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Date ....24.6.05....

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Constraint Models for Multiple Interference in the Channel Assignment Problem

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A thesis supervised by Dr R. M. Whitaker and Prof. S. Hurley, sponsored by the EPSRC (www.epsrc.ac.uk) and Ofcom (www.ofcom.org.uk), and submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the University of Wales.

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ABSTRACT

For the channel assignment problem, the adequacy of binary channel separation constraints based on the single interferer assumption and/or a constant re-use distance has been questioned by several authors. The single interferer assumption is convenient for channel assignment purposes as it leads to a generalised graph-colouring model which is simple to formulate and very popular. However, it is desirable to approximate the operational criteria more closely than a single interferer assumption model allows, by modelling the effects of *multiple simultaneous interferers.* This thesis addresses the problem of modelling multiple interferers in channel assignment using constraints, with a view to finding an efficient and convenient approach which offers resilience against multiple interference whilst minimising additional spectral requirements.

Motivated by a discussion of the literature concerning single and multiple interference, the thesis analyses the coverage failure as progressively higher numbers of multiple simultaneous interferers occur, characterising those interferers which lead to coverage reduction. A hybrid sequential and simulated annealing heuristic is applied which obtains optimised channel assignments for analysis, created under the single interferer assumption, for two-hundred-and-forty problem cases. The library of test cases is created using a purpose-built problem generator which is applied to create problems with differing randomised distributions of transmission sites.

The analysis informs the consideration of methods for the reduction/elimination of multiple interferer effects. A multiple interference model based on higher order constraints called co-channel set constraints is assessed. Results concerning the theoretical properties of these constraints, and their satisfaction, are presented. An alternative way forward is then considered, which involves challenging the commonly applied assumption that the multiple interferer assumption implies constraints are necessarily non-binary. New methods are introduced that incorporate multiple interference into the generalised graph-colouring formulation i.e. *binary constraints.* The methods are tested using the test problem library; optimised assignments are made and their resilience against multiple interference and the spectral requirements are used to evaluate the approaches. Evidence is provided that one of the methods provides an improved model for channel assignment with multiple interference and can be recommended for use to provide constraints which perform well under the multiple objectives concerned.

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In memory of Beryl Lewis, Glyn Lewis and Phyllis Howells.

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Chapter 1

Introduction to Channel Assignment

1.1 Context

Radio communication technologies have become ubiquitous and the demand for radio based services is now such that finding suitable available frequencies for additional wireless services is a significant technical problem. Channel re-use and sharing of the medium are required to maximise the use of the radio spectrum, a finite natural resource. This requires careful management and coordination, considering issues of quality of service and interference on users. Improving the techniques for modelling interference and assigning channels is the focus of this thesis.

Cellular networks are arguably the most prominent form of radio communication system today. Cellular networks are used for applications including television and radio broadcasting, telephony, private mobile radio (PMR) networks and broadband fixed wireless access. The problem of interference on users is particularly important when there are many simultaneous users communicating in a local geographic area. The channel assignment problem (CAP), also known as the frequency assignment problem (FAP), is concerned with assigning channels to transmitters such that problems are minimised whilst efficient use is made of the available spectrum.

Conventional approximations to the channel assignment problem, which use the *single interferer assumption*, consider interference from each source in isolation. This thesis considers channel assignment in the context of multiple simultaneous interferers. Specifically, it addresses:

- the advantages of considering multiple simultaneous interferers over using the single interferer assumption;
- the quantification of service coverage when channel assignments created under the single interferer assumption are analysed under multiple interference conditions;
- the characteristics of channel assignments causing performance degradation under multiple interference;
- the use and properties of higher order channel separation constraints for modelling multiple interference;
- the comparison and evaluation of new techniques for generating binary channel separation constraints which offer increased resilience against multiple interference.

This chapter introduces the radio spectrum available for communications use and the need for careful effective and efficient management of this resource. It considers issues involved in designing an effective cellular radio communications network which will provide high-quality communication links to users whilst also satisfying the requirements of other interested parties such as operators and regulators. The chapter defines the channel assignment problem and its objectives, introducing the operational criteria involved and the use of constraints in modelling the channel assignment problem. The chapter concludes by explaining the structure of the remaining thesis chapters.

1.2 FUNDAMENTALS OF RADIO COMMUNICATIONS

1.2.1 ELECTROMAGNETIC SPECTRUM FOR COMMUNICATIONS

When an electric charge oscillates or is accelerated, energy-carrying disturbances are produced which are partly electric and partly magnetic and travel at 3×10^8

metres per second—the speed of light, denoted c. These electric and magnetic field variations, known as electromagnetic radiation or electromagnetic waves, pass through space in the form of sinusoidal waves and can be defined by their wavelength or frequency (or energy).

The wavelength λ of a sinusoidal wave is the distance between successive peaks or crests in the wave, measured in metres; the frequency f represents the temporal separation between successive peaks and is measured in cycles per second (Hz) in SI units. The two are inversely proportional via the relationship

$$f = \frac{c}{\lambda}$$

The electromagnetic spectrum is a continuum of all electromagnetic waves, arranged according to wavelength and often divided into named subsections: radio, the longest waves; microwave; infrared; visible light; ultraviolet; x-rays; and gamma waves, the shortest waves. (This division of the continuum is in a certain sense arbitrary, as most of the boundaries are only vaguely defined.)

Visible light was the first electromagnetic wave to be used to transmit long-distance telegraphic communication signals, in the semaphore system which originated in 18th century France. All wireless communication systems use electromagnetic waves to carry their signals. Today, wireless communication makes huge use of the particular region of the electromagnetic spectrum containing radio waves.

The radio region of the electromagnetic spectrum can be further divided into named subsections according to wavelength. These are shown in figure 1.1 and described by Schiller [1, chapter 2]. The UHF band (containing radio waves with frequencies in the range 300MHz–3GHz) is most commonly used in cellular systems, with the lower frequencies in this range having propagation characteristics which make them preferable.

Sections of the radio spectrum available for communications are allocated to specific purposes or operators. These radio spectrum allocations are then subdivided into discrete small bands which are normally equally sized and are called *channels*. The bandwidth between the frequencies at the centre of adjacent bands is known as guard space. Transmitters in a network operate on specific channels belonging to that network's allocation. The process of deciding which transmitter should operate on which channel is called *channel assignment*.

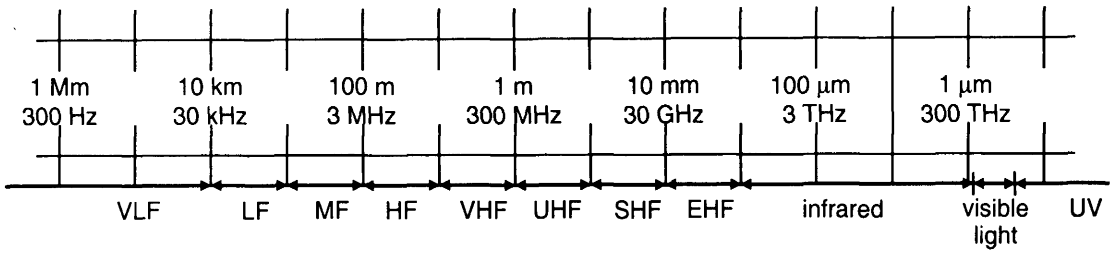

Figure 1.1: Frequencies for communications

The chapters which follow discuss channel assignment and how it can be performed in an efficient and effective manner. The need for channel assignment to be studied arises due to interference effects experienced by users of a radiocommunication network. This thesis is based around the idea of using effective channel assignment to minimise such problems.

1.2.2 SIGNAL, INTERFERENCE AND NOISE

The use of radio waves to provide communication links without the use of wires introduces inherent interference problems which are a topic of importance in wireless transmission.

Radio waves are sinusoidal waves of form, for example,

$$g(t) = A\sin(\omega t + \beta) \tag{1.1}$$

where $\omega = 2\pi f$, A is the amplitude of the wave and β is the phase shift.

The sinusoidal nature of electromagnetic waves, and their ability to travel over long distances, means that they can be used as the carrier for a communications system. The sinusoidal carrier wave undergoes modulation in such a way as to provide a physical representation of the information required to be sent. In essence, the information signal is 'piggy-backed' onto the carrier signal. Three properties of the wave can be varied to carry the signal, leading to three types of modulation: amplitude modulation, frequency modulation and phase modulation. The introduction of frequency modulation was a major step in the advancement of radio communications technology and this is currently in widespread use in analogue systems. Amplitude modulation gives lower bandwidth than frequency modulation, and phase modulation is more difficult to implement.

The user equipment in a cellular network has to be able to distinguish its own signal from that of other users. This is further complicated by the way in which electromagnetic waves interact when they meet each other; determined by the principle of superposition. The result of this interaction can vary between the two extremes of total destructive interference, where the waves effectively cancel each other out, and total constructive interference, where the peaks combine to give a wave whose amplitude is the sum of the amplitudes of the two constituent waves [2, 3].

Definitions of the term *interference* vary greatly by context. Any interaction of waves is called interference, but the word is often used more specifically for unwanted interaction which reduces radio reception to unacceptable levels, and there are even more specific definitions in use in areas of engineering.

The transmitter emitting the signal which is desired at a point in the network is called the *serving transmitter* and is usually the transmitter offering the strongest signal. A transmitter which is known to produce an interfering signal at a point in a network is referred to as an *interferer*.

Interference is considered here to be radio emissions from transmitters other than the serving transmitter; these impede reception of the desired signal by the recipient. In other words, one recipient's signal is other recipients' interference and vice versa.

The extent to which a transmitter is an interferer is largely dependent on the following factors:

- the proximity in terms of channel separation between the interferer's transmission and that of the serving transmitter;
- the geographical distance involved;
- the respective received powers of the transmissions.

These factors will be discussed later. For now it is useful to note that if the interferer and serving transmitter are operating using the same channel from the radio spectrum, then the interferer is called a *co-channel interferer*; otherwise it is known as an *adjacent channel interferer*. Note that the term 'adjacent channel' refers to both immediately adjacent and non-immediately adjacent channels and as such denotes any separation other than the co-channel case of no separation.

Interference is distinguished from *noise* in that the latter is defined to be an incoherent emission, from a natural or a man-made source, of a character unlike that of the desired signal and usually having a broad spectral content [4].

The term *multiple interference* is used to refer to interfering signals being received from multiple sources simultaneously, and the resulting cumulative interference effects experienced by users in the network.

1.2.3 SIGNAL-TO-INTERFERENCE RATIO (SIR)

Signal-to-interference ratio (SIR) is a measure which can be used to determine the quality of service (QoS) in a radio communications network. The SIR is the ratio of wanted to unwanted signal strengths at a receiver and is usually quoted in decibels (see below). In this thesis, the SIR is calculated using only knowledge of transmissions within the network i.e. without considering noise external to the system (section 3.1.3.2).

A threshold SIR associated with some given level of quality is often quoted for a network and denoted σ. If the SIR being experienced at a given point within the network is at or above this service threshold, i.e. SIR $> \sigma$, then the QoS is considered to be satisfactory at that point. The SIR can be calculated throughout the network, thus providing for a QoS measure for the network as a whole.

A receiver will always experience interference of some kind; the use of the SIR helps to establish whether the effect of the interference is sufficiently large to mean that the receiver can no longer successfully demodulate the desired signal. The ability of a receiver to successfully decode its wanted signal in the presence of interference is sometimes referred to as the *capture effect* [6] and the minimum SIR value at which this can occur is then the *capture ratio.* The minimum value that σ will take is equal to the capture ratio, but σ may be given a higher value to achieve a given level of quality of service.

Power and signal strengths are measured in Watts (W) where

$$\begin{aligned} 1 \text{ Watt} &= 1 \text{ joule per second} \\ &= 1 \text{ kg m}^2/\text{s}^2 \end{aligned}$$

An SIR is the ratio of two quantities which are each measured in Watts. This means that SIR is dimensionless. The SIR is usually converted into decibels (dB), a dimensionless unit with a logarithmic scale. When a threshold SIR value for quality of service is given, it too is given in decibels, thus facilitating the comparison of the two.

For two quantities P_0 and P_1 expressed in Watts, their ratio in terms of decibels is defined as

$$10\log_{10}\left(\frac{P_0}{P_1}\right) \quad \text{dB}$$

For SIR calculations, P_0 measures the strength of the wanted received signal, and P_1 measures interfering signal strength:

$$\text{SIR} \;=\; 10\log_{10}\left(\frac{\text{wanted signal strength}}{\text{interfering signal strength}}\right) \quad \text{dB} \tag{1.2}$$

1.2.4 Signal propagation

The way that radio waves of certain frequencies propagate leads directly to their usefulness in communication systems. For example, our use of radio for long-distance communication is so great because of the way that the radio waves used propagate in the upper atmosphere—'bending' round the earth's surface.

The term *propagation* refers to the motion of waves through or along a medium. Propagation of electromagnetic waves may occur in a vacuum as well as in material media.

If an omnidirectional signal were transmitted with power P_t by an ideal source in a vacuum, then the signal would propagate outwards from the source, detectable on the surface of a sphere centred at the source and increasing in size. The surface area of this sphere increases as the distance from the source increases, meaning that the power is effectively 'spread out' with uniform distribution over the sphere. At distance d from the source the received power would be

$$\frac{P_t}{4\pi d^2}$$

This received power is lower than the transmitted power, due to *free space loss*—the effect of the distribution over the expanding sphere. In a realistic situation, the received power is altered by other effects due to

- the equipment being used;
- the medium through which the wave is travelling;
- obstacles encountered by the wave on its path between transmitter and receiver;
- environmental factors.

These effects are collectively known as *propagation loss* or *path loss*; the term *attenuation* is used for path loss which is due to factors other than geometric spreading.

Because in a realistic situation electromagnetic waves are attenuated as they travel, and so that received powers, signal strengths and SIRs can be calculated as accurately as possible, path loss must be incorporated into the model in some way. One way of incorporating these factors is to take signal strength measurements throughout a network. This can be impractical, in terms of the logistics of gathering, storing and publishing data, and measured signal strengths will alter in the presence of, for example, heavy rain or other electrical equipment. Hence, mathematical propagation models are often used as an alternative to provide values for received signal strength and fulfil this function of incorporating propagation factors into the network model. Each of these propagation models [7, 8] represents an attempt to predict radio propagation as it is affected by real-world conditions. The model chosen for use in this thesis is discussed in section 3.1.3.1.

1.3 CELLULAR NETWORKS

This thesis is concerned with channel assignment and interference in the context of *cellular* networks. Rather than studying a particular cellular system such as GSM, the thesis considers the general context of cellular systems and their generic characteristics.

A cellular network has a structure in which radio signals are transmitted by transmitters at base stations and received by users. A *cell* is the geographical area around a transmitter position in which the service is provided by that transmitter. A cellular system enables the user equipment to communicate directly with a base station, which connects each call to the backbone cable network.

To provide capacity by enabling sharing of the medium by multiple communications occurring in a network, and to do this without creating too many problems such as interference or call-blocking, cellular systems commonly use a mixture of four different multiplexing schemes: Frequency Divisional Multiple Access (FDMA), Space Divisional Multiple Access (SDMA), Time Divisional Multiple Access (TDMA) and Code Divisional Multiple Access (CDMA). In cellular systems the use of SDMA is implicit within the structure of the network: the geographical space is divided into cells. Within cells, FDMA, TDMA and occasionally CDMA are employed to enable many calls to take place simultaneously. The use of FDMA is closely related to the channel assignment problem in that frequencies may be used to distinguish between the communications of different users.

There are two types of cellular system: broadcast, which has *downlink* transmission only; and bi-directional transmission, in which communication links are established in two directions:

- uplink: from user to base station;
- downlink: from base station to user.

The work presented in this thesis is applicable to both types of system (section 3.1.2).

Due to the manner in which radio waves propagate, cells cannot tessellate, and are required to have overlapping regions if service is to be widely available on a geographical basis. This overlap facilitates the seamless transfer of calls as user equipment moves from one cell to the next. Automated *handover* protocols enable this to occur without interruption to the call underway.

Wherever it is located in the network, the user equipment must be able to distinguish between the wanted and unwanted signals it is receiving. This is particularly acute in regions of overlap or where transmitter density is high. At the infrastructure design phase, decisions can be made which can assist in the management of interference. If cells are placed in such a way as to allow the required handover but avoid further overlap of cells that would lead to interference effects, then the process of assigning channels to the transmission equipment will be aided in advance.

1.4 THE CHANNEL ASSIGNMENT PROBLEM

By careful re-use of the channels assigned to transmitters for transmission purposes, the interference users receive is controlled. Re-use must only occur in cells that are far enough from each other not to cause unacceptable interference effects. One of the ways to model this situation is to define constraints which impose requirements on the separation of the channels used by certain subsets of transmitters (section 1.5.2).

A channel assignment is a mapping from a set of transmitters to a set of channels, usually referred to by consecutive integer channel numbers.

The general channel assignment problem can be stated as follows:

Given

- a set of transmitters $\mathrm{Tx} = \{t_1, \ldots, t_n\}$
- a set of channels $F = \{f_1, \ldots, f_k\}$
- a set of *domains* $D = \{D_1, \ldots, D_n\}$, where $D_i = \{f_j \in F : f_j$ is permitted to be assigned to $t_i\}$
- a *demand vector* (or *requirements vector*) $C = (c_1, \ldots, c_n)$, where c_i is the number of distinct channels to be selected from D_i for assignment to t_i
- a set of channel separation constraints which restrict the permitted assignments to subsets of more than one transmitter
- an objective function O (see section 1.5)

construct a mapping $A : \mathrm{Tx} \longrightarrow F$ such that to each $t_i \in \mathrm{Tx}$ are assigned c_i channels from D_i and the objective O is optimised.

The *order* of a channel assignment is then the number of channels used in total;

the *span* of a channel assignment is the difference between the two extreme channel numbers used in the assignment. For example, the diagram shown in figure 1.2 illustrates an assignment to seven transmitters, encompassing eleven consecutive channels. This assignment (which is a zero-violation assignment for the graph in figure 2.3) has span=10 and order=6. For any assignment, order $\leq$ number of variables, due to channel re-use, and order $\leq$ span+1.

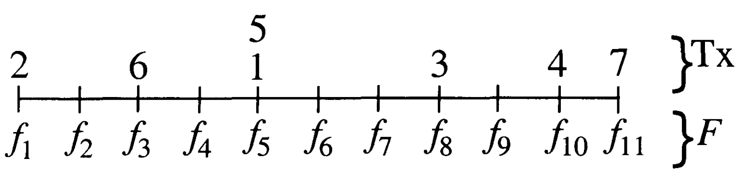

Figure 1.2: A channel assignment, mapping Tx $\rightarrow$ F

1.5 MODELLING CHANNEL ASSIGNMENT

The process of modelling the channel assignment problem involves three distinct aspects:

▷ Operational criteria: Measurement of quality of service in relation to SIR experienced by users in the network. (Section 1.5.1.)

▷ Constraints: Imposing restrictions on assignments to avoid poor QoS. (Section 1.5.2.)

▷ Objectives: Function to be optimised to give quality of service over the network as a whole. Note that the objectives may be defined directly as a function of operational criteria, considering SIRs experienced across the network, or abstracted and expressed as a function of the satisfaction of the constraints imposed. (Section 1.5.3.)

An assignment whose operational criteria are acceptable (e.g. SIR for all users is above a desired threshold) is called an *interference-free* assignment. An assignment satisfying all the constraints imposed is called a *zero-violation* assignment. Note that these two terms are not synonymous as a given set of constraints may not fully encapsulate the operational criteria. This disparity is a focus of the thesis, which has among its aims finding a *convenient and effective constraint representation of operational criteria.*

1.5.1 OPERATIONAL CRITERIA

Traditionally, the *single interferer assumption* has been used to encapsulate the channel assignment problem in *binary channel separation constraints* (section 2.2.2). Such constraint generation considers interference from each single interferer, calculates the geographical distance or SIR between the serving transmitter and that interferer and formulates a constraint between that transmitter *pair* if the required value for service is not met.

In reality, a user is likely to experience interference from multiple interferers i.e. from several sources simultaneously. The combined effect of several transmitters may cause sufficient interference for a constraint to be required in a situation where individual transmitters would not be considered problem enough to introduce a binary channel separation constraint in the above manner.

For example, the mobile user equipment communicating with transmitter C in figure 1.3 may not receive sufficient interference from each transmitter A, B, D or E individually to cause a problem, but the combined effect of all four could be enough to render its communication impossible.

The generation of binary channel separation constraints under the single interferer assumption does not account for interfering signals emanating from multiple non-serving sources simultaneously. It can be seen in chapter 2 that few authors consider the effects of multiple combined interferers i.e. employ the *multiple inter-*

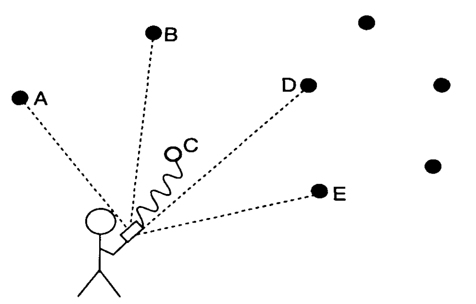

Figure 1.3: Experiencing interference from multiple simultaneous sources

ferer assumption.

When multiple interference is considered, the signal strengths of the interferers must be combined in some way to enable calculation of SIRs. For example, Dunkin and Jeavons [9] use a root-sum-square technique to combine the received interfering signal strengths. The assumptions used to incorporate multiple interference in the model in this thesis are outlined in chapter 3.

Note that whenever the term 'single interferer assumption' is used in this thesis, it refers to the assumption that each interferer is considered singly. SIRs are calculated using the two signal strengths from the serving and interfering transmitters respectively and no other terms. Other single interferer assumptions exist, such as the assumption that only the most dominant single interferer needs to be considered; when a different single interferer assumption is used in the thesis, this is stated.

1.5.2 CONSTRAINTS

The channel assignment scenario has been modelled as a mapping from a set of transmitters to a set of available channels, the details of which are established by somehow encapsulating the operational criteria into computable entities called *constraints*. These entities are abstracted from the real-life scenario but, if they are well-chosen, the solution of the constraints directly translates to a solution of the network problem.

A constraint is essentially a rule which limits in some way the choices that can be made in creating the channel assignment mapping. This may include forbidden combinations of values for some subset of the variables, a mandatory value for a variable, or a mathematical formula which defines a required relationship between some of the values. Once this formulation has been accomplished, the channel assignment problem becomes a specific instance of a constraint satisfaction problem (CSP) [10, 11].

Binary constraints are constraints which define restrictions between the channels assigned to pairs of transmitters i.e. the subset which is being simultaneously restricted has cardinality 2. The use of a specific type of binary constraint, called a *binary channel separation constraint* and introduced in section 2.2.2, will be discussed further in the chapters which follow. These constraints are the most commonly used and relate directly to graph-colouring (section 2.2.3). Binary channel separation constraints have the advantage of being simple to generate and relatively easy to solve and are supported by a significant amount of literature from discrete mathematics. However, constructing a set of binary channel separation constraints which successfully encapsulate the operational criteria (particularly the presence of multiple interferers) remains a challenge.

A possible alternative to the use of binary constraints is the introduction of higher order, or non-binary, constraints which simultaneous constrain the assignment made to a number of transmitters. The suggestion has been made (e.g. [12, 13]) to use higher order constraints to facilitate the more accurate expression of the operational criteria. In practice, binary constraints are the more tractable, making a binary constraint representation preferential wherever such a representation is possible.

1.5.3 Objectives

Recall that the objectives of channel assignment may be defined directly as a function of operational criteria or abstracted and expressed as a function of the satisfaction of the constraints imposed. Different objectives are required for different channel assignment problems; these are dependent on the availability of channels and the operators' requirements.

Occasionally a problem is presented which requires an interference-free assignment to be found using a given allocation of channels only. Such problems are called *fixed spectrum problems*. It is not always possible to achieve the main objective of creating a zero-violation/interference-free assignment using only the given channels and in this case a secondary objective is given, such as the minimisation of some cost function of the interference unavoidably remaining. When a CSP has a set of constraints which together are tight enough to mean that a zero-violation assignment is impossible, the problem is said to be over-constrained [10]. This is often the case in CSPs motivated by practical applications and in this situation the problem becomes a partial constraint satisfaction problem [14].

Four objectives are described in this section, using a classification employed by Koster [15] and others. The latter two are examples of fixed span problems. Some authors use objectives which cannot be placed into one of these categories directly, but these are usually based on varying or combining these four.

(a) Minimum Order Problem

In the minimum order channel assignment problem, frequencies should be assigned in such a way that:

- The assignment is interference-free/zero-violation;
- The total number of occupied channels (the order) used in the assignment is minimised.

This was the first of the four problem type categories to be discussed in the literature, with many authors citing Metzger [16] as the researcher who first described it.

(b) Minimum Span Problem

In the minimum span channel assignment problem, frequencies should be assigned in such a way that:

- The assignment is interference-free/zero-violation;
- The span of the channels used in the assignment is minimised.

The minimum span required to achieve this objective is notated *spn*.

(c) Minimum Interference Problem

In the case of a fixed spectrum problem where an interference-free/zero-violation assignment cannot be found, the minimum interference version of the channel assignment problem involves assigning frequencies in such a way that:

- A given cost function of the interference remaining in the network is minimised.

(d) Minimum Blocking Problem

In a network, a request for a communication link may be denied due to strain on the network's resources at that particular time. This is called call-blocking. The blocking probability for a network is the probability that a request will be denied i.e. a call will be blocked.

In the case of a fixed spectrum problem where an interference-free/zero-violation assignment cannot be found, the minimum blocking version of the channel assignment problem involves assigning frequencies in such a way that:

- A partial assignment is interference-free/zero-violation
- The blocking probability of the whole network is minimised.

1.6 Thesis structure

The remainder of the thesis consists of chapters arranged as follows:

Chapter 2 looks at existing models for channel assignment and at attempts in the literature to incorporate multiple interference into these models. It considers solution techniques, including meta-heuristics, which have been applied and their degree of success in producing solutions with respect to the operational criteria and constraints.

Chapter 3 introduces the modelling assumptions used in the remainder of the thesis, including those to incorporate multiple interference, along with the methods used to create assignments from sets of binary channel separation constraints. The discrete nature of channel separations is noted to be important. A library of test problems is presented to provide for a statistically validated investigation; suitable problem generation techniques are developed and described in detail.

Chapter 4 develops methods for analysing multiple interferer effects, considering the tractability of generating interferer sets for analysis under the multiple interferer assumption. The chapter analyses the effects of multiple simultaneous interferers in terms of cumulative SIR received by users in the network. The analysis also reveals the dominant causes of interference in this situation in terms of channel separation with the server. The conclusions drawn from the analysis motivate the consideration of different types of constraint to mitigate the effects observed.

Chapter 5 investigates the potential use of higher order constraints to incorporate multiple interference into the model for channel assignment, introducing a specific type of higher order constraint and supporting theoretical observations pertaining to its use. The inherent complexity of working with higher order constraints as opposed to binary constraints is considered.

Chapter 6 considers the potential use of binary constraints which are generated in such a way as to consider multiple interferers rather than employing the single interferer assumption. New techniques to generate such constraints are discussed; tested, using meta-heuristic techniques; and compared. The resulting assignments are evaluated against those produced under the single interferer assumption.

Chapter 7 consolidates the findings from the preceding chapters and provides suggestions as to how the conclusions may be applied to incorporate multiple interference into the channel assignment problem.

Chapter 2

Literature Review

2.1 Introduction

This chapter describes how different authors approach the modelling and solving of channel assignment problems. The discussion focusses on whether authors consider multiple interference effects. The ways in which authors incorporate and/or analyse these effects are presented, along with the conclusions reached. Attention is also given to the application of channel sharing, including re-use distances and co-channel interference effects.

It is common practice to formulate representations of real world problems to facilitate the implementation of automated solution techniques. Modelling techniques are employed which aim to encapsulate the *operational criteria* of channel assignment into computable structures, often *channel separation constraints*, and *objectives*. The translation from real world to representation can often result in the loss of elements present in the original problem, meaning that choices pertaining to this abstraction must be carefully made so that solutions remain valuable for application in the operational situation. Techniques to model channel assignment must be chosen in such a way that assignments, made to solve the modelled situation and objectives, can be related back to the operational situation with reasonable accuracy and success.

Constraint generation is not performed in a single unique manner; there are many different alternative methods. This thesis considers the choice of constraint generation method, as this is crucial to the operational quality and relevance of solutions. There is a trade-off between capturing all operational criteria and ensuring that constraints are reasonable in terms of generation time, storage and tractability.

2.2 SINGLE INTERFERENCE MODELLING

2.2.1 THE SINGLE INTERFERER ASSUMPTION

Of the authors in the literature who refer to the generation of the constraints used, most appear to employ a *single interferer assumption* (e.g. [18, 19, 20, 21]). This involves considering interference from each source in isolation, and is based on the assumption that the total interference received at a point in a network can be approximated by the interference received from the most dominant interferer without a loss in accuracy of the model. This section discusses the fundamentals of this popular model for channel assignment, introducing the concepts of re-use distance, binary channel separation constraints and the analogy between the channel assignment, when formulated in this way, and graph theory.

Figure 2.1 shows a serving transmitter and several interfering transmitters. The single interference assumption would consider each of the interferers individually, as illustrated, but would not account for the more realistic situation which considers these simultaneously.

The single interferer assumption is frequently selected for use, as it involves modelling only binary interactions. This means that only pairs of transmitters (i.e. the serving and each interfering transmitter) are considered when decisions are made in relation to channel separation. It is assumed in this thesis that SIRs calculated under the single interferer assumption use the two signal strength powers from the serving and interfering transmitters respectively and no other terms.

If a channel assignment is robust to single interference then, at each point in the network, the unwanted signal from each interferer in isolation will not cause an SIR value below the required QoS threshold. This is not realistic, however, as in the operational scenario multiple simultaneous unwanted signals will be received, which may cause the SIR to fall below the required QoS threshold.

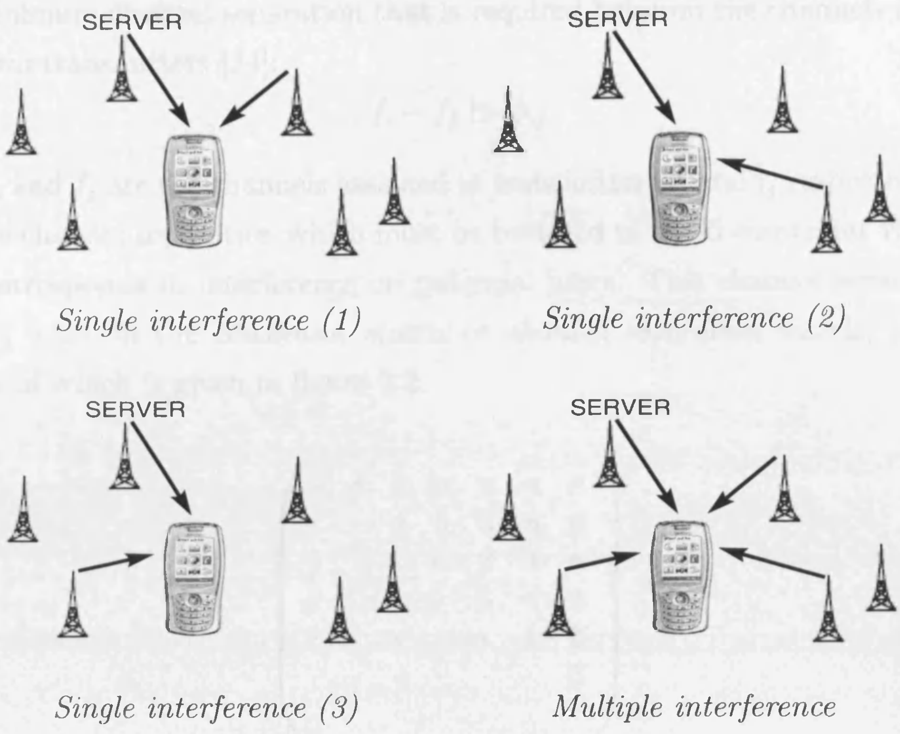

Figure 2.1: Single and multiple interference

Closely related to the single interferer assumption is the concept of *re-use distance*, a term often used in the literature (e.g. [19, 22, 23]). This may be applied by use of a model which defines a re-use distance d, constant throughout the network, at or beyond which a channel may be re-used. The further away an interferer is, compared to the serving transmitter, the better the quality of SIR. However, using constant distances throughout the network simplifies the situation (section 2.3.1).

2.2.2 Constraints for single interference

Constraints for single interference are involved with the channel separation between pairs of serving and interfering transmitters and are therefore binary constraints. (Although that is not to say that binary constraints cannot be used in other situations (section 2.3.3).)

Binary channel separation constraints are often used to model channel assignment as they are simple to generate and tractable in relation to their solution. They constrain the assignment made to the pair of transmitters involved by stating a minimum channel separation that is required between the channels assigned to the two transmitters [24]:

$$| f_i - f_j | > \phi_{ij}$$

where f_i and f_j are the channels assigned to transmitters t_i and t_j respectively and ϕ_{ij} is the channel separation which must be bettered to avoid constraint violation, which corresponds to interference on potential users. This channel separation is the (i, j) entry of the *constraint matrix* or *channel separation matrix*, (ϕ_{ij}), an example of which is given in figure 2.2.

$$\begin{pmatrix} \cdot & 3 & 2 & n & n & n & n \\ \cdot & \cdot & 4 & 5 & 3 & n & n \\ \cdot & \cdot & \cdot & n & 2 & n & n \\ \cdot & \cdot & \cdot & \cdot & 3 & 4 & n \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 5 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Figure 2.2: Channel separation matrix

The entry n in the constraint matrix occurs when no constraint exists between that particular pair of transmitters. The constraint matrix is usually written in upper triangular form as the constraint between t_i and t_j is considered to be the same as that between t_j and t_i; if these differ in a practical situation, the stronger constraint is used to symmetrise the situation and the weaker will be satisfied implicitly. There are no separation entries on the diagonal in the example in figure 2.2. Non-n entries on the diagonal can be used to specify the channel separation required for co-sited transmitters.

2.2.3 GRAPH-THEORETIC MODEL

There is a one-to-one mapping between a given set of binary channel separation constraints and a finite, simple, undirected graph [25, 26] $G = (V, E)$ in which

- vertex $v_i \in V$ represents the transmitter t_i;
- an edge $(v_i, v_j) \in E$ exists if and only if there is a binary channel separation constraint between t_i and t_j;

- edge (v_i, v_j) has a positive integer label which is the entry ϕ_{ij} from the constraint matrix.

Zoellner and Beall [24] cite Metzger [16] as the first to use this approach, recognising it as a "breakthrough in frequency assignment technology" [17].

The constraint matrix shown in figure 2.2 would lead to the constraint graph representation shown in figure 2.3.

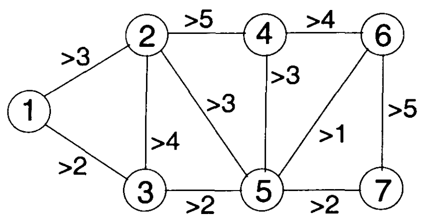

Figure 2.3: Graph G to represent binary channel separation constraints

The channel assignment problem is then analagous to a generalised form of the graph-colouring problem [27]. The assignment mapping $A : V(G) \rightarrow F$ gives a zero-violation assignment if and only if it satisfies all conditions

$$| f(v_i) - f(v_j) | > \phi_{ij}$$

for all pairs $(v_i, v_j) \in E$. Graph-theoretic models are often employed in channel assignment [28], and the analogy means that results from graph theory can be applied directly to the channel assignment problem.

2.2.4 CO-CHANNEL CONSTRAINTS

Constraints restricting the transmitters which are allowed to operate on the same channel are referred to as co-channel constraints. If $\phi_{ij} = 0$, the associated constraint is a *binary co-channel constraint*, which forbids the sharing of a channel by a particular pair of transmitters. Binary co-channel constraints give rise to a graph representation consisting of co-channel relationships only. These can be represented by an unlabelled constraint graph G, where

- vertex $v_i \in V$ represents the transmitter t_i;
- an edge $(v_i, v_j) \in E$ exists if and only if there is a binary co-channel constraint between t_i and t_j.

In these circumstances, the channel assignment problem becomes the well-studied traditional vertex-colouring problem [27, 29] from discrete mathematics which requires each vertex in V to be coloured in such a way that no edge in E is monochromatic. Note also that re-use distances can be used to generate a graph-based representation of the problem, again consisting of co-channel relationships only (see e.g. [30]).

2.3 MULTIPLE INTERFERENCE MODELLING

The incorporation of multiple interference into a model for channel assignment was considered as long ago as 1986, when Whitehead [31] formulated a signal-to-interference ratio using the sum of unwanted signal strengths as the cumulative strength of interference in the ratio. This idea was largely put aside in favour of the consideration of binary relationships, until authors began to call into question its omission in the following decade.

When Aardal, van Hoesel, Koster, Mannino and Sassano [32] surveyed the models and solution techniques available for channel assignment at the end of 2001, they found that most models consider interference between pairs of transmitters only. They refer to only two notable exceptions to this situation:

- Fischetti, Lepschy, Minerva, Romanin-Jacur and Toto [33], who choose to consider total cumulative interference when developing constraints in their model, stating two different SIR threshold values: one value for pair-wise interference and another, lower value for cumulative interference.
- Dunkin *et al.* [34], who follow up a series of papers motivating the use of higher order constraints by beginning to consider methods of formulating and solving such constraints (see sections 2.3.1 and 2.3.2).

Fischetti *et al.* [33] also note that the consideration of cumulative interference effects in models is rare.

In the formulation of any operational problem into a model or set of constraints, some aspects present in the original problem may not be translated fully into the new representation. Whether these aspects are important or not needs to be considered carefully for each type of problem [35].

This section discusses the ideas put forward by the authors who question the absence of multiple interference in models for channel assignment, and the modelling decisions, results and conclusions of authors who do incorporate this operational aspect.

2.3.1 EVALUATING THE SINGLE INTERFERER ASSUMPTION

2.3.1.1 Evaluation criteria

Several authors evaluate the use of the single interferer assumption by analysing the assignments produced under this assumption against two operational criteria:

▷ Efficiency of spectrum use: Criteria on the spectral requirements of the assignment—the number (order) or span of channels used is to be minimised;

▷ Coverage: Criteria on the performance of the assignment—some measure of coverage (the proportion of the network where satisfactory SIR is experienced) is to be maximised.

When the latter is used, a measure of coverage and a level of coverage which is deemed satisfactory must be defined. Some authors require 100% coverage whist others accept a lower coverage level. For example, Dunkin and Jeavons [9] require total coverage and measure this by requiring the SIR to be above the threshold SIR at all points in the network, whereas Haas, Winters and Johnson [18] deem a coverage of 80% to be satisfactory, measured in terms of the percentage of mobiles (whose locations are assumed to be known and on a uniform grid or uniformly random distribution) with SIR above threshold.

An ideal assignment would have both total coverage and minimal spectral requirements. The simultaneous optimisation of both criteria is not trivial, as the two criteria conflict; use of the spectrum can be optimised by sacrificing coverage and vice versa. A careful and effective balance is desired, and representations which facilitate this.

2.3.1.2 Evaluations in the literature

McEliece and Sivarajan [36, 37] and Sarkar and Sivarajan [30] begin to question the coverage capabilities of the traditional graph coloring model in a series of papers published in the 1990s, comparing a multiple-interferer-based hypergraph model with the traditional single-interferer-based graph-theoretic model as described in section 2.2.2. The comparison is made for both systems with uniform traffic and systems with non-uniform traffic (in which traffic is more dense closer to the centre of the region). The instances used contain regular hexagonal cells, but the authors state that their observations extend to irregular instances. They illustrate that the graph-theoretic model has some inadequacies in relation to channel sharing, but point out that there is a cost in terms of computational complexity when instead adopting a hypergraph model.

Meanwhile in 1993, Carlsson and Grindal [38] note that the performance of a system should be measured by taking into account combined interference from all interferers. However, they choose a model definition which approximates the combined interference by the interference from the strongest interferer (a single interferer assumption). In 1994, Haas, Winters and Johnson [18] also evaluate the single interferer assumption against a coverage criteria. Their work considers a fixed re-use distance model which permits or denies channel sharing depending on distance. The evaluation method involves combining in some way the signal strengths from all co-channel interferers and comparing this with the wanted signal strength to establish whether the QoS threshold is achieved. The probability of a mobile receiving inadequate SIR when co-channel interference is considered is calculated as 0.2 when σ=9dB, 0.1 when σ=10.5dB and 0.05 when σ=12dB. The authors conclude that these probabilities are relatively small and justify the single interferer assumption by stating that any errors due to its use may be considered negligible. When considering the spectral requirements criteria, Haas *et al.* [18] consider the traditional model to have potential inadequacies, stating that "the fixed re-usability factor may be too pessimistic".

Dunkin and Allen [35] and Dunkin and Jeavons [9] present illustrative examples of situations in which binary channel separation constraints based on re-use distance have inadequacies. The authors analyse assignments made using binary channel separation constraints generated under the constant re-use distance model.

These assignments are compared with a 'global constraint' approach, which involves finding good assignments via a naïve backtracking algorithm and evaluating them directly against operational criteria. Total coverage is required i.e. at all points in the network the SIR received must be above a given threshold. The conclusion is drawn that the single interferer assumption is not the best model to use, as equal or greater coverage can be achieved via other methods. The inadequacies observed in the model under investigation are due to its inability to represent different types of co-channel effects other than the simple permission/ denial for sharing via re-use distance. Dunkin and Jeavons [9] suggest the single interferer assumption is abandoned and non-binary constraints are used to represent the problem. Dunkin and Allen [35] compare the 'global constraint' approach with a *tuned* binary constraint representation from Gower and Leese [39] and discover that even these optimised constraints use more spectrum than is in practice necessary to achieve total coverage.

Jeavons, Dunkin and Bater [12] discuss the inadequacies of binary distance constraints using the examples from [35] and an additional finite hexagonal grid instance. Considering the combined effects of multiple interferers, they conclude that the situation requires non-binary constraints which simultaneously constrain the assignment made to more than two transmitters.

In [34], Dunkin *et al.* find that an assignment which satisfies all binary co-channel distance constraints between adjacent cells in fact gives adequate coverage (SIR $> \sigma = 15$dB) at only 68% of points throughout the regular hexagonal network instance considered. The points which receive unsatisfactory SIR are at or near the borders of the hexagonal cells, at points which are approximately equidistant from two or more transmitters. In this paper, the authors again suggest the use of higher order constraints and continue by considering ways of formulating such an approach.

Bater, Jeavons and Cohen [40] optimise sets of binary constraints by finding the minimum re-use distances for adequate coverage, in a similar manner to the tuning performed by Gower and Leese [39]. The calibrated constraints are compared against a global constraint formulation, as in [35], and the discovery made that even these calibrated constraints are not as spectrally efficient as is operationally possible for the instances considered.

Following the work of Dunkin and Jeavons [9], Smith, Allen, Hurley and Watkins

[17] compare binary channel separation constraints generated under the single interferer assumption with a form of non-binary constraint. A constraint-free approach, which involves testing assignments directly against operational criteria, is used to facilitate the evaluation of the results from the different constraint approaches. The evaluation of the binary constraint representation leads the authors to state that "it would be more accurate to consider interference from all potentially interfering transmitters when deciding whether the receiver met the required minimum signal-to-interference ratio" [17], thus advocating the use of a *multiple interferer assumption* to replace the single interferer assumption.

Re-use distance based binary channel separation constraints created under the single interference assumption cannot be used to encapsulate channel loading problems as presented by Hurley *et al.* [41] and Whitaker *et al.* [22]. The constraints created would be either ineffective or inefficient: too small a value for d does not sufficiently limit channel sharing, leading to constraints which are too weak to satisfy the QoS, and excess co-channel interference experienced in the operational situation; too high a value for d introduces constraints which are over-engineered and result in an assignment which uses more channels than needed.

Hurley, Whitaker and Smith [42, 43] investigate the single interferer assumption using instances from the Benchmark Generator of [44], measuring coverage in terms of the percentage of reception points, placed at Voronoi points, falling below threshold SIR. Channel assignments are created using the traditional model and analysed considering multiple simultaneous interferers. For each of the eight problem instances (four geographical instances from the Generator [44], each assigned for two values of σ), coverage is found to be lower than is claimed by the single interferer assumption. Co-channel interferers are found to cause a substantial proportion of the interference experienced.

Due to the potentially high cost of additional spectrum, Montemanni, Smith and Allen [45] motivate the consideration of multiple interference to avoid unnecessary excess channel separation in assignments. In their ANTS algorithm for the minimum span channel assignment problem (section 2.4), they choose to incorporate multiple interference by considering an additive combined interfering signal strength.

2.3.1.3 Summary

Assignments created from binary channel separation distance constraints may have lower coverage than is attainable in other ways, or use more spectrum than is actually required, depending on the re-use model selected. An assignment which is zero-violation, in that it satisfies a given set of binary channel separation constraints, may not be an interference-free assignment when operational criteria are directly considered. The majority of authors who analyse the single interferer assumption conclude that the incorporation of multiple interference effects into the model would have benefits in terms of the operational criteria and should be pursued.

Many authors put forward the suggestion that higher order constraints be used towards this end (section 2.3.2). Other authors suggest alternatives using either binary constraints generated in new, more innovative ways (section 2.3.3) or the avoidance of constraints in favour of constraint-free approaches (section 2.3.4) based on the operational criteria.

2.3.2 USING HIGHER ORDER CONSTRAINTS FOR MULTIPLE INTERFERENCE

2.3.2.1 Higher order constraints suggested in the literature

The majority of authors who consider constraints for multiple interference suggest the use of *higher order constraints* (HOCs), also known as non-binary constraints.

Dunkin and Allen [35] suggest the use of a hybrid constraint representation, which would involve using binary constraints in less dense regions and HOCs in dense areas. They point out that this would, however, require new solution techniques and they do not take the idea forward to the selection of possible constraint formulations. In [12] and [34], Dunkin *et al.* carry forward the suggestions made in their previous publications by beginning to investigate possible constraint representations for the channel assignment problem.

In [12], the analysis of the binary channel separation constraint based on distance leads to the recommendation that HOCs be used to better represent the operational criteria in constraints. They note that, as signal strengths decrease with distance, the effect of an interfering transmitter can be arbitrarily small when further than a chosen distance D away from a serving transmitter. Circles of radius D are constructed around the transmitters, and HOCs formed which constrain the assignment made to all transmitters falling within that circle. The authors briefly describe a possible HOC representation which involves the storage of tuples of channel *offsets* from the central transmitter which are permitted at other transmitters or, more compactly, the storage of lists of minimal offsets. In [34], the authors formulate higher order constraints on subsets of more than two transmitters in two ways: tuples of mandatory separations between transmitters' channels and that of some reference transmitter; tuples of minimal relative separations, formalising the approach suggested by Bater.

The constraints used have the capability to represent co-channel effects that cannot be represented by the traditional model. Consider for example a constraint stating that only three of six transmitters, all equidistant from a central transmitter, may share a channel with that central transmitter. Under the usual distance constraint representation, the binary constraints created between the central transmitter and each of these equidistant transmitters would be identical, meaning that either all six would be permitted to re-use the central transmitter's channel, or none of the six.

Because no software capable of solving their new constraints is available to the authors of [34], they develop their own *high arity constraint solver* which combines simple backtracking with heuristic pruning methods. Coverage is improved, with no detrimental effect on spectral usage. The authors of [43] also discover that introducing non-binary constraints can increase coverage with very little expense in terms of the span.

The non-binary constraints employed in [17], and also in [46], represent different combinations of binary constraints between members of a set of more than two transmitters, whose satisfaction leads to above threshold SIR at a test point in the network when the summed interference is considered. Although Smith *et al.* [17] use a higher order constraint representation for the purpose of analysing the single interferer assumption, they conclude that the introduction of HOCs may not be the way to proceed in practice. Rather, they suggest solving the original binary channel separation distance constraints, then employing *constraint strengthening* to give an improved assignment (section 2.3.3).

The analysis performed by Hurley *et al.* in [42] motivates their introduction of a type of higher order constraint which restricts channel sharing. These *co-channel set constraints* are discussed further in chapter 5. A tabu search is used to solve a combination of binary and co-channel set constraints, and reveals an improvement in coverage with a marginal or no increase on the span of channels. Hurley and Whitaker [41] also motivate the introduction of co-channel set constraints in relation to problems of *channel loading* in private mobile radio (PMR) networks, choosing this particular type of constraint because their use restricts channel sharing.

2.3.2.2 Summary

The majority of authors who consider that multiple interference needs to be considered in the model suggest the use of higher order, non-binary constraints to achieve this. Few authors move beyond the suggestion of HOCs to actual implementation to formulate HOCs and apply them to the channel assignment problem.

HOCs can improve the model for channel assignment by incorporating multiple interference and channel loading effects whilst increasing spectral requirements only marginally. The introduction of higher order constraints can, however, lead to potential difficulties: authors point out that new methods are required to generate the constraints, process them, calculate bounds (section 2.5), and solve to find assignments; and that computational complexity and higher memory requirements for such constraints may be prohibitive [17].

2.3.3 USING BINARY CONSTRAINTS FOR MULTIPLE INTERFERENCE

The majority of authors who consider multiple interference are in agreement that constraints introduced to manage these effects will be non-binary constraints. However, a small number of authors note the possibility of using binary channel separation constraint approaches which take into account multiple interference effects. Binary constraints have the advantages of being simple to generate and tractable, and solution techniques for these constraints are well-known. The use of binary constraints for multiple interference takes two forms in the literature:

(a) binary channel separation constraints are generated under the single interferer assumption and the assignments improved upon taking into account the operational criteria of SIRs which use combined signal strengths;

(b) binary channel separation constraints are generated in a non-traditional manner which allows them to reflect the multiple interference aspects of the operational criteria.

Constraint strengthening, as described in [17] and applied by Watkins, Hurley and Smith in [47], and with Allen in [46], is a method which may improve upon the binary constraint model, without resorting to potentially intractable higher order constraints. The method involves progressively adding selected constraints to the original set of binary channel separation constraints from the traditional model. Each new set of constraints is analysed under the multiple interferer assumption against operational criteria until 100% coverage is achieved. This increase in coverage is achieved at a cost in terms of increased span, emphasising the need for careful balancing of these two criteria.

In a 2002 report, Hodge, Hurley and Smith [13] analyse binary constraints generated by considering at each RTP cumulative interference from

(a) those transmitters that cause inadequate SIR by themselves (*selected interference model*)

(b) all possible interfering transmitters, as suggested in [17] (*complete interference model*)

These models are compared with the single interferer assumption model, evaluating assignments under complete interference.

The selected interference model provides a small increase in coverage eight out of twelve times when compared with the single interferer assumption model, with the lower bound being increased in only three of these cases. However, the coverage increases observed are small and a decrease is observed in one of the twelve cases, so the method provides no guarantees of improvement upon the single interferer assumption model.

The complete interference model guarantees 100% coverage, but at a huge cost in terms of increase in span, especially as the number of transmitters in the test case under consideration increases.

Based on their analyses, Hodge *et al.* suggest, but do not implement, a 'middle-ground' approach, to achieve improved coverage without making unnecessary sacrifices in terms of the span of channels needed. Instead of only selecting interferers that cause interference above threshold on their own, a larger set of interferers would be considered, but of limited cardinality i.e. not as many as in the complete interference model, in the hope of achieving good coverage improvements without requiring extreme increases to span. This interference set would incorporate into the constraint generation those interferers which contribute relatively highly to the overall interference, but are not significant enough to cause failure individually.

The generation of binary constraints for multiple interference in chapter 6 of this thesis successfully provides such a compromise between the single interference and complete interference models.

2.3.4 Constraint free assignment for operational criteria

Smith *et al.* [17] and Dunkin *et al.* [9] create assignments without the use of constraints, via a naive algorithm which makes assignments and checks them directly against the operational criteria—looking for satisfactory SIR at RTPs throughout the network. For example, the cost function in [17] is a sum of (even, integer) powers of the amount by which threshold SIR is missed. This type of assignment has a requirements-based focus, and removes some of the issues pertaining to modelling and abstracting elements of the operational situation. A constraint free approach would also avoid "the need to identify, select and store the non-binary constraints" [17].

These authors use constraint-free assignment purely as a tool for comparison with assignments made from various constraint approaches. They do not suggest its use as a solution technique, mainly due to the computational time involved [9]. However, a constraint-free approach could be advantageous if developed in such a way as to be tractable.

[48] advocates the use of a constraint-free approach, as detailed in [49], as a solution technique for channel assignment. This incorporates multiple interfer-

ence effects without the use of channel separation constraints—neither binary nor higher order. The approach involves the analysis of assignments in terms of the operational criteria under multiple interference, and perturbations of the assignment performed according to the results.

The approach presented in [48] outperforms the traditional graph colouring model, as the optimisation process is solving a more directly representative problem. The assignment process involves SIRs directly, and therefore remains very close to the operational criteria rather than being abstracted.

2.4 SOLUTION TECHNIQUES

This section discusses some of the solution techniques applied to the channel assignment problem, including their applicability to different constraint representations and to the single and multiple interference assumptions. It is not an exhaustive survey of channel assignment solution techniques, but aims merely to exemplify the solution techniques available. Aardal *et al.* [32] evaluate the field and provide an in-depth survey of the models and solution techniques available, performed under the DONET project [50] and based in part on the work of Koster [15]. Various aspects of current research in the field of radio channel assignment are explored in [51], which includes chapters on the development of state-of-the-art computational algorithms for channel assignment.

Many computational solution techniques for channel assignment are suggested, applied and analysed in the literature; some are designed specifically with channel assignment in mind and others originate in different fields of research. For example, the analogy between channel assignment and graph theory means that many results and algorithms from that field can be applied almost directly to solving channel assignment problems (see e.g. [24, 28, 52, 53, 54, 55]). Methods from mathematical programming have also been successfully applied to the channel assignment problem, often for bounding techniques, by many authors (e.g. [15, 33, 44, 56, 57, 58]). Borndörfer *et al.* [59], for example, develop several heuristics for use by the German telephone system provider *eplus*, and draw their algorithmic ideas from both mathematical programming and the T-colouring of graphs.

Solution techniques for channel assignment include a number of non-exhaustive search techniques, employed due to the complexity of the problem, which is known to be NP-hard [16, 33, 60]. These methods of solution must be tailored to the problem under consideration, by careful choice of cost functions and objectives for optimisation.

2.4.1 SEQUENTIAL METHODS AND ORDERING OF VARIABLES

In the 1960s and 1970s, exact methods and sequential methods for channel assignment were developed (see e.g. [24, 53]). However, as the channel assignment problem is NP-hard, exact methods are only feasible on small problem instances and are therefore not used in practice. Leung [61] successfully applies the idea of partial backtracking to channel assignment, but this type of algorithm is much less flexible than a good heuristic [17].

Sequential methods use simple greedy functions to build a reasonable solution iteratively. These methods have the advantage of being easy to implement but must be designed for each specific problem and can often result in a poor-quality solutions as they are myopic. Sequential algorithms for channel assignment assign channels with no reassignment possible, using different orderings of transmitters and channels. A reasonably good assignment can sometimes be achieved using a sequential algorithm, but in general assignments created in this way are used as starting positions to be improved upon by (meta-)heuristic algorithms [17].

The current alignment of the UHF 2 band in the UK, licensed by the former Radiocommunications Agency, suffers from inefficiencies because the assignment procedure was essentially sequential. Having evaluated the current operational assignment, Whitaker, Hurley and Smith [43] conclude that heuristic methods should be employed for significant improvement, in terms of reducing the number of channels without reducing QoS.

For (meta-)heuristic algorithms (section 2.4.2) as well as for sequential methods, the ordering of the entities to be considered is often crucial to the success of the approach. Possible such orderings of transmitters and channels were introduced by [62]. In fact, it has been proven [63] that sequential techniques can always obtain a minimum span (optimal) solution from binary channel separation constraints, when provided with the appropriate ordering in which to perform the assignment.

Dunkin and Allen [35] provide a brief example showing the influence that the order in which transmitters are considered can have on the results obtained. Sivarajan, McEliece and Ketchum [52] exploit this concept by ordering cells and calls each in two different ways and applying two different search strategies, thus providing eight different frequency assignment algorithms. As these algorithms are fast, each running in time $O(n^2)$, the authors apply all eight to a solution instance and can then select the best from the varying results. It is not always the same version which gives the best results, but at least one of them gives a solution equal or close to the lower bound for each of the tests performed.

2.4.2 NEIGHBOURHOOD SEARCHES AND META-HEURISTICS

A neighbourhood search or local search method starts from some initial solution and moves to a better neighbouring solution until it arrives at a solution which has no better neighbour—a local optimum. Such searches are easy to implement and usually reach local optimality in a short computational time, but simple such searches can result in poor quality by being unable to escape from, and terminating at, local optima which are not global optima.

To implement a local search, a neighbourhood function is required. This is usually defined by the set of solutions generated by a *move* which changes one or more attributes of the current solution. A neighbourhood must be chosen in such a way as to be capable of leading to good solutions without making the search to complex—a large neighbourhood is hard to explore. A strategy via which to then search the neighbourhood is required, along with an evaluation function which will determine how good a solution has been located and an acceptance criterion which will decide whether a neighbourhood solution should replace the current solution. The problem of getting trapped in local minima may be overcome by repeating the method many times with different starting conditions.

An example of a simple neighbourhood search algorithm is hill-climbing. This starts from a random assignment in the solution space and replaces that assignment by a neighbouring assignment if this move reduces the cost function. This process is continued until all neighbouring assignments have higher cost function values.

Meta-heuristic techniques [64, 65] are approximate methods, designed to solve difficult combinatorial optimistation problems by finding good solutions from large search spaces. Meta-heuristic algorithms use problem-specific knowledge to guide an underlying heuristic/local search, allowing it to escape from local optima and to explore other (better) areas of the solution space for global optima. When designing a meta-heuristic algorithm, the choice of neighbourhood and the choice of cost function must be carefully made. If meta-heuristics are to be effective in the evaluation of large search spaces, they require cost functions which can be quickly evaluated [17].

