dynamics better serve the purposes of composition to be developed below in Section 4.1.3. Broadly speaking, the danger of appeal to optimal methods at any one level of a deep specification hierarchy is their parochial nature. [28]

4.1.3 Programming

This section considers the prospects for a functional programming language at the most basic level of a robot's interface to the environment. Section 4.1.3.1 discusses the availability of grounded compositions for the grounded task specification primitives just introduced (§4.1.2.2). Section 4.1.3.2 explores the availability of a type theory capable of treating such compositions as formal combinators [29] whose evaluation has an operational semantics in the formal properties of the task specifications so composed. [29]

4.1.3.1 Composition of task specifications A far wider realm of tasks than the motion planning setting of Section 4.1.2.2 seems to require specification not merely in terms of set-theoretic goals but also with respect to reference dynamics. Many examples from biology suggest that animals solve the degrees-of-freedom problem [202], (§C.1.1) by using low dimensional abstract templates [203] (§C.2.2). More broadly still, chain-grounded goal symbols (§4.1.2.2) represent intrinsically steady-state behavior whereas much of a robot's work can be expected to entail transitional maneuvers. Thus, at the very least, the agenda of programming work seems to require a syntax for building up specifications of behaviors entailing compositions of dynamical primitives. To require that these compositions be grounded is to specify which formal properties of constituent task primitives are inherited by the results of the composition.

Construing the construction and embedding of reference dynamics (§C.2.2.3) as a hierarchical composition (§C.2.3.1) instantiates the concept of an anchored template [203]—a module of behavior that can be represented and composed via a symbol grounded in a physically embodied sensorimotor behavior (§4.1.2.2). Here, the guaranteed property of the composition is that the behavior in the resulting higher-dimensional anchoring space converges toward a lower-dimensional subspace whose dynamics is a change of coordinates away from that of the template [204, 205].

From the perspective of the agenda pursued by this article, Raibert's hoppers [113] contributed the most important of any advances to twentieth-century robotics. On the one hand, their reliance on dynamical equilibria (stable limit cycles rather than mere point attractors) underscored the primary role of energy in robot tasks. On the other hand, they pioneered empirical candidates for parallel composition (§C.2.3.1). From a purely formal point of view there is nothing simpler: The parallel composition of two functions simply takes as domain and codomain the Cartesian products of the indivduals' corresponding sets and evaluates them independently. A suitably elaborated version of this idea can be used to define products over the hybrid dynamics category [167]. However, in robotics, as in any setting of mechanical circuitry [39], conjoining two physical systems inevitably entails cross talk. An urgent problem is to express more relaxed [206] parallel compositions that can distinguish safe from inimical cross talk [207, 208] in a categorical setting (§C.2.3.1). Here, the grounding requirement is that the steady-state dynamics of the product system are guaranteed to be included in the product of the constituents' steady-state dynamics.

A version of sequential composition using Lyapunov sublevel sets [184] has been expressed categorically as well [167]. However, the present working version is defined only for basins associated with the attractors arising from chain analysis rather than basins associated with hyperbolic attracting sets established via smooth

[28]Consider the succinct assessment "premature optimization is the root of all evil (or at least most of it) in programming" [3, p. 671]. The problem is similarly eloquently addressed from a historical perspective in the context of the opposition to compositional methods evinced by layer-level optimality specialists on the eve of the VLSI revolution [201].

[29] The reader will observe that this is a far cry from the deep layers of specification languages urged in Section 2.2.2. In response, the author will again plead the immaturity of a young field against the considerable challenges outlined there. Potentially, the language emerging at this level of direct environmental interface will come to be seen as analogous to the assembly languages of computers upon which much more abstract and useful—but still grounded—languages can be built (§C.2.3.2).

 https://doi.org/10.1146/annurev-control-080320-011601

analysis, exemplified in Figure 1 in the Supplemental Appendix (§C.2.3.1) and the powerful, exquisitely detailed applications [209] of bifurcation theory [210] that follow. This is a major motivation to understand how and under what conditions the hybrid dynamics category of Reference [167] can be refined to work with higher-resolution (but more rigid) hyperbolic attracting manifolds (§A.3). Here, the grounding requirement is that the steady-state dynamics of the paired sequence are included in the steady state dynamics of the second system of the pair. Enforcing this property using the sublevel sets of the associated energy landscapes yields an effective means of stringing together highly energetic transitional maneuvers [211].

4.1.3.2 Grounded type theory The compositional framework just discussed now invites a reprise of the conceptual passage from modules to functional programs briefly highlighted in Section 2.2.2 suggesting a transplant from computer science into an analogous development for robotics (§C.2.3). [30] Crucially, the purpose of modules is to be reused in varied compositions, promoting the construction of more complex behaviors from simpler constituents with known, reliable properties. As hinted just above, this amounts to the requirement for a category-theoretic [216] treatment of the primitives and their compositions. That requirement arises in the form of an unavoidable link whereby types emerge intrinsically from and at once intrinsically define categories [217]. In other words, the physical grounding of the type theory is established through its category-theoretic model.

Type theories for robotics have long been proposed but their associated categories typically remain unspoken and hence ungrounded. For example, the signals of functional reactive programming [218] take their domain in the reals with an unspecified (apparently arbitrary) codomain; therefore the combinators applied to compose their user-accessible signal functions have no physically specified operational semantics. In consequence, the applicability of their appealingly broad, elegant type theories [219] to any particular robot architecture in any class of environments is indeterminate.

Conversely, a long-developing literature on physically grounded symbolic specification of robot motion control compositions [220, 221, 222] yields a version that presents as a context free grammar [223]. Locating the complexity of a specification language in the Chomsky hierarchy is surely important for managing trade-offs between ease of expression and cost of evaluation. However, it is not a substitute for characterizing the behaviors specified. Absent some categorical analysis of the mathematical objects constructed by such grammars, one guesses that the operational semantics are roughly comparable to those of arbitrary hybrid systems whose most interesting qualitative properties are typically undecidable [224].

A contrasting approach to behavioral specification obtains from appeal to purely linguistic representation. For the price of translating the description of coupled hybrid robot–environmental dynamics given the first equation of Section B.1.1into modal logic, recourse to model checking yields computationally effective behavioral verification (i.e., correctness guarantees or failure diagnostics) of plans and controllers written out in the same syntax [225]. Moving up the Chomsky hierarchy, an analogous verification tool has been developed for models, tasks and policies written out in a more expressive but computationally costlier context-free grammar [226]. Formal interfaces to structured natural language [227] and probabilistic interfaces to human natural language [228] further enrich the ease of expression. A substantial advance in such approaches to robot programming is an explicit representation of the entire problem triple: architecture, task, and environment. A central challenge is the grounding gap—the computational complexity [229] and rigidity [230]—facing any linguistic representation of the physical world [231].

Eminently practicable functional programming environments have been developed for popular contemporary robot operating systems [232], and increasingly powerful autonomous high-level task planners have been built with more formal versions of such tools [233]. It remains to exercise them with physically grounded categorically generated type theories (§C.2.3.2). Here arises a new challenge to ensure that the formalism serves purposes of safe and expressive task specification rather than generating sterile, impracticable nostrums. This can be achieved only by a constant interplay between the hypotheses of the designs and the refutations of their empirical examination.

4.2 Building and Deepening Collaborations

This article has focused on the need to reach more deeply into the physical, mathematical and teleological foundations that underlie robotics. It seems fitting to end with a glimpse of the need to reach out. Accord-

[30] A similarly analogous conceptual passage may be detected in animal motor science which has progressed from identifying muscle synergies [212] and then multiple modules [213] to forming compositional hypotheses [214] and, ultimately, the prospect of their clinical application [215].

 https://doi.org/10.1146/annurev-control-080320-011601

ingly, Section 4.2.1 outlines the importance to robotics of its interdisciplinary relationships (including the importance of establishing an explicit departmental identity) within the university and Section 4.2.2 urges using that platform to lead the charge for greater equity and diversity in science and society.

