Investigation into Learners' Progression in Algebra from Grade 9 to Grade 11

King Wun Vincent Leung

Supervisor: Dr Craig Pournara

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Declaration

I declare that this research report is my own work. It is submitted for the Degree of Master of Education in the University of the Witwatersrand, Johannesburg, South Africa. It has not been submitted before for any degree or examination in any other University. I further declare that this research report has not been submitted previously for any degree or examination at any other university.

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(Signature of candidate)

5th day of September, 2014

Abstract

This study investigates learners' progression in algebra from grade 9 to 11, in terms of the levels developed in the ICCAMS (Increasing Students' Competence and Confidence in Algebra and Multiplicative Structures) diagnostic test. Three key questions are posed with regard to learners' progression: Do learners progress to higher ICCAMS levels from grade 9 to grade 10? For those who do not progress from grade 9 to grade 10, do they progress to higher ICCAMS levels from grade 10 to grade 11? For those who make no progress on ICCAMS levels from grade 9 to 11, what errors hinder their progression? Two sets of test scripts of 34 learners from a secondary school in Gauteng were analysed to determine their levels in grades 9 and 10. Approximately half the sample progressed to a higher level from grade 9 to grade 10. However, almost the same number of learners improved in the number of correct responses, but failed to progress to a higher level. Individual interviews were conducted with five learners from this group, in their grade 11 year. The results show that four of these learner had progressed to a higher ICCAMS level in grade 11. However, one learners did not achieve a higher level. Typical errors that impact their progress include negative numbers and conjoining.

Dedication

To my beloved mother and father, Ching Wan Tang and Hon Shek Leung, my sister Wing Sze Vingin Leung and the entire Leung family.

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Abbreviations

University of Witwatersrand - Wits

Wits Maths Connect Secondary project – WMC-S

Third International Mathematics and Science Study – TIMMS

Annual National Assessment – ANA

Department of Basic Education – DBE

Increasing Students Competence and Confidence in Algebras and Multiplicative Structures – ICCAMS

Concepts in Secondary Mathematics and Science – CSMS

Multiple Choice Question – MCQ

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Chapter 1 – Introduction and Rationale

1.1 - Introduction

This study was aimed at examining the gains in algebraic performance made by a sample of learners. Learners from grade 9 year to their grade 11 year were analysed in their performance in algebra, after being assessed by means of a test. The nature of the test will be discussed later. This study offers a detailed look into what gains (if any) learners made in their algebraic performance, and the nature of progression in algebra in the South African context. This study attempts also to inform the larger Wits Maths Connect Secondary project (WMC-S) by providing insight into learner gains in algebra, hence allowing the project to make informed decisions regarding future such interventions. It has also sought to add to the emerging literature regarding gains in learner mathematical performance, in algebra, within the South African context.

1.2 – Background

At present the state of Mathematics in South Africa is at an undesirably low level. Learners are performing at much lower levels compared to others internationally (Moloi and Strauss, 2005; Howie, 2007, cited in Taylor, 2008). As Makgato & Mji (2006) stated, South Africa conducted the Third International Mathematics and Science Study (TIMMS) in 1995, 1999, and 2003. All of them indicated poor improvement in learners' performance over the years. Furthermore, in the WMC-S project the grade 8 performances on the pre-test were also poor. In addition, according to the Annual National Assessment Report (ANAs) in 2012, the average of grade 9 learners across the country was 12 7% (DBE, report on ANA, 2012), and in 2013 the average of grade 9 learners across the country was 14% (DBE, report on ANA, 2013). All these showed that South Africa has a very poor performance in Mathematics, a matter that needs urgent attention.

1.3 – Rationale and Purpose

Besides the problem of slow progression in performance and low scores in Mathematics, there is also very little literature regarding what gains learners are being made, and what the nature of these gains are, at a detailed level, in the South African context.

In light of the state of Mathematics in South Africa, this study aimed to identify what gains in learners' mathematical performance are being made. Reddy, Van Der Berg, Janse Van Rensburg, & Taylor (2012) highlight the notion that passing mathematics in Matric is strongly influenced by their performance in grade 8, where foundations are meant to be laid.

This shows the importance of diagnosing learners' performance every year, and recording their performance. This motivates me to examine the progression (if any) learners have made from grade 9 to 11.

The WMC-S project, led by Professor Jill Adler, is involved in interventions intended to improve performance in Mathematics. It is a five-year project, assessing the Mathematics of a cohort of learners from 11 schools across Johannesburg. In 2010 WMC-S Project set an annual test for all project schools, and a large number of data was recorded. The purpose in collecting large numbers of data initially was to have a baseline of learners' actual performance in algebra and function. However, the data was reviewed in terms of the project schools in general, but not at the level of individual learners. Hence the purpose of this study is to investigate individual learners' progression in detail.

1.4 – Contribution to field

In light of the lack of literature regarding learner gains, this study seeks to add to the emerging of literature regarding the nature of gains in learners' performance in algebra in the South African context.

Reddy et al. (2012) highlights the importance of diagnostic assessment, because of the direct influence of poor grade 8 performances leading to poor Matric results. The ICCAMS framework of levelling a learner can facilitate the tracking of learners' performance and progression.

Furthermore, this study can help to identify the change in the kinds of errors that learners make, and factors that may account for them. These two findings may help to direct the focus for teachers during teaching.

1.5 – Critical questions

WMC-S annual test is not the full ICCAMS items, since the ICCAMS instrument contains more than just algebra. WMC-S only made used of the algebra items. The WMC-S annual test comprised three components: curriculum –related questions for the appropriate year, a small selection of TIMMS items, and algebra items from ICCAMS. I focus only on the ICCAMS section in this study. The learners in this study had passed grade 9 and 10, so a progression in performance would be expected, discerned by comparing their current results with those of previous grades. We can assume learners have progressed if there is an improvement in ICCAMS level (refer to Chapter 2.2), or they make fewer errors compared to the previous year, or both these.

In order to investigate the progression made by learners, the main criterion is the progression in learners' performance in algebra from grade 9 to grade 11. This main question will be explored by following three guiding questions:

1. Do learners progress to higher ICCAMS levels from grade 9 to grade 10?
2. For those who do not progress from grade 9 to grade 10, do they progress to higher ICCAMS levels from grade 10 to grade 11?
3. For those who make no progress on ICCAMS levels from grade 9 to 11, what errors hinder their progression?

In order to observe the progression in their performance from grade 9 to grade 10 I analysed the difference in the total number of correct responses and the difference in ICCAMS level. During the analysis I realised that even when a learner improved in the total number of correct responses he/she did not necessarily progress to the next level. More than one-third of the learners, 12 learners out of 34, improved in the number of correct responses, but remained at the same level. In view of this I decided to focus on these learners who did not progress in level despite getting more correct answers. I decided to focus on these learners because they have achieved more correct answers, yet did not progress in level, so it is important to investigate what is holding them back. Moreover, they are more likely to progress when they have more correct answers. When I refer to a learner who did not progress it means that he/she maintained his level, but did not regress. This will be discussed further in chapter 5.

Chapter 6 presents the analysis of the interviews with five learners. The interview instrument was designed to determine whether learners who maintained ICCAMS level in grades 9 and 10 had progressed into a higher ICCAMS level in grade 11. Obstacles to learners' progression are discussed in this chapter.

Chapter 2 – An overview of Increasing Students' Competence and Confidence in Algebra and Multiplicative Structure (ICCAMS)

2.1 – Introduction

In this chapter I have included the ICCAMS history and the ICCAMS level of understanding, to provide an overview of what ICCAMS and ICCAMS levels actually are. In the ICCAMS history section, I have discussed where and how ICCAMS was developed. I also discuss how to determine whether a learner has progressed or not, and how to identify learners' errors by using Hart's framework. In the ICCAMS level of understanding section, I have discussed how the ICCAMS levels were established, and the difference between each level.

2.2 – ICCAMS history

Hart, Brown, Kerslake, Küchemann, & Ruddock (1985) developed a diagnostic test for algebra as part of the Chelsea Diagnostic Mathematics Tests to identify learners' levels and the common errors that learners make. They have since been refined in the Increasing Students Competence and Confidence in Algebra and Multiplicative Structures framework (ICCAMS). One of the research aims in the mathematics component of the Concepts in Secondary Mathematics and Science (CSMS) was to provide information for teachers on 'levels of understanding' in secondary school mathematics by giving codes to the different responses of the learners (refer to Chapter 4.3).

The development of Hart's framework into the ICCAMS provides both a tool and a framework with which to identify what level learners are at in terms of their performance, and also possible improvement levels. Keeping this framework in mind one can form a concept of the nature of possible progression in learner performance. With a better understanding of how to assign different levels to learners, how to code their responses, and how to interpret responses to questions on ICCAMS, one can use this information to see what learners can or cannot do and what errors they make. Hart et al. (1985) also go into detail of how codes can be used for interpreting the learners' performance, which can lead to identifying their obstacles and misconceptions in algebra.

Moreover, Hart et al. (1985) address the matter of identification of errors. For them, Chelsea Diagnostic mathematics tests were designed not just to diagnose learners' performance, but also to critically identify their errors. These codes described the types of errors that learners were making. However, some of these codes, such as codes 3 to 7 in the ICCAMS coding scheme (Appendix G), were not included in this study, because very few learners got those codes, hence it is not very useful. However, the coding was part of WMC-S, so it was nevertheless included in the Appendix. Even though the coding was not very useful, the manual provides a framework which diagnoses learners' performance, which may be helpful to identify learners' errors. The procedure of the ICCAMS test is the same as any other diagnostic test, i.e. written under exam conditions, needing only a pen or a pencil as basic equipment, filling in the information requested on the front page, specific time limits set for the whole test, and working out to be done on the test paper itself, not on separate scrap paper, with no calculator allowed.

2.2 – ICCAMS levels of understanding

As Küchemann (1981) noted "children's responses were classified into different levels of understanding" (p. 105). The ICCAMS section of the WMC-S annual test contains 30 questions, and most are assigned different levels. Depending on the number of correct answers, the learners can be assigned to a specific level. In these 30 questions, Hart et al. (1985) assigned six questions to level 1, seven questions to level 2, eight questions to level 3, and nine questions to level 4. These questions were levelled based on learners' achievement. For example, $5 + a = 9$ is the type of question that many learners got correct, hence this question was labelled as level 1. However in the ICCAMS instrument, some of the questions were not levelled, due to items which showed insufficient differentiation. The descriptions of these levels are based on Küchemann's (1981) work where he identifies six different ways in which learners might interpret letters: letter evaluated, letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalised number, and letter used as a variable (refer to Chapter 3.3).

A brief explanation of the above levels:

- ***Level 1*** – learners can answer level 1 items correctly without having to treat letters as unknowns. The letters are treated as objects or ignored. An example of a task where the letter can be treated as an object is: $5 + a = 9\ then\ what\ is\ a$? The main feature here is that a can simply be evaluated to be 4. An example of a task where the letter can be ignored is: $a + b = 41, a + b + 5 = \ldots$. Here learners can ignore the letters and see that the left hand side of the equation has been increased by 5.
- ***Level 2*** – this has increased in complexity from level 1, and requires greater familiarity with algebraic notation, though the letters have to be evaluated or used as objects. In

other words, learners at level 2 would be able to apply some of the rules of algebraic operations and algebraic conventions. For example, tasks can also be solved by only evaluating the letter, or ignoring the letter, or using the letter as an object, but in a more complex structure, such as $if\ h = 3g + 1, and\ g = 5, what\ is\ h$? In this task it is required that learners know that $3g$ means 3 multiplied by the letter g, and also multiplication takes precedence over addition. Another example: the learners were given a task to calculate the perimeter of a pentagon with the lengths of sides labelled a, a, a, b, b and asked to determine the perimeter. In this task learners were required to add the three letters a and two letters b, to derive $3a + 2b$, and to know that these two letters do not necessarily represents the same unknown.

- ***Level 3*** – These are tasks with a complex structure where letters need to be treated as specific unknowns or numbers that are generalised. For example, $if\ m + n = 8,$ $then\ what\ is\ m + n + k =$? In this task learners have to cope with the lack of closure of the expression $8 + k$.They have to accept and be able to see this expression as an answer, and not as an instruction to do something. Another example: write down the perimeter of a shape with n sides, all of length 4. In this task learners must be able to see the letter n is representing a specific or general unknown number. It cannot be treated as an object, such as name or label.
- ***Level 4*** – Here letters must be seen as variables, or treated as specific unknowns, or the letters represent the numbers of the object, but not the object itself. A level 4 task is also a complex structure, but it also requires 2 dimensions of interpretation of letters, such as: *State whether $a + b + c = a + b + d$ is sometimes, always or never true?* Here learners need to recognise that it is true if $c = d$, hence this statement is sometimes true. Another example: Multiply $p + 2\ by$ 5. In this task learners are required to see both p and 2 are to be multiplied by 5.

Generally as the learner progresses in school grades, he/she tends to achieve higher ICCAMS levels. This is similar to Piaget’s (1964) theory that learners cannot learn beyond their capabilities, and a learner cannot learn if he/she hasn’t reached a particular stage of development. Küchemann (1981) tried to link Piagetian stages of development (i.e., below late concrete, late concrete, early formal and late formal) with the four levels. However, Küchemann realized that in secondary school mathematics, Piaget’s stages of development were not very useful as “it is difficult to establish a direct link between the algebra levels and

Piaget's stages of cognitive development" (Küchemann, 1981, p. 117). However according to Küchemann (1981) there was still some value in trying to establish the link at the time, as "it puts the analysis of children's understanding into a more general framework which might apply to other areas of mathematics and to areas outside mathematics. Also the framework might be familiar to the reader" (p. 117).

In this chapter I have discussed the ICCAMS section of the WMC-S annual test as a diagnostic test, which can diagnose learners' performance and progression. The ICCAMS instrument categorises learners into four levels. Level 1 represents the lowest level of understanding in algebra and level 4 as the highest. Identifying learners' levels can indicate their performance, and by comparing their performance across different periods can reflect their progression. Even though Hart's manual provides a framework to diagnose learners' performance which may be used to identify learners' errors, yet more issues in algebra, and a review of their inter-relationship is needed to explore learners' errors. These issues will be discussed in the next chapter.

Chapter 3 – Theoretical Framework and Literature Review

3.1 - Introduction

In most of the research investigating learners' performance, researchers, such as Reddy et al. (2012), tend to use only TIMMS as a framework to measure learners' performance. TIMSS is a multiple-choice question (MCQ) assessment, used for diagnostic purposes. It can highlight the aspect of learner performance, but it cannot answer the question of the nature of gains, due to the MCQ being too broad to identify specific errors.

In this chapter I discuss four issues related to algebra, and review their inter-relationship. The first two issues are symbolic interpretation, and minus symbol interpretation. These will help to provide a basic understanding of how learners interpret symbols in algebra. These two issues will lead to the third - the ICCAMS levels of understanding, where interpretation of symbols forms the basis of classification of levels. The last issue is errors in algebra, which highlights the common errors made by learners.

3.2 – Theoretical Framework

The constructivist Piaget (1964) proposed that a person's mental development process is developed by increasing the number and complexity of schemata during learning. Piaget's theory explains how knowledge is constructed in learners' minds when new ideas or knowledge come into contact with existing knowledge developed by experience.
As Olivier (1989) states, a constructivist views on learning is that the concepts are not taken directly from experience, but the ability that the learner has, and what he/she learns from an experience depends on ideas that he/she has been able to link into his experience. In other words, knowledge is not just from experience, but from connecting between experience and knowledge. As Stacey & MacGregor (1997) describe it, it is natural and healthy that learners interpret new ideas in terms of their experience.

However, in Piaget's view intellectual growth is a process of adaption to the world, but learners' experience can sometimes go with or against the new ideas or knowledge. Piaget (1964) uses assimilation, accommodation, and equilibrium to describe the learners' process of adapting to the world.

A brief definition of assimilation or accommodation by Olivier (1989):

- ***Assimilation**: If some new, but recognisably familiar, idea is encountered, this new idea can be incorporated directly into an existing schema that is very much like the new idea, i.e. the idea is interpreted or re-cognised in terms of an existing (concept in a) schema. In this process the new idea contributes to our schemas by expanding existing concepts, and by forming new distinctions through differentiation.*
- ***Accommodation**: Sometimes a new idea may be quite different from existing schemas; we may have a schema which is relevant, but not adequate to assimilate the new idea. Then it is necessary to reconstruct and re-organise our schema. Such re-construction leaves previous knowledge intact, as part or subset or special case of the new modified schema (i.e. previous knowledge is never erased).*

(Olivier, 1989, p.3)

Hatano (1996) noted that is important to know the link between assimilation, accommodation and equilibration, as it can help learners to reconstruct new knowledge, new ideas and existing knowledge. As Hatano (1996) states, "…students construct knowledge by themselves not by swallowing ready-made knowledge from the outside…" (p.211). Thus, learners should not just assimilate the knowledge, but accommodate it with their prior knowledge, and equilibrate between the new and existing knowledge. Hence, learners would interpret the knowledge in relation to their prior knowledge. In other words, as Piaget (1964) explains, when children deal with new ideas or knowledge, assimilation and accommodation work hand in hand. For example, the learner uses his existing schema to deal with a new idea or knowledge (assimilation), and the learner's existing schema can deal with most of the new idea or knowledge then equilibrium has occurred. However, if the learner's existing schema cannot deal with a new idea or knowledge (disequilibrium), then equilibration will drive the learning process and seek to restore balance by changing to deal with the new idea or knowledge (accommodation). Once the new idea or knowledge has become part of the existing schema, the process of assimilation continues until the next time an adjustment needs to be made.

3.3 – Symbolic Interpretation

As Naidoo (2009) highlighted, "the interpretation of letters refers to the minimum meaning needed to be given to the letter to solve the task" (p. 38). Küchemann (1981) noted that learners interpret letters in a specific order, from low level of response to high. He then assigns learners' interpretation to six categories, from low level response to high.