The most widely used meta-heuristic techniques are tabu search, genetic evolutionary algorithms and simulated annealing. The meta-heuristic algorithms which have been used in the attempt to find the best possible channel assignment for a given network of transmitters are briefly described in [17], compared in [66] and more recently reviewed in [51, chapter 3] and [67, 68]. Meta-heuristics for channel assignment are tailored to the type of problem under consideration by defining objectives either against the operational criteria or against the constraints being used.

A (meta-)heuristic algorithm needs an initial solution from which to begin its search. This could be generated randomly, or from a sequential method, or by creating a partial assignment. For example, [58] uses a sequential method to create a starting configuration, and Dunkin and Allen [35] begin by assigning difficult subgraphs (cliques), fixing the assignment to these transmitters, and then assigning the whole graph via a heuristic. Smith, Hurley and Thiel [69] investigate the potential of assigning subgraphs first in this manner, pointing out the advantages and limitations and applying the technique to produce the first optimal solutions to some Philadelphia instances [70].

Tabu search, derived from the ideas of Glover in the 1970s and 1980s and described by Glover, Taillard and Werra in [71], exploits forms of flexible memory to control the search process. From the current solution, it moves to the best admissible solution in the neighbourhood, taking into account tabu restrictions and aspiration criteria. Tabu restricted moves are those which form part of a set of moves which are forbidden, dependent on the short- and long-term history of the states previously encountered. So that tabu restrictions may be overridden in certain circumstances, such as when allowing the move would obtain the best solution found so far, aspiration criteria are employed.

Tabu search was first used for frequency assignment by Lanfear [55], who used it to solve a graph colouring model for radio relay networks. Boyce, Dimitropoulos, Scheidt and Taylor [72] show that tabu performs best on small tightly constrained problems. Tabu search has problems when used on large problems, as neighbourhood evaluations have to be re-calculated at each iteration, exemplifying and confirming that a cost function which can be quickly evaluated is essential [17], however it is robust and can be easily tuned [73]. Hao, Dorne and Galinier [74] present a powerful and competitive tabu search, suggesting that parallel and distributed versions of the tabu search be used to overcome the difficulties of optimising large networks. Castelino and Stephens [75] present an efficient and effective variation on tabu thresholding, which differs from tabu search in being less dependent on memory structures. Smith, Taplin and Hurley [68] apply tabu search to problems consisting of several types of constraints and find that this method, with a starting assignment generated by a sequential algorithm, is fast and effective, noting that a fast method of evaluating assignments is essential. Montemanni, Moon and Smith [73] apply tabu search to fixed spectrum channel assignment problems, improving upon the best known assignments for some of the COST 259 instances [76]. These authors concur with the general opinion that tabu search, aside from its limitations in terms of solution speed, is the most effective algorithm for channel assignment problems.

Evolutionary algorithms mimic biological processes concerning life, growth, survival, evolution and natural selection. These algorithms start from a population of candidate solutions and generate new solutions via recombination (a new solution from a pair of solutions in the current population) and mutation (a new solution from one solution in the current population), which are then selected by use of a fitness function.

Crompton, Hurley and Stephens [77] use a genetic algorithm which is parallelised, with different nodes running their own genetic algorithms and occasional exchanges of chromosomes with other parallel genetic algorithms taking place. In a variation on the use of sequential methods to find a starting configuration for meta-heuristic algorithms, Hurley, Smith and Valenzuela [19] use a permutation based genetic algorithm to find an ordering of transmitters which is then used in a sequential assignment technique, equalling the contemporary performance of tabu search and simulated annealing based techniques.

Simulated annealing is derived from statistical mechanics and is analogous to the way metals cool and anneal in thermodynamics. The application of these ideas to optimisation problems was first documented by Kirkpatrick, Gellat and Vecchi [78]. The parameters used by a simulated annealing algorithm must be finely tuned to achieve the best results.

Several authors have since applied simulated annealing to optimisation in the channel assignment problem, formulating cost functions from the operational criteria and carefully designing neighbourhoods to maximise solution quality while keeping run time reasonable [79, 80]. Hurley, Thiel and Smith [66] compare simulated annealing, tabu search and genetic algorithms for a set of realistic instances and gain the best results in terms of numbers of constraints violated from the simulated annealing approach.

2.4.3 Solutions techniques for multiple interference

Solution techniques which are available or suggested for multiple interference in the literature include: those which are are not designed for, but can be applied to, higher order constraints; those which are designed to solve higher order constraints (see also section 2.3.2); and those which are designed with the operational criteria of multiple interference directly in mind (see also sections 2.3.3 and 2.3.4).

The meta-heuristic algorithms described in [17] for binary constraints can be, and are, used with types of non-binary constraints. The authors state that usually all the adaptation required is a change to an appropriate cost function. In [34], discussed in section 2.3.2, the authors develop their own high arity constraint solver, for the purpose of creating assignments which consider operational criteria, but they do not suggest its use as a solution technique, because of the computational time required. [43] documents a *non-binary solver* (NBS) which extends the capabilities of FASoft [49], a frequency assignment software package for binary constraints. This NBS uses tabu search techniques to solve co-channel set constraints and deal with constraints which have weights associated with their violation. If multiple interference is incorporated into the model via the use of co-channel set constraints (chapter 5) then the hypergraph model and analogy of [30] means that literature from hypergraphs can be used for channel assignment for multiple interference.

The constraint-free assignment approaches of [48] and [47] (section 2.3.4) use an algorithm based on simulated annealing, in which neighbouring assignments are generated by randomly changing the assignment made to the transmitter serving the RTP receiving the worst SIR under the current assignment. [47] applies this approach iteratively in the attempt to find a minimum span assignment—each time attempting to find a satisfactory assignment when the span of channels permitted is reduced by one.

Using the terminology of Lau and Leung [6], who also note that the single interferer assumption results in an optimistic model, Capone and Trubian [5] apply a tabu search meta-heuristic to a model for channel assignment which incorporates multiple interference. Capone and Trubian note that whilst such a multiple interference model increases problem dimension and model complexity, it is necessary towards the aim of overcoming the drawbacks of the classical approach and achieving the channel assignment objective of maximum system capacity.

Montemanni, Smith and Allen [45] develop a heuristic for the minimum span problem which considers multiple interference in its design. Their method is inspired by the way that ant colonies function and provides promising results which in many cases outperform algorithms previously presented in the literature.

2.5 BOUNDS

Upper and lower bounding techniques are used to find theoretical bounds against which assignments may be evaluated to discover their proximity to optimality. Bounding techniques are also employed to establish the effectiveness of new algorithms and techniques, providing a more valuable evaluation than the simple comparison of different results against one another, and indicating whether further improvement is possible. If a lower bound and an upper bound can be proven which coincide, then a description of an optimal situation is provided. However, the methods may not provide detail of how this optimality should be achieved in practice.

For example, the employment of channel re-use means that for any assignment, the number of transmitters is a loose upper bound on the number of channels used in the assignment. A simple upper bound U_B on *spn* can be found by making a satisfactory assignment in $U_B + 1$ channels.

Lower bounding techniques for the minimum span of assignments satisfying binary channel separation constraints are well-known [17, 81] and techniques based on the constraint graph formulation or using mathematical programming techniques are presented in [35, 44] and [51, chapter 4]. When a lower bound on the minimum span and the span of an assignment for the same problem are equal to one another then optimality has been achieved. However, when a lower bound and an assignment span differ, it is difficult to know whether it is the bound or the assignment span achieved which is weak [17], and more work is needed to assess the quality of the bounding and assignment techniques. Allen, Smith and Hurley [82] provide algorithms which generate lower bounds and can effectively assess the quality of assignments made by other methods such as meta-heuristic algorithms.

Lower bounding techniques for fixed span problems are less well-studied and more difficult to derive [17, 44]. The bounds presented in the literature, most of which are suitable for specific types of problem and not for general purposes, are listed in [83] and reviewed in the doctoral thesis of Montemanni [57], who goes on to contribute some novel lower bounding techniques which estimate global lower bounds for this problem from local lower bounds of subproblems. Montemanni, Smith and Allen [83], building on their previous work [84], have produced a technique to quickly predict quality lower bounds for the fixed span problem, even for large instances. The authors suggest that variations on the technique be considered, to optimise the results possible for different types of problem.

Bounds for higher order constraints remain open. Hurley, Whitaker and Smith [41] propose a bound for the *co-channel set constraint* which is presented in section 5.7. The applicability of known bounds when multiple interference is considered is briefly explained in section 3.1.3.3.

2.6 Conclusions

Many very good high performance solution techniques are available, especially for use with binary constraints. However, the model to which the solution technique is applied may be improved, for example by the abandonment of the single interferer assumption in favour of the consideration of more realistic multiple interference effects.

Several authors question the traditional graph-theoretic constraint model based on re-use distance. The re-use distances used lead to either lower performance systems (in terms of coverage) or higher spectral usage than the best known assignments. Other methods can produce assignments which provide a better balance between the two criteria of maximising coverage and minimising spectral requirements.

Few authors have considered multiple interference and a recent summary of the situation [45] confirms that work which considers multiple interference remains scarce. Of those authors who do evaluate the effect of multiple interference, all but one conclude that this should be incorporated in some way into a model for channel assignment. The majority of authors who suggest a multiple interference assumption extend their conclusions to suggest that HOCs are used for this purpose.

Haas, Winters and Johnson [18] calculate that up to 20% of mobiles may experience inadequate interference when co-channel interferers only are considered and use their data to justify the single interferer assumption. The data given may in fact lead to the opposite conclusion, depending on the level of coverage deemed acceptable.

The chapters which follow analyse the single and multiple interferer assumptions, including the sources of multiple interferer effects, and illustrate the use of HOCs. The investigation aims to better inform models and solution techniques for channel assignments including the formulation of constraints which both maintain tractability and improve accuracy.

Chapter 3

Assumptions for Modelling, Problem Generator and Methods of Assignment

3.1 Assumptions for modelling channel assignment with multiple interferers

To facilitate further investigation into multiple interference effects, this chapter firstly presents the assumptions and definitions used in the remainder of the thesis. Different aspects of modelling channel assignment are included and discussed as follows:

- ▷ Network assumptions: the geography of transmitters, receivers and interferers;
- ▷ Channel assignment assumptions: the adaptation of the general formulation of the channel assignment problem in section 1.4 for use in this thesis;
- ▷ Signal and interference assumptions: assumptions pertaining to propagation and SIR calculations;
- ▷ Objective assumptions: the qualities required of a channel assignment and how these are assessed.

3.1.1 NETWORK ASSUMPTIONS

A radio communications network can be modelled in general by a region in which are a set of transmitters and a set of receivers. A transmitter is defined by the following data: its geographical position; the channel it is using for transmission; the power of the transmission; the directional distribution of the transmission. A *reception test point* (RTP) is a point in the region at which a receiver could potentially be located, and at which signal strengths can be measured or calculated. An RTP is defined by two items of data: its geographical position; its serving transmitter. RTPs may share a location but these will be distinguishable by their differing serving transmitters.

The following assumptions, similar to those made in e.g. [12, 17, 40], are made about the network:

(a) the geographical position of each transmitter is fixed and known;

(b) a transmitter uses a single channel for transmission;

(c) all transmitters use the same transmission power;

(d) transmitters are omnidirectional i.e. their signal has a uniform directional distribution;

(e) the geographical position of each RTP is fixed and known (the locations and properties of the RTPs used in this thesis are further explained in section 3.3);

(f) at an RTP, all transmitters other than the serving transmitter are considered to be interferers;

(g) the network is situated on a two-dimensional plane in which factors such as terrain, clutter and the environment are not modelled, except to the extent that this is incorporated into the propagation model chosen.

3.1.2 CHANNEL ASSIGNMENT ASSUMPTIONS

A bi-directional (dual) communication link requires one channel for downlink and a second channel for uplink. If downlink transmission takes place using a particular radio channel f_i, then it can be assumed [41] that the channel used for uplink is $f_i + K$ for some fixed constant K which is large enough that the sets of channels used for downlink and uplink respectively do not overlap. This assumption is made throughout the relevant literature. Assignment of a downlink channel to the transmitter only is considered, as the uplink assignment is simply an offset copy of the same assignment, translated through K. This effectively halves the number of assignments which need to be made.

In this thesis, it is assumed that:

(a) t_i is a transmitter being assigned channel f_i for downlink;

(b) each transmitter requires one channel [35] i.e. the demand vector C has $c_i = 1$ $\forall\ i$ (complying with the network assumption that transmitters use a single channel for transmission);

(c) the channel assigned to transmitter t_i is selected from among all the channels in the network's allocation i.e. the domains are $D_i = F\ \forall\ i$;

(d) channels are numbered consecutively beginning at 1 i.e. $F = \{1, \ldots, k\}$.

3.1.3 SIGNAL AND INTERFERENCE ASSUMPTIONS

3.1.3.1 Propagation model

The propagation model chosen here is that advanced by for example Gower and Leese [39] and Wang and Rappaport [85]. This model is deemed sufficient by Hodge *et al.* [13] for work of a similar type to that presented in this thesis. This model could easily be replaced by another propagation model, however the choice of RTPs representative of the network would have to be reassessed (section 3.3.2.2).

If an RTP r_i is served by transmitter T_k then the wanted signal strength S_i at r_i is assumed to be given by

$$S_i = \frac{P_k}{(d_{ik})^\gamma} \tag{3.1}$$

where P_k is the power at which T_k transmits, d_{ik} is the distance between receiver r_i and transmitter T_k, and $2 < \gamma \leq 4$. The parameter γ represents the attenuation of signal strengths with distance, and is usually given a value in the upper region of the stated range, e.g. $\gamma = 3.8$ or $\gamma = 4$, to give a sufficiently steeper signal strength fall off than free space loss.

Similarly, the interfering signal strength from the non-serving transmitter T_h is

$$I_{ih} = \frac{P_h}{(d_{ih})^\gamma}\theta \qquad (3.2)$$

where

$\theta = 10^{\frac{-\alpha(1+\log_2 df)}{10}}$ if $df \neq 0$ (adjacent channel)

$\theta = 1$ if $df = 0$ (co-channel)

df is the channel separation between the serving transmitter and the interfering transmitter; α is an attenuation factor for adjacent channel interference.

Values of the parameters used in the propagation model are selected to be: $\gamma = 4$ and $\alpha = 15$. This choice of values is used frequently in the literature e.g. [17, 39, 43, 45]. Tabulating the factors by which received signal strength reduces as channel separation increases, for this propagation model and those used by different operators (who request anonymity), as in table 3.1, shows that this choice of model is appropriate.

Channel	Attenuation in dB		
separation	Model	GSM Operator 1	GSM Operator 2
0	0.00	0	0
1	15.00	9	18
2	30.00	41	50
3	38.77	48	58
4	45.00	–	–
5	49.83	–	–

Table 3.1: Signal attenuation with channel separation

3.1.3.2 SIR with multiple interference

In this thesis it is assumed that multiple interfering signal strengths (in Watts) are summed to give the total interfering signal strength to be used in the SIR. This choice is consistent with that made in for example [12, 43, 45, 86, 87] and is a conservative choice of method due to the way that electromagnetic waves interact with one another.

When multiple interference is considered, the total interference at RTP r_i, tuned to transmitter T_k, is then given by

$$I_i = \sum_{\substack{j=1 \\ j \neq k}}^{n} \frac{P_j}{(d_{ij})^\gamma} \theta_j \tag{3.3}$$

where n is the number of transmitters and the attenuation θ is determined for each individual interferer.

Noise which is external to the system is ignored in this thesis and the interference effects modelled are caused only by other transmitters within the network. This choice is often made, although noise may be considered; Katzela and Naghshineh [86], for example, add an additional constant term N_0 to the denominator of the calculation to represent environmental noise.

The assumption of equal transmission powers means that powers cancel out in the SIR calculations. Hence, incorporating the assumptions and chosen propagation model parameters, the SIR at RTP r_i, served by transmitter T_k, is calculated via

$$\frac{S_i}{I_i} = \left[(d_{ik})^4 \sum_{\substack{j=1 \\ j \neq k}}^{n} \left(\frac{1}{(d_{ij})^4} \theta_j \right) \right]^{-1} \tag{3.4}$$

This ratio of Watts is then converted to decibels for comparison with the threshold SIR in use.

3.1.3.3 Lower bounds

If σ is the threshold value in dB with which the SIR can be compared when converted to dB, then the equivalent threshold value with which the Watts ratio may be compared directly is

$$10^{\frac{1}{10}\sigma}$$

Denoting this value ψ, it is necessary for satisfactory SIR that

$$\frac{S_i}{I_i} \geq \psi \qquad \text{so} \qquad I_i \leq \frac{S_i}{\psi}$$

Clearly if

$$\sum_{\substack{j=1 \\ j \neq k}}^{n} \frac{P_j}{(d_{ij})^{\gamma}} \theta_j \leq \frac{S_i}{\psi}$$

then, because each term in the summation is positive, it must be the case that

$$\frac{P_a}{(d_{ia})^{\gamma}} \theta \leq \frac{S_i}{\psi} \tag{3.5}$$

for each single interferer a. This means that an assignment which gives adequate SIR at all RTPs under the multiple interferer assumption will also satisfy the binary co-channel separation constraints generated using threshold σ, the same set of RTPs and the single interferer assumption. This in turn means that lower bounds formulated using the traditional binary constraints still apply for use with multiple interference, as noted in [46, 47]. The proximity of the lower bound to optimality is discussed in [46] by Smith, Allen, Hurley and Watkins, who go on to introduce techniques for the improvement of these bounds.

3.1.3.4 Service and interference ranges

It is assumed that when a wanted signal has reduced in signal strength by some amount, due to propagation losses, the signal becomes indecipherable to the receiver, even in the absence of any interferers. The received signal strength required for a serving signal to be decipherable is assumed to be operator-defined in a particular network.

This effect is represented in the modelling assumptions by defining a *service radius*, outside of which a receiver cannot successfully receive a signal. Due to the assumption of uniform directional propagation distribution, the distance between a transmitter and the edge of its service radius is then a constant, abbreviated by SR and illustrated in figure 3.1.

Interference effects may still be experienced further than the service radius distance away from a transmitter, although these will be small due to attenuation with distance and will likely contribute little to the combined interfering signal strength. No limit on the interference range is given.

Due to the assumption of equal power and the choice of a uniform directional propagation distribution, the service radius and transmitter power are effectively

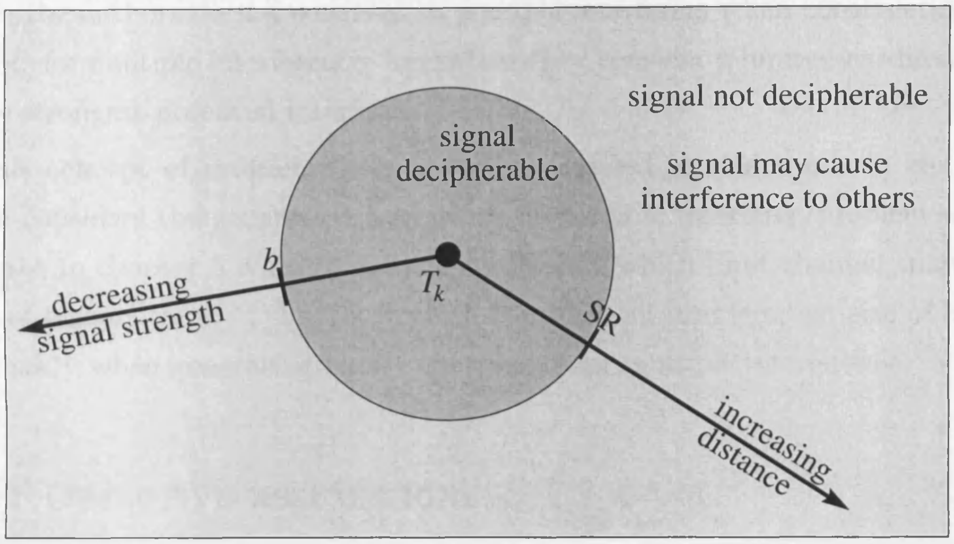

Figure 3.1: Service and interference ranges

homologous. This means that power is included implicitly in the model, even though a value is not explicitly given due to the assumptions and the cancellation of powers in the SIR calculation. If the received signal strength required for a serving signal to be decipherable is denoted b (figure 3.1) then the relationship between the service radius and the power of the serving transmitter T_k is given by

$$P_k = b(\mathrm{SR})^4 \tag{3.6}$$

Note that this idea is not homologous with the constant re-use distance d. Although the service radius considered is a constant distance, channel re-use depends on calculations (e.g. SIRs) which involve more than two transmitters and the underlying re-use model is therefore more complex.

Jeavons, Dunkin and Bater [12] use a concept of interference range when suggesting a method for construction of higher order constraints for multiple interference. They define a distance outside of which the interference from a particular transmitter is considered negligible. A circle of this radius is constructed around a given central transmitter, and all transmitters which lie within this circle are made into a local region. The number of transmitters appearing together in a local region varies from 2 to 6. HOCs, of arity between 2 and 6, are generated within all these local groups of transmitters, leading to a set of HOCs for the whole network.

Hence the authors do not consider all possible interferers when constructing constraints for multiple interference, but rather they consider a limited-cardinality set of the strongest potential interferers.

This concept of interferer sets is carried forward in this thesis in chapter 4 which considers the generation by various methods of interferer/problem sets for analysis; in chapter 5 which considers constraints which limit channel sharing by a set of transmitters; in chapter 6 which considers an interferer set size of limited cardinality when generating binary constraints for multiple interference.

3.1.4 OBJECTIVE ASSUMPTIONS

The objective adopted in this thesis is as follows:

> *Frequencies should be assigned such that the assignment is interference-free when multiple interference is considered and network service coverage is evaluated. The assignment process should attempt to minimise the span of channels used in the resultant assignment.*

Network service coverage represents the proportion of user locations in the network at which operational criteria are satisfied. The choice is made here to require 100% coverage, a choice made by the authors of e.g. [9, 42, 43]. Coverage is measured in terms of SIRs: the SIR as calculated by equation (3.4) (converted into dB) must be above the QoS threshold value σ throughout the network i.e.

$$-10\log_{10}\left((d_{ik})^4\sum_{\substack{j=1\\j\neq k}}^{n}\left(\frac{1}{(d_{ij})^4}\theta_j\right)\right)\geq\sigma\quad\forall i \tag{3.7}$$

3.2 IMPLEMENTATION SPECIFICATION

The computer used to run all programs is an Intel Pentium 4 3GHz IBM compatible PC with 1GB of RAM running Windows XP Professional and Sun's Java SDK, version 1.4.2. All algorithms presented in this thesis and other programs used in the course of the work are implemented in Java. Any exceptions to this are stated.

3.3 GENERATION OF PROBLEM INSTANCES FOR CHANNEL ASSIGNMENT

This section presents the creation of a new problem generating program which can produce many useful test cases in reasonable time. This is then used to generate a library of test problems of the type required for the investigations which follow.

Six test problems, consisting of transmitter and RTP locations have been made available from [44], with 8, 15, 27, 45, 95 and 458 transmitters respectively. To successfully analyse assignments, statistically validate results and draw conclusions in relation to multiple interference effects, it is necessary to consider results over many assignments. Hence, many network scenarios are desirable, from which constraints and/or assignments may be produced. Other benchmark instances for channel assignment are available, but these are not suitable here for various reasons. For example, the Philadelphia minimum span instances, which were among the earliest to be discussed [70], consider twenty-one cellular phone base station sites around the city of Philadelphia, modelled on a hexagonal grid. Each site demands a certain number of frequencies and constant re-use distance values are employed. Further instances can be generated from these by introducing different demand vectors, as was done in [52]. However, the network assumption that all transmitters require a single channel means that this idea is not applicable here. The regular hexagonal nature of the problems is another limitation, as more realistic transmitter distributions need to be considered. Several of the other benchmark instances available are unsuitable because they do not allow for the generation of constraints by different methods or the consideration of operational criteria directly. The GRAPH benchmarks [90], for example, involve sets of constraints generated directly, without considering an operational network scenario. A CSP exercise is not required here, but rather it is necessary to see the relationship between the results and the network situations to which they relate, as the investigation is into multiple interference which is a phenomenon present in the operational system. The use of geographical instances means that different constraint generation methods can be considered and compared.

The test problems being generated are of a type similar to that available from [44] and used in [17, 46]. The instances firstly consist of transmitter locations on the two-dimensional network plane. Given such a set of transmitter locations, corresponding RTPs are then placed. These have locations in the plane and, in addition, a serving transmitter identified from the set of transmitters. Each RTP is assumed to be served by the geographically closest member of the transmitter set. The theory and algorithms for different methods of transmitter placement are presented and an RTP set is produced which contains service points in the network at which interference problems are likely to be encountered by users. A library of test cases classified by the type and density of the transmitter placement is produced for use in the channel assignment problem.

3.3.1 Transmitter locations

Transmitter locations are selected within a square region on a two-dimensional plane in three distinct ways, which may approximate ways in which transmitters and cells could be geographically dispersed:

(a) uniform square grid (a regular, somewhat artificial test problem);

(b) pseudo-random, uniformly distributed (to represent areas of evenly spread population);

(c) pseudo-random, becoming more dense towards the centre (to represent urbanisation).

Each of these methods is detailed in this section.

In addition, the program can superimpose, for example, a centred 'town' onto a random or uniform grid, as well as 'stitch' together 'patches' of these different types to represent a wider area containing differing population distributions.

3.3.1.1 Square grid

The algorithm is given parameters `side` and g, both in metres, and produces a square grid accordingly, with this `side` and in which g is the minimum distance between two grid-points. The first transmitter location is placed at $(0, 0)$ and a grid can be produced with or without points on the two far boundaries when

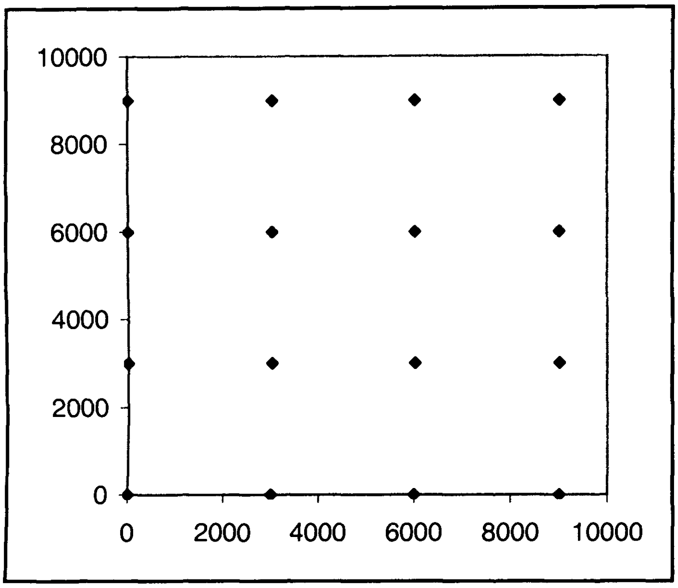

Figure 3.2: Regular grid with `side`=10000m, g=3000m

$g|$`side`. For example, the parameter pair input `side`=10000m and g=3000m would produce the points plotted in figure 3.2.

The parameter g used by the algorithm and the density are homologous in the following way:

$$\text{mean density} = \frac{10^6}{g^2} \text{ trans. per sq. km} \tag{3.8}$$

3.3.1.2 Uniformly random

The algorithm is given values for `side` and g once again. It produces the same number of points as would appear in a uniform grid with the same parameters, but with x and y co-ordinates each produced by a uniformly distributed random number generator. This allows for specific results to be reproduced by use of the same recorded seed. An example of random transmitter locations using the same parameters as those in figure 3.2 can be seen in figure 3.3.

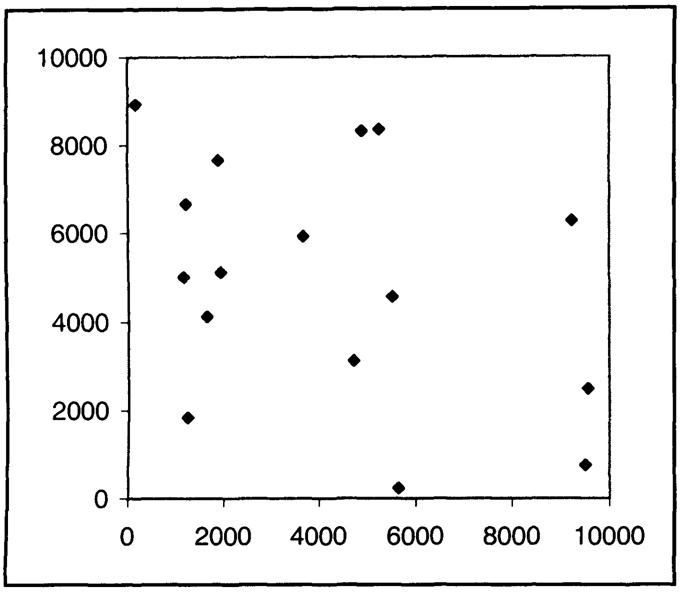

Figure 3.3: Uniformly random with side=10000m, g=3000m

3.3.1.3 Centred

The same number of points are produced as in the previous two methods. A random point is generated then accepted or denied with a probability dependent on the distance d of the point from the centre of the square region under consideration. The probability is given by

$$\exp\left(-\frac{d^2 p}{\texttt{side}^2}\right) \tag{3.9}$$

where p is a parameter which can be varied to give a range of results between 'random-looking' and 'very centred'.

The probability that a point at distance d from the centre is accepted is given by

$$\exp\left[-p\left(\frac{d}{\texttt{side}}\right)^2\right]$$

Because this involves the ratio $d/\texttt{side}$, the distribution of points is relative and scalable i.e. a 10m square region will 'look like' a 10km square region.

For illustrative purposes, the probabilities of a transmitter location being accepted at certain positions are now considered. These positions are shown in figure 3.4, where C is the centre of the square.

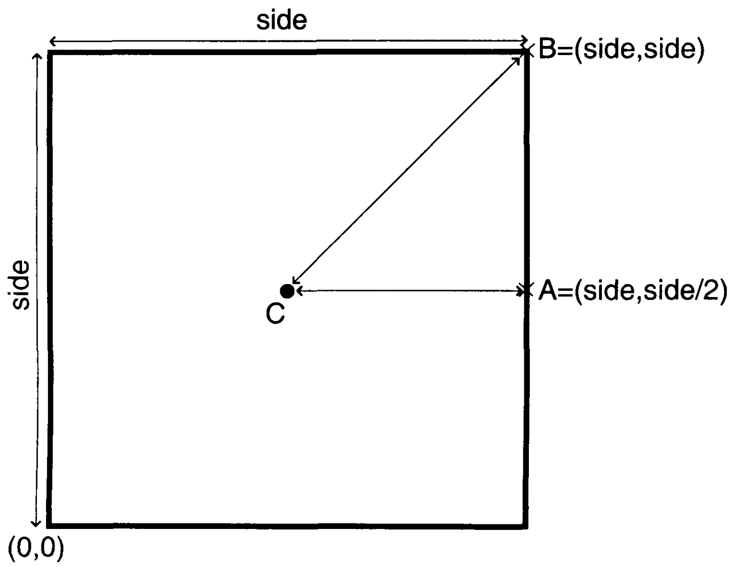

Figure 3.4: Accepting certain locations

Firstly if a location is generated at the centre itself, it will be accepted with probability $\mathrm{P(C)} = e^0 = 1$ regardless of the value of p.

In the diagram of figure 3.4,

$$\begin{aligned} \mathrm{AC} &= \frac{\texttt{side}}{2} \\ \mathrm{AC}^2 &= \frac{\texttt{side}^2}{4} \end{aligned}$$

giving the probability of acceptance at A to be

$$\begin{aligned} \mathrm{P(A)} &= \exp\left[-\frac{\mathrm{AC}^2 \times p}{\texttt{side}^2}\right] \\ &= \exp\left[-\frac{p}{4}\right] \end{aligned}$$

Similarly

$$\begin{aligned} \mathrm{BC}^2 &= \mathrm{AC}^2 + \mathrm{AB}^2 \\ &= 2\left(\frac{\texttt{side}}{2}\right)^2 \\ &= \frac{\texttt{side}^2}{2} \end{aligned}$$

and

$$\begin{aligned} \mathrm{P(B)} &= \exp\left[-\frac{\mathrm{BC}^2 \times p}{\mathtt{side}^2}\right] \\ &= \exp\left[-\frac{p}{2}\right] \end{aligned}$$

Table 3.2 shows the probabilities of acceptance (to four decimal places) for sample values of p, and figure 3.5 illustrates the distribution for $p = 15$, the value chosen for problem generation in the library presented in section 3.3.4.

p	P(A)			P(B)		
5	$e^{-\frac{5}{4}}$	=	0.2865	$e^{-\frac{5}{2}}$	=	0.0821
10	$e^{-\frac{10}{4}}$	=	0.0821	$e^{-\frac{10}{2}}$	=	0.0067
15	$e^{-\frac{15}{4}}$	=	0.0235	$e^{-\frac{15}{2}}$	=	0.0006
20	$e^{-\frac{20}{4}}$	=	0.0067	$e^{-\frac{20}{2}}$	=	0.0000
25	$e^{-\frac{25}{4}}$	=	0.0019	$e^{-\frac{25}{2}}$	=	0.0000

Table 3.2: Effect of parameter p on acceptance

Finally, for any point whose Cartesian co-ordinates are (x, y),

$$\begin{aligned} d^2 &= \left(x - \frac{\mathtt{side}}{2}\right)^2 + \left(y - \frac{\mathtt{side}}{2}\right)^2 \\ \mathrm{P}((x,y)\ \text{accepted}) &= \exp\left[-\frac{p\,d^2}{\mathtt{side}^2}\right] \\ &= \exp\left[-p\left(\left(\frac{x}{\mathtt{side}} - \frac{1}{2}\right)^2 + \left(\frac{y}{\mathtt{side}} - \frac{1}{2}\right)^2\right)\right] \\ &= \exp\left[-p\left(\frac{x^2+y^2}{\mathtt{side}^2} - \frac{x+y}{\mathtt{side}} + \frac{1}{2}\right)\right] \end{aligned}$$

The generation of transmitter locations by this method can be seen in algorithm 3.1, and a sample of the locations produced in figure 3.6. (Lines 1–3 of the pseudocode may be included or omitted according to whether points on the far boundaries are required.)

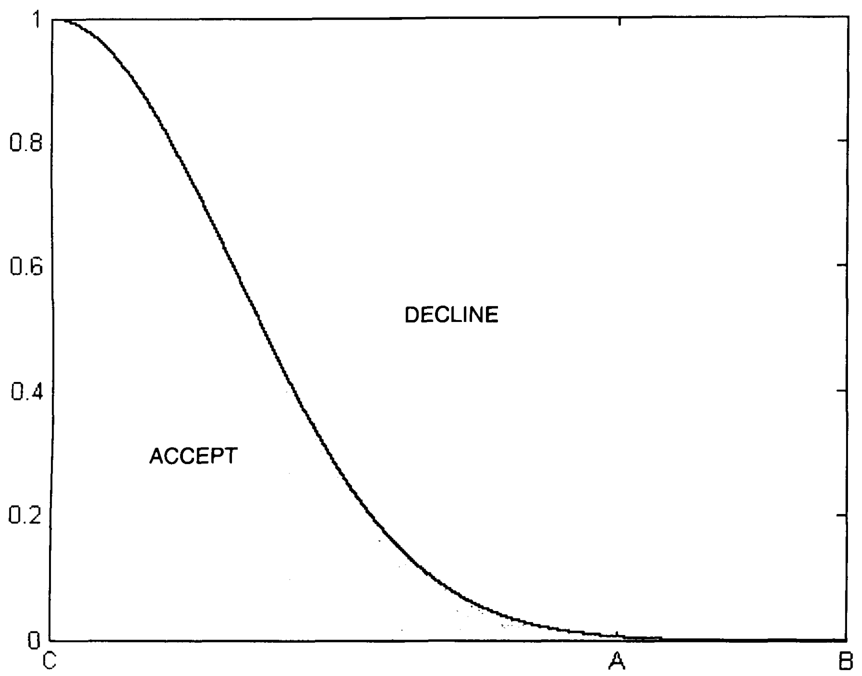

Figure 3.5: Effect of distance from centre on acceptance, p=15

```
if g is factor of side then
    Set side ← side-1 (to avoid points on far boundaries)
end if
Set numPoints ← (side/g + 1)^2
Set numFound ← 0
Set p ← 1.5
while numFound<numPoints do
    Generate a random point (x, y)
    Set d ← distance of point from centre
    Generate random number rand between 0 and 1
    if rand < exp(− p d^2 / side^2) then
        Accept the point
        Increment numFound
    end if
end while
```

Algorithm 3.1: Algorithm to generate centred locations

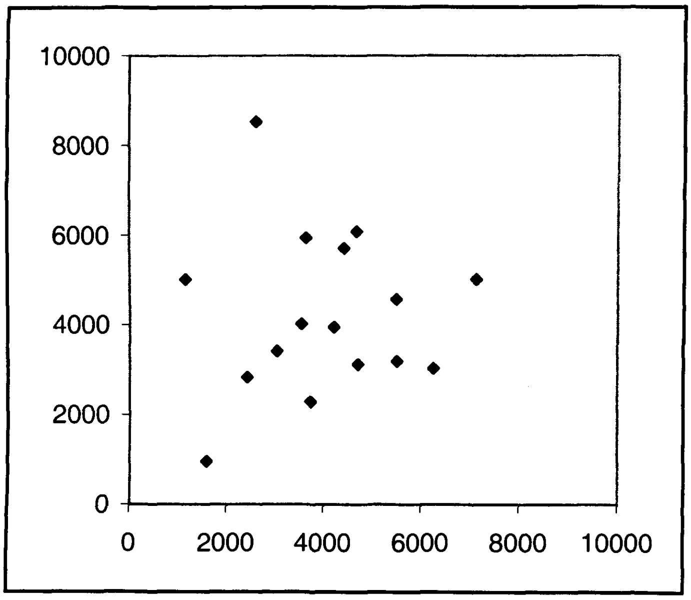

Figure 3.6: Centred with side=10000m, g=3000m, p=15

3.3.1.4 Thinning

Thus far, the transmitter locations produced may be at any distance from one another. In a realistic situation this would not be the case. For this reason, a 'thinning' method is introduced (algorithm 3.2). The location sets shown in figures 3.2, 3.3 and 3.6 had not undergone this process.

A radius around a transmitter within which another transmitter may not appear is used, along with a co-site distance which allows exceptions to this rule. Transmitters in close proximity are allowed to remain if the distance between them is less than the co-site distance. This exception takes into account the possibility of, for example, transmitters at either end of the roof of a building.

```
Input transmitters
for the number of runs required do
    Choose a transmitter at random
    for all other transmitters do
        Set d ← distance between current and chosen transmitters
        if coSiteDist < d < radius then
            Remove this transmitter
        end if
    end for
end for
```

Algorithm 3.2: Thinning algorithm

Algorithm 3.2 is not exhaustive; instead, a certain number of locations are chosen at random and all other locations which are within a disallowed distance of each chosen location removed. Note that the algorithm may choose a location more than once, resulting in an iteration of the loop which essentially does nothing, but it may not choose an already removed location. An alternative exhaustive version of this algorithm considers in turn all the locations produced and, if two are found to be within a disallowed distance of one another, selects at random which of the two to remove.

For the purpose of illustrating this thinning algorithm only, figure 3.7 shows an example in which the algorithm is run 200 times with a co-site distance of 8m and a radius of 1600m.

As thinning affects the number of transmitters and hence the density, the radius

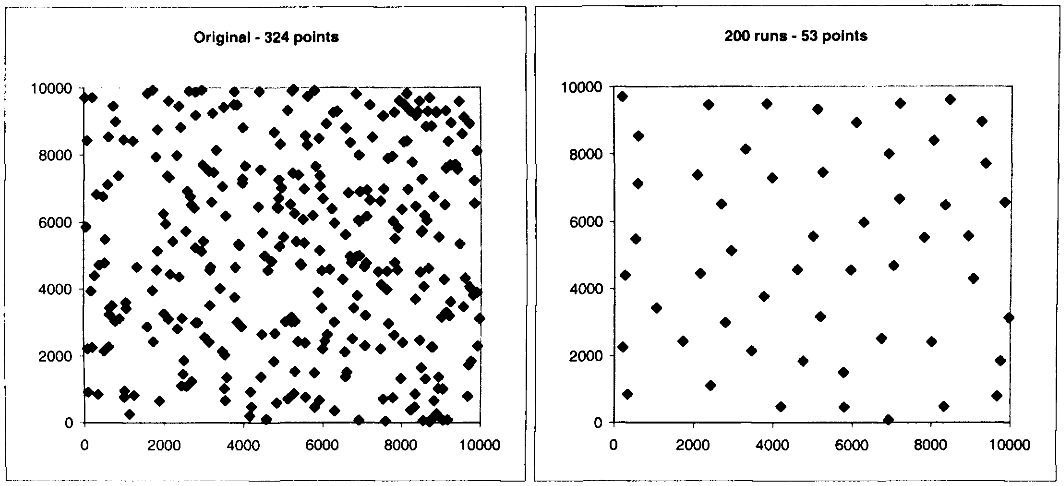

Figure 3.7: Illustrative example of thinning

and co-site distance must be chosen carefully. After thinning, the density given in equation (3.8) will no longer be accurate and, if required, the new density may be calculated from the `side` (in metres) and the number of transmitters remaining by

$$\text{mean density} = \frac{10^6 \times \text{no. trans. remaining}}{\texttt{side}^2} \quad \text{trans. per sq. km} \tag{3.10}$$

3.3.2 RTPs TO CHARACTERISE A NETWORK

3.3.2.1 Mesh points, service points and RTPs

To generate RTPs from a given set of transmitter locations (either generated by the transmitter generation presented here, or generated elsewhere and provided with an appropriate file format), a uniform grid of points is first produced. The grid of points has `side` equal to that used in producing the transmitter locations, but grid points are placed every t metres rather than every g metres, where $t < g$. If $t = \frac{1}{m} \times g$, then the density of this grid is m^2 times the density of the transmitters. The set of points produced by this fine grid are referred to as *mesh points*. Any mesh points which lie within the service radius (section 3.1.3.4) of at least one transmitter can be served by the network and are referred to as *service points*. The constant service radius distance is denoted SR.

An RTP set is a subset of the service points, selected in such a way as to be representative of regions in the network where interference problems are likely to be experienced by users. Networks can be assigned and evaluated without the computational time required to consider every service point in the network, and the nature of the RTPs means that these assignments/evaluations should reflect the situation in the whole network.

$$\{\text{RTPs}\} \subset \{\text{service points}\} \subseteq \{\text{mesh points}\}$$

Each of the mesh points is considered in turn to establish whether it should be kept as an RTP i.e. is at such a position that it will be useful in creating constraints which characterise the problem. The method by which this selection is performed can be seen in algorithm 3.3, where the parameter SR performs the function of limiting RTPs to service points.

3.3.2.2 RTPs for the assumptions in use

The placement of RTPs which characterise the service points is dependent on the propagation, transmitter and interference assumptions used. An alternative method of RTP generation would be needed if these assumptions changed, so that the new set of RTPs may characterise the new problem. The RTPs produced here are intended to characterise a situation in which omnidirectional transmitters of equal transmission power and a uniform directional propagation are assumed, and in which multiple interferers are considered and a service radius used.

A cell is the geographical area served by a transmitter. The assumptions that

- RTPs are served by the transmitter providing the strongest signal
- all transmitters use equal transmission power
- the area served by a transmitter is limited by the service radius

mean that a cell is the maximal set of points with the best server in common. Cells are effectively bounded by Voronoi polygons [91, 92] surrounding the transmitters, except where these would be outside the service radius, when the cell boundary is constrained by the circle of radius SR around the transmitter.

In a radio engineering scenario, the most sensitive areas in terms of interference effects are in the handover regions between cells. Signal strength degrades with distance and in these areas the best server is less dominant and other transmitters may be providing almost as much signal strength. Therefore, RTPs to characterise the network's service points should belong to the set of service points which are at cell boundaries.

The RTPs produced in the Benchmark Generator of [44] are placed at the vertices of Voronoi polygons constructed using the transmitters as lattice points. This type of RTP generation has two disadvantages pertaining to their use here:

(a) service radius is not considered (see figure 3.8);

(b) binary interactions (i.e. single interference) are implicit.

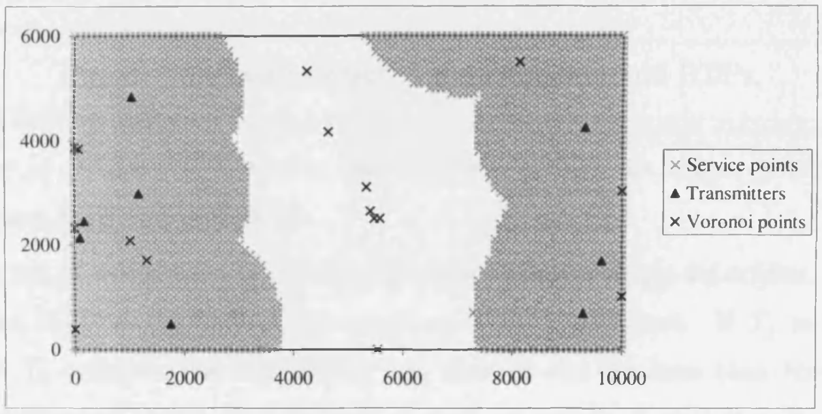

Figure 3.8: Benchmark Generator

The second issue can be explained by considering the diagram in figure 3.9, which shows three transmitters and the resultant local Voronoi points in part of an imaginary network instance. On the straight line from V1 to V2, including the end-points, the SIR experienced when considering the pair-wise relationship between transmitters T_j and T_k would be the same, whichever point along this line was chosen. The same is true for the pair-wise SIR experienced from transmitters T_i and T_j at any point along the line between and including V2 and V3. Therefore any point on the line may be selected to be a representative RTP for single interference. Voronoi points as RTPs are a good choice as they minimise the size of the RTP set size whist characterising the service area for single interference by producing a suitable representative RTP for each pair-wise interaction. Point V2 would be an appropriate RTP for the SIR caused by transmitters T_i and T_k. When multiple

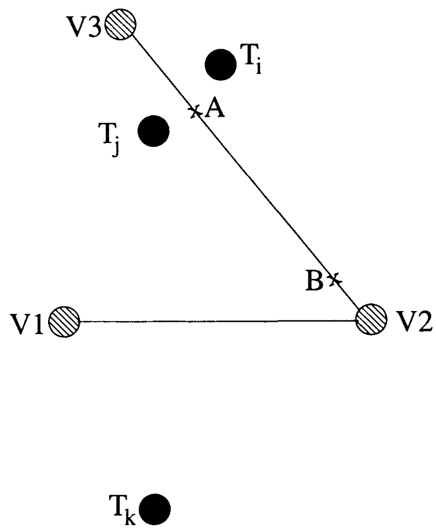

Figure 3.9: Transmitters, Voronoi points and RTPs.

interference is considered, however, the interactions are no longer pair-wise and a more complex RTP set is needed.

When a set of interferers is considered rather than a single interferer, the placement of the RTP is dependent on angle as well as distance. If T_j is the server and T_i and T_k comprise an interfering set, then it can be seen that the SIR with additive interfering signal experienced at each of points A and B will not be the same. As different interfering sets are to be considered, an RTP set is produced containing all service points on cell boundaries.

RTPs are produced at points which will experience interference effects between the server and other cells which have boundaries with the serving cell. The edge-of-cell RTPs to be produced are then of two types:

Handover RTPs are chosen at points which are equidistant (to the precision permitted by mesh points) from two transmitters and within distance SR of both of those transmitters (section 3.3.3.1).

Edge-of-service RTPs are chosen at points which are on the edge of the network service radius, within SR of one transmitter only (section 3.3.3.2).

There is a trade-off between the two conflicting objectives of computational time and representation/characterisation. The set contains those points at which there is a high probability of interference on users. The RTP set generated here is more conservative, containing more points than the set of Voronoi points, whilst keeping the number significantly lower than the total number of service points.

3.3.2.3 Co-sited RTPs

When the authors of [40] come across an RTP which is equidistant from two transmitters, they choose which transmitter is the server arbitrarily. When a handover RTP is produced here, a second co-sited RTP tuned to the other transmitter may also be produced when the distances from the RTP to each transmitter are exactly equal or differ by a small enough amount that they can be considered equidistant.

To summarise:

- edge-of-service RTPs are always tuned to the geographically nearest transmitter;
- a handover RTP is tuned to the nearest transmitter of the two transmitters under consideration at the time;
- a second handover RTP tuned to the other transmitter may be produced when the two transmitters are the same distance away to some given precision.

RTPs with the same location but tuned to different serving transmitters are distinct. Note that 'repetition' refers to identical RTPs i.e. with the same location *and* tuned transmitter. (Occasionally, identical RTPs may be produced twice. This occurs either because an RTP is produced at position A tuned to the nearest transmitter i when transmitter j is being considered as a possible interferer, and again when considering transmitter k, in a region where the test point is in service radius of all three transmitters i, j and k. It also occurs due to the creation of double RTPs mentioned above. For example, RTPs at both A and B may be produced, then later, when considering a transmitter again, RTPs at both B and C, thus resulting in repetition at B. For this reason, any repetition is removed at the end of the algorithm.)

3.3.2.4 Evaluation points

RTPs are produced to characterise the problem by being effectively the worst case service points in terms of SIR under the assumptions adopted. Such points are useful in creating constraints and/or assignments, as they save computation by characterizing the whole network in a smaller set of points which can then be considered in computation.

A set of test points is often used to evaluate a network. These *evaluation points* may or may not be the same points as were selected to be RTPs, depending on the measure of coverage being employed. For example, [17] uses Voronoi point RTPs in creating assignment but evaluates assignments using mesh points; [45] uses Voronoi points for evaluation; [18] evaluates at known mobile positions, which may or may not include worst case SIR positions.

As well as producing files containing transmitters and RTPs respectively, the problem generation program can also produce a file of all service points to be used for evaluation. This third output file is, however, quite large. As the evaluation points to be used depend on the coverage measure chosen, this issue will be discussed further when coverage evaluation is considered in chapter 4.

3.3.3 RTP selection criteria

3.3.3.1 Handover RTP selection

If the minimum distance between two mesh points is t, then there will be an RTP which should be selected, which is as close as possible to being equidistant from two transmitters, within distance $t/2$ of the line of equidistance between those two transmitters.

Figure 3.10 shows two transmitters and a potential RTP. The RTP is at distance d_1 and d_2 from the two transmitters respectively, and a perpendicular distance of a from the line of equidistance between the two transmitters. If this RTP is one to be selected, then

$$a \leq \frac{1}{2}t \tag{3.11}$$

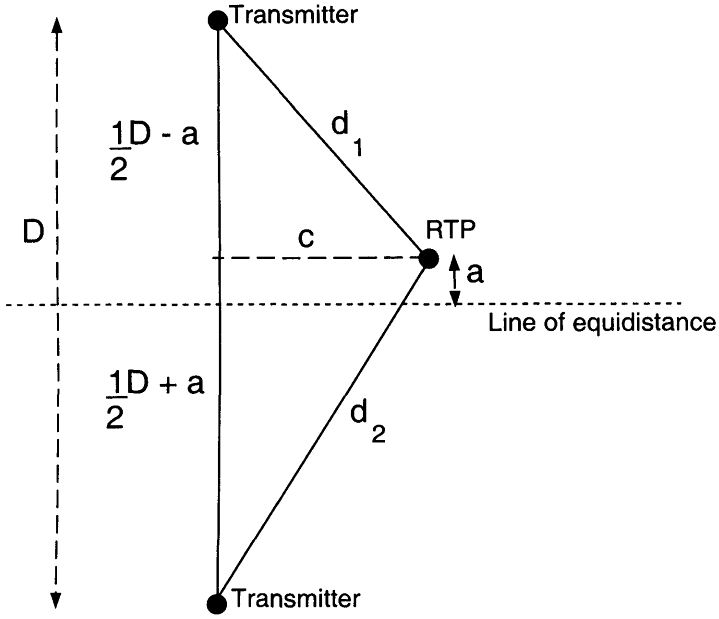

Figure 3.10: Conditions for RTP selection

Using Pythagoras' theorem in the two right-angled triangles of figure 3.10 gives

$$d_1^2 = c^2 + \left(\frac{D}{2} - a\right)^2$$

and

$$d_2^2 = c^2 + \left(\frac{D}{2} + a\right)^2$$

Then

$$\begin{aligned} d_2^2 - d_1^2 &= \left(\frac{D}{2} + a\right)^2 - \left(\frac{D}{2} - a\right)^2 \\ &= 2Da \end{aligned} \tag{3.12}$$

and combining (3.11) and (3.12) gives the following condition on the distances

$$d_2^2 - d_1^2 \leq Dt$$

Using the above, if a mesh point is at distance d_n from the nearest transmitter n and within the service radius of another transmitter i which is d_i away from the mesh point and D from the nearest transmitter, then a handover RTP is placed at that mesh point when the following conditions are all satisfied:

$$\boxed{\begin{aligned} d_n &\leq \text{SR} \\ d_i &\leq \text{SR} \\ {d_i}^2 - {d_n}^2 &\leq Dt \end{aligned}} \tag{3.13}$$

where SR is the service radius, t is the minimum distance between two mesh points, and all distances are in metres.

Figure 3.11 shows an example (using the transmitter locations of the fifteen transmitter instance from the Benchmark Generator [44]) of transmitters and their corresponding handover RTPs. It can be seen that the RTPs trace out would-be Voronoi polygons around the transmitter locations, but are constrained to the area of service.

3.3.3.2 Edge-of-service RTP selection

If a mesh point is at distance d_n from the nearest transmitter n then an edge-of-service RTP is placed at that mesh point when

$$\boxed{\begin{aligned} d_n &\leq \text{SR} \\ \min_{i \neq n}(d_i) &> \text{SR} \\ d_n &> \text{SR} - \frac{\sqrt{2}}{2}t \end{aligned}} \tag{3.14}$$

Equation (3.14) can also be written $\text{SR} - \frac{\sqrt{2}}{2}t < d_n \leq \text{SR} < \min_{i \neq n}(d_i)$ and a simple proof constructed using Pythagoras' theorem.

Figure 3.12 shows the example from figure 3.11 with all RTPs including edge-of-service RTPs. When edge-of-service RTPs are included, any straight line constructed between any pair of transmitters will pass through an RTP somewhere on its path, or more accurately within $t/\sqrt{2}$ of an RTP, where t is the minimum possible distance between two RTPs, due to the discrete grid-point nature of the RTPs.

```
Input parameters: array of transmitters, side, service radius SR, mesh size t
Create grid of potential RTPs with parameters side and t
Store distances from each trans to each RTP
Store distances from each trans to each other trans
for each potential RTP do
  Find the nearest transmitter to the RTP
  Set nearestDist accordingly
  if nearestDist≤ r (RTP within service radius of nearest) then
    Set alone ← true (remains true if RTP in range of one trans only)
    for each other trans do
      Set thisDist ← distance from the other trans to this RTP
      if thisDist≤ r (RTP within service radius of this trans too) then
        Set alone ← false (as have found handover region)
        Set D ← distance between the two trans under consideration
        if thisDist²−nearestDist² ≤ D×t then
          Create handover RTP tuned to nearest trans
          if |thisDist-nearestDist| < t/100 then
            Create second handover RTP tuned to other trans
          end if
        end if
      end if
    end for
    if alone AND nearestDist> r−(√2/2)t then
      Create edge-of-service RTP tuned to nearest trans
    end if
  end if
end for
Remove any repetition of RTPs
Create RTP file
```

Algorithm 3.3: RTP generation algorithm

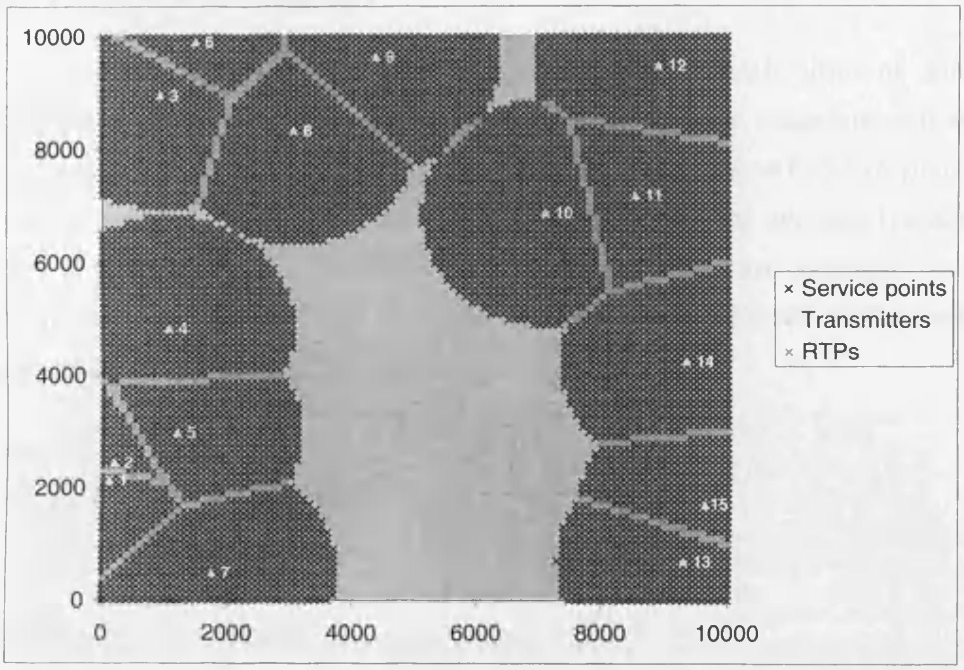

Figure 3.11: Handover RTPs

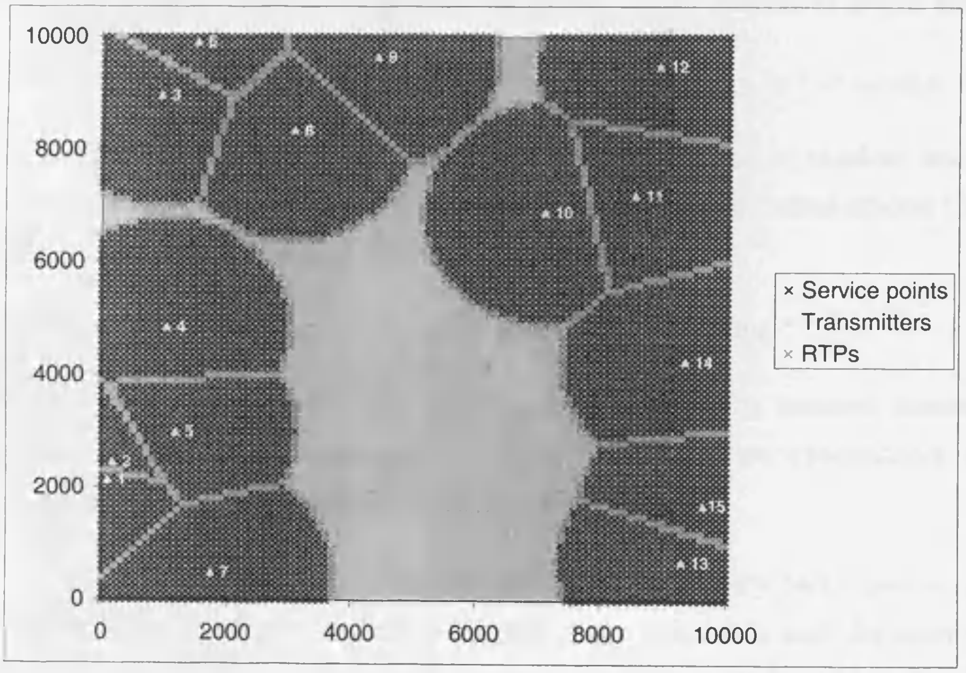

Figure 3.12: Handover and edge-of-service RTPs

3.3.4 TEST CASE LIBRARY

A library of these problem instances of various types, i.e. with different densities and distributions, is generated. This library consists of sixty transmitter files and their corresponding RTP files as shown in figure 3.3. Transmitters are placed by the various methods on a 10km sided square region, having average transmitter densities ≤ 4 trans/sq.km. Note that the densities quoted are averaged over the region and therefore that higher densities will occur, for example at the centre of normally distributed examples.

g (metres)	500	750	1000	1500	2000
Density (per sq. km)	≤ 4	$\leq 16/9$	≤ 1	$\leq 4/9$	$\leq 1/4$
Grid	$\times 1$	$\times 1$	$\times 1$	$\times 1$	$\times 1$
Random	$\times 5$	$\times 5$	$\times 5$	$\times 5$	$\times 5$
Centred	$\times 5$	$\times 5$	$\times 5$	$\times 5$	$\times 5$
Combined	Various types/densities $\times 5$				

Total = 60 test problems

Table 3.3: Library of test problems

Other parameters used in the generation of the library instances are as follows:

- The service radius used in the generation of the library is $r = \max(g, 1000)$.
- Thinning of transmitters is performed for 100 iterations in random and centred instances, with a co-site distance of 30 and thinning radius of $\min(1000,g)$ for random instances and 200 for centred instances.
- Mesh points are placed on a square grid every $t = 100$m.
- The instances produced here have second RTPs in the manner mentioned earlier if the respective distances from the RTP to the two transmitters under consideration at the time differ by $< t/100$.