4.2.1 Institutions

The foregoing discussions have amply rehearsed the crucial role that other disciplines have to play in robotics. Indeed, today's prevailing conception of the field seems to be that of a technological breeding ground where electrical, mechanical and materials engineers can all collaborate with computer scientists to spawn wonderful contraptions. Surely, it is clear to these other disciplines how valuable a role robotics can play in motivating their favorite specialties. For reasons touched upon in Section 3.2, absent an explicit departmental identity in the universities, the affection of other units does not necessarily advance the field.

It is striking to revisit Gorn's [234] proposal from more than half a century ago for a discipline called "the computer and information sciences." That tripartite argument prescribed the intellectual focus (mechanical languages), characterized the boundaries by listing its adjacent disciplines (electrical engineering, linguistics, mathematics, philosophy, and psychology) and strategized the politics of departmental startup (escape from a neighboring department fueled by demand for its service courses). [31] As just remarked, this article focuses nearly exclusively on the first concern. Gorn's second line of argument seems deserving of its own independent treatment at a time closer to the widespread establishment of robotics departments. There are many other adjacent and more distal disciplines that both contribute to and benefit from robotics. Beyond the obvious adjacent disciplines of computer science, engineering and mathematics, throughout this article there has been constant mention of the vital interplay with biology. [32] There are a similarly deep connections to be established with history and philosophy. The reach of robotics clearly extends far deeper into the humanities, encountering the arts as well. All of these connections will be needed to establish the discipline of robotics.

4.2.2 People

Attracting the best and brightest young minds is of course the most essential driver of any field. But contemporary robotics, drawing its researchers largely from those established disciplines most notoriously homogeneous with respect to gender, race and ethnicity, will continue to suffer a consequently diminished pace of innovation [235] until more strenuous effort achieves greater diversity. The enduring, paradoxical [236] and pernicious [237] nature of inequitable access [33] has been documented at every step [239] along the leaky STEM pipeline, including disparities in mentorship even at the doctoral level [240], reaching increasingly disproportionate numbers at successively higher ranks [241]. Accumulating evidence that neither specific public school redesign [242] nor even recourse to competition in eliciting it [243] has substantially broadened access to achievement prompts a growing chorus seeking newly vigorous, intentional recruitment of existing social [244] and legal [245] structures to break apart the many interlocking obstacles lying beyond the reach of mere educational reform.

But robotics portends a unique social impact that the discipline must embrace and deliver for human benefit. On the one hand, urgent moral issues [246] compounding dramatic 50-year declines in manufacturing jobs and the [seemingly consequent [247]] decoupling of income from GDP growth [248] oblige the creators of automation technologies to recognize their implicit policy concomitants [249] and become more thoughtful about their social impacts [250]. [34] On the other hand, the technology is irresistibly fascinating. Thus, of all disciplines, robotics is charmed in its visceral appeal as evidenced by millennia of human dreams and nightmares about the inanimate made vital. Robotics researchers owe it to their field as well as their society to leverage that popular fascination in the recruitment, retention and promotion to secure the participation of presently under-represented groups. Beyond the promise of intellectual advance, we will need the broadest

[31] It appears that this third argument, Gorn's wise appeal to service courses as the basis for a declaration of independence from colonializing neighbors, remains a tantalizing fantasy, awaiting greater penetration of robot technology into society.

[32] A great deal of confusion concerns the relationship of animals to robots, and the role their juxtaposition has to play in advancing both sciences. A proper commentary would require at least a dedicated essay of comparable length to this one.

[33] These inequities have been carefully studied largely with respect to the US domestic population. The powerful intellectual advantages conferred by [now imperiled [238]] historical US access to the elite of international STEM populations and the innovation-pumping cultural diversity they bring does not mitigate the damage incurred by missing out on the underserved local talent.

[34] Other observers emphasize the centuries-long experience of technology-driven job disruption leading to greater productivity and accelerated creation of new jobs [251]. Nevertheless, social rejection might still be triggered not by a wholesale retreat of employment but rather by an inability to educate the workforce to track the rising high-end demand [252].

 https://doi.org/10.1146/annurev-control-080320-011601

possible diversity of disciplinary experts to collectively balance the benefits of automation against the future of human work while ensuring that our burgeoning technological prowess works to counter social injustice and spread those benefits across cultures and classes.

5 CONCLUSION

The Future Issues list below summarizes the overarching next steps that this article proposes as particularly urgent to advance the foundations of robotics. But the article is first of all offered in the hope of helping new researchers hone their proposals by exploiting alignments, exploring connections or exposing fallacies introduced by the observations and arguments made along the way. Even better would be if some pronouncement here influences the introduction—and best of all, imaginably, the conclusion—of someone's next scientific paper to support or refute one of those many opinions. For such readers, the Supplemental Appendix may hopefully offer much more usefully specific targets of attack and the Summary Points list that leads it off is intended to outline the connections between the conceptual questions raised here and the technical machinery involved in addressing them. More broadly, this article will have achieved a large part of its purpose if it spurs other researchers, young and old, to articulate coherent foundations of robotics that improve upon, contest, or outright reject and replace this account in a manner that better promotes actionable fundamental research. In the long run, a synthetic science of robotics will emerge anyway, and it may be at least historically interesting to look back at the concerns that attracted and the confusions that bedeviled one toiling denizen of its pre-disciplinary era.

DISCLOSURE STATEMENT

As the text indicates, this review reflects the author's personal view of the entire field of robotics——not necessarily as it exists today but as it might come into clearer form as an intellectual discipline. The author asserts that the discussion of all articles cited is accurate, fair, and unbiased and is not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of their treatment in this review.

ACKNOWLEDGMENTS

The time and thought necessary to complete this article was underwritten by Office of Naval Research grant N00014-16-1-2817, awarded as a Vannevar Bush Faculty Fellowship sponsored by the Basic Research Office of the Assistant Secretary of Defense for Research and Engineering. The material was originally conceived and delivered as the 2011 Henry Paynter Lecture to the faculty of the Department of Mechanical Engineering at the Massachussetts Institute of Technology. I am extremely grateful to Professor Hari Asada and his MIT colleagues who invited the address and provided lively feedback throughout the day and evening (and indeed, over the course of many discussions before and since) that significantly advanced my thinking about these ideas. A next chance to develop the themes was occasioned by my delivering the 2012 Goldman Lecture to the faculty of the Technion Autonomous Systems Program Faculty at the Israel Institute of Technology. I thank Professor Elon Rimon and his Technion colleagues for their kind hosting throughout another very stimulating week (and, again, many years prior and subsequent) of exchanges related to these ideas.

FUTURE ISSUES

1. Scientific foundations necessary to advance the capabilities and safety of emerging robot technologies will require the identification of fundamental physical limits as well as models that characterize types of environments in which types of machines can be expected to operate appropriately.

2. Intellectual advances necessary to undergird those technologies with these foundations require a constant interplay between hypothesized type theories for assigning tasks to architectures in environments and empirical study of physical machines in uncurated settings that can support such theories or refute them.

3. Disciplinary developments necessary to foster such intellectual advances include the creation of robotics departments capable of systematic collaboration with adjacent disciplines and populated by the greatest diversity of thinkers that the age-old human fascination with robots promises to achieve.

 https://doi.org/10.1146/annurev-control-080320-011601

Those having even minimal familiarity with my work will already know that the source of any good ideas to be found here or elsewhere in my writing lies in the individual lessons taught to me and the collective culture created around me by the amazing group of PhD students and postdoctoral fellows I have had the wondeful good fortune to grow up with over the course of a nearly four-decade career. Many of my phenomenal collaborators (students, peers, and elders alike) are responsible for specific technical insights discussed here, and I have tried to acknowledge this by appropriate citation. [35] My teacher, Professor K. S. Narendra, taught me how to think.