A brief explanation of children's interpretations of the letters:

- ***Letter evaluated*** – the learner assigns a numerical value to the letter. For example, $a + b = 6$, *then what is* $a + b + c$? Here the learner would say the answer is 9 because $3 + 3 + 3 = 9$, or they might replace the alphabetical letter c with 3 because the letter c is in the 3rd place in the alphabetical order.
- ***Letter not used*** – learners here are ignoring the letter or acknowledging the letter's existence, but not giving it any meaning. For example, $2a + 1 = 3a$: the learner could either ignore, or not give meanings to the letter a in the operation of addition, and only combine the expression with the familiar number - for which the answer will be $3a$.
- ***Letter used as an object*** – the learner cannot see the number represented by the letters, so they create objects for the letter. For example, $a + b + a = 2\ apples\ and\ 1\ banana$: the learner can interpret the letter only as a physical object. In many situations this would work as follows, for example*: If I buy 4 cakes and 3 buns, what does* $4c+3b$ *stand for?* Here the learner could say the letter c stands for the number of cakes and letter b stands for the number of buns. This is incorrect, for it is supposed to be the price of the four cakes and three buns. Learners may not be aware of their error since their interpretation appears to make sense.
- ***Letter used as a specific unknown*** – the learner refers to the letter as a specific, but unknown number which he/she tries to solve algebraically. For example, *what is* $a + 1 = \ldots$*?* Here the answer will stay the same, because letter a is a specific unknown number with which nothing can be done.
- ***Letter used as a generalised number*** – the letter as generalized number is a more advanced interpretation than letter as a specific unknown number. Here learners must see that the letter is able to take on more than a value, for example *state whether the equation* $L + M + N = L + P + N$ *is true or not*. Here the learner must recognise the conditions under which the equation will be true, that $M = P$.
- ***Letter used as a variable*** – the learner sees the letter as a variable, and understands that it can represent a value that varies. For example, $which\ is\ larger$: $2n\ or\ n + 2$?: the learner knows that the letter can take on a range of values.

3.4 – Minus symbol interpretation

As researchers such as Vlassis (2004), Gallardo & Rojano (1994), and Halley (2011) have shown, minus symbols can have several meanings depending on the structure of the expression. Although in this study only 'sign rule' and 'operating and choosing signs' appeared, they have identified learners' interpretation of minus symbols into different categories.

A brief explanation of learner's interpretation of minus symbols:

- ***Right-to-left reasoning*** – the learner operates the expression from right-to-left, the reverse order of the operation between two or more numbers (Vlassis, 2004). For example, $4-9$ would be treated as $9-4$. Sometimes it could be correct, if the sign and operation are the same. For example, $4+9$ could be treated as $9+4$.
- ***Brackets reasoning*** – the learners insert imaginary brackets around parts of an expression. These brackets are to be solved before simplifying the rest of the expression (Vlassis, 2004). For example, $8x-4x-4-1$ would be simplified to $4x-3$, which the learner correctly treats $8x-4x$ as a pair, and similarly, the other is $4-1$.
- ***Signs rule*** – the learner uses multiplicative sign rules of '$- \times -= +$, and $+ \times -= -$' in questions that involve only addition and subtraction (Vlassis, 2004). For example,$-4x-3x=7x$, and $+4x-3x=-7x$. This applying multiplicative sign rules is not always incorrect (Halley, 2011). For example, $4x-(-3x)$ can be written as $4x+3x$ and $4x+(-3x)$ can be written as $4x-3x$. This interpretation of the sign rule may lead into the next categories of minus symbols interpretation: "too many signs".
- ***Too many signs/symbols*** – the learners choose to ignore adjacent symbols after the operation (Gallardo & Rojano, 1994). For example, he interprets $5x-(-2x)\ as\ 5x-2x$. Thus the learner interprets the symbol after $5x$ as a minus operation, and the second symbol in the bracket is not needed.
- ***Incorrect operation*** – the learner might be confused between multiplication and addition in the expression (Halley, 2011). For example, $(-3x)(-2x)=-5x$ or $(4x)(-5x)=-x$, which shows the learner simply adding the expression. Another example,$-3-2=+6$ or $4-5=-20$, shows the learner multiplying. Thus the learner is confused about when to use multiplication or addition.
- ***Operating and choosing signs*** – this interpretation is very similar to 'sign rules', but the learner uses the sign of the number to decide the sign of the answer (Halley, 2011). For example,$-4x+(-3x)=+7x$: the learner thought that the negative number and a negative number gives a positive, and the plus symbol means they need to add.

Similarly, $-4x - (-3x) = +x$: the learner thought that the negative number and a negative number gives a positive, and the minus symbol means they need to subtract.

- ***Duality of the minus symbol*** – confusion between subtraction and negative because the one symbol represents both the sign and operation (Halley, 2011). For example, $-4 - 8$: the learner might view it as negative 4 and negative 8, which shows that the minus symbol could be interpreted as an operation or as a sign. Thus there is a double meaning to the minus symbol.

This list shows the different kinds of error that learners might have for minus, but I deal only with the sign rule and operating and choosing appropriate signs in this work. This will appear in Chapter 6.

3.5 – Notion of misconception and errors

Olivier (1989) distinguishes between slips, errors and misconceptions as follows:

- Slips do not happen systematically, but rather now and then, and can happen to both experts and novices. The wrong answer of slips is due to processing, which can easily be detected and corrected.
- Errors happen systematically, and often apply in the same circumstances. The wrong answer is due to planning, which Olivier (1989) explains: "Errors are the symptoms of the underlying conceptual structures that are the cause of those errors" (p.3).
- Olivier (1989) defines misconceptions as "underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors" (p.3).

Nesher (1987) points out that a misconception is part of learning, and that it is not only behind errors, but also embedded in many cases of correct performance. Furthermore, many other authors such as Olivier (1989) and Li & Li (2008), believe that misconceptions are valuable in teaching and learning because they form part of their structure and will affect the way they interpret new concepts and new learning. This negative path rests on the misconceptions generating errors. Hence misconceptions are closely linked with errors.

3.6 – Errors in algebra

Küchemann (1981) classified learners' interpretation of algebraic letters into two divisions. The first is letter-evaluated: the letter not used or used as an object, which is referred to as lower division. The second division is a letter used as a specific unknown, or a generalised number, or a variable, which is referred to as higher division. The interpretations of the lower division are misconceptions. It is considered as a misconception as the learner has a correct understanding of the letter, which is a variable of the unknown, whose value changes.

Furthermore, MacGregor & Stacey (1997) point out some of the typical errors in algebra such as conjoining, i.e. the letter = 1 belief, and the letter in different quantities. Conjoining is seen as joining terms during addition. For example, $a + b = ab$: the learner that has an understanding of addition will find the sum of the two terms, but does not understand the difference between value and letter. Another type of error is where a learner thought that the letter is equal to one. For example, $2 + a + 1$: the learner would give an answer of $4a$ or 4 because he interpreted the value of the letter as 1. This error can be explained through two sets of reasoning: misinterpretation of the coefficient, and the power of a variable. For example, a variable always has a coefficient of one ($a = 1a$), and the learner misinterprets $a = a^0$: all variables to the exponent of 0 are equal to 1. Moreover, errors that are normally in word problems are also when the learner uses one letter in different quantities. For example $a + b = 8$: the error here is that the learner does not know when the two letters are equal, or he/she randomly equates the two letters to make a sum of 8.

Essien & Setati (2006) point out another typical error in algebra, which is misinterpretation of an equal sign. They that learners in grades 8 & 9 misinterpret the equal sign as a symbol of finding the answer or as unidirectional. Learners with a symbol of finding the answers may struggle with the understanding of the algebraic equation. For example, $x + y = ?$: because, according to the learner's understanding, he/she needs to find the answer. However, there is nothing to be done, unlike arithmetic addition such as $1 + 2 = ?$.

Unidirectional symbols are similar to the symbol of finding the answer except that the learner sees the equal sign as symbolising one direction, such as $1 + 2 = 3 + 4 = 7$. This will only make sense in one direction, and not in reverse or double movement. Learners with a symbol of unidirectional movement may struggle to accept a statement such as $? = 1 + 2$ due to the equal sign being in the incorrect place.

This chapter reviews four issues: symbolic interpretation, minus symbol interpretation, and error in algebra. The first two issues are about how learners interpret symbols in algebra.

This gives a good basic understanding of interpretation of symbols in the learner's lens. This was used to construct probing questions in the interview and analyse the data, whereas the last issues, error in algebra, was formed in the process of knowledge being constructed in different learners. The last issues suggested some possible reasons as to why some the learners were not progressing into higher ICCAMS levels. However, even though it suggested some possible reasons, we cannot neglect the notion of misconception and errors. Therefore error in algebra and notion of misconception and errors provides a more in-depth view, for those who still do not progress, what errors they make that hinder their progression. In short, this chapter is the foundation of Chapter 4, Research design and Methodology.

Chapter 4 – Research Design and Methodology

4.1 – Introduction

In this chapter I explain the framework that has assisted in highlighting learners' progression. I also discuss the selection of the school and methodology of the study. Thereafter, discussions centre on issues of sample, instrument, data collection, and methodology of analysis for the first and second phases. In the latter part of this chapter I discuss ethics considerations, reliability, validity, and limitation of the study.

The difference in results between grade 9, conducted in October 2011, and those of grade 10 in October 2012, are compared and analysed using the ICCAMS framework in order to determine the progression made. As the test is consistent with the ICCAMS framework, the ICCAMS is the levelling system with which to identify the progression noted in the data set (refer to Chapter 2). The main benefit in this diagnostic test is, apart from being able to diagnose learners' performance, assigning them to levels. It is also able to identify learners' symbolic interpretations (refer to Chapter 3) through error coding.

This framework has assisted in highlighting learners' progression. Using coding, a system has helped to select the sample for interview purposes, meaning the learners that show a strong relation in this study. Furthermore, coding can assign learners to a specific level, which helps to determine whether the learners progress or not, and enables me to generalise learners' responses into categories to assess the nature of the potential gain. I have two phases in collecting data: test scripts and interviews. I took learners' responses from the tests already conducted, and used them to produce questions for the interviews. This helped me to identify the reasoning behind learners' errors.

4.2 – Selection of school

Dragonhill Secondary School (this is a pseudonym to avoid direct or indirect link to the actual school) is one of the eleven schools in the WMC-S project. I chose this school as my project school since the teachers and learners were very willing to participate in my study and were active participants in the project.

The school is located in a township with a shortage in land, and hence homelessness and a high rate of unemployment, and has quintile ranking as 2, i.e., not fee-paying (schools that are rated as quintile ranking 1 or 2 are referred to as 'poor' and the school will allow learners to enrol without paying fees). This school has a feeding scheme (school feeding is a small part of the integrated food security strategy for South Africa). In the two years before my study the school had an acting head of the Mathematics department. The matric overall pass rate between 2007 and 2012 ranged from 34 25% to 73 7%. In those 6 years the numbers of learners in matric has ranged between 181 and 18. The highest enrolment had the lowest pass rate of 34 25%, but the lowest enrolment did not have the highest pass rate.

4.3 – This study's sample is an extension of WMC-S's sample

In 2011 WMC-S conducted their annual test with selected learners in grade 9 at all project schools. This was repeated in 2012 as the cohort moved to grade 10. The test scripts from both years were available for me to use in this study.

The WMC-S annual test results were coded by different markers in the different years. The coding scheme used in 2011 was modified slightly in 2012 and then used as is in 2013. It was thus necessary to recode all 34 scripts from grades 9 and 10 to ensure the reliability of the coding. I recoded my sample based on the 2012 coding scheme. I chose that one because the main change in the 2012 scheme were to streamline the number of codes, particularly code 9. For example, in question 1.6 there were more code 9 options (incorrect answer) in the 2012 coding scheme. Another example: from question 3.1 to question 4.1 rendered some of code 1 (correct answer) as code 2 (ambiguous), whereas in 2011 there was no code 2 or separation of code 1 into 1(a) and 1(b).

An example of how the sample was recoded: in question 2.2, learners were asked which is larger, $2n$ or $n + 2$? If the learner did not attempt the question, it would be coded as 0. If the learner stated that 'it depends', with an adequate reason, such as depending on the value of n, then it would be coded as 1. If the learner stated that 'it depends' without an adequate reason, then it would be coded as 2, which means the answer is ambiguous. However, if the learner chose either of the two, or says both are equal, then it would be coded as 8, which means the answer is premature closure. Within code 8 we have categories a, b, and c to distinguish between the different possible solutions. If the solution was incorrect then it would be coded as 9, but within code 9 are included all the common incorrect solutions. For example, if the learner said $2n$ is larger because multiplication makes it bigger, then it would be coded as 9a, whereas if learner said $n + 2$ is larger because addition makes it bigger, then it would be coded as 9b.

Based on the analysis I designed a task-based interview to find out if the learners had progressed, even though they attained within the same ICCAMS level, compared to performance in the previous year. Thus I collected two sets of data, first was the annual test, and the second derived from the interview on the selected sub-sample from the main sample.

I had a set of test scripts for phase and from them I selected learners for phase two (Interview). Thus the sample for the test was much larger than the sample for the interview. Furthermore, as these learners were selected in grade 9 and tracked through, most of these learners studied Mathematics, with some doing Mathematics Literacy. The learners selected for the interview include some doing Mathematics and some doing Mathematical Literacy but I did not seek to balance these numbers.

This study uses several methods of analysing data, and is both quantitative and qualitative in nature; hence it falls into the category of mixed methods.

4.4 – Phase one

4.4.1 – Sample

I began with a sample of 113 grade 9 learners in 2011 but some of the learners either left the school or failed grade 9 in that year. Hence my sample was reduced to 75 learners in grade 10 in 2012. Of the entire grade, in 2011 and 2012, who wrote the annual test only 34 learners were chosen. Thirty-four learners were selected under two conditions: firstly, I needed to ensure the chosen learners all still attended the school, and had not been retained in their previous grade. Secondly, in their grade 10 year in 2012, learners had to have achieved at least 30% of correct responses in the ICCAMS section. The percentage was calculated by taking the number of questions that the learners got correct and dividing it by the number of questions that were in the ICCAMS section. There is a total of 33 questions in the ICCAMS section from the annual test. I choose 30% of the number of correct responses as a minimum bench mark for selection for this study because the focus of the study was regarding progression. If learners that got below 30% of the number of correct responses in their grade 10 year, did likewise in 2012, then it shows that they have progressed little compared to their previous grade performance. I chose not to focus on balancing the issue of gender and race because is not relevant to the study, and the 34 learners were not from the same class.

4.4.2 – Instrument

The test was set by WMC-S, and is known as the “annual test”. The WMC-S annual test consists of most items from the ICCAMS algebra but not all (refer to Appendix A). The WMC-S test contains 33 questions, most of which are assigned to different levels. The questions are ranged as follows: six questions in level 1, six questions in level 2, five questions in level 3, five questions in level 4, and 11 questions have not been identified in terms of a particular level. Moreover, the test comes with the coding scheme which includes the different possible solutions, which provides an overview of the expectation from the learners. Apart from providing an expectation of learners’ responses, it also indicates learners’ interpretation of symbols.

4.4.3 – Methodology

I needed to analyse learners’ responses from their grade 9 to their grade 10 year and compare performance in two different stages, in ICCAMS levels and learners’ percentage. Firstly, I had to compare their ICCAMS level (refer to Chapter 2) to see whether or not there is any significant progression in order to answer my first research question. In order to identify learners’ ICCAMS levels, the following was required (note that it is not related to learner marks):

Table 4.1 – Minimum requirement for each level

Level	Total number of questions	Number of questions required to be correct
1	6	4
2	6	4
3	5	3
4	5	3

When comparing their grade 9 ICCAMS levels with those of grade 10 these results has helped me to answer the question of whether there was a change in level, such as progression, based on the ICCAMS instrument. In order to identify the progression, I had to compare the ICCAMS level between the two grades. To do so, I excluded all the questions that were not levelled. In other words, I only focused on the 22 questions that were levelled out of the 33 questions. After eliminating non-levelled questions I grouped the rest of the questions by level. I counted the number of correct responses according to the levels and used the WMC-S requirement (refer to Table 4.1) to identify the learners’ level.

Table 4.2 – Example of levelling a learner

Level	**L1**	**L1**	**L1**	**L1**	**L1**	**L1**	**Total Correct answers**
Question	**1.1**	**5.1**	**6.1**	**8.2**	**9**	**10.1**	
Learner 1	1	8	9	1	1	1a	4
Learner 2	9a	9	1	9b	1	1a	3

In table 4.2, learner 1 got four out of six level 1 questions correct, and thus would be considered as level 1. Learner 2 only got three out of six level 1 questions correct, so cannot be considered as level 1, so I referred to it as level 0 (zero). After identifying the learners' levels in that year, I then compared their levels between the two years, so I could identify whether the learner had progressed, maintained or regressed. This will be discussed in detail in Chapter 5, the learner's performance from grade 9 to grade 10.

In order to get the first set of data I needed to analyse the learners' annual test scripts. In the process of analysing them I had two objectives: firstly, to identify the learners' current level under the ICCAMS framework, comparing them to their previous grade ICCAMS level. This was coded from the WMC-S annual test marker, and in order to ensure the coding I had recoded all the scripts. This was to identify possible shifting of levels in the learners' performance, which would help me to answer my first research question.

The second objective was the total number of correct answers in the ICCAMS to see whether the learner made fewer errors compared than in their previous grade performance. This has helped me to identify the learners who had improved in the number of correct responses but maintained in ICCAMS level, for the second phase of the study. Moreover, comparing their number of total correct answers would highlight the improvements that the learner may have made, while remaining in the same level. The reason why I chose to compare the correct answers was to give learners a percentage of correct responses for the ICCAMS section, as the ICCAMS only works with levelling the learner according to his responses, which do not fully reflect progression. For example, learners may stay on the same ICCAMS level, (due to the learner not getting enough correct questions for the next level as described above), which may appear to indicate that they have not progressed at all. For when learners make fewer errors on the test we can also refer to it as an improvement in performance. By comparing their correct responses we gain a more detailed view of performance than by simply comparing the levels of ICCAMS. This comparison of the correct answers has not been done before in the WMC-S project. It has be followed by analysing learners' responses at a much deeper level by looking at the common errors in algebra. This is to check whether there is improvement, even though the learner may remain at the same level. The learner making fewer errors, however, is a sign of improvement. This will be discussed further in Chapter 5.