The transmitter location and RTP files of the sixty library test cases are generated in 48 seconds. Table 3.4 contains a list of the test cases and the number of transmitters each contains. Figures 3.13 and 3.14 show sample output files from the library. Figures 3.15– 3.19 show the transmitter and RTP locations for the test cases from the library.

Test case directory	Number of transmitters in network	Test case directory	Number of transmitters in network
GridExample0	400	CentredExample0	329
GridExample1	196	CentredExample1	331
GridExample2	100	CentredExample2	313
GridExample3	49	CentredExample3	328
GridExample4	25	CentredExample4	320
RandomExample0	229	CentredExample5	160
RandomExample1	216	CentredExample6	165
RandomExample2	215	CentredExample7	169
RandomExample3	231	CentredExample8	160
RandomExample4	210	CentredExample9	159
RandomExample5	79	CentredExample10	87
RandomExample6	80	CentredExample11	90
RandomExample7	78	CentredExample12	85
RandomExample8	77	CentredExample13	88
RandomExample9	81	CentredExample14	88
RandomExample10	40	CentredExample15	43
RandomExample11	40	CentredExample16	46
RandomExample12	40	CentredExample17	48
RandomExample13	42	CentredExample18	43
RandomExample14	44	CentredExample19	45
RandomExample15	24	CentredExample20	25
RandomExample16	28	CentredExample21	24
RandomExample17	27	CentredExample22	24
RandomExample18	26	CentredExample23	25
RandomExample19	30	CentredExample24	23
RandomExample20	19	CombinedExample0	164
RandomExample21	16	CombinedExample1	178
RandomExample22	19	CombinedExample2	162
RandomExample23	16	CombinedExample3	162
RandomExample24	17	CombinedExample4	164

Table 3.4: The sixty test cases available in the library are tabulated, showing the number of transmitters contained in each (the problem dimension).

326.00	4506.00	1
3762.00	7002.00	2
8613.00	6376.00	3
2630.00	9058.00	4
8986.00	2916.00	5
9171.00	1521.00	6
5006.00	6804.00	7
7658.00	5701.00	8
41.00	7926.00	9
2605.00	1627.00	10
6364.00	7866.00	11
6618.00	1922.00	12
1081.00	8077.00	13
385.00	5824.00	14
708.00	3155.00	15
8798.00	5362.00	16
977.00	1160.00	17
7514.00	8160.00	18
2717.00	4811.00	19
9968.00	8382.00	20
3556.00	63.00	21
4917.00	5465.00	22
9233.00	7606.00	23
8280.00	205.00	24
9925.00	6419.00	25
4851.00	8054.00	26
4171.00	3779.00	27

Figure 3.13: Contents of file 'Tco_ord27' which contains the transmitter locations for test problem 'RandomExample17'. This problem consists of randomly placed transmitters in a square region of side 10km.

0.00	100.00	1	17
0.00	2000.00	2	17
0.00	3700.00	3	1
0.00	5200.00	4	14
0.00	6800.00	5	14
0.00	9400.00	6	9
100.00	0.00	7	17
100.00	2100.00	8	15
100.00	3700.00	9	15
100.00	5200.00	10	14
100.00	6900.00	11	9
100.00	9400.00	12	9
200.00	2100.00	13	15
200.00	3700.00	14	15
200.00	5200.00	15	14
200.00	6900.00	16	9
200.00	9400.00	17	9
	.		
	.		
	.		
	.		
9800.00	200.00	874	6
9800.00	2300.00	875	6
9800.00	4100.00	876	5
9800.00	4300.00	877	16
9800.00	5400.00	878	16
9800.00	7100.00	879	25
9800.00	7800.00	880	23
9900.00	2300.00	881	6
9900.00	4100.00	882	5
9900.00	4400.00	883	16
9900.00	5300.00	884	16
9900.00	7200.00	885	23
9900.00	7200.00	886	25
9900.00	7700.00	887	23
10000.00	300.00	888	6
10000.00	2300.00	889	6
10000.00	4000.00	890	5
10000.00	4500.00	891	16
10000.00	5200.00	892	16
10000.00	7300.00	893	23
10000.00	7600.00	894	23

Figure 3.14: Partial contents of file '`Rco_ord27`' which contains the corresponding handover and edge-of-service RTPs for the transmitter locations in figure 3.13.

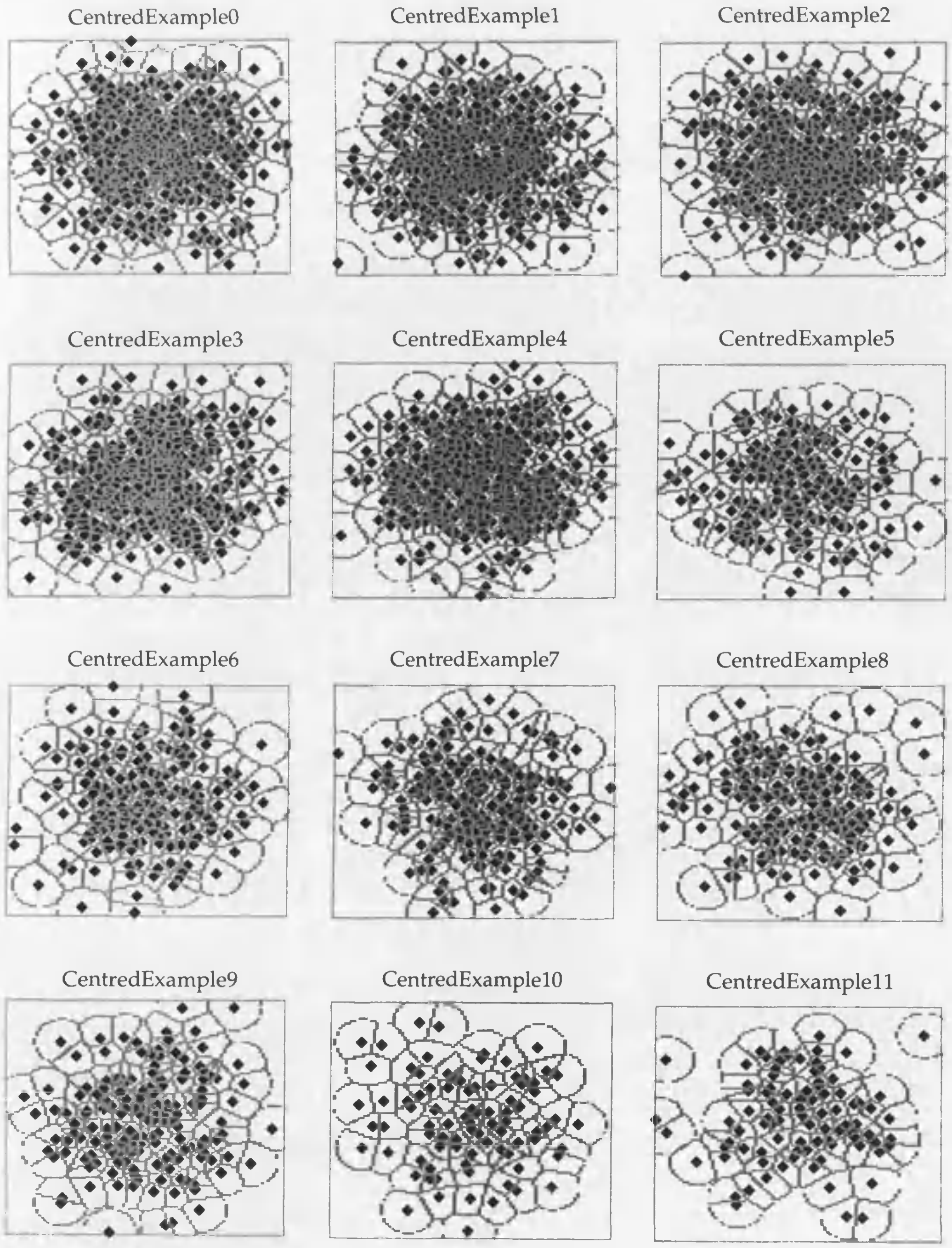

Figure 3.15: Library instances CentredExample0–CentredExample11, showing the transmitter locations and the corresponding RTP set

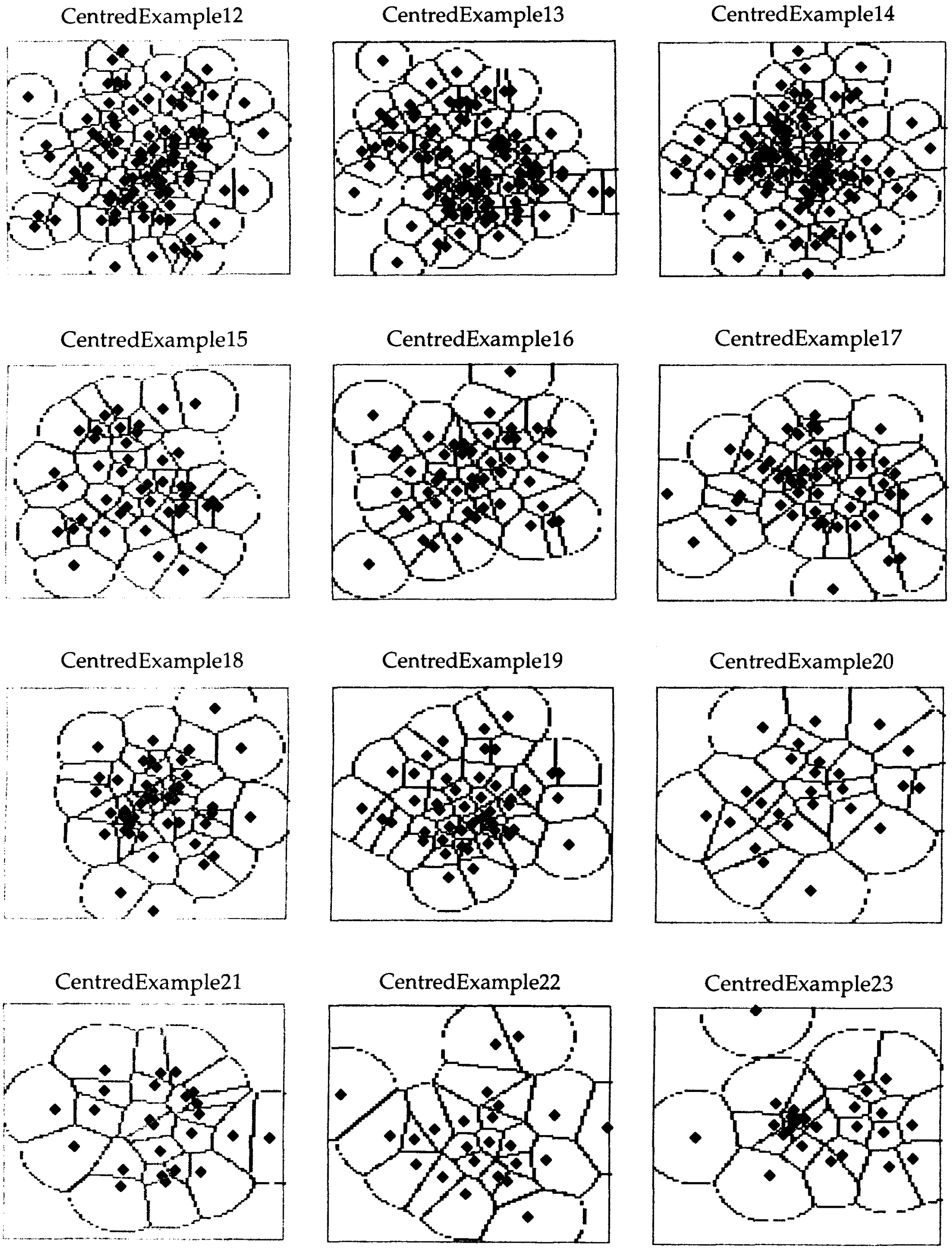

Figure 3.16: Library instances CentredExample12–CentredExample23, showing the transmitter locations and the corresponding RTP set

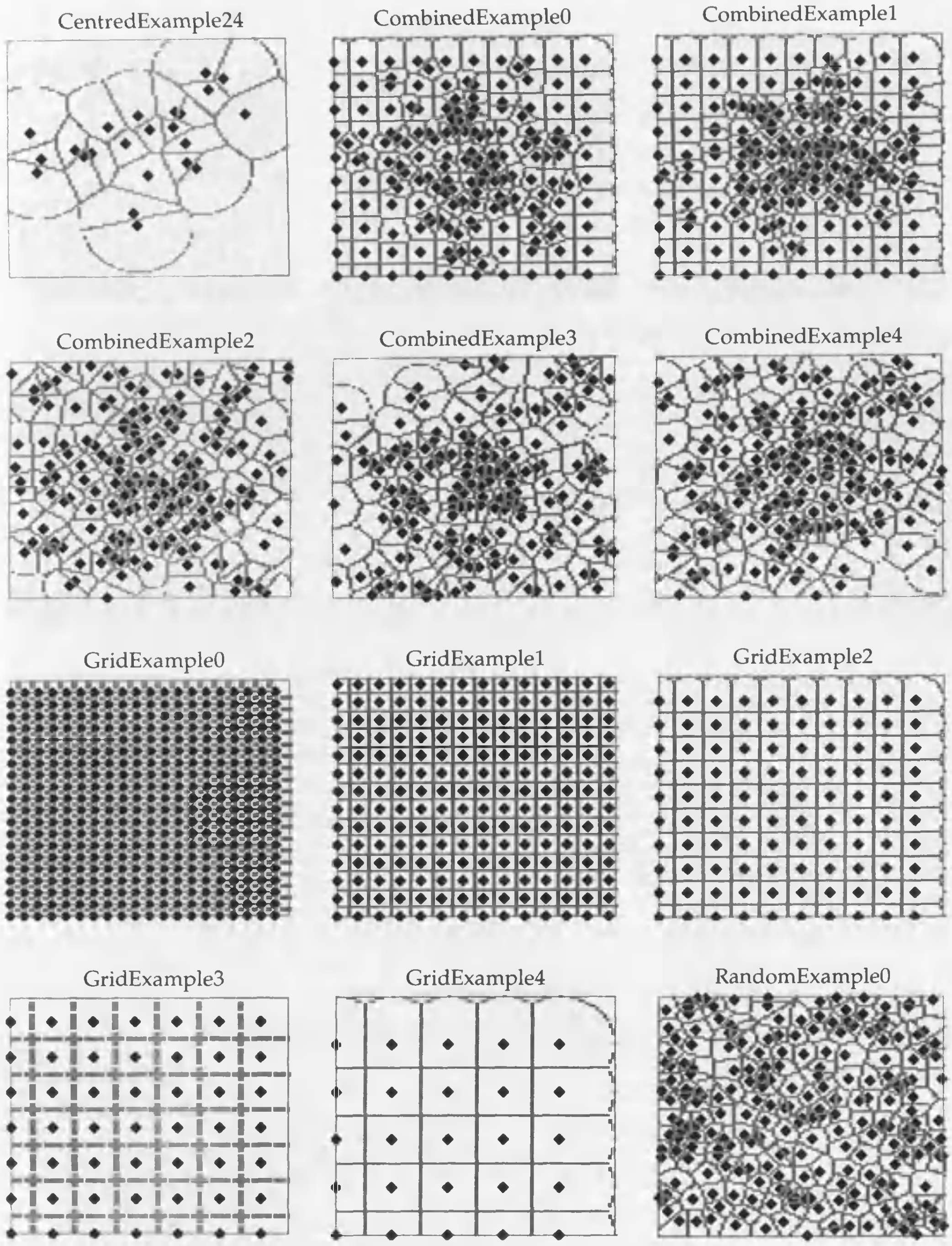

Figure 3.17: Library instances CentredExample24, CombinedExample0–CombinedExample4, GridExample0–GridExample4 and RandomExample0, showing the transmitter locations and the corresponding RTP set

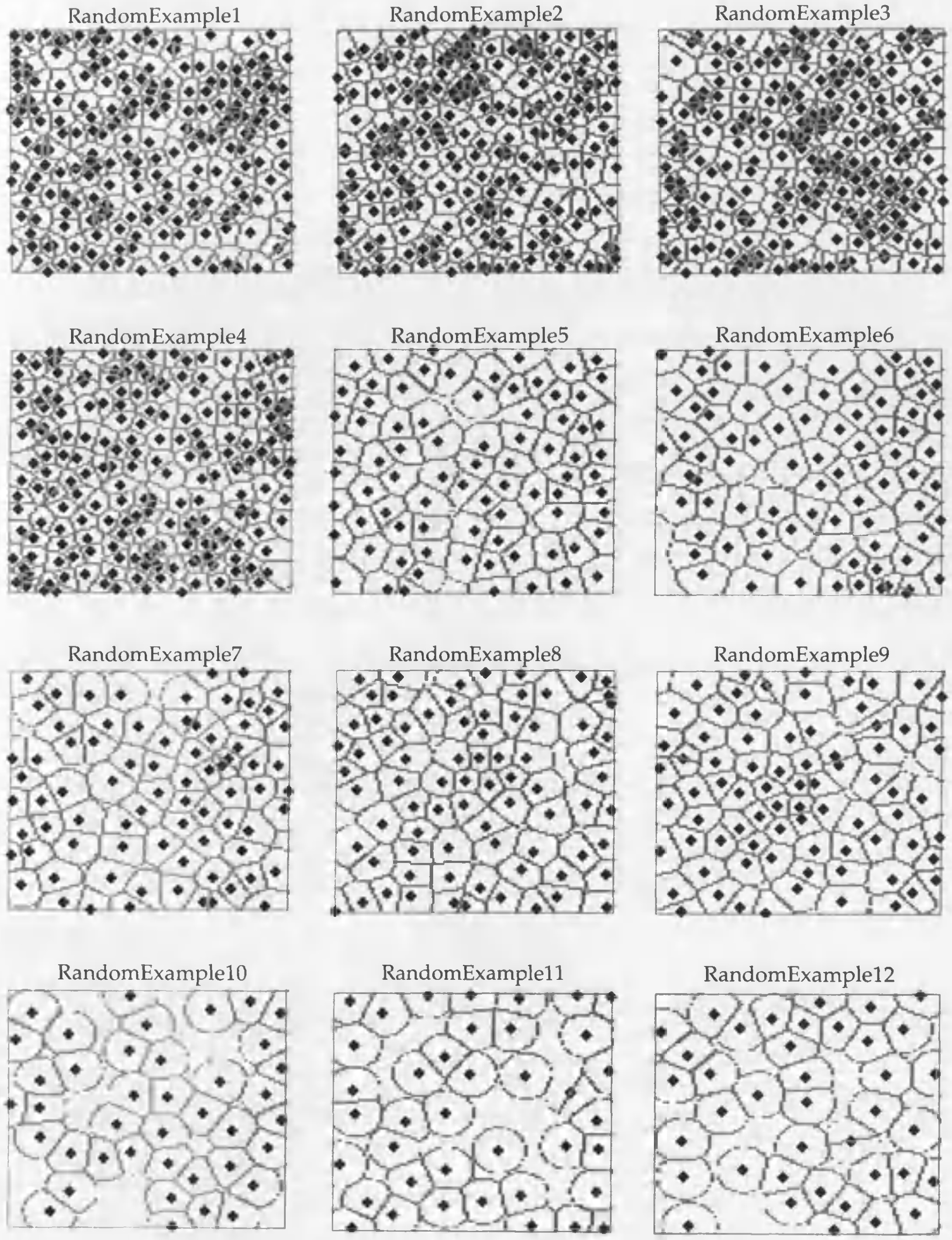

Figure 3.18: Library instances RandomExample1–RandomExample12, showing the transmitter locations and the corresponding RTP set

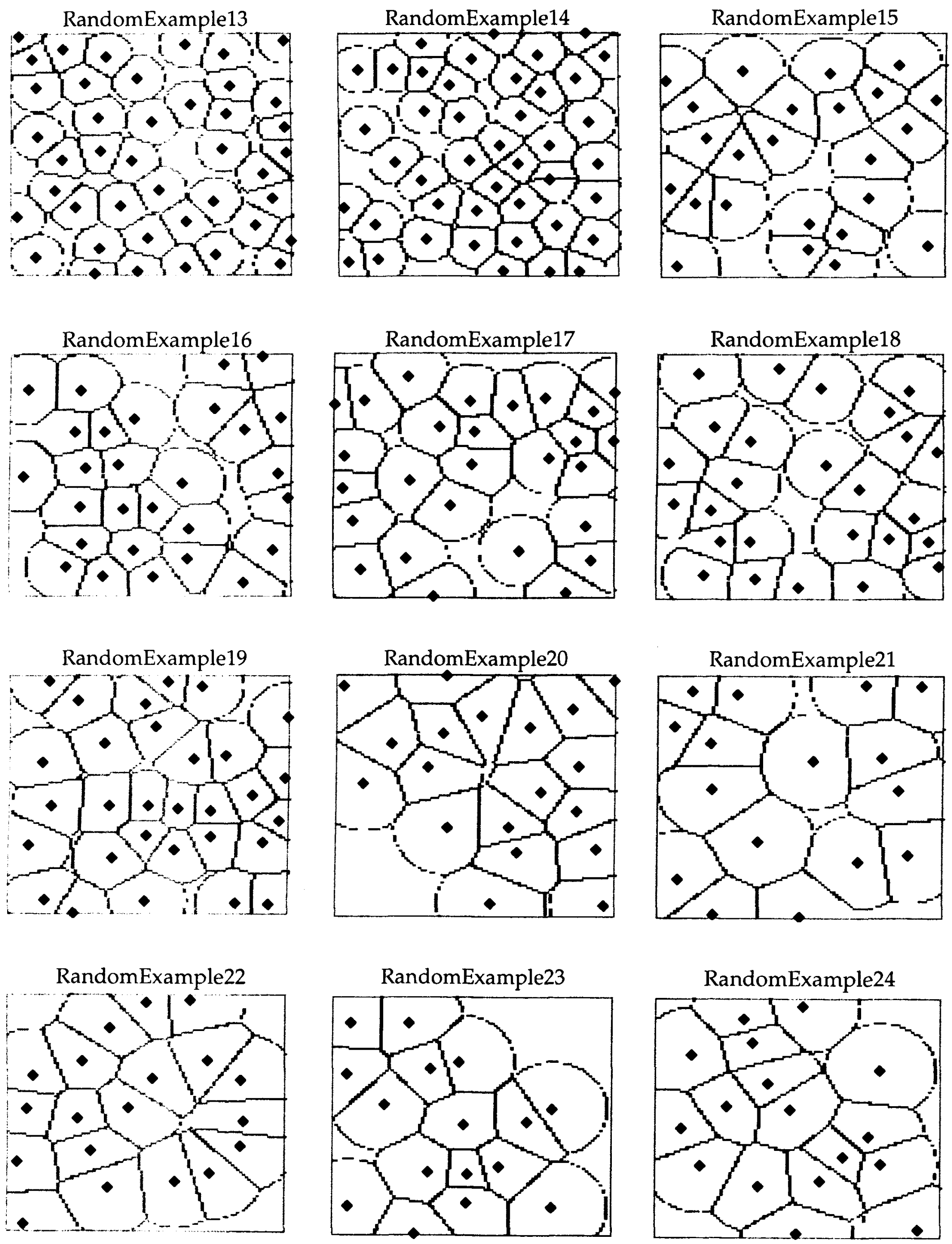

Figure 3.19: Library instances RandomExample13–RandomExample24, showing the transmitter locations and the corresponding RTP set

3.4 ASSIGNMENTS TO SATISFY BINARY CHANNEL SEPARATION CONSTRAINTS

In this thesis, binary channel separation constraints are generated in several ways (section 4.2, chapter 6). Assignments are required which satisfy all of these constraints and which, in addition, minimise the spectral requirements. Although solution techniques for channel assignment are not the focus of this thesis, assignments for different sets of binary channel separation constraints are required and a solution method for these constraints must therefore be selected and employed. This section introduces the assignment algorithms used in the subsequent chapters of the thesis, and section 3.5 briefly looks at their performance in terms of constraint satisfaction and spectral usage.

Because the channel assignment problem modelled by the generalised graph-colouring formulation is NP-hard, exhaustive techniques are often impractical for use in finding a channel assignment for a given constraint matrix and meta-heuristic methods are employed (section 2.4). In this thesis, assignments are created by an exhaustive method in the case of 'small' problems (those which have < 40 transmitters and $\mathrm{SIR}_a < 15\mathrm{dB}$) in chapter 4, and a hybrid simulated annealing and sequential algorithm (see section 2.4.2) for other problems.

3.4.1 EXHAUSTIVE ASSIGNMENT METHOD

The exhaustive method results in an assignment satisfying all of the binary constraints for single interference (zero-violation assignments) and using the minimum span of channels possible. Having found an assignment for span q by forward-checking, the assignment algorithm attempts to find a zero-violation assignment with span $q-1$, halting when such an assignment cannot be found. Transmitters are ordered by their generalized degree, which can be calculated by halving the sum of the entries in that transmitter's corresponding column in the computed symmetric constraint matrix (ϕ from algorithm 4.1, or Φ in chapter 6).

3.4.2 HYBRID ASSIGNMENT METHOD

In the hybrid algorithm used for larger problems, a zero-violation assignment is firstly generated by the sequential method. Transmitters are ordered by their generalized degree, as described above, and channels are ordered lowest frequency first. The next transmitter is selected as per this transmitter ordering, and the first acceptable channel is assigned. Transmitters which have been assigned the highest used channel by this method are then randomly assigned a lower channel. This reduces the span of channels used but the reassignment is likely to introduce constraint violations.

A simulated annealing algorithm for fixed spectrum assignment is employed with the reduced span to attempt to eliminate the constraint violations introduced and hence find a zero-violation assignment with this smaller span. This process is continued until the algorithm cannot resolve the violations created. This algorithm is available (coded in C) in the package 'FASOFT' [49, 88]. Dunkin and Allen [35] note that, towards minimising the span of assignments, the heuristic methods of this package work best as a hybrid method as used here.

The resulting assignment provided by this hybrid assignment method is zero-violation for the constraints used and has minimal (or good sub-optimal) span. This performance is discussed in section 3.5.

3.4.3 SIMULATED ANNEALING PROCEDURE

The simulated annealing procedure is shown in algorithm 3.4. A *configuration* is an array of indices representing an assignment. For example, the configuration (1,3,1,...) would mean transmitter t_1 is assigned the first channel in the ordered list of channels, t_2 the third and t_3 the first again etc. A new configuration differs from the old configuration whose neighbour it is by having one of the indices altered; the transmitter whose assignment is altered appeared in a constraint which failed under the old configuration. In algorithm 3.4, n is the number of transmitters and the cooling schedule used is that presented by Hurley and Smith [89] in which

$H(t_k)$ is evaluated

$$H(t_k) = \frac{1}{n}\left[n\sum_{i=1}^{n}(E_i^k)^2 - \left(\sum_{i=1}^{n} E_i^k\right)^2\right]^{1/2} \tag{3.15}$$

where E_i^k is the cost function value for the assignment obtained at iteration i at temperature t_k. This cooling schedule is used by Hurley, Thiel and Smith [66] when they compare simulated annealing, tabu search and genetic algorithms for a set of realistic instances and gain the best results in terms of numbers of constraints violated from the simulated annealing approach. The cost function (energy function) used here is the sum of the amounts by which constraints are violated. The algorithm is halted if the while loop has accepted no new configurations ten times (has encountered ten 'frozen' temperatures).

```
k ← 0
t_k ← 0.1
t_min ← 0.01
Get starting configuration X_old
while t_k > t_min do
    for n loops do
        Generate new configuration X_new from X_old
        Calculate new energy E_new
        Calculate ΔE = E_new − E_old
        if ΔE < 0 or (random ∈ [0,1]) < e^(−ΔE/t_k) then
            X_old ← X_new
            E_old ← E_new
        end if
    end for
    t_{k+1} ← t_k (1 + ln(1+δ)t_k / 3H(t_k))^(−1)
    k ← k + 1
end while
```

Algorithm 3.4: Simulated annealing procedure for fixed spectrum channel assignment

3.5 ASSIGNMENT ALGORITHM PERFORMANCE

The assignments created are required to be zero-violation assignments, which satisfy each of the given constraints. Such an assignment would be easy to create were the spectral resources available unlimited. Whether the algorithm provides an efficient assignment depends on multi-fold issues; the objective of violating no constraints when combined with the requirement to use spectrum appropriately defines the performance.

3.5.1 CONSTRAINT SATISFACTION

The assignment algorithms used (which are given sufficient initial span) always result in a zero-violation assignment for the given constraints. The assignments created do not violate any of the binary channel separation constraints produced and hence give 100% QoS when the network is evaluated for the same assumptions as were used to generate the constraints. (The potential discrepancy between zero-violation and interference-free assignment, due to the assumptions made in constraint generation, means that the assignments created may not be interference-free when evaluated against operational criteria.)

3.5.2 EFFICIENCY OF SPECTRUM USE

When the exhaustive method of assignment is employed, minimality of the span of channels used is confirmed. The heuristic methods used also aim to minimise the span of channels used for the given constraint matrix, but by their nature do not guarantee optimality. Their performance must therefore be assessed, either against assignments provided by other algorithms or against theoretical bounds, to determine that their use of the spectrum is not excessive.

When the 'small' problems were assigned both exhaustively (guaranteeing a minimum span assignment) and using the heuristic method, it was found that the spans achieved were the same by either method, meaning that the heuristic method is proven to provide optimal solutions in these cases.

Section 3.5.3 compares the spans achieved by the heuristic method with the theoretical lower bounds for one-hundred-and-eighty different constraint matrices but firstly a simple illustrative example of bounding is presented.

Consider the following system of eleven binary channel separation constraints on seven transmitters, in which f_i represents the channel assigned to transmitter t_i:

$$
\begin{array}{lcl}
|f_1 - f_2| & > & 0 \\
|f_1 - f_3| & > & 0 \\
|f_2 - f_3| & > & 0 \\
|f_2 - f_4| & > & 0 \\
|f_2 - f_5| & > & 0 \\
|f_3 - f_5| & > & 0
\end{array}
\qquad
\begin{array}{lcl}
|f_4 - f_5| & > & 0 \\
|f_4 - f_6| & > & 0 \\
|f_5 - f_6| & > & 0 \\
|f_5 - f_7| & > & 0 \\
|f_6 - f_7| & > & 0
\end{array}
$$

A value is required for the minimum span of channels needed to provide a zero-violation assignment for these constraints. The analogy with graph theory can often be used to provide bounds for binary constraints. Each of these eleven constraints is a co-channel constraint, so an unlabeled graph G can be constructed to represent the problem, using the following model:

- vertex $v_i \in V$ represents the transmitter t_i;
- an edge $(v_i, v_j) \in E$ exists if and only if there is a binary co-channel constraint between t_i and t_j.

Figure 3.20 shows the graph which represents these binary co-channel constraints (edges) and transmitters (vertices).

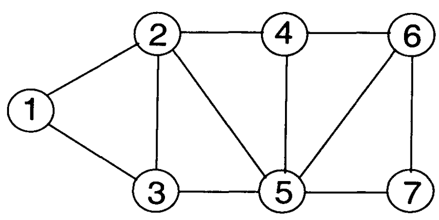

Figure 3.20: Graph G to represent binary co-channel constraints

Clearly, the greatest number of channels that can be used in any assignment to this network is seven (one per transmitter), and as all the constraints are co-channel, these seven channels may be consecutive, meaning that a naïve upper bound on the minimum span is $spn \leq 7$.

Finding a minimum span assignment for these constraints is then equivalent to finding a colouring of the vertices of G which uses the minimum possible number of colours. Using results from graph theory [25, 26], stronger upper and lower bounds on the number of colours, $\gamma(G)$, needed in this case can be stated. For example,

- $\gamma(G) \leq 4$ as G is planar (by the *Four Colour Map Theorem* [93, 94]);
- $\gamma(G) \geq 3$ as G contains a triangle (clique of three vertices).

From these bounds, it is known that $3 \leq \gamma(G) \leq 4$. Using consecutive channels, this means that $2 \leq spn \leq 3$.

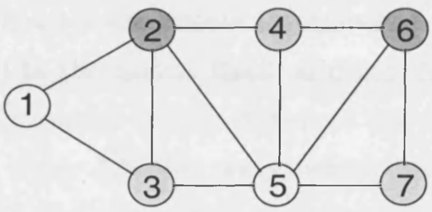

Figure 3.21: Optimal three-colouring

In fact, the colouring of G can be performed with three colours (channels) as shown in figure 3.21, providing a strengthened upper bound. This colouring is equivalent to assigning transmitters 1 and 5 the same channel; 2 and 6 another channel; and 3, 4 and 7 a third channel. In this example the minimum span, assuming that consecutive channels are used, is therefore *spn*=2, as a three-colouring can be provided and proven to be optimal as the lower and upper bounds are now equal.

3.5.3 Comparing spans achieved with Prim's lower bound

The use of lower bounds in determining proximity to optimality is described in section 2.5 and illustrated above. It involves a comparison between upper bounds, often found by making assignments, and lower bounds, established by theoretical calculations using the properties of the constraints involved.

These techniques are important for a numbers of reasons. When the lower bound is equal to the upper bound, it is known that the minimum span has been attained.

The lower bound can be used to benchmark the performance of meta-heuristics, although in cases where the lower bound and upper bound do not coincide it is not possible to determine where the discrepancy lies; in the lower bound, the upper bound or both bounds.

From the perspective of this thesis, lower bounds are used to ensure that the span of channels for assignment is 'reasonable' relative to *spn*. For example, if excessive span is made available it will be possible to satisfy *all* constraints, irrespective of what they mitigate against, and consequently the performance of different interference modelling techniques (and the associated constraint matrices) will not be fully assessed.

To facilitate the required comparison, constraint matrices are required from which to calculate lower bounds and create assignments. Note that the proximity to optimal span is related to the matrix itself, and not directly to the operational situation. The sixty geographical library instances are used, and constraints are generated from them in three different ways, which are selected to give matrices whose assignments require spans of channels which represent the range seen throughout the thesis:

- under the single interferer assumption requiring an SIR of 9dB at all RTPs, using algorithm 4.1;
- under the single interferer assumption requiring an SIR of 17dB at all RTPs, using algorithm 4.1;
- under multiple interference requiring an SIR of 17dB at all RTPs, using algorithm 6.4.

These one-hundred-and-eighty matrices are then assigned via the hybrid method of section 3.4.2 and a lower bound calculated as described in the following section. Note that, because of the ways the constraint matrices mentioned above are generated, the bound is being applied to cases of both single interference and multiple interference respectively.

3.5.3.1 Prim's bound

Algorithm 3.5 shows how a lower bound can be calculated for a given constraint graph G, whose adjacency matrix is the constraint matrix under consideration.

The bound used is presented by Smith and Hurley [81] and explained by Dunkin and Allen [35]. Due to the use of Prim's algorithm (algorithm 3.6) during the calculation of the bound, Dunkin and Allen [35] refer to it as 'Prim's bound'.

A level p clique of G is a maximal complete subgraph of G having all edge labels $\geq p$. In algorithm 3.5, ${C_p}'$ refers to the graph with edge and vertex sets equal to those of C_p but in which the weight of each edge is given by

$$w(v_i v_j) = 1 + (\text{label of edge } v_i v_j \text{ in } C_p)$$

The notation $S({C_p}')$ used in algorithm 3.5 refers to the total weight of a minimal spanning tree of ${C_p}'$. The minimal spanning tree used is found using Prim's algorithm [95, 96] which is shown in algorithm 3.6.

Set $p \leftarrow 0$
Set LB $\leftarrow 0$
repeat
    Set $C_p \leftarrow$ level p clique of G
    if $S({C_p}') >$ LB **then**
        Set LB $\leftarrow S({C_p}')$
    end if
    Increment p
until $|V(C_p)| = 1$
LB is now the lower bound for the constraint matrix

Algorithm 3.5: Generation of a lower bound on *spn* for a given constraint matrix

Initialise tree T containing a single vertex chosen arbitrarily from G
Initialise set F as empty
while T does not span G **do**
    Make F contain all edges of G which have one vertex in the current T and the other vertex not in the current T
    Add to T the edge of minimum weight in F
end while
T is now a minimum spanning tree for G

Algorithm 3.6: 'Growing' a minimum spanning tree T of a connected weighted graph G using Prim's algorithm

Although lower bounds are available which are potentially stronger than Prim's minimum-spanning-tree-based bound as used here, this bound has the advantage that the minimum spanning tree may be found by a greedy algorithm in low polynomial time [35]. It can therefore quickly establish whether the use of spectrum by a particular assignment is reasonable, although without necessarily providing a definitive value for *spn*.

3.5.3.2 Results of the comparison

The results of the comparison of the heuristically determined span to the lower bound give a guide to how the algorithm is performing but this is not an absolute. As noted in section 2.5, when an assignment span and lower bound are not equal, it may be that the assignment is not optimal, or that the bound was not sufficiently strong. The assignment span provides an upper bound and, when combined with the lower bound, a range within which it is known the optimal span falls, but the *spn* has not been determined explicitly.

Tables 3.6, 3.7 and 3.8 show the discrepancies, where they exist, between the lower bound calculated and the span of the assignment produced in the one-hundred-and-eighty constraint matrix cases being considered. Table 3.5.3.2 summarises the average results of the comparisons in tables 3.6, 3.7 and 3.8. It can be seen that the discrepancy is low and remains low as the problem dimension increases. Figures 3.22 and 3.23 illustrate this.

It is seen that the span of channels used by the assignments created using the hybrid heuristic assignment method is on average within approximately ten percent of the lower bound, meaning that the spectral usage is sensible/appropriate and hence that the assignment algorithms provide zero-violation assignments efficiently. The quality of assignments produced in this way is deemed amply sufficient for this method to be carried forward and used throughout the chapters which follow.

	Table 3.6	Table 3.7	Table 3.8
average discrepancy (number of channels):	0.033	1.850	2.533
average span used (number of channels):	5.617	12.467	20.683
average discrepancy per channel span:	0.006	0.148	0.122

Table 3.5: Summarising the proximity of the span used to the lower bound

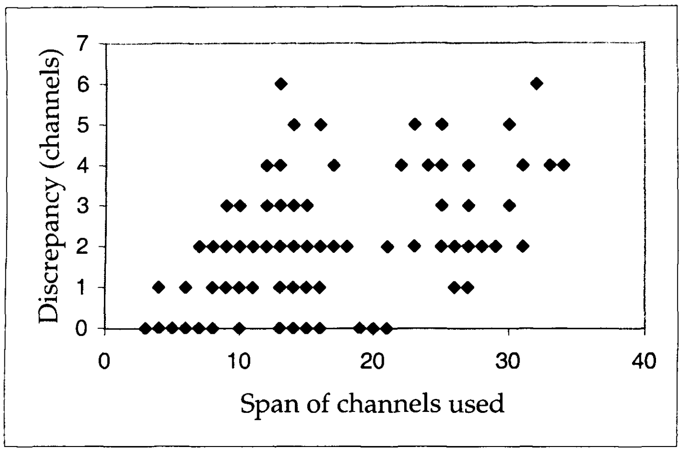

Figure 3.22: The discrepancy between the lower bound calculated and the span of the generated assignment is plotted against the span used for each case in which such a discrepancy exists. (Those cases where the span used is equal to the lower bound and therefore optimality is proven are not plotted.)

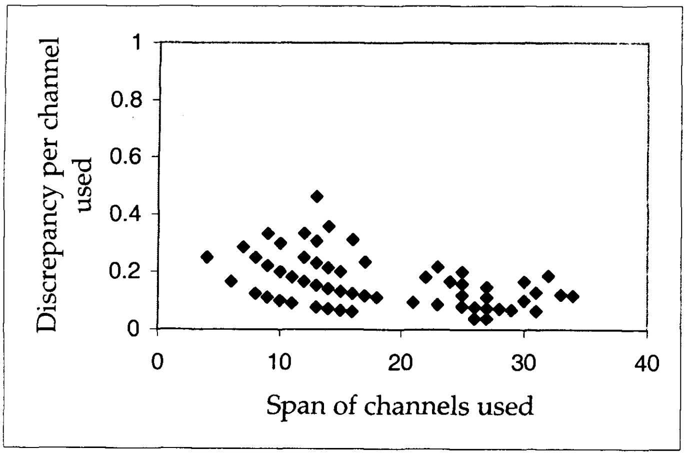

Figure 3.23: The discrepancy between the lower bound calculated and the span of the generated assignment, divided by the span used, is plotted against the span used for each case in which such a discrepancy exists. (Those cases where the span used is equal to the lower bound and therefore optimality is proven are not plotted.)

Table 3.6: For each of the sixty library test cases, a constraint matrix is created for 9dB SIR under single interference. Assignments are created and the spans of these assignments are tabulated (overleaf) along with the lower bound. Discrepancies between the lower bound and the span achieved are shown.

Test case	Number of trans	Lower bound	Span used	Discrepancy	Test case	Number of trans	Lower bound	Span used	Discrepancy
CentredExample0	329	6	6	-	GridExample0	400	3	3	-
CentredExample1	331	8	8	-	GridExample1	196	3	4	1
CentredExample2	313	8	8	-	GridExample2	100	3	3	-
CentredExample3	328	7	7	-	GridExample3	49	3	3	-
CentredExample4	320	7	7	-	GridExample4	25	3	3	-
CentredExample5	160	7	7	-	RandomExample0	229	7	7	-
CentredExample6	165	5	6	1	RandomExample1	216	6	6	-
CentredExample7	169	7	7	-	RandomExample2	215	7	7	-
CentredExample8	160	8	8	-	RandomExample3	231	7	7	-
CentredExample9	159	7	7	-	RandomExample4	210	6	6	-
CentredExample10	87	7	7	-	RandomExample5	79	5	5	-
CentredExample11	90	7	7	-	RandomExample6	80	5	5	-
CentredExample12	85	6	6	-	RandomExample7	78	4	4	-
CentredExample13	88	6	6	-	RandomExample8	77	5	5	-
CentredExample14	88	6	6	-	RandomExample9	81	5	5	-
CentredExample15	43	7	7	-	RandomExample10	40	4	4	-
CentredExample16	46	6	6	-	RandomExample11	40	3	3	-
CentredExample17	48	7	7	-	RandomExample12	40	4	4	-
CentredExample18	43	8	8	-	RandomExample13	42	3	3	-
CentredExample19	45	6	6	-	RandomExample14	44	4	4	-
CentredExample20	25	6	6	-	RandomExample15	24	5	5	-
CentredExample21	24	6	6	-	RandomExample16	28	4	4	-
CentredExample22	24	5	5	-	RandomExample17	27	4	4	-
CentredExample23	25	8	8	-	RandomExample18	26	4	4	-
CentredExample24	23	6	6	-	RandomExample19	30	4	4	-
CombinedExample0	164	6	6	-	RandomExample20	19	6	6	-
CombinedExample1	178	6	6	-	RandomExample21	16	4	4	-
CombinedExample2	162	6	6	-	RandomExample22	19	6	6	-
CombinedExample3	162	7	7	-	RandomExample23	16	4	4	-
CombinedExample4	164	8	8	-	RandomExample24	17	4	4	-

Table 3.7: For each of the sixty library test cases, a constraint matrix is created for 17dB SIR under single interference. Assignments are created and the spans of these assignments are tabulated (overleaf) along with the lower bound. Discrepancies between the lower bound and the span achieved are shown.

Test case	Number of trans	Lower bound	Span used	Discrepancy	Test case	Number of trans	Lower bound	Span used	Discrepancy
CentredExample0	329	14	15	1	GridExample0	400	5	6	1
CentredExample1	331	15	16	1	GridExample1	196	6	8	2
CentredExample2	313	14	16	2	GridExample2	100	7	13	6
CentredExample3	328	14	15	1	GridExample3	49	6	9	3
CentredExample4	320	15	16	1	GridExample4	25	7	10	3
CentredExample5	160	14	15	1	RandomExample0	229	11	14	3
CentredExample6	165	13	15	2	RandomExample1	216	11	14	3
CentredExample7	169	15	15	-	RandomExample2	215	12	14	2
CentredExample8	160	13	14	1	RandomExample3	231	13	15	2
CentredExample9	159	12	15	3	RandomExample4	210	10	13	3
CentredExample10	87	11	14	3	RandomExample5	79	8	12	4
CentredExample11	90	12	14	2	RandomExample6	80	8	12	4
CentredExample12	85	13	14	1	RandomExample7	78	8	12	4
CentredExample13	88	12	15	3	RandomExample8	77	8	12	4
CentredExample14	88	12	14	2	RandomExample9	81	9	12	3
CentredExample15	43	13	13	-	RandomExample10	40	7	8	1
CentredExample16	46	12	14	2	RandomExample11	40	5	7	2
CentredExample17	48	14	15	1	RandomExample12	40	6	9	3
CentredExample18	43	14	16	2	RandomExample13	42	7	8	1
CentredExample19	45	14	14	-	RandomExample14	44	8	10	2
CentredExample20	25	14	14	-	RandomExample15	24	7	9	2
CentredExample21	24	14	14	-	RandomExample16	28	7	9	2
CentredExample22	24	13	13	-	RandomExample17	27	8	9	1
CentredExample23	25	16	16	-	RandomExample18	26	8	9	1
CentredExample24	23	14	14	-	RandomExample19	30	9	11	2
CombinedExample0	164	11	14	3	RandomExample20	19	9	10	1
CombinedExample1	178	11	14	3	RandomExample21	16	6	8	2
CombinedExample2	162	11	14	3	RandomExample22	19	10	10	-
CombinedExample3	162	14	15	1	RandomExample23	16	8	9	1
CombinedExample4	164	11	14	3	RandomExample24	17	8	9	1

Table 3.8: For each of the sixty library test cases, a constraint matrix is created for 17dB SIR under multiple interference using the 'spread' method of chapter 6. Assignments are created and the spans of these assignments are tabulated (overleaf) along with the lower bound. Discrepancies between the lower bound and the span achieved are shown.

Test case	Number of trans	Lower bound	Span used	Discrepancy	Test case	Number of trans	Lower bound	Span used	Discrepancy
CentredExample0	329	27	31	4	GridExample0	400	11	13	2
CentredExample1	331	29	31	2	GridExample1	196	9	14	5
CentredExample2	313	29	33	4	GridExample2	100	11	16	5
CentredExample3	328	30	34	4	GridExample3	49	9	13	4
CentredExample4	320	26	32	6	GridExample4	25	10	13	3
CentredExample5	160	27	30	3	RandomExample0	229	18	22	4
CentredExample6	165	25	27	2	RandomExample1	216	18	23	5
CentredExample7	169	27	29	2	RandomExample2	215	20	24	4
CentredExample8	160	26	28	2	RandomExample3	231	21	25	4
CentredExample9	159	25	30	5	RandomExample4	210	19	21	2
CentredExample10	87	22	25	3	RandomExample5	79	14	16	2
CentredExample11	90	22	25	3	RandomExample6	80	14	16	2
CentredExample12	85	23	27	4	RandomExample7	78	13	17	4
CentredExample13	88	24	26	2	RandomExample8	77	15	17	2
CentredExample14	88	24	26	2	RandomExample9	81	16	18	2
CentredExample15	43	21	23	2	RandomExample10	40	9	11	2
CentredExample16	46	23	25	2	RandomExample11	40	8	10	2
CentredExample17	48	25	26	1	RandomExample12	40	9	12	3
CentredExample18	43	26	27	1	RandomExample13	42	9	11	2
CentredExample19	45	23	25	2	RandomExample14	44	10	13	3
CentredExample20	25	20	20	-	RandomExample15	24	10	13	3
CentredExample21	24	21	21	-	RandomExample16	28	12	13	1
CentredExample22	24	21	21	-	RandomExample17	27	10	12	2
CentredExample23	25	21	21	-	RandomExample18	26	10	13	3
CentredExample24	23	19	19	-	RandomExample19	30	13	14	1
CombinedExample0	164	20	24	4	RandomExample20	19	12	13	1
CombinedExample1	178	20	25	5	RandomExample21	16	8	10	2
CombinedExample2	162	24	27	3	RandomExample22	19	14	14	-
CombinedExample3	162	23	27	4	RandomExample23	16	10	11	1
CombinedExample4	164	22	25	3	RandomExample24	17	12	13	1

3.5.4 NOTE ON THE DISCRETE NATURE OF CHANNEL SEPARATION

Channel separations are made to ensure that SIRs above the service threshold are achieved under given assumptions. Consider an RTP currently receiving 8.5dB SIR in a network whose service threshold requirement is 9dB. A channel separation will be implemented to cause this SIR to increase above the service threshold SIR. Due to the discrete nature of channel separations, the SIR achieved will not be exactly 9dB but will in fact be much higher as the interference is significantly attenuated by the additional channel separation (table 3.1). A fractional channel separation leading to an SIR exactly equal to the threshold value cannot be made. This means that assignments, made to satisfy a given level of SIR quality, may achieve much higher SIRs and therefore give much better coverage than designed for, by 'coincidence'. Equally, an assignment may only marginally satisfy the service threshold throughout the network. This is due to the discrete nature of channel assignments and the continuous nature of the service coverage values attainable.

An assignment designed to achieve a given SIR throughout the network under single interference may achieve that SIR at many test points when evaluated under multiple interference, on an *ad hoc* or random basis. Thus when the performance of single interference assignments (and other assignments) is evaluated, it is beneficial to look at coverage results over many cases, to 'smooth out' this element of chance. Due to these issues it is desirable to consider many network instances when investigating multiple interference effects, and either discount the most extreme results or take averages.

In table 4.5, for example, it can be seen that the average network when assigned and evaluated for 15dB performs worse in terms of coverage than those same networks when assigned and evaluated for other SIRs, higher or lower than 15dB. This is not to say that a 9dB assignment provides better SIR to users than a 15dB assignment or that a 17dB assignment is easier to make than a 15dB assignment. Rather, an assignment is made to satisfy certain criteria and evaluated under these same criteria, comparing like with like.

This type of effect is noted by Whitaker *et al.* [43] who make assignment to satisfy binary constraints only. They discover that these assignments may also satisfy some of the non-binary constraints generated to improve the model, even though they have not been engineered to do so. This means that comparisons need to be drawn with caution. Constraints are needed which can guarantee a given level of coverage, rather than providing it on a random basis.

3.6 SUMMARY

The assumptions which will be used in the remainder of the thesis have been presented. These included assumptions about the the geographical nature of the networks, the manner of incorporating multiple interference into signal and interference calculations and the objectives to be used. Issues of generating test problems were discussed and a library of test cases created for use in the investigations which follow. The algorithms that will be used when assignments are required have been presented. These assignments are zero-violation for the given constraint system, and the algorithm aims to minimise the span of channels used.

Chapter 4

Analysis of Multiple Interference Effects

4.1 Introduction

In this chapter the single and multiple interferer assumptions are investigated in detail; quantifying firstly the total effects of additional sources of interference on downlink coverage, and also the proportion of these effects caused by interferers which are co-channel with the server or separated from it by a certain number of channels. The chapter aims to

(a) develop a technique for the analysis of multiple interference effects that can be applied to large networks and is conservative;

(b) investigate the reduction in coverage provided by assignments made under the single interferer assumption as more interferers are included in assignment evaluation i.e. are considered to be simultaneously active;

(c) investigate the effects of different types of interference (co-channel, adjacent channel) on downlink service coverage.

An investigation of this type has not been conducted previously. The most detailed previous investigation [43] considers the reduction in coverage provided by assignments made under the single interferer assumption when a complete interference model is considered i.e. when all transmitters other than the serving transmitter are considered to be active interferers at an RTP. In practice it is unlikely that all interferers will be active simultaneously throughout the day. It is also the case that some interferers will be sufficiently far away as to cause negligible interfering signal strength at an RTP. In their investigation, Whitaker *et al.* [43] analyse an assignment for the 95 transmitter test case from [44], made under the single interferer assumption. They consider all transmitters to be active and display a profile of those interferers which cause $> 5\%$ of the cumulative interference received at Voronoi points whose SIR value under this complete interference model falls below the service threshold σ. It is discovered that all such interferers are either co-channel or first adjacent channel with the serving transmitter, with co-channel interference being the dominant source of the problems encountered. The authors also locate the geographical positions of co-channel interferers appearing in this profile, finding that they reside in a middle distance range, between the region of locality in which binary constraints prevent channel sharing and the region of far distance in which geographical distance makes the signal strength negligible (i.e. $< 5\%$ of total interference received).

There is no unique way of analysing the effects of multiple interference. Analysis techniques must be developed in accordance with the requirements of researchers or projects. The chapter proceeds by presenting three analysis techniques, each of which motivates refinements to its successor.

Firstly, investigations are carried out to assess the tractability of producing potential sets of interferers for analysis. The first investigation (section 4.3) produces, via backtracking, all sets of transmitters for subsequent analysis under multiple interference. This technique is found to be computationally expensive, leading to the development of a refined second method which aims to remove from consideration some of the sets which were found to be redundant (see section 4.3.5) in the first investigation. This second method (section 4.4) generates, via backtracking, sets of transmitters which, if simultaneously active, cause excess cumulative interference. Transmitter sets are generated via backtracking once again, but are subject to a preliminary analysis upon their generation to determine whether they are *problem sets* under the multiple interferer assumption. Supersets of known problem sets are not generated for explicit consideration. This second method also has the capability to limit the arity of sets considered. With these refinements, tractability problems still arise, even for small numbers of interferers.The tractability issues prohibit the investigation of moderate to large test problems via this kind of interferer set generation.

To avoid generating all potential interferer sets for analysis, the final investigation technique (section 4.5) applies conservative assumptions which lead to the consideration of only the worst case interferer set of a given size at any time as the investigation proceeds. The determination of these worst sources involves considering the channel separations in the assignment under consideration as well as the geography of the scenario. The worst interferers may be co- or adjacent channel with the server. Further advantages of this approach are that it permits analysis of interferer effects in greater depth and can reveal the effects individual interferers have on the system.

4.1.1 NOTATION, TERMS AND ABBREVIATIONS

An RTP which receives adequate SIR under given assumptions is referred to as *satisfied* or *covered*; an RTP which does not, *failed*.

The abbreviation ISS is used for *interferer set size*; the number of potential interferers that are considered to be active in addition to a serving transmitter. So the situation in which the single interferer assumption is used to produce binary channel separation constraints effectively has ISS = 1.

During the course of the investigations which follow, the concept of SIR is used in several different manners. Thus, to avoid confusion, the following notation will be used:

- SIR_a is the *a*ssignment SIR; the SIR service threshold used to create the constraints from which the assignment under investigation was created.

- SIR_e is the *e*valuation SIR, the minimum SIR that is required for coverage, and is a parameter of the analysis.

- SIR_c is the SIR that is being achieved *c*urrently as calculations within the investigation are underway.

The following terms are defined for use when describing interference from different numbers of sources:

- *single interference* considers interference from each source individually and no cumulative interference effects;
- *multiple interference* considers interference from multiple simultaneously active interferers and their cumulative interference effects. The following abbreviated terms are also used for convenience:
  - *double interference* refers to interference from two simultaneous sources and their cumulative interference effects;
  - *triple interference* refers to interference from three simultaneous sources and their cumulative interference effects;
- *complete interference* considers all possible interferers to be simultaneously active i.e. cumulative interference from $(n-1)$ interfering transmitters where n is the number of transmitters in the network.

4.2 GENERATING BINARY CHANNEL SEPARATION CONSTRAINTS UNDER THE SINGLE INTERFERER ASSUMPTION

In this chapter, the strategy for experimentation involves generating binary channel separation constraints subject to the single interference assumption, and analysing zero-violation assignments for these constraints when different levels of multiple interference are considered. This section explains how such constraints are generated.