[35]I am particularly grateful to Sam Burden, Jean-Luc Cambier, Paul Gustafson, Matt Kvalheim, Matt Mason, Lisa Miracchi, and Vassilis Vasilopoulos for detailed comments and advice that substantially improved the quality of this article.

 https://doi.org/10.1146/annurev-control-080320-011601

Supplemental Appendix

This material is supplementary to the main text of

Koditschek DE. 2021. What is robotics? Why do we need it and how can we get it? Annu. Rev. Control Robot. Auton. Syst. 4. In press. https://doi.org/10.1146/annurev-control-080320-011601

and intended to provide a somewhat more technically focused overview of concepts and bibliographical resources underlying the following issues identified in the body of the article:

Specifically, this supplement offers a supporting (but necessarily extremely abbreviated) sketch of ideas from dynamical systems theory (§A), some details on models of a robot's interface to its environment (§B) and further details (§C) supporting the speculation on how to advance the discipline of robotics.

SUMMARY POINTS

1. Research urgently needed on materials properties includes physical experiments and reasoning about: Co-limits of power, information rate, and grip (§4.1.1.1); and consistent mechanics models for hybrid systems (§B.1.4).
2. Research urgently needed to help advance the development of information interface models (§B.2) includes developing necessary or sufficient conditions: For internal environmental representation in robot architectures (§B.2.2); computationally effective abstract models of the environment's geometry or work capacity (§4.1.1.2); and computationaly effective tests guaranteeing qualitative properties of contemporary learning technologies (§B.2.3.2).
3. Research urgently needed on hybrid systems properties includes questions about which models support what versions of classical properties such as: Conley's Fundamental Theorem (§4.1.2.1); topological perplexity (§4.1.2.2); structural stability (§A.1); smooth Lyapunov functions (§A.2); and orbit decompositions and hyperbolicity (§A.3).
4. Research urgently needed on categorical properties includes the enlargement of applicability needed to capture more physically effective specification such as: Relaxed parallel composition with cross-talk (§4.1.3.1); hierarchical composition with asymptotic phase (§C.2.3.1); and increasing the palette of hybrid dynamics compositional operators to achieve grounded type theories that meet the greater expressiveness of more abstract higher level task specification languages (§C.2.3.2) .

A Dynamical Systems

A dynamical system ideally describes evolutionary processes via the group action of a model of time on a model of space [253]. For all dynamical systems theory variants of interest to robotics, there are contrasting views of internal state evolution and the associated input/output (behavioral [254] or linguistic [8]) properties (§B.1.2). This section focuses on the internal model viewpoint.

The three principal dynamical systems theories of robotics vary in their models of both space and time. Discrete finite automata that underly theories of computation [8] entail a discrete model of time and symbolic (i.e., finite or at most countable) model of space that boasts a precisely calibrated accompanying input–output behavioral representation. [e] Lagrangian systems [255] of working kinematic chains [67] entail a continuous model of time and space that has a topologically endowed canonical decomposition into symbols (§A.1) but lacks a well developed corresponding input–output theory (§B.1.2). Hybrid dynamical systems, arising in robotics from the making and breaking contact with the environment [169], superimpose an induced family of dynamics that entail a discrete model of time marked by events, discussed in Section B whose qualitative properties remain a very active area of research.

Section A.1 briefly outlines a coarse but very robust global steady state theory that relies on topological tools. [a] Section A.2 sketches a crucial consequence of

[a] Initially focused on neighborhoods and their continuous transformation, the further abstractions of topology continue to spill over into ever broader application domains [216], recently engaging computer science to return a new type theoretic foundation for mathematics itself [256].

that theory: dynamical systems give rise to an energy-like Lyapunov function that has been found in a large and growing array of the many variant internal state dynamics—including a candidate for hybrid robot dynamics (§B.1). Section A.3 contrasts this with a more rigid, finer-resolution steady state notion equipped with a looser energy-like Lasalle function whose extension to robot hybrid dynamics (§B.1.1) is not yet well understood. For ease of exposition the body of the article blurs these important details and proceeds speculatively as though both sorts of tools are available and under the assumption that the associated scalar valued functions can be selected so as to afford reasonable manifestations of physical energy (as is true for all classical Lagrangian systems [171], but only conjecturally so for their robotic hybridizations presented in Section B.1. [c]

A.1 Chains

This section sketches out the theory underlying Conley's insight that "rough models" requite "rough terms" of study [166]. His [166] *Fundamental Theorem* [257] introduces the analysis of chains (§4.1.2.1) to guarantee that any well-behaved classical dynamical system admits a partition (i.e. a cover whose components are all mutually disjoint) into a countable collection of transient open *basins* disjoint from their closed *attracting sets* that comprise their respective steady state fate as well as a final complementary closed component of *repelling sets*. Specifically, chain analysis yields a decomposition of a classical dynamical system into a closed, steady state *chain recurrent* set and its transient complement [166]. A component of the chain recurrent set is termed an attracting set exactly when it lies in the interior of its basin. [b] Section 4.1.2.2 proposes that a *goal* be expressed as the union of attracting sets: its symbolic meaning is then given by the union of their associated basins.

In this coarsest analysis of "flows with errors," the dynamics on the attracting sets is washed away: any sequence of states selected from it can be approximated by some chain through it [166, Ch. 1.8.3]. However, the great advantage of this enforced nondeterminism is its structural stability: a basin's topology (in the sense of homotopy type) persists under small perturbations of the model—a seeming consequence of Conley index continuation [166, Ch. IV.1.4], [216, Thm. 7.14]. In contrast, the deterministic study of orbits (the topology of flows with no error permitted) affords generic structural stability only for systems of dimension two or less [260]. Moreover the notion of hyperbolicity central to that classical theory depends upon smoothness assumptions which will not generally prevail for the pinched and creased "hybrifolds" [261, 262] arising from dynamics models of closed loop robotics. Thus, it seems urgent as well to establish what sort of structural stability guarantees may be possible for hybrid extensions of Conley theory such as [168].

A.2 Lyapunov Functions

Conley brings a generalization of Lyapunov theory, guaranteeing the existence of a continuous scalar valued function which is constant on the chain recurrent set and strictly decreasing along the flow on the complement. The importance for robotics of this global converse Lyapunov theorem is analogous to that of classical stability theory for control [263] and related systems applications [264]. Moreover, while there is no canonical classical procedure [265], energy plays a central role in the construction of Lyapunov functions for mechanical systems [195, 171]. Hence it is not unreasonable to speculate that at least "naturally controlled" robots (e.g., whose feedback forces and torques are designed to mimic potential–dissipative physics) [266] will admit global Lyapunov functions that encode the energetic cost of attraction, offering the prospect of grounding codesign (§4.1.1) to fundamental limits (§2.2.1.1). [c] This article adopts that speculative hypothesis as a conveniently tidy refuge from niggling technical hedges. It seems urgent to develop a more constructive formal insight that can guarantee or preclude the existence of an energy-Conley function for a robot hybrid dynamics.

[b] When that component is chain transitive (i.e., there is a chain included within it between any two of its elements) then it might be called an *attractor* [258, Def. 3.4.22]. The asymptotic flow of open sets of initial conditions into such a component of the chain recurrent set isolated within the interior of its basin is a notion coincident with asymptotic stability in the sense of Lyapunov [259] to be discussed in Section A.3. Traditionally, the term *attractor* has been more narrowly reserved for attracting sets that contain a dense *orbit* [210]. The appropriateness of one or the other definition seems best determined by the perceptual model as discussed in Section 4.1.2.2.