4.5 – Phase two

4.5.1 –Sample

I selected 12 learners who had improved in number of correct response but maintained their level as my interview sample (refer to Chapter 4 for further detail). Of the 12 learners, ten were in level one, one in level two, and one was in level three. With the majority of the learners in level 1, I decided to focus my interview sample on level 1. However, I reduced my sample from ten learners to eight, as two of them had left the school. Eight learners is still an ideal numbers of learners for the interview (two learners for pilot and six learners as part two of the sample).

Even though the number of learners matched my ideal number, the interview did not go as well as planned. One learner was absent due to transport problems. Hence my supervisor and I decided to exclude this learner and only interview seven (two learners for pilot and five for interview as part two of the sample). The two learners from piloting were only used to refine my interview questions, and hence not used for analysis.

After interviewing the two pilot learners my supervisor and I realised that I needed to refine some of my questions from part one of the interviews to avoid misleading questions. For example, in questions 1 and 2 I changed the variable from z to q to avoid the learners misreading 2 for z in their own hand writing, which may have led to an incorrect answer. Another example: in question 5.2 I changed the variables from $k \times m$ to $n \times m$ to avoid the confusion of km as an abbreviation of kilometre.

Moreover, apart from refining questions from part one of the interview I had also to refine the questions from part two of the interview so I could bring the probing into better focus. For example in questions 2 and 3 of part two of the interview I highlighted the expression of $p + 3$ in bold so that the learner would know they needed to add or multiply $p + 3$ as a whole expression, and not as a process of: 'first add or multiply p then $+3'$ (refer to Appendix B part two). The bracket was not considered for highlighting $p + 3$ as a whole expression, due to student use of the bracket as an operation, such as distribution, more than grouping terms.

4.5.2 – Instrument

The ICCAMS instrument is a levelling system that sees levels as assessment markers of learners' responses that satisfy a certain set of criteria at a particular time. Hence in the interview I adapted ICCAMS style of questions based on findings from learners' responses taken from the test scripts. I used these to elicit learners' responses regarding their reasoning in their errors.

This interview involves a task-based instrument, and was designed to be similar to the 2008 CSMS and/or ICCAMS algebra section (refer to Appendix B). I have selected the nature of the questions in ICCAMS based on different levels according to learners' current level, to identify learners that are moving toward the next level. For example, if a learner is on level 1 then I have asked them questions on levels 1 and 2.

This interview consisted of two parts: Part one is to have the written responses from three sections: Section A – arithmetic types of algebra (three questions), Section B – areas and perimeter of shapes (six questions), Section C – more arithmetic types of algebra, but at a higher level (three questions). All the questions in part one were adapted from ICCAMS level 1 and level 2 questions. In part two, questions were designed to elicit learners' reasoning from their responses in part one (refer to Appendix B part two).

In part one of the task-based interview, there was two types of task-based questions. The interviewees were all in level 1 in their previous grade, so I used level 1 and level 2 questions, yet probing differently in order to see how the learner responded to higher-level questions. Using their level 1 questions would help to assess whether they were still on the same level, and using level 2 questions would help to assess whether they are capable of level 2 questions. If the learner were not in level 2 then he/she would make errors, for example in question 2 of part one in the interview, learners were given a level 2 question to simplify $3q + 6y + q$. A learner not on level 2, would give a response of $10qy\ or\ 10y$ as their answer.

All the level 1 and level 2 questions were taken from the ICCAMS instrument, changing the numbers and variables but retaining the format. The reason for adapting the ICCAMS instruments instead of adopting it, was to avoid the impact on the data collection of WMC-S annual assessment. The same annual test was used every year by WMC-S for tracking learners' performance, which was repeated in 2013. Therefore adaptation was necessary.

Table 4.3 – Comparison between original ICCAMS and adapted questions

Original questions	**Adapted questions**
1.1. simplify $2a + 5a$	1. simplify $2z + 6z$
1.4. simplify $2a + 5b + a$	2. simplify $3z + 6y + z$

Table 4.3 shows how the ICCAMS questions were adapted, with the format kept the same, but the coefficients and variables changed. Apart from this minor change, the order of questions was also changed. Tables 3.1 and 3.2 show the similarity of the questions across the two years.

Table 4.4 – Equivalent level 1 questions in two years

2011 and 2012 questions	**2013 questions**
1.1. Simplify $2a + 5a$.	1. Simplify $2q + 6q$.
5.1. $Add\ 4\ onto\ n + 5$	3. Add 6 onto p.
6.1. Find a if $a + 5 = 8$	4. Multiply 6 by p.
8.2. What is the areas of the shape? *Given a rectangle shape with 6 and 10 as the value of the sides.*	5.1 What is the areas of the shape? *Given a rectangle shape with 4 and 12 as the value of the sides.*
9. Work out the perimeter of this shape. *Given a quadrilateral shape with 10, 1, 9, and 2 as the value of the sides.*	6. Work out the perimeter of this shape. *Given a quadrilateral shape with 11, 9, 2, and 6 as the value of the sides.*
10.1. Find the perimeter of the shapes. *Given a Triangle shape with e, e, and e as the value of the sides.*	7.1. Find the perimeter of the shapes. *Given a Triangle shape with k, k, and k as the value of the sides.*

Table 4.4 shows that questions from 2011 and 2012 were very similar to those from 2013. For example, in question 1.1 from 2011 and 2012 the formats are very similar to question 1 from 2013, except that the number and variables are changed. Similarly so for Table 4.5:

Table 4.5 – Equivalent level 2 questions in two years

2011 and 2012 questions	2013 questions
1.4. Simplify $2a + 5b + a$.	2. Simplify $3q + 6y + q$.
7.1. If $u = v + 3$ and $v = 1$, find the value of u.	8. If $h = i + 8$ and $i = 6$, find the value of h.
7.2. If $m = 3n + 1$ and $n = 4$, find the value of m.	9. If $f = 5g + 2$ and $g = 8$, find the value of f.
8.3. What is the area of the shape? *Given a rectangle shape with m and n as the value of the sides.*	5.2 What is the area of the shape? *Given a rectangle shape with m and n as the value of the sides.*
10.2. Find the perimeter of the shapes. *Given a Pentagon shape with h, h, h, h, and t as the value of the sides.*	7.2. Find the perimeter of the shapes. *Given a Pentagon shape with c, c, c, c, and d as the value of the sides.*
10.3. Find the perimeter of the shapes. *Given a Pentagon shape with u, u, 5, 5, and 6 as the value of the sides.*	7.3. Find the perimeter of the shapes. *Given a Pentagon shape with e, e, 7, 7, and 9 as the value of the sides.*

Question 4 in 2013 and question 6.1 in 2011 and 2012 are not equivalent to each other. In 2011 and 2012 question 6.1 learners were asked to determine a if $a + 5 = 8$, whereas in 2013 learners were asked to *multiply* $6\ by\ p$. Thus one question was asked to reverse an equation, while the other was to multiply a number with a variable. The two questions were aligned together, because both them were on the same level. These were the only questions that not to have the same format compared to the previous years. These two questions could only be accurate in comparing for level purpose and would therefore not be used in deeper analysis.

Part two of the interview was intended to elicit learner's reasoning for their responses that were given in part one after their written responses. Thus in designing the probing instrument I needed to include the possibility of the learner getting part one of the interview question correct or incorrect. Hence for each probing question I needed two sub-questions, one for a correct, and the other for an incorrect response.

Of course, each learner would only answer one of the sub-questions. For example, in the probing, question 2 is a follow-up of question 3 from part one of the interview, which contains two sub-questions, in order to follow up from the correct or incorrect responses. The follow-up question for the correct responses is labelled 2.1, and that for incorrect response 2.2. Thus each learner answered only one of the sub-questions. However, except in the probing question 4, learners were required to answer both sub-questions, due to it being designed to see the difference between positive and negative numbers. For example, in the probing question 4.1 learners were asked: *if* $h = i - 7$ *and* $i = 6$*, then find the value of* h. Similarly, in the probing question 4.2 learners were asked, *if* $h = i - 7$ *and* $i = -9$*, find the value of* h. The questions have the same format, but were set to see the difference between positive and negative value.

In order to represent these sub-questions more easily, I used a tree diagram for each of the probing questions to present the sub-questions (refer to Appendix B part two). In order to differentiate visually between the two different sub-questions I used different lines in the tree diagram, for example, a solid line to represent the probing question for correct response in part one of the interview, and a dotted line for that for the incorrect response.

When a learner got a right answer for the probing question I would ask a similar question at a more difficult level to confirm that he/she really was capable of doing that type of question, or if it was simply embedded with other errors. Similarly for incorrect answers, I would ask other similar questions at an easier level to confirm if the learners really did not understand the concept, or if it was just a slip. For example, when Lebo gave the correct response for question 9 in part one: *find the value of* f *if* $f = 5g + 2$ *and* $g = 8$*,* I asked him a similar question but at a more difficult level: *find the value of* f *if* $f = 5(g + 2)$ *and* $g = 8$. Likewise, when Mpho gave the incorrect response for question 9 in part one, I asked her a similar question but at an easier level: *find the value of* f *if* $f = 3 + g - 1$ *and* $g = 8$. In these probing questions the aim is to probe the learner's responses in arithmetic types of algebra (refer to Appendix B part two). The probing questions also include questions involving area and perimeter of shapes, but these were not included in the analysis, since area and perimeter were not the focus of the study.

After interviewing the two pilot learners my supervisor and I realised that I needed to refine some of the questions from part one of the interviews to avoid misleading questions. For example, in questions 1 and 2 I changed the variable from z to q to avert the possibility of the learners misreading 2 for z in their own handwriting. Another example: in question 5.2 I changed the variables from $k \times m$ to $n \times m$ to avoid the confusion of km as an abbreviation of kilometre.

Moreover, apart from refining questions from part one of the interview, I also had to refine the questions from part two in order to bring the probing into better focus. For example, in questions 2 and 3 from part two, I highlighted the expression of $p + 3$ in bold so that the learner would know to add or multiply $p + 3$ as a whole expression, and not by a process of first add or multiply $p,$ and then $+3$ (refer to Appendix B part two).

4.5.3 – Methodology

For accurate analysis I adapted the coding scheme from 2012 for my part one of the interview questions (refer to Appendix G). The format of the interview questions was the same as 2012 WMC-S ICCAMS section of the annual test, and the coding scheme for 2013 was very similar to that of 2012. Hence I decided to adapt the 2012 coding scheme to code 2013's data.

For example, in 2012 the coding scheme for question one was $2a + 5a$:

Table 4.6 – Coding scheme for question one 2012

Learner's responses	Numbers of codes	Meaning of the codes
7a	1	Correct response
7	3	Letter evaluated
8a	8	Premature closure
$7a^2$	9a	Algebraic error
Other incorrect response	9b	Other error

Similarly in 2013 the coding scheme for question two was $2q + 6q$:

Table 4.7 – Coding scheme for question two 2013

Learner's responses	Numbers of codes	Meaning of the codes
8q	1	Correct response
8	3	Letter evaluated
9q	8	Premature closure
$8q^2$	9a	Algebraic error
Other incorrect response	9b	Other error

From this example it can be seen that the 2013 coding scheme is the same as for 2012, with that the coefficients and variables were adapted according to the question. This was done in every question in 2013 part one.

In the analysis of the interview, in order to see the learners' progression at a detailed level, responses for each question over the three-year period needed to be compared. Even though the 2013 questions were not exactly the same as in previous years, the questions were adapted from them, so were similar. They were almost identical except that the numbers or letters were changed. Thus it was still possible to compare their responses to the various kinds of questions to see whether the learner had progressed over the period of three years. Table 6.2 (see Chapter 6 in section 6.3) aligns the same kinds of question across the three years, and indicates which question from 2013 was an equivalent question to 2011 and 2012 for easier comparison. Then, with learners who still have not progressed, we can refer to their reasoning and identify the obstacle to progression.

4.5.4 – Data Collection

The interviews were conducted individually. Each learner was given a maximum of 15 min to answer part one, with black ink, and I coded their responses with red ink on a separate piece of paper immediately afterwards. I deliberately used the coding system to identify their incorrect response so as to avoid the learner seeing that their responses were incorrect, which might affect their reasoning. For each of the questions learners were asked to explain how they derived their response. Depending on their responses I had refer to the tree diagram of probing questions. Thereafter I asked the learner to write down their responses to my probing questions in green ink, and then I asked the learner to explain how they derived their response. The written questions were adapted from the WMC-S annual test, which has helped me to identify the learner current level. This has answered my second research question of ‘do they progress to higher ICCAMS level from grade 10 to grade 11.’ Moreover, throughout the interview I was constantly probing learners about their responses, which has helped me to understand their reasoning behind their answer. This has answered my third research question of the nature of the errors they make.

4.6 – Ethics consideration

In this research two ethical clearances were required, one for learners’ written scripts and the other for the interviews. Since the learners’ written scripts were part of WMC-S project data, ethics clearance had already been obtained and there was no need to reapply. Whereas the ethics clearance for the interview was required and was approved (Protocol number: 2013113M). After ethic clearance had been obtained, I approached the school for permission to conduct the research. Two letters were sent out. The first was the ‘Letter of Information’ sheet regarding my research which was sent to the school principal (refer to Appendix C), participants’ parents/guardians (refer to Appendix D), the participants themselves (refer to Appendix E), and a permission form to the learners for audio recording of the interviews (refer to Appendix F). The permission form informs them about the research and undertakes to keep their information confidential and anonymous. Furthermore, I also sent out consent letters to the participants in the research and permission for audio recording of the interviews (refer to Appendix G). I have ensured that the information collected through document WMC-S is stored and locked in a safe place. Pseudonyms have been used in place of learner names in audio recordings, to protect the learners' identity. In terms of the ethical behaviour during the interview, learner identity was assured not to be revealed throughout the interview. No real name were used in the audio recording, and no physical action was mentioned that could identify the learner. Furthermore, with learners not performing at grade 11 level, in the interview I did not embarrass the learner by questioning them further after they admitted not knowing the answer. When the learner had a silent moment, I would rephrase my question, and if still no answer was forthcoming I refrained from pushing further.

4.7 – Reliability and Validity

As Bell (2005) has mentioned, an unreliable item is always invalid, but an item that is reliable is not necessarily valid. ICCAMS is widely recognised as a reliable and valid instrument, together with its coding scheme. Since it was designed for the English context, it was necessary to check its validity in the South African context. WMC-S piloted the annual test in 2010, and the results suggested the test was valid in South Africa too.

Creswell (2009) describes qualitative reliability as 'indicating that the researcher's approach is consistent across different researchers and different projects' (p.190). The detailed coding scheme promotes reliability across projects. As noted earlier, I recoded all scripts using the 2012 coding scheme. This ensured reliability across the grade 9 and 10 scripts. Where there was a conflict in coding between my code and the original coder, I took advice from WMC-S management to resolve the matter.

As Bell (2005) mentioned, reliability is 'the extent to which a test or procedure produces similar results under constant conditions on all occasions' (p.117). In this study both annual tests (grade 9 and grade 10) had the same procedure and questions; each learner was issued the identical test (except for the curriculum items which were grade specific), with the same set of instructions. In grade 11 learners did not write a test and the interview was a one-on-one interaction, but it was comparable to the tests of grade 9 and grade 10. Even though the procedures in grade 11 were different from grade 9 and grade 10, the questions were adapted based on the annual test and thus it is comparable. Having different procedures does have the disadvantage of only being able to compare indirectly, but it has a some advantage for the study, as it deals with learners' errors at a more detailed level. To do so, an interview is the best way to access learners' thinking at a deeper level since the test did not elicit some of the responses that learners gave as a result of my probing in the interviews.

According to Creswell (2009) qualitative validity is upheld when 'the researcher checks for accuracy of the findings by employing certain procedures' (p.160). This was achieved by recoding learners' test responses.

WMC-S used data collection instruments such as learner's written work. In addition to that I also interviewed the learners. I ensured validity by collecting data from different sources, such as learners' written responses and verbal responses, notes from the interview, and audio recording. Learners' written responses provide physical evidence of what learners were doing, but not the errors that are embedded in the learner's mind, hence my use of notes during the interview. Notes may record all the interesting events that happen during an interview, but it is recorded very briefly. The audio recording is thus valuable in supplementing them. Audio recording allows one to capture all the conversation in the interview. However, it does not capture what the learners are *doing* during the interview, and hence the need for the learner's written response. So, each source has it limitation, but taken together they complement one another.

In order to further ensure validity of the study, I used different strategies suggested by Merriam (1998), such as triangulation and peer examinations. The triangulation strategy is to re-use different sources, investigations and methods to confirm the finding. This strategy was referring to my interview, after given test questions to the learners and then I probed the learner. The performance on the test questions before probing suggested a particular level. Then the probing either confirmed this or not. Counting correct responses is not the same as levels but both indicate some kind of improvement. Thus, I used different methods such as counting correct answers to identify learners' progression, and not only relying on the ICCAMS levels. Peer examination concerns asking colleagues to comment on the findings, this examination appeared during the comparison between different codes across the different coding scheme and recoded results with the original marker. During this comparison, my supervisor, WMC-S members, and my fellow-Masters students have constantly given comments on my findings.

4.8 – Limitations

Time was the severest constraint, as it limits this study's range to a small scale. This study was limited to a single school and involved only 34 learners. Such a sample is too small to be considered generally representative.

Furthermore, the progression of the learners' performance could be caused by a number of extraneous factors, such as different teachers, learning interventions from outside the school, or even family issues.

From analysis of the interview it is impossible to make any statement about why learners jumped levels, such as the learner that jumped from level 1 to level 3. Moreover, from the analysis of the interview I cannot make any statement whether the learners who were clearly on level 2 have progressed to level 3, because my focus of the interview was level 1 and level 2.

4.9 – Summary

In this chapter I have discussed the setting of the study and the reasons for choosing the setting, which includes the willingness of the school, teachers, and learners to participate in WMC-S. I also have discussed the approach and the methodology used to capture the data. I have described the approach from quantitative (34 learners' test scripts) to qualitative (interview of the five learners), which is a mixed method. Including ethics considerations, reliability, validity and limitations in the discussion was important. In the next two chapters I discuss the analysis of my data. Chapter 5 is the analysis of the test scripts and Chapter 6 is the analysis of the interviews.