Binary channel separation constraints are generated under the single interferer assumption by considering interactions between pairs of transmitters only. Many reception test points RTP_i within a certain area containing transmitters t_j are considered, each of which has a serving transmitter (usually the transmitter offering the strongest signal). Assume that equipment at RTP_2 in figure 4.1 is required to receive a signal transmitted by t_4. The equipment will also be receiving signals from several interfering transmitters e.g. t_1. All the transmitters from which a signal is being detected are considered in turn. The signal strengths that would be received and hence the SIR for each server-interferer pair are calculated. Using these ratios, the minimum channel separation required between transmitters such that the SIR is adequate at each RTP is found.

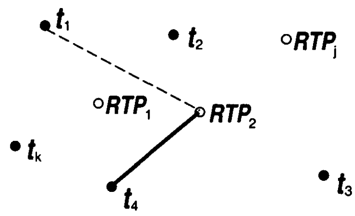

Figure 4.1: Reception test points receiving signal and interference

Input network (transmitters and RTPs)
Set $\phi_{jk} \leftarrow 0 \quad \forall \quad j, k$
for all RTP_i **do**
    $t_k \leftarrow \text{RTP}_i$'s serving transmitter
    for all t_j, $j \neq k$ **do**
        Set $\delta f \leftarrow 0$
        while $\text{SIR}(i, j) <$ desired threshold SIR **do**
            Increment δf
        end while
        if $\delta f > \phi_{jk}$ **then**
            Set $\phi_{jk} \leftarrow \delta f$
            Set $\phi_{kj} \leftarrow \delta f$ *(to ensure symmetry)*
        end if
    end for
end for

Algorithm 4.1: Generation of binary channel separation constraints

Algorithm 4.1 shows an algorithm for generating binary channel separation constraints under the single interferer assumption. In the algorithm, δf is the channel separation and is incremented as needed in each case for adequate SIR, and the overall required separations are stored in a constraint matrix ϕ. As described in section 2.2.2, the constraint between t_i and t_j is considered to be the same as that between t_j and t_i, so the constraint matrix ϕ is made symmetric by using the higher separation of the two that would appear in an asymmetric constraint matrix.

Note that the entry 0 in the computed matrix means that no separation is required and therefore the separations stored represent constraints of the form $| f_i - f_j | \geq \phi_{ij}$ where the use of $\geq$ makes this matrix equivalent to that in figure 2.2 in which n means no separation. Constraints of the form $| f_i - f_j | > \phi_{ij}$ may be produced by subtracting 1 from the computed entries representing required separations on the right hand side of the constraints.

4.3 EXHAUSTIVE CALCULATION OF INTERFERER SETS

This investigation aims to assess in terms of its tractability the possibility of producing interferer sets for analysis. An interferer set is defined to be a set of transmitters which causes, somewhere in a network, cumulative interference SIR higher than the service threshold when the assignment made under the single interferer assumption is evaluated for multiple interference.

A number of methods could potentially be used to calculate interferer sets. This section explains how sets can be generated for analysis by producing, via backtracking, all sets of transmitters for subsequent analysis under multiple interference. Once these subsets of transmitters are generated, they are considered in an RTP-centric manner to determine whether they are interferer sets.

4.3.1 TEST CASE ASSIGNMENT AND MEASUREMENT OF COVERAGE

Constraints are generated under the single interferer assumption via algorithm 4.1 for each of the six problems available from the Benchmark Generator of [44]. Assignments are produced for each problem which satisfy all of the binary constraints for single interference (zero-violation assignments) whilst minimising the span of channels used.

Coverage is measured in this investigation by the percentage of the Voronoi points (provided by the Generator along with the transmitter file) which receive SIR above the evaluation threshold under the current assignment and assumptions.

4.3.2 SIR CALCULATION

Computational time is saved by calculating and storing the signal strengths received from each transmitter at each RTP as these strengths are used repeatedly. For each RTP-transmitter pair, the actual signal strength is not stored, but rather the value of θ/d^γ where d is the distance between the RTP and transmitter under consideration. (Note that storing this value for the RTP and its serving transmitter is consistent, as the channel separation between the server and itself is zero, giving $\theta = 1$, multiplying by which does not alter the stored value.) Unnecessary calculation is also avoided by use of an alternate version of the cumulative SIR calculation as follows. The desired SIR threshold for coverage is denoted σ. At each RTP it is required that

$$\begin{aligned}
10\log_{10}\left[\left(\frac{\theta_k}{(d_{ik})^\gamma}\right) \div \left(\sum_{j\neq k}\frac{\theta_j}{(d_{ij})^\gamma}\right)\right] &> \sigma \\
\left(\frac{\theta_k}{(d_{ik})^\gamma}\right) \div \left(\sum_{j\neq k}\frac{\theta_j}{(d_{ij})^\gamma}\right) &> 10^{\sigma/10} \\
\left(\frac{\theta_k}{(d_{ik})^\gamma}\right) \div 10^{\sigma/10} &> \sum_{j\neq k}\frac{\theta_j}{(d_{ij})^\gamma} \\
M(i) &> \sum_{j\neq k}\frac{\theta_j}{(d_{ij})^\gamma} \qquad (4.1)
\end{aligned}$$

where $M(i)$ is a constant that is calculated for each RTP r_i under the assignment being investigated. The summation on the right hand side is halted once the inequality is no longer true, as it is then known that the desired service threshold cannot be achieved. (If the explicit SIR achieved is required specifically, then the summation in its entirety is completed.)

4.3.3 INTERFERER SET CALCULATION METHOD

Combinations of numbers (excluding the serving transmitter's number) of a particular cardinality are produced via backtracking. This exclusion of the serving transmitter number occurs by generating combinations from the integers in the range $[1, n-1]$, where n is the number of transmitters, then adding 1 to all the generated numbers equal to or greater than the tuned transmitter number.

Transmitters with these numbers then form transmitter sets which may be interferer sets. Once these sets are generated, analysis as in algorithm 4.2 can test whether each potential interferer set breaks the required SIR threshold under the current assignment and is indeed an interferer set.

At each RTP, all potential interferer sets are considered i.e. all subsets of transmitters other than the serving transmitter at that RTP. If any transmitter set causes an SIR lower than the required threshold, then that RTP *fails* and the transmitter set is then known to be an interferer set.

Algorithm 4.2 shows the method for a single ISS parameter value and a single SIR_e parameter value. The algorithm is called firstly with $SIR_e = SIR_a$ and ISS = 1. This must lead to 100% coverage in terms of satisfactory RTPs, as the assignment satisfied all binary constraints and is being analysed for the criteria for which it was guaranteed i.e. no interferer sets of cardinality 1 will be found. The algorithm is then repeatedly called with increasing SIR_e thresholds and ISS values to establish the interferer sets under each new test situation. A measure of coverage can also be found via a count of how many RTPs fail. The algorithm is called with ρ interferer set sizes, incremented by 1, and c SIRs, incremented by 1.0dB.

Output consists of files containing the information for each ISS and SIR_e pair, as well as an extra file containing overall results. Figures 4.2 and 4.3 show the type of output: figure 4.2 shows the individual output file for the 15 transmitter test instance, with $SIR_e = SIR_a = 9$dB and ISS = 2; figure 4.3 shows the overall output file for this same instance when $SIR_a = 9$dB and the algorithm is called for $\rho = c = 4$.

This chapter is concerned only with the effects of changing the ISS parameter and therefore does not consider further the results of analyses which use $SIR_e \neq SIR_a$.

```
Input network and assignment
Initialise SIR_e and ISS as required
Set problemCount ← 0
for each RTP do
  Set problemFound ← false
  Get combinations (size ISS, excl. tuned transmitter no.)
  for each combination found do
    if this transmitter set causes SIR_c < SIR_e at this RTP then
      Output the interferer set as required
      Set problemFound ← true
    end if
  end for
  if problemFound then
    Increment problemCount
  end if
end for
Coverage measure given by 100-(problemCount/numRTPs)*100
```

Algorithm 4.2: Exhaustive calculation of interferer sets

```
Using 9.0 dB SIR and interferer sets of size 2

RTP 23
--------------
Tuned to transmitter number 5 Problems caused by: { 4 6 }

RTP 30
--------------
Tuned to transmitter number 14 Problems caused by: { 3 5 } { 3 7 }

RTP 35
--------------
Tuned to transmitter number 13 Problems caused by:{ 1 9 }{ 7 9 }{
9 11 }

RTP 37
--------------
Tuned to transmitter number 13 Problems caused by: { 7 9 }

94.44444444444444% of RTPs succeed.
```

Figure 4.2: 15 transmitter test case assigned for 9dB SIR under single interferer assumption; output showing RTPs at which SIR is unsatisfactory when evaluated for 9dB SIR and two simulatneous interferers

```
Results for interferer set size 1 to 4 (rows), SIRs from 9.0dB in
increments of 1.0dB (columns) n.b. percentage values are truncated
```

100	95	88	84
94	86	84	81
88	84	84	80
88	84	84	79

Figure 4.3: 15 transmitter test case assigned for 9dB SIR under single interferer assumption; output showing percentage of Voronoi points covered when assignment is evaluated for different ISS and SIR_e parameter values

4.3.4 PRELIMINARY COVERAGE RESULTS

Whilst producing interferer sets, algorithm 4.2 also counts the numbers of RTPs at which problems were encountered, thus giving a measure of the coverage provided by this assignment. Figure 4.4 shows a sample of the coverage results produced for the 45 transmitter test case. The assignment made for SIR_a = 9dB under the single interferer assumption is evaluated when more than one simultaneous interferer is considered to be active. Figure 4.4 shows the percentage of Voronoi points receiving SIR above the required service threshold when ISS values from 1 to 5 are considered.

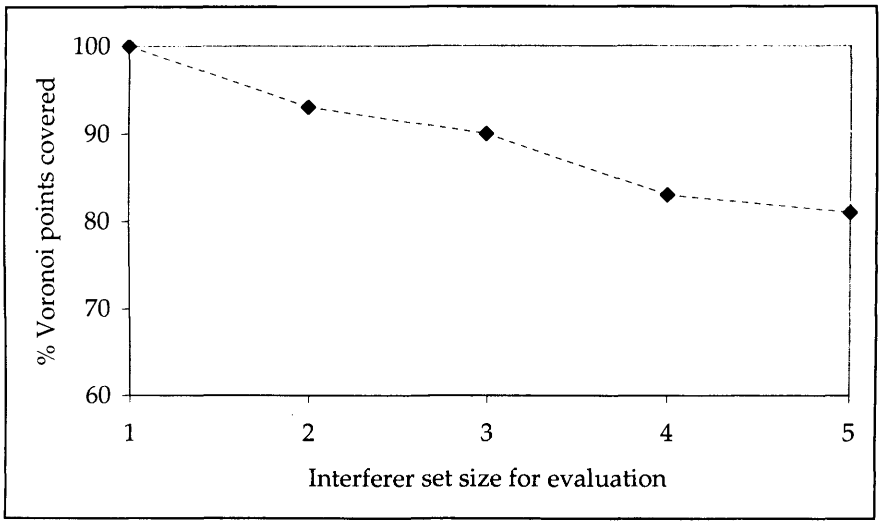

Figure 4.4: 45 transmitter test case assigned for 9dB SIR under single interferer assumption; percentage of Voronoi points covered when assignment is evaluated for 9dB SIR as the number of simultaneous interferers increases

The results achieved when assignments for the six test cases are investigated in this way can be seen in table 4.1, in which percentage values are truncated to integer values. Because the same required threshold SIR is used in assignment and evaluation ($SIR_e = SIR_a$), all coverage reduction effects seen in table 4.1 are due to the discrepancies between the single and multiple interferer assumptions. Results show that increasing the interferer set size leads to a reduction in the coverage provided by the assignment i.e. the single interferer assumption leads to an assignment with reduced operational coverage. Considering double interference immediately gives a decrease of at least 5% in these test case scenarios, and success drops towards 80% when ISS = 5.

% of RTPs covered	8 trans	15 trans	27 trans	45 trans	95 trans	458 trans
1 interferer	100	100	100	100	100	100
2 interferers	100	94	95	93	90	95
3 interferers	100	88	92	90	84	-
4 interferers	100	88	88	83	-	-
5 interferers	100	88	87	81	-	-

Table 4.1: Benchmark Generator test cases each assigned for 9dB SIR under single interferer assumption; percentage of Voronoi points covered when assignment is evaluated for 9dB SIR as the number of simultaneous interferers increases

Approx. runtime	8 trans	15 trans	27 trans	45 trans	95 trans	458 trans
1 interferer	< 1 sec	< 1 sec	< 1 sec	< 1 sec	<1 sec	5 sec
2 interferers	< 1 sec	< 1 sec	1 sec	1 sec	10 sec	20 mins
3 interferers	< 1 sec	< 1 sec	2 sec	17 sec	6 mins	-
4 interferers	< 1 sec	< 1 sec	12 sec	3 mins	-	-
5 interferers	< 1 sec	< 1 sec	1 min	27 mins	-	-

Table 4.2: Benchmark Generator test cases each assigned for $SIR_a = 9dB$ under single interferer assumption; runtimes when assignment is evaluated for $SIR_e = 9dB$ and $ISS = 1, \ldots, 5$

4.3.5 Evaluation

Table 4.2 shows the time taken for algorithm 4.2 to produce each of the results in table 4.1. Assignments created with $SIR_a = 9dB$ were evaluated for $SIR_e = 9dB$ using the version of the algorithm which does not calculate the SIRs explicitly but stops the calculations as soon as SIR_c is known to be over threshold (section 4.3.2).

Several experiments were not completed due to their memory requirements and therefore have no entries in tables 4.1 and 4.2. This intense use of memory, even for small problems, shows that the method of generating all transmitter sets in this way needs to be improved upon if analyses of medium to large test scenarios are to be performed.

If a set of transmitters is known to cause excess interference at an RTP, then any superset of that set will cause equal or greater interference at that RTP and will also be an interferer set. It is therefore wasteful to explicitly consider such supersets, as these supersets are effectively redundant. In this method, each run (ISS=h) does not know the results of the preceding run (ISS=$h-1$). This results in supersets of known problem sets being wastefully considered when they are already guaranteed to be interferer sets.

4.3.6 Conclusion

This method successfully produced interferer sets which could be analysed to give specific information about how and where problems occur within the network under the assignment being investigated. However, it has drawbacks in terms of computational expense and the repetition of calculation. This intractability leads to the development of a refined second method which aims to improve upon these aspects of the first method.

4.4 Refined calculation of problem sets

This investigation aims to improve upon the tractability problems encountered in the first investigation by analysing transmitter sets as they are produced, providing additional information to subsequent steps of the process, and by limiting the arity of sets considered.

A problem set is defined to be a set of transmitters which causes, somewhere in the network, coverage reduction due to the assignment made to the transmitters it contains. The set generation of this second investigation method aims to remove from consideration some of the sets which were found to be redundant in the first investigation. This is done by disallowing the generation of subsets of known problem sets.

4.4.1 Problem set calculation method

Test case assignment, coverage measurements and SIR calculation are as described in sections 4.3.1 and 4.3.2 and used in the first investigation.

A backtracking method (algorithm 4.3) is used to dynamically create combinations of numbers of different sizes from the (entire) set of transmitters. Each set produced is tested to see if it is a problem set and, if so, it is output and none of its

```
Input network and assignment
Set posn = 0
Set limit = min(num. trans, input limit on set size)
Initialise array of length limit to contain all -1s
while not finished do
  while posn < limit do
    while array up to posn is not all valid numbers in increasing order do
      Increment array[posn]
    end while
    if array contains no number > num. trans then
      if  array up to posn is a problem set then
        Store the problem set
        Remove any already occurring supersets from storage
        Increment posn
        break out of inner while (so as not to consider supersets)
      end if
    end if
    Increment posn
  end while
  Decrement posn
  Increment array[posn]
  while array contains a number > num. trans AND not finished do
    if posn is 0 then
      Finished
    else
      Set array[posn] = −1
      Decrement posn
      Increment array[posn]
    end if
  end while
end while
```

Algorithm 4.3: Refined calculation of problem sets supersets considered. However, the problem of discovering unnecessary supersets is not completely overcome, as will be explained.

A `limit` parameter is introduced which is an upper limit on the size of problem sets used and is employed for two reasons:

(a) To make computational time reasonable for the larger test problems. Considering every possible combination of interferers up to the size of the network is implausible for all but the smallest problem cases (see also table 4.3).

(b) To allow the investigation of specific characteristics of the assignment: to find out about double interference, limit to size 3 problem sets; limit 2 is equivalent to single interference.

When a transmitter set is generated by algorithm 4.3, each transmitter within the current transmitter combination is in turn set to be the tuned transmitter, and the remaining transmitters become an interferer set. For each RTP served by this tuned transmitter, it is investigated whether the assignment made results in an SIR below the required threshold caused by this interferer set. A problem caused by any of these (RTP, tuned, interferer set) triples makes the current transmitter set, as a whole, a problem set.

4.4.2 Evaluation

When assignments are made under the single interferer assumption, they may not be satisfactory when multiple interference effects are considered. The subsets of transmitters whose assignment causes the problems encountered are identified by this method. This information could be applied to altering the assignment in a manner similar to constraint-strengthening, or provide a set of transmitters between which a higher order constraint should be generated. This method improves upon the first by replacing repetitive, RTP-centric, calculation with redundant elements to determine interferer sets with calculation which is based more directly upon the sets themselves.

Table 4.3 shows the time taken for algorithm 4.3 to produce problem sets of cardinality up to a given limit for the six assignments under consideration. Assignments created with SIR_a = 9dB were evaluated for SIR_e = 9dB using the version of the algorithm which does not calculate the SIRs explicitly but stops the calculations as soon as SIR_c is known to be over threshold (section 4.3.2). The need for the upper limit on ISS can be seen in table 4.3: it would be unreasonable to allow the 458 investigation to run looking at set sizes up to 458.

Number of Transmitters	Limit	Approx. Runtime
8	8	<1 second
15	15	3 seconds
27	4	2 seconds
27	6	1 minute
45	4	21 seconds
45	6	half an hour
458	3	3 hours

Table 4.3: Benchmark Generator test cases each assigned for $SIR_a = 9dB$ under single interferer assumption; runtimes when assignment is evaluated for $SIR_e = 9dB$ and ISS $\leq$ given limit

This method avoids the issue of unwanted results to a certain extent, but not entirely. Redundant sets still occur if the algorithm finds a superset first as in figure 4.5. In this figure, the only problem set desired to be output is $\{6, 7, 8\}$ but, due to the order in which the transmitters appear in the input file, this subset of cardinality 3 is encountered *after* all of its supersets. Line 12 ("remove any already occurring supersets from storage") in algorithm 4.3 is introduced to avoid such output, but does not address its calculation in the first place.

4.4.3 CONCLUSION

This refined technique has advantages but these are outweighed by its drawbacks in terms of tractability, even for relatively small cases, and the redundancy issue is not entirely resolved. The sets produced by this technique and/or by the technique in section 4.3 can be further analysed to provide information on how many interferers are co-channel with the server etc., but this is not pursued due to the tractability problems.

```
Problem sets found:
 1   2   3   4   5   6   7   8
 1   2   3   4   6   7   8
 1   2   3   5   6   7   8
 1   2   3   6   7   8
 1   2   4   5   6   7   8
 1   2   4   6   7   8
 1   2   5   6   7   8
 1   2   6   7   8
 1   3   4   5   6   7   8
 1   3   4   6   7   8
 1   3   5   6   7   8
 1   3   6   7   8
 1   4   5   6   7   8
 1   4   6   7   8
 1   5   6   7   8
 1   6   7   8
 2   3   4   5   6   7   8
 2   3   4   6   7   8
 2   3   5   6   7   8
 2   3   6   7   8
 2   4   5   6   7   8
 2   4   6   7   8
 2   5   6   7   8
 2   6   7   8
 3   4   5   6   7   8
 3   4   6   7   8
 3   5   6   7   8
 3   6   7   8
 4   5   6   7   8
 4   6   7   8
 5   6   7   8
 6   7   8
```

Figure 4.5: Sample output illustrating generation of unwanted supersets *before* subsequent minimal problem set

4.5 Multiple interference analysis with conservative generation of interferer sets

The tractability issues arising in the techniques in sections 4.3 and 4.4 prohibit the investigation of moderate to large test problems via this kind of interferer/problem set generation. The preliminary results provided by the preceding investigations motivate further investigation into multiple interferer effects as operational coverage reduction can be experienced when single interference assignments are used. Further investigations are required which

- generate interferer sets in a more conservative manner to resolve issues of tractability
- consider more test problems for statistical validation;
- go on to reveal the details and sources of interference problems in the assignments evaluated.

The investigation presented in this section avoids the generation of all potential interferer sets for analysis by applying conservative assumptions which lead to the consideration of only the worst case interferer set of a given size at any time as the investigation proceeds. The determination of these worst sources involves considering the channel separations in the assignment under consideration as well as the geography of the scenario. The worst interferers may be co- or adjacent channel with the server; this approach permits analysis of interferer effects in greater depth and revelation of the effects of individual interferers on the system.

The investigation strategy involves the analysis of assignments which are zero-violation for binary channel separation constraints generated under the single interferer assumption. The library of test cases generated and documented in section 3.3 is employed to allow for statistical validation of results over many instances. Binary channel separation constraints for the library instances are produced via the single interferer assumption and algorithm 4.1. Assignments are then created to solve these constraints. All channel separations used in the evaluations are those which are present in the assignment under investigation and are not taken directly from the binary constraints matrix. Coverage is analysed for different numbers of active interferers, beginning with ISS = 1, whilst the SIR threshold remains at $SIR_e = SIR_a$.

The analysis strategy for a particular test case is described and results over the library of test cases with different parameters given. The assignment investigation strategy employed in this section hinges on a value called *saturation* (defined in section 4.5.2), which is calculated for each individual RTP in a network scenario. All results are then given as percentages of this value, and are therefore dimensionless, and commensurable. The strategy includes analyses of interference at individual RTPs and the average RTP as well as the effect of individual transmitters and the average interferer at failed RTPs. These results can be then averaged over the whole network and profiles built up to show the effects of numbers of interferers on coverage etc.

The test case analysis strategy for an assignment involves addressing:

- what happens at each individual RTP (section 4.5.4.2);
- what happens at the average RTP (section 4.5.4.3);
- what effect each transmitter has at each RTP (section 4.5.4.4);
- the average effect of each interferer over failed RTPs (section 4.5.4.4);
- the effect of the average interferer on the average failed RTP (section 4.5.4.5).

This technique allows analysis of assignments from several perspectives, extending the capabilities of work previously presented in the literature. The measures calculated can be averaged across many test case networks, to build up a picture of the effects of increasing downlink transmitter activity (increasing ISS) on interference experienced, and interferer potency. The measures used have not been applied previously and give a deeper insight into the effects of multiple interference.

4.5.1 Test case assignment

The investigation strategy involves the analysis of the performance, when multiple interferers operate, of channel assignments which are zero-violation for single source interference, over a range of test problems.

For each of the sixty problems available from the test case library documented in section 3.3, binary channel separation constraints are produced via algorithm 4.1. In each case, four sets of constraints are generated, requiring service threshold SIRs of SIR_a = 9dB, 12dB, 15dB and 17dB respectively. Typical QoS threshold values for GSM are 9dB and 14dB [48]. Zero-violation assignments are produced for each of the constraint sets, effectively making available two-hundred-and-forty assignment instances for analysis. These assignments violate none of the binary channel separation constraints produced and give satisfactory QoS at 100% of RTPs when one interferer in isolation is considered (single interference).

The assignments are created in the manner described in section 3.4.2. An exhaustive method is used in the case of small problems (those which have < 40 transmitters, SIR_a < 14dB), and a hybrid simulated annealing and sequential algorithm for other problems.

4.5.2 TEST CASE ANALYSIS

Algorithm 4.4 shows the method for analysing a test case assignment for an ISS and SIR_e parameter pair. The procedure of algorithm 4.4 is repeated over many test cases and with different parameters to complete the whole investigation. The steps and measures involved in the method are explained below.

STEP 1 : Take a zero-violation assignment.

This assignment has been made to violate none of the binary channel separation constraints generated by requiring $\text{SIR}_c \geq \text{SIR}_a$ at all RTPs, under the single interferer assumption.

STEP 2 : Specify an ISS for evaluation.

This is an integer value in the range $[1, n\text{-}1]$ where n is the number of transmitters in the network.

STEP 3 : Specify a value of SIR_e.

For the current investigation, this will remain at $\text{SIR}_e = \text{SIR}_a$. An alternative value can be used in future for other purposes.

```
Input network and assignment
for each RTP do
    Set saturation ← max. tolerable interference in Watts
    Find the ISS worst sources of interference
    Set totalInterferenceAtRTP ← 0
    for each worst source do
        Calculate Watts interference from this source
        Add this to totalInterferenceAtRTP
        Store this transmitter's interference as pc/sat at this RTP
    end for
    if totalInterferenceAtRTP > saturation then
        Record failure of this RTP
        Determine cause of failure (what pc/sat of each type of interference and
        from how many of this type of interferer)
    end if
end for
Average over all failed RTPs (what pc/sat of each type of interference and from
how many of this type of interferer, on average)
for each transmitter do
    for each failed RTP do
        if this transmitter ∈ this RTP's worst interferers then
            Determine effect this interferer has at this RTP (as what type of inter-
            ferer, providing what pc/sat)
        end if
    end for
    Average this interferer's effect over the system (as what type of interferer,
    providing what pc/sat, on average)
end for
Average over all interferers (how much of saturation is provided by the average
interferer of each type at the average RTP at which it contributes to failure)
```

Algorithm 4.4: Analysis of interference under a given assignment

STEP 4 : Work out the saturation *value at each RTP.*

Given the serving signal strength, *saturation* point interference is the maximum amount of interference, in Watts, which can be tolerated whilst $\text{SIR}_c \geq \text{SIR}_e$. For an RTP to be covered, it is required that

$$\text{SIR}_c \geq \text{SIR}_e$$

$$10\log_{10}\frac{\text{serving signal strength}}{\text{interference}} \geq \text{SIR}_e$$

So, at saturation point,

$$\begin{aligned} \text{SIR}_e &= 10\log_{10}\frac{\text{serving signal strength}}{\text{saturation}} \\ 10^{\frac{\text{SIR}_e}{10}} &= \frac{\text{serving signal strength}}{\text{saturation}} \\ \text{saturation} &= \frac{\text{serving signal strength}}{10^{\frac{\text{SIR}_e}{10}}} \end{aligned} \tag{4.2}$$

Taking this saturation value to be the 100% interference level, and pc/sat to abbreviate percentage of saturation point interference: at failed test points, >100 pc/sat will be experienced; at satisfied test points, ≤100 pc/sat.

STEP 5 : Find the ISS worst *sources of interference at the RTP under consideration.*

This covers the worst case interferer set scenario without wasting computational time on other sets. The determination of these worst sources involves considering the channel separations in the assignment under consideration as well as the geography of the scenario. The worst interferers may be co- or adjacent channel with the server.

The interaction between the two types of separation, channel and geographical, is illustrated by example 1 (see also [43]).

EXAMPLE 1

Under the propagation model presented in section 3.1.3.1, a co-channel signal from a transmitter 20m away will give signal strength

$$6.25 \times 10^{-6} \times \text{transmitter power}$$

However, if this distance is reduced to only 2m but the channel separation increased to 4, the signal strength would be

$$1.975 \times 10^{-6} \times \text{transmitter power}$$

Although the transmitter in the second instance is 10% of the first distance away, i.e. significantly closer geographically, the larger channel separation reduces the received signal strength greatly, resulting in the second situation having the lower received signal strength.

STEP 6 : Determine whether the RTP fails.

Calculate the interference received from each of the ISS worst sources of interference as a percentage of saturation point interference. The summation of these received signal strengths (in Watts) will determine whether the RTP is covered under this assignment.

STEP 7 : Count how many RTPs fail over the network.

The coverage may be expressed as a percentage of the total number of RTPs (see also section 4.5.3).

STEP 8 : For each failed RTP, analyse the interference received.

Calculate, as a percentage of the saturation value at that RTP,

- total excess interference
- total co-channel interference
- total first adjacent interference
- total second adjacent interference
- total interference from other (further) channel separations

This represents the combined effects of interferers, classified by channel separation with the server.

STEP 9 : Average over RTPs.

Across all failed RTPs, average the above measures and determine the standard deviation.

STEP 10 : Consider the contribution of each transmitter at each RTP.

Quantify the interference from each transmitter, as a percentage of the saturation value at the RTP, noting whether it occurs at that RTP as a

- co-channel interferer
- first adjacent interferer
- second adjacent interferer
- other type of interferer

STEP 11 : Consider the composite contribution of individual interferers.

This will facilitate the determination of the average effect that interferers have on RTPs at which they contribute to failure, classified by channel separation with the server.

Each transmitter has some measurable effect at each RTP. A particular interferer belongs to the set of ISS worst interferers of some RTPs. At some of these, it occurs as a co-channel interferer; at others, first adjacent, etc. The potency of a transmitter in its role as each type of interferer is pertinent here.

STEP 12 : Average over interferers.

Average the above measures across all interferers for failed RTPs.

4.5.3 MEASURING COVERAGE

4.5.3.1 Percentage of RTPs

Coverage is firstly measured as the percentage of RTPs which receive satisfactory SIR under the current assignment and evaluation assumptions. The objective is 100% coverage by this measure (section 3.1.4). This objective is appropriate because RTPs are selected to be representative of the whole network, and when 100% of RTPs are covered, then QoS is satisfactory throughout the network.

If a percentage of RTPs is used to measure failure over the system, the measure is dependent on the method used to place RTPs. For example, the method used in the benchmark generator of [44] produces fewer RTPs than that used in generating the test case library. Thus the same qualitative failure would give a significantly different percentage; potentially misleading if used for comparison. An alternative measure of failure, which is independent of RTP placement, is therefore used in addition.

4.5.3.2 Percentage of transmitters

This second measure involves using a percentage of transmitters to illustrate the coverage achieved. The number of *transmitter failures* is defined to be the number of transmitters which serve at least one failed RTP. In this way, a transmitter is considered to have *failed* if it does not fulfil its required function i.e. cover the RTPs it is required to serve. A similar concept of using a transmitter-centric evaluation is used by Whitaker *et al.* [43] who describe a transmitter as *violating* if it is included in a constraint which is violated by the assignment being evaluated.

The percentage of transmitters which fail can then be used as a measure of coverage. Although transmitters cannot be 'covered', the term *transmitter coverage* is used and is defined via the relationship

$$\text{percent coverage} = 100 - \text{percent failure}$$

The transmitter coverage then demonstrates what proportion of the network's transmitters successfully serve all their respective tuned RTPs.

The objective is 100% coverage by this measure. This objective is equivalent to the objective of 100% RTPs coverage, as each measure gives the objective value of 100% under the same conditions. If all transmitters serve all their tuned RTPs then all RTPs are covered and vice versa.

The measurement of transmitter coverage is especially useful as assignments are made to transmitters, and this measure shows how many of those assignment choices are successful in facilitating the transmitters' fulfilment of their requirement to cover all tuned RTPs.

Assuming RTPs are placed well enough to characterise the situation, this measure is independent of their actual method of placement, and it is therefore useful as an additional measurement for comparison purposes.

4.5.4 SAMPLE TEST CASE ANALYSIS RESULTS

This section presents an example of the type of results obtainable from the analysis of an individual problem instance assignment.

4.5.4.1 Case study example

The test case used in this section is the 15 transmitter case from the Benchmark Generator [44], which consists of 15 transmitters and the corresponding RTPs. The assignment produced exhaustively from binary channel separation constraints generated under the single interferer assumption for $\text{SIR}_a = 9\text{dB}$ is shown in table 4.4. This example shows evaluation of the assignment with $\text{SIR}_e = \text{SIR}_a = 9\text{dB}$ and $\text{ISS} = 6$.

Transmitter		Channel	Transmitter		Channel
1	⟶	8	9	⟶	7
2	⟶	10	10	⟶	1
3	⟶	5	11	⟶	8
4	⟶	3	12	⟶	3
5	⟶	4	13	⟶	7
6	⟶	4	14	⟶	5
7	⟶	6	15	⟶	9
8	⟶	2			

Table 4.4: Zero-violation assignment for the 15 transmitter case from the Benchmark Generator, made under the single interferer assumption

4.5.4.2 Analysis at an RTP

Analysis is first performed at RTPs. At each failed RTP, the output is the saturation value at the RTP, the actual interference experienced, and by what pc/sat the RTP fails. Also displayed is a breakdown of this interference by channel separation with the server, including the numbers of interferers causing each type of interference. At a failed RTP, the detailed output is as exemplified in the sample given in figure 4.6.

Figure 4.7 shows the types of interference occurring at each failed RTP. Also of interest is from how many individual interferers the total amount of each type of interference originates. This is plotted in figure 4.8 for the same case.

```
Looking at RTP 22, tuned to transmitter 4
------------------------------------------
        Saturation:  7.0230E-16 Watts
        Total interference (worst 6 interferers):  7.4440E-16 Watts
        RTP FAILS (with 5.9 pc/sat excess interference)
    At the failed RTP 22, interferences are as follows:
    66.0 pc/sat co-channel (1 interferers)
    38.8 pc/sat 1st adjacent (3 interferers)
    1.0 pc/sat 2nd adjacent (2 interferers)
    0.0 pc/sat further (0 interferers)
```

Figure 4.6: Output of detail at individual RTP

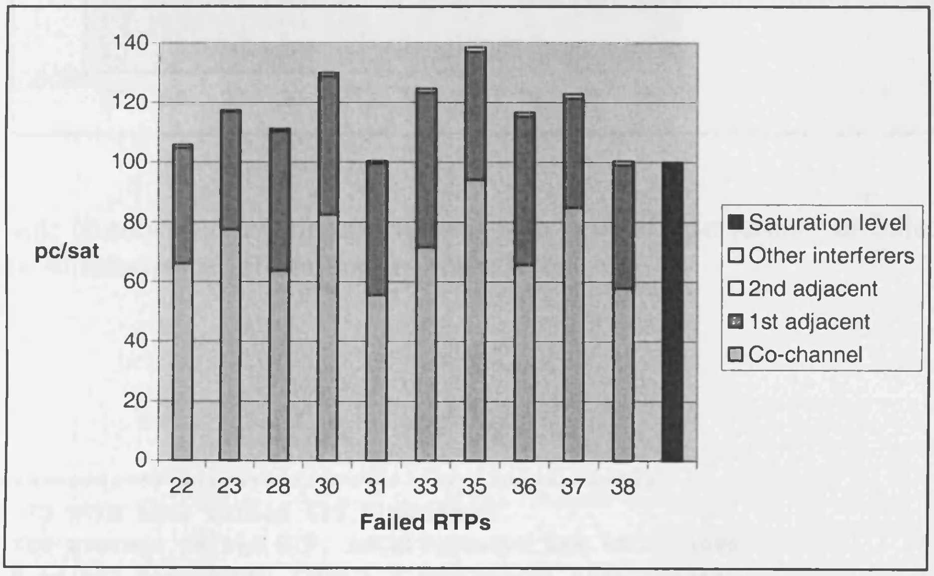

Figure 4.7: Quantifying different types of interference at failed RTPs when six simultaneous interferers are considered

4.5.4.3 Analysis of failures

The detailed output displays the profile of each failed RTP, followed by the profile of the average failed RTP. The average amount of interference of each type is displayed, along with the standard deviations. This file also contains failure rates in terms of the number of failed RTPs and the number of failed transmitters. A sample of this output appears in figure 4.9, which does not show the output of individual RTPs, but displays the average failed RTP profile and the failure rates.

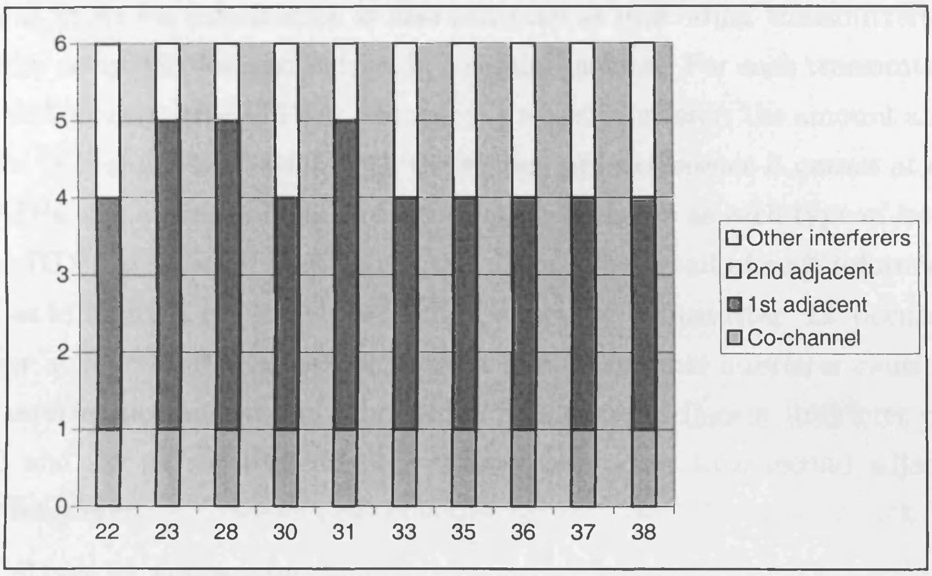

Figure 4.8: Numbers of interferers causing each type of interference at failed RTPs when six simultaneous interferers are considered

```
***************************************************
 10 of 72 RTPs have failed (13.8 percent)
   At the average failed RTP, interferences are as follows:
   71.9 pc/sat co-channel from 1.0 co-channel interferers
   43.8 pc/sat 1st adjacent from 3.3 1st adjacent interferers
   1.0 pc/sat 2nd adjacent from 1.7 2nd adjacent interferers
   0.0 pc/sat further from 0.0 further interferers
   Standard deviations:
   11.8 pc/sat co-channel
   4.6 pc/sat 1st adjacent
   0.3 pc/sat 2nd adjacent
   0.0 pc/sat further
***************************************************

***************************************************
 4 of 15 transmitters have 'failed'
 this is 26.6 percent of the network
***************************************************
```

Figure 4.9: Averaging over RTPs

4.5.4.4 Analysis of an interferer

As well as at RTPs, information is also collected at individual transmitters, when they occur as interferers, and output in a similar fashion. For each transmitter, the output will contain: the RTPs at which it is a worst interferer; the amount and type (in terms of channel separation with the server) of interference it causes at each of those RTPs; the average amount of interference it causes as each type of interferer over the RTPs at which it contributes to failure. The detail of such information is output as in figure 4.10. In this particular example, transmitter '12' occurs as an interferer at four RTPs. At one of these it is a co-channel interferer causing 66.0 pc/sat interference and at the other three it is a first adjacent interferer causing 2.0, 2.3 and 2.9 pc/sat respectively. It does not occur as a second adjacent or further interferer.

```
Looking at effect of transmitter 12
-------------------------------------
   At RTP 22, this is a co-channel interferer causing 66.0 pc/sat interference
   At RTP 23, this is a 1st adjacent interferer causing 2.0 pc/sat interference
   At RTP 28, this is a 1st adjacent interferer causing 2.6 pc/sat interference
   At RTP 31, this is a 1st adjacent interferer causing 2.3 pc/sat interference
 This transmitter causes an average of 66.0 pc/sat interference as a co-channel interferer
 This transmitter causes an average of 2.3 pc/sat interference as a 1st adjacent interferer
 This transmitter does not occur as a 2nd adjacent interferer
 This transmitter does not occur as a further interferer
```

Figure 4.10: Output of detail at individual interferer

A plot of the type and amount of interference caused by the transmitters in this example reveals the dominant interferers as in figure 4.11.

4.5.4.5 The average interferer

The pc/sat caused by the average interferer of each type is also recorded; or that there are no interferers of that type. The average interferer is displayed as in figure 4.12.

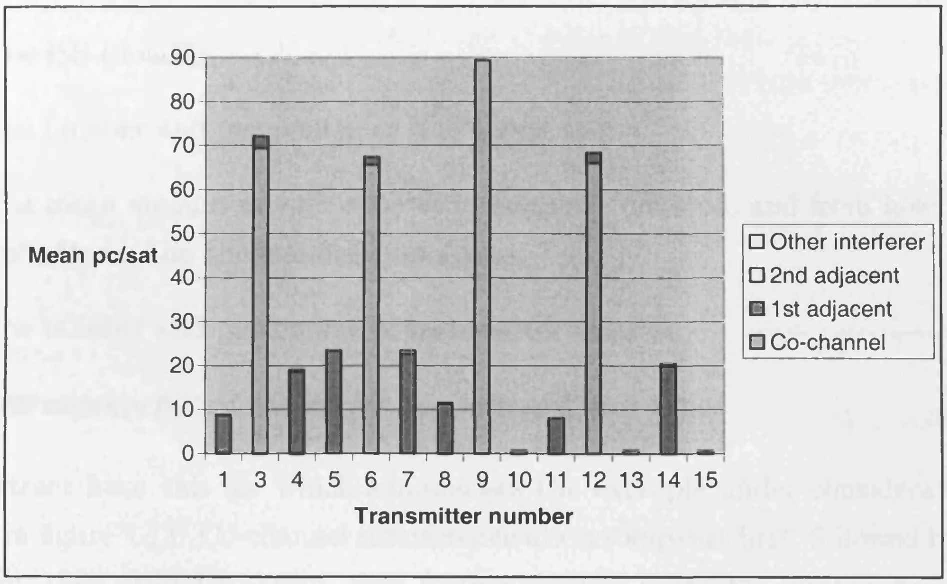

Figure 4.11: Mean effect of interferers over failed RTPs

```
****************************************************
The average co-channel interferer causes 71.9 pc/sat interference
The average 1st adjacent interferer causes 13.2 pc/sat interference
The average 2nd adjacent interferer causes 0.6 pc/sat interference
There are no further interferers
****************************************************
```

Figure 4.12: Averaging over interferers

4.5.4.6 Summary output

All the averaged results are also output to a summary file which contains a section for each ISS value evaluated. Each section contains:

- the ISS ('load');
- the number and percentage of RTPs that fail;
- the mean amount of each type of interference received, and from how many interferers, and the standard deviation;
- the number and percentage of transmitter failures;
- the average pc/sat caused by interferers of each type.

The extract from this file which summarises the example under consideration is shown in figure 4.13. Co-channel interference always appears first, followed by first adjacent, then second adjacent, then further channel separated interference.

```
6               load
10              RTPs failed
13.8            %
71.9            mean
43.8            mean
1.0             mean
0.0             mean
1.0             of type
3.3             of type
1.7             of type
0.0             of type
11.8            stdev
4.6             stdev
0.3             stdev
0.0             stdev
4               trans failures
26.6            % of network
71.9            pc/sat
13.2            pc/sat
0.6             pc/sat
0
```

Figure 4.13: Extract from overall output file

4.6 DISCUSSION OF RESULTS FOR LIBRARY TEST CASES

4.6.1 TRACTABILITY

This investigation has removed the tractability problems seen in the techniques used in sections 4.3 and 4.4 by applying conservative assumptions and generating only worst case interferer sets of given sizes. This improvement is demonstrated by the runtimes of the analysis process and the fact that assignments for the larger instances of the test case library have been successfully analysed.

- It takes seven minutes to run the analysis on all two-hundred-and-forty library problems (sixty networks each assigned for four SIR_a values), each for $ISS = 1, \ldots, 6$.
- It takes eleven and a half minutes to run the analysis on all two-hundred-and-forty library problems (sixty networks each assigned for four SIR_a values), each for $ISS = 1, \ldots, 10$.
- It takes seven and a half minutes to run the analysis on all two-hundred-and-forty library problems (sixty networks each assigned for four SIR_a values), each for $ISS = n - 1$ where n is the number of transmitters in the network being analysed i.e. evaluation is for complete interference.

4.6.2 COVERAGE PROVIDED BY SINGLE INTERFERER ASSUMPTION ASSIGNMENTS

This investigation has among its aims the quantification of the coverage provided by assignments, which are zero-violation for binary channel separation constraints generated under the single interference assumption, when multiple simultaneous interferers are considered active and cumulative interference effects taken into account.

Each of the library cases assigned for single interference is evaluated for complete interference. This means that cumulative interference effects from all transmitters other than the serving transmitter are considered at any point in the network. Evaluation is also performed considering cumulative interference from different numbers of simultaneous interferers (values of ISS).

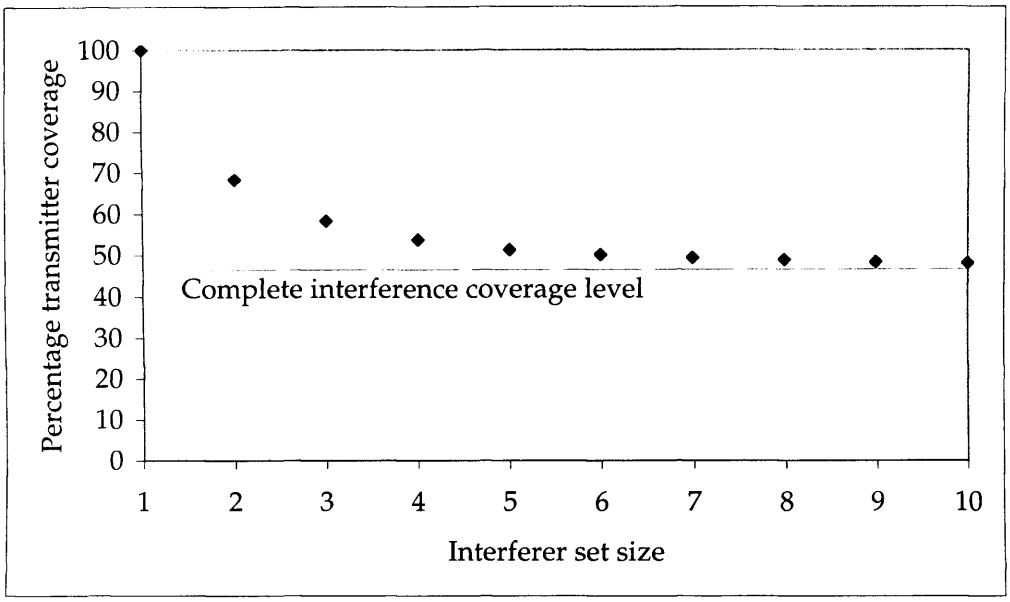

Figure 4.14: Analysis of the single interferer assumption when multiple interference is considered. The plot shows the transmitter coverage as the ISS increases, averaged over 240 library test cases, and the equivalent coverage level when all possible interferers are active.

Figure 4.14 plots coverage values when different numbers of interferers are considered active, averaged over the two-hundred-and-forty test case assignments. In figure 4.14, the coverage level achieved when complete interference is evaluated is shown as a dashed line. Coverage values, when evaluations are performed for different interferer set sizes, reveal that even at ISS = 2, the drop in coverage from that claimed under the single interferer assumption is not negligible. The decrease continues, tending to level out as the ISS value evaluated increases, as the 'next worst' interferers being added to the scenario become less significant. The complete interference coverage level in figure 4.14 shows 45.0% of transmitters covering all their RTPs when all interferers are considered to be simultaneously active. The percentage of transmitters covering all their RTPs is 49.2% when cumulative interference from six simultaneous interferers is considered; a difference of 4.2% from the complete interference value. Each further increase in the interferer set size alters the coverage measure by less than 1%.

Note that although the two measures of coverage used (section 4.5.3) give different percentage values, they each give the objective value of 100% under the same conditions. The trend of the decrease in coverage is essentially the same whether

RTP or transmitter coverage is used. Figure 4.14 showed average values over the two-hundred-and-forty test case assignments evaluated via transmitter coverage. The equivalent plot for coverage as a percentage of RTPs is given in figure 4.15. The values of percentage coverage achieved, with results averaged over the assignments for each SIR_a, are tabulated in table 4.5. In general, results are also consistent over the different problem types (distributions of transmitter locations) (figure 4.16). The maximum and minimum values of coverage achieved often occur in the regular grid examples. This is because regular instances compound the issue raised in section 3.5.4; if SIR values achieved in one region are very close to the threshold value, then this is likely to be repeated across the regular network, giving an assignment which is 'only just' satisfying the assumptions under which it was generated and which will therefore perform poorly under stronger assumptions; likewise, if SIR values are high in one region, they are likely to be so throughout the regular network, giving an assignment which may perform very well under stronger assumptions than those for which it was designed.

As figure 4.16 classifies the results by the different transmitter distributions, so figure 4.17 classifies them by service threshold SIR. Figure 4.17 requires careful interpretation bearing in mind the note of section 3.5.4; the 15dB cases on average give worse coverage as ISS increases than occurs at the other three threshold SIRs. This is not to say that assigning for a 17dB threshold is 'easier' than assigning for 15dB. The assignments under consideration were created to solve constraints whose generation considered a single interferer. The resulting assignments lead to satisfactory SIR (i.e. at or above threshold SIR) at all test points in the network. The threshold SIR may have been met exactly or slightly exceeded, or it may have been substantially bettered. If the achieved SIR was close to threshold, then coverage will fall off steeply as it is evaluated under stronger assumptions; if the achieved SIR was much higher than threshold, the assignment will be more robust to stronger interference conditions (higher ISS) and therefore coverage levels will be higher even though this was not directly engineered or guaranteed.

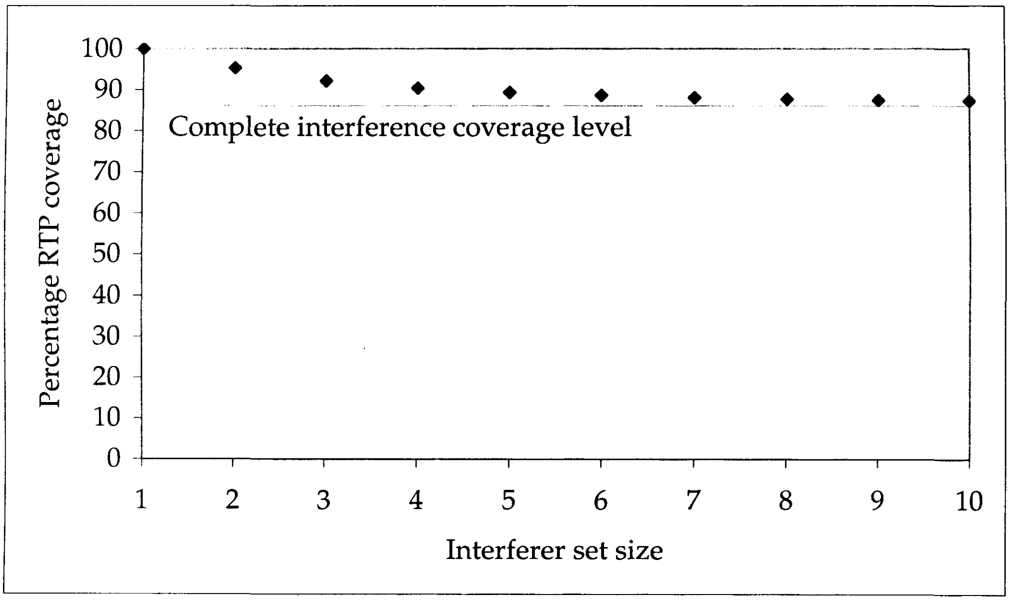

Figure 4.15: Analysis of the single interferer assumption when multiple interference is considered. The plot shows the RTPs coverage as the ISS increases, averaged over 240 library test cases, and the equivalent coverage level when all possible interferers are active.

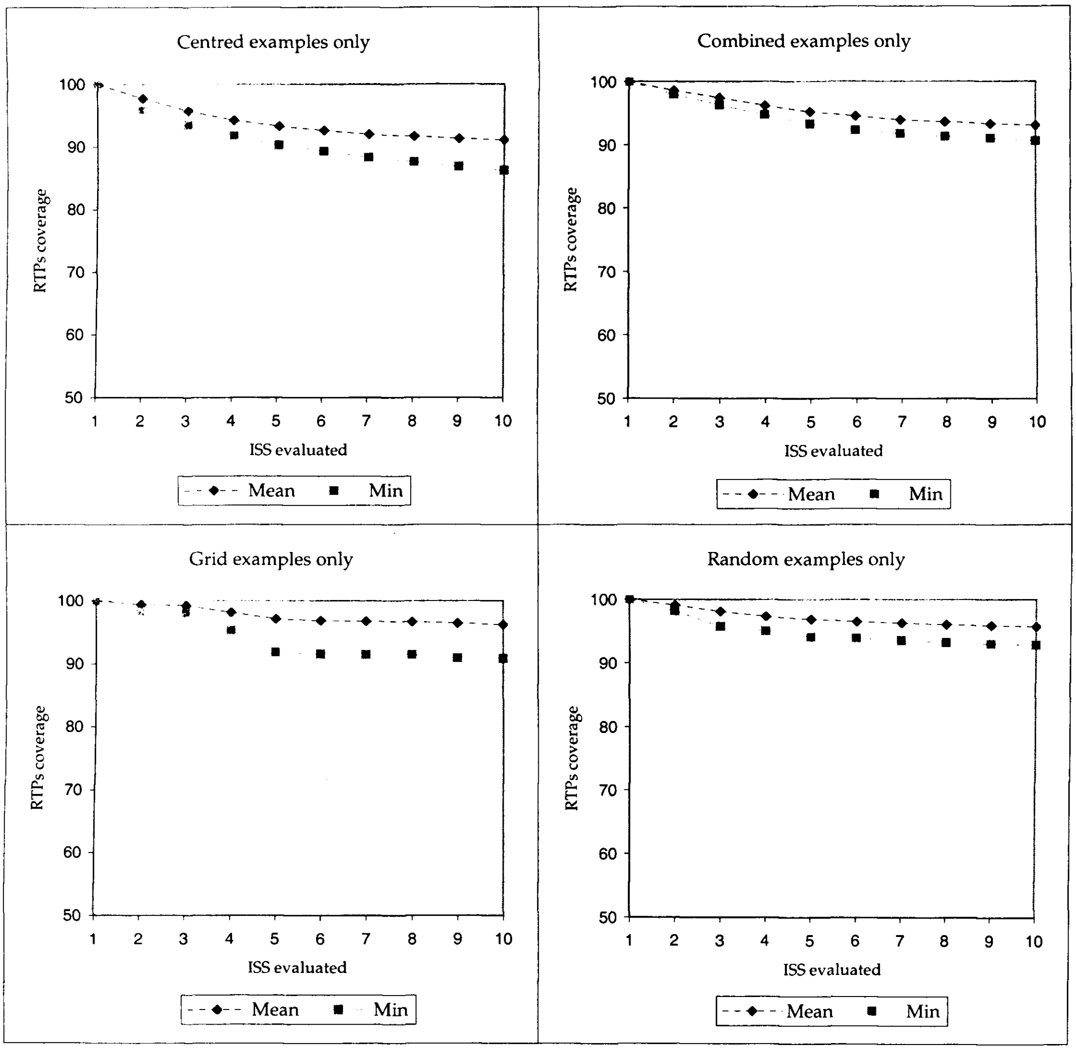

Figure 4.16: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 9dB service threshold SIR. Mean RTP coverages are plotted, classified by test case transmitter distribution when the assignment is evaluated for $SIR_e = SIR_a$ and ISS=1, ..., 10.

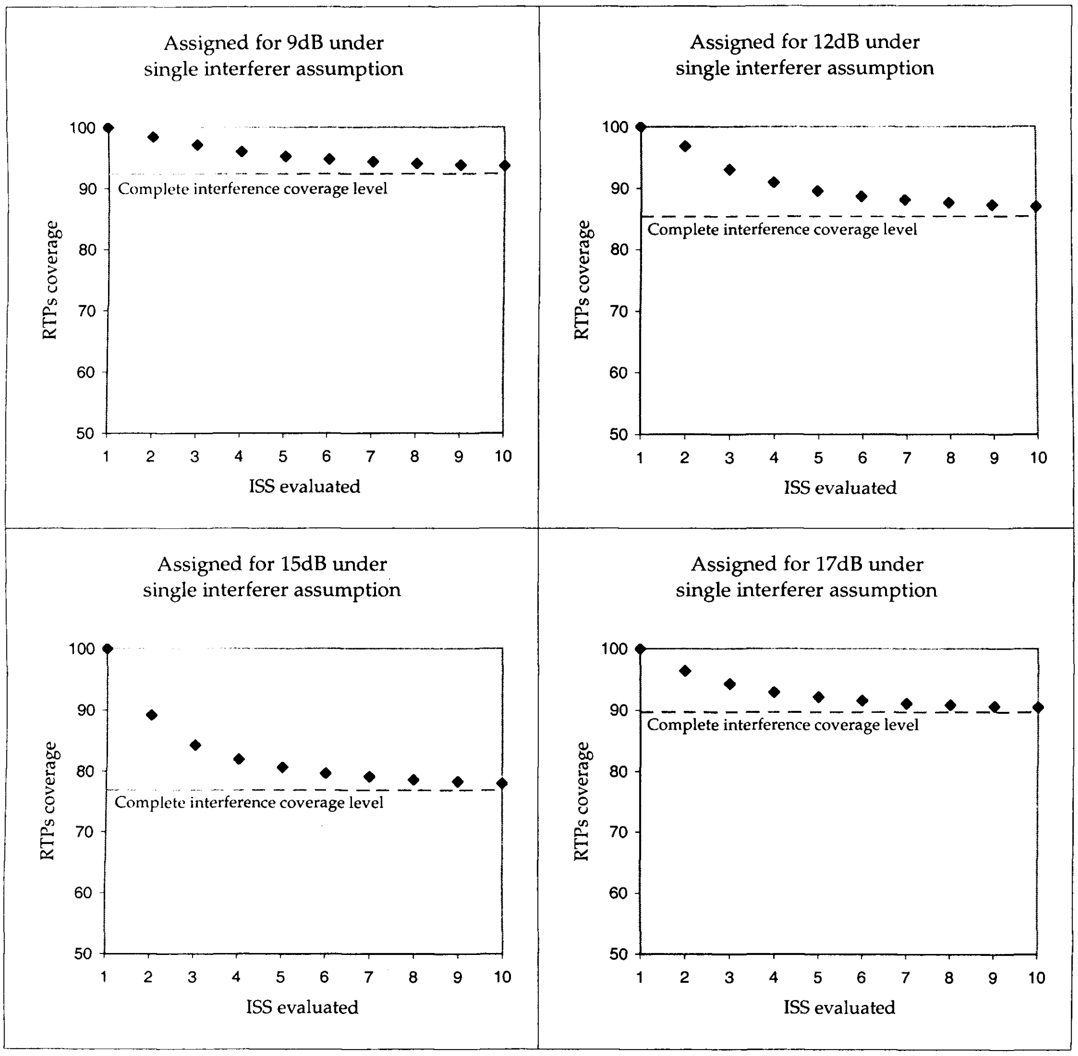

Figure 4.17: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 9dB, 12dB, 15dB and 17dB service threshold SIR. Mean RTP coverages are plotted for each, when the assignment is evaluated for $SIR_e = SIR_a$ and ISS$=1, \ldots, 10$.