[c] This is not only true for "natural control" [267]—a straightforward generalization to the nonlinear robotics setting of the familiar proportional–derivative feedback regulator. Considerably more general feedback policies, for example, the graph error controller (eqn.4) and its generalizations, will also yield Lyapunpov functions that express the expenditure of mechanical energy.

A.3 Hyperbolicity and LaSalle Functions

The topological theory just reviewed offers a crucial toolbox for robot hybrid systems (§B.1) whose abruptly guarded resets (eqn.2) may preclude the application of more conventional tools (calculus and linear algebra) of smooth dynamical systems theory [168]. That said, the symbols arising from these tools as proposed in Section 4.1.2.2 are necessarily coarse—very robust, but likely to miss the degree of precision required by the sort of task specification symbols proposed in Section 4.1.2.2. In contrast, smooth dynamics introduces via the notion of hyperbolicity a rich account of local structure capturing key aspects of transient behavior [210] that escape the notice of purely topological methods underlying Conley's theorem [258]. Accompanying this local analysis is a potent, decades-enriched theory of *bifurcations* (qualitative changes in the topology and or stability of the attracting sets) [210] which can be used to induce exquisitely detailed behavioral structure in a nearly model-agnostic manner [209]. Moreover, a substantial compositional systems theory [263] has been built using the smooth Lyapunov functions associated with this class of models [268]. While such methods extend to compact invariant sets [269], it is often the case that the transient behaviors one seeks to encode via the template dynamics introduced in Section C.2.2.3 are most simply represented by restrictions to unbounded invariant sets as in the example below (§C.2.2.3). In such cases, one turns to LaSalle functions [270] to play the role of the scalar landscapes that capture the qualitative features of interest. This presumes the commitment to a metric that truly encodes physical energy [271, pp. 425–426].[d]

For example, the critically damped unit sping-mass system whose phase portrait is plotted in Figure 1 has a singleton at the orgin that comprises the entire attracting set in the sense of Conley. However, it is also the case that the reference dynamics—the scalar potential field on the sole proper invariant subspace spanned by the unique real eigenvector, $[1, -1]$ — is actually itself attracting in the sense that the velocity "graph error," $V(x_1, x_2) := |x_2 - f_{\text{ref}}(x_1)|^2$ plays the role of a LaSalle function for $f_{\text{ref}}(x_1) := -x_1$. A finer decomposition affording such dynamically specified goals relies on the smooth structure of differential topology, affording calculus and linearized analysis via local coordinate charts [272]. In this classical setting of smooth dynamical systems [210] the collapse of dimension onto reference dynamics is built into the folklore of the field and serves as a ready made paradigm for hierarchical composition [203].

Fortunately, in many settings [273, 206], the closed loop hybrid systems can be shown to be at least locally conjugate to classical dynamics and the full power of classical dynamical systems theory (e.g., hyperbolicity, bifurcation theory, averaging, and so on) [210] may likely be applicable, at least in the neighborhood of the steady state behavior. Thus, another matter of urgent interest concerns the extent to which this is true, and what more general classes of hybrid dynamics will admit this finer toolset from classical dynamical systems theory. Once again, the relevance of this point of view seems contingent on the availability of a sufficiently refined perceptual system as to afford the view of orbits rather than chains (§A.1). However, for purposes of this article, it is now convenient to proceed as if those necessary constituents are all in place: for example, Section B.2.3.2 speculates [k] that the required sensorimotor coherence of symbol grounding Footnote 16 may be achieved by recourse to learning in a generalization of newly emerging work [274, 275].

B Robot–Environment Interface Models

A still unsettled but rough consensus model concerning the mechanics of robot–environment work exchange presented in Section B.1 invites comparison with the many important, extensively researched and used, but still quite disparate accounts of the information interface discussed in Section B.2.

B.1 The Work Interface: Robot Hybrid Dynamics

This section sketches the manner in which (eqn.1) gives rise to a class of hybrid dynamical systems following roughly the development of [169]. After introducing a family of environmental models similar to (eqn.1) (§B.1.1), and contrasting the prospects for an accompanying behavioral model (§B.1.2), attention turns to the discrete transition mechanics of guards and resets (§B.1.3), concluding with a sketch of the inevitable trade-off between accuracy and tractability of such models (§B.1.4).

A satisfactory version of Conley's theorem (§A.1) has been developed [168] for a somewhat tamed version of [169] but it remains to be seen how much of the classical hyperbolic theory (§A.3) can generalize to this setting. Eventually it seems likely that some suitable Morse decomposition [276] into perceptually

[d]Thanks to Matt Kvalheim for pointing out this reference.

 https://doi.org/10.1146/annurev-control-080320-011601

grounded [k] attractor basins will finally superimpose a suitably fitting discretizataion of space to yield a physically grounded discrete finite automaton representation relevant to robotics.

B.1.1 Robot–Environment Mechanics

From the perspective of both manipulation [67] as well as locomotion [68], the consensus view of the robot–environment work interface embraces the Lagrange–d'Alembert formulation of Newton's laws [277] to posit a robot system of the form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \Upsilon(q,\dot{q},\tau,\lambda_v) \tag{1}$$

where $q \in \mathcal{Q}$ denotes some local coordinate representation of the configuration of a mechanism, $\mathcal{Q}$, typically a Lie Group, acting through its internally actuated torques or forces, τ, coupled to its environment by contact forces, λ_v, through which the world can constrain, receive, or deliver energy. Models that account for actuator limits must posit a further internal model for the actuation input signal, τ. The simplest of these introduces a speed–torque model [75], but the recent introduction of substantial computational resources into motor drive electronics encourages much more intricate representations of the manner in which information interacts with power [76, 77].

Given its unsettled status, present purposes of exposition seem best served by completing the description of the "environment" as a family of complementary systems of the form (eqn.1). Each is defined over some "external" family of configurations, $\tilde{q}_v \in \tilde{\mathcal{Q}}_v, v \in V$, each potentially capable of asserting its own torques or forces, $\tilde{\tau}_v$, while bonded to the robot through the mutual constraints $\tilde{\lambda}_v(q,\dot{q},\tilde{q}_v,\dot{\tilde{q}}_v)$, defining an V-indexed family of differential-algebraic dynamical controlled systems with states $x_v := (q,\dot{q},\tilde{q}_v,\dot{\tilde{q}}_v)$ constrained within some appropriately defined total space, $\mathcal{X}_v$ typically called the *mode* .

B.1.2 Internal State vs. Behavioral Models

In the long run, the viewpoint of ports [278] will be an essential tool in specifying and understanding the flows of work exchanged through the interface specified in (eqn.1) and its extended differential algebraic dynamical system over a the states of given mode $x_v := (q,\dot{q},\tilde{q}_v,\dot{\tilde{q}}_v) \in \mathcal{X}_v$. At present, the relationship between such behavioral models [254] and their internal state concomitants (eqn.1) is still being worked out for the dynamical systems of interest here—a promising start to a general theory of physically meaningful compositions [279]. [e]

The (largely unspoken) consensus view in robotics is that these coupled robot–environment modes on $\mathcal{X}_v$ admit the description as a well-posed control system with input forces and torques from actuators that can be arbitrarily assigned functions of the full history of output signals reported by the sensors—with slight adjustments made for whether the "environment" to be worked upon includes the robot's body (locomotion) or not [68]. This view may turn out to be insufficiently general in the long run, pending the adoption of a carefully specified information interface.

B.1.3 Guards and Resets

Keeping track of physical circumstances under which occur the crucial transitions between modes is given by the *event* graph, $\mathcal{E} := (V,E)$ for some $E \subset V \times V$ by means of *resets*

$$r_{(v,\tilde{v})} : \mathcal{G}_{(v,\tilde{v})} \to \mathcal{X}_{\tilde{v}} \tag{2}$$

from the *guards,* $\mathcal{G}_v \subset \mathcal{X}_v$, of the initiating mode. A formal account of this general setup can be found in [280], accompanied by a judiciously brief discussion of the huge antecedent literature and results.