Chapter 5 – Interpretation and Analysis of Test Scripts

5.1 – Introduction

In this chapter I have used ICCAMS levels of understanding and indicators of progression framework (refer to chapter 2), to describe learners' performance in test and written responses of the interview. Analysis of the written responses of the interview was an important step, as it will indicate the level in grade 11, and determine whether or not the learner progressed over the three years - which answers the first research question (refer to Chapter 1.6). Analysis of the verbal response is an important step in exploring the learners' interpretation, misconceptions and errors in algebra, which answers the second and third research questions (refer to Chapter 1.6).

In order to investigate to what extent there is progression in learners' performance in algebra two sets of data must be analysed: the first set of data is from the ICCAMS section of the annual test and the second set of data is conducted from the interview. The interview sample is an extract from the main sample. Before proceeding to the interview we first need to identify the learners that are progressing from grade 9 to grade 10 from their performance in total number of correct answers, and their ICCAMS levels.

5.2 – Performance: number of correct answers

To highlight the learners' progression on the test I decided to compare the number of code 1 (correct responses) in both grades. To do so, I counted the total number of correct responses for each learner in both grades, and then calculated the percentage of those correct responses out of 33 questions. I then noted the percentage for each learner (Table 5.1).

Table 5.1 – Comparison between Gr9 and Gr 10 performance

- Negative value represents the performance/ level decreased.
- Positive value represents the performance/ level improved.
- Zero value represents the performance/level maintained.

Learner	Gr 9 Correct Responses	Gr 10 Correct responses	Difference	Gr 9 level	Gr 10 level	Difference in level
1	13	26	13	1	4	3
2	10	11	1	1	2	1
3	14	21	7	1	3	2
4	10	19	9	1	2	1
5	10	12	2	1	2	1
6	18	13	-5	2	1	-1
7	13	17	4	2	1	-1
8	7	14	7	1	2	1
9	7	11	4	1	1	0
10	8	11	3	1	1	0
11	15	20	5	1	1	0
12	4	12	8	0	2	2
13	11	12	1	1	1	0
14	9	14	5	0	1	1
15	8	12	4	1	2	1
16	9	12	3	1	2	1
17	8	13	5	1	1	0
18	11	24	13	1	3	2
19	12	11	-1	1	2	1
20	10	12	2	1	1	0
21	15	11	-4	3	1	-2
22	6	11	5	1	1	0
23	10	11	1	1	1	0
24	11	14	3	1	1	0
25	17	23	6	2	3	1
26	19	21	2	3	3	0
27	8	14	6	1	2	1
28	15	20	5	1	3	2
29	12	11	-1	1	2	1
30	14	12	-2	2	0	-2
31	16	18	2	2	2	0
32	17	14	-3	2	2	0
33	12	13	1	1	1	0
34	3	8	5	0	1	1

In order to summarise the data I grouped the learners' percentage of improvement into different ranges (Table 5.2).

Table 5.2 – Summary of the difference in percentage from the tests

	Decreased	**Improved by 1% to 20%**	**Improved by 21% to 40%**
Numbers of learners	6	22	6

Table 5.2 is a summary of comparison between learners' improvement between grade 9 and grade 10, in percentage. This table shows that six learners decreased and 28 who improved. Of those 28 learners, six showed a significant improvement (21% to 40%), and the others an improvement of 1% to 20%. Thus, 28 out of 34 learners improved in terms of getting more correct responses than in their previous years. In an ideal world a learner who has improved in the percentage of correct responses should also progress in his/her level. However, this was not the case, and similarly, the learners who decreased in the percentage of correct responses did not necessarily regress in their level. This will be explained in more detail in the next section.

5.3 – Performance in ICCAMS levels

Comparison of the two grade levels can show us the progression of the learners' levels, which relates to the first research question: 'is there a change in performance level based on the ICCAMS instrument?' For this, I have categorised the learners' performance into three different categories: Progress, Maintain and Regress. If the learner has moved at least one or more levels higher compared to their previous year, I classified the learner as having progressed. Similarly if the learner has moved at least one or more level lower, I classified them as having regressed. Those that remained at the same level as their previous year will be classified as 'maintained'.

Table 5.3 – Summary of progression in ICCAMS level

	Level					**Level**					**Level**					**Level**					**Level**				
Gr 9	0					1					2					3					4				
Gr 10	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
No. of learners		2	1				10	9	3	1	1	2	2	1			1		1						

Table 5.3: Summary of the 34 learners' performance in the number of correct responses.

This table shows the learners' performances in terms of levels from grade 9 to grade 10, and helps to categorize the learners' performance. For example, the second shaded column shows six learners who were levelled as level 2 in grade 9. Then in grade 10 they performed differently: three dropped their level into either level 1 or level 0, two retained the level of previous years, and one learner had improved his/her level into level 3.

- Regressed in level from grade 9 to grade 10: four learners)
- Maintain level from grade 9 to grade 10: 13 learners)
- Improved in level from grade 9 to grade 10: 17 learners)

In table 5.3 the category of the six learners who decreased in percentage of correct responses was included in order to see what impact decreasing in percentage had on their levels. Three of the six learners who dropped between two to three correct responses regressed at least one level, and one of the learners who dropped 3 correct responses maintained his/her level. The other two learners who dropped one correct response progressed from level 1 to level 2. This table shows that a drop in percentage of correct responses does not affect their level of performance. Initially I expected the larger the decrease in percentage of correct responses the more the learner would regress in level. That learners who dropped in percentage of correct responses could maintain or even progress to a higher level calls for further investigation in other research.

5.3.1 – Progress in levels

In order to be considered as having progressed in level, learners need to progress at least one level compared to their grade 9 year. A total of 17 learners progressed. There were three categories of progression: progress by one level, or two, or, surprisingly, three levels (level 1 to level 4).

Learners that progressed by one level: Table 5.1 shows two learners progressing from level 0 to level 1, nine from level 1 to level 2, and one from level 2 to level 3. All these learners progressed one level at a time, which is expected. As Hart mentioned, learners progressed at a level by first achieving success at a lower level.

Then there were five learners who progressed more than one level in a year. All of the five were coded again to confirm their progression of more than one level at a time. The learners who progressed more than one level at a time progressed to the level above within the year. For example, in the process of a learner moving from level 0 in grade 9 to level 2 in grade 10 he also achieved level 1 in grade 10. However, among these five learners were two who progressed unexpectedly, appearing to skip a level. Thus one of the learners progressed from level 1 in grade 9 to level 3 in grade 10, skipping level 2. Similarly, another learner progressed from level 1 in grade 9 to level 4 in grade 10 skipping level 2 and level 3. It would be interesting to interview this learner.

5.3.2 – Maintain and Regress in level

As mentioned above, only seventeen learners progressed, 50 percent of the thirty-four learners did not progress. Of the seventeen who did not progress, there were thirteen who maintained their level and four learners who regressed. Among the thirteen who maintained their level ten learners were in level 1, two in level 2 and one in level 3. As table 4.3 showed most of the thirteen learners who maintained their level had improved in terms of number of correct answers. Interviewing them about why they did not progress will be discussed further in the next section. I excluded the learners who regressed in level from my possible interview sample because this study is focused on progression only.

5.4 – Comparison between the Percentage progression and ICCAMS Level progression

An interesting observation for me was: Table 5.1 shows that 28 out of 34 learners improved in percentage, yet only 17 progressed in the ICCAMS level. Table 5.4 below is a summarized version of Table 3, excluding the four learners who regressed in level.

Table 5.4 – Comparison between percentage and level progression

		Number of correct responses	
		Decreased	Improved
Level	Maintained	1	12
	Progressed	2	15

Table 5.4 shows that seventeen learners progressed in level: fifteen of these were expected, as they improved in number of correct responses. However, two dropped in number of correct responses, yet still progressed in level. Referring these two performances to table 5.1 showed that both dropped one correct response in the test: not a large decrease. Analysis at a more detailed level, by viewing their performance in individual questions, showed they got more levelled questions correct, and unlevelled questions incorrect. Similarly for the learner who dropped in correct answers, but maintained level. He dropped three correct responses, and most of those questions were unlevelled. This suggests that the unlevelled questions affected learners' improvement in number of correct responses. However, this is not my focus of this study, but this is sufficient to show that further research needs to be done to reduce the influence of this factor.

5.5 – Improved in number of correct responses but maintained level

In the remainder of the study I have focused only on the twelve learners who maintained their level. Of particular interest were these questions: Why did not these learners change level from grade 9 to grade 10 as the number of correct responses increased? Did they change their level in grade 11? Is there a certain type of question that the learners are getting wrong?

In order to answer these questions I first isolated the twelve learners that maintained level to make the analysis more focused, and then looked into the individual answers to each question across the two years (refer to Table 5.3). This might indicate why these learners did not change level over the two years, and whether there was a specific question that the learners were getting wrong.

In table 5.5, there are 12 learners, and each had a response for grade 9 and grade 10, so 24 responses for each question. The last row of the table shows the number of incorrect responses out of the 24 responses. Most of the learners maintained level one, and so the high number of incorrect responses in level 3 and level 4 would be expected. However, the high numbers of incorrect responses in level 2 were unexpected. As mentioned before, these are the learners who improved in the number of correct responses in the test, but only maintained their level. All these learners would be expected to have achieved more on level 1, and be moving towards level 2 in grade 10. The high number of specific incorrect responses in Table 5.5 show that the learners are having difficulty with a particular level 2 question, such as question 1.4, question 7.2, or question 10.3. To achieve level 2, learners must get at least four level 2 questions correct, and without these questions correctly answered they could not progress to level 2. Moreover in Table 5.5 I noticed that in the level 2 questions learners were more likely to get grade 10 responses incorrect if their grade 9 responses were also incorrect; especially so with the questions that the learner was finding difficulty with. In the analysis of the interview, to determine whether the learners are still experiencing problems with these questions would thus be significant.

Table 5.5 – Individual responses to the levelled questions for the twelve learners who maintained level

		Level 1						Level 2						Level 3					Level 4				
Question no.		**1.1**	**5.1**	**6.1**	**8.2**	**9**	**10.1**	**1.4**	**7.1**	**7.2**	**8.3**	**10.2**	**10.3**	**1.2**	**1.8**	**3.3**	**5.3**	**10.4**	**1.5**	**2**	**4.2**	**8.4**	**11**
Learners	**Year**																						
9	**2011**	1	1	1	9b	1	9	1	8	9b	9c	9b	9	8	9e	9a	3	9	9f	9e	7b	9	5
	2012	1	1	1	1	1	9	9b	8	9d	9c	9b	9	9b	9e	7a	4b	9	9b	9e	1a	9	5
10	**2011**	9a	1	1	9b	1	1a	9a	1	1	9c	2	9	8a	9e	7a	9	8	9c	8a	7b	9	5
	2012	9a	1	1	1	1	1a	9a	1	1	1a	2	9	8a	9e	7b	4b	9	9c	9d	9	7a	3
11	**2011**	1	1	1	1	1	2	1	1	9a	9c	9b	8c	1a	9c	9a	4b	8	9d	8b	7b	7a	5
	2012	1	1	1	1	1	1a	1	1	1	1a	1a	1c	8a	1	7a	1	1a	9b	8c	7b	7a	5
13	**2011**	1	1	9d	1	9	1a	8a	9b	9d	1a	1a	9	8a	9e	1	8	8	9f	8c	7a	7	1
	2012	9a	1	1	1	1	1a	9c	1	1	1a	9b	8c	8a	9e	9b	8	4	9f	9e	7b	7b	1
17	**2011**	1	1	9a	1	1	1a	9c	0	0	9c	9a	9	1a	9e	9b	9	9	9f	8b	7b	7b	5
	2012	1	9	1	1	1	1a	1	9a	9d	9c	1a	1b	1a	9e	1	9	4	9d	9e	7b	9	5
20	**2011**	1	1	1	9a	1	9	9c	1	9d	0	8a	1b	8a	9e	9	1	3	9f	9d	9	7b	1
	2012	9a	1	1	1	1	1a	9a	9b	9d	1a	1b	1a	8a	9d	7a	4b	8	9c	9d	9	0	5
22	**2011**	1	9	1	9b	1	1a	9e	9b	9d	9c	1a	9	8a	9e	1	9	9	9f	9a	9	9	5
	2012	1	1	1	1	1	1a	9c	1	9d	9a	1b	9	8a	9e	9a	4b	3	9f	9b	7b	7a	5
23	**2011**	1	1	1	9a	1	1a	1	8	9b	0	2	2a	1a	1	9b	3	9	9f	8b	9	7b	0
	2012	1	1	9d	1	1	3	9c	1	9c	1a	9b	9	1a	9e	7a	3	9	9d	9b	7a	7a	1
24	**2011**	1	1	9d	9b	1	1a	9c	1	9d	9c	1a	9	8a	1	7a	4b	1a	9d	9c	7b	9	9
	2012	9a	1	1	9b	1	1a	9c	1	1	1a	1a	1c	8a	9e	7a	4d	9	9a	9c	7b	9	5
26	**2011**	1	9	1	1	1	1a	1	1	1	1a	1a	1a	1a	1	7a	1	9	9a	8a	9	7a	5
	2012	9b	1	1	1	1	1a	1	1	1	1a	1a	1a	1a	9e	7a	1	1a	9a	8c	1a	7a	1
31	**2011**	1	1	1	9a	1	1a	1	1	9d	1a	1a	1a	1a	9e	1	3	4	9f	9e	9	7b	1
	2012	1	1	1	1	1	1a	1	8	9d	1a	1a	1a	1a	1	7a	4d	9	9a	9b	2a	7b	5
33	**2011**	1	1	1	1	1	1a	9c	1	9d	9c	8a	2a	8a	9e	7a	3	3	9c	8a	7b	7b	8
	2012	1	1	1	1	1	1a	8a	9b	9d	1a	1a	9	8a	9e	7b	4b	9	9f	9e	2a	7a	5
Total of incorrect responses		**6**	**3**	**4**	**8**	**1**	**5**	**15**	**9**	**16**	**10**	**8**	**14**	**15**	**19**	**20**	**20**	**21**	**24**	**24**	**22**	**23**	**18**

Table 5.6 – Total number of incorrect responses for level 1 questions across 2011 and 2012

	Level 1					
Year	**1.1**	**5.1**	**6.1**	**8.2**	**9**	**10.1**
2011	1	2	3	7	1	4
2012	5	1	1	1	0	1

A further interesting observation from Table 5.6 is that even though all these learners progressed to level 1, there were a number of incorrect responses in level 1 questions. These learners only just met the minimum requirement of level 1, and had not fully achieved on level 1, i.e. not getting all the level 1 questions correct. Table 5.6 suggests that the learners are experiencing particular difficulty with certain questions, which presented an obstacle to fully achieving level 1. Table 5.6 showed question 1.1 to be that obstacle question, as the incorrect responses to this questions came mainly from the grade 10 response. This means that the learners had particular difficulty with this question in grade 10, either only in grade 10 or in both grade 9 and grade 10. In question 1.1 learners were asked to *simplify* $2a + 5a$, and all but one of the incorrect responses were coded as 9a. This meant that learners gave $7a^2$ as an answer: a conjoining error. Similarly, the learner who was coded as 9b gave an answer of $8a^2$. This could be caused by the new learning of exponent in grade 10: when two identical variables are multiplied together, the exponents must be added. Question 8.2 had a higher number of incorrect responses than question 1.1 but it is not considered an obstacle question, as the responses were mainly from grade 9, and corrected grade 10.

5.6 – Conclusion

In this chapter I have identified three types of performance in the number of correct responses and the ICCAMS level. Learners could either drop, maintain, or improve in their number of correct responses, and so too for their ICCAMS level. A key finding in this chapter is that a learner could get more answers correct but still be dropped a level. However, this study is focused on progression in terms of level, hence dropping or regression in performance was eliminated. Furthermore, the majority of learners who improved in the number of correct responses did progress by least one level. However, there were nearly as many (about a third of my data from phase one) who got an increase in number of correct responses yet maintained the same level. Thus I decided to focus for the remainder of the study on learners who improved in the number of correct responses and maintained the same level. Moreover, for these learners there are certain level 1 and level 2 questions that cause particular difficulty, which will be discussed in Chapter 6.

Chapter 6 – Interpretation and Analysis of Interview (Phase two of Data)

6.1 – Introduction

In the previous chapter we discussed 12 learners who improved in number of correct responses, but did not progress to the next level. Of those 12 learners, ten were in level 1. Again, of those ten only seven learners were available for interview, for various reasons. Of the seven learners, five were used for analysis and two learners for piloting. The focus of the analysis in part one of the interview is to determine whether the learner has progressed in terms of level; three of the five learners had done so. The focus in part two of the interview was to determine the obstacle(s) to progression in the case of the two learners who did not progress.

In the analysis in this Chapter, I determined the learners' level in their grade 11 year, and I also investigated the obstacle(e) to progressing to higher level. In order to determine the learners' level I have used the data from part one of the interview, followed by questioning to determine if the learners' response has correct reasoning behind it. This will be discussed in section 6.3. However, during the probing of the learners' reasoning, most of the learners changed their initial response; this will be discussed in section 6.2. Furthermore, I used part two of the interview to elicit learners' reasoning from part one, to determine their obstacle in progression. This will be discussed in detail in section 6.4.

6.2 – Changing responses in part one of the interview

During the probing of the learners' reasoning, most of the learners were constantly changing their written or verbal responses. Fortunately the changes did not influence the analysis of the verbal responses, since their purpose was to elicit factors that may account for any changes and their common errors. The changes were captured while analysing the verbal responses. On the other hand, the changing of written responses had a great impact on analysis of the data, since some of the learners had progressed to the next level after changing their responses. Table 5.6 shows the responses that the learners changed during part two of the interview. For example, Lebo changed 3 incorrect answers to correct, as a result of which he achieved level 2. Another example: Mpho changed from three correct responses to four in level two questions, which meet the minimum requirement for being in level 2. As for Simon, even though he did not change level, his getting more level 2 questions correct indicates a progression compared to his previous year.