Sixty library cases assigned and evaluated for SIR	% transmitter coverage			
	Single interferer assumption	Multiple interference with:		
		2 interferers	6 interferers	$n-1$ interferers
9dB	100	84.6	64.4	58.3
12dB	100	71.0	45.4	40.7
15dB	100	39.8	27.5	25.1
17dB	100	74.8	59.5	55.9

Sixty library cases assigned and evaluated for SIR	% RTP coverage			
	Single interferer assumption	Multiple interference with:		
		2 interferers	6 interferers	$n-1$ interferers
9dB	100	98.5	94.8	92.3
12dB	100	96.8	88.6	85.4
15dB	100	89.2	80.0	76.9
17dB	100	96.4	91.5	89.7

Table 4.5: Analysis of the single interferer assumption when multiple interference is considered. The tables show transmitter coverage and RTPs coverage respectively for single interference, double interference and six simultaneous interferers, averaged over the sixty library test cases at each SIR used, and the equivalent coverage level when all possible interferers are active.

Figure 4.18 shows that, even when only one additional interferer is considered, the assignment made for single interference is already causing failure to some degree at all but two of the sixty library cases assigned for 17dB. (Note that when plots are given for the sixty library cases at a certain $\text{SIR}_a = \text{SIR}_e$ this choice of SIR is for illustration purposes only, as all two-hundred-and-forty cases cannot be successfully displayed on some of the plots used. Trends are the same throughout the experiments performed and any differences are stated.)

Note that although the bars in figure 4.18 seem to suggest that in some cases one of the two interferers causes $> 100\%$ of saturation point interference, this is not the case, as this would mean failure caused by a single interferer. This is an average over all failures, some of which are caused by a pair of one type of interferer, thus giving the average of one type the ability to be greater than 100%. No single interferer causes $>$ 100pc/sat interference on its own under the assignment being considered: if it did, it would have motivated the generation of a binary constraint under single interference, been mitigated for by the assignment, and not appear here.

4.6.3 Sources of interference

Having quantified total received interferences, the chapter also aims to discover from where the problems encountered originate i.e. from interferers separated by what number of channels from the server.

Looking at individual cases such as the example seen in section 4.5.4 reveals that the majority of interference is co-channel (figure 4.7), and that usually a single co-channel interferer is the source of a large proportion of the interference (figure 4.8).

Figure 4.11 also emphasises that the dominant interferers are co-channel with the server. It can be seen in figure 4.18 that all failures are caused by pairs of interferers which are either both co-channel with the server, both first adjacent, or one of each. Second adjacent and further interferers do not appear at all. In fact, on average over the failures, the interference of 119 pc/sat is attributed to 82.8 pc/sat co-channel interference combined with 36.2 pc/sat first adjacent and zero contribution from any other type.

In figure 4.19, it can be seen that the main contributors to failure are again co-channel and first adjacent interferers, with second adjacent and further interferers having almost no effect. The interferers further away than first adjacent only appear because display of the worst six interferers is being insisted upon. Comparison with figure 4.18 emphasises that it is the co-channel and first adjacent interference which is influential. Note that for each library test case, the worst two of the six interferers in figure 4.19 will be the two which appear in figure 4.18. To facilitate this comparison, the same scale is used on the pc/sat axes of these two figures.

Plotting the numbers of interferers causing each type of interference shows that co-channel interferers are in the majority (e.g. figure 4.20). These numbers of interferers are averaged over the library cases in figures 4.21 4.22, showing the dominance of co-channel interferers, the secondary presence of first adjacent interferers and the much less significant role of second adjacent and further separated interferers. The interferer type most likely to cause interference is co-channel with the server.

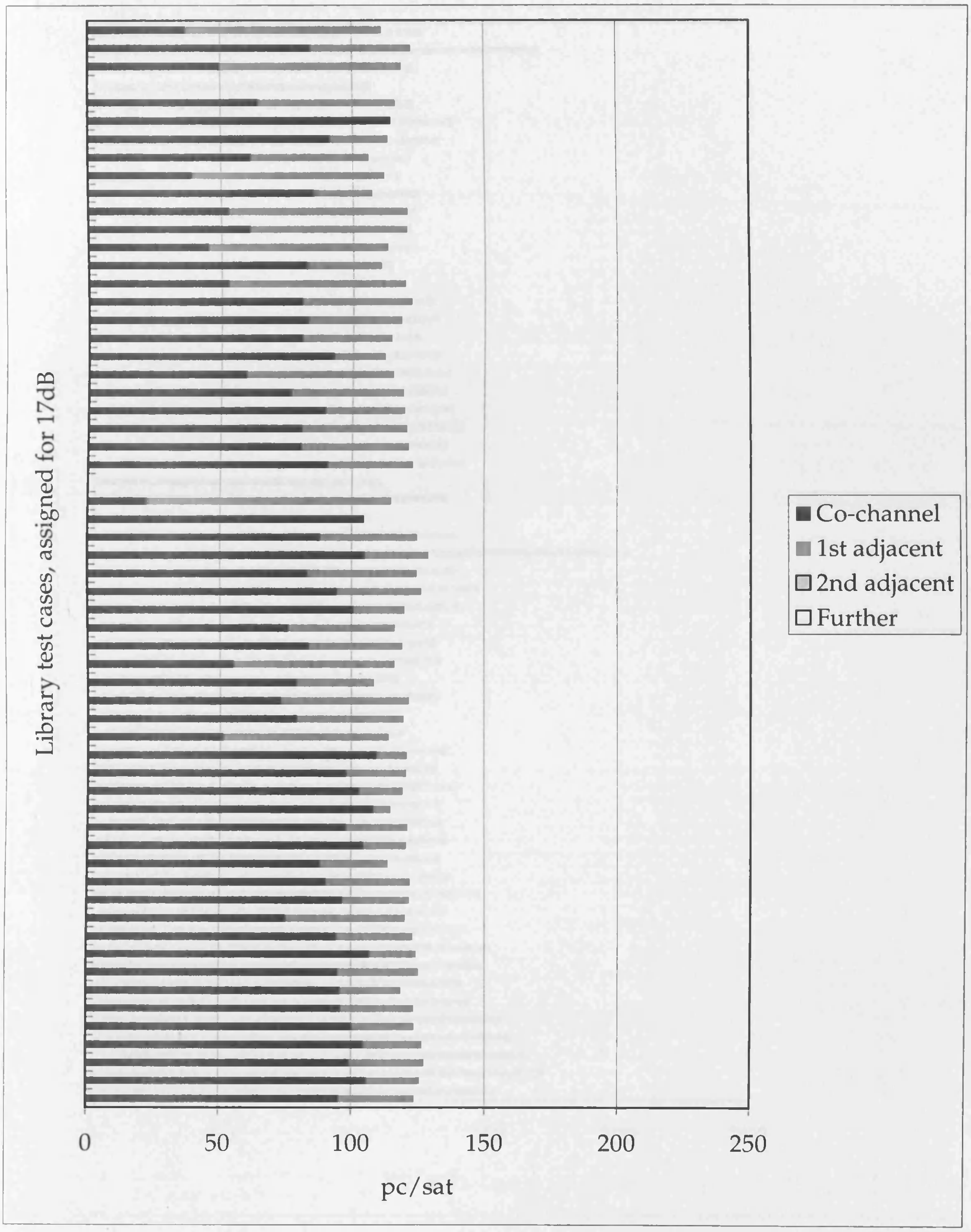

Figure 4.18: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; interference received by the average failed RTP from its worst two interferers is plotted

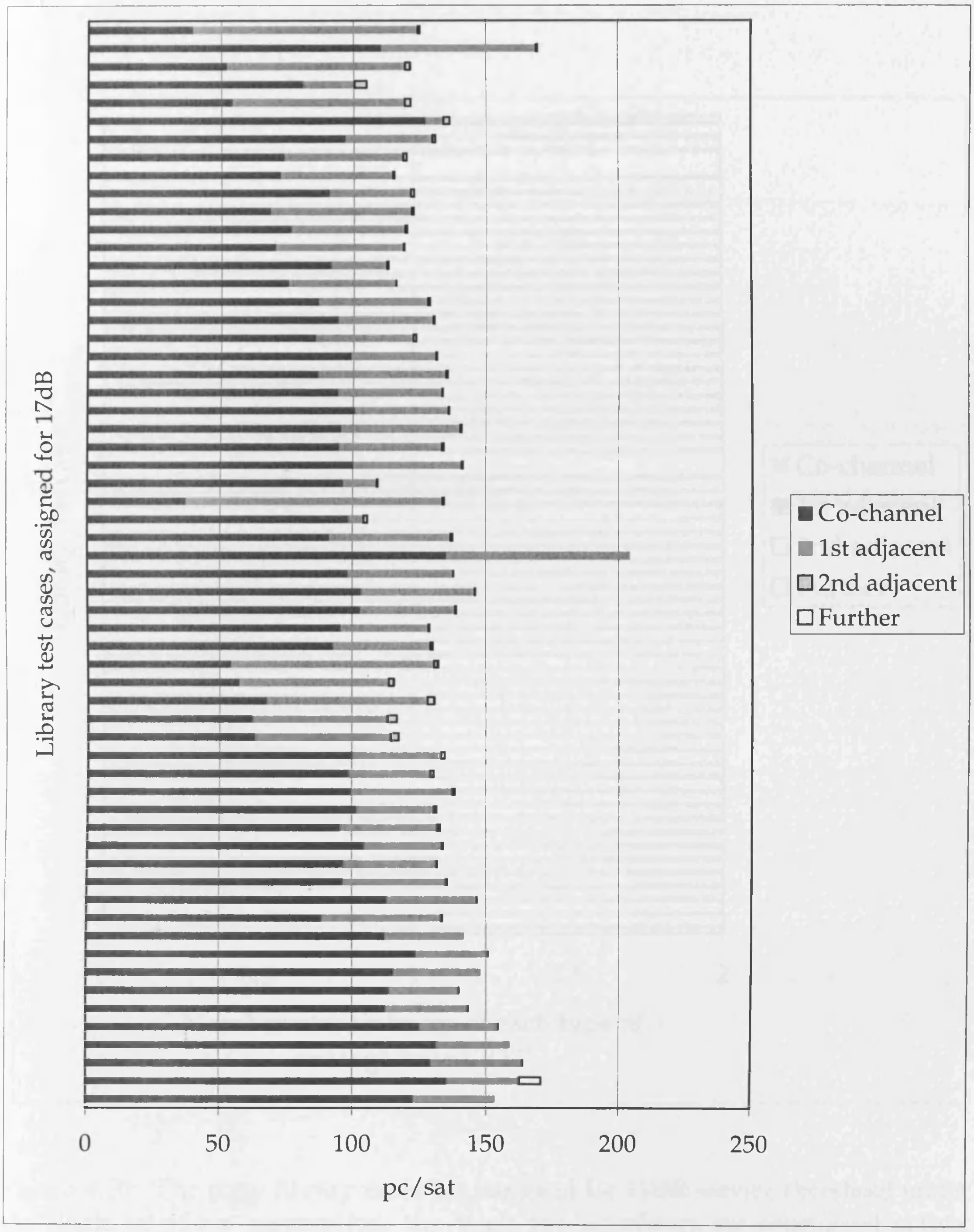

Figure 4.19: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; interference received by the average failed RTP from its worst six interferers is plotted

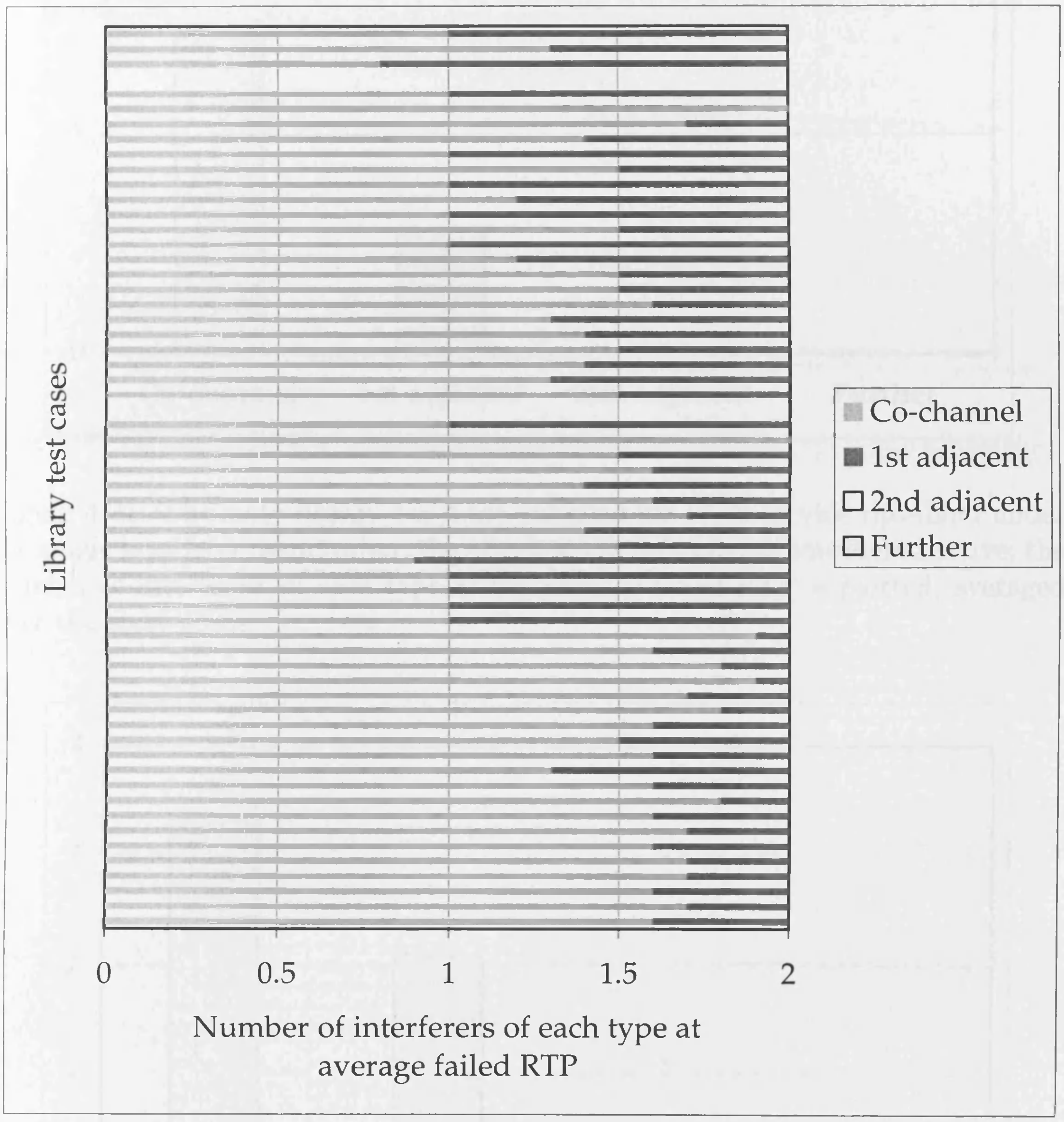

Figure 4.20: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; the worst two interferers are considered active; the number of interferers of each type at the average failed RTP in each case is plotted

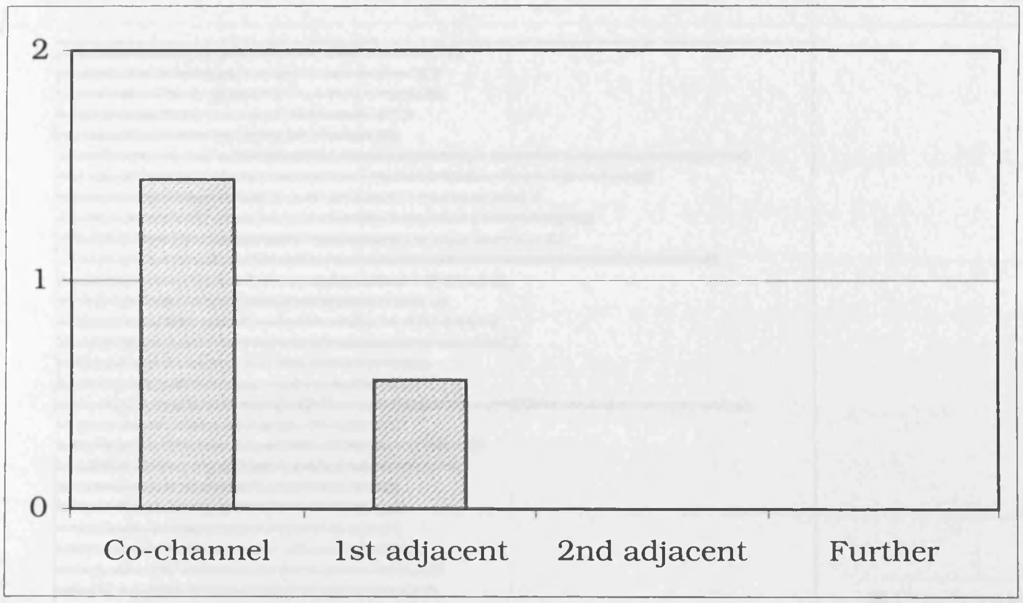

Figure 4.21: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; the worst two interferers are considered active; the number of interferers of each type at the average failed RTP is plotted, averaged over the sixty cases

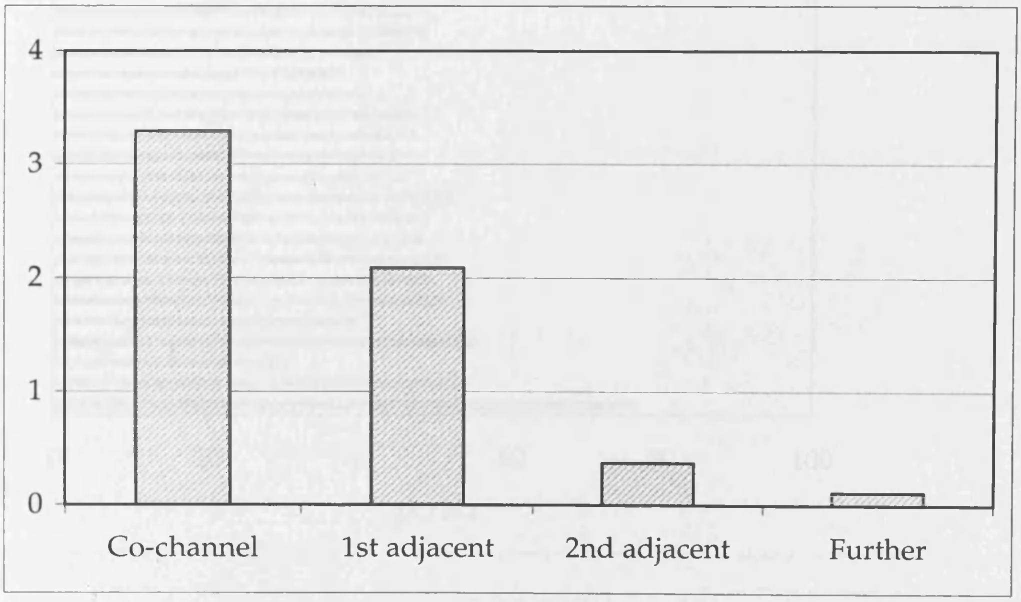

Figure 4.22: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; the worst six interferers are considered active; the number of interferers of each type at the average failed RTP is plotted, averaged over the sixty cases

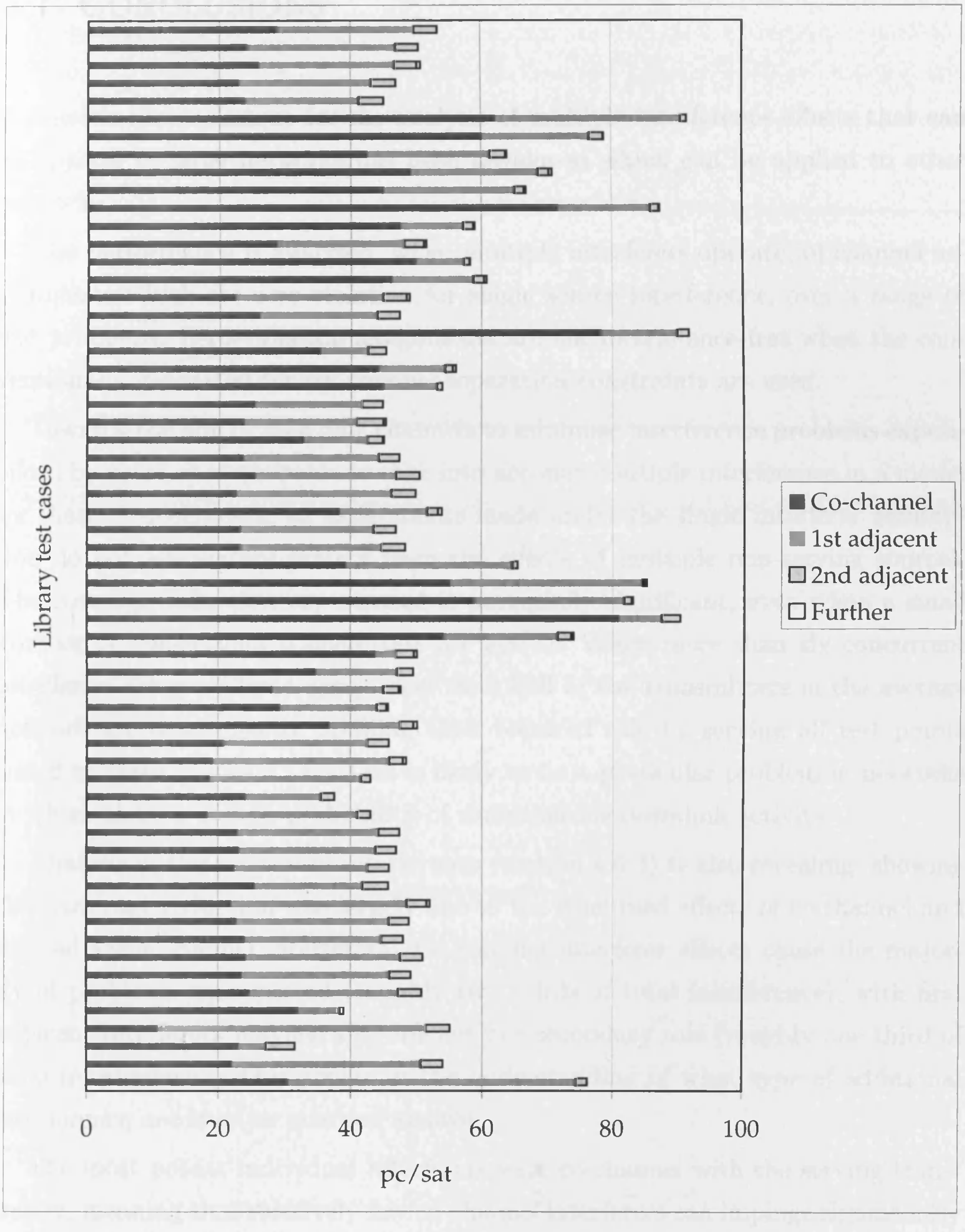

Figure 4.23: The sixty library cases are assigned for 17dB service threshold under the single interferer assumption; the worst six interferers are considered active; displayed is the interference caused by the average interferer at the average failed RTP at which it contributes to failure by being one of the worst six interferers

4.7 CONCLUSIONS

A conservative technique for the analysis of multiple interference effects that can be applied to large networks has been developed which can be applied to other networks.

The performance is analysed, when multiple interferers operate, of channel assignments which are zero-violation for single source interference, over a range of test problems. Zero-violation assignments are not interference-free when the conventionally generated binary channel separation constraints are used.

Towards the aim of assigning channels to minimise interference problems experienced by users, it is desirable to take into account multiple interference in a model for channel assignment, as assignments made under the single interferer assumption do not adequately protect from the effects of multiple non-serving sources. The coverage reduction experienced is potentially significant, even when a small number of non-serving transmitters are active. When more than six concurrent interferers are considered active, less than half of the transmitters in the average network are satisfactorily fulfilling their required role by serving all test points tuned to them (figure 4.14). This is likely to be a particular problem in networks in which there is a high probability of simultaneous downlink activity.

Analysis of the sources of interference (section 4.6.3) is also revealing; showing that coverage reduction was largely due to the combined effects of co-channel and first adjacent channel interferers. Co-channel interferer effects cause the majority of problems encountered (roughly two thirds of total interference), with first adjacent interferers playing a significant but secondary role (roughly one third of total interference). This assists in the understanding of what type of additional interference needs to be guarded against.

The most potent individual interferers were co-channel with the serving transmitter, meaning that relatively few co-channel interferers can impinge significantly on coverage. It would therefore be advantageous to consider co-channel interferer effects specifically in new solution techniques and models, as this is the most significant type of interference and of a manageable nature, relatively simple to incorporate.

It is not practical in computational terms, nor necessary, as noted in [13], to consider all possible interferers; at a particular RTP in the network, some interferers will have negligible effect, due to their geographical distance or channel separation from the server. This is an important consideration, rarely utilised. Taking into account just one additional interferer, i.e. assigning for double rather than single interference, would likely improve the model significantly. The consideration of larger sets of interferers, although small enough not to make computational time unreasonable, would be ideal. From the investigations here, the suggestion is made to consider at least three interferers to improve the model; preferably six or more; with each increase beyond seven becoming less and less significant. This is a general 'rule of thumb' based on the average results from the two-hundred-and-forty particular instances used here and would realistically be dependent on the characteristics of a particular network, especially the density of its transmitters. Jeavons, Dunkin and Bater [12] find that, for their particular instances, sets of interferers ranging in number from 2 to 6 should be considered, and higher order constraints generated of these arities.

A general formula for the number of interferers to consider has not been provided, as this would not be of use to a network operator. Rather, methods and ideas have been provided which can be taken by an operator and applied to their particular network. In general, the choice of a small interferer set size for consideration in assignment is desirable for computational reasons, although this must be balanced against selecting a set size which is large enough to be representative of complete interference to the degree required by an operator. To characterise the problem without wasting computational time, it is most efficient to consider sets of the selected cardinality which contain the non-serving sources providing the highest interfering signal strengths at an RTP.

The results and conclusions presented here assist in the deeper understanding of this issue which are valuable towards developing solution techniques and models to resolve the problem of multiple interference in channel assignment. One way to aim for the mitigation of multiple interference effects in channel assignment is to consider new types of constraint, or new generation methods. A good constraint model would mean the terms zero-violation and interference-free were necessary and sufficient for one another. The chapters which follow look at potential constraint representations to incorporate multiple interference into the model for channel assignment at the constraint generation phase.

Chapter 5

Higher Order Constraints for Multiple Interference

5.1 Introduction

Multiple interference problems are associated with multiple transmitters, leading logically to the suggestion that higher order constraints, which simultaneously restrict the assignment to more than two transmitters, be used to represent such problems. (This does not have to be the case, as will be seen in chapter 6.) Co-channel effects were found to be the majority source of interference causing coverage reduction in chapter 4, so a constraint successfully restricting channel sharing would be of use as it would have the capability to mitigate much of the effect of multiple interference on users.

This chapter discusses the use of higher order constraints for channel assignment, illustrating the discussion by introducing a particular type of higher order constraint called a co-channel set constraint, which restricts channel sharing by transmitters. Observations relating to the properties of these constraints are made and the suggestion to use higher order constraints is considered.

5.2 Higher order constraints

Unary constraints restrict the assignment made to a single transmitter, often by stating a mandatory channel it should use. Binary constraints simultaneously restrict the assignment made to two transmitters. Section 2.2.2 discussed the use of a particular type of binary constraint in modelling channel assignment problems. A CSP is not restricted to unary and binary constraints and, in practical situations, not all constraints occurring in channel assignment have to be unary or binary constraints. Higher order constraints, also known as non-binary constraints, simultaneously constrain the assignment made to a number of transmitters and occur in a variety of forms. For example:

An 'intermodulation product' [68] such as $3f_i - f_j \neq 2f_k$ constrains how the assignments made to three transmitters can relate to one another; it says that the frequency assigned to transmitter j subtracted from three times that assigned to transmitter i must not be double the frequency assigned to transmitter k.

An 'all-different' constraint [97] such as *all-different*(f_i, f_j, f_k) simultaneously constrains the assignments made to three transmitters; it says that the frequencies assigned to transmitters i, j and k respectively must all be different from one another.

Considerable attention has been paid to modelling and solving channel assignment problems using unary and binary constraints (see e.g. [21], and chapter 2). Several authors have suggested the use of higher order constraints for channel assignment (section 2.3.2), usually motivated by the shortcomings of the previously used binary channel separation constraint, as described in section 2.2.2, in relation to multiple interference, re-use distance and co-channel interference effects. These authors believe HOCs could better approximate the operational criteria. The analysis performed in this thesis confirms that it would be advantageous to improve the incorporation of these aspects into the model for channel assignment, to facilitate the more accurate expression of the operational criteria being encapsulated.

5.3 Co-channel set constraints

Higher order constraints can be formulated in many ways, using restrictions between different transmitters and involving different types of interference. *Co-channel set constraints* are a particular type of higher order constraint which focus on one type of interference: co-channel interference. This type of constraint has the advantages of being

(a) of a simple formulation;

(b) known to have direct relevance to multiple interference.

Co-channel set constraints, a specific type of non-binary constraint, can be imposed to incorporate multiple interference [42], and also channel loading [41, 22] problems, into the channel assignment problem. These constraints restrict the assignment made to a (potentially large) subset of transmitters rather than just a pair, by limiting channel sharing by transmitters; they are represented as unordered subsets of (two or more) transmitters which cannot all share the same channel. The choice of this type of constraint is motivated by the knowledge that co-channel effects are the dominant cause of multiple interference on users. Wu and Wey [87] restrict the consideration of multiple interference to cumulative effects from co-channel interferers only. Whitaker *et al.* [43] choose to use a type of non-binary constraint which focusses on co-channel effects, and select co-channel set constraint for use, stating that it is the simplest type of non-binary constraint for use in mitigating multiple interference.

A co-channel set constraint has form

$$\{t_1, \ldots, t_k\} \qquad (k \geq 2)$$

where each t_i is a transmitter within a network and it is stipulated that not all of $t_1, \ldots, t_k$ can be assigned the same channel.

A co-channel set constraint with two elements can be written in the form $\{t_1, t_2\}$. This is exactly equivalent to $\mid f_1 - f_2 \mid > 0$; a binary co-channel constraint. Each says that the frequencies assigned to transmitters 1 and 2 must not be identical.

Note that when solving for an assignment via a set of constraints containing co-channel set constraints only, the minimum span and minimum order objectives become equivalent to one another. The constraints merely state whether certain channels should be different from one another; they do not state by how much (i.e. the channel separation required). Thus the span and order are essentially homologous and related by span = order − 1, *spn* = minimum order − 1.

5.4 HYPERGRAPH MODELLING

The graph theoretical approach described earlier (section 2.2.3) cannot be used directly for non-binary co-channel set constraints. For example, given three transmitters and the constraint $\{1, 2, 3\}$, it is possible to draw vertices as before, but not to state where an edge should be placed. Any of the three diagrams in figure 5.1 would be valid, but this does not satisfy the requirement for a *unique* representation.

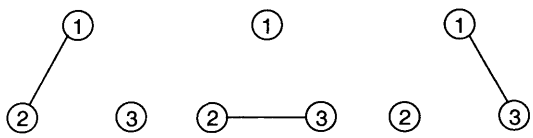

Figure 5.1: Edge ambiguity

Sarkar and Sivarajan [30] champion the use of a hypergraph model for channel assignment, as introduced by McEliece and Sivarajan [36, 37]. A hypergraph [29] $H = (V, E)$ is defined similarly to a graph, with the exception that an edge is no longer restricted to being a pair of vertices, but may also be a larger subset of V. The following terms are defined by Sarkar and Sivarajan [30]:

- A set of transmitters forms a *forbidden set* if those transmitters may not all use the same channel simultaneously;
- An *independent set* is a set of transmitters which is not forbidden;
- A *minimal forbidden set* is a forbidden set in which the removal of any one element would result in the set being independent.
- A *maximal independent set* is one to which no more transmitters can be added whilst the set remains independent.

This model is directly related to the co-channel set constraint: a forbidden set is equivalent to a co-channel set constraint, and the co-channel set constraints remaining after reduction by lemma 1 (see below) are minimal forbidden sets.

In the same way as there is a bijection between a system of binary co-channel constraints and a corresponding graph (section 2.2.3), so can a system of co-channel set constraints be mapped onto a hypergraph, the analogy being made as follows:

- vertex $v_i \in V$ represents the transmitter t_i;
- an edge $e \in E$ is a subset of V which forms a minimal forbidden set i.e. co-channel set constraint.

For example, just as the constraint $\{1, 2\}$ was represented in a graph as in figure 5.2, the constraint $\{1, 2, 3\}$ can be represented in a hypergraph as in figure 5.3, where the ellipse is a *hyperedge* joining 1, 2 and 3.

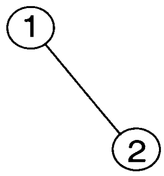

Figure 5.2: $\{1, 2\}$ as an edge

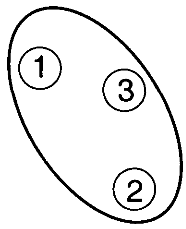

Figure 5.3: $\{1, 2, 3\}$ as a hyperedge

The channel assignment problem is then akin to vertex-colouring the hypergraph in such a way that no hyperedge is monochromatic.

Sarkar and Sivarajan [30] compare this hypergraph model for channel assignment with the graph-theoretic model based on re-use distance. The conclusion is reached that hypergraph modelling enables better exploitation of channel re-use than the simple re-use distance idea. However, the computational complexity involved in finding maximal independent sets is exponential in the number of transmitters; potentially prohibitive in large systems.

5.5 PROBLEM MAGNITUDE

5.5.1 REDUNDANCY OF CONSTRAINTS

Given a set of generated constraints, the aim is to satisfy them all (or as many as possible in the case where a zero-violation solution is not attainable) to minimise problems experienced by users in the operational situation represented by the constraints. The number of constraints which need to be considered explicitly and hence the complexity and computations required can sometimes be reduced. Smith *et al.* [17] note that a set of higher order constraints may contain some *redundant* constraints. If the satisfaction of a particular constraint is implied by the satisfaction of one (or more) of the other HOCs, then that particular constraint is redundant. Its removal from the system does not have an impact on the solution of the constraint system and there is no need to consider it in computation. The redundancy of certain constraints within a system is considered for different types of constraint in for example [12, 46, 81].

A generated system of co-channel set constraints may contain redundant constraints. For example, as co-channel set constraints contain unordered transmitters, $\{1, 3, 4\}$ and $\{4, 1, 3\}$ are the same constraint. This means that one of them can be removed if they appear together in a constraint system. Consider a set of constraints, two of which are $\{1, 2\}$ and $\{1, 2, 3, 4\}$. If the first constraint is satisfied, then transmitters 1 and 2 must have been assigned different channels, so the second constraint is satisfied automatically; regardless of the assignment made to transmitters 3 and 4, the four transmitters are guaranteed to not all share the same channel. The second constraint can therefore be removed from the list of constraints which need to be considered. In general,

- any constraint which is repeated may be removed immediately;
- any constraint which has a proper subset appearing in the constraint system is redundant and may be removed.

This idea of redundancy for co-channel set constraints is formalised in lemma 1:

LEMMA 1 (REDUCTION LEMMA)
If C_i and C_j are co-channel set constraints such that $C_i \subseteq C_j$ then

$$C_i \textit{ satisfied} \Rightarrow C_j \textit{ satisfied}$$

PROOF: If C_i is satisfied, then ≥ 2 channels have been assigned to the transmitters which appear in C_i. Since all these transmitters also appear in C_j, ≥ 2 channels have been used here also. Therefore C_j is satisfied. (Conversely, if C_j is not satisfied, then all transmitters appearing in C_j must have been assigned identical channels. Since $C_i \subseteq C_j$, all channels in C_i must also be identical. Therefore C_i is not satisfied.) □

In the situation in lemma 1, C_j is redundant and can be removed from the set of constraints to be considered, hence reducing the problem size and potential computations required in the assignment process. The process of *reduction* removes redundant constraints as described above, and leaves a reduced set of constraints with which to continue working towards the aim of creating best possible assignments. This reduced set of constraints contains minimal forbidden co-channel sets.
COROLLARY 2 (TO THE REDUCTION LEMMA)
(1) Given a network of n transmitters, a constraint of cardinality n is redundant, except in the case where it is the only constraint.
(2) A set of distinct constraints of identical size can be reduced no further at this stage.

PROOF: (1) The 'n-constraint' contains all of the n transmitters. If any other constraint is present, it must be a subset of the set of transmitters, and therefore also a subset of the n-constraint. The n-constraint can therefore be removed by the reduction lemma. (2) No constraint here is a subset of another. Removing a constraint at this point would result in a different CSP not equivalent to the original. □

Note that reduction of the problem can be continued further during the process of creating a channel assignment. For example, consider a network of four transmitters and the constraints $\{1, 2, 3\}$ and $\{4, 1, 2\}$. By corollary 2, this problem can be reduced no further. However, if in the course of creating an assignment the consideration of the first constraint results in transmitters 1 and 2 being assigned different channels, then the second constraint is automatically satisfied and need not be considered explicitly by the remainder of the assignment algorithm.

5.5.2 APPLICATION OF EXTREMAL SET THEORY

If a family of distinct sets F is such that no member of F is contained in any other, i.e.

$$A, B \in F \Rightarrow A \not\subset B \text{ and } B \not\subset A$$

then it is called a *Sperner family.* The theory of Sperner families can be applied to co-channel set constraints: a set of these constraints once reduced by lemma 1 is a Sperner family of sets.

THEOREM 3 (SPERNER'S THEOREM)

If X is a set with n elements and F is a Sperner family of subsets of X then

$$| F | \leq \binom{n}{\lfloor \frac{n}{2} \rfloor}$$

where the floor function is evaluated using

$$\left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n-1}{2} & \text{if } n \text{ is odd} \end{cases}$$

and the binomial coefficient is evaluated using

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

For a proof of this theorem, see [98, p101].

Sperner's theorem has application to the reduction of co-channel set constraint problems in that it can give the maximum potential size of the reduced set, i.e. the maximum number of constraints remaining after redundant constraints have been removed. X in the theorem is equivalent to the set of n transmitters, and F to the reduced set of constraints.

COROLLARY 4 (TO SPERNER'S THEOREM)

For equality, i.e. $| F | = \binom{n}{\lfloor \frac{n}{2} \rfloor}$, *$F$ consists of all subsets of X of size $\lfloor \frac{n}{2} \rfloor$ or all* *subsets of X of size $\lceil \frac{n}{2} \rceil$ (which are the same in the case of even n).*

This corollary is also proven in [98].

Lemmas 5 and 6 which follow are derived from corollary 4 and apply to a system of n transmitters.

LEMMA 5

If the set of constraints contains

$$\binom{n}{\lfloor \frac{n}{2} \rfloor}$$

distinct constraints of cardinality N where

$$N = \frac{n}{2} \text{ for even } n$$

and

$$N = \frac{n+1}{2} \text{ or } \frac{n-1}{2} \text{ for odd } n$$

then all constraints of cardinality $> N$ can be removed immediately.

PROOF: By implication from the corollary, the $\binom{n}{\lfloor \frac{n}{2} \rfloor}$ constraints of cardinality N are all possible subsets of this size of the set of transmitters. Any larger constraint will therefore contain at least one of these N-constraints, and can be removed by lemma 1. □

After this lemma has been applied, the set of constraints will consist of the N-constraints along with any constraints of cardinality $< N$ and reduction can be continued. If there are no constraints with $< N$ elements, the set of constraints has been reduced as far as possible (part 2 of corollary 2) and is the maximum size given by Sperner's theorem.

LEMMA 6

If there exists a constraint of cardinality $< \lfloor \frac{n}{2} \rfloor$ then the reduced set of constraints will contain $< \binom{n}{\lfloor \frac{n}{2} \rfloor}$ co-channel set constraints.

PROOF: Equality occurs only when all sets of size N (as defined above) are present. In the case where all sets of size N are present, the smaller constraint in

this lemma must be a subset of at least one of the N-constraints, thus reducing the number of constraints by the reduction lemma and giving the stated result.
□

THEOREM 7

Given a set of n transmitters, and $1 < l < m < n$, a constraint of size m can always be removed from the problem if there are greater than

$$\binom{n}{l} - \binom{m}{l}$$

distinct constraints of size l present.

PROOF: By the reduction lemma, a constraint may be removed if there is another constraint in the set which is a subset of the first. Given a constraint of size m, there are $\binom{m}{l}$ subsets of size l which can be used to remove the constraint. The total number of subsets of transmitters of size l is $\binom{n}{l}$. There are therefore $\binom{n}{l} - \binom{m}{l}$ possible subsets of transmitters of size l which are not also subsets of the m-constraint. So if more than this number of l-constraints are present, at least one of them must be a subset of the m-constraint which can therefore be removed. □

Note that theorem 7 contains a sufficient but not necessary condition—reduction may still be possible when this condition does not hold (i.e. when there are fewer constraints of size l present).

There follow several examples to illustrate the removal of redundant constraints and the application of theorems 3 and 7.

EXAMPLE 2 (OF REDUCTION AND USING SPERNER'S THEOREM)

Consider a network of five transmitters and the following set of twelve constraints:

$A = \{1,2,3,4\}$	$B = \{1,3,5\}$	$C = \{2,3,4,5\}$	$D = \{1,2,3\}$
$E = \{1,2,4\}$	$F = \{1,2,3,4,5\}$	$G = \{1,4,5\}$	$H = \{1,2,5\}$
$I = \{1,3,4\}$	$J = \{2,4,5\}$	$K = \{2,3,4\}$	$L = \{3,4,5\}$

Sperner's theorem says that the reduced set of constraints in a situation with 5 transmitters will contain $\leq \binom{5}{2} = 10$ *constraints.*

Notice firstly that $K \subset C$ means constraint C can be removed from the list. Next, $D \subset A \subset F$, so A and F may also be removed. The constraints which remain at this stage are

$B =$	$\{1,3,5\}$	$D =$	$\{1,2,3\}$	$E =$	$\{1,2,4\}$
$G =$	$\{1,4,5\}$	$H =$	$\{1,2,5\}$	$I =$	$\{1,3,4\}$
$J =$	$\{2,4,5\}$	$K =$	$\{2,3,4\}$	$L =$	$\{3,4,5\}$

This is a set of distinct constraints of identical size and can therefore be reduced no further (corollary 2).

The constraints which remain are a reduced set of magnitude $9 \leq 10$ as predicted by Sperner's theorem.

EXAMPLE 3 (OF APPLICATION OF THEOREM 7)

Consider a network of 5 transmitters and the constraints

$\{1,2,3,4\}$ $\{1,2,5\}$ $\{1,3,5\}$ $\{1,4,5\}$ $\{2,3,5\}$

$\{2,4,5\}$ $\{3,4,5\}$ $\{1,2,3\}$ $\{2,3,4\}$ $\{4,5\}$

Theorem 7 says that a 4-constraint may be removed if there are

$$> \binom{5}{3} - \binom{4}{3} = \frac{5*4*3}{3*2*1} - \frac{4*3*2}{3*2*1} = 6$$

3-constraints present. Here there are $8 > 6$ 3-constraints. Six of these 3-constraints are not subsets of the 4-constraint, but two are. Either of these can be used to reduce the set.

The presence of the constraint $\{4,5\}$ means that the set can in fact be reduced further. All the 3-constraints which contain both transmitters 4 and 5 can be removed. The reduced set in this case would be

$\{1,2,5\}$ $\{1,3,5\}$ $\{2,3,5\}$ $\{1,2,3\}$ $\{2,3,4\}$ $\{4,5\}$

EXAMPLE 4 (TO ILLUSTRATE ONE-WAY IMPLICATION IN THEOREM 7)

- *Consider another network of 5 transmitters and the constraints*

  $\{1,2,3,4\}$ $\{1,2,5\}$ $\{1,3,5\}$ $\{1,4,5\}$ $\{2,3,5\}$ *In this case the condition in theorem 7 is not satisfied and reduction is not possible.*

- *Now consider the 5 transmitters but with constraints*

  $\{1,2,3,4\}$ $\{1,2,3\}$ $\{4,5\}$ *Again the condition is not satisfied, but reduction* is *possible:* $\{1,2,3\} \subset \{1,2,3,4\}$ *so* $\{1,2,3,4\}$ *can be removed.*

5.5.3 COMPARATIVE PROBLEM SIZE

Sperner's theorem provides a mathematical expression for the maximum number of co-channel set constraints which need be considered for a problem with n transmitters. An expression for the number of constraints when binary constraints only are used would be

$$\binom{n}{2}$$

For comparison purposes, table 5.1 computes the numbers of each type of constraint which could potentially be encountered for problems of different sizes.

In practice, the numbers of constraints would usually be lower than those in the table, because the constraints come from a geographical problem and it is unlikely that a transmitter on one extreme of the area would appear in a constraint with a transmitter from the other extreme and so on. However, the table gives an idea of the relative sizes of problems involving binary and non-binary constraints, and the computational problems implied.

Transmitters	Maximum potential number of constraints	
	Binary co-channel	Total co-channel set
n	$\binom{n}{2}$	$\binom{n}{\lfloor \frac{n}{2} \rfloor}$
3	3	3
6	15	20
10	45	252
30	435	155117520
100	4950	$\sim 1.0 \times 10^{29}$
300	44850	$\sim 9.4 \times 10^{88}$
1000	499500	$\sim 2.7 \times 10^{299}$

Table 5.1: Problem size comparison

Whitaker *et al.* [43], who develop solution software for co-channel set constraints, find that they must restrict the arity of co-channel set constraints in larger cases to ensure the constraint sets are generated in reasonable time and can be used for solution. The authors find the results using these restricted arity constraints disappointing in terms of the network coverage achieved.

The numbers of higher order constraints and their potential solution quickly become intractable, leading to a need for other methods which do not involve the generation, storage and management of such constraints.

5.6 Problem representation

Different diagrammatic representations of a system of constraints can provide different information about the system and lead to techniques such as bounds and solutions. In section 5.4 it was seen that higher order co-channel set constraints cannot be represented by graphs in the way that binary channel separation constraints were so represented. However, other graph-theoretic representations may be possible, and may lead to information about the corresponding constraint system. Some such possible graph-theoretic representations are introduced in this section. The (hyper)graph representations used previously have transmitters as vertices, but other representations may have constraints as vertices, or both constraints and transmitters as vertices.

5.6.1 Bipartite graphs

The vertices of a *bipartite graph* can be partitioned into two sets V_1 and V_2 such that any edge joins a vertex $v_i \in V_1$ to a vertex $v_j \in V_2$. Bipartite graphs can be used to represent non-binary co-channel set constraints by letting V_1 be the set of transmitters and V_2 the set of constraints. An edge is drawn between $t \in V_1$ and $C \in V_2$ if and only if transmitter t appears in constraint C. For example, the constraints $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$ and $C = \{1, 2, 4\}$ can be represented by figure 5.4. This formulation provides a *unique* graph representation; there is no ambiguity in deciding where edges should be drawn.

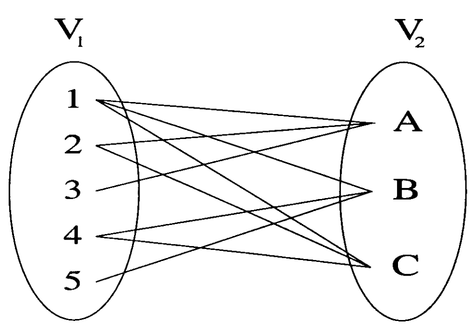

Figure 5.4: Bipartite graph

An equivalent diagrammatic representation of this example is shown in figure 5.5.

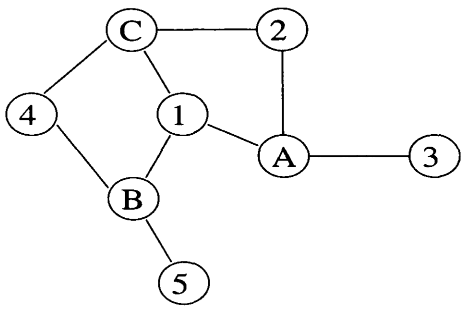

Figure 5.5: Equivalent representation

This representation allows which constraints have elements in common to be seen at a glance. In this particular example, transmitter 1 is an element common to all three constraints A, B, and C.

5.6.2 WEIGHTED OVERLAP GRAPH

The ways in which co-channel set constraints overlap form the fundamental properties of an individual problem of this type. It is therefore useful to be able to see which transmitters constraints have in common. The *weighted overlap graph* presented here illustrates the possibility of a representation as a graph with constraints as vertices. This is not a representation of the entire system but rather of its complexity.

The weighted overlap graph $G_\Omega = (V, E)$ is constructed from the co-channel set constraints as follows:

- vertex $v_i \in V$ represents the constraint C_i;
- an edge $(v_i, v_j) \in E$ exists if and only if $C_i \cap C_j \neq \emptyset$;
- edge (v_i, v_j) has a positive integer label which is $|C_i \cap C_j|$ and is called the *overlap weight.*

Again using the constraints $A = \{1, 2, 3\}$, $B = \{1, 4, 5\}$ and $C = \{1, 2, 4\}$, G_Ω is as shown in figure 5.6, which shows clearly that constraints A and B overlap by having one transmitter in common. In fact, this common transmitter is transmitter number 1, as can be seen from the constraints. The representation is not deigned to encapsulate this fact, but is abstracted further than *which* transmitters to rather demonstrate *how many* transmitters. Constraints B and C overlap by two transmitters (1 and 4) and A and C have two shared transmitters (1 and 2).

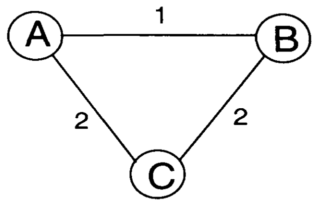

Figure 5.6: Weighted overlap graph G_Ω

For each pair of constraints (edge), an overlap weight $\Omega_{weight}(C_i, C_j)$ is defined, and for each individual constraint (vertex), an *overlap degree* $\Omega_{deg}(C_i)$ which is the sum of labels on that vertex's incident edges. For example, in figure 5.6,

- $\Omega_{weight}(A, C) = 2$
- Constraint C would have the largest overlap degree of $\Omega_{deg}(C) = 2 + 2 = 4$
- A and B would each have overlap degree 3

The following sections provide short examples of how this type of representation may prove useful.

5.6.2.1 Towards an ordering

Solution techniques for constraints are often dependent on the ordering of the entities to be considered. The idea of overlap between constraints can lead to possible orderings and hence possible solution techniques.

For example, when assigning frequencies, the constraints with highest overlap degree could be considered first, in relation to the constraints which gave the highest overlap weight when paired with the constraint under consideration.

In the small example under investigation, constraint C would be considered first, in relation to constraint A or B (as the pairs (C, A) and (C, B) both have equal overlap weight of 2). If C were to be considered in relation to A, it would be seen that these two have the subset $\{1, 2\}$ in common, so transmitters 1 and 2 would be assigned different frequencies e.g. $1 \to f_1$ and $2 \to f_2$. Next, C would be considered in relation to B, and it would be seen that the subset they share is $\{1, 4\}$. Transmitter 1 has already been assigned f_1, so if the assignment $4 \to f_2$ is made, all the constraints have been satisfied. The remaining transmitters can be assigned arbitrary channels from among those already used and an assignment has been made with *spn* = 1.

Note that it is not always possible to produce an optimal assignment using the above exemplified method only. More intelligent solution techniques would be required for larger examples, but the idea of ordering constraints by overlap degree could be employed by such techniques.

5.6.2.2 Problem complexity

The weighted overlap graph and its labels could be applied to showing how 'tangled' a system of constraints is.

For example

$$\Omega_{mean} = \left\lceil \frac{\sum_{\substack{i,j \\ i<j}} \Omega_{weight}(C_i, C_j)}{n} \right\rceil$$

where n is the number of transmitters appearing in the set of constraints and the ceiling is taken to maintain whole number results. This coefficient gives a *mean overlap weight* for the network by dividing by the number of transmitters.

Division by the number of transmitters makes it easier to compare systems of different sizes, but this is not the only factor needed to know how difficult it is to actually assign a system. It is also desirable to consider the number of constraints, as a system with many constraints is going to be more difficult to assign than a small system in general. However, a system with large numbers of almost independent constraints may be easier to assign in practice than a much smaller more 'tangled' system. The relative importance of the factors

(a) overlap

(b) number of constraints

(c) number of transmitters

must be investigated and the coefficient weighted to reflect this if it is to be employed.

For example, a problem with 2 distinct constraints on 1000 transmitters is an easy one, as it is known (see lemma 11) that any system with only two constraints may be solved with $spn = 1$; here the fact that $n = 1000$ plays no part in the complexity of the problem, as the majority of assignment choices are unconstrained and can be made arbitrarily.

These ideas could be refined to provide a metric useful in determining how difficult it is to create an assignment for a particular network of transmitters and constraints.

5.7 BOUNDS FOR CO-CHANNEL SET CONSTRAINTS

Lower bounds which are calculated for single interference can be used directly for multiple interference when multiple interfering signals are assumed to be additive (see section 3.1.3.3 and [46]). Many existing bounds are dependent on the binary constraint graph however, making them unsuitable for direct application to higher order co-channel set constraints. Currently only one method has been presented in the literature which is known to give a lower bound on *spn* for co-channel set constraints: Whitaker's *independence set bound* [41, 43].

THEOREM 8 (INDEPENDENCE SET BOUND)
If T is a set of transmitters and $max(t)$ is the size of the largest subset of T which can be assigned the same channel as t while violating no constraints. Then, calculating each $max(t)$ by inspection,

$$spn \geq \sum_{t \in Tx} \frac{1}{max(t)} - 1$$

For a proof of this bound, see [41].

Unlike the majority of lower bounds for the minimum span problem, this bound has the advantage of being independent of a constraint graph. The bound does, however, involve the computation of maximal independent sets, a non-trivial problem for which the time required grows exponentially with the number of transmitters [30, 43]. Whitaker *et al.* [43] generate valid maximal independent sets using a method adapted from [99] but find this to be very slow on problems with many transmitters or a high required service threshold.

5.7.1 TWO-COLOURABILITY

Using terminology from the hypergraph analogy, a system of co-channel set constraints which can be entirely satisfied by a channel assignment with span= spn = 1, i.e. using two channels only, is referred to as *two-colourable.* Determining whether a given hypergraph is two-colourable is NP-complete [100].

The following system of co-channel set constraints is an example of a system where two-colouring is *not* possible:

$$\{1,2,3\} \quad \{3,4,5\} \quad \{1,5,6\} \quad \{2,4,6\} \quad \{1,7,4\} \quad \{2,7,5\} \quad \{3,7,6\}$$

An assignment using two channels only cannot satisfy all seven of these constraints; one will remain unsatisfied unless a third channel/colour is introduced, thus increasing spn to 2. Note that this system is *edge-critical*: if any one of the seven constraints (edges) is removed from the system, then it becomes two-colourable.

There are, however, several situations in which a system can definitely be said to be two-colourable. This section focusses on the issue of two-colourability and a discussion of two-colourable situations is begun below (lemmas 11–15) but first, some essential elementary results are stated in lemmas 9 and 10.

LEMMA 9 (FUNDAMENTAL LEMMA)
To satisfy any co-channel set constraint problem, ≥ 2 channels are required.

PROOF: If only one channel is used, all transmitters are co-channel and no co-channel set constraint can be satisfied. Therefore, the channels must number more than one. □

From lemma 9, a lower bound on spn is simply 1. It will be seen shortly that it is possible to satisfy some problems using exactly two channels and therefore attain this lower bound. As lemma 9 gave an elementary lower bound on the number of channels, so lemma 10 states an elementary upper bound on the same quantity.

LEMMA 10

The number of channels required is

$$\leq \max\{2, \min(\textit{number of constraints, number of transmitters})\}$$

PROOF: The worst case scenario is that all transmitters require different channels. Each time a constraint is added to a system, either 0 or 1 extra channel is required (see section 5.7.3.1). This leads to there being no more channels than constraints needed, except in a system with one constraint only, as lemma 9 says that the minimum number of channels is 2. □

LEMMA 11

Any problem involving two non-binary constraints only is two-colourable.

PROOF: Where the two constraints have elements in common, an assignment which gives one of the common elements one channel and all other elements a second channel will always be a zero-violation assignment. Where the constraints do not overlap, each constraint is partitioned into two non-empty subsets. This is always possible as constraints with one element only do not occur. In each constraint, one subset is given the first channel, and the other subset the second. Thus both constraints are satisfied. □

(Note that the methods in this proof are not the only ones which will lead to an assignment with two channels for any particular problem, and are not suggested as solution methods. They are methods which are always possible and not specific to individual constraints, and therefore used for the purpose of proof.)

LEMMA 12

If $C_1, C_2, \ldots, C_k$ are co-channel set constraints such that for $i \neq j$, $C_i \cap C_j = \emptyset$ then the system of constraints is two-colourable.

PROOF: Each C_i requires ≥ 2 channels, otherwise C_i is violated (as in the Fundamental Lemma). The channels assigned to transmitters in C_i can be 'reused' for those in C_j $(i \neq j)$ without restriction, as C_i and C_j have no elements in common. Therefore no more than 2 channels are needed. □

LEMMA 13

If $C_1, C_2, \ldots, C_k$ are co-channel set constraints such that for $i \neq j$, $C_i \cap C_j = \{t_m\}$, where transmitter t_m is fixed, then the system of constraints is two-colourable.