B.1.4 Negotiating the Accuracy–Tractability Trade-Off

As soon as we countenance the crucial making and breaking of contact through reversible bonds introduced in Section 2.2.1.2, extension of the consensus model (eqn.1) through the *guards* and *resets* (eqn.2) of a hybrid dynamical systems model (§B.1.3) is threatened by the limitations of both rigid body mechanics

[e] Contrast this with the exact correspondence between the Chomsky hierarchy of languages (classes of "behaviors" in the sense of [254]) and the memory capacity and access of discrete finite automata (state representations in the sense of [254]) achieved for the discrete time discrete space dynamical systems studied in the Turing branch of computer science [8] discussed in Section 2.1.2.

 https://doi.org/10.1146/annurev-control-080320-011601

as well as the (surely but only slowly) developing field of dynamical systems theory (§A). Whereas classical dynamical systems converged on a broadly accepted set of foundational definitions and consequences in the middle of the previous century [253, 210, 257], different formulations of hybrid dynamics over the past two decades [281, 170, 282] enjoy their own strengths and support somewhat distinct groups of practitioners, with little study of their formal relationships or comparative empirical applicability. At the same time, negotiating a rigid body's conflicting and paradoxical idealizations [283, 284] has proven confusing enough that a demonstrably practicable model—e.g., affording such ubiquitious simplifications as plastic impact, persistent contact, massless limbs, and so on—has only recently been developed with the guarantee of a formally consistent hybrid dynamics [169]. Adding in a similarly practicable yet formally consistent model of friction remains an urgently important active area of research [285, 286].

B.2 The Information Interface: A Fragmentary Account

Despite the central importance of perception in the history of the field [287, 81, 288] and the obvious manner in which computation fuels its advances, robotics has yet to develop a distinctive theory of information processing. Clarifying the fundamental role of bit streams in specifying and accomplishing work is surely best promoted by research avenues focused on closed loop interaction with the environment such as active perception [82, 289, 80], navigation [290, 291], and manipulation [185, 91, 292, 293]. This section will review the prospects for achieving a consensus representation of a robot's information interface in Section B.2.1, before distinguishing in Section B.2.2 advances related to the robot from those in Section B.2.3 related to representations of the environment. The obvious primacy of information—its use in internal computation and its collection, processing and dissemination through ports and channels—impels a better understanding of this resource, particularly given the burgeoning contemporary interest in data-driven methods (§B.2.3.2).

B.2.1 Modeling the Information Interface

In rough correspondence to the making and breaking of physical ports, perceptual interactions come and go in the course of robot–environment work exchange. This is obviously the case for proprioceptive sensing and associated signal processing [294, 146, 151, 147]. But exteroceptive modalities are also subject to such intermittencies, famously plagued by the correspondence problem [79], substantially compounded by the need for semantic (i.e., robustly and repeatably recognized named) [295] *channels*. Even given enough prior knowledge of structure that semantic channels can be presumed to operate reliably, mobility—whether of the robot [274] or the environment [296]—implies impermanence and the possible combinations of perceptually available channels will impose an additional dimension of hybrid dynamics [275].

Type theories that include named channels supporting information exchange between multiple, mobile computational threads undergird functional calculi called process algebras [69]. These universal programming languages [5, Ch. 41.7] are broadly accepted as underlying any effective representation of the disorderly world of concurrent computation and interaction [297]. An early effort to define the information interface and associated programming model for robotics in such terms [298] might be argued to still represent the field's best attempt yet made to identify those aspects specific to the exchange of work. Why, after decades of further advances in robot perception and algorithm design, is there no obvious information processing theoretic counterpart to the mechanical work interface formulation of (eqn.1)?

A central reason this may not be straightforward is that symbols and signals seem to mix indiscriminately at the information interface. The recently introduced notion of a combinatorial filter [299] exemplifes the benefit of articulating explicitly a robot's symbolic sensorium. When grounded in the lattice of perceptual equivalence classes [300, 301], symbolic filtering offers a principled framework for posing and assessing resource requirements of perceptual processing problems [302] as well as insight into the aspects of the physical sensorium [303]. [f] But analog communication and computation play a major role in a robot's information exchange with its environment.

For example, preflexes [304]—feedback loops whereby data affecting output forces and torques, τ (eqn.1), is gathered, processed, disseminated and executed through materials properties of the body—are intrinsically analog and crucial for animal [162, 305, 163, 306] as well as robot [51, 307, 308, 208] control and coordination. Proprioceptive sensing in the actuators [73, 294, 151] is similarly analog. Moreover, a great deal of internal computation addresses the signal flows (positions, velocities, forces, and so on) through the interface using models of the architecture and its environment developed from (eqn.1). While the field has grown up using

[f] There does not yet seem to be a full expression in terms of a process calculus.

 https://doi.org/10.1146/annurev-control-080320-011601

conventional digital computers to implement such signal processing and control, evidence of an emerging Moore's law for power density in commercial mixed analog and digital computation [309], suggests that analog hardware might eventually be developed to the point of claiming its natural role here [41].

In sum, the central challenges for representing a robot's information interface seem to lie in developing a perceptually grounded model of symbolic processing [300, 301] that encompasses analog communications and computation. Such a model lays the groundwork for a process algebraic type theory for programming [69].

B.2.2 Architecture

In turn, a central challenge of a robot's information processing architecture arises from the need to relate the interface model and associated programming languages of *channels* just described with those arising from the work interface model of *ports* in Section B.1. That they are intricately related is intuitively compelling. For example, appropriately grounded symbols can dramatically change the computational complexity associated with a task. [g] One way in which these models urgently need to be related regards the familar but largely still unresolved question of what internal model of the environment is necessary or sufficient support a task–environment pairing (§2.2.2.2).

Mounting interest in analog computation from synthetic biology and neuromorpic engineering [311] has fueled the development of promising abstractions [312] including type theory and associated programming lanuages for compilation of dynamical systems specifications into physical substrates that can compute them [42]. A less charted challenge is presented by the need to compile symbolic behavioral descriptions onto analog signal processors and controllers at the preflex [304] level of the work flows modeled in Section B.1 intersecting the problem of codesign discussed in Section 4.1.1.2. An interesting proposal for more general encoding of information in the hybrid dynamics of work (eqn.1) is the suggestion to build controllers with enough nonlinear internal state dynamics as to enable direct symbol representation via the homotopy class of a pulse family [313]. More broadly, there seems no literature extending any such specifications of internal dynamics to the type theoretic framework of communicating and mobile processes [69].

Still more broadly, there remains the challenge introduced in Section A.3 of developing sufficiently fine perceptual information to enable hybrid dynamics decomposition resolvable by the tools of normal hyperbolicity. For reasons bearing on the intrinsic robustness discussed in Section 4.1.2.1, long term behavior at the level of chain recurrence will be generically manifest to any sensor suite. But that level of resolution is too coarse to capture the template/anchor compositions advocated in Section 4.1.3.1. For these must encompass transient phenomena governed by local linearized dynamics that are typically key to the success of rapidly transitional maneuvers [211, 314]. [h] In contrast, when Conley's theorem fails, there may well be no persistently structured behavior, raising a different perceptual challenge. For example, a recent study of certain paradigmatic transitional maneuvers [192, 193] reveals a complicated but topologically regular relationship (a simplicial complex) between the combinatorial choices of limb recruitment and the resulting behavior. For our purposes, the chief virtue of this emerging account of regularity in purposively exercised hybrid dynamical systems is the impact of topology on the prospects for advances in learning.