Table 6.1 – The effect of changing responses in part one of the interview

Level of the question		**Level 1**						**Level 2**						**Learner's Final Level**
Learners' Name	**Question number 2013**	**1**	**3**	**4**	**5.1**	**6**	**7.1**	**2**	**8**	**9**	**5.2**	**7.2**	**7.3**	
Lebo	**Before**	1	1	9b	9b	1	1a	9c	1	1	1a	1	9	2
	After			1				1					1	2
Mpho	**Before**	1	1	1	9b	1	1a	1	8	9d	9c	1a	1a	1
	After								9b	1				2
Simon	**Before**	1	1	1	1	1	1a	9c	8	9d	1a	1b	9	1
	After							9c		1				1

- **Before** status is the initial response that the learner gave.
- **After** status is the final response that the learner gave.

During part two of the interview, Lebo, Mpho and Simon changed their written responses from part one of the interview. The reason for changing was either 'I didn't see', or 'I didn't read the instruction', or simply 'I made a mistake'. It seems that the learners either realized their mistake or that they thought more deeply when re-addressing the question. Lebo is a good example of a learner realizing his mistake when re-addressing the question. When he did so he immediately admitted his mistake and corrected it before I had begun to question him (refer to the transcription below).

Transcription of Lebo's interview for question 2 and question 7.3	
Speaker	***Utterance***
In the interview, for Question 2: Simplify $3q + 6y + q$, Lebo gave an answer of $10q$.	
King	*In question 2 you said $3q$ plus $6y$ and plus q is equal to $10q$ so could you explain how did you get $10q$?*
Lebo	*No…no I did a mistake I didn't see the right here (pointed at the y). It should be $3q$ plus a q is $4q$ and plus $6y$ on a side.*
Later in the interview, for question 7.3, Given a Pentagon shape with $c, c, c, c,$ and d as the value of the sides. Lebo gave an answer of $2e + 21$.	
King	*This question over here (I pointed at question 7.3) could you explain to me how did you get this (pointed at the answer)?*
Lebo	*Okay from here (pointed at 7.3 picture), I got two e and two 7 and 9. And then right here I said err… right here I say two e together will be $2e$ and here I said 7 plus 7 which is equal to 14 and which mean here I did a mistake and I put 7 here. I suppose to put 9. So err… that will be giving me an answer of 23. So it is supposed to be $2e$ plus 23, so I did a mistake.*

By contrast, Mpho and Simon first tried to convince me that his/her answer was correct, and only when they saw that their responses contradicted their explanation did they change their responses (refer to the transcription below).

Transcription of Mpho's interview for question 9	
Speaker	***Utterance***
In the interview, for Question 9, If $f = 5g + 2$ and $g = 8$, find the value of f. Mpho first gave an answer of 7 and then changed to a 42.	
King	*Could you explain what did you did over here (I pointed at question 9)?*
Mpho	*Okay the equation was equation was this (pointed at $f = 5g + 2$) so I divided the…this (pointed at 5g+2) by g. Then I remain with 5+2 which is 7.*
King	*Where did the division come from?*
Mpho	*I thought is better to do like that?*
King	*Why do you think is better?*
Mpho	*So I can remain this here (pointed at the h).*
King	*But why do you need to remove the letter g?*
Mpho	*When finding an unknown we must only focus on one letter.*
King	*Okay then what must we do with this information (pointed at g=8)?*
Mpho	*It tells us g is equal to 8…hmmm…okay…I think I am wrong.*
King	*Okay, why do you think you are wrong?*
Mpho	*To get this (pointed at 5g) we take 5 times g so g equal 8, then is 5 times 8. Which is 40 and then plus 2.*

Similar in Simon response in the interview, for question 9, If $f = 5g + 2$ and $g = 8$, find the value of f. First Simon gave an answer of 40 then changed the answer to 42.	
King	*Could you explain to me how did you get this (pointed at his answer)?*
Simon	*I wrote in my data, 5g plus 2 and g is equal to 8 and f is a question mark like I said before to show that I am looking…then I multiply 5 times 8 and I got 40.*
King	*But what happened to the 2?*
Simon	*5g plus 2…if f is equal to $5g + 2$ …and…g…I did not write the 2.*
King	*So is the answer 40?*
Simon	*No, it will be 40 plus 2 so is 42.*

I believe Lebo's mistake was cause by rushing through the question, which Olivier (1989) would refer it as a 'slip', since it did not happened systematically. In the case of Mpho and Simon changing their responses, by contrast, I would argue that they were just using the word 'mistake' as an excuse for the contradiction between their response and reasoning. Olivier (1989) would refer this contradiction as a 'misconception'. Stacey & MacGregor (1997) mention that misconception occurs when learners interpret new ideas with their experience. Thus I believe that there might be new ideas of some kind that have entered their schema, which they are trying to accommodate.

In terms of changing responses, it is interesting to see Mpho and Simon only changed the responses from a level 2 question, whereas in the level 1 question they were very sure that the answer they gave was the final answer, and most of the time they were correct. I believe this happened because these learners were fully in level 1, hence they are very sure about their responses, but were still progressing toward level 2, and so were unsure about the level 2 question. Piaget (1964), Olivier (1989) & Hatano (1996) would argue that these learners are in the process of accommodating (refer back to chapter 2.5) level 2 questions, which makes them less comfortable about the way of viewing letter that requires. This might be an indication of learners that are still in the process of progressing from level 1 to level 2.

6.3 – Levelling learners' response using ICCAMS coding scheme

The analysis of part one in the interview was used to determine learners' level, achieved by levelling the learners according to the number of correct responses (refer to section 4.4.3). Next I compared the learners' responses with those of their previous year. This comparing process of the analysis in part one of the interview is similar to the analysis in Chapter 4. The analysis of part one of the interview answered my second research question, which is: 'do those who do not progress in level progress to a higher ICCAMS level from grade 10 to grade 11?' The focus of the analysis in the part two of the interview is to determine learners' reasoning behind their responses. The analysis of part two of the interview answered the third research questions, which is: 'what errors do those who still do not progress make which hinder their progression?'

As mentioned in chapter 4, in order to compare learners' responses with those of their previous year I need to align the equivalent questions (refer to 4.5.2). After aligning the questions from different years that are equivalent to each other, I coded learners' grade 11 responses with the adapted version of the ICCAMS coding scheme (refer to Appendix G). Table 6.2 below shows the codes from the learners' responses across the three years. Using this table, I can start comparing the learner's specific responses for the specific type of questions across the three years. I categorised my interviewed learners into two groups: those that progressed in level, and the group that progressed in the total number of correct responses. This made it possible to see the difference between the two types of progression.

In Table 6.2 Lebo, Thandi, and Wazi had progressed into level 2. Lebo is a good example of a learner that progressed from level one to level two. Even though he got one more incorrect response in a level 1 question compared to his previous years, the level 2 question showed a very good progression. Consider, for example, his level 2 question performance: in grade 9 Lebo attempted only four of six questions but only got two correct. In grade 10 he attempted all six questions but only got three correct, and in grade 11 he progressed from three correct responses to five, which qualifies him for level two. Another good example of level 2 progression: even though Thandi did not progress the level every year, every year gives indications of moving towards level 2. For example, in grade 9 this learner only had one correct response on level 2, and in grade 10 she got three. In grade 11 she got five correct responses, indicating a good progression across the three years: from one correct response to five correct responses. As mentioned in the previous chapter, progress in level or response are expected as they progress in grades. An example of what was unexpected was the progression of Wazi. Normally when a learner progress he/she would achieve more and more correct responses every year. However in Wazi case, she first regressed and then progressed. In grade 9 this learner got two correct responses, and in grade 10 she got none, and then in grade 11 suddenly showed a steep progression.

Lebo, Thandi, and Wazi reached the minimum requirement of being level 2, and thus they should be able to explain correctly how the solutions were derived. I, therefore considered the questions which produced the highest rate of incorrect level 2 responses from Table 5.4 in the previous Chapter. There were two: question 7.2 and question 10.3 from grade 9 and grade 10, equivalent to question 9 and question 7.3 from grade 11. Question 1.4 from grade 9 and grade 10 was excluded since they were different from the question in grade 11.

Table 6.2 – Learners' performance across the three years

Learners Name		**Level 1**						**Level 2**						**Learner's Final Level**
	Question number 2011&2012	**1.1**	**5.1**	**6.1**	**8.2**	**9**	**10.1**	**1.4**	**7.1**	**7.2**	**8.3**	**10.2**	**10.3**	
	Question number 2013	**1**	**3**	**4**	**5.1**	**6**	**7.1**	**2**	**8**	**9**	**5.2**	**7.2**	**7.3**	
	Year of response													
Lebo	**2011**	1	1	9a	1	1	1a	9c	0	0	9c	9a	9	1
	2012	1	9	1	1	1	1a	1	9a	9d	9c	1a	1b	1
	2013	1	1	1	9b	1	1a	1	1	1	1a	1	1	2
Thandi	**2011**	1	1	1	1	1	1a	9c	1	9d	9c	8a	2a	1
	2012	1	1	1	1	1	1a	8a	9b	9d	1a	1a	1b	1
	2013	1	1	1	1	1	1a	1	1	9d	1a	1a	1a	2
Wazi	**2011**	1	1	1	1	1	2	1	1	9a	9c	9b	8c	1
	2012	1	1	1	1	1	1a	8a	8	9d	9c	9a	9	1
	2013	1	1	1	1	1	1a	1	1	1	1a	1a	1a	2
Mpho	**2011**	1	1	1	9b	1	9	1	8	9b	9c	9b	9	1
	2012	1	1	1	1	1	9	9b	8	9d	9c	9b	9	1
	2013	1	1	1	9b	1	1a	1	9b	1	9c	1a	1a	1
Simon	**2011**	9b	1	1	0	1	9	9c	1	9b	0	9a	9	1
	2012	9a	1	1	1	1	1a	9a	1	9e	1a	9a	8c	1
	2013	1	1	1	1	1	1a	9c	8	1	1a	1b	9	1

Transcription of Lebo's interview for question 7.3 and question 9	
Speaker	***Utterance***
In the interview, for question 7.3, given a pentagon shape with $e, e, 7, 7, and\ 9$ as the value of the side. Lebo gave an answer of $2e + 21$.	
King	*This question over here (I pointed at question 7.3) could you explain to me how did you get this (pointed at the answer)?*
Lebo	*Okay from here (pointed at 7.3 picture), I got two e and two 7 and 9. And then right here I said err…right here I say two e together will be $2e$ and here I said 7 plus 7 which is equal to 14 and which mean here I did a mistake and I put 7 here. I suppose to put 9. So err…that will be giving me an answer of 23. So it is supposed to be $2e$ plus 23, so I did a mistake.*
Later on in the interview, for question 9, If $f = 5g + 2$ and $g = 8$, find the value of f. Lebo gave an answer of 42.	
King	*How did you get this answer?*
Lebo	*I replaced g with 8 and then times 5 and 8, which is 40 and then plus the 2. It would give me 42.*
King	*Okay what if I changed the question into f equal to 5 open bracket g plus 2 close bracket and g is equal to 8, how will you solve it? (I wrote the equation of $f = 5(g + 2) and\ g = 8$).*
Lebo	*As I was telling you earlier, this is factorizing so I would factorized 5 times g is 5g. Then 5 times 2 is plus 10 and then…Oooo no no no…Okay they said g is equal to 8 right? So in this g I replaced 8 and then I would show it 5, 8, and plus…errr…2. Here like this and then I would say errr…5 times 8 which is 40 and then 5 times 2 which is 10. So with this (pointed at number 40 and number 10) would give me 50.*

Although Lebo got question 7.3 incorrect the first time, from his response we could see he corrected it immediately, while he was explaining his response. He knew that only like terms can be added together. Moreover in the probing question of question 9, apart from using the word 'factorising' incorrectly, Lebo was able to solve the question by evaluating the letter and not using the letter or using the letter as an object in a complex structure. Thus I am convinced that he is a level 2. Moreover with him getting six level 2's correct, he has clearly achieved level 2 even though he did not give six correct responses for level 1 questions.

Transcription of Thandi's interview for question 7.3 and question 9.	
Speaker	***Utterance***
In the interview, for question 7.3, given a pentagon shape with $e, e, 7, 7, and\ 9$ as the value of the side. Thandi gave an answer of $2e + 23$.	
King	*Okay what about question 7.3?*
Thandi	*These are numbers (pointed at the constants) and these are letters (pointed at variable e). So I say 2e and I added 7 plus 7 and plus 9.*
Later on in the interview, for question 9, If $f = 5g + 2$ and $g = 8$, find the value of f. Thandi gave an answer of 47.	
King	*Okay and this question here (pointed at question 9)?*

Thandi	*You take 8 and you put it here (pointed at the letter g) and times 5 and then add 2.*
King	*Okay what is 8 times 5?*
Thandi	*45*
King	*Okay what if I changed the equation to f is equal to 3g minus 1, and g will still be 8? How will you solve it?*
Thandi	*Errrr…it will be… it will be…it will be 23 because 3 times 8 is 24 and minus 1.*
King	*Okay can you write it down?*
Thandi	*Okay*
King	*Okay what if I changed the equation to f is equal to 3 plus g minus 1, and g is still 8? Can you solve f for me?*
Thandi	*Yes, f will be equal to 3 plus 8 minus 1 which is equal to 11 and minus 1 which is 10.*

In question 7.3 Thandi was able solve the question without evaluating the letter e, and she knew that numbers and letters cannot be added together. Although she got question 9 incorrect, it was caused by incorrect multiplication. This can be confirmed by the first probing question, which was almost identical to question 9, and she got it correct. This shows that Thandi had no problem in the structure of the question, though might have problems with multiplying with larger values, such as 5 times 8. In the second probing question she had a good understanding of solving this type of question in a different structure.

Transcription of Wazi's interview for question 7.3 and question 9.	
Speaker	***Utterance***
In the interview, for question 7.3, given a pentagon shape with $e, e, 7, 7, and\ 9$ as the value of the side. Thandi gave an answer of $2e + 23$.	
King	*Okay and this one over here? (pointed at question 7.3)*
Wazi	*Well, they are the same (pointed at the letters e) so you add them together and you add the numbers.*
King	*Okay great! But can we add numbers and letters together?*
Wazi	*No, because they are not the same.*
Later on in the interview, for question 9, If $f = 5g + 2$ and $g = 8$, find the value of f. Wazi gave an answer of 42.	
King	*Okay and this question here (pointed at question 9)?*
Wazi	*You putted 8 for g and then times the 5, then add the 2.*
King	*Okay what if I changed the equation over here to, f is equal to open bracket g plus 2 and g is still 8? Can you solve f for me?*
Wazi	*Let me…(start solving the variable on the paper)*
King	*Okay so did you substituted the 8 in the letter and then solve the bracket?*
Wazi	*Yes, and then I times 5.*

Wazi’s response to question 7.3 showed that she knew that numbers and letters are not the same, hence cannot be added together. That shows that she understood that letters are an unknown. In both questions, question 9 and the probing question, it has been shown that Wazi was able to solve the question by evaluating the letter and not using the letter or using the letter as an object in a complex structure, which indicates that she achieved level 2. Moreover she had all 12 responses correct for levels 1 and 2, which mean she had fully achieved levels 1 and 2.

The analysis showed that Lebo, Thandi, and Wazi were all in level 2, and most of them got all the level 1 and level 2 questions correct. Thus there is no need for me to discuss their response further, and so for the remainder of the study I have focused on Mpho’s and Simon’s responses.

Table 6.2 showed that Mpho and Simon progressed in the total number of correct responses. Simon is a good example of progression in total number of correct responses in level 1 and 2 questions. In grade 9 he got only four correct responses out of 12 questions. In the following year he increased to seven, and in grade 11 he further improved to eight correct responses. Even though Simon was not in level 2 yet, he showed a very good progression by increasing the total number of correct responses. Another example of progression in the total number: in grade 9 Mpho got five correct responses out of 12 questions, and in grade 11 he improved to eight. Although from grade 9 to grade 10 she did not make a progression in terms of the total number of correct responses in level 2, she nonetheless was more competent in level 1, rather than just meeting the requirement of being in level 1. Although she had just reached the minimum requirement as a level 2, her response consisted of many errors, so it does not seem that she had achieved level 2 yet. Mpho’s error will be discussed with Simon’s, because both have some similar errors. These are discussed in the next section.

6.4 – Eliciting Mpho’s and Simon’s progression obstacles

In this section I discuss the errors that appeared from Mpho’s and Simon’s interview. Although identifying errors was not a part of the research questions, it is important to identify these errors, as they are obstacles to progression. In order to identify the error I have focused on the questions that learners got incorrect. According to Table 5.6, after Mpho and Simon changed their answers, it appears that most of the incorrect responses were from the level 2 questions. Hence I analysed each of their incorrect level 2 questions, in the course of which some unexpected reasoning from level 1 questions came to light. The discussion deals first with individual, and then common, errors.

Mpho had achieved level 1 and the minimum of level 2. She only had two level 2 questions incorrect, (questions 8 and 9). Both of the transcripts for question 8 and also the probing question showed integer errors. In question 9 and its probing question showed an error in equations.