PROOF: Each constraint must contain t_m and at least one other element. An assignment which assigns one particular channel to transmitter t_m and a second channel to all other transmitters in the set will therefore always be a possible zero-violation assignment i.e. provide a two-colouring. □

This may be extended as follows:

LEMMA 14
If $C_1, C_2, \ldots, C_k$ are co-channel set constraints such that for $i \neq j$, $C_i \cap C_j = T$, where T is any fixed subset of the set of transmitters and $| T |> 1$, then the system of constraints is two-colourable.

PROOF: T contains at least two elements. An assignment which gives one element of T one particular channel, and all other elements of T a second channel will always be a possible zero-violation assignment. The assignment for other transmitters is then arbitrary as all constraints have been satisfied. Note that this proof is still valid if one of $C_i \equiv T$. □

LEMMA 15
If $C_1, C_2, \ldots, C_k$ are co-channel set constraints such that each C_i contains some transmitter t_j that does not appear in any other constraint, then the system of constraints is two-colourable.

PROOF: Such an assignment can be made using one channel for the 'once-only's and another channel for all other transmitters. □

5.7.2 APPLICATION OF RESULTS FROM THE THEORY OF HYPERGRAPHS

Just as results from graph theory can be used to solve for binary channel separation constraints, so can results from hypergraph theory [101, 100] be applied to the solution of systems of co-channel set constraints. This section briefly exemplifies results that can be applied.

5.7.2.1 Bounding the required order

Work done by Seymour in [102] implies the simple result shown in lemma 16.

LEMMA 16

A hypergraph is two-colourable if the number of edges is less than the number of vertices (in those edges).

This means that any counterexample to two-colourability will have least as many constraints as there are transmitters appearing in those constraints. The counterexample to two-colourability provided earlier in section 5.7.1 contained seven co-channel set constraints on seven transmitters:

$\{1,2,3\}$ $\{3,4,5\}$ $\{1,5,6\}$ $\{2,4,6\}$ $\{1,7,4\}$ $\{2,7,5\}$ $\{3,7,6\}$

When any one edge is removed from this set of constraints, the system becomes two-colourable, because the condition in lemma 16 is then satisfied.

The chromatic number of a hypergraph, introduced in [103], is defined similarly to that of a graph. The chromatic number is notated $\chi(H)$ and is the smallest number of colours needed to colour the vertices of H so that no (hyper)edge of H is monochromatic. Any results that bound $\chi(H)$ immediately correspond to a bound on the order (number of channels) of an assignment, because of the hypergraph analogy outlined in section 5.4.

The degree of a vertex in a hypergraph is the number of edges (of size ≥ 2) of the hypergraph which contain that vertex. An initial bound on $\chi(H)$, which shows a relationship between the colourability of a hypergraph and its degrees, is presented in lemma 17. Lemma 18 also uses the degrees of the hypergraph to provide a bound on the chromatic number. Lemmas 17 and 18 are both corollaries to a theorem of Tomescu [104] and their proofs can be found in [29]. The upper bounds stated in each can be attained using hypergraphs with certain conditions on rank and cliques.

LEMMA 17

If H is a hypergraph of maximum degree d_0 then

$$\chi(H) \leq d_o + 1$$

LEMMA 18

Here, x denotes a vertex of H and $d(x)$ its degree. If q is a positive integer such that

$$\mid \{x \mid d(x) \geq q\} \mid \leq q$$

then

$$\chi(H) \leq q$$

This says that if the number of vertices (transmitters) of the hypergraph with degree $\geq q$ is $\leq q$, then so is the chromatic number.

5.7.2.2 Solution techniques

The hypergraph analogy can also be used to find solution techniques for co-channel set constraints. For example, although section 5.7.1 discussed ways of determining whether a hypergraph is two-colourable, this does not provide a method for actually generating such a two-colouring. Beck [105] introduced an algorithm to approach this task, and many authors have since introduced refinements to it (e.g. [106, 107, 108, 100]).

5.7.3 ADDITIONAL HIGHER ORDER CONSTRAINTS

This section presents bounds on the change in order (number of channels needed) when one or more higher order co-channel set constraints are added to a system of binary co-channel constraints only which has been given an optimal (zero-violation and minimum span) assignment already. Note that because all constraints are co-channel, the change in span and change in order both refer to the number of extra channels required.

5.7.3.1 Initial bounds on the change in *spn*

Firstly, because the given assignment for the binary constraints is optimal, it is known that the number of channels needed cannot be reduced by adding HOCs (the binary constraints remain and, reducing the number of channels would cause at least one of them to be unsatisfied) so

$$0 \leq \text{num. extra}$$

If one constraint is added to a problem and there is no way of satisfying the new problem with the channels already used, assigning an element of the new constraint to a new channel will ensure that the new constraint can be satisfied without altering the satisfiability of the constraints already present. So one channel is the most that will be needed each time a constraint is added to the system. Hence

$$\text{num. extra} \leq \text{num. HOCs added}$$

The number of extra channels required therefore lies in the range

$$0 \leq \text{num. extra} \leq \text{num. HOCs added}$$

5.7.3.2 Achieving the lower bound

This section provides a simple illustration of a situation in which the lower bound of 0 additional channels given above is achieved. Transmitters will be represented by numbers, and channels by letters.

Consider the constraints $\{1, 2\}$, $\{3, 4\}$ and $\{5, 6\}$ and assume the assignment has been made as follows:

A	B
1	2
3	4
5	6

Now introduce the higher order constraint $\{1, 3, 5\}$. This is not satisfied by the above assignment but can be satisfied without the use of additional channels by performing a swap between transmitters 3 and 4 (which will be notated $3 \longleftrightarrow 4$). The assignment is now

A	B
1	2
4	3
5	6

The binary constraints are still satisfied, but the HOC is also satisfied. This same swap would also satisfy the addition of the constraint $\{2, 4, 6\}$. No *single* HOC can be added which isn't satisfied by either the original assignment or the exchanged version.

If two constraints were added to the original situation which were $\{1,3,5\}$ and $\{2,3,6\}$, the swap made above would not work, but an alternative pair could be swapped e.g. $1 \longleftrightarrow 2$.

5.7.3.3 Refining the upper bound

In this section, a necessary refinement to the upper bound is introduced.

Consider the same example, beginning with constraints $\{1,2\}$, $\{3,4\}$ and $\{5,6\}$ and the assignment

A	B
1	2
3	4
5	6

Now add to the situation all the possible 3-constraints which have a single element from each of the binary constraints. (Note that any other 3-constraints would have been removed during the reduction process as they would contain, as a subset, at least one of the 2-constraints.) The following constraints are added:

$$\begin{array}{cccc} \{1,3,6\} & \{1,3,5\} & \{1,4,6\} & \{1,4,5\} \\ \{2,4,5\} & \{2,3,5\} & \{2,4,6\} & \{2,3,6\} \end{array}$$

There are eight of these constraints, so the current upper bound implies that eight additional channels are needed in the worst case. In fact, the system involves six transmitters and the worse case scenario would be that all the transmitters have different channels i.e. the increase is from 2 to 6 channels and therefore uses 4 extra channels. It is for the reason illustrated here that the following refinement to the bounding inequality is introduced:

$$\begin{aligned} 0 &\leq \text{num. extra} \\ &\leq \min(\text{num. HOCs added, num. trans. - binary assignment channels}) \end{aligned}$$

5.7.3.4 Achieving the upper bound

It has been ascertained that the lower bound can be reached. The following example shows that the upper bound may also be attained. The example illustrates once again the importance of interdependence and overlap when dealing with these constraints.

Begin with the constraints

$$\{1,2\} \quad \{2,3\} \quad \{3,4\} \quad \{4,5\} \quad \{5,6\}$$

Here (because of the chain between the hyperedges in the order they appear) there are no arbitrary choices in assignment. To two-colour the transmitters, they must be partitioned as follows:

A	B
1	2
3	4
5	6

If for example $\{1,3,5\}$ is introduced, the problem cannot be solved in two channels by performing swaps: swapping $1 \longleftrightarrow 2$ would satisfy the HOC but would mean the original $\{2,3\}$ is no longer satisfied. An attempt to fix this by swapping $3 \longleftrightarrow 4$ would in turn lead to the requirement to swap $5 \longleftrightarrow 6$. Then the assignment is

A	B
2	1
4	3
6	5

The assignment has in fact been completely switched, so the partition is the same and the HOC is unsatisfied once again. There is no choice but to introduce a third channel e.g.

A	B	C
3	2	1
5	4	
	6	

In this example, one extra channel was needed and

$$\text{num. extra} = \text{num. HOCs added}$$

therefore attaining the upper bound and showing that the inequalities must both be '$\leq$' and that the number of extra channels required is bounded by

$$\begin{aligned} 0 &\leq \text{num. extra} \\ &\leq \text{min(num. HOCs added, num. trans. - binary assignment channels)} \end{aligned}$$

5.8 DECOMPOSITION OF NON-BINARY CONSTRAINTS

Most of the literature on constraints deals with binary constraint systems. This is for two main reasons:

(a) binary constraints are easier to work with than more complex constraints;

(b) non-binary constraints can, in theory, be translated into a set of binary constraints [109].

Sometimes, this translation to binary constraints can be performed using the same variables as used in the higher order system; sometimes new variables have to be introduced. In practical cases, this translation can be infeasible due to underlying computational and memory costs.

A set of higher order constraints which can be represented by a set of binary constraints on the same variables is called *network decomposable* [110]. For example, the all-different constraint seen in section 5.1 can be decomposed as follows:

$$\textit{all-different}(f_i, f_j, f_k) \iff \begin{cases} |f_i - f_j| > 0 \\ |f_j - f_k| > 0 \\ |f_i - f_k| > 0 \end{cases} \tag{5.1}$$

The single HOC says that the frequencies assigned to transmitters i, j and k respectively must all be different from one another. The system of binary constraints on the right hand side is exactly equivalent to the HOC.

The co-channel set constraint cannot be directly decomposed in this way, as was shown in figure 5.1 and is exemplified by e.g. Dunkin and Allen [35, chapter 3]. This is one reason why more attention is now being paid to non-binary constraints and their potential in relation to real-life applications. Channel sharing is an important aspect of the operational situation which cannot be successfully approximated by the binary channel separation constraint generated under the single interferer assumption.

Although the system of co-channel set constraints representing multiple interference cannot be simply decomposed into a set of binary constraints, that is not to say that binary constraints cannot be used for multiple interference. Rather than generating higher order constraints and trying to solve these directly, or to decompose them into a system of binary constraints, generating binary constraints directly for multiple interference could be considered. This idea is carried forward in chapter 6.

5.9 CONCLUSIONS

Several authors (see chapter 2) suggest the use of higher order constraints for multiple interference; the effects caused by multiple interfering transmitters are directly constrained by the restriction of the assignment to more than two transmitters. Few have successfully applied the idea of higher order constraints to channel assignment. The numbers and inherent complexity of higher order constraints can limit their tractability.

Whenever a new type of constraint is introduced, the need arises to find solutions to several theoretical and practical problems which arise when working with the constraints. For example

- methods for generating the constraints
- theoretical results such as bounds on the number of channels required in an assignment to satisfy certain conditions
- methods to assist assignment including pre-processing, reduction of problem magnitude, orderings etc.
- algorithms for assignment
- useful representations of problems including diagrams, mathematical objects and metrics

- adapting known methods, where possible, for when these constraints are used.

This chapter discussed a particular type of higher order constraint: the co-channel set constraint. This type of constraint was selected as it is of a simple form, dealing with one type of interference only, and co-channel effects are known (section 4.6.3, [43]) to be a significant part of multiple interference problems. Co-channel set constraints also have the advantage of being able to be mapped onto a corresponding hypergraph, thus allowing for the application of hypergraph-theoretic results to the channel assignment problem formulated in a system of co-channel set constraints.

Although these advantages make co-channel set constraints attractive for use, these constraints have drawbacks in terms of computational requirements. This chapter has highlighted a number of these issues. The numbers of these constraints quickly become intractable, and the generation of maximal independent sets is exponential in the number of transmitters in a network. This leads to a preference for other methods which do not involve the computationally intensive generation, storage and management of such constraints.

If a suitable binary constraint approach could be found which provides results on coverage equalling those potentially available from the use of HOCs, implicitly satisfying the underlying co-channel set constraints, then such an approach would be preferable, as it would avoid the need for all of these areas to be researched. Due to the findings in this and the previous chapters, this thesis pursues the possibility of generating *binary* constraints to efficiently mitigate multiple interference, and doing so without considering all possible interferers at each RTP.

Chapter 6

Binary Constraints for Multiple Interference

6.1 Introduction

This chapter considers the possibility of using binary constraints to encapsulate the channel assignment problem when multiple interference is considered.

Binary constraints are tractable and bounding and solution techniques for them are well-studied (chapter 2); higher order constraints prove difficult to work with. Hence if a binary constraint representation is possible, it is likely to be preferable. This motivates the consideration of the possibility of generating binary constraints for multiple interference. The constraints are no longer based on the single interferer assumption, but improve upon this model, whilst enabling solution by existing techniques which have been developed for use with binary constraints. If multiple interference can be characterised successfully by binary rather than higher order constraints, then all the existing methods and bounds which deal with such constraints can be applied directly, giving binary constraints a clear advantage over higher order constraints, which would require much more new work to be done in these areas.

The inadequacies of binary channel separation constraints produced using the single interferer assumption and/or a constant re-use distance model have been discussed in previous chapters. Most authors who consider multiple interference conclude that a binary constraint representation should be abandoned and a higher order constraint representation used, or even a constraint-free approach (section 2.3). However, it is not necessary that the single interferer assumption be encapsulated in binary constraints and the multiple interferer assumption in higher order constraints. Notwithstanding how this may be achieved, it is entirely permissible for single interference to be represented by non-binary constraints; multiple interference by binary (section 2.3.3).

This chapter looks at the encapsulation of a channel assignment problem in the binary channel separation constraint format discussed previously (section 2.2.2), but by use of novel generation methods which incorporate the consideration of multiple interference. The achievement of such an encapsulation means that existing systems which take binary channel separation constraints as input can be used to create assignments for these constraints, thus allowing the extension of the use of tools previously applied for the single interferer assumption to solution of the multiple interferer problem.

To distinguish between constraints generated in different manners, the following notation will be used:

ϕ is a constraint matrix produced for the single interferer assumption by considering each possible interferer in turn and placing a separation in the matrix if the SIR requires it. The generation of ϕ is shown in algorithm 4.1.

Φ is a new constraint matrix produced using the new methods described in this chapter, which take multiple sources of interference into consideration.

Different ways of generating the constraint matrix Φ are considered, evaluated and compared. Resulting assignments are analysed in two dimensions: coverage provided and spectral requirements. These two operational criteria when considered together describe the trade-off between the two conflicting objectives of maximising coverage whilst using spectrum efficiently (section 2.3.1).

6.2 USING BINARY CONSTRAINTS FOR MULTIPLE INTERFERENCE

Methods are required which produce binary channel separation constraints which are more robust to the effects of cumulative interference than those produced under the single interferer assumption. It is likely that a constraint matrix Φ generated under the multiple interferer assumption will require more channel separation than the matrix ϕ generated under the single interferer assumption. It is desirable that such separations ensure the SIR levels required for coverage, but do so without over-engineering excess separation which could lead to large *unnecessary* increases in spectral requirements.

The constraints generated in this chapter consider sets of potential interferers, rather than each single interferer, moving beyond the scope of the single interferer model to provide for more robust constraints. The algorithms developed are required to control cumulative interference effects from more than one transmitter so, in each algorithm, sets of interferers are formed and their assignment restricted by the imposition of appropriate binary constraints. The set of interferers used in constraint generation is of limited cardinality i.e. does not contain as many transmitters as the $n-1$ potential interferers of the complete interference model. The analysis and results of chapter 4 are used to inform the choice of cardinality of the sets considered in constraint generation. Results on coverage show that generating constraints to mitigate for the cumulative interference effects from sets of relatively few transmitters can lead to assignments which are effective in terms of the coverage they provide.

Three related new methods of constraint generation are applied and evaluated in this chapter. These algorithms impose constraints which lead to a better quality of solution than the single interferer assumption model. This type of constraint generation has not been performed previously. This section discusses the algorithms' common attributes and explains how assignments can be produced for the generated systems of constraints and how the performance they provide is evaluated. The sections which follow go on to define each method more closely, providing pseudo-code and highlighting the differences between the methods, and looking at how the performance of each compares to that of the other methods and of the single interferer assumption model.

6.2.1 Generation of matrix ϕ for single interference

It is useful to note that algorithm 4.1 can be performed, with the same resulting ϕ matrix, in the manner shown in algorithm 6.1. Instead of calculating the required

Input network
Set $\phi_{jk} = 0 \quad \forall \quad j,k$
RTP_i's 'wanted' transmitter is t_k
for all RTP_i **do**
  for all t_j, $j \neq k$ **do**
    while SIR$(i,j) <$ SIR$_a$ (under current ϕ) **do**
      increment ϕ_{jk}
      increment ϕ_{kj}
    end while
  end for
end for

Algorithm 6.1: Generation of matrix ϕ for single interference

separation δf and then comparing it to the current state of ϕ, this method starts from the current state of ϕ at any point and builds on it, incrementing the entries where necessary. This idea of building upon the current state of Φ is used in the algorithms presented by this chapter.

6.2.2 GENERATION OF MATRIX Φ FOR MULTIPLE INTERFERENCE

The constraint generation algorithms presented in this chapter require knowledge of the transmitters and corresponding RTPs to be used. They are given a desired threshold SIR for QoS in the network, as was required for generation using the single interferer assumption and algorithm 4.1. An additional parameter N is required, specifying the number of interferers to be considered. This parameter can be in the range $1 \leq N <$ number of transmitters. When N=1, the situation is equivalent to a single interferer assumption. The analysis and results of chapter 4 are used to inform the choice of value for N. It is seen later that good levels of coverage can be achieved with $N \ll$ number of transmitters. A further parameter, `maxSep`, which is introduced in section 6.5, is also used.

When the algorithm is first invoked, the matrix Φ is the zero matrix. The algorithm considers the signal strengths that would result were an assignment to be created with separations exactly equal to those in Φ at a particular time during the generation. An assignment does not exist at this point, but measures of signal strength are calculated directly from the separations in Φ and the geography of the network.

Each time Φ is altered, the signal strengths being received at RTPs from transmitters will change also, as these are dependent on the channel separations in force. For this reason, whenever Φ is updated, the signal strengths must be updated as well. Only the signal strengths at the RTP under consideration are needed at any time during the constraint generation so, to save unnecessary calculation, a single column of strengths is used and updated. At any point during the algorithm, the ith entry of this column will contain the strength at the RTP under consideration from the ith transmitter under the current Φ. After an increment occurs, the strengths are recalculated under the current Φ and the algorithm continues.

At each RTP, the algorithm finds the set of the N worst potential interferers under the current Φ at that point in the calculations. Φ may still contain all zeros, or some entries may have been incremented in the consideration of a previous RTP. The worst interferers selected are those that would hypothetically be the worst interferers if an assignment were made with channel separations equal to those appearing in the current, possibly incomplete, version of Φ. If this set of worst potential interferers causes unsatisfactory SIR at the RTP under consideration (combining the interference from this set of transmitters in the manner described in section 3.1.3.2, and ignoring all other interferers), an appropriate increment in Φ is desirable.

Three versions of the algorithm are considered, and detailed in the following sections. These are employed, analysed and compared with one another and with the single interferer assumption model. The evaluation, using the analysis method of section 4.5, considers the coverage provided by assignments created for the constraints generated using each method, and looks also at the spectral requirements.

The methods introduced are

- fixed set method (section 6.3);
- recalculation method (section 6.4);
- spread method (section 6.5).

6.2.3 CREATING ASSIGNMENTS

Once binary channel separation constraints have been generated in Φ by the different algorithms under consideration, assignments are required which violate none of these constraints. Such assignments are produced via the hybrid simulated annealing and sequential algorithm described in section 3.4.2.

As noted in section 4.2, the entry 0 in the computed matrix means that no separation is required and therefore the separations stored represent constraints of the form $| f_i - f_j | \geq \Phi_{ij}$. No constraint is output when the entry in the computed matrix Φ is 0. Binary channel separation constraints of the form $| f_i - f_j | > \Phi_{ij}$ are produced by subtracting 1 from the computed entries representing required separations.

For example if the computed matrix is

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

which implies that the separations between two pairs of transmitters are required to be constrained and the other separations may be ≥ 0 i.e. are unconstrained, then the constraints of the form $| f_i - f_j | > \phi_{ij}$ which are output are

$$\begin{aligned} | f_1 - f_2 | &> 0 \\ | f_2 - f_3 | &> 1 \end{aligned}$$

6.2.4 MEASURING PERFORMANCE

The performance of an assignment can be assessed by measuring the coverage provided and the span used to achieve this coverage. This assessment can in turn indicate how successful the constraints were at characterising the operational situation towards the aim of achieving improved coverage in relation to other constraint generation methods.

Ideally, a method of increasing coverage without increasing span is required. However, an improvement in coverage is likely to come with a necessary increase in spectral requirements. These two conflicting objectives must be balanced according to the requirements of the network under consideration. For example, a particular network operator may choose to be satisfied with the coverage provided by a single interferer assumption to avoid the increase in span that a multiple interferer assignment may require, whereas a second network operator may prefer to have maximal coverage as their highest priority.

The measures for coverage are those introduced in section 4.5.3 and employed throughout section 4.5.

Coverage as a percentage of RTPs: The percentage of RTPs which receive satisfactory SIR under the current assignment and evaluation assumptions. The objective is 100% coverage by this measure.

Coverage as a percentage of transmitters: The percentage of transmitters which have all their RTPs covered under the current assignment and evaluation assumptions. The objective is 100% coverage by this measure.

The objective of 100% RTPs coverage and that of 100% transmitter coverage are equivalent: if all transmitters serve all their tuned RTPs then all RTPs are covered, and vice versa.

The objective for spectrum use is to minimise the span of channels used. Both upper bounds, found by creating assignments, and lower bounds, found using the technique outlined in section 3.5.2, are used to determine performance in terms of this second objective.

6.3 FIXED SET METHOD

6.3.1 ALGORITHM

Algorithm 6.2 shows the pseudo-code for this, the simplest constraint generation algorithm presented, which involves at each RTP finding the worst N potential interferers, and making separations in Φ to mitigate for the cumulative interference from that set of transmitters. Once found at an RTP, the set `F` of worst interferers is fixed throughout the duration of the procedure at that RTP. This differs from the succeeding versions of Φ generation in which the set of worst interferers is recalculated with each change to Φ. When N=1, this algorithm becomes equivalent to calculating constraints under a single interferer assumption considering only the single most dominant interferer (under the current Φ) at each RTP, rather than each single interferer in turn as is the case in algorithm 4.1.

Input network
Set $\Phi_{jk} = 0 \quad \forall \quad j, k$
Initialise N as required
for all RTP_i **do**
    Store signal strengths from all transmitters at this RTP under current Φ
    Find the set F of N worst inter. at this RTP with current Φ
    while F causes failure **do**
        Order F, worst first
        Increment first separation
        Recalculate strengths under current Φ
    end while
end for

Algorithm 6.2: Generation of matrix Φ by fixed set method

At each iteration, the current Φ is built upon. Consider two neighbouring RTPs and assume that the current Φ has all zero entries. When the first RTP is considered, the N worst interferers will be the closest N to that RTP in terms of geographical distance (as there are no channel separations in place). The cumulative effect of these N interferers will be mitigated for by increments made to Φ in this iteration. When the neighbouring RTP is considered, several or all of the interferers which would have been dominant under the initial Φ will no longer be dominant due to the separations already made. Therefore the set of worst N interferers will be a different set, thus allowing the consideration at each RTP to build upon the information already gained in previous iterations.

The set of worst interferers is always sorted in order, worst first. When an increment is called for, the algorithm chooses to increase the separation between the serving transmitter and the worst interferer of the N interferers in the set F. Note that although the elements of the set F remain constant once the set has been constructed, their order will change so that F is always sorted worst first under the current Φ.

This method cannot guarantee 100% coverage, even when resulting assignments are evaluated under single interference. When the method is used with N=1, it considers interference from the single most dominant interferer at any point. This can give coverage levels lower than those provided by assignments produced for constraints generated under the single interferer assumption by considering all individual interferers in turn. When a multiple interferer set of three interferers is considered, for example, it may be possible that these three, and a fourth, would each have been single interferers requiring a separation in ϕ. The resulting assignment may not mitigate interference from *all* sets of three interferers, nor even *all* single interferers. In general, running this algorithm with $N=m$ cannot guarantee that there will be no cumulative interference effects from any interferer set of size m, or indeed from any sized interferer set. This method has the potential to be better or worse than the single interference ϕ in different network situations, depending on factors such as transmitter density.

6.3.2 Coverage results and spectrum use

Assigned for		N=1		N=3		N=6	
Evaluated with		ISS=1	ISS=$n-1$	ISS=3	ISS=$n-1$	ISS=6	ISS=$n-1$
RTP cov. provided	mean	98.5	88.4	99.6	97.6	99.9	99.7
	min	93.5	69.0	96.8	87.7	99.2	96.6
Trans. cov. provided	mean	86.6	47.3	95.7	83.1	99.1	97.7
	min	63.8	1.0	80.2	28.0	94.0	84.5

Table 6.1: The sixty library cases are assigned for 9dB via constraints which are generated by the fixed set method considering the number of interferers specified; transmitter coverages and RTP coverages are displayed when the assignments are evaluated for (a) ISS=N (b) ISS=$n-1$ i.e. complete interference

Table 6.1 shows the coverage results achieved when the sixty library cases are assigned for 9dB via constraints which are generated by the fixed set method considering 1, 3 and 6 interferers respectively. These assignments are evaluated with the number of active interferers, ISS, set equal to the value of parameter N used to create them, and then with ISS set to the maximum possible (i.e. complete interference evaluation). The coverages provided are displayed in this table, using both measures (transmitter and RTP coverage). The transmitter coverage provided under complete interference evaluation when N takes values from 1 to 6 for 9dB assignments can be seen in figure 6.1.

Note that when evaluation has ISS=m, evaluation is for the worst m interferers under the final Φ and the assignment produced using it, whereas generation considered the worst m potential interferers under Φ as it stood at the time. These are not necessarily the same, meaning that an assignment made for $N=m$ may evaluate below 100% coverage for ISS=m.

This method provides good levels of coverage over the library test cases. When the method is used with N=6, the transmitter coverage provided by assignments made for SIR_a=9dB and evaluated under complete interference is on average 97.7% (figure 6.1). This assignment solves constraints generated for the six most dominant potential interferers, but provides high coverage even under complete interference evaluation, when all interfering transmitters are simultaneously active. Figure 6.2 highlights how increasing the parameter N improves coverage, even when evaluation is for ISS>N. As N rises, the coverage provided converges well towards that of a complete interference generation model even though $N \ll n$.

The average transmitter coverage provided by the assignments made using N=6 is 97.7% under complete interference evaluation, as opposed to 58.1% when the single interferer assumption assignment is evaluated under the same conditions. In fact, over the sixty cases, the worst case transmitter coverage provided by such an assignment is 84.5% (table 6.1), meaning that even the worst assignment produced by the fixed set method with N=6 provides better coverage than the average single interferer assumption assignment.

Although excellent coverage was provided by this method for the higher values of N used (figure 6.2), table 6.1 reinforces the drawback that using the algorithm with N=m cannot guarantee 100% coverage even when evaluated for ISS=m. Twenty-eight of the sixty cases assigned for 9dB via the fixed set method with N=3 give 100% coverage when evaluated for ISS=3 (table A.6). Thirty-nine of the sixty cases assigned for 9dB via the fixed set method with N=6 give 100% coverage when evaluated for ISS=6 (table A.7). This limitation is overcome in the recalculation method presented in section 6.4.

Figure 6.3 shows the additional span used when using the fixed set method with N=6 as opposed to the single interferer model when creating assignments for 9dB SIR. It is seen that the increase in the span used is significant when balanced against the fact that the method provides no coverage guarantees. These issues are discussed further in section 6.4.

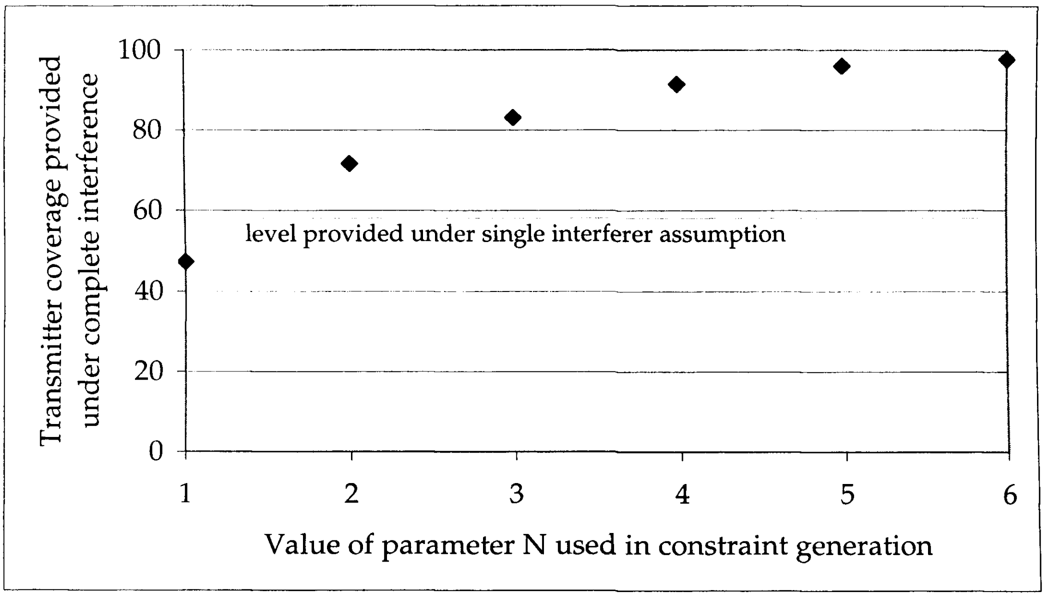

Figure 6.1: The sixty library cases are assigned for 9dB via constraints which are generated by the fixed set method for different values of the parameter N; plotted are the transmitter coverages provided by these assignments when evaluated under complete interference

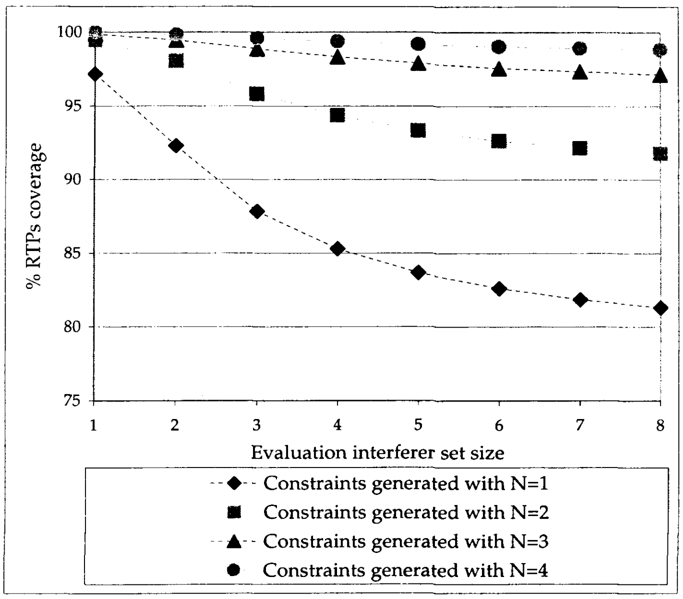

Figure 6.2: All sixty library cases are assigned for 12dB service threshold SIR via constraints generated by the fixed set method with N=1, N=2, N=3 and N=4; evaluation is for ISS=1, . . . , 8 and the percentage RTPs coverage is plotted.

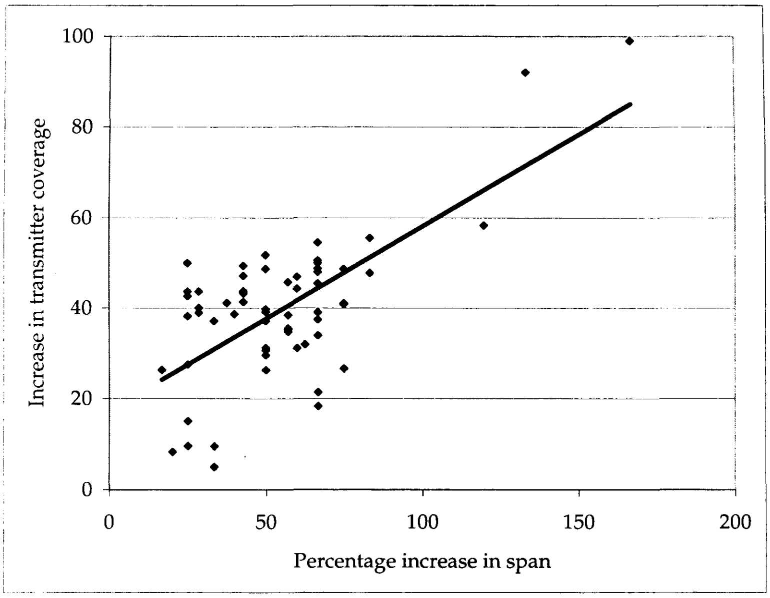

Figure 6.3: The sixty library cases are assigned for 9dB service threshold using constraints produced (a) under the single interferer assumption (b) using the fixed set method with N=6. The change in transmitter coverage is plotted against the percentage change in span, with each point representing a single test case.

6.4 Recalculation method

6.4.1 Algorithm

This method greatly improves upon the fixed set method of section 6.3, having the important advantage that an assignment created for constraints generated by this method with N=m can be guaranteed to give 100% coverage when evaluated for ISS=m. Algorithm 6.3 shows the pseudo-code for this constraint generation algorithm. Algorithm 6.3 causes the set R of the current N worst potential interferers to be *recalculated* whenever the Φ is updated, rather than keeping the same set of N once it is determined at a particular RTP (as in algorithm 6.2), meaning that at each RTP *all* potential interferer sets of size N are mitigated against.

The set of worst interferers is always sorted in order, worst first. Note that the elements of the set R change as Φ is updated, as does their ordering, so that R always contains the worst N interferers under the current Φ, sorted worst first. When an increment to the matrix is called for, the algorithm chooses to increment the separation between the serving transmitter and the first interferer in R i.e. the worst interferer.

If algorithm 6.3 is used, the multiple interference constraint generation method becomes directly analagous to the original single interferer assumption binary ϕ generation algorithm given in algorithm 4.1. The single interferer binary con-

Input network
Set $\Phi_{jk} = 0 \quad \forall \quad j,k$
Initialise N as required
for all RTP_i **do**
  Store signal strengths from all transmitters at this RTP under current Φ
  Find the set R of N worst inter. at this RTP with current Φ
  while R causes failure **do**
    Order R, worst first
    Increment first separation
    Recalculate strengths under current Φ
    Recalculate R with current Φ
  end while
end for

Algorithm 6.3: Generation of matrix Φ by recalculation method straints generated by considering each single interferer in turn guarantee that any one interferer will not cause failure; this algorithm will ensure that any set of N will not cause failure. N=1 gives a set of binary channel separation constraints for single interference which is qualitatively the same, and often identical to, the set of constraints produced by algorithm 4.1. (Any slight differences are caused by the order in which transmitters are considered: this algorithm considers them worst first; the original algorithm, in numerical order.)

6.4.2 COVERAGE RESULTS AND SPECTRUM USE

Figure 6.5 shows the coverage achieved in terms of the percentage of RTPs covered when the library cases are assigned from constraints generated for 12dB with N=1, N=2, N=3 and N=4 (i.e. considering single, double and triple interference and four simultaneous interferers). When the assignment is made using constraints designed to mitigate triple interference effects, it in fact also gives 99.72% coverage when eight simultaneous interferers are considered by the evaluation (ISS=8). When the effects of four simultaneous interferers are mitigated for (N=4), this is 99.99%. Due to the direct relationship between the methods, the points on the graph for the recalculation method with N=1 provide an indication of the coverage levels provided by the single interferer assumption model.

The two main advantages of the recalculation method over the fixed set method are that

(a) the recalculation method with N>1 always provides improved coverage in comparison with the single interferer assumption model whereas the fixed method may not lead to a coverage level as high as that provided under single interference; this can be seen in figure 6.7 which combines figures 6.1 and 6.4 and provides a comparison of the coverage provided by these methods.

(b) when the recalculation method is used to generate constraints with $N=m$, 100% coverage is guaranteed when assignment evaluation is performed with ISS=m; the fixed set method provides no such guarantee, and this is illustrated by tables 6.1 and 6.2 and figure 6.5.

Figure 6.5 plots the percentage of RTPs covered when assignment is made for constraints using the recalculation method with N=1, N=2, N=3 and N=4, averaged over the sixty library cases with SIR_e=12dB. When N=5 and N=6 in this same situation, all coverages when assignment evaluation was for ISS=1, ..., 8 were 100%. This figure (6.5) shows that assigning for double interference significantly increases coverage and when triple interference is considered, situations with interferer sets of up to 8 interferers have almost 100% coverage, without consideration of these numbers of interferers in generation.

When an assignment created to solve the constraints generated by the recalculation method with $N=m$ is evaluated with ISS=m, the coverage achieved is always 100%. Over the sixty library cases assigned at 9dB, the recalculation method with N=6 also gives 100% coverage in 56 of 60 assignments evaluated under complete interference; the other cases have 99.9% RTPs coverage. Over the sixty library cases assigned at 17dB, the recalculation method with N=6 in fact gives 100% coverage under every one of the 60 cases when evaluated under complete interference (figures 6.13 and 6.16), highlighting the superfluity of moving from a single interference model directly to consideration of complete interference during constraint generation for multiple interference.

It is unnecessary over the library instances used to consider more than six interferers in constraint generation to achieve excellent coverage, and significant coverage improvements over the single interferer assumption model may be achieved with as few as two, three or four concurrent interferers brought into consideration during constraint generation. The number of interferers to consider in a particular case can be selected according to preference for the balance achieved in the trade-off between coverage and span.

For example, figures 6.9–6.16 show that over the sixty 17dB cases, the recalculation method with N=3 gives almost as much coverage improvement over the single interferer assumption as does the recalculation method with N=6, but with approximately half the increase in span necessary.

Although the increase in span when moving from the single interference model to use of the recalculation method (e.g. figure 6.6) is not insignificant, the extra span used is not much more than for the fixed set method, with the crucial difference that the recalculation method guarantees the coverage being achieved by this increase in span. The trade-off between coverage and spectrum use is also illustrated for N=6 in figure 6.8. Figures 6.17 and 6.18 illustrate the profile of the trade-off between span and coverage as the parameter N is varied.

Figure 6.6 displays the change in span when moving from the single interferer assignment to the assignment from the recalculation method with N=6. Figures 6.9–6.16 demonstrate that, for the instances considered in this thesis, it may be more appropriate to employ an even lower value of N, thus reducing the necessary increase in span. The recalculation method with N=3 leads in some cases to 100% coverage under complete interference evaluation (see appendix A), so there is no need to consider N any higher than this for these cases.

	Assigned for	N=1		N=3		N=6	
	Evaluated with	ISS=1	ISS=$n-1$	ISS=3	ISS=$n-1$	ISS=6	ISS=$n-1$
RTP cov.	mean	100	92.3	100	99.1	100	100.0
provided	min	100	81.6	100	94.3	100	99.9
Trans. cov.	mean	100	58.1	100	91.5	100	99.9
provided	min	100	1.0	100	28.0	100	98.5

Table 6.2: The sixty library cases are assigned for 9dB via constraints which are generated by the recalculation method considering the number of interferers specified; transmitter coverages and RTP coverages are displayed when the assignments are evaluated for (a) ISS=N (b) ISS=$n-1$ i.e. complete interference

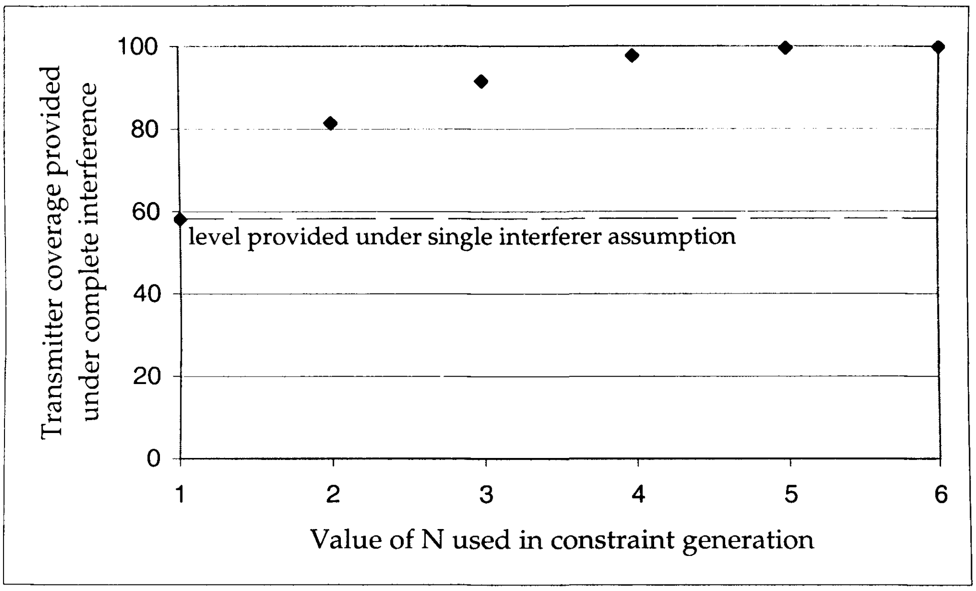

Figure 6.4: The sixty library cases are assigned for 9dB via constraints which are generated by the recalculation method for different values of the parameter N; plotted are the transmitter coverages provided by these assignments when evaluated under complete interference

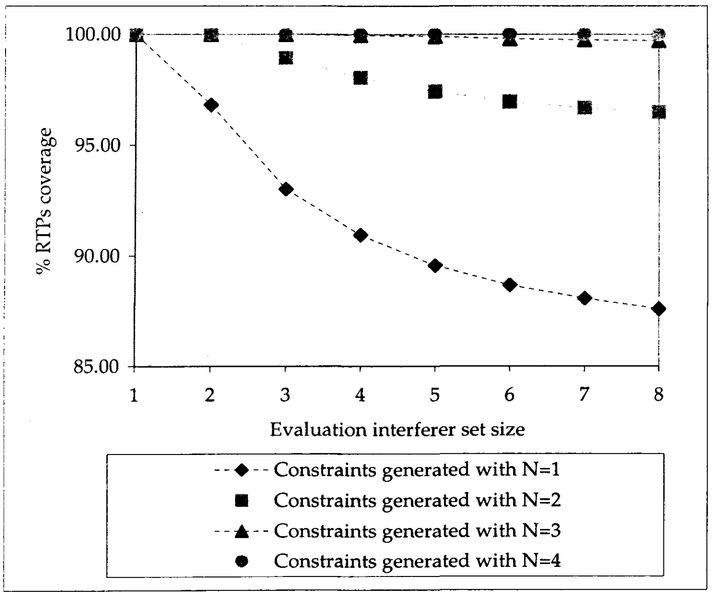

Figure 6.5: All sixty library cases are assigned for 12dB service threshold SIR via constraints generated by the recalculation method with N=1, N=2, N=3 and N=4; evaluation is for ISS=1, ... , 8 and the percentage RTPs coverage is plotted.

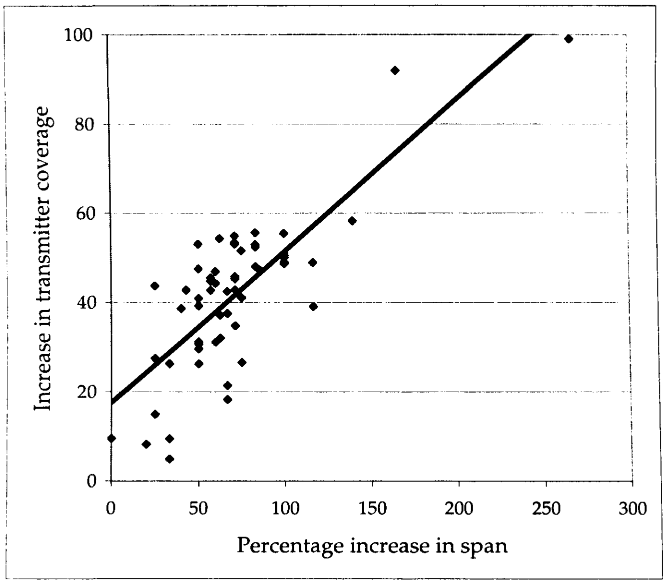

Figure 6.6: The sixty library cases are assigned for 9dB service threshold using constraints produced (a) under the single interferer assumption (b) using the recalculation method with N=6. The change in transmitter coverage is plotted against the percentage change in span, with each point representing a single test case.

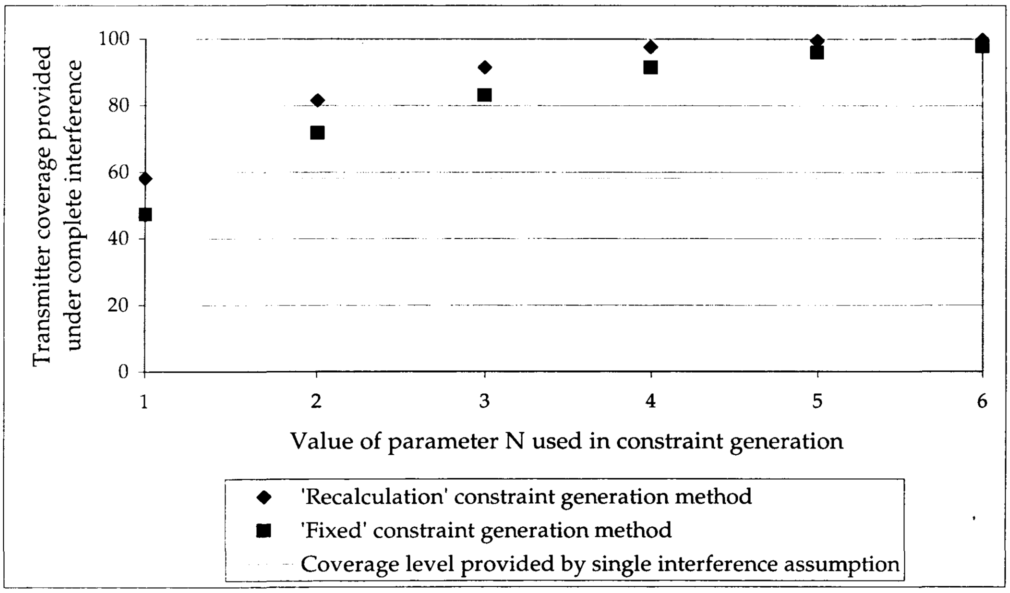

Figure 6.7: The sixty library cases are assigned for 9dB via constraints which are generated by the fixed set and recalculation methods for different values of the parameter N; plotted are the transmitter coverages provided by these assignments when evaluated under complete interference; the level of coverage provided by the assignment made under the single interferer assumption is also shown.

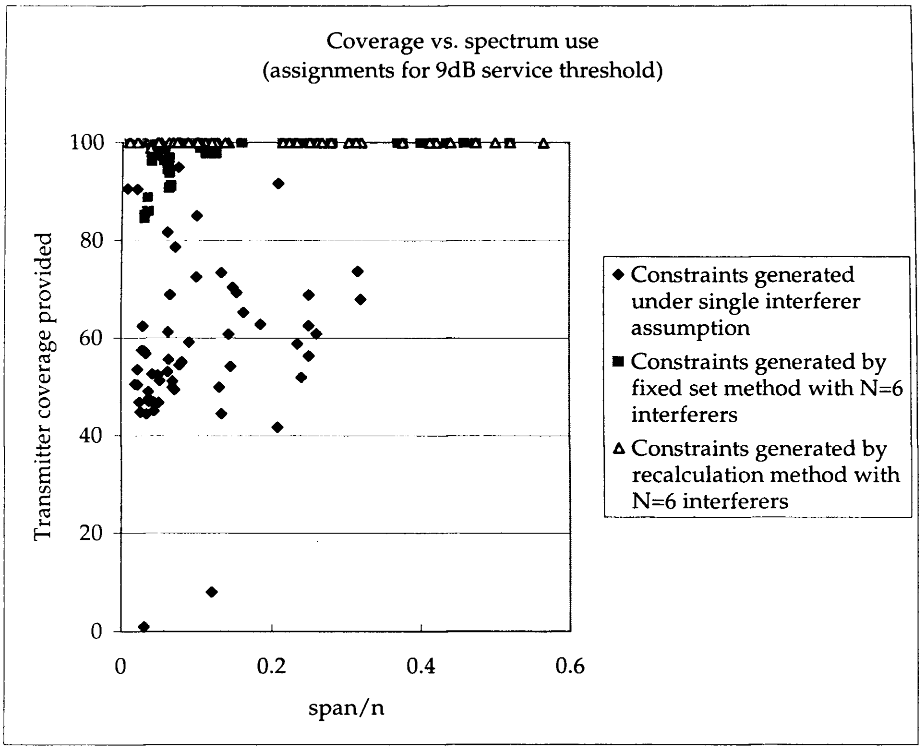

Figure 6.8: Assignments for 9dB service threshold SIR are made via constraints generated under (a) the single interferer assumption (b) fixed set method with N=6 (c) recalculation method with N=6; the transmitter coverage provided when evaluation is for complete interference is plotted against span/n where n is the number of transmitters in that network

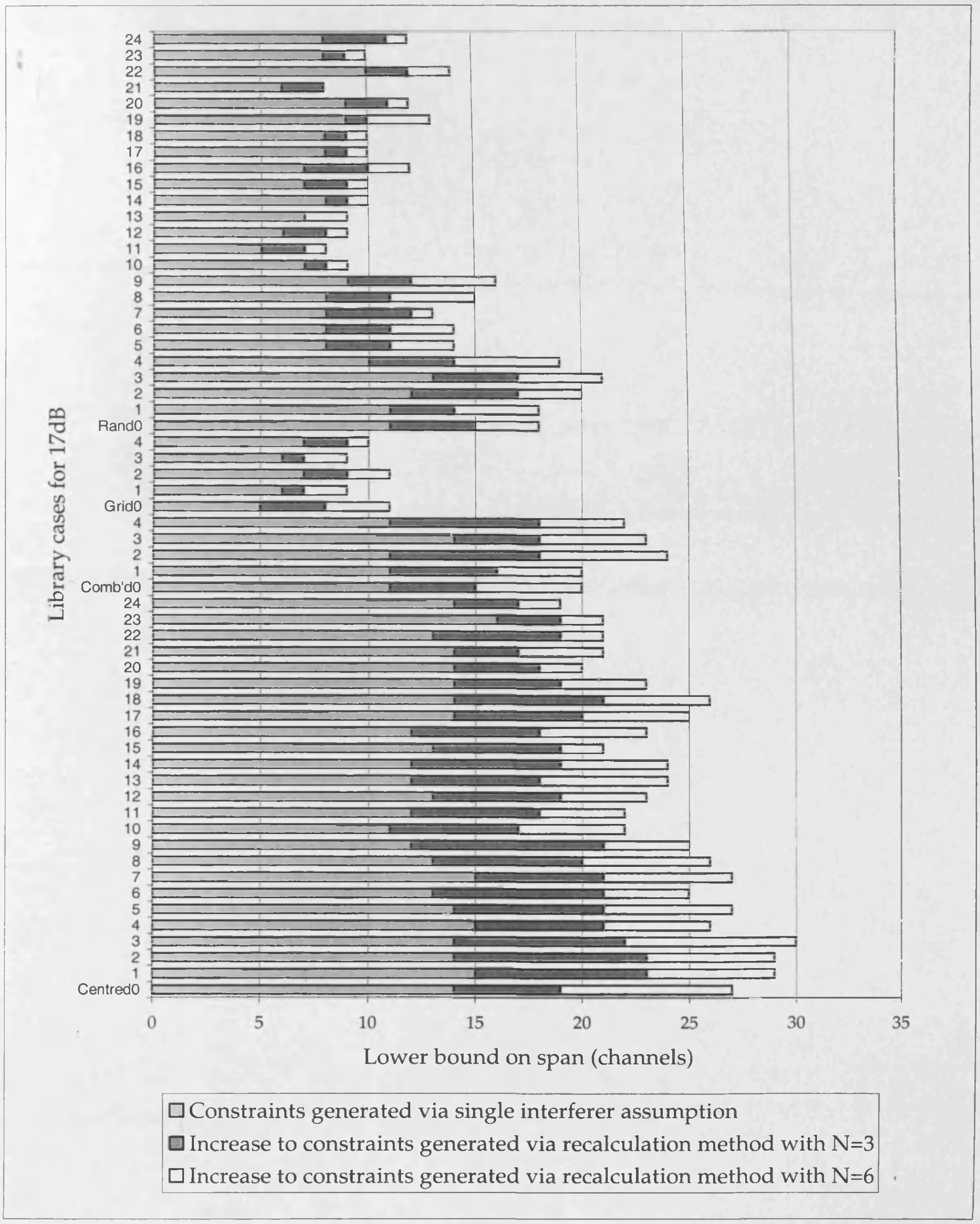

Figure 6.9: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The lower bounds are shown for each case, illustrating the potential required increase in span when the different constraint methods are used.

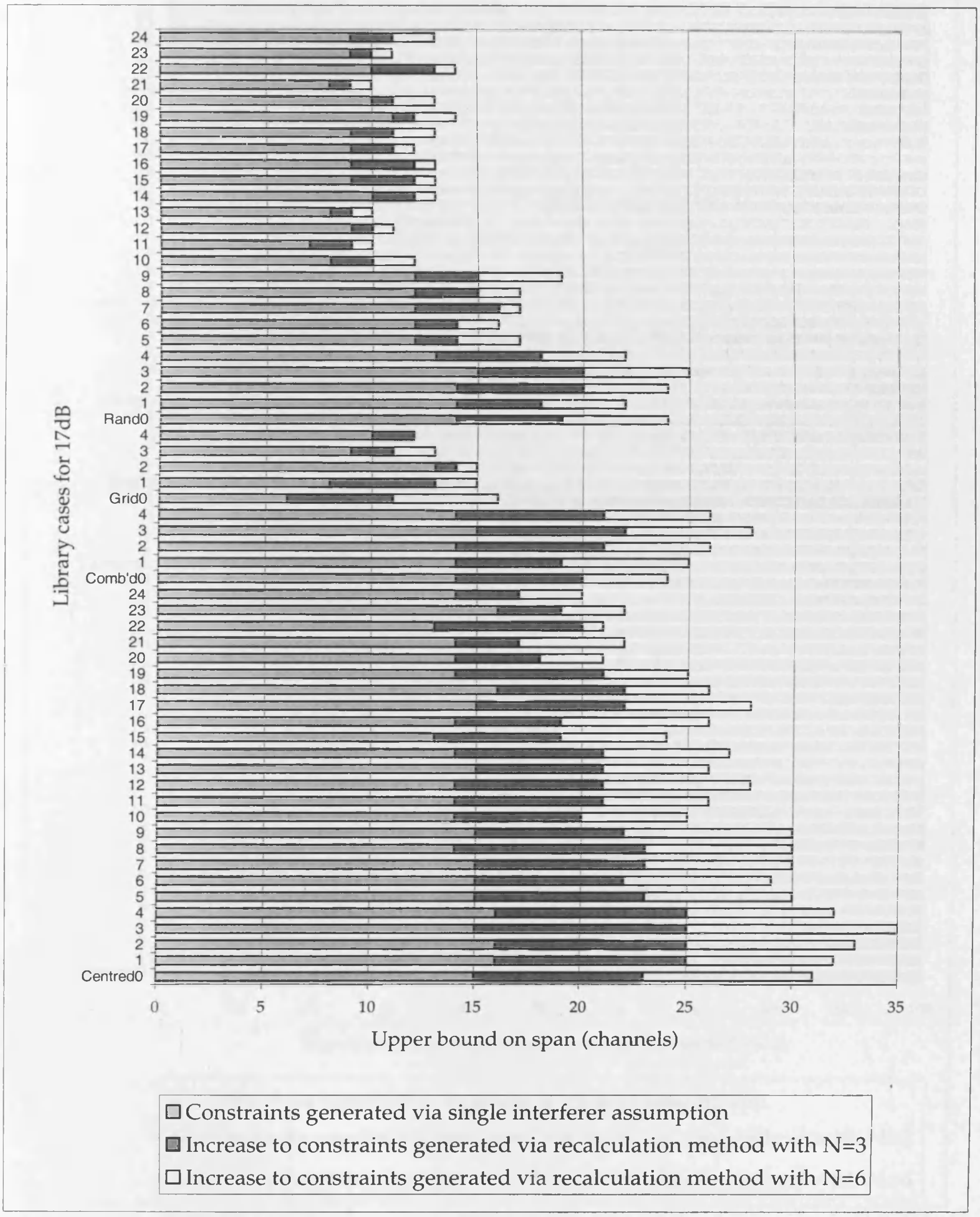

Figure 6.10: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The upper bounds are shown for each case, illustrating the observed increase in span when the different constraint methods are used.