B.2.3 Environment

Generically, a robot can be said to have achieved a task by acting to bring into a specified goal condition the projection onto its sensorium of the environment's perceptible features. An important extreme obtains when its designers' environmental model is sufficiently accurate and its work interface provides sufficient affordance that task achievement can be deduced from purely open loop interaction from any conjoined robot–environment initial condition [92]. There are at least two important dimensions of relaxation from this extreme: task–environment pairings relative to which more or less sensory information is required by a fixed architecture [91]; and architecture–environment pairings relative to which more or less active materials are required to achieve a fixed task by preflexive engagement [71]. Along either dimension, a model of the

[g] Significant advances in sampling-based motion planning now begin to permit the computation of plans for manipulation of moveable objects by mobile robots [310]. However, the explicit decomposition into the individual strata (robot-object-in-contact) on the boundary of the freespace incurs huge growth in computational cost as the number of objects grows. In contrast, discrete combinatorial complexity can be mitigated by imposing topology (adjacency relations) on the symbol spaces inherited from the grounding relation [190].

[h] There are paradigmatic examples of robot hybrid dynamics that can be shown to yield no wilder behavior than exhibited by (formally, they are locally topologically conjugate to) classical dynamical systems [273, 206].

 https://doi.org/10.1146/annurev-control-080320-011601

environment is required to support such reasoning, and it seems clear that a great deal more empirically thorough research will be needed to advance such urgently needed inquiry [315].

B.2.3.1 Designed Models The highly abbreviated sketch of the last half century's development of robotic tasks recounted in Section 4.1.2.2 entails environments characterized by their fixed geometry. Taking that perspective—navigation fundamentally a topological problem requiring computationally effective representation—motivates a workspace expressed in terms of tame topology [216, Ch. 3.5]. [i] Tasks involving making and breaking contact with such merely inert, geometrically conceived environments, require complicated models still under very active development [286], while moveable objects entail notorious modeling challenges, even for putatively "rigid" bodies [284]. It has proven harder still to develop computationally effective (i.e. lumped parameter, finite dimensional) environmental models that capture the dynamics of work exchange. The recent emergence of terradynamics [142] and its blossoming into a broader pursuit of "robophysics" [320] substantially boosts the prospects for a discipline of robotics. The proposal to develop a representation of the environment's work capacity in terms of low dimensional energy landscapes [143] represents a particularly important new advance in this endeavor. Stochastic representations [321] can be used to reason about safety and passage through cluttered environments.

The tautological need for environmental models facing robot designers who wish to reason about their designs of course holds no such sway over the designs themselves. How much of an internal model a robot requires, how much can be achieved through accumulating experience, and what sort of description must be endowed at the time of its design remain fundamental questions about which relatively little progress has been made beyond the insights from the last century of control theory. [14] Internal descriptions of geometrically and topologically simple environments [180] can be replaced by realtime measurements with no loss of navigation capabilities [322]. Geometric details used to deform real environments into such simple topological models [323] can be learned from experience and instantiated using realtime measurements with the same formal navigation guarantees assuming correctness of the topological model [275]. These formally correct navigation architectures exhibit good empirical performance in a diversity of physical environments that lie well out of scope of the formal guarantees [274]. An urgent question concerns what sort of new information about the environment should the robot seek to learn in the face of a navigation failure that logically entails some out of scope aspect of the environment.

B.2.3.2 Learned Models These last examples suggest a broader need to investigate the prospects for integrating into disciplinary robotics the recently exploding capabilities of learning architectures [324, 95] [j] that have long been shown to have great promise for nonlinear control applications [96]. A crucial but slowly advancing literature is emerging that can offer structured architectures [97] or computationally effective tests [325] to guarantee that such function approximators enjoy desirable formal qualitative properties.

The navigation architectures [274, 275] just discussed (§B.2.3.1) introduce a complement of deep learning modules [326, 327] that serve as sensory "oracles," presumed to perceptually ground certain essential superlevel sets [n] (specifically, the anti-goals [m]) of the task specification energy landscape. The "scope" of environments relative to which the architecture can be proven sufficient to the task amounts to those populated by objects that the oracles have successfully "learned" to label. [k] An urgent need for future research concerns the possibility of extending qualitative guarantees such as achieved in [325] to criteria on landscapes sufficient to guarantee the approximation of their sublevel sets to specified accuracy.

[i] This is the study of sets defined by systems of inequalities shown to permit a topological decomposition [316] reminiscent of Collins' cylindrical algebraic decomposition [317] with similar computational complexity [318]. The defining equations need not be merely polynomial but, by virtue of o-minimal geometry [319], can include suitably restricted analytic functions.

[j] For the last few years, and likely a few more to come, the splicing of these technologies for data driven function approximation onto the robot technology project has consumed the academic field. This has the potential benefit of substantially increasing the empirical capabilities of buildable robots. To date, the literature on learning in robotics appears largely hostage to the technological predilection for anecdotal demonstrations in place of experiments designed to refute or support clearly stated hypotheses.

[k] A potentially compelling (but still very speculative) generalization of this idea might achieve the crucial coherence between dynamically and perceptually grounded symbols (a notion first introduced at the point of Footnote 16). Namely, it is irresistable to imagine that the new learning technologies [324] might be more broadly trained not simply to recognize anti-goals but potentially general (perhaps even arbitrary) classes of the sub-level sets [n] arising from the controls-generated energy landscape.

 https://doi.org/10.1146/annurev-control-080320-011601

C Speculative Problem Solutions: Toward a Type Theory for Work

C.1 Toward Problem 1

C.1.1 Degrees of Freedom: Toward a Dynamical Landauer Limit

Clearly, packing deeper smooth basins into the fixed volume of (a compact) space must result in steeper barriers. These high magnitude gradients, in turn, appear in the power landscape (since, irrespective of the specific vector field realized by a given controller, the chain rule dictates that the power landscape take the form of its inner product with the energy landscape gradient). Interpreting attractor basins as the underlying behavioral symbols of the mechanical agent and requiring that noise-defying barriers separate them now gives a compelling motivation for building high DoF bodies of a kind we see in the animals: higher dimensional spaces offer exponentially growing volumes for the same base length scale; hence mechanical "versatility" (rapid transitions between distinct behaviors) [328] incurs greater power burden or more distributed bodies-—or both. The foregoing (essentially quasi-static) arguments promise to bite more sharply in higher energy regimes where the dynamics is not overdamped and trajectories follow more closely the level sets (rather than steepest descent down the sub-level sets) and potential barriers must be still higher (to repel high energy initial conditions near obstacles as laid out generally in [171] and exemplified by the desiderata of navigation functions [180]). Critically, it seems clear that energy landscape steepness (i.e., the magnitude of the gradient)-—again, relative to the noise level-—is the key parameter in determining the power cost of switching between stable behaviors. Note that as derivatives are involved, the impact of the tail behavior of the noise power spectrum increases.

In general, as the complexity of a behavior increases, the body will need to recruit multiple simultaneous combinations of basins, e.g. as in fully spatial bipedal running [329]. Now, versatility requires parallel channels of both coordinated and asynchronously switching basins, each channel incurring its own noise–power trade-off. This intuition yields the dynamic analogue of the Landauer bound: there is a minimal energetic cost per channel that can only be diminished at the expense of pushing down the noise floor (greater information). At the same time there is a maximal speed of transmission (speed of sound for proprioceptive data from the periphery through the structural vibrations; slower depending upon the degree of compliance; much faster for ions over axons; the speed of light for bits over wires). Working out the details of these bounds (the lower-bounded numerator over the upper-bounded denominator) over full range of spatiotemporal scales yields the dynamic Landauer limit and would seem to justify a major effort of understanding in robotics.

C.1.2 Codesign and Morphological Computation

Codesign is a general term that has come to designate a framework for yoking by formal (or perhaps simply computational) means specified aspects of the design triple (§2.1.3) to each other and/or to the available physical resources (§2.2.1).