Transcription of Mpho's interview for question 8.	
Speaker	***Utterance***
In the interview for question 8, If $h = i + 8$ and $i = 6$, find the value of h Mpho gave an answer of 2 and then later changed to -2	
King	*How did you get your answer? Why did you changed $h = i + 8$ to $h + i = 8$?*
Mpho	*In order to solve the equation we must take all the letter to one side.*
King	*Okay, if you put all the letter on one side, then why the sign remain the same?*
Mpho	*Mmm … I don't know… (She then quickly gave the second solution).*
King	*Okay, then how did you get negative 2 in your final answer?*
Mpho	*Because of the sign rule.*
King	*What sign rule?*
Mpho	*When there is a negative with negative is positive; when there is a positive with a positive is positive.*
King	*Okay, what if there is a positive with a negative or a negative with a positive?*
Mpho	*It will be a negative.*
I then moved onto the first probing question: find the value of h if $h = i - 7$ and $i = 6$, her response was -1.	
King	*How did you get negative 1?*
Mpho	*You replace the i with a 6. So you will have 6 minus 7, which is negative 1*
King	*Okay, so you didn't changed the letter to the other side?*
Mpho	*No I just replace the letter with that number (pointed at number 6) and solve it.*
I then moved onto the second probing question: find the value of h if $h = i - 7$ and $i = -9$, her response was 2.	
King	*Okay, how did you get this answer (pointed at her final answer)?*
Mpho	*I did the same for this question (pointed at the previous probing question). I replaced this letter (pointed at i) with negative 9, and then get the answer.*
King	*Okay, and is negative 9 minus 7 a positive 2?*
Mpho	*Yes, because negative and a negative is a positive.*

Mpho's response to question 8 was significant. By just viewing her final answer across grade 9 and grade 10 it was difficult to identify the change in the kinds of error, especially as the final answer is given with no working out. In grade 11 Mpho happened to show her full working out, which showed that she substituted the value in the letter correctly. However, for some unknown reason, she changed the whole equation while solving it. Thus the error seems to have persisted from grade 9 to grade 11.

Figure 6.4.1

A **B**

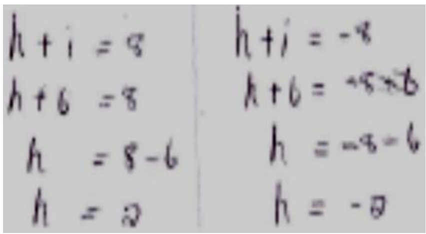
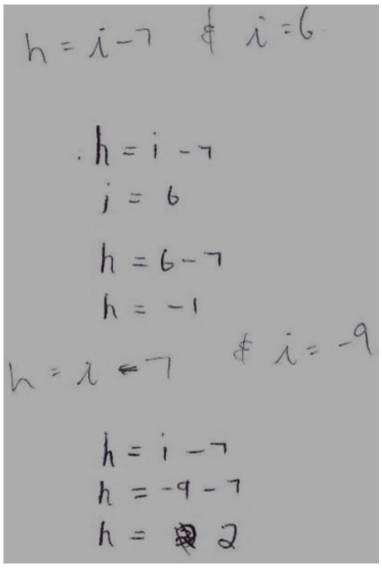


Figure 6.4.1 on the left showed Mpho's actual responses.

What grabbed my interest was not that Mpho changed the equation from $h = i + 8$ to $h + i = 8$, but still has difficulties with integers in grade 11.

According to Mpho, "to solve the equation we must take all the letters to one side". This explains why she moved all the variables on one side, but does not explain why the sign of the i stayed the same when changing sides. She moves letters, but does not work with additive inverse. I had a follow-up question about why the sign does not change when changing sides, and she replied "I don't know". Perhaps she was only focusing on putting all the variables on one side (while neglecting that the sign also changes), and only when substituting the value in the variable then realised the sign must change. Another possible reason is I gave her a hint that she was incorrect without changing the sign. Her response also show an integer error.

Figure 6.4.2



In both of the probing questions, Mpho seemed to stop treating $h = i - 7$ as an equation: this could be caused by my questioning technique. Perhaps I questioned in such a way that it give an indication to Mpho that treating $h = i - 7$ as an equation is incorrect. However in her first response Mpho simply substituted the variable and then applied the operation. The correct answer this might be due to the question not involving negative numbers. In the second probing question her response showed an integer problem again. According to her she was following the sign rule, which is an error from minus sign interpretation. It is interesting to see integer error at a grade 11 level, though it is not the focus of my research.

Transcription of Mpho's interview question 9	
Speaker	***Utterance***
In the interview, question 9: If $f = 5g + 2$ and $g = 8$, find the value of f. Mpho first gave an answer of 7 and then changed to a 42.	
King	*Could you explain what did you did over here (I pointed at question 9)?*
Mpho	*Okay the equation was this (pointed at $f = 5g + 2$) so I divided the this (pointed at $5g + 2$) by g. Then I remain with 5+2 which is 7.*
King	*Where did the division come from?*
Mpho	*I thought is better to do like that?*
King	*Why do you think is better?*
Mpho	*So I can remain this here (pointed at the h).*
King	*But why do you need to remove the letter g?*
Mpho	*When finding an unknown we must only focus on one letter.*
King	*Okay then what must we do with this information (pointed at $g = 8$)?*
Mpho	*It tells us g is equal to 8…hmmm…okay…I think I am wrong.*
King	*Okay, why do you think you are wrong?*
Mpho	*To get this (pointed at $5g$) we take 5 times g so g equal 8, then is 5 times 8. Which is 40 and then plus 2.*
I then moved to the first probing question: find the value of f if $f = 3g - 1$ and $g = 8$. Mpho gave an answer of 23.	
King	*Okay now in this question, how did you get this (pointed at her answer)?*
Mpho	*This says $3g$ so I put 8 here (pointed at the g) and then I times and then I minus this one.*
I then moved to the second probing question: find the value of f if $f = 3 + g - 1$ and $g = 8$. Mpho gave an answer of 10.	
King	*And this answer (pointed at her answer)? How did you get it?*
Mpho	*I take the 3 plus this 8 (pointed at the question $g = 8$) and then minus the one.*

In this question 9 no real equation thinking was required, but Mpho seemed to have treated it as equation solving again. In grade 9 she gave an answer of $m = 1$, which is a common response. Learners had often answered the question: $m = 3n + 1$, and $if\ n = 4$, with $m = 4 + 1$, thus $m\ must\ be\ 1$. In grade 10 Mpho gave an answer of $m = 4$, and just by viewing her grade 10 response it would be difficult to identify her error. However, if combined with grade 11 responses the grade 10 error can be explained. In grade 10 Mpho cancelled the 'n' variable and was left with $3 + 1$, Thus the final answer $m = 4$. This *method* is the same as in grade 11, hence in this case, both years were represented by the same code. In her grade 11 response she was trying to cancel the g variables by dividing by g, such as $f = \frac{5g+2}{g}$. She explained that "when finding an unknown we must only focus on one letter", so she cancelled g to focus on the f variable. Later she said "I think I am wrong", and gave the correct answer. This suggests that she does know some of the content. However the probing question confirms whether she had actually mastered this type of question.

There are three probing questions for question 9: when the learner gets question 9 correct he/she was asked the first probing question. Only if learner gave an incorrect response then they will proceed to the second and third probing question. These probing questions are similar to those for question 8: they also requires the learner to substitute the value into the variable and then find the value of g, but it involves more mathematical syntax.

Even though Mpho corrected herself in her explanation of question 9, this could be because my questioning gave her a direction of how to solve it. So I put the second and third probing questions,to her, both of which asked for the value of f, but the structure of the questions were different. In the second probing question was $f = 3g - 1$ and $g = 8$, whereas the third probing question was $f = 3 + g - 1$ and $g = 8$. Applied to Mpho, these probing questions showed she was able to do this type of question. She seemed to have an understanding of this type of question, which means that the error she initially made in question 9 might be just a slip, or she could have learnt her mistake during the questioning.

Moreover besides the integer error in the level 2 question, Mpho was still having difficulty with level 1 questions. She might have given the correct response, but her reasoning was incorrect. For example, in questions 3 and 4 she got both questions correct, but her reasoning was incorrect.

Transcription of Mpho's interview for question 4	
Speaker	***Utterance***
In the interview, in question 4: multiply 6 by p. Mpho gave an answer of 6p.	
King	*Is this p an exponent (pointed at her second step of her answer?*
Mpho	*Yes.*
King	*Is there a difference between 6 times p and 6 to the exponent of p?*
Mpho	*No.*
King	*Okay is 2 times 3 the same as 2 to the power of 3?*
Mpho	*No is not the same.*
King	*Okay if I replace 3 with any letter? For example if I replace 3 with a letter a?*
Mpho	*2 times a and 2 to the power a is the same.*

Figure 6.4.3

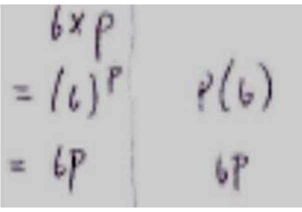


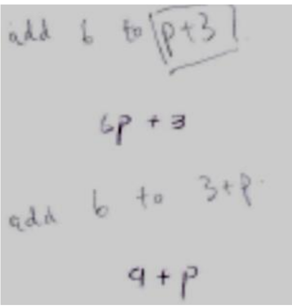
Figure 6.4.3 on the left shows her actual response. Mpho thought that $6 \times p$ can be written in two ways: $(6)^p$ or $p(6)$, and that either way it produces the same answer. Then I asked her: "is 2 times 3 the

same as 2 to the power of 3?", and she said "No, it is not the same". But as soon as I replaced it with letter, she said it is the same. This shows that she can only tell the difference when dealing with numbers, and not variables**.**

Similarly, in question 3, Mpho gave the correct response, but in the probing question her reasoning was incorrect. In question 3 of the interview learners were asked to add 6 to p and she gave the correct answer with the correct reasoning. However, in the probing question, the reasoning she gave was unexpected at a grade 11 level.

Transcription of Mpho's interview for question 3's probing question	
Speaker	***Utterance***
In the interview, in this probing question: add 6 to $p + 3$. Mpho gave an answer of $6p + 3$.	
King	*Okay how did you get $6p + 3$?*
Mpho	*I know number and letter can't add together but $p + 3$ is one term so when we add 6 to this (pointed at $p + 3$), it must be $6p + 3$.*
King	*Can the answer be $p + 9$?*
Mpho	*No.*
King	*Why not $p + 9$?*
Mpho	*I think it depends on the order.*
King	*Okay what if I changed the question to this (I wrote add 6 to 3+p), then what is the answer?*
Mpho	*Then it will be this (she wrote $9 + p$).*

Figure 6.4.4



Mpho's response was "$6p + 3$", an error of 'conjoining'. She explained, "I know number and letter can't add together but $p + 3$ is one term so when we add 6 to it, it must be $6p + 3$". Then I questioned her "why not $p + 9$?" She answered "it depends on the order" of the question. Figure 6.4.4 shows Mpho's example of the order matters.

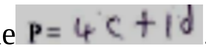
Mpho has achieved the minimum requirement on level 2, but the integer error and the error of equations is preventing her from achieving more level 2 questions correctly. Moreover, even though she has achieved level on level 1, there is still a conjoining error hindering her progression to full level 1 in terms of her reasoning.

Simon had achieved level 1 completely, but not yet level 2, due to questions such as questions 2,7.3 and 8. The transcripts for question 2 and its probing question, showed that

Simon had conjoining errors. Furthermore, the questions 7.3 and 9 showed that my questioning may affect learners' responses. Moreover, the transcript of question 8 showed that Simon also had integer errors.

Transcription of Simon's interview for question 2.	
Speaker	***Utterance***
In the interview, for question 2: Simplify $3q + 6y + q$. Simon gave an answer of $10qy$ and then he changed his response to $10y$.	
King	*How did you get $10qy$?*
Simon	*When two different letters are adding we must put them together.*
King	*So you are saying when two different variables are adding together, you must just put it together.*
Simon	*No, it supposed to be this (changed his answer from $10qy$ to $10y$).*
King	*Okay, how did you get $10y$?*
Simon	*… (didn't reply my question).*
King	*Where did the letter q go?*
Simon	*… (also didn't reply my question).*
I then moved onto the probing question ($Simplfy\ 2x + y + x$), his response was $4x$.	
King	*How did you get $4x$? Why can't we put two variables together with an addition operation?*
Simon	*You can only put it together if is adding a number with a letter.*
King	*Okay, can you explain more?*
Simon	*What I know is I must put all the number and variable together, but two variable can't put it together.*
King	*Okay but why did you dropped out the y variable and not the x variable?*
Simon	*I think it doesn't matter, it can be x or y.*

Simon thought that "when two different letters are adding, we must put them together", which means he has put the variable q and y together to get an answer of $10qy$, a conjoining error. If he meant multiplying when he said "put them together" then he would have multiplied the coefficients as well. However, after a minute or so, he said "No, it is supposed to be this" and changed his answer from $10qy$ to $10y$, but he was unable to explain why, or where the 'q' had disappeared to. It seems that he had ceased the 'conjoining' of two different letters together from an addition operation, but the reason for this change was unknown. This seems different compared to his answers from the previous years. In grade 9 and grade 10, asked a similar question, to *simplify* $2a + 5b + a$, his grade 9 response was $3a + 6b + b$ and in grade 10 his response was $7a^2b$. In grade 9 he increased the a and b variables by one and changed the third term from b to a. It seems that he kept the format of the question, and counted in ascending order. He increased all the coefficients by one, and when there was no coefficient he increased it according to alphabetical order. This shows no conception of like terms. If he had added the a variables because they were like terms, then he would also have added the b variables. In grade 10 he showed the error of 'conjoining' (MacGregor & Stacey, 1997). He combined all the coefficients and variables in the question as his answer. Thus in grade 10 and grade 11 Simon was conjoining, but in a different way.

In the probing question most of the learners answered the same question, because all the learners got question 1 and question 2 correct in the written interview, except Simon. Because Simon only got question 1 correct and question 2 incorrect, his probing questions were different. His question was to *simplify* $2x + y + x$, which requires one to add like terms, and his response was $4x$. This is another example of Simon's conjoining error. In his question 7.2 response he had convinced me that he had stopped 'conjoining' with two letters. For example 

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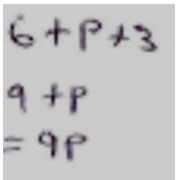
To confirm it I asked "why we can't put two letters together with an addition sign?" he replied, "You can only put it together if is adding a number with a variable". This means Simon has stopped the 'conjoining' with letters, though still conjoins with a number and a letter. When he was asked to explain in more detail, he said, "I must put all the numbers and letters together, but two letters can't put it [be put] together". I asked further, "why did you drop the letter y and not the letter x ?" and he replied "… it doesn't matter … it can be x or y". This shows that his thought in a question consisting of two different letters we must follow the operation, but two letters cannot be combined, so you can drop one of them. This explanation in the probing question may explain why he changed his response from $10qy$ to $10y$. Later on, in the probing question for question 3, learners were asked to $add\ 6\ to\ p$, and then in the probing question they were asked to $add\ 6\ to\ p + 3$.

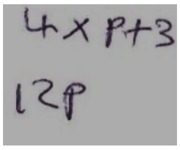
Figure 6.4.5



In Simon's response, his conjoining error appears again. Even though this question has only one variable, Simon's conjoining error can still be seen. He first added the numbers together, and then combined the number with the variable.

A yet further example of Simon conjoining numbers with letters: in question 4 learners were asked to $multiply\ 4\ by\ p$, then in probing question leaner were asked to $multiply\ 4\ by\ p + 3$.

Figure 6.4.6 In his response Simon explained that "because it says $p + 3$, so is $3p$ and I times it by 4, the answer is $12p$". Combining this explanation with that of question 3's probing question, it is certain that Simon is still 'conjoining'.



However, this 'conjoining' only occurs with numbers and letter, and not with letters only.

Transcription of Simon's interview for question 7.3.	
Speaker	***Utterance***
In the interview, for question 7.3: Given a Pentagon shape with e, e, 7, 7, and 9 as the value of the sides. Simon gave an answer of 2e, 24, 9, then later he changed his answer to 2e+23.	
King	*How did you get your answer?*
Simon	*I am not sure what to do here.*
King	*Is there a difference between question 7.2 and question 7.3?*
Simon	*No (then he changed his answer to 2e+23)*
King	*Okay, how did you get 24 in your first answer?*
Simon	*No, it's a mistake. It's 14 not 24.*

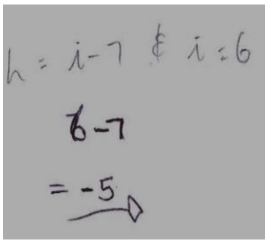
In question 7.3 learners were asked to determine the perimeter of a certain shape, and if the length of each side has variables as well as numbers. Simon's response was $p = 2e, 24, 9$. It seems that he added all the identical sides only, and then listed all the identical sides without operations. However, it does not explain that 24: 24 is not the answer of 7×7 *or* $7 + 7$. Hence I believe this might be a slip. Apart from this 'slip' I was not sure why he left out all the operational signs. I asked him how he obtained his answers, and he replied "Not sure what to do here". I was surprised, because questions 7.2 and 7.3 were almost identical, I asked him "is there a difference between question 7.2 and question 7.3?" and he replied "no" and then immediately gave the correct answer. I am not sure why his answer did not have any operational sign; perhaps my question suggested to him that he was wrong. However, when compared to his previous response from grade 10 there is an improvement. In grade 10 similar questions were asked, and his response was "number with number and letters with letters added separately, and then combine the numbers with variables together". For example, $P = 6 + 5 + 5 + 2u$, which is equal to $16 + 2u$, and his final answer was $18u$. Even though probing area and perimeter it is not within my focus, this question shows that my questioning may affect learner's final responses.

Transcription of Simon's interview for question 8.	
Speaker	***Utterance***
In the interview, for question 8: find the value of h if $h = i + 8$ and $i = 6$. Simon gave an answer of 2.	
King	*How you explain how did you get $h = 2$?*
Simon	*We have to find h right? i is 6, is given h we need to find so is this (pointed at a question mark). So you take 8 minus 6. Oh…is plus…eish I got the question wrong.*
King	*Okay, so you are saying you copied the question incorrectly. You thought the question is h minus i, instead of h plus i?*
Simon	*Yes.*
King	*Okay.*
I then moved onto the first probing question, find the value of $h = i - 7$ and $i = 6$, his response was – 5.	
King	*How did you get negative 5?*
Simon	*I got a problem with negative.*
King	*It's okay, I just want to know how you got your answer.*
Simon	*They said i is 6 so I put the 6 in the i. Then it is negative because 7 is bigger than 6 so I must take that sign…but I don't know the answer so I put 5.*
King	*Okay, so you are saying the sign of the answer must follow the sign of a bigger number from the question?*
Simon	*Yes.*
I then moved onto the second probing question, find the value of h if $h = i - 7$ and $i = -9$, his response was -2.	
King	*How did you get an answer of negative 2?*
Simon	*I put negative 9 in the i but I know the answer is a negative because 9 is bigger than 7 and then the difference is 2.*
King	*Okay, I understand where the negative sign come from but didn't you also state the difference in this question (pointed at the first sub-question).*
Simon	*Ai…I don't know.*

Question 8 requires the learner to first substitute the letter with the value and then find the value of h by adding the 8 with the value. All the learners had substituted the letter with the given value correctly, but not all gave the correct responses. For example, Simon made the mistake of copying the operational sign incorrectly, changing from addition to subtraction. This mistake was corrected almost immediately while the learner was explaining his answer. Thus it seems to be a slip, but that needed to be confirmed by the probing question.