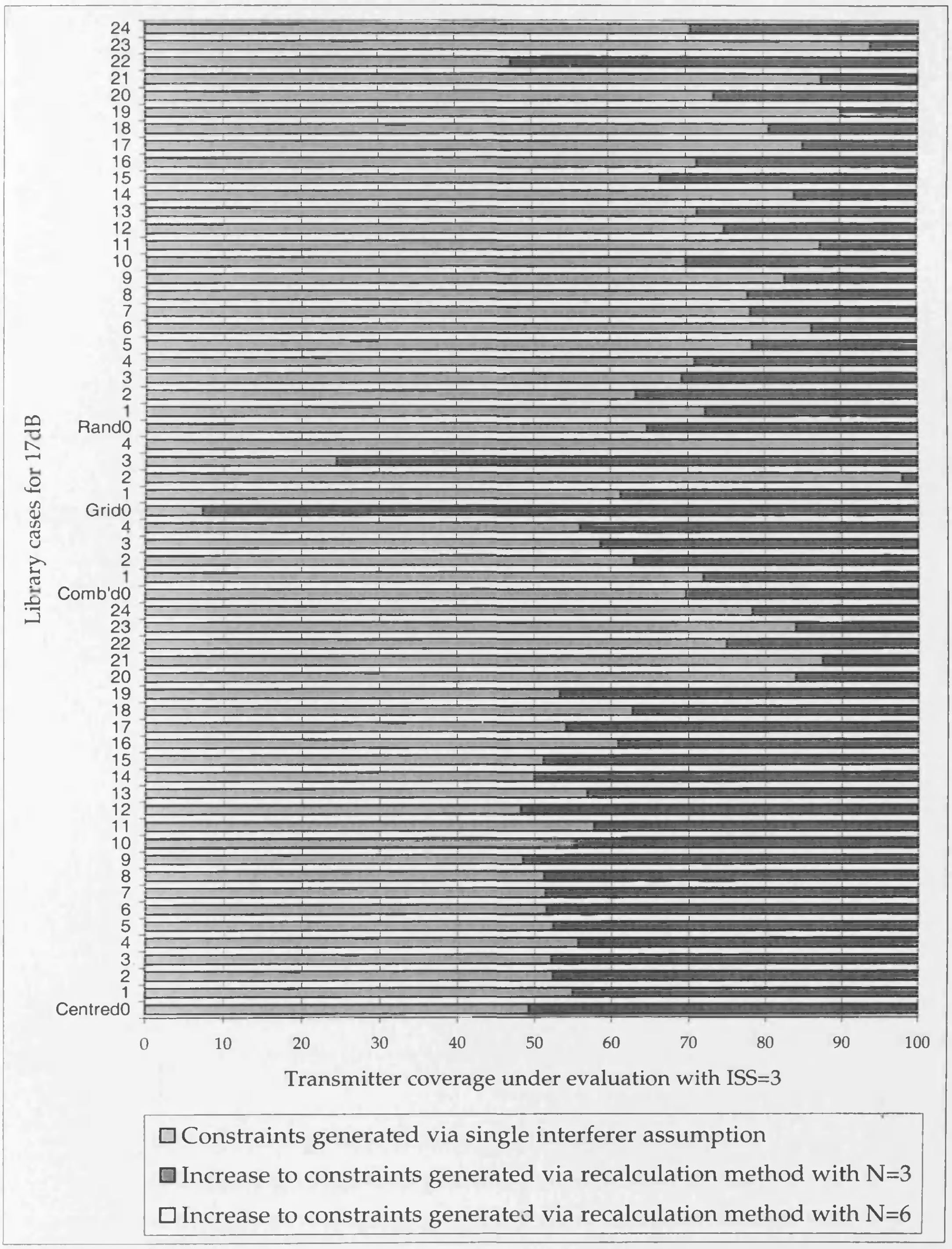

Figure 6.11: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage transmitter coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed for ISS=3.

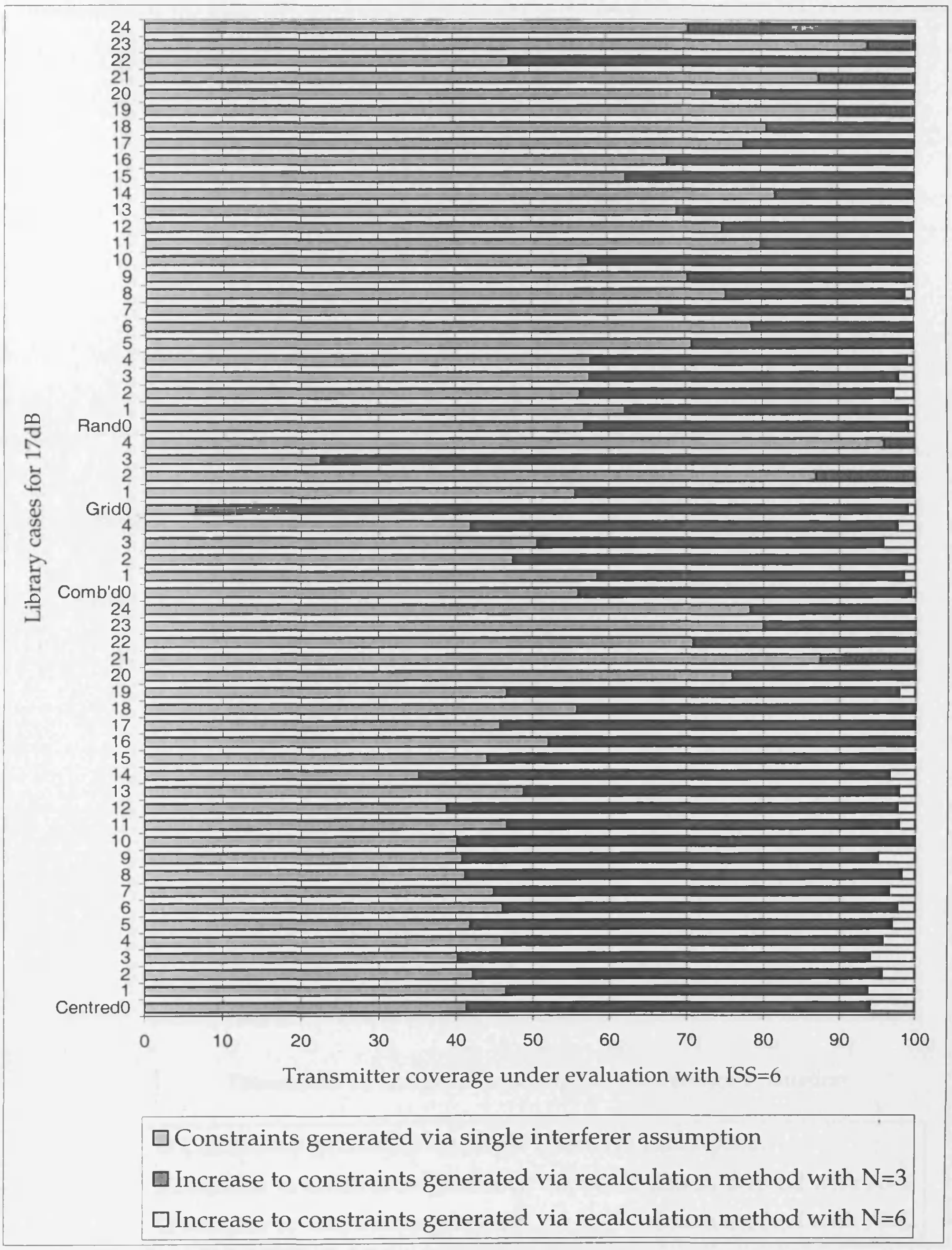

Figure 6.12: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage transmitter coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed for ISS=6.

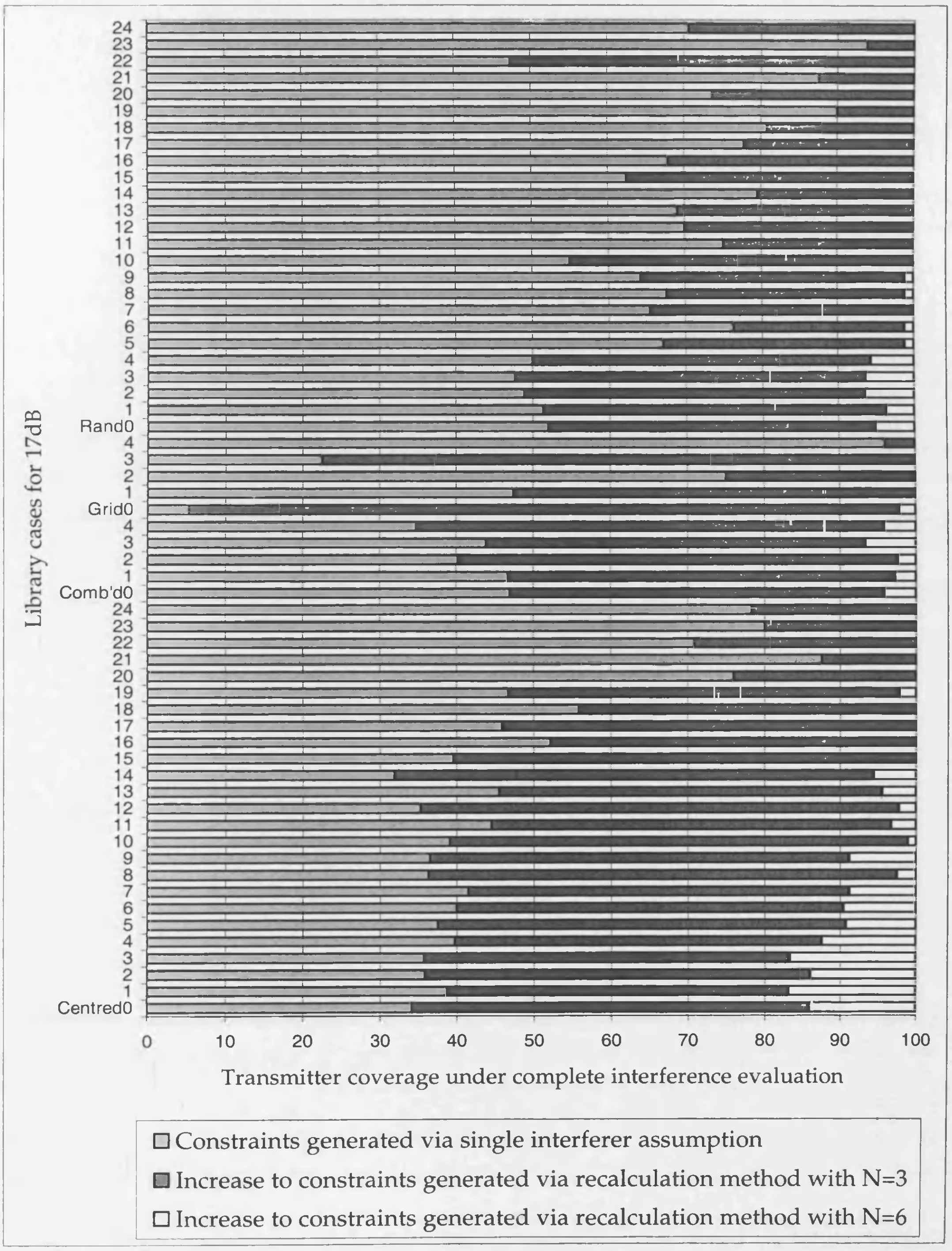

Figure 6.13: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage transmitter coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed under complete interference.

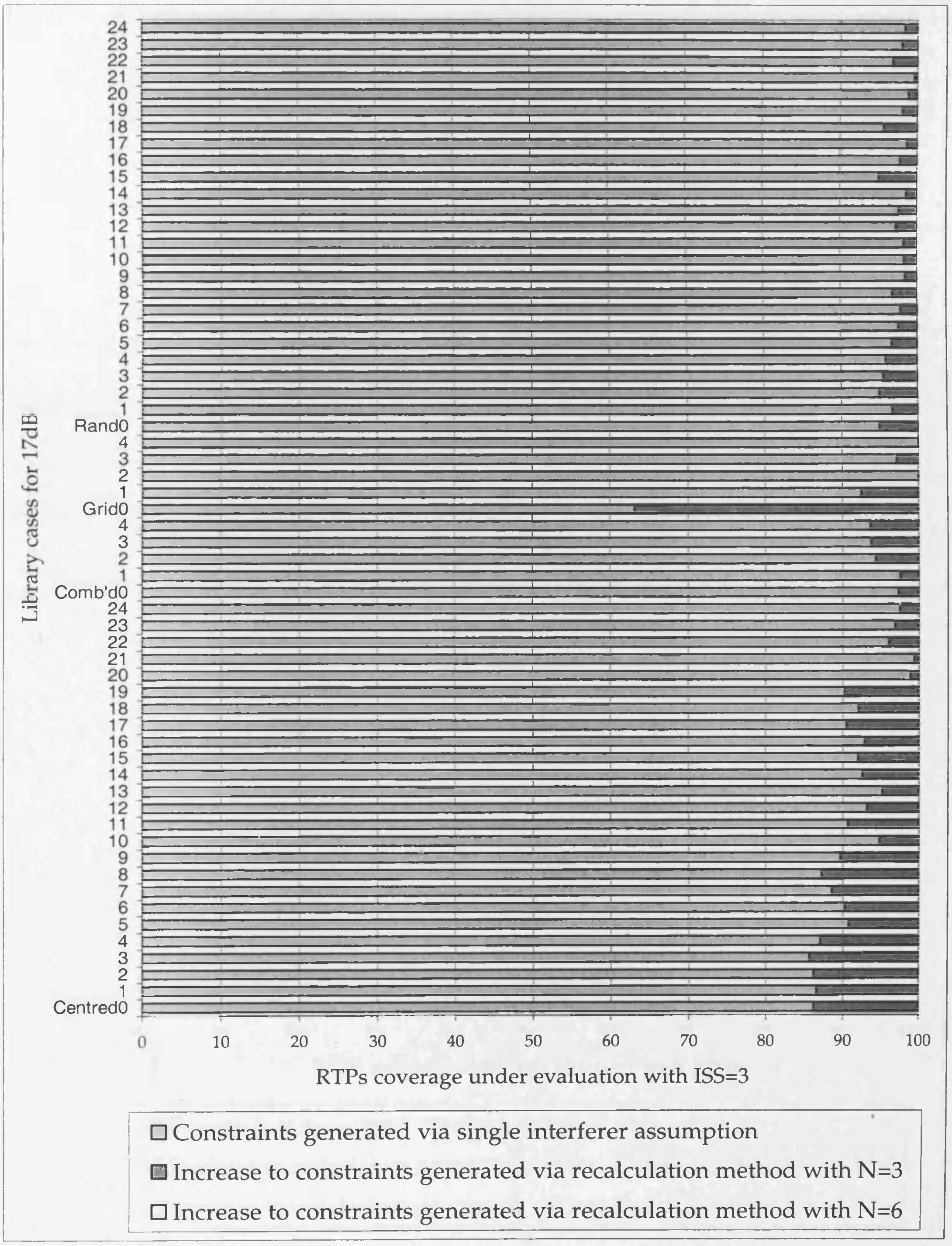

Figure 6.14: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage RTP coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed for ISS=3.

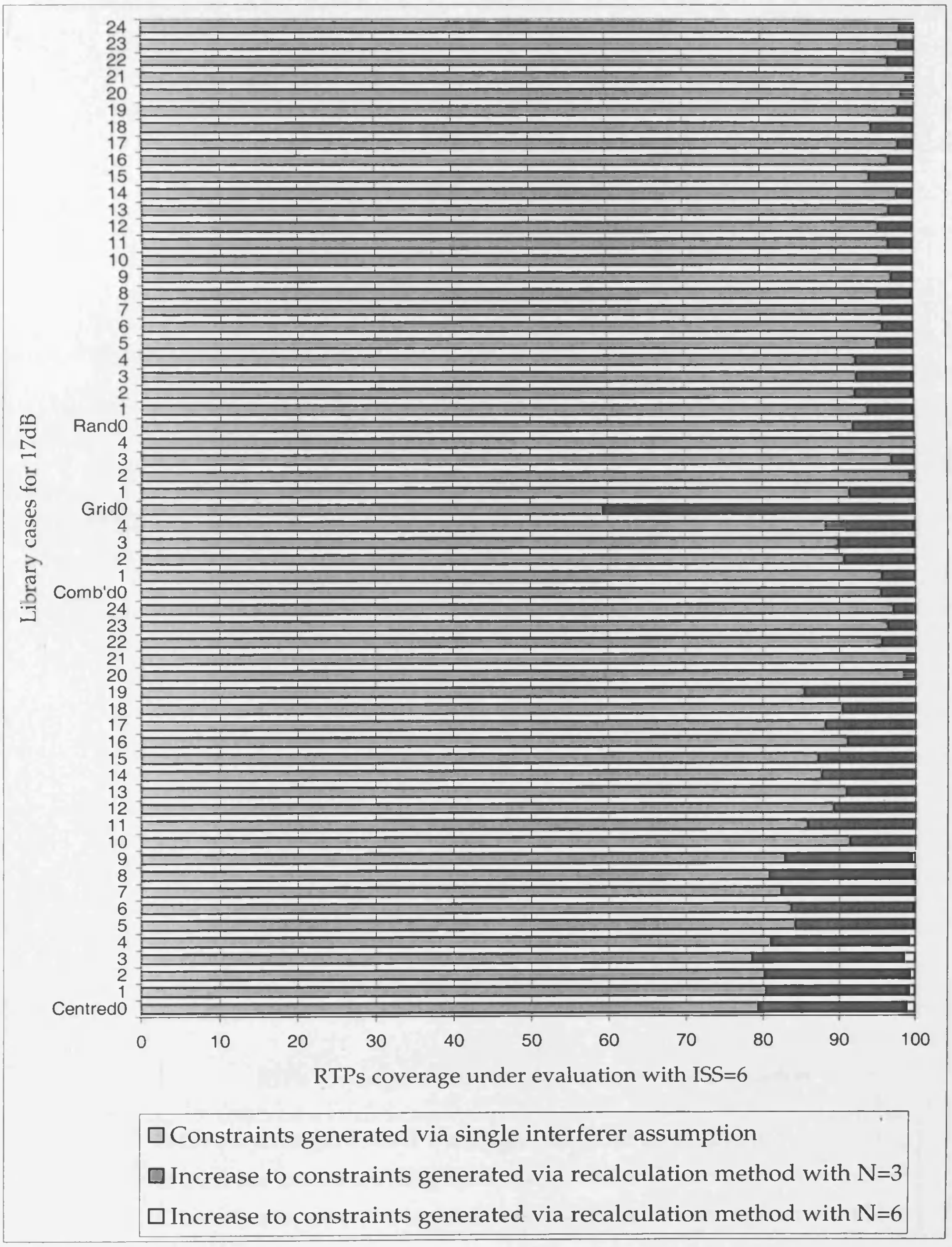

Figure 6.15: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage RTP coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed for ISS=6.

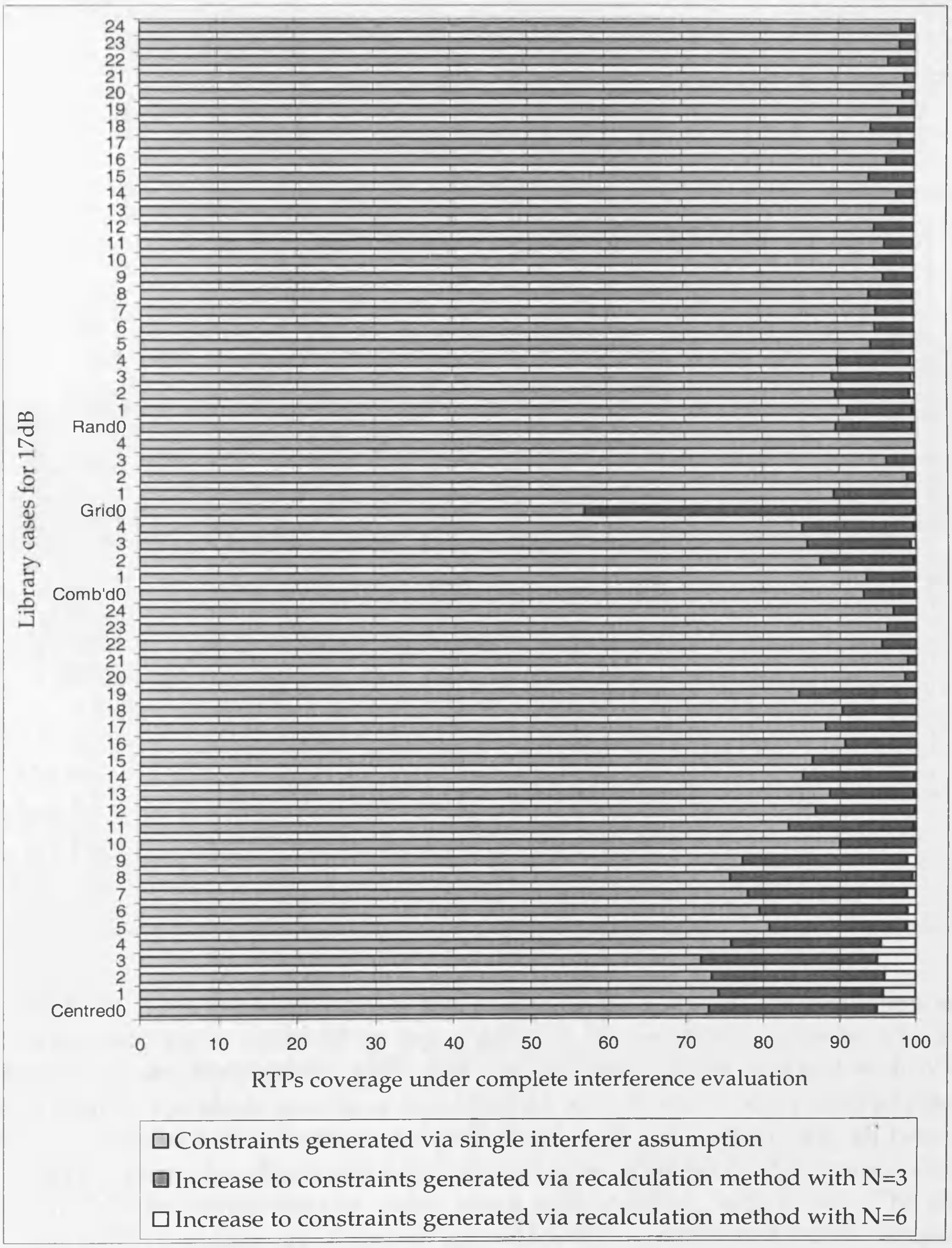

Figure 6.16: The sixty library cases are assigned for 17dB SIR via (a) the single interferer assumption (b) the multiple interferer assumption using the recalculation method with N=3 (c) the multiple interferer assumption using the recalculation method with N=6. The percentage RTP coverages are shown for each case, illustrating the observed increase in coverage when the different constraint methods are used. Evaluation is performed under complete interference.

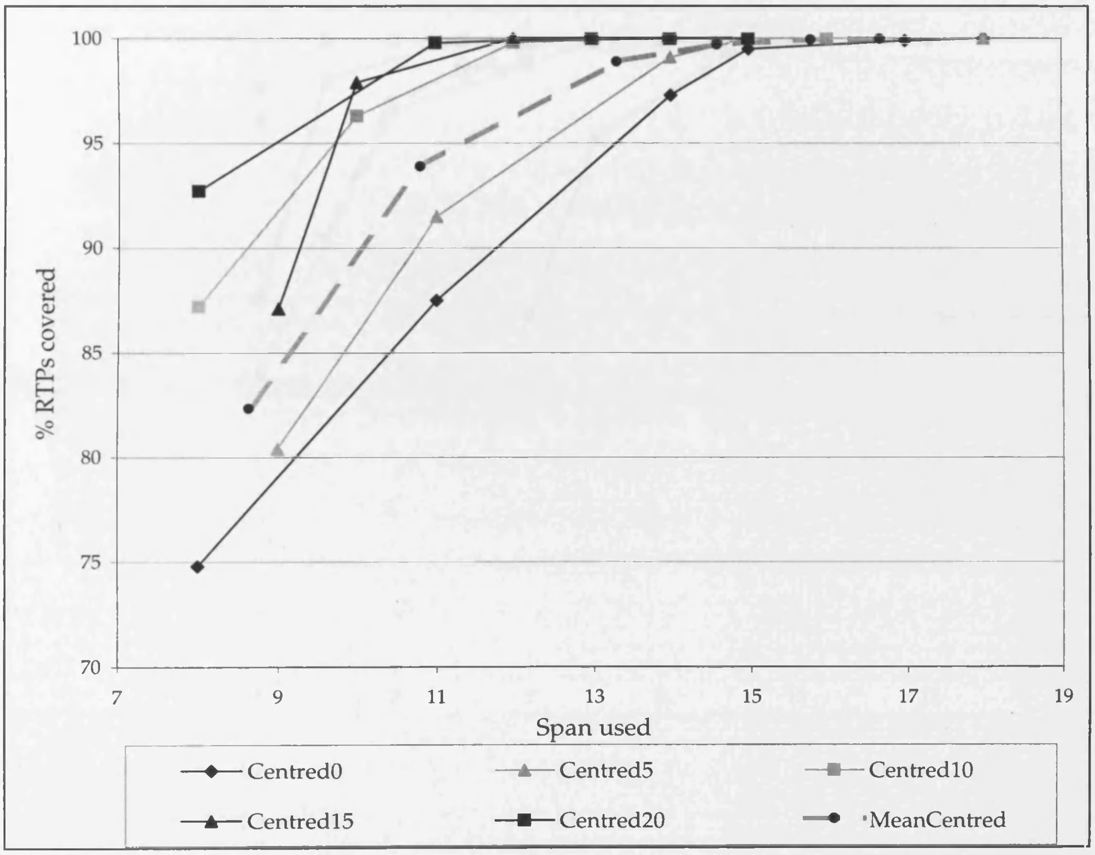

Figure 6.17: This figure illustrates the profile of the trade-off between span and coverage that can be achieved by using different values of the parameter N. The library cases are assigned for 12dB SIR via the recalculation method with N=1 (equivalent to the single interferer assumption), and N=2, . . . , 6. A centred example of each density classification is shown, along with the average over all twenty-five centred examples. Each point represents the assignment of that instance with different N, the lower-leftmost point being that achieved with N=1. The percentage RTP coverages are shown when evaluation is performed under complete interference, and the span shown is the upper bound span achieved by the assignment.

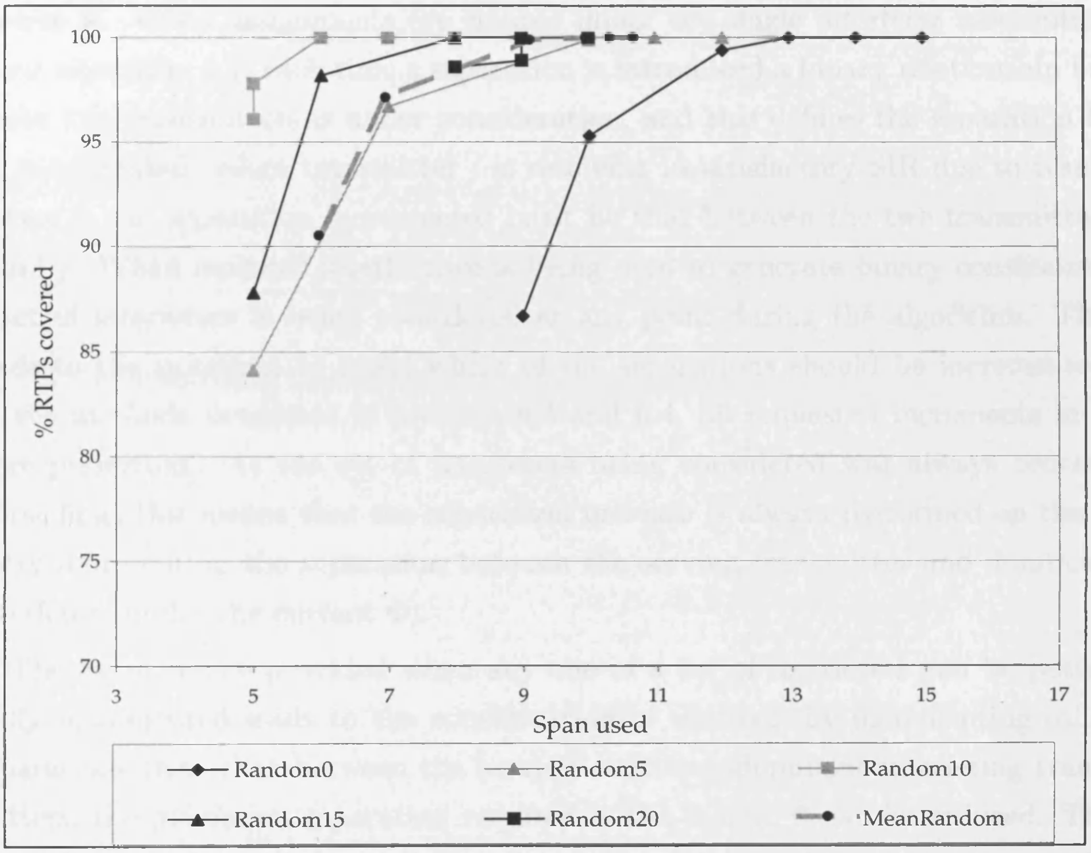

Figure 6.18: This figure illustrates the profile of the trade-off between span and coverage that can be achieved by using different values of the parameter N. The library cases are assigned for 12dB SIR via the recalculation method with N=1 (equivalent to the single interferer assumption), and N=2, ..., 6. A random example of each density classification is shown, along with the average over all twenty-five random examples. Each point represents the assignment of that instance with different N, the lower-leftmost point being that achieved with N=1. The percentage RTP coverages are shown when evaluation is performed under complete interference, and the span shown is the upper bound span achieved by the assignment.

6.5 Spread method

As multiple interference is taken into account from more sources than under the single interferer assumption, more separations are necessarily introduced into the matrix Φ. When assignments are created under the single interferer assumption using algorithm 4.1, each time a separation is introduced a binary relationship between two transmitters is under consideration, and this defines the separation to be incremented: when transmitter i is receiving unsatisfactory SIR due to transmitter j, the separation incremented must be that between the two transmitters i and j. When multiple interference is being used to generate binary constraints, a set of interferers is being considered at any point during the algorithm. This leads to the potential to *select* which of the separations should be incremented. In the methods presented in sections 6.3 and 6.4, all requested increments in Φ were permitted. As the set of interferers being considered was always ordered worst first, this means that the separation increase is always performed on the Φ entry representing the separation between the serving transmitter and dominant interferer (under the current Φ).

The extra choice provided when any one of a set of interferers can be potentially incremented leads to the consideration of whether, by incrementing other separations than that between the serving and most dominant interfering transmitters, the maximum separation required in the matrix Φ can be reduced. The 'spread' method (algorithm 6.4) aims to minimise the maximum separation in Φ. It attempts to replace the constraints generated by the recalculation method by constraints which will be more numerous but each requiring a lower separation.

This section performs experimentation towards determining whether the spread method can indeed influence the separations occurring in Φ and, if so, what impact this has on the two evaluation criteria of coverage and span.

6.5.1 Algorithm

Algorithm 6.4 shows this third method of constraint generation. As well as focussing on the mitigation of interference from sets of N potential interferers under SIR_a, as in the recalculation method, it also considers the separations used, trying to minimise the largest separation appearing in Φ. This is done by use

```
Input network
Set Φ_jk = 0  ∀  j, k
Set maxSep= 1
Initialise N as required
while an increment in Φ took place do
  for all RTP_i do
    Store signal strengths from all transmitters at this RTP under current Φ
    Find the set R of N worst inter. at this RTP with current Φ
    while R causes failure do
      if a possible Φ increment remains then
        Order R, worst first
        Increment first allowed separation
        Recalculate strengths under current Φ
        Recalculate R with current Φ
      else
        Start again with the next maxSep
      end if
    end while
  end for
end while
```

Algorithm 6.4: Generation of matrix Φ by spread method

of the parameter maxSep. This additional parameter is the maximum separation allowed in the matrix Φ produced. When the algorithm is first invoked, the matrix Φ is the zero matrix. When the process is finished, the Φ matrix produced will have elements with values in the range $0 \leq \Phi_{ij} \leq$ maxSep.

When an assignment is created to satisfy a Φ matrix, the primary objective is to violate no constraints; the secondary objective is to minimise the span of channels used. The minimum possible span is defined by the properties of the constraints. It is possible that use of the spread method to minimise the maximum separation may allow the assignment to be performed with reduced span. However, the 'smearing out' of the required separations over the matrix entries may introduce more complicated relationships between the constraints, such as larger cliques, and this may result in the span not being reduced, and perhaps increasing.

Whenever the set of worst interferers causes unsatisfactory SIR at the RTP under consideration (combining the interference from this set of transmitters in the manner described in section 3.1.3.2, and ignoring all other interferers), an appropriate increment in Φ is desirable. When an increment to the matrix is called for, the algorithm firstly checks if any increments are possible: possible increments remain if at least one separation in Φ between the RTP's tuned transmitter and the members of worst set is < `maxSep`. The set of worst interferers is always sorted in order, worst first and the algorithm chooses to increment the first possible separation in the set of N. This may not be the separation between the current dominant interferer and the serving transmitter as it has been previously, but may be between the serving transmitter and a different interferer with a smaller contribution to the cumulative interference causing failure. For example, if the worst interferer and the serving transmitter are already separated by `maxSep`, this separation cannot be incremented, so it moves on to attempt to increment the second worst interferer and so on. If `maxSep` is sufficiently large, then the separation incremented will be that between the serving transmitter and the dominant interferer under the current Φ, as previously.

The algorithm first attempts to mitigate all the multiple interference required of it whilst using `maxSep`=1, producing a matrix of 0s and 1s only i.e. unconstrained separations and co-channel constraints. As soon as an RTP is found which cannot be satisfied by this limited Φ, the algorithm begins again with `maxSep` increased by 1. The computation is then attempted of a matrix which will contain only 0s, 1s and 2s as entries representing separations. This continues until a loop of the algorithm manages to produce a Φ which means no failure is caused by sets of size N.

Note that in cases where the recalculation and spread methods result in matrices whose respective maximum occurring separations do not differ, the matrices will be identical as the spread method has concluded by performing the exact calculations performed by the corresponding recalculation method.

The recalculation method and spread method are performed on the same problem instances for the same values of N. Sixty network instances are each assigned for four SIR_a values, each using values of N from 1 to 6. Thus, the comparison between the two methods is made over 1440 cases.

6.5.2 RESULTS OF SPREAD METHOD

The spread method successfully decreases the maximum separation from that produced by the recalculation method 102 times over the 1440 cases for which the comparison is made. All of these reductions are by 1. A breakdown of when this occurs is shown in table 6.3. The instances for which the spread algorithm has an effect are distributed across the problem networks in terms of their transmitter distribution and number of transmitters. The occurrences are enumerated in table A.11.

	SIR_a =9dB	SIR_a =12dB	SIR_a =15dB	SIR_a =17dB
$N = 1$	0	0	0	0
$N = 2$	0	9	1	0
$N = 3$	0	1	1	0
$N = 4$	1	1	0	0
$N = 5$	50	1	0	0
$N = 6$	36	1	0	0

Table 6.3: The recalculation and spread methods are used to generate constraints for the same instances and parameters, giving a total of 1440 constraint systems for each method. These are compared to establish in which cases the spread method successfully reduces the maximum separation appearing in Φ. Tabulated are the numbers of times this reduction occurs for different parameter values of N and SIR_a.

Note that the spread method can never alter the Φ produced by the recalculation method when N=1, as there is no freedom in choosing between separations to alter as the number of interferers under consideration is one, and the separation incremented must be that between the serving transmitter and interfering transmitter pair.

As N increases, the number of interferers in the interfering set increases, creating more freedom of choice in terms of which separation may be incremented. This means that the probability of the spread method reducing the maximum separation increases with N. Note however, that this probability will not continue to increase: as the number N of interferers in the interferer set increases, each additional interferer becomes less potent, so increasing the separation between this last interferer and the serving transmitter will have little effect. The spread method can have an effect when other members of the interferer set than the most dominant are still sufficiently interfering that their increased separation from the server can have a not negligible impact upon the cumulative interference caused by the set.

In table 6.3, it can be seen that the spread method has most effect when SIR_a is at its lowest i.e. 9dB. No maximum separations of Φ can be reduced for the library cases when SIR_a=17dB. Whenever the same network is assigned for both 9dB and 17dB threshold SIR from constraints in a matrix with equal maximum separation, the 17dB Φ matrix will have necessarily more increments. This means that there is more scope for change in the 9dB Φ but that the 17dB Φ is likely to be tight and would fail to attain the required SIR levels if the maximum separation were reduced.

Table A.11 shows the cases for which the spread method has an effect on the maximum separation in the matrix Φ. In 102 of the 1440 attempts, the spread method achieves a reduction in the maximum separation. This change to the Φ matrix may lead to an increase or decrease in the span of channels used in the assignment made to satisfy the binary channel separation constraints. The set of constraints produced under the recalculation and spread methods respectively both give 100% coverage when assignments are evaluated for the SIR and number of interferers for which they were generated i.e. SIR_e=SIR_a and ISS=N. The coverage provided when evaluated under complete interference may potentially be reduced or improved.

Of the 102 cases in which the maximum separation occurring in Φ is reduced by application of the spread method, those for which a change in upper or lower bound is observed are detailed in table A.12. Only two of the reductions in maximum separation lead to a corresponding reduction to the lower bound. Twelve cases experience a reduction in the span used (upper bound) when the new constraint matrix is assigned. Of these twelve decreases in span, eleven cases experience no detrimental effect on coverage even under complete interference evaluation. In one case, the span is decreased by three channels without a negative impact on the coverage provided by the assignment.

For the library cases used here, the spread method leads to a better assignment, i.e. a decrease in span without a decrease in coverage, than that provided by the equivalent recalculation method in 11 of 1440 attempts. However it must be noted again that when the lower bound and upper bound are not equal, it is not known whether this decrease in upper bound is due to a corresponding decrease in *spn* or simply the heuristic providing a different assignment.

The application of this method was found to increase the lower bound 23 times of the 102 times in which the maximum separation was affected, and the upper bound in 12 cases; more increases are therefore observed than decreases. The spread method is also costly in terms of computational time relative to the recalculation method. However, as the method does provide an improved assignment in some cases, albeit a small proportion, it would be worth creating and evaluating an assignment via this method to see if an improvement could be made if the highest priority is to minimise the span and the computational time is less important. Note also that the majority of constraints in force throughout this thesis require low separations and therefore there is not much scope for decreasing the maximum separation in the matrices. The method may prove more useful for cases in which high separations are observed from the recalculation method.

6.5.3 COMPUTATIONAL TIME

Each of the three methods presented in this chapter requires more computational time than its preceding method. The recalculation method has the same structure as the fixed set method but requires additional recalculation of the interferer sets. The for loop of the spread method is the same as that of the recalculation method, meaning that the spread method repeatedly performs the recalculation algorithm with increasing `maxSep`. However, different calculations occur within the loop due to the limiting capacity of the `maxSep` parameter and the resultant need to increment different separations (and potentially to increment many more separations due to the lesser contribution of each). This means that the spread method involves greater computational time than the recalculation method. Table 6.4 shows the time taken to perform constraint generation by each of the three methods for each of the sixty library instances, each for $N = 1, \ldots, 6$, each at $\text{SIR}_a = 9, 12, 15, 17\text{dB}$ (i.e. $60 \times 6 \times 4 = 1440$ constraint sets generated via each method).

Method	Fixed set	Recalculation	Spread
Time	2 hours 45	2 hours 55	13 hours

Table 6.4: Time to generate constraints for all sixty library cases each for $N = 1, \ldots, 6$ and each at $\text{SIR}_a = 9, 12, 15, 17\text{dB}$ i.e. $60 \times 6 \times 4 = 1440$ constraint sets generated.

than under a complete interference model.

The spread method provides no improvement to assignments at considerable computational cost in the majority of cases, however 11 cases saw improved usage of span without sacrificing coverage levels. This method can therefore be used if time allows, as it is possible an improved assignment can be found. The spread method experiment showed that the recalculation algorithm achieves the desired coverage results with the minimum possible maximum separation appearing in Φ in the vast majority of cases, meaning that the assignments aren't 'over-engineered' in that sense.

The results of this chapter have shown that it is not necessary to abandon binary constraints in favour of a higher order constraint model when considering multiple interference rather than single interference. The recalculation method provided a binary constraint model for multiple interference which is much less costly than a complete interference model and provides improved coverage over single interferer assumption assignments with only necessary increases in span. The parameter N allows for the balance between the two objectives of coverage and span to be selected according to requirements and priorities.

Chapter 7

Conclusions

The channel assignment problem is a vital problem in wireless communication systems. Channels need to be re-used in a channel assignment as much as possible, subject to the assignment providing a tolerable level of interference at test points in the network. As revealed in chapter 2, much of the related literature has developed computational algorithms to assign channels subject to constraints based on the generalised graph-colouring formulation. This consideration of solution algorithms is an important pursuit as the general channel assignment problem is NP-hard. However, it is equally important to consider *how* channel separation constraints are generated; specifically how the effects of interfering signals on users are incorporated. This is crucial because it affects two fundamental issues:

(a) the quality of coverage (signal-to-interference ratio) throughout the network;

(b) the efficiency of spectrum use by the channel assignment.

Chapter 2 showed that limited previous work has been undertaken to quantify the extent to which multiple interfering signals affect service coverage. It is also apparent that there is no widely preferred single methodology for modelling the combined effects of interferers. Generally speaking, the proposed models to incorporate multiple interference effects have moved away from the generalised graph-colouring model, on which most computational techniques developed for channel assignment have been based. Various schools of thought have emerged including the application of higher order constraints and constraint-free approaches, which avoid explicit modelling via constraints. These findings motivated the subsequent chapters which proceeded by analysing the degradation and causes of service coverage failure when the single interferer assumption is modelled and assigned using the generalised graph-colouring model and evaluation considers increasing numbers of multiple interferers.

Due the discrete nature of the channel assignment problem and the continuous nature of the service coverage evaluation, is it prudent to investigate the effects of multiple interference using a library of test problems for channel assignment. Chapter 3 described the assumptions applied, and the methodology behind the problem generator which used different probability distributions to emulate various characteristics seen in the geographical dispersion of transmitters. Techniques were developed to identify subsets of reception test points which would be particularly susceptible to the effects of multiple interferers. A library of sixty test case networks from this generator has been used for investigation purposes.

In chapter 4, the effects of multiple interference have been analysed using the test case library. This involved formulating binary channel separation constraints under the signal interferer assumption, and determining zero-violation channel assignments, optimised to use a minimal span of channels. The hybrid sequential and simulated annealing search procedure (as described in section 3.4.2) was used to determine channel assignments, and assignments were determined to use span reasonably by applying lower bounding techniques. This was repeated for each test problem case at a range of target signal-to-interference ratio values (9dB, 12dB, 15dB and 17dB), resulting in a total of two-hundred-and-forty channel assignments. The aim of analysis was to identify the decline in service coverage for each assignment, as more and more interferers were incorporated. A number of different techniques are introduced in chapter 4 to perform this analysis. It was found that, even with refinements, the exhaustive calculation of sets of interferers quickly became intractable. Consequently, a conservative approach was adopted in which only the worst interferer sets are considered. In section 4.5, analysis methodology was introduced to assess the effects of interferer sets in a range of different ways. This analysis has been performed across the range of test problems to smooth out the effects of discretisation in channel assignment. The results of this analysis provide a number of useful findings. The results show a rapid decline in coverage quality as interferer set size increases. This quickly converges to that experienced under complete interference, indicating that the inclusion of a small number of dominant interferers will closely approximate the inclusion of all interferers across the randomised test problems considered. This is particularly useful since inclusion of all interferers is significantly more computationally expensive than incorporating a small subset (e.g. of 6–10 interferers). Furthermore, from assessing failed reception test points, interference from co-channel transmitters was found to dominate, with contributions from first adjacent transmitters being significant but secondary, and other interferers contributing only marginally.

In chapter 5, consideration is given to the most simple form of higher order constraint. The results from chapter 4 indicate that controlling the effects of co-channel interferers is important, and consequently, co-channel set constraints are considered, which are a natural logical extension to controlling multiple simultaneous interferers. Despite the simple formulation of these constraints, there is inherent complexity which is manifested in different ways, exposing the potential problems of attempting to model multiple interference with non-binary constraints. Attention is paid to issues of problem representation and methods to determine channel assignment bounds for co-channel set constraints using results concerning two-colourability of hypergraphs. Problems of constraint redundancy and number of constraints are addressed, and extremal set theory is applied to bound the maximal number of non-redundant constraints of this form. These issues impede the pre-processing and explicit generation of such constraints for channel assignment. However it is important to realise that any model to mitigate against the effects of multiple interference must implicitly satisfy the underlying co-channel set constraints.

Chapter 5 closes by highlighting that binary channel separation constraints (equivalent to the generalised graph-colouring formulation) could be formulated so that the underlying co-channel set constraints are automatically satisfied. Binary channel separation constraints with this property would offer a higher degree of resilience to multiple interference whilst well-established and successful heuristic optimisation techniques, developed to create assignments subject to binary constraints, could be applied. This motivated the development and comparison of new techniques to create binary channel separation constraints, as considered in chapter 6.

The techniques for generating binary channel separation constraints in chapter 6 were developed with a view to improving the quality of the assignments under multiple interference, while minimising increases in minimum span. The test problem library was used to conduct experimentation for three alternative methods for creating binary constraints which incorporate multiple interference. The approaches are distinguished by the way in which the contributions from sets of multiple interfering sources are incorporated. The algorithms were compared by making minimum span assignments to each set of constraints, and then analysing the assignments under the criteria and methodology developed in chapter 4. This was performed at a range of SIRs, and with a range of parameters, resulting in the assignment and analysis of 1440 assignments under each of three methods.

The results of this chapter indicated that the recalculation method presented overcame many of the disadvantages of other models: it allowed tractable binary constraints to provide resilience to multiple interference, improving coverage with only necessary increases in span; it provides the opportunity for a network operator to balance their priorities in terms of coverage and spectral usage (using profiles of the trade-off like those seen in figures 6.17 and 6.18); constraints can be generated which provide excellent coverage without resorting to considering all possible interferers. The spread method provided two cases of reduced lower bound and eleven instances of maintained coverage levels using less span. Although leading to an improved assignment in a very small proportion of cases, this method can be used to attempt to provide improved assignments when computational time is less of a priority than optimising span as far as possible.

Consequently, it has been shown that effective and efficient ways of modelling multiple interference can be obtained via binary channel separation constraint generation. This is a new contribution that offers a number of practical advantages: it avoids more complex non-binary models which are unlikely to be adopted and fully understood by radio engineers, while ensuring that the extensive development of heuristic techniques for optimisation of the generalised graph-colouring problem can be deployed to mitigate multiple interference. In the course of this development, a deeper understanding of multiple interference in the channel assignment problem has been attained. This has quantified the effects of considering multiple interference over single interferers, in terms of coverage and characteristics of channel assignments. It has also addressed the use and properties of higher order constraints and highlighted the range of complexities involved with such models.

The work presented provides analysis and recommendations that can be adopted to improve the model for channel assignment, to incorporate multiple interference effects in constraints without unnecessary additional spectrum and computational time requirements; the algorithms are simple to implement and computationally undemanding in general. The model suggested provides a constraint representation which more closely approximates the operational situation.

7.1 FUTURE WORK

All the techniques described in this thesis can be taken forward and employed using models of the operational criteria which are more realistic. This includes temporal issues such as traffic and usage patterns at different times and physical issues such as extending to three-dimensional geographical instances and more realistic propagation models or data.

Although this work has focussed on constraint representations, it was noted that an alternative is to formulate a method of constraint-free assignment. This would likely require an evaluation technique, and a means of determining perturbations to be made to assignments to improve the performance. Chapter 4 may be used to inform the development of such a constraint free approach, as it provides an evaluation technique which determines coverage performance and reveals the dominant interferers, which transmitters' assignment could then be altered when creating a neighbouring assignment solution within a heuristic solution technique.

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All URLs are correct at 22nd September 2004

GLOSSARY

The multi-disciplinary nature and rapid evolution of the field of wireless communication leads on occasion to confusing or even contradictory jargon. Terms may originate in computer networking, or wireless engineering, or mathematics, and come together somewhat uneasily. This glossary contains short explanations of selected terms used in the thesis. These terms are chosen for inclusion here because they are terms or abbreviations which are used without definition within the text; or because they are used repeatedly and it is useful to have a definition available for reference; or because they may be used slightly differently in other contexts and clarification is required.

See also [1, 4] for mobile communications terms and [29, 95] for terms from graph and hypergraph theory.

Dictionaries and other sources used in producing the glossary:

- Merriam-Webster Online
  `www.m-w.com`
- *The American Heritage Dictionary of the English Language*
  Fourth Edition, Houghton Mifflin Company, 2000
- *The Free On-line Dictionary of Computing* Edited by Denis Howe
  `www.foldoc.org`
- WordNet 1.6 Copyright 1997 by Princeton University. All rights reserved.
- Schiller *Mobile Communications* [1]
- Telecomm Glossary 2000 [4]
- Berge *Graphs and Hypergraphs* [29]

A

(Channel) Allocation

The region of the radio spectrum allocated to a particular use or operator. This is a collection of channels, whose specific use is then decided by the process of channel assignment.

Antenna

Metallic apparatus for sending or receiving electromagnetic waves; may be part of base station or receiver equipment.

Attenuation

The decrease in intensity of a signal or wave as a result of absorption of energy along the path between the transmitter and the detector, but not including the reduction due to geometric spreading.

B

Bandwidth

The rate at which data can be passed along a communications link.

Base station

All the radio equipment located at one fixed location that is used for serving a cell.

C

Call-Blocking

The denial of a request for a communication link leading to a failed attempt to set up a call, usually due to strain on the network's resources at that particular time.

Call-Dropping

The abortion of a call in progress because the system cannot maintain the communication link, perhaps due to failed handover protocols as a mobile moves from one cell to another.

Capacity

How many calls or how much data a cell or network can handle.

Chain, cycle

In a hypergraph H a *chain of length* q is a sequence $(x_1, e_1, x_2, e_2, \ldots, e_q, x_{q+1})$ such that

(a) $x_1, \ldots, x_q$ are distinct vertices of H

(b) $e_1, \ldots, e_q$ are distinct edges of H

(c) $x_k, x_{k+1} \in e_k$ for $k = 1, \ldots, q$

If $q > 1$ and $x_{q+1} = x_1$, then the chain is a *cycle of length* q. The definition also holds for graphs, in which edges contain two vertices only.

Clique

A clique of a graph is a complete subgraph i.e. a subgraph in which each pair of vertices is connected by an edge.

Clutter

The buildings etc. encountered by a wave between the transmitter and the receiver, which lead to propagation loss.

E

ETSI The European Telecommunications Standards Institute. A standardisation organisation of the telecommunications industry in Europe, with influence worldwide.

F

Free space

A theoretical concept of space devoid of all matter; implies remoteness from material objects that could influence the propagation of electromagnetic waves.

G

GSM

Originally the *Groupe Spéciale Mobile*, founded to create a mobile phone standard; now used for *Global System for Mobile communication*, the Europe-wide standard created by this group and ETSI, and currently used as standard for mobile telephony in Europe and some other parts of the world.

H

Hand-off

See Handover.

Handover

The process of transferring a phone call in progress from one cell to another, without interrupting the call.

I

Interference

The interruption or degradation of reception by unwanted signals.

M

Modulation

Superimposing a user or subscriber signal onto a carrier signal by some method.

N

Noise

The interruption or degradation of reception by something of a different nature to the required signal.

P

PMR

Private Mobile Radio. Simple 'push-to-talk' networks in which all users hear all conversations and there is no hand-over between cells. The international standard for digital PMR is TErrestrial Trunked RAdio (TETRA), an ETSI standard first published in 1995. This is the 'walkie talkie' standard used by police, ambulances and the military.

R

Roaming

The use of a mobile phone outside of the service provider's tariffed geographic area; the phone 'roams' onto another service provider's network and additional charges usually apply.

S

Signal

Detectable transmitted energy being used to carry information.

SI units

Système International (d'Unités). The modern coherent and rationalised system of measurement, founded on seven SI base units for seven base quantities assumed to be mutually independent: metre, kilogram, second, ampere, kelvin, mole, and candela.

T

Telegraphy, telegraph

A communication system between distant points whose apparatus transmits and receives simple unmodulated impulses, especially one in which the transmission and reception stations are directly connected by wires and the signal transmitted by electrical action. Intelligence is communicated by visible or audible signals representing words or ideas, or by means of words and signs. Wireless telegraphy is telegraphy carried on by radio waves and without connecting wires.

Telephony, telephone

A communication system between distant points whose apparatus transmits and receives sound or speech, especially by radio or telephone, with or without connecting wires. The term telephony is used to indicate transmission of the voice, as distinguished from telegraphy, radio teletypewriter transmission (or frequency shift keying) and facsimile.

Traffic

The information moved over a communications channel is called traffic. Traffic density in a telecommunications system is measured by a unit called the Erlang. The term also refers to a measure of the total use of a network occurring at a specified time.

Transmitter

Equipment that produces a single signal; this is stationary and located at a base station in the middle of a cell. (Mobile phones and similar devices will contain apparatus for both transmission and reception of signals; use of the word transmitter is avoided in this context.)