Possibly the first computationally posed version of the problem coparametrized a body and controller design so as to allow an evolutionary algorithm to shape them simultaneously in tandem toward improved dynamical locomotion in a simulated 3D flatland [330]. An important recent advance develops a formalism for trading physical resources against task capabilities that yields sufficient conditions (and a computationally effective procedure) for determining infeasibility or an optimal design [331]. A computational framework for specifying and executing body reconfigurations of a modular robot system in response to navigation tasks in a parametrized family of physical environments [159] has been extended to allow autonomous planning and execution of environmental reconfiguration to render it entirely navigable [160].

An important but far more constrained version of the codesign problem arises from the hope to systematize the clever use of resources such that the same material simultaneously fulfills multiple functions as in "preflexes" (§B.2.1). Design of preflexes often results from bioinspiration [307, 208]. Such design challenges are so central to robotics that they have begun to accumulate an important and specialized literature, approximately gathered under the rubric of "Morphological Computation" [332, 333].

C.2 Toward Problem 2

C.2.1 From Signals to Grounded Symbols

Take as working definition of a lexicon of *symbols* over a space, the elements of a countable cover (i.e., a countable collection of subsets whose union yields that space). Say that a lexicon over the state space of a robot's hybrid system (eqn.1) is *dynamically grounded* if its symbols can be expressed in terms of topological operations (i.e. unions, intersections and complements) on invariant sets. In this view, Conley's Theorem (§A.1) offers an intrinsic lexicon of symbolic goal primitives: attracting sets and their associated basins. [m]

Say that a lexicon is *perceptually grounded* if each of its symbols can be defined by some computation applied to its sensorium and memory. Clearly, a well-designed robot would be programmed using symbols that are both dynamically as well as perceptually grounded. Absent an established information interface model as discussed in Section B.2.1 it seems fruitless for the time being to hope for a proposal of how to do so by reasoning from first principles. Instead, it will be convenient for purposes of this article to simply declare that the relevant Lyapunov sublevel sets [n] are perceptually grounded—as are the anti-goals in [274, 275] and, more generally, via learning [k]—and proceed.

C.2.2 Specification of Goal Primitives

A tutorial account of the passage outlined in Section 4.1.2.2 from path planning [173] to motion planning [109] to kinodynamic planning [196] to vector field planning [178] is partially laid out in [174]. For purposes of this article, these distinctions can be illustratively caricatured in the trivial setting of a 1 DoF unit point mass "robot, " $x \in \mathbb{R}$ subject to a user commanded force, u, modeled as a double integrator $\ddot{x} = u$ as follows.

C.2.2.1 Reference Motions Suppose the robot is required to reach the origin from some initial configuration, $x_0 \in \mathbb{R}$. The path planning problem is solved by the line segment joining x_0 to the origin. The generic motion planning problem is solved by the time parametrized function $m(t) := (1-t)x_0$. A kinodynamic version of the problem additionally imposing the requirement of zero velocity at each end point is solved by $k(t) := (1 + 4t^3 - 3t^4)x_0$, while further restrictions, (e.g., on allowed accelerations [334]) would entail higher order spline solutions. All of these approaches require a controller to force the actual robot to track the proposed motions, achievable.

For example, in the classical setting—e.g. within one mode of (eqn.1)—this can readily be achieved by an inverse dynamics controller, $u := \ddot{r} + \kappa_d(\dot{r} - \dot{x}) + \kappa_p(r - x)$ where the role of r might be played by m or k depending upon the criteria. In the hybrid setting the problem of motion specification is considerably more complicated. A recent well conceived example of this approach for planning the motion of an agile legged robot is presented in [335]. An offline motion planner selects continuous mode knot points that select subgoal pairs in the guard sets of stance modes to be joined and subject to constraints inherited by the dynamics (eqn.1). A cubic spline (one algebraic degree lower than the quartic reference spline k above) is selected under the assumption that a "lower level" stance controller will track it, e.g. using an inverse dynamics scheme.

From the view of iterated re-planning introduced in Section 4.1.2, a weakness of all such motion reference schemes is the absence of a global recovery strategy. When the actual execution remains close to the specified reference then the presumption is that no recovery is needed—the lower level controller will continue to act until the errors vanish. However, if the controller fails dramatically, perhaps because the underlying planning model was faulty, the only recourse is to start up the offline planning procedure with the updated information about the environment.

[m] We are typically interested in both goals (desired target states) and "anti-goals" (states to be avoided). In classical systems, the latter role is played by repelling sets dual to their paired attracting set (in the sense that they attract in reverse time solutions that approach the attracting set in forward time). But the hybrid systems of robotics generally support only forward time uniqueness properties (different guards might have intersecting reset images). Now, the repelling set dual to a given attracting set is taken to be the complementary forward limit set [168, Footnote 4]. Hence, it is convenient to use Lyapunov function superlevel sets to define anti-goals.

[n] The *sublevel* sets of a scalar-valued function $V : \mathcal{X} \to \mathbb{R}$ are its pre-images over intervals, $V^{-1}[(-\infty, c)] := \{x \in \mathcal{X} \mid V(x) < c\}$ for $c \in \mathbb{R}$ and their complements are called *superlevel* sets. For convenience, this article will sloppily use the term to refer to the entire algebra of sets so generated—i.e. their smallest closure under the operations of union and complement. The reader might imagine unions of "level bands" such as $V^{-1}[(a, b)] := \{x \in \mathcal{X} \mid a \lhd V(x) \lhd b\}$ for $a < b \in \mathbb{R} \cup \{\pm\infty\}$ and $\lhd \in \{<, \leq\}$ to be the typical exemplars.

 https://doi.org/10.1146/annurev-control-080320-011601

C.2.2.2 Optimal Dynamics Optimization methods can be used to generate goals taking the form of reference trajectories (with a resulting methdology similar to that just sketched above) or feedback controllers. Alternatively, a major inheritance of robotics from control theory is the idea that a designer's goals should be expressed in terms of scalar valued cost functions whose suitably discounted future accumulation along the motions of a dynamical system should be minimized by the feedback control policy. Of the many variants on this idea developed in the late twentieth century, the most enduring—and with arguably the greatest value for robotics—is the so-called *model predictive control (MPC)* framework [200] wherein costs are only accumulated over a finite horizon and the next control action is recomputed over the next iteration of that horizon, realizing the popular intuition that plans should be made for the long term but executed via frequently updated actions over the short term.

There is a long tradition of stability analysis for the closed loop systems resulting from such methods [200] and an active literature continues to pursue its extensions to the robot hybrid dynamics model of Section B.1. [o]

C.2.2.3 Reference Dynamics In contrast, a vector field planner can be specified by the potential function, $\varphi(x) := x^2/2$, yielding the gradient field,

$$f_{\mathrm{ref}}(x) := -\nabla\varphi(x) = -x. \tag{3}$$

The solutions of the associated reference dynamics $\dot{r} = f_{\mathrm{ref}}(r)$ solve the motion planning problem from any particular initial configuration, $r_0 := x_0$ and can be used in the inverse dynamics controller defined above. Alternatively, any potential dissipative controller,

$$u := -\kappa_p\dot{x} + \kappa_p f_{\mathrm{ref}}(x)$$

applied to the plant will solve the problem directly from any initial configuration and velocity, $(x_0, \dot{x}_0)$ [171]. For purposes of further discussion below, it is useful to single out a particular version of this controller taking the form of a damped "graph error" controller [197]

$$u = -2(\dot{x} - f_{\mathrm{ref}}(x)) - \nabla\varphi(x) \tag{4}$$

yielding the critically damped linear time invariant system depicted in Figure 1.

The motivation for and construction of such embedded transient properties was initially proposed in [197] and has re-appeared throughout the vector field planning literature, e.g., [198, 199, 337].