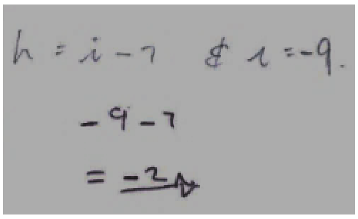
In the probing question for question 8 there were two sub-questions, both of which required learners to substitute the value into the variable and then find the value of h. Simon followed the correct procedure of substituting the value into the variable, but the final answer was incorrect, due to integer problem.

Figure 6.4.7



While the learner was trying to solve this question, he told me "I have a problem with negative", which should not appear at grade 11 level. He got the answer of "-5" because "it is negative, because 7 is bigger than 6 so I must take that sign … but I don't know the answer so I put 5". This seems like the error of 'Operating and choosing signs' (Halley, 2011), which is to use the sign of the number to decide the sign of the answer.

Figure 6.4.8



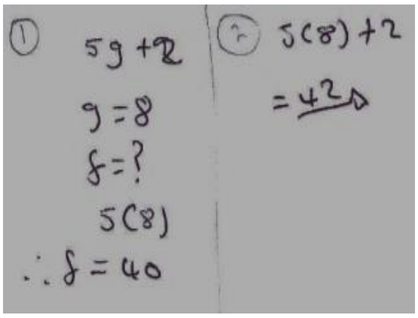
The second sub-question is slightly more difficult than the first, due to it involving negative numbers, so I was expecting more integer problems. In Simon's response, he first followed the sign of the bigger number, and then stated the difference between the two numbers.

However, apart from the integer errors in this question, it shows he is able to substitute correctly without changing the operation of the equation. Thus it seems like question 8 might be just a slip of copying the operation incorrectly, but Simon does have an integer error. Operating and choosing sign is a minus sign interpretation error.

Transcription of Simon's interview for question 9.	
Speaker	***Utterance***
In the interview, in question 9: find the value of f if $f = 5g + 2$ and $g = 8$, first Simon gave an answer of 40 then changed the answer to 42.	
King	*Could you explain to me how did you get this (pointed at his answer)?*
Simon	*I wrote in my data, 5g plus 2 and g is equal to 8 and f is a question mark like I said before to show that I am looking…then I multiply 5 times 8 and I got 40.*
King	*But what happened to the 2?*
Simon	*5g plus 2…if f is equal to $5g + 2$…and…g…I did not write the 2.*
King	*So is the answer 40?*
Simon	*No, it will be 40 plus 2 so is 42.*
I then moved to the probing question, find the value of f if $f = 5g + 2$ and $g = 8$. Simon gave an answer of 42.	
King	*Okay, how did you get 42?*
Simon	*You take 5 times 8 and then add this (pointed at the 2) 2.*
King	*Okay, is there a difference between this (I pointed at 5(8+2)) and this (I pointed at question 9).*
Simon	*No...oh ya there is…I did it wrong (and he changed to the correct answer).*

Question 9 required learners to substitute and find the value of f. It requires more mathematical syntax than Question 8, such as requiring the learner to multiply the number after substituting the value into the letter. In grade 9 and grade 10 the question in the test was, $if\, m = 3n + 1$ *and* $n = 4$, *find the value of* m, whereas in grade 11 the interview question was, if $f = 5g + 2$ and $g = 8$, then find the value of f. Three of the four learners gave an incorrect response the first time, but at the end everyone gave the correct response. Simon was one of those who gave an incorrect response the first time and then corrected himself afterward, but his reason was different from Thandi and Mpho. His response from grade 9 can be explains as follows: $if\ m = 3n + 1\ but\ n = 4\ then\ m = 4 + 1.$ In order to calculate m we must reverse the procedure: $= 3n - 1\ then\ m = 4 - 1\ which\ is\ 3.$ In grade 10 Simon did something similar, but added a -2 on the m side, which is difficult to interpret. What interested me is that in grade 11 he seemed to be using the number '8' as a letter to solve the variable.

Figure 6.4.9



In Simon's grade 11 response he took the 8 as the unknown variable, and while he was explaining his reasoning, he changed his answer. It could be a slip that he forgot to add the 2, or it also could be the way I asked the question that gave him a hint of direction. However I do need to confirm this slip by the probing question.

I followed up with the probing question. In the first one Lebo and Wazi used two different methods to solve it. Lebo first distributed the 5 into the bracket, before adding the two numbers inside the bracket, whereas Wazi followed the BODMAS procedure, added the numbers inside the bracket first and then distributed. For example:

Figure 6.4.10

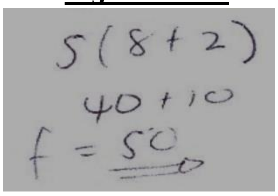
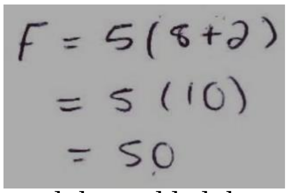


Figure 6.4.10 (left) was Lebo's response.

Figure 6.4.11 (right) was Wazi's response.

Figure 6.4.11



However Simon only distributed the first number inside the bracket and then added the number 2 afterwards, which gave him an answer of 42. But this answer was changed to 50 while I questioned him further, and then he said "I did it wrong". It seems that he realised his mistake and then corrected it. However with regard to why he changed the answer, it could be he was getting tired from the interview and did not read the question carefully, or my questioning could have given him a hint. Probing Simon showed that he did know what to do in the question, but there was a possibility of Simon not knowing the difference between $5(8 + 2)$ and $5(8) + 2$.

In short, Simon has fully achieved level 1, and is still in the process of achieving level 2. Throughout the analysis has shown that his obstacles to progression are the errors of conjoining and integers.

6.5 – Conclusion

All five of the participants took Mathematics instead of Mathematical Literacy as a subject, hence all learners have been exposed to abstract mathematics, and particularly algebraic manipulation, beyond grade 9. However, this chapter has shown that only Lebo, Thandi, and Wazi progressed to level 2, both quantitatively and qualitatively. Quantitatively, almost all the learners have got level 1 and level 2 questions correct. Qualitatively, they all were able to answer level 2 questions correctly with correct reasoning. These were the same kinds of question that they had had difficulties with in the previous year, and were the obstacle questions to progression.

Although Mpho has achieved the minimum of level 2, she has errors in integer and in equations, which are the obstacles of her progression to being fully on level 2. She still has incorrect reasoning, such as conjoining error for level 1 questions, even though she gave the correct answer. It is debateable whether Mpho is fully in level 1 or not. Simon has fully achieved on level 1, but he suffers the same kinds of errors as Mpho, namely conjoining and integer errors. Although Mpho and Simon have the same kinds of errors, Simon’s error is much the more critical. Mpho only conjoins when there are three or more terms, whereas Simon conjoins when numbers are added to the variable. As I have mentioned in Chapter 3.6, conjoining is a typical algebra error, identified by MacGregor & Stacey (1997). According to them, there are two different types of conjoining: the first is adding two letters together, which shows the learner does not understand the difference between value and letter. The second is adding a number and a letter together, in which the learner interprets the letter as having a value of 1. In this study both of these conjoining errors were apparent. Mpho’s integer error was caused by confusion of the sign rule, which does not apply to addition operation. Simon, on the other hand, chose the sign with the bigger number. In general, all participants improved compared to their previous years. Perhaps maturity and a greater exposure of Mathematics could have played a part in the result.

Chapter 7 – Findings, Reflections and Conclusion

7.1 – Summary of the project

In the analysis of Chapter 5, I found that not all the learners had progressed to a higher ICCAMS level from grade 9 to grade 10. Of the 34 learners only 15 had improved the number of correct responses and progressed to a higher ICCAMS level, and the rest had either regressed or maintained their level. Almost 50 percent of the learners progressed. However 12 learners improved in number of correct responses but maintained their level.

In the second phase I only focused on the five learners who had improved in number of correct responses but maintained their level from grade 9 to grade 10. Of those five, four learners progressed into a higher ICCAMS level in grade 11. However, one of the four learners who progressed did not provide convincing reasoning for level 2 items. Thus I analysed her interview in more detail, together with the learner who had not progressed, to determine the obstacles to their progression.

7.2 – Findings

In this section I discuss the answers to my research questions. Theoretically, as the number of correct responses increase, the learners' ICCAMS level should also increase. However, my study showed that this is not necessarily the case for all learners. Chapter 5 showed different combinations of performance between the ICCAMS level and number of correct responses, such as learners who maintained ICCAMS level but improved in the number of correct responses, which was continued into the next phase of the study.

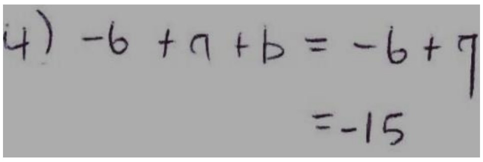
The second phase showed four of the five learners who had previously maintained ICCAMS level, but improved in number of correct responses, had now progressed to a higher ICCAMS level. However, one of them did not give enough convincing reasons in her responses for me to be confident in placing her at level 2 since she made a lot of mistakes in her responses in the interview. Thus I decided to analyse her responses further, together with those learners who still had not progressed, to view the obstacles to their progress, as well as the number of errors.

The obstacles hindering Simon from progressing were mainly conjoining errors and minus sign interpretation errors. However the conjoining error in grade 11 was slightly different to that of grade 9 and grade 10. In the previous year he said two letters could be put together. For example, in question 1.4, his answer for $2a + 5b + a$ was $7a^2b$, whereas in grade 11 he said "he must put all the numbers and letters together, but two letters cannot be put together”. Thus in question 2 of the interview, his initial response for *simplify* $3q + 6y + q$ was $10qy$ but was changed to $10y$. This shows that the conjoining error still existed, but in a different form. Moreover he also had integer problems, and tended to use the sign of the bigger number as the sign of the solution. For example, in the interview, he gave the answer $-9 - 7 = -2$. He chose the sign of the greater number and then subtracted 7 from 9 to produce an answer of -2, which is an operating and choosing signs error.

Mpho’s main error was also conjoining. However her type of conjoining involved both numbers and letters. This is very different from Simon’s type of conjoining. For example, in the interview she was required to add 6 to an expression and he gave the following responses $6 + (p + 3) = 6p + 3$ and $6 + (3 + p) = 9 + p$. Apart from the conjoining error, Mpho also had integer error. She applied the multiplication of sign in a subtraction situation. For example, her answer for $-9 - 7$ was 2, which she reasoned as negative times negative to give her a positive and then subtract 7 from 9.

There are data from the study which are not reported that show a substantial problem with integers. Although this study did not elaborate much on integer error, this error has a substantial impact on algebra. For example, if $a + b = 9$ *then find the value of* $a + 7 + b$ *and* $-6 + a + b$. Learners dealing with $a + 7 + b$ were all able to find the value. However, when dealing with $-6 + a + b$, then most of them had difficulties. Most of them got it wrong, and their procedures were all the same. For example, they substituted both ‘a + b’ with the value of 9 and then subtracted 6 (refer to Figure 7.3.12).

Figure 7.2.1



Wazi and Mpho had the same response and their explanations were that “negative and a positive will have a negative”, a clear ‘sign rules’ error.

Moreover, Simon had a similar answer to Wazi and Mpho (Figure 7.3.13), but with a positive sign. He said “…the sign of the bigger number tells me the sign of the answers”. This is an error of ‘operating and choosing signs’, learners using the sign of the number to decide the sign of the answer (Halley, 2011).

Figure 7.2.2

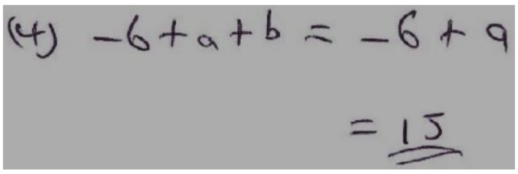
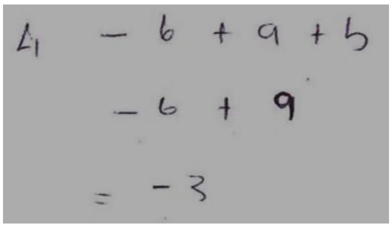


Figure 7.2.3



Similarly, Figure 7.3.14 shows Tandi's response. Initially this response seems like a 'sign rule' error, but instead it was the same as Simon's error, 'operating and choosing sign'. In this error Thandi shows a different reasoning from Simon's. Thandi has a negative in her answer because negative appeared first in the question.

There is also two findings from other learners which give insight into their thinking. Firstly, some learners use examples as guidelines to determine their solution. For example, in question 3 learners were asked to $add\ 6\ to\ p$, where the answers should remain the same as the question, but in an expression form, such as $6 + p$. Even though all of the learners gave the correct response, but there were two different strands of reasoning. Most of the learners were able to explain that there is "no like term so we must keep it like that", but Lebo said "I wasn't sure what to do so I followed the example". The example was very similar to the question: *4 added to n can be written as* $n + 4$. Thus it is debatable whether Lebo knows how to work with the algebraic rules.

The second interesting finding arose while exploring learners' reasoning in the question of calculating area (question 5.2). Thandi, Wazi, and Mpho said "$m \times n$ can only be written as mn and not nm". Two reasons were given: Thandi said "the order is different in this question…m must be first…m is the length". This suggests she thought that the order of the answer depended on the order of the question. Wazi and Mpho said similarly that *m* must be first, not because 'm' was written first in the question, but because of the alphabetical order, "m must always be first because m always comes before n in the alphabetical order".

Another type of order error appeared in the analysis, slightly different from the previous one. It was embedded with conjoining errors. In question 5.2 the matter of order applied to the multiplication situation, such as $m \times n = mn$ and not nm. The probing question of question 3 in part one applied to addition, such as $6 + (p + 3) = 6p + 3$ and $6 + (3 + p) = 9 + p$. It appears that learners who struggle with making sense of algebra would assume order of question matter.

7.3 – Recommendations

In this section I recommended four issues that need further research: conjoining error, the integer error, example, and order matter. In terms of the conjoining error, in grade 9 and grade 10 most of the learners were conjoining but in grade 11 almost all the learners had stopped conjoining. However, we do not know what it is that results in the overcoming of the conjoining error. One possible reason may be better teaching of algebra by grade 11 but this needs further research. Furthermore, in the literature on conjoining there was no difference between conjoining variables with variables and numbers with variables. My research shows that there is a difference between the two kinds of conjoining. Conjoining numbers with variables might be a progression from conjoining variables with variables. However this needs further research.

The research on negative number has focused on grades 6 to 9 (Vlassis, 2004; Gallardo & Rojano, 1994; Halley, 2011).,My research shows demonstrates the integer problem that learners experience even when they are in grade 11. Therefore further research is necessary to find ways of helping learners to overcome such errors even in grades 10 to 12.

During the interview it became clear to me that the examples were not necessarily functioning as the ICCAMS designers had intended. While Lebo said, “The example helps me to get the answer because it gives a clue”, by contrast Thandi said, “I use example to answer the question but I don’t even understand what the word perimeter means”. This suggests that the example will only help if learners have a sense of what it is illustrating. It would be interesting to investigate further how the example can affect learners’ responses.

In this study, it has emerged that there are an issue related to the matter of order of letters. Some learners determined their solution based on the given order of letters in the question or the alphabetical order of the given letters. This needs further research.

7.4 – Reflection on the test and interview as an instrument for data collection

In this section I reflect on the process of collecting data, from the test scripts to the interview. I have shown that the test script itself can only provide an overview of a learner’s ICCAMS level. In order to have more insight into the learner’s ICCAMS level it must be supplemented with an interview to confirm their levels. There are cases where the learners should progress into higher ICCAMS level, but slips in their responses caused them to maintain their level. In contrast, there are learners who got the question correct, but only coincidentally. For example, Thandi achieved level 2 but some of her responses to level 2 questions were correct only because she was just following the given examples in the test, and not because she understood the concepts. Thus, individual interviews are needed to follow up their written response, in order to determine their ICCAMS level. However, I acknowledge that this is not feasible for a large sample.

Furthermore, even the ICCAMS level itself can only provide an overview of the learner's performance. Again, only an interview can give us sufficient insight into the learner's performance, which enables us to have a more detailed level of analysis. For example, learners could achieve a higher ICCAMS level than in previous grades, but make more errors. In contrast, they could maintain a level but improve in the number of correct responses. Moreover, they could get ICCAMS questions correct but with a hidden error, such as Mpho, who achieved a higher ICCAMS level, and only in the interview did the embedded conjoining error appear. For example, Mpho gave the correct answer, but this depended on the order of the question. i.e. $6 + (p + 3) = 6p + 3$ and $6 + (3 + p) = 9 + p$. Similarly she gave the correct answer for $6 - 7 = -1$, but only in the second probing question did her embedded sign rule error appear, i.e. $-9 - 7 = +2$. This suggests that we should be cautious in making claims about what learners' know and can do based on test performance alone. Interviews provide the possibility to gain more insight into learners' thinking.

Moreover in chapter 6 I have shown Simon is still on level 1, due to his conjoining and integer error. However there is additional data which I have not reported in this study, involving level 3 questions such as *find the value of* $a - 4 - b$ *if* $a + b = 9$. Surprisingly Simon got this question correct, whereas other learners I interviewed got it incorrect. But given that I ultimately chose to focus the study on level 2 questions, this data has not been included.