Appendix A

Selected Tables of Results

Table A.1: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 9dB service threshold SIR. RTP coverages are tabulated when the assignment is evaluated for ISS=1, ..., 10 and ISS=$n-1$. (The case names are abbreviated.)

	no. simultaneous interferers for evaluation										
	1	2	3	4	5	6	7	8	9	10	$n-1$
Centred0	100	96.8	93.5	91.8	90.3	89.3	88.3	87.6	86.8	86.2	81.6
Centred1	100	97.4	95.3	93.3	91.8	90.7	89.6	89	88.3	88	81.7
Centred2	100	96.9	94.8	93	91.4	90.2	89.1	88.6	88.1	87.7	82.5
Centred3	100	96.7	94.4	91.9	90.6	89.5	88.4	87.8	87.1	86.6	81.6
Centred4	100	97.2	95.1	93.1	91.6	90.5	89.8	89.3	88.8	88.5	84.4
Centred5	100	98	95.9	93.8	92.3	91.5	90.7	90.5	89.8	89.5	86.8
Centred6	100	97.7	95.4	94	92.9	91.8	90.9	90.4	89.8	89.3	85.6
Centred7	100	97.5	95.4	93.6	92.6	91.8	91.2	90.3	90	89.5	86.6
Centred8	100	98.4	96.4	95	93.9	92.7	91.9	91.4	91	90.6	87.2
Centred9	100	97.8	95.3	93.6	92.6	91.7	90.8	90.3	89.9	89.5	87.3
Centred10	100	99.5	98.5	98.2	97.6	97.1	96.5	96.3	96	95.7	95.2
Centred11	100	98	96.7	95.6	94.9	94.6	94.1	93.7	93.5	93.3	92.4
Centred12	100	97.7	96	93.8	93.3	92.6	92.1	91.7	91.5	91.3	90.3
Centred13	100	98.3	96.6	95	93.9	93.3	93.1	92.4	92.2	92.1	90.8
Centred14	100	98.3	96.7	95.8	94.7	93.9	93.6	93.1	92.7	92.3	91
Centred15	100	97.5	96.2	95.3	94.9	94.1	93.7	93.5	93.4	93.3	93
Centred16	100	98.9	97	95.6	94.7	94.3	93.7	93.5	93.3	93.2	92.9
Centred17	100	97.9	95.7	93.9	92.8	91.9	91.8	91.6	91.2	90.6	89.9
Centred18	100	99.3	97.7	95.9	95.4	95.1	94.7	94.5	94.5	94.5	94.2
Centred19	100	97.7	95.4	94	92.8	91.8	91	90.5	89.8	89.7	89.1
Centred20	100	96.7	93.8	92.6	91.8	91.8	91.7	91.7	91.6	91.6	91.6
Centred21	100	98.2	97.1	96.3	96.1	96	96	96	96	96	96
Centred22	100	97.4	95.4	93.4	92.4	92.4	92.4	92.3	92.2	92.2	92
Centred23	100	96	93.8	93	92.6	92.3	92	91.9	91.9	91.9	91.9
Centred24	100	97.5	95.8	95.1	94.8	94.7	94.2	94	94	93.9	93.9
Combined0	100	99	98.3	97.3	96.3	95.7	95.1	94.9	94.6	94.5	92.9
Combined1	100	99.3	98.3	97.5	96.6	95.9	95.2	95.1	94.7	94.5	92.9
Combined2	100	98	97	95.6	94.8	94.3	93.5	93.1	92.6	92.2	90.5

Combined3	100	98	96.3	94.8	93.3	92.3	91.8	91.3	91	90.6	88.3
Combined4	100	98.7	97.3	96	95.3	94.6	94.2	93.8	93.7	93.7	92.1
Grid0	100	98.3	98.1	98.1	98.1	98.1	98.1	98.1	97.5	97	96.7
Grid1	100	100	100	100	100	100	100	99.9	99.7	99.6	98.3
Grid2	100	99.6	99.3	95.3	91.8	91.5	91.4	91.4	90.9	90.8	86.8
Grid3	100	100	100	100	100	100	100	100	100	100	98.8
Grid4	100	99.2	98.6	97.5	96	94.7	94.4	94.2	94.1	93.8	93.4
Random0	100	98.4	96.7	95.6	94.5	94	93.5	93.2	93	92.8	91.7
Random1	100	98.4	97.1	95.9	95.1	94.5	93.7	93.4	93.2	92.9	91.8
Random2	100	98.6	96.9	95.8	95.1	94.4	94	93.6	93.2	92.9	91.7
Random3	100	98.2	96.7	95.7	94.9	94	93.6	93.3	93	92.9	91.5
Random4	100	98.7	97.3	96.4	95.8	95.4	95.1	94.9	94.6	94.4	93.1
Random5	100	98.4	98	97.2	96.6	96.1	95.7	95.3	95	94.9	94
Random6	100	99.5	98.9	97.9	97.1	96.5	96.2	96	95.8	95.7	94.5
Random7	100	99.2	98.1	96.8	95.9	95.3	94.5	94	93.6	93.1	91.6
Random8	100	98.8	97.8	96.8	96.2	95.7	95.4	94.8	94.7	94.3	93.5
Random9	100	99	97.7	96.5	95.9	95.2	94.6	93.9	93.9	93.7	92.5
Random10	100	100	99.3	97.8	97.2	97	96.8	96.6	96.5	96.5	96.4
Random11	100	100	100	100	100	100	100	99.9	99.9	99.9	99.9
Random12	100	99.8	99.3	99	99	99	99	99	99	98.9	98.9
Random13	100	99.8	99.3	98.7	98.4	98.4	98.2	98.1	98	97.8	97.6
Random14	100	99.6	99.4	99	98.9	98.7	98.5	98.4	98.2	98	97.8
Random15	100	100	99.8	99.6	99.6	99.6	99.6	99.6	99.6	99.6	99.6
Random16	100	99.4	98.2	96.9	96.5	95.9	95.5	95.4	94.9	94.9	94.5
Random17	100	99.4	99.2	98.9	98.7	98.7	98.7	98.7	98.7	98.7	98.6
Random18	100	99	97.8	96.9	96.3	95.8	95.2	95	94.6	94.4	94.3
Random19	100	98.9	97.9	97.5	97.2	96.8	96.7	96.5	96.5	96.3	96.1
Random20	100	99.1	98.5	98.2	97.9	97.9	97.8	97.8	97.8	97.8	97.8
Random21	100	98.2	96.5	95	94.4	94.3	93.9	93.8	93.8	93.6	93.5
Random22	100	98.7	98.1	97.7	97.7	97.7	97.7	97.7	97.7	97.7	97.7
Random23	100	98.2	95.7	95	94	93.9	93.9	93.9	93.9	93.9	93.9
Random24	100	99.6	98.4	98.3	98.1	98.1	97.8	97.8	97.8	97.8	97.8
mean	100	98.5	97.2	96.0	95.3	94.8	94.3	94.1	93.8	93.6	92.3

Table A.2: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 12dB service threshold SIR. RTP coverages are tabulated when the assignment is evaluated for ISS=1, ..., 10 and ISS=$n-1$. (The case names are abbreviated.)

	no. simultaneous interferers for evaluation										
	1	2	3	4	5	6	7	8	9	10	$n-1$
Centred0	100	94.9	90	86.9	84.8	83	81.8	81.1	80.3	79.7	74.8
Centred1	100	94.4	89.8	86.5	84.4	82.8	82	81.4	80.5	80	74.5
Centred2	100	94.3	89.2	86.1	84.2	83.4	82.4	81.6	80.8	80.1	75.7
Centred3	100	94.1	89	85.7	83.2	81.9	80.6	79.7	79.2	78.6	73.9
Centred4	100	94	88.5	86	84.4	83	82	81.2	80.4	80	76.1
Centred5	100	96	91.8	89.8	87.9	86.1	85.1	84.3	83.7	83.2	80.4
Centred6	100	95.8	91.8	89.2	87.1	85.5	84.2	83	82.4	82.1	78.6
Centred7	100	95	90.8	87.7	85.6	83.9	83	82.2	81.2	80.6	77.6
Centred8	100	95.5	90.7	88.1	86.1	84.3	83.3	82.3	81.7	81.2	78.6
Centred9	100	96	91.8	88.8	86.5	85.3	83.9	83.1	82.2	81.6	78.4
Centred10	100	97.3	94.1	92	90.8	89.9	89.1	88.8	88.5	88.2	87.2
Centred11	100	96.8	93.4	91.8	90.2	89.2	88.4	87.3	87.3	87.1	86.2
Centred12	100	96.4	93.1	90.3	88.7	87.7	86.8	85.8	85.6	85.6	84.2
Centred13	100	96.7	93.2	90.4	88.9	88.3	87.9	87.5	87.1	86.9	85.8
Centred14	100	96.6	92.7	89.9	87.8	86.9	86.4	85.6	85	84.7	83.2
Centred15	100	96.4	92.9	89.9	89	88.2	88	87.9	87.7	87.5	87.1
Centred16	100	93.5	88.9	86	83.8	82.4	81.5	80.8	80.6	80.4	79.6
Centred17	100	94.8	89.2	86.2	84.7	83.8	83.4	83.1	82.9	82.9	82.2
Centred18	100	95.5	89.7	87.1	85.8	84.9	84.5	84.1	83.4	83.3	83.2
Centred19	100	96.9	90.8	87.5	86.4	85.1	84.5	83.6	83.2	83	82.6
Centred20	100	96.9	94.3	93.3	92.9	92.9	92.8	92.7	92.7	92.7	92.7
Centred21	100	96.2	92.4	89.3	86.7	86.6	86.3	86.3	86.2	86.2	86.2
Centred22	100	95.1	90.2	88.6	87.8	87.5	87.3	87	86.9	86.8	86.7
Centred23	100	94.9	90.4	89.7	89	88.9	88.7	88.7	88.7	88.7	88.7
Centred24	100	98.3	95.4	93.1	92	91.6	91.6	91.5	91.5	91.5	91.4
Combined0	100	97.1	93.8	92	90.8	89.9	89.1	88.5	88.1	87.9	85.7
Combined1	100	95.8	92.2	89.7	88.1	87.1	86.3	85.7	85.1	84.7	81
Combined2	100	96.7	93.8	91.4	90	89	88.7	88	87.7	87.4	85.7

Combined3	100	95.9	92.2	90	88.5	87.7	87	86.6	86.3	85.7	83.5
Combined4	100	97.5	93.6	91.7	90.5	89.9	89.1	88.4	88	87.9	85.5
Grid0	100	100	100	99.3	99.2	98.9	98.9	98.8	97	96.7	87.2
Grid1	100	100	93.9	92	88.2	85.2	84.2	83.8	83.3	83	79.9
Grid2	100	92.2	81.2	77	75.5	74.1	72.6	71.3	70.7	70	66.2
Grid3	100	99	87.4	84.7	81.3	78.7	77.4	76.6	76.4	76	73.9
Grid4	100	94.1	82.2	78.3	76	75.1	74.3	74	73.3	73.2	73.1
Random0	100	96.7	93.3	91.5	90.4	89.9	89.6	89.2	88.7	88.5	86.7
Random1	100	97.1	94.2	92.3	91	90.2	89.6	89.1	89	88.8	86.6
Random2	100	96.8	93.7	92.5	91.4	90.7	90.1	89.5	89.3	89.1	87.7
Random3	100	96.8	93.3	91	89.8	89.2	88.4	87.9	87.5	87.2	85.3
Random4	100	97	94.1	92.4	90.7	89.9	89.4	88.9	88.4	88.2	86.6
Random5	100	97.2	93.6	91.1	89.2	88	86.9	86.2	85.9	85.7	84.1
Random6	100	96.9	94	92.1	90.7	89.8	89.1	88.6	88.2	87.9	86.2
Random7	100	97.7	94.6	92.7	91.3	90.4	89.5	89	88.7	88.4	87.1
Random8	100	98.1	95.9	94	92.5	91.8	91.5	90.9	90.6	90.3	89
Random9	100	97.8	94.8	92.5	90.8	89.8	89.4	89.1	88.5	88.2	86.8
Random10	100	98.9	98.1	97.3	96.8	96.5	96.3	96.3	96.3	96.2	96.1
Random11	100	99.5	98.8	98	97.7	97.4	97.3	97.2	97.1	97.1	96.8
Random12	100	98.3	97.4	96.6	95.9	95.7	95.6	95.2	94.9	94.8	94.5
Random13	100	99.1	97.8	96.8	95.9	95	94.6	94.3	94.2	94.1	93.8
Random14	100	98.1	95.8	94	92.9	92.3	91	90.6	90.3	90.2	89.4
Random15	100	95.8	93.1	91.4	90.1	89.2	88.5	88.2	88.1	88.1	87.8
Random16	100	97.6	93.4	91.5	90.1	89.2	88.9	88.7	88.7	88.7	88.6
Random17	100	99.7	98.7	97.1	96.7	96.5	96.2	96.1	96.1	96	95.9
Random18	100	97.9	94.6	93	91.7	91.3	90.8	90.7	90.6	90.6	90.5
Random19	100	97.6	94.6	92	90.4	89.2	88.6	88.3	88.1	88	87.6
Random20	100	99.9	99.3	98.6	98.6	98.6	98.6	98.6	98.6	98.6	98.6
Random21	100	99	96.6	95.4	94.6	94.3	94.3	94.3	94.3	94.3	94.3
Random22	100	98.1	95.4	94.2	93.7	93.5	93.2	93.2	93.2	93.2	93.1
Random23	100	98.2	96.4	95.7	95.6	95.6	95.6	95.6	95.6	95.6	95.6
Random24	100	99.4	97.6	96.9	96.2	96.2	96.2	96.2	96.2	96.2	96.2
mean	100	96.8	93.1	90.9	89.5	88.6	88.0	87.6	87.2	87.0	85.4

Table A.3: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 15dB service threshold SIR. RTP coverages are tabulated when the assignment is evaluated for ISS=1, ..., 10 and ISS=$n-1$. (The case names are abbreviated.)

	no. simultaneous interferers for evaluation										
	1	2	3	4	5	6	7	8	9	10	$n-1$
Centred0	100	89.7	83.2	80	77.6	75.5	74.6	73.7	72.8	72.3	68.4
Centred1	100	88.6	83	80	77.8	75.9	75.2	74.5	73.7	73.1	69.4
Centred2	100	88.7	82.6	79.2	77.6	76.2	75.2	74.3	73.4	72.7	68.8
Centred3	100	86.3	80.2	77	75.2	73.7	72.5	71.6	70.9	70.3	66.7
Centred4	100	86.6	81.5	78.6	76.6	75.4	74.6	74	73.4	72.9	69.5
Centred5	100	87.7	81.5	79.2	77.4	75.8	74.6	73.8	73.3	72.8	70.3
Centred6	100	89.4	84.8	81.9	79.3	78	77	76.2	75.7	75	72
Centred7	100	90.4	85	82.1	80.1	78.3	77.1	76.4	76	75.3	72.7
Centred8	100	87.8	81.2	77.2	75.3	74.4	73.4	72.6	71.9	71.4	69.1
Centred9	100	89.2	84.2	80.8	78.9	77.7	76.3	75.7	75	74.6	71.9
Centred10	100	91.6	87.7	84.3	82.5	81.7	81.3	80.5	80	79.7	78.6
Centred11	100	89.9	84.6	81.7	79.1	78.3	77.1	76.2	75.7	75.2	74.4
Centred12	100	92.3	87.9	85.8	84.6	83.8	83.4	82.9	82.5	82.3	81.6
Centred13	100	92.1	87.3	84.8	83.4	82.5	81.6	81.2	80.6	80.3	79.4
Centred14	100	89.5	84.4	81.5	79.7	78.2	77	76.4	76	75.7	75.2
Centred15	100	90.2	85.8	83.2	82.2	81.5	81.3	81	81	80.9	80.9
Centred16	100	90.1	85.3	82.7	81.3	80.7	80.4	80	79.9	79.8	79.7
Centred17	100	82.9	76.8	74.9	73.2	72.8	72.5	72.3	72.2	72	71.5
Centred18	100	88.3	83.3	81.1	80	79.6	79.1	79	78.9	78.9	78.7
Centred19	100	88.8	83.5	79.3	77.9	76.9	76.7	76.5	76.2	75.9	75.5
Centred20	100	89.5	86.6	86.1	86	85.7	85.7	85.7	85.6	85.6	85.6
Centred21	100	86.1	83	81.9	81	80.7	80.4	80.4	80.4	80.4	80.2
Centred22	100	89.3	86.8	85	84.4	83.9	83.4	83.3	83.3	83.3	83.2
Centred23	100	89.3	82.6	81	80.7	80.3	80.2	80.2	80.2	80.2	80.2
Centred24	100	91.9	88.6	87.9	87.6	87.5	87.5	87.5	87.5	87.5	87.4
Combined0	100	96.3	94.5	93.1	92.3	92	91.6	91.3	90.8	90.5	89.1
Combined1	100	95.8	93.4	91.7	90.8	89.9	89.2	88.8	88.3	87.9	86.8
Combined2	100	88.9	84.6	82.2	81	79.4	78.6	77.8	77.3	76.9	75.6

Combined3	100	88.8	84.2	81.9	80.5	79.9	79.3	78.7	78.5	78.1	76.6
Combined4	100	91.4	87	84.6	83.2	82.3	81.6	81	80.7	80.4	79.1
Grid0	100	97.9	66.2	61.1	59.7	58.7	58.2	57.6	57.4	57.1	55.6
Grid1	100	73.4	58.8	53.4	46.7	44.7	43.4	42.5	41.6	40.6	36.4
Grid2	100	100	100	100	100	100	100	100	100	100	100
Grid3	100	77	66.7	61.9	58.5	56.8	55.9	55.1	54.5	54.2	52.9
Grid4	100	100	100	100	100	100	100	100	100	100	100
Random0	100	89.2	85.6	83.1	81.9	80.7	80	79.7	79	78.8	77.2
Random1	100	89.2	85.5	83.5	81.9	80.7	80.1	79.6	79.2	78.8	76.9
Random2	100	88.8	84.2	81.6	80.4	79.4	78.7	78.3	78	77.7	75.6
Random3	100	89.2	85.7	83.4	82	81.3	80.8	80.2	79.9	79.5	77.6
Random4	100	90.1	85.8	83.2	81.5	80.5	79.8	79.4	78.9	78.7	76.7
Random5	100	88.9	84.9	82.4	80.5	79.4	78.8	78.2	77.5	77.4	76.7
Random6	100	87.3	83	81	80	78.7	77.7	77.1	76.5	76.3	75.3
Random7	100	89.1	85.3	82.2	80.7	79.4	78.5	77.8	77.5	77	75.6
Random8	100	89.6	85.1	83.6	82.5	82.1	81.5	80.9	80.5	80	79
Random9	100	90.1	87	85.1	83.8	82.5	81.7	81.2	81	80.6	80
Random10	100	85.9	80.8	78	77.2	75.7	74.9	74.3	73.6	73.1	71.9
Random11	100	90.4	84.5	82.6	81.1	79.9	79.5	79.1	78.5	77.9	77
Random12	100	90.6	86.6	84.6	83.5	82.2	81.7	80.8	80.3	80	79.5
Random13	100	88	82.6	79.4	78.1	76.1	74.9	74	73.4	73.2	71.7
Random14	100	87.4	83.3	81.7	80.2	79.1	78.6	78	77.6	77.6	77.2
Random15	100	92.6	87.4	85.4	84.6	84.6	84.5	84.3	84.1	84.1	84.1
Random16	100	84.9	80	76.4	74.3	72.9	72.7	72.7	72.4	72.3	72.2
Random17	100	85.7	81.9	80.1	79	78.6	78.1	78	77.9	77.9	77.9
Random18	100	85.7	80.3	78.3	76.8	76.6	76.3	76	75.8	75.8	75.7
Random19	100	85.1	81	79.4	78.4	78	77.9	77.8	77.7	77.6	77.6
Random20	100	91.3	89	88.2	87.9	87.9	87.9	87.9	87.9	87.9	87.9
Random21	100	84.7	82.1	80.6	80	79.5	79.5	79.5	79.5	79.5	79.5
Random22	100	93.1	91.1	90.9	90.6	90.6	90.6	90.4	90.4	90.4	90.4
Random23	100	89	87.1	86.7	86	86	86	86	86	86	86
Random24	100	91.4	90	89.6	89.4	89.3	89.2	89.2	89.2	89.2	89.2
mean	100	89.2	84.3	82.0	80.6	79.7	79.1	78.7	78.3	78.0	76.9

Table A.4: All sixty library cases are assigned for binary constraints generated under the single interferer assumption for 17dB service threshold SIR. RTP coverages are tabulated when the assignment is evaluated for ISS=1, ..., 10 and ISS=$n-1$. (The case names are abbreviated.)

	no. simultaneous interferers for evaluation										
	1	2	3	4	5	6	7	8	9	10	$n-1$
Centred0	100	91.2	86.2	82.7	80.9	79.4	78.2	77.2	76.3	75.8	72.9
Centred1	100	91.6	86.6	83.7	81.7	80.4	79.5	78.8	78.2	77.5	74.2
Centred2	100	91.5	86.2	83.5	81.6	80.2	79	78.1	77.6	76.8	73.3
Centred3	100	91.1	85.7	82.4	80.1	78.6	77.3	76.6	75.8	75.4	71.9
Centred4	100	92.1	87.1	84.4	82.6	81	79.9	79.2	78.6	78.3	75.8
Centred5	100	94.9	90.8	87.7	85.5	84.2	83.5	82.9	82.4	82.2	80.8
Centred6	100	94.7	90.3	87.2	85.3	83.7	83	82.4	81.8	81.3	79.5
Centred7	100	93.9	88.6	85.9	83.5	82.4	81.2	80.5	80	79.6	78
Centred8	100	92.2	87.3	84	82.4	80.9	79.8	79.1	78.3	78	75.7
Centred9	100	94.7	89.7	86.9	84.8	82.9	81.2	80.2	79.6	79.1	77.3
Centred10	100	97	94.8	92.9	92.2	91.4	90.8	90.5	90.4	90.2	90
Centred11	100	94.5	90.7	88.6	86.8	85.9	84.9	84.5	84.1	84.1	83.3
Centred12	100	96.5	93.2	91.6	90.3	89.3	88.8	88.3	87.8	87.6	86.8
Centred13	100	97.7	95.2	93.1	91.9	91	90.3	89.9	89.7	89.4	88.7
Centred14	100	95.5	92.6	90.3	88.5	87.7	87.1	86.6	86.4	86.1	85.2
Centred15	100	94.6	92	89.8	88.1	87.3	87	86.8	86.7	86.6	86.4
Centred16	100	96.3	92.9	92	91.6	91.1	90.8	90.8	90.7	90.7	90.7
Centred17	100	94	90.5	89.2	88.6	88.2	88.2	88.2	88.2	88.2	88.1
Centred18	100	94.6	92.1	91.4	90.7	90.4	90.3	90.3	90.3	90.3	90.3
Centred19	100	94.2	90.3	87.2	86.7	85.5	85.2	84.8	84.7	84.7	84.7
Centred20	100	99.2	98.9	98.8	98.5	98.5	98.5	98.5	98.5	98.5	98.5
Centred21	100	99.7	99.4	99.2	99.2	98.9	98.9	98.9	98.9	98.9	98.9
Centred22	100	97.2	96.1	95.7	95.6	95.6	95.5	95.5	95.5	95.5	95.5
Centred23	100	99.1	96.9	96.8	96.6	96.4	96.2	96.2	96.2	96.2	96.2
Centred24	100	98.2	97.5	97.2	97.1	97.1	97.1	97.1	97	97	97
Combined0	100	98.5	97.3	96.5	96	95.5	95	94.7	94.3	94.2	93.1
Combined1	100	98.8	97.6	96.8	96.1	95.6	95.3	95	94.6	94.4	93.5
Combined2	100	97.1	94.4	92.7	91.4	90.7	90.3	89.9	89.5	89.1	87.5

Combined3	100	97	93.8	92.2	91	90	88.9	88.2	87.7	87.4	85.8
Combined4	100	96.3	93.7	91	89.3	88.3	87.8	87.5	87	86.8	85.1
Grid0	100	66	63.2	61.2	60	59.4	59.1	59	58.8	58.6	57.1
Grid1	100	94.4	92.5	92.2	91.7	91.4	91.2	91.1	90.9	90.5	89.3
Grid2	100	100	100	99.8	99.6	99.4	99.3	99.2	99.1	99.1	98.9
Grid3	100	97.4	97.2	97.1	97.1	97	96.7	96.4	96.3	96.3	96.2
Grid4	100	100	100	100	100	100	99.9	99.9	99.9	99.9	99.9
Random0	100	97.2	95	93.7	92.8	92	91.6	91.4	91.1	90.7	89.6
Random1	100	97.9	96.7	95.5	94.5	93.9	93.3	92.9	92.7	92.6	91.1
Random2	100	97.1	95	93.7	93	92.3	91.6	91.2	90.8	90.4	89.6
Random3	100	97.4	95.6	94.3	93.3	92.5	92	91.6	91.2	90.9	89.2
Random4	100	97.7	95.9	94	93.1	92.4	92	91.7	91.4	91	89.9
Random5	100	97.5	96.7	96	95.6	95.2	94.9	94.7	94.6	94.6	94.3
Random6	100	98.5	97.6	97	96.4	96	95.5	95.3	95.3	95.2	94.9
Random7	100	98.8	97.9	96.9	96.3	95.9	95.7	95.5	95.3	95.3	95
Random8	100	98.3	96.8	96.1	95.6	95.4	95.1	94.9	94.6	94.6	94.1
Random9	100	99.2	98.5	97.8	97.3	97.2	97	96.7	96.5	96.5	96
Random10	100	99.4	98.4	96.8	95.8	95.6	95.5	95.3	95.1	95.1	94.9
Random11	100	99.5	98.3	97.5	97.1	96.8	96.8	96.5	96.3	96.2	96.2
Random12	100	98.9	97.3	96.5	95.7	95.5	95.2	95.2	94.9	94.9	94.9
Random13	100	99.1	97.7	97.4	97.1	96.9	96.7	96.5	96.3	96.3	96.3
Random14	100	99.2	98.6	98.4	98.1	97.9	97.8	97.7	97.7	97.7	97.7
Random15	100	97.3	95.1	94.8	94.6	94.3	94.2	94.1	94.1	94.1	94.1
Random16	100	99.3	97.8	97.2	97	96.8	96.5	96.5	96.5	96.4	96.4
Random17	100	98.9	98.7	98.5	98	98	98	98	98	98	97.9
Random18	100	97.5	95.7	95.2	94.7	94.5	94.3	94.3	94.3	94.3	94.3
Random19	100	98.8	98.1	98	98	98	97.9	97.9	97.8	97.8	97.8
Random20	100	99.2	98.9	98.6	98.4	98.4	98.4	98.4	98.4	98.4	98.4
Random21	100	100	99.6	99.1	99	99	98.8	98.8	98.6	98.6	98.6
Random22	100	97.7	96.8	96.8	96.8	96.6	96.5	96.5	96.5	96.5	96.5
Random23	100	98.5	98	98	98	98	98	98	98	98	98
Random24	100	98.7	98.4	98.3	98.3	98.1	98.1	98.1	98.1	98.1	98.1
mean	100	96.4	94.2	93.0	92.1	91.5	91.1	90.8	90.6	90.4	89.7

Table A.5: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the fixed set method with N=1. RTP coverages are tabulated when evaluation is for ISS=1, ..., 8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	94.2	89.4	86.2	83.6	82	80.4	79.2	78.5	71.6
Centred1	93.8	88.9	85.5	82.2	80.3	79.2	78.7	77.9	72.4
Centred2	93.5	89	85.5	82.7	80.6	79.4	78.6	77.9	71.8
Centred3	94.3	88.5	84.8	81.6	79.3	78.1	77.1	76	69
Centred4	94.8	90.1	86.2	83.7	81.9	80.5	79.3	78.5	72.1
Centred5	95.4	91.4	88.2	85.9	84.6	83	82.3	81.4	76.3
Centred6	96.8	92.9	90.3	88.6	86.8	85.5	84.7	84.1	79.2
Centred7	96.6	93.3	90.7	88.1	86.8	85.5	84	83.5	79.6
Centred8	97.3	94.1	91.2	88.7	86.4	85.2	84	82.2	76.8
Centred9	97.2	93.3	90.7	87.6	85.8	84	83.1	82.3	77.1
Centred10	97.8	94.4	92.5	91	90	89.5	89.1	88.6	87.1
Centred11	97.4	93.8	91.1	89.7	88.8	88.2	87.3	87.1	83.7
Centred12	98.5	95.3	92.7	91.3	90	88.6	87.8	87.1	84.9
Centred13	97.1	93.6	91.2	89.6	88.2	87.3	86.9	86.4	84.3
Centred14	98.1	95.6	93.8	92	90.4	89.7	88.8	87.6	83.8
Centred15	99.4	97	96.1	93.9	92.1	91.2	90.7	90.2	89.5
Centred16	99.5	98.2	96	94.4	92.7	91.8	91.3	90.3	88.7
Centred17	99.3	95	91.7	90.1	89.1	87.6	86.6	86	85.1
Centred18	99.3	96.4	94.9	93	92.5	92	91.4	91.2	90.2
Centred19	99	96.1	93.8	91.4	89.9	89.2	88.2	88	87.4
Centred20	99.3	96.5	93.6	92.3	91.7	91.6	91.3	91.3	91.3
Centred21	99.3	97.3	95.1	93.2	92.4	91.9	91.8	91.6	91.1
Centred22	99.8	97.3	96.1	95.1	94.3	94.1	93.7	93.5	93.4
Centred23	98.3	92.6	88.7	86.7	86	85.9	85.6	85.3	85.1
Centred24	99.2	96.2	93.1	91.1	90.3	89.9	89.6	89.2	89.1
Combined0	98.8	97.2	95.8	94.3	92.9	92.1	91.4	91.1	88
Combined1	98.9	97.4	96.2	94.8	93.7	93.2	92.6	91.8	88.7
Combined2	98	94.6	92.6	90.5	89.3	88.7	87.7	86.9	83.8
Combined3	97.2	93.3	90.7	88.8	87.6	86.5	85.8	85.1	81.3
Combined4	97.3	94.2	92.5	90.6	89.7	88.8	88.1	87.6	84.9

Grid0	95.5	94.8	94.7	94.7	94.7	94.5	94.5	94.5	94.2
Grid1	99.2	98.5	98.4	98.4	98.4	98.4	98.4	98.3	97.6
Grid2	100	99.6	99.3	95.3	91.8	91.5	91.4	91.4	86.8
Grid3	100	100	100	100	100	100	100	100	98.8
Grid4	100	99.2	98.6	97.5	96	94.7	94.4	94.2	93.4
Random0	97.8	95	93	91.6	91	90.2	89.7	89.3	87.4
Random1	98.4	95.7	93.6	92	91	90.3	89.7	89.3	86.7
Random2	97.1	94	92.1	90.6	89.4	88.6	88.2	87.7	84.9
Random3	97.9	95.4	93.5	91.9	90.8	89.8	89	88.5	85.7
Random4	98.7	96.4	95.4	94.3	93.4	92.9	92.5	92	89.6
Random5	100	98.1	96.6	95.3	94.3	93.6	93.1	92.5	91.2
Random6	99.9	98.6	97.4	96.4	95.6	94.6	94.2	93.9	92.2
Random7	99.7	98.7	97.3	96.3	95.4	95	94.6	94.5	92.5
Random8	99.7	97.9	97.4	96.3	95.6	95.1	94.7	94.3	92.6
Random9	99.9	99	98.1	97	96.2	95.9	95.5	95.1	94
Random10	100	100	99.3	97.8	97.2	97	96.8	96.6	96.4
Random11	100	100	100	100	100	100	100	99.9	99.9
Random12	100	99.1	98.2	97.6	97.5	97.5	97.5	97.5	97.4
Random13	100	99.8	99.3	98.7	98.4	98.4	98.2	98.1	97.6
Random14	100	99.6	99.3	99	98.9	98.7	98.5	98.4	98
Random15	100	100	99.8	99.6	99.6	99.6	99.6	99.6	99.6
Random16	100	99.4	98.2	96.9	96.5	95.9	95.5	95.4	94.5
Random17	100	99.4	98.4	97.7	97.3	96.9	96.8	96.8	96.7
Random18	100	99.6	97.7	96.3	95.2	93.9	93.5	92.8	92.2
Random19	100	99.5	98	97.1	96.7	96.2	96	95.9	95.7
Random20	99.9	99.3	97.5	97.1	96.5	96.1	96.1	96	95.8
Random21	99.9	97.7	96.1	94.6	93.9	93.8	93.3	93.2	92.9
Random22	100	99.3	98.6	98.6	98.5	98.5	98.5	98.5	98.5
Random23	100	98.2	95.7	95	94	93.9	93.9	93.9	93.9
Random24	100	99.2	98	97.7	97.4	97.2	97	97	97
mean	98.5	96.2	94.4	93.0	92.0	91.2	90.8	90.4	88.4
minimum	93.5	88.5	84.8	81.6	79.3	78.1	77.1	76	69

Table A.6: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the fixed set method with N=3. RTP coverages are tabulated when evaluation is for ISS=1,...,8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	99.8	98.8	97.6	96.4	95.8	95.4	94.8	94.3	90
Centred1	99.6	99	98	96.9	96.1	95.2	94.3	93.9	87.8
Centred2	99.3	97.9	96.8	95.6	94.8	94.2	93.7	93.2	87.7
Centred3	99.5	98.8	97.7	96.7	95.8	95.3	94.4	93.7	87.9
Centred4	99.9	99.4	98.4	97.2	96.3	95.6	94.3	93.7	88.6
Centred5	99.8	99.4	98.9	98.4	97.9	97.6	97.5	97.2	95.6
Centred6	100	99.8	99.2	98.7	98.3	98.1	97.7	97.5	95.5
Centred7	99.9	99.2	98.7	98.1	97.4	96.8	96.5	96	93.6
Centred8	100	99.9	99.5	99.2	98.8	98.2	97.7	97.2	95.5
Centred9	99.9	99.6	98.9	98.4	97.7	97.3	96.5	96.2	94.2
Centred10	99.9	99.7	99.5	98.9	98.6	98.6	98.4	98.3	97.9
Centred11	99.9	99.3	98.5	98.3	97.8	97.5	97.3	97	95.6
Centred12	100	99.7	99.6	99.3	98.9	98.8	98.6	98.4	97.8
Centred13	100	99.8	99.2	98.7	98.3	98	98	97.5	97
Centred14	100	99.8	99.5	99	98.8	98.6	98.4	98.2	97.5
Centred15	100	100	100	100	100	100	100	99.9	99.9
Centred16	100	100	99.9	99.7	99.5	99	99	98.8	98.7
Centred17	100	99.9	99.9	99.5	99.3	99.1	98.9	98.9	98.9
Centred18	100	100	99.7	99.4	99.2	98.8	98.8	98.6	98.5
Centred19	100	100	100	100	100	99.9	99.7	99.7	99.6
Centred20	100	100	100	100	100	100	100	100	100
Centred21	100	100	100	100	100	100	100	100	100
Centred22	100	100	100	100	100	100	100	100	100
Centred23	100	100	100	100	100	100	100	100	100
Centred24	100	100	100	100	100	100	100	100	100
Combined0	100	100	99.9	99.5	99.3	99.1	99	98.9	98
Combined1	100	100	99.9	99.5	99.2	99.1	98.9	98.7	98.1
Combined2	100	99.9	99.5	98.9	98.4	98.1	97.7	97.5	96.3
Combined3	100	99.4	98.8	98.4	98.2	97.7	97.4	97.2	96
Combined4	100	99.9	99.6	99.1	98.6	98.4	98	97.7	96.7

Grid0	100	100	100	100	100	100	100	100	100
Grid1	100	100	100	100	100	100	100	99.9	98.3
Grid2	100	100	100	98.5	97.4	96.6	96.3	96.1	95.1
Grid3	100	100	100	100	100	100	100	100	98.8
Grid4	100	100	100	99.2	98.7	98	97.8	97.7	97.2
Random0	100	99.8	99.3	98.8	98.4	97.9	97.6	97.4	96.1
Random1	100	99.8	99.7	99.5	99.2	99.1	98.9	98.9	98
Random2	99.9	99.7	99.2	98.8	98.3	97.9	97.8	97.5	96.8
Random3	99.9	99.5	99.1	98.8	98.5	98.2	98	97.9	97.1
Random4	100	99.7	99.5	99.2	99	98.8	98.7	98.6	97.5
Random5	100	100	100	99.9	99.9	99.8	99.8	99.8	99.7
Random6	100	100	99.9	99.7	99.5	99.4	99.3	99.3	98.8
Random7	100	100	100	100	100	100	100	100	99.9
Random8	100	100	100	100	100	99.9	99.9	99.6	99.6
Random9	100	100	100	99.9	99.9	99.9	99.9	99.7	99.4
Random10	100	100	100	100	100	100	100	100	100
Random11	100	100	100	100	100	100	100	100	100
Random12	100	100	100	100	100	100	100	100	100
Random13	100	100	100	100	100	100	100	100	100
Random14	100	100	100	100	99.7	99.7	99.7	99.7	99.7
Random15	100	100	100	100	100	100	99.9	99.9	99.9
Random16	100	100	100	100	100	100	100	100	100
Random17	100	100	100	100	100	100	100	100	100
Random18	100	100	100	100	100	99.9	99.9	99.9	99.8
Random19	100	100	100	100	100	100	100	100	100
Random20	100	100	100	100	100	100	100	100	100
Random21	100	100	100	100	99.9	99.7	99.7	99.7	99.7
Random22	100	100	100	100	100	100	100	100	100
Random23	100	100	100	99.7	99.7	99.7	99.7	99.7	99.7
Random24	100	100	100	100	100	100	100	100	100
mean	100.0	99.8	99.6	99.3	99.0	98.8	98.6	98.5	97.6
minimum	99.3	97.9	96.8	95.6	94.8	94.2	93.7	93.2	87.7

Table A.7: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the fixed set method with N=6. RTP coverages are tabulated when evaluation is for ISS=1,...,8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	100	99.9	99.9	99.8	99.5	99.3	99.1	99.1	97.5
Centred1	100	100	99.9	99.8	99.7	99.5	99.3	99.2	96.6
Centred2	100	100	99.8	99.6	99.3	99.2	99.1	99	97.7
Centred3	100	100	100	100	99.7	99.5	99.3	99.2	97.5
Centred4	100	100	100	99.9	99.7	99.5	99.2	99.1	97.2
Centred5	100	99.9	99.8	99.7	99.7	99.7	99.6	99.5	99
Centred6	100	100	100	99.9	99.9	99.9	99.9	99.9	99.6
Centred7	100	100	99.9	99.8	99.7	99.6	99.6	99.5	98.8
Centred8	100	100	100	100	100	100	100	99.9	99.7
Centred9	100	100	99.9	99.8	99.8	99.8	99.8	99.7	99.1
Centred10	100	100	100	100	100	100	100	100	100
Centred11	100	100	100	100	100	100	100	100	99.9
Centred12	100	100	100	100	100	100	100	100	100
Centred13	100	100	100	100	100	100	100	99.9	99.8
Centred14	100	100	100	100	100	100	100	100	100
Centred15	100	100	100	100	100	100	100	100	100
Centred16	100	100	100	100	100	100	100	100	100
Centred17	100	100	100	100	100	100	100	100	100
Centred18	100	100	100	100	100	100	100	100	100
Centred19	100	100	100	100	100	100	100	100	100
Centred20	100	100	100	100	100	100	100	100	100
Centred21	100	100	100	100	100	100	100	100	100
Centred22	100	100	100	100	100	100	100	100	100
Centred23	100	100	100	100	100	100	100	100	100
Centred24	100	100	100	100	100	100	100	100	100
Combined0	100	100	100	100	100	100	100	100	100
Combined1	100	100	100	100	100	100	100	100	100
Combined2	100	100	100	100	100	100	100	100	99.7
Combined3	100	100	99.9	99.9	99.9	99.9	99.8	99.8	99.6
Combined4	100	100	100	100	100	99.9	99.8	99.8	99.6

Grid0	100	100	100	100	100	100	100	100	100
Grid1	100	100	100	100	100	100	100	100	100
Grid2	100	100	100	100	100	100	100	100	100
Grid3	100	100	100	100	100	100	100	100	100
Grid4	100	100	100	100	100	100	100	100	100
Random0	100	100	100	100	100	99.9	99.9	99.9	99.8
Random1	100	100	100	100	100	100	99.9	99.9	99.7
Random2	100	100	100	100	100	100	100	100	99.9
Random3	100	100	100	100	99.9	99.9	99.9	99.9	99.7
Random4	100	100	100	100	100	100	100	100	100
Random5	100	100	100	100	100	100	100	100	100
Random6	100	100	100	100	100	100	100	100	100
Random7	100	100	100	100	100	100	100	100	100
Random8	100	100	100	100	100	100	100	100	100
Random9	100	100	100	100	100	100	100	100	100
Random10	100	100	100	100	100	100	100	100	100
Random11	100	100	100	100	100	100	100	100	100
Random12	100	100	100	100	100	100	100	100	100
Random13	100	100	100	100	100	100	100	100	100
Random14	100	100	100	100	100	100	100	100	100
Random15	100	100	100	100	100	100	100	100	100
Random16	100	100	100	100	100	100	100	100	100
Random17	100	100	100	100	100	100	100	100	100
Random18	100	100	100	100	100	100	100	100	100
Random19	100	100	100	100	100	100	100	100	100
Random20	100	100	100	100	100	100	100	100	100
Random21	100	100	100	100	100	100	100	100	100
Random22	100	100	100	100	100	100	100	100	100
Random23	100	100	100	100	100	100	100	100	100
Random24	100	100	100	100	100	100	100	100	100
mean	100	100.0	100.0	100.0	99.9	99.9	99.9	99.9	99.7
minimum	100	99.9	99.8	99.6	99.3	99.2	99.1	99	96.6

Table A.8: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the recalculation method with N=1. RTP coverages are tabulated when evaluation is for ISS=1,...,8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	100	96.8	93.5	91.8	90.3	89.3	88.3	87.6	81.6
Centred1	100	97.4	95.3	93.3	91.8	90.7	89.6	89	81.7
Centred2	100	96.9	94.8	93	91.4	90.2	89.1	88.6	82.5
Centred3	100	96.7	94.4	91.9	90.6	89.5	88.4	87.8	81.6
Centred4	100	97.2	95.1	93.1	91.6	90.5	89.8	89.3	84.4
Centred5	100	98	95.9	93.8	92.3	91.5	90.7	90.5	86.8
Centred6	100	97.7	95.4	94	92.9	91.8	90.9	90.4	85.6
Centred7	100	97.5	95.4	93.6	92.6	91.8	91.2	90.3	86.6
Centred8	100	98.4	96.4	95	93.9	92.7	91.9	91.4	87.2
Centred9	100	97.8	95.3	93.6	92.6	91.7	90.8	90.3	87.3
Centred10	100	99.5	98.5	98.2	97.6	97.1	96.5	96.3	95.2
Centred11	100	98	96.7	95.6	94.9	94.6	94.1	93.7	92.4
Centred12	100	97.7	96	93.8	93.3	92.6	92.1	91.7	90.3
Centred13	100	98.3	96.6	95	93.9	93.3	93.1	92.4	90.8
Centred14	100	98.3	96.7	95.8	94.7	93.9	93.6	93.1	91
Centred15	100	97.5	96.2	95.3	94.9	94.1	93.7	93.5	93
Centred16	100	98.9	97	95.6	94.7	94.3	93.7	93.5	92.9
Centred17	100	97.9	95.7	93.9	92.8	91.9	91.8	91.6	89.9
Centred18	100	99.3	97.7	95.9	95.4	95.1	94.7	94.5	94.2
Centred19	100	97.7	95.4	94	92.8	91.8	91	90.5	89.1
Centred20	100	96.7	93.8	92.6	91.8	91.8	91.7	91.7	91.6
Centred21	100	98.2	97.1	96.3	96.1	96	96	96	96
Centred22	100	97.4	95	93	92	91.9	91.8	91.6	91.2
Centred23	100	96	93.8	93	92.6	92.3	92	91.9	91.9
Centred24	100	97.5	95.8	95.1	94.8	94.7	94.2	94	93.9
Combined0	100	99	98.3	97.3	96.3	95.7	95.1	94.9	92.9
Combined1	100	99.3	98.3	97.5	96.6	95.9	95.2	95.1	92.9
Combined2	100	98	97	95.6	94.8	94.3	93.5	93.1	90.5
Combined3	100	98	96.3	94.8	93.3	92.3	91.8	91.3	88.3
Combined4	100	98.7	97.3	96	95.3	94.6	94.2	93.8	92.1

Grid0	100	98.3	98.1	98.1	98.1	98.1	98.1	98.1	96.7
Grid1	100	100	100	100	100	100	100	99.9	98.3
Grid2	100	99.6	99.3	95.3	91.8	91.5	91.4	91.4	86.8
Grid3	100	100	100	100	100	100	100	100	98.8
Grid4	100	99.2	98.6	97.5	96	94.7	94.4	94.2	93.4
Random0	100	98.4	96.7	95.6	94.5	94	93.5	93.2	91.7
Random1	100	98.4	97.1	95.9	95.1	94.5	93.7	93.4	91.8
Random2	100	98.6	96.9	95.8	95.1	94.4	94	93.6	91.7
Random3	100	98.2	96.7	95.7	94.9	94	93.6	93.3	91.5
Random4	100	98.7	97.3	96.4	95.8	95.4	95.1	94.9	93.1
Random5	100	98.4	98	97.2	96.6	96.1	95.7	95.3	94
Random6	100	99.5	98.9	97.9	97.1	96.5	96.2	96	94.5
Random7	100	99.2	98.1	96.8	95.9	95.3	94.5	94	91.6
Random8	100	98.8	97.8	96.8	96.2	95.7	95.4	94.8	93.5
Random9	100	99	97.7	96.5	95.9	95.2	94.6	93.9	92.5
Random10	100	100	99.3	97.8	97.2	97	96.8	96.6	96.4
Random11	100	100	100	100	100	100	100	99.9	99.9
Random12	100	99.8	99.3	99	99	99	99	99	98.9
Random13	100	99.8	99.3	98.7	98.4	98.4	98.2	98.1	97.6
Random14	100	99.6	99.4	99	98.9	98.7	98.5	98.4	97.8
Random15	100	100	99.8	99.6	99.6	99.6	99.6	99.6	99.6
Random16	100	99.4	98.2	96.9	96.5	95.9	95.5	95.4	94.5
Random17	100	98.8	97.6	96.7	96	95.5	95.4	95.4	94.9
Random18	100	99	97.8	96.9	96.3	95.8	95.2	95	94.3
Random19	100	99.5	98	97.1	96.7	96.2	96	95.9	95.7
Random20	100	99.1	98.5	98.2	97.9	97.9	97.8	97.8	97.8
Random21	100	98.2	96.5	95	94.4	94.3	93.9	93.8	93.5
Random22	100	99.3	98.6	98.6	98.5	98.5	98.5	98.5	98.5
Random23	100	98.2	95.7	95	94	93.9	93.9	93.9	93.9
Random24	100	99.6	98.1	97.2	96.7	96.6	96.6	96.6	96.6
mean	100.0	98.5	97.1	96.0	95.2	94.7	94.3	94.0	92.3
minimum	100.0	96.0	93.5	91.8	90.3	89.3	88.3	87.6	81.6

Table A.9: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the recalculation method with N=3. RTP coverages are tabulated when evaluation is for ISS=1, ..., 8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	100	100	100	99.9	99.8	99.6	99.3	99	95.9
Centred1	100	100	100	99.8	99.5	99	98.7	98.3	94.3
Centred2	100	100	100	99.9	99.6	99.3	99	98.7	96.2
Centred3	100	100	100	99.9	99.7	99.3	99.1	98.8	94.3
Centred4	100	100	100	100	99.8	99.4	99.1	98.9	95.4
Centred5	100	100	100	99.9	99.8	99.7	99.7	99.5	98.7
Centred6	100	100	100	100	100	99.9	99.8	99.7	98.7
Centred7	100	100	100	100	99.6	99.4	99.2	99.1	97.9
Centred8	100	100	100	100	99.8	99.5	99.2	99	97.9
Centred9	100	100	100	99.9	99.8	99.5	99.3	99.2	98.3
Centred10	100	100	100	100	99.9	99.9	99.8	99.8	99.7
Centred11	100	100	100	99.9	99.7	99.5	99.4	99.2	98.8
Centred12	100	100	100	99.9	99.8	99.8	99.8	99.8	99.8
Centred13	100	100	100	100	100	99.7	99.6	99.5	99.5
Centred14	100	100	100	99.9	99.8	99.8	99.8	99.8	99.6
Centred15	100	100	100	100	100	100	100	100	100
Centred16	100	100	100	100	100	100	100	100	100
Centred17	100	100	100	99.7	99.5	99.5	99.5	99.3	99.3
Centred18	100	100	100	100	100	100	100	100	100
Centred19	100	100	100	100	100	100	100	100	100
Centred20	100	100	100	100	100	100	100	100	100
Centred21	100	100	100	100	100	100	100	100	100
Centred22	100	100	100	100	100	100	100	100	100
Centred23	100	100	100	100	100	100	100	100	100
Centred24	100	100	100	100	100	100	100	100	100
Combined0	100	100	100	99.9	99.7	99.6	99.5	99.4	99
Combined1	100	100	100	99.9	99.6	99.4	99.3	99.2	98.6
Combined2	100	100	100	99.9	99.8	99.6	99.4	99.4	98.8
Combined3	100	100	100	100	99.9	99.6	99.5	99.3	98.8
Combined4	100	100	100	100	99.8	99.6	99.5	99.5	99.1

Grid0	100	100	100	100	100	100	100	100	100
Grid1	100	100	100	100	100	100	100	99.9	98.3
Grid2	100	100	100	98.5	97.4	96.6	96.3	96.1	95.1
Grid3	100	100	100	100	100	100	100	100	98.8
Grid4	100	100	100	99.2	98.7	98	97.8	97.7	97.2
Random0	100	100	100	100	99.9	99.9	99.9	99.8	99.3
Random1	100	100	100	100	100	100	99.9	99.8	99.4
Random2	100	100	100	100	99.8	99.7	99.6	99.6	99.1
Random3	100	100	100	100	100	99.9	99.9	99.7	99.3
Random4	100	100	100	100	99.9	99.9	99.8	99.7	99.4
Random5	100	100	100	100	100	100	100	99.9	99.7
Random6	100	100	100	100	100	100	100	100	99.9
Random7	100	100	100	100	100	100	99.9	99.9	99.5
Random8	100	100	100	100	100	100	99.9	99.9	99.8
Random9	100	100	100	100	99.9	99.8	99.7	99.7	99.6
Random10	100	100	100	100	100	100	100	100	100
Random11	100	100	100	100	100	100	100	100	100
Random12	100	100	100	100	100	100	100	100	100
Random13	100	100	100	100	100	100	100	100	100
Random14	100	100	100	100	100	100	100	100	100
Random15	100	100	100	100	99.9	99.9	99.8	99.8	99.8
Random16	100	100	100	100	100	100	100	100	100
Random17	100	100	100	100	100	100	100	100	100
Random18	100	100	100	100	100	100	100	100	100
Random19	100	100	100	100	100	100	100	100	100
Random20	100	100	100	100	100	100	100	100	100
Random21	100	100	100	100	100	100	100	100	100
Random22	100	100	100	100	100	100	100	100	100
Random23	100	100	100	100	100	100	100	100	100
Random24	100	100	100	100	100	100	100	100	100
mean	100.0	100.0	100.0	99.9	99.8	99.7	99.7	99.6	99.0
minimum	100.0	100.0	100.0	98.5	97.4	96.6	96.3	96.1	94.3

Table A.10: All sixty library cases are assigned for constraints for 9dB service threshold. The method used is the recalculation method with N=6. RTP coverages are tabulated when evaluation is for ISS=1,...,8 and ISS=$n-1$.

	no. simultaneous interferers for evaluation								
	1	2	3	4	5	6	7	8	$n-1$
Centred0	100	100	100	100	100	100	100	100	99.9
Centred1	100	100	100	100	100	100	100	100	99.9
Centred2	100	100	100	100	100	100	100	100	99.9
Centred3	100	100	100	100	100	100	100	100	100
Centred4	100	100	100	100	100	100	100	100	99.9
Centred5	100	100	100	100	100	100	100	100	100
Centred6	100	100	100	100	100	100	100	100	100
Centred7	100	100	100	100	100	100	100	100	100
Centred8	100	100	100	100	100	100	100	100	100
Centred9	100	100	100	100	100	100	100	100	100
Centred10	100	100	100	100	100	100	100	100	100
Centred11	100	100	100	100	100	100	100	100	100
Centred12	100	100	100	100	100	100	100	100	100
Centred13	100	100	100	100	100	100	100	100	100
Centred14	100	100	100	100	100	100	100	100	100
Centred15	100	100	100	100	100	100	100	100	100
Centred16	100	100	100	100	100	100	100	100	100
Centred17	100	100	100	100	100	100	100	100	100
Centred18	100	100	100	100	100	100	100	100	100
Centred19	100	100	100	100	100	100	100	100	100
Centred20	100	100	100	100	100	100	100	100	100
Centred21	100	100	100	100	100	100	100	100	100
Centred22	100	100	100	100	100	100	100	100	100
Centred23	100	100	100	100	100	100	100	100	100
Centred24	100	100	100	100	100	100	100	100	100
Combined0	100	100	100	100	100	100	100	100	100
Combined1	100	100	100	100	100	100	100	100	100
Combined2	100	100	100	100	100	100	100	100	100
Combined3	100	100	100	100	100	100	100	100	100
Combined4	100	100	100	100	100	100	100	100	100

Grid0	100	100	100	100	100	100	100	100	100
Grid1	100	100	100	100	100	100	100	100	100
Grid2	100	100	100	100	100	100	100	100	100
Grid3	100	100	100	100	100	100	100	100	100
Grid4	100	100	100	100	100	100	100	100	100
Random0	100	100	100	100	100	100	100	100	100
Random1	100	100	100	100	100	100	100	100	100
Random2	100	100	100	100	100	100	100	100	100
Random3	100	100	100	100	100	100	100	100	100
Random4	100	100	100	100	100	100	100	100	100
Random5	100	100	100	100	100	100	100	100	100
Random6	100	100	100	100	100	100	100	100	100
Random7	100	100	100	100	100	100	100	100	100
Random8	100	100	100	100	100	100	100	100	100
Random9	100	100	100	100	100	100	100	100	100
Random10	100	100	100	100	100	100	100	100	100
Random11	100	100	100	100	100	100	100	100	100
Random12	100	100	100	100	100	100	100	100	100
Random13	100	100	100	100	100	100	100	100	100
Random14	100	100	100	100	100	100	100	100	100
Random15	100	100	100	100	100	100	100	100	100
Random16	100	100	100	100	100	100	100	100	100
Random17	100	100	100	100	100	100	100	100	100
Random18	100	100	100	100	100	100	100	100	100
Random19	100	100	100	100	100	100	100	100	100
Random20	100	100	100	100	100	100	100	100	100
Random21	100	100	100	100	100	100	100	100	100
Random22	100	100	100	100	100	100	100	100	100
Random23	100	100	100	100	100	100	100	100	100
Random24	100	100	100	100	100	100	100	100	100
mean	100	100	100	100	100	100	100	100	99.99
minimum	100	100	100	100	100	100	100	100	99.90

Table A.11: The 102 cases in which the maximum entry in Φ is reduced by the application of the spread method are tabulated, showing the change in maximum separation and the type of constraints which result.

Test case	n	N	SIR_a	reduction in max Φ separation	resulting constraints
CentredExample2	313	2	12	1	co-channel
CentredExample4	320	2	12	1	co-channel
CentredExample5	160	2	12	1	co-channel
CentredExample14	88	2	12	1	co-channel
CentredExample15	43	2	12	1	co-channel
RandomExample1	216	2	12	1	co-channel
RandomExample4	210	2	12	1	co-channel
RandomExample6	80	2	12	1	co-channel
RandomExample9	81	2	12	1	co-channel
GridExample0	400	2	15	1	co-channel
GridExample1	196	3	12	1	co-channel
GridExample0	400	3	15	1	co-channel
CombinedExample1	178	4	9	1	co-channel
GridExample0	400	4	12	1	co-channel
CentredExample0	329	5	9	1	co-channel
CentredExample1	331	5	9	1	co-channel
CentredExample2	313	5	9	1	co-channel
CentredExample4	320	5	9	1	co-channel
CentredExample5	160	5	9	1	co-channel
CentredExample6	165	5	9	1	co-channel
CentredExample7	169	5	9	1	co-channel
CentredExample8	160	5	9	1	co-channel
CentredExample10	87	5	9	1	co-channel
CentredExample11	90	5	9	1	co-channel
CentredExample12	85	5	9	1	co-channel
CentredExample13	88	5	9	1	co-channel
CentredExample14	88	5	9	1	co-channel
CentredExample15	43	5	9	1	co-channel
CentredExample16	46	5	9	1	co-channel
CentredExample17	48	5	9	1	co-channel
CentredExample18	43	5	9	1	co-channel
CentredExample19	45	5	9	1	co-channel
CentredExample20	25	5	9	1	co-channel

CentredExample21	24	5	9	1	co-channel
CentredExample22	24	5	9	1	co-channel
CentredExample23	25	5	9	1	co-channel
CentredExample24	23	5	9	1	co-channel
CombinedExample2	162	5	9	1	co-channel
CombinedExample4	164	5	9	1	co-channel
GridExample2	100	5	9	1	co-channel
GridExample4	25	5	9	1	co-channel
RandomExample0	229	5	9	1	co-channel
RandomExample1	216	5	9	1	co-channel
RandomExample2	215	5	9	1	co-channel
RandomExample3	231	5	9	1	co-channel
RandomExample4	210	5	9	1	co-channel
RandomExample5	79	5	9	1	co-channel
RandomExample6	80	5	9	1	co-channel
RandomExample7	78	5	9	1	co-channel
RandomExample8	77	5	9	1	co-channel
RandomExample9	81	5	9	1	co-channel
RandomExample10	40	5	9	1	co-channel
RandomExample11	40	5	9	1	co-channel
RandomExample12	40	5	9	1	co-channel
RandomExample13	42	5	9	1	co-channel
RandomExample14	44	5	9	1	co-channel
RandomExample15	24	5	9	1	co-channel
RandomExample16	28	5	9	1	co-channel
RandomExample18	26	5	9	1	co-channel
RandomExample19	30	5	9	1	co-channel
RandomExample20	19	5	9	1	co-channel
RandomExample21	16	5	9	1	co-channel
RandomExample22	19	5	9	1	co-channel
RandomExample24	17	5	9	1	co-channel
GridExample0	400	5	12	1	co-channel
CentredExample5	160	6	9	1	co-channel
CentredExample6	165	6	9	1	co-channel
CentredExample13	88	6	9	1	co-channel

CentredExample14	88	6	9	1	co-channel
CentredExample15	43	6	9	1	co-channel
CentredExample16	46	6	9	1	co-channel
CentredExample17	48	6	9	1	co-channel
CentredExample18	43	6	9	1	co-channel
CentredExample19	45	6	9	1	co-channel
CentredExample20	25	6	9	1	co-channel
CentredExample22	24	6	9	1	co-channel
CentredExample24	23	6	9	1	co-channel
CombinedExample4	164	6	9	1	co-channel
GridExample2	100	6	9	1	co-channel
GridExample4	25	6	9	1	co-channel
RandomExample4	210	6	9	1	co-channel
RandomExample5	79	6	9	1	co-channel
RandomExample6	80	6	9	1	co-channel
RandomExample7	78	6	9	1	co-channel
RandomExample8	77	6	9	1	co-channel
RandomExample9	81	6	9	1	co-channel
RandomExample10	40	6	9	1	co-channel
RandomExample11	40	6	9	1	co-channel
RandomExample12	40	6	9	1	co-channel
RandomExample13	42	6	9	1	co-channel
RandomExample14	44	6	9	1	co-channel
RandomExample15	24	6	9	1	co-channel
RandomExample16	28	6	9	1	co-channel
RandomExample17	27	6	9	1	co-channel
RandomExample18	26	6	9	1	co-channel
RandomExample19	30	6	9	1	co-channel
RandomExample20	19	6	9	1	co-channel
RandomExample21	16	6	9	1	co-channel
RandomExample22	19	6	9	1	co-channel
RandomExample23	16	6	9	1	co-channel
RandomExample24	17	6	9	1	co-channel
GridExample0	400	6	12	1	co-channel

Table A.12: Of 102 cases in which the maximum entry in Φ is reduced by the application of the spread method, those where a change in lower or upper bound is observed are tabulated, showing the change in the bound and the resulting percentage of RTPs covered when evaluation is for complete interference. (When coverage is evaluated for ISS=N, both the recalculation and spread methods will give 100% coverage, as they are designed to do.)

Test case	n	N	SIR_a	Recalc. LB	Spread LB	Change	Recalc. UB	Spread UB	Change	Recalc. RTPs cov.	Spread RTPs cov.	Change
Centred4	320	2	12	11	12	+1	11	12	+1	86.4	86.6	+0.2
Random1	216	2	12	9	9	–	9	10	+1	95.3	95.4	+0.1
Grid0	400	2	15	5	8	+3	6	8	+2	64.5	68.4	+3.9
Grid1	196	3	12	4	4	–	7	5	−2	94.6	93.6	−1
Grid0	400	3	15	6	12	+6	9	12	+3	99.5	99.5	–
Centred0	329	5	9	11	12	+1	11	12	+1	99.5	99.6	+0.1
Centred1	331	5	9	12	12	–	13	12	−1	99.5	99.6	+0.1
Centred5	160	5	9	11	12	+1	12	12	–	99.8	99.9	+0.1
Centred15	43	5	9	10	11	+1	10	11	+1	100	100	–
Centred16	46	5	9	10	12	+2	10	12	+2	100	100	–
Centred17	48	5	9	11	12	+1	11	12	+1	100	100	–
Centred23	25	5	9	12	13	+1	12	13	+1	100	100	–
Centred24	23	5	9	10	12	+2	10	12	+2	100	100	–
Grid4	25	5	9	6	6	–	8	7	−1	100	100	–
Random0	229	5	9	10	9	−1	10	10	–	100	100	–
Random3	231	5	9	10	10	–	11	10	−1	100	100	–
Random5	79	5	9	7	7	–	8	7	−1	100	100	–
Random6	80	5	9	6	7	+1	7	7	–	100	100	–
Random7	78	5	9	7	7	–	8	7	−1	100	100	–

Test case	n	N	SIR_a	Recalc. LB	Spread LB	Change	Recalc. UB	Spread UB	Change	Recalc. RTPs cov.	Spread RTPs cov.	Change
Random8	77	5	9	7	7	–	8	7	−1	100	100	–
Random13	42	5	9	4	4	–	5	4	−1	100	100	–
Random19	30	5	9	7	6	−1	7	7	–	100	100	–
Grid0	400	5	12	5	6	+1	6	6	–	96.7	98.5	+1.8
Centred5	160	6	9	12	13	+1	12	13	+1	100	100	–
Centred13	88	6	9	11	11	–	12	11	−1	100	100	–
Centred15	43	6	9	11	12	+1	12	12	–	100	100	–
Centred16	46	6	9	11	11	–	12	11	−1	100	100	–
Centred17	48	6	9	12	13	+1	12	13	+1	100	100	–
Centred18	43	6	9	13	14	+1	13	14	+1	100	100	–
Centred20	25	6	9	11	12	+1	11	12	+1	100	100	–
Centred24	23	6	9	13	14	+1	13	14	+1	100	100	–
Grid2	100	6	9	6	6	–	11	8	−3	100	100	–
Grid4	25	6	9	6	6	–	8	7	−1	100	100	–
Random6	80	6	9	7	7	–	7	8	+1	100	100	–
Random7	78	6	9	7	8	+1	8	8	–	100	100	–
Random8	77	6	9	7	8	+1	8	8	–	100	100	–
Random14	44	6	9	6	7	+1	6	7	+1	100	100	–
Random21	16	6	9	5	5	–	5	6	+1	100	100	–
Random23	16	6	9	5	6	+1	6	7	+1	100	100	–
Grid0	400	6	12	5	6	+1	7	7	–	100	99.9	−0.1