C.2.3 From Composition to Category

The control policy (eqn.4) presents a very simple instance of hierarchical composition in the form of a one dimensional *template* dynamical system f_{ref} (eqn.3) that has been *anchored* in the two dimensional system plotted in Figure 1. This collapse of dimension notion (trajectories of the two dimensional system quickly come to be nearly indistinguishable from those of the one dimensional system) is deeply rooted in the folklore of classical dynamical systems theory [338] and the robust version of the concept envisioned in [203] has been stated with great generality in [205] for hyperbolic classical dynamical systems (§A.3). [30] A version of the construction has been proposed for the categorical presentation of robot hybrid dynamics (§B.1) in [167].

Section C.2.3.1 describes two more dynamical compositions that emerge from the template/anchor formalism illustrated by Figure 1. Section C.2.3.2 describes recent advances in developing a hybrid dynamical systems category that might eventually subserve the type theory urged in Section 4.1.3.2.

[o] Optimal methods derive the feedback law from the cost function whereas the template methods of Section C.2.2 work oppositely, inducing a Lyapunov function from the dynamics. In principle, the point of view developed in Section 4.1.2.2 is agnostic with respect to the origins of the closed loop dynamic to be composed. In practice, the template methods are less precise but more robust because they typically do not rely on exact plant models [336]. Moreover, they may be easier to design with for low DoF templates since the gain parameters and the control terms so parameterized directly effect the forces and torques—in contrast to the parameters of a cost function which are once-removed from the physics. Surely the direct designs are more amenable to online adaptation because their controls are parametrized directly from the task–environment pairing whereas the optimal feedback laws are computed numerically. Finally, the traditions of optimal control are to pose monolithic "end-to-end" expressions of a task rather than develop methods of modularization and reuse as advocated in Section 4.1.2.2. However, carefully developed optimal methods such as MPC [200] offer the substantial advantage of automating the production of stabilizing controllers in contrast to the "hand built" template designs. Surely, in the long run some appropriate mix of these direct and indirect methods of control design will emerge in the corpus of robotics best practices.

 https://doi.org/10.1146/annurev-control-080320-011601

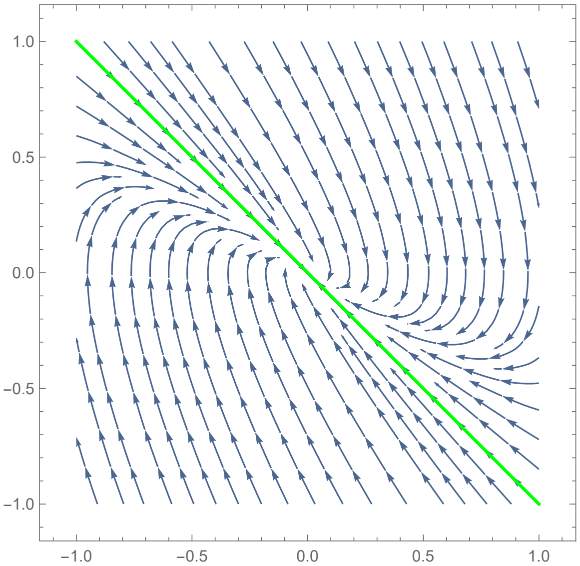

Figure 1: Phase portrait of the critically damped 1 DoF linear unit spring-mass system, $\ddot{x} + 2\dot{x} + x = 0$.

C.2.3.1 Composition of Templates Raibert pioneered the efficacy of parallel composition in robotics. He famously showed that dynamically complicated pogo sticks could be reliably regulated by completely decoupled, computationally simple, energetically aware 1 DoF controllers [113]. Hopping height was managed by regulating radial energy along the leg—provably correctly in isolated vertical position [339]—simultaneous with PD control of orientation during stance. These decoupled stance policies alternated with an independent stepping policy to regulate fore-aft speed—again provably correctly in isolation [340], with some sufficient conditions for more realistic models [341]. Work continues to the present day enlarging understanding of why such decoupled policies work in the presence of such highly coupled dynamics [206], and how much analogous cross talk can be ignored in more general settings of parallel composition [207, 208]. It seems an urgent matter to try to build into an emerging hybrid dynamics category formalism weaker, hence, more broadly practicable notions of parallel composition that can countenance some degree of non-deleterious cross talk [206].

In contrast, the ubiquitous presence of Lyapunov functions in dynamical systems enables robotics to immediately encode the crucial operation of sequential composition [185] in terms of sublevel sets [184]. This operation can be formulated categorically [167] when the sublevel set isolates a single (chain-transitive) component of the attracting set in the sense of Conley, but the finer structure of classical dynamical systems theory is not yet encoded in so doing.

To illustrate the practical implications of this gap, consider hierarchical composition by template–anchor pairs arising in that categorical formulation whose templates are encoded as attracting sets in the sense of Conley [167]. For example, a classical phase oscillator is formally anchored in a vertically constrained hopper of the kind discussed in [329] and, thus in the unconstrained two DoF pogo-stick resulting from Raibert-like parallel composition of vertical hopping and horizontal stepping. Unfortunately, while the entire template (a radially embedded copy of the vertically constrained hopping model) can be shown to comprise an attracting invariant subsystem of the resulting parallel hybrid dynamics, it is not defined on a Conley attracting set. Roughly speaking, the point attractor of Figure 1 corresponds to the anchored limit cycle while the one dimensional invariant subspace corresponds to the constrained vertical hopper. A structurally stable formulation of the finer view of attraction discussion in Section A.3 suitable for the more broadly construed notion of templates and anchors has been laid out for the classical setting [205]. It seems an urgent matter to find general conditions under which this finer view can admit expression in a more refined hybrid category.

C.2.3.2 Toward a Category of Hybrid Dynamical Systems The hybrid dynamical systems analysis of [342] represented an important first step toward a physically grounded type theory for robot programming. Recasting the classical dynamical systems notion of semi-conjugacy [257] in category theoretic terms via the process-algebraic [69] notion of bisimulation afforded extensions to both controlled and hybrid systems.

 https://doi.org/10.1146/annurev-control-080320-011601

Introducing additional structure yields categories more specifically robotics-oriented, yielding categorical accounts of hybrid dynamics [343] even affording representation of both sequential as well as hierarchical composition [167]—albeit still limited to the chain-based steady state view as discussed in Section 4.1.3.1. The landmark proposal of approximate bisimulation relations for classical systems [344] via Lyapunov-like comparison functions now begs for a category theoretic extension along the lines of [342]. On the one hand, a suitably relaxed version of the category theoretic extension to hybrid systems networks [280] of the original controlled systems category [342] offers the potential for a formalized version of the classical systems theory expounded in [263]. On the other hand, approximate bisimulation using energy landscapes (§4.1.2.2) suggests a route to "pulling back" the desirable finer resolution steady state theory of orbits (§A.3) from "naive" smooth templates onto their less regular hybrid dynamical anchors (§C.2.3.1).

At the same time, it is enticing to speculate about the prospects for using an energy-grounded (§C.2.1) type theory to ground in turn some of the higher level formal languages that have been proposed for robotics. For example, modal logic [225] or context free grammars [226] might be endowed with extensions to a type theory that prescribes their interface to the hybrid dynamics categories in a manner that partially or wholly removes the requirement for the subsequent verification step. A more natural grounding interface is likely for functional programming languages with abstract but explicitly defined operational semantics in robotic actions such as the manipulation framework of [233] whose linear logic formulation anticipates the (likely) unavailability of full categorical products in the energy-grounded setting [167]. This latter example presents a compelling argument for the importance of greater abstraction by its impressive application of computationally tractable proof theories to partially automate the generation of detailed task specification [233].

 https://doi.org/10.1146/annurev-control-080320-011601

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 https://doi.org/10.1146/annurev-control-080320-011601