In reflection of the interviews, I need to consider my questioning technique. While probing the learners' responses there were times when I might have been giving hints, which led to the learners changing or correcting their responses. For example, with Simon's responses in question 7.3 of part one, my questioning affected his response, and possibly caused him to change from an incorrect to a correct response. When he did not know what to do in that question, I asked him if there was a difference between question 7.2 and question 7.3. By doing so, I was indirectly hinting that he must follow question 7.2 to get the answer for question 7.3.

7.5 – Conclusion

At the end of the investigation of progression in learners' performance in algebra from grade 9 to grade 11, all three guiding questions helped to answer the main question: Do learners progress to higher ICCAMS levels from grade 9 to grade 11? This study showed that half of the sample progressed into higher ICCAMS levels from grade 9 to grade 10, whereas the other half either dropped or maintained their ICCAMS level. However, some of the learners who dropped or maintained their ICCAMS level actually progressed in terms of achieving more correct answers. This study investigated only learners who maintained their ICCAMS level and progressed in the total number of correct responses, and showed that most of the learners did progress to a higher ICCAMS level in grade 11. The interviews revealed two types of error that may appear to hinder learners' progression: integer error and algebra error. The integer error concerns incorrect interpretation of the minus symbol, such as the sign rule, and operating and choosing sign. The algebra error involves different types of conjoining errors.

From this study I have learnt both about research and teaching. In terms of research, I have learnt to code learner's responses during the interview. This would help to avoid the discouragement of seeing learners' response as incorrect, and at the same time it helps to highlight the same kinds of errors. Furthermore, I have also learnt that diagnostic tests do not always reflect accurately what learners know and can do, and so interviews are needed to probe further.

Learners at grade 11 level still need to be helped to correct their errors, such as conjoining and integers. Thus the policy in teaching of mathematics needs to initiate interventions, especially on these errors, with learners at a much earlier stage in order to progress properly in their algebra. MacGregor & Stacey (1997) stated that "misinterpretations lead to difficulties in making sense of algebra, and may continue for several years if not recognised and corrected in time" (p.15). Moreover Smith, DiSessa, & Roschelle (1993) claim that if learner misconceptions are not corrected, they would affect their accepting of new knowledge. Thus, if a learner's misconceptions are not reconstructed/refined in his/her mind the learner may not be able to accept new knowledge nor, of course accommodate it with the existing knowledge. This shows the importance of highlighting learner errors during the detailed investigation of individual learners' progression.

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Appendices

Appendix A – ICCAMS section of the Annual test from WMC-S

Section B ICCAMS

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1. $a+3a$ can be written more simply as $4a$.

Simplify each of the following, where possible:

1.1 $2a+5a$ = ____________________

____

1.2 $2b+5b$ = ____________________

____

1.3 $(a+b)+a$ = ____________________

____

1.4 $2a+5b+a$ = ____________________

____

1.5 $(a-b)+b$ = ____________________

____

1.6 $3a-(b+a)$ = ____________________

____

1.7 $a+4+a-4$ = ____________________

____

1.8 $3a-b+a$ = ____________________

____

1.9 $(a+b)+(a-b)$ = ____________________

____

2. Which is larger, $2n$ or $n+2$? ____________________

Explain: ____

____

3. **4 added to *n*** can be written as **n+4**.

Add 4 to:

3.1 8 ____________ 3.2 $n+5$ ____________ 3.3 $3n$ ____________

4. ***n* multiplied by 4** can be written as **4*n***.

Multiply each of these by 4:

4.1 8 ____________ 4.2 $n+5$ ____________ 4.3 $3n$ ____________

5.1 If $a+b=43$,

then $a+b+2=$ __________

5.2 If $n-246=762$,

then $n-247=$ __________

5.3 If $e+f=8$,

then $e+f+g=$ __________

6.1 Find a if a + 5 = 8 ______________________________

6.2 Find b if b + 2 = 2b ______________________________

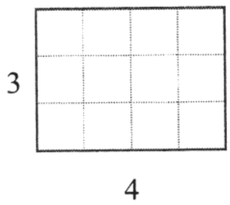
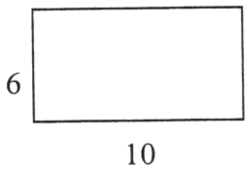
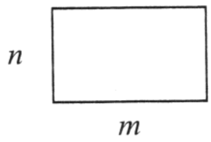
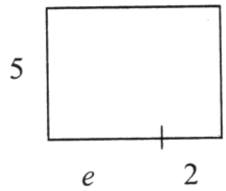
7.1 If $u=v+3$ and

$v=1$, find the value of u ______________________________

7.2 If $m=3n+1$ and

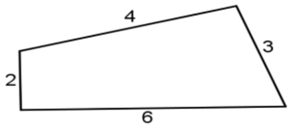
$n=4$, find the value of *m* ______________________________

8. What are the areas of the following shapes?

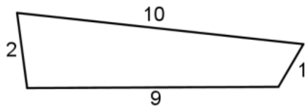
8.1	8.2	8.3	8.4
			
A =	A =	A =	A =

9. The perimeter of this shape is equal to

$6 + 3 + 4 + 2$, which equals 15.

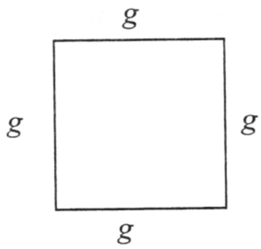


9.1 Work out the perimeter of this shape:



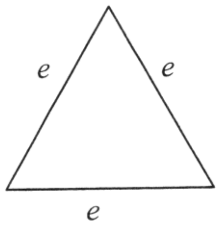
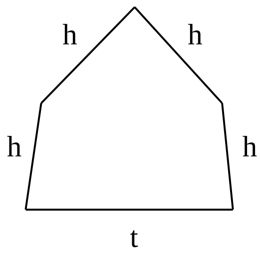
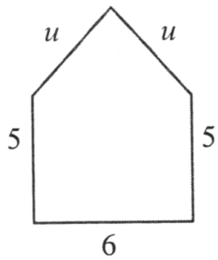
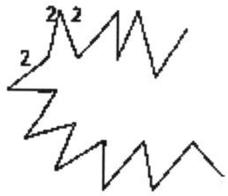
____

10. This square has sides of length *g*.



So, for its perimeter, we can write $p = 4g$.

Find the perimeter for each of the shapes:

10.1	10.2	10.3	10.4
			 *Part of this figure is not drawn. There are n-sides altogether all of length 2.*
P =	P =	P =	P =

11. Cakes cost *c* rand and buns cost *b* rand each.

If I buy 4 cakes and 3 buns, what does $4c + 3b$ stand for?

____

The End

Thank you!

Appendix B – Task-based interview for the learner

Part one – Written part of the interview

Name: ____ **Date:** _____________

Surname: ____

Part one

Section A

1) $a + 3a$ can be written more simply as $4a$,
simplify $2q + 6q$.

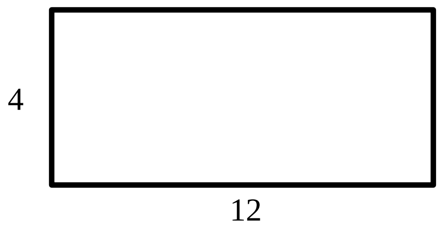
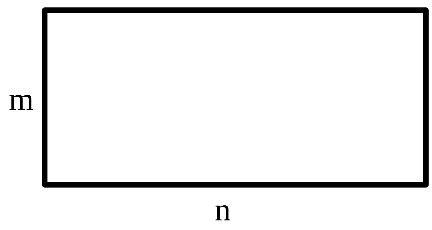
2) $a + 3a$ can be written more simply as $4a$,
simplify $3q + 6y + q$.

3) 4 added to n can be written as $n + 4$,
add 6 to p.

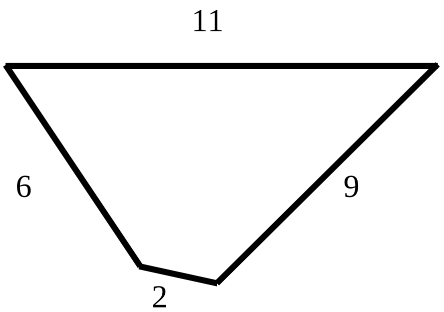
4) n multiplied by 4 can be written as $4n$,
multiply 6 by p.

Section B

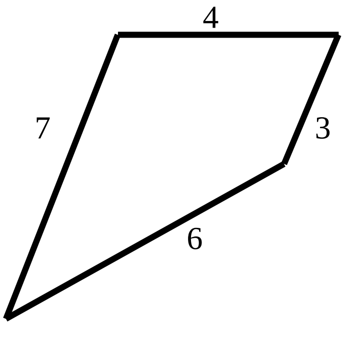
5) What are the areas of the following shapes?

5.1	5.2
	
A =	A =

6) The perimeter of this shape is $6 + 11 + 9 + 2$, which equals 28

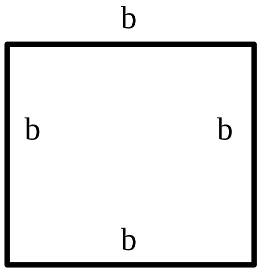


Work out the perimeter of this shape:

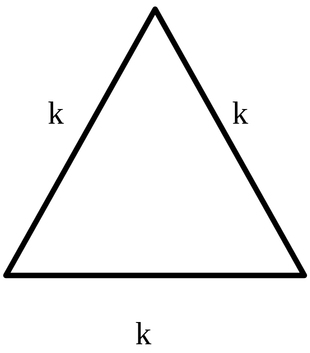
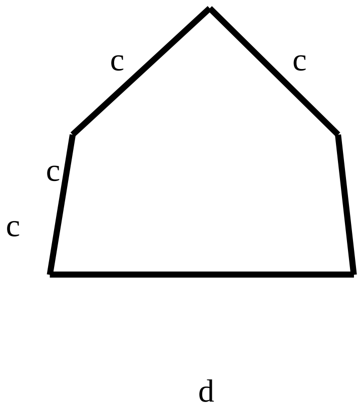
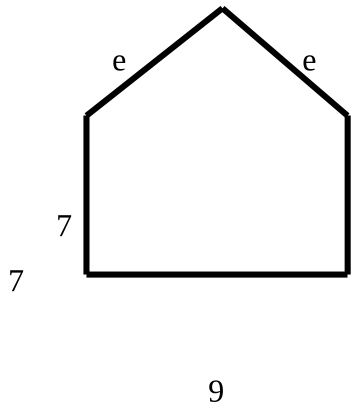


7) This square has sides of length b.

So, for its perimeter, we can write $P = 4b$.



Find the perimeter for each of the shapes

7.1	7.2	7.3
		
P =	P =	P =

Section C

8) If $h = i + 8 \; and \; i = 6$, then find the value of h.

9) If $f = 5g + 2 \; and \; g = 8$, then find the value of f.

Part one – Solution of the written part of the interview

Solution for Part one – Written Response

1) $8q$

2) $4q + 6y$

3) $6 + p$

4) $6p$

5.1) $A = 48$

5.2) $A = mn$ or nm

6) $P = 20$

7.1) $P = 3K$

7.2) $P = 4c + d$

7.3) $P = 2e + 23$

8) 14

9) 42

__Part two – Probing learner’s responses from part one__

——— *Correct response*

- - - - - *Incorrect response*

1) **Question 1 & 2**

- Question 1
  - ——— Question 2
    - ——— $3a - b + a$
    - - - - $2x + y + x$
  - - - - Question 2
    - ——— $6x + 2x + 3y$
    - - - - $2x + x$

2) **Question 3 - If 4 added to n can be written as $n + 4$, then add 6 to p.**

- Question 3
  - ——— Same format but add 6 to $\boldsymbol{p + 3}$.
  - - - - Same format but add 6 to a.

3) **Question 4 – If n multiply by 4 can be written as $4n$, then multiply 4 by p.**

- Question 4
  - ——— Same format but multiply 4 by $\boldsymbol{p + 3}$.
  - - - - Same format but multiply 4 by u.

4) **Question 8 – If $h = i + 8$ *and* $i = 6$, then find the value of h.**

Question 8 – Same format but change the equation to $h = i - 7$ and $i = 6$.

– Same format but change the equation to $h = i - 7$ and $i = -9$.

5) **Question 9 – If $f = 5g + 2$ *and* $g = 8$, then find the value of f.**

- Question 9
  - ——— Same format but change the equation to $f = 5(g + 2)$.
  - - - - Same format but change the equation to $f = 3g - 1$.
    Same format but change the equation to $f = 3 + g - 1$.

Appendix C – Letter to the Principal

Protocol number: 2013113M 12th November 2013

Dear Sir/ Madam

My name is King Wun Vincent Leung. I am a student in the School of Education at the University of the Witwatersrand.

I am doing research on "**An Investigation into Learners' Progression in Algebra from Grade 9 to Grade 11"**

Since 2010 Wits Maths Connect Secondary has been tracking learner performance through tests at the end of each year. This research is led by Professor Jill Adler.

My study is a follow-up of this data. I would like to invite some of your learners to participate in interviews in this regard. My research involves interviews with about eight learners in grade 10 and grade 11. The interviews will not be done during instruction time at the school and will therefore not disrupt teaching and learning at your school. The interviews will be conducted after school hours at a time suitable for the selected learners.

The reason that I have chosen your school is because the WMC-S annual tests were conducted at your school from 2010 and your school is one of the most active schools within the WMC-S project.

I was wondering whether you would mind if I interviewed eight of your grade 11 learners in November.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed five years after completion of the project.

Please let me know if you require any further information or have any enquiries. You can also contact my supervisor. I look forward to your response as soon as is convenient.

Yours sincerely

SIGNATURE:

King Wun Vincent Leung

king.wun.vincent@gmail.com

076 045 6465

Supervisor : Dr. Craig Pournara

Craig.Pournara@wits.ac.za

011 717 3253

Appendix D – Information Sheet for Parents/Guardian and Learners

INFORMATION SHEET for PARENTS/GUARDIAN and LEARNERS

04 November 2013

Dear parent/guardian

My name is King Wun Vincent Leung and I am registered at the University of the Witwatersrand for a Master in Education degree in Mathematics Education. As part of my studies I am conducting research into Learners' Progression in algebra from grade 9 to grade 11. I am focusing on grade 11 my supervisor is Dr Craig Pournara.

Your child ….……………………………………………………………………………………… is invited to be part of my research project.

My research forms part of the Wits Maths Connect Secondary Project which is funded by the First Rand Foundation, Department of Science and Technology, and managed by the National Research Foundation, and directed by Prof Jill Adler.

In my research I want to try to understand how learner progress in algebra from grade 9 to grade 11.

During the interview I will give your child some mathematics tasks to complete. Then I will ask her/him to explain to me how s/he got her/his answers. I will audio-record the interview in order to have an accurate record of the discussion. I will also collect the written work that your child produces during the interview. The duration of the interview will be approximately 45 minutes.

The findings that come out of my research will assist teacher educators to design programs that address learning challenges that learners encounter when learning algebra. Thus, teachers coming from University into our schools are likely to bring along ways of teaching that address learning difficulties associated with algebra.

HOW WILL THE INFORMATION BE USED

I will use the data to explore your child's progression in algebra. I will write a report for my Master degree. I also hope to present my research at conferences and publish in journal articles. The data will be used for the duration of the Wits Maths Connect Secondary Project and stored for a further five years. Thereafter all data will be destroyed.

YOUR RIGHTS AND THE RIGHTS OF YOUR CHILD

We will not use your child's name in any reports or articles.
The research is completely separate from your child's school work. All information obtained for research purposes will not affect your child's assessment in school.

There will also be no problem if you do not want your child to take part in the research. If you choose that your child do not participate, this will not affect your child in any way.

If you decide that your child should no longer continue participating in the research, you are free to withdraw this consent at any time. You should then inform me. My contact details are given below.

If you wish to discuss the research further, feel free to contact me.

My contact details are:

King Wun Vincent Leung, 076 045 6465, king.wun.vincent@gmail.com

My supervisors' contact details are:

Dr Craig Pournara , 011 717 3253, craig.pournara@wits.ac.za

Appendix E – Learner consent form for participating in this research project

Ref no: 2013113M

04 November 2013

LEARNER CONSENT FORM FOR PARTICIPATING IN RESEARCH PROJECT

Researcher: King Wun Vincent Leung

Topic: Investigating into Learners' Progression in algebra from Grade 9 to Grade 11

If you are happy for your child to take part in the research, please sign below.

I am happy for my child to be interviewed as part of the research.

I understand that:

- My child's name will not be used in any reports or articles.
- The research will not affect my child's assessment in school.
- There will also be no problem if I do not want my child to take part in the research.
- I may withdraw permission at any stage for my child to participate in the research.

Signed by parent/guardian

Signed ……………………………………………………………………

Date ……………………………………………………………………

Name of parent/guardian …………………………………………………..

Name of learner …………………………………………………………….

Signed by learner

Signed …………………………………………………………………….

Date ……………………………………………………………………..

Name of learner ……………………………………………………………

Return date for the consent form is on the 6th of November 2013.

Appendix F – Learner consent form for audio recording

Ref no: 2013113M

04 November 2013

LEARNER CONSENT FORM FOR RESEARCH PROJECT - CONSENT FORM FOR BEING AUDIOTAPED

Researcher: King Wun Vincent Leung

Topic: Topic: Investigating into Learners' Progression in algebra from Grade 9 to Grade 11

If you are happy for the interview with your child to be audio-taped, please sign below.

I am happy for my child to be audio-taped during the interview.

Signed by parent/guardian

Signed …………………………………………………………

Date …………………………………………………………

Name of parent/guardian ……………………………………….

Name of learner …………………………………………………

Signed by learner

Signed ……………………………………………………….....

Date ………………………………………………………….

Name of learner …………………………………………………

Return date for the consent form is on the 6th of November 2013.

Appendix G – 2013 Adapted version of ICCAMS coding scheme

ICCAMS

Ques	**Code 0** Missing	**Code 1** Correct	**Code 2** Ambiguous	**Code 3** Letter Evaluated	**Code 4**	**Code 5** Letter as Object	**Code 6**	**Code 7** Letter not used	**Code 8** Premature Closure	**Code 9** Wrong
1	✔	**8q**		8					9q	9a) $8a^2$ 9b) Other
2	✔	**4q+6y**							8a) **10qy** 8b) 9qy 8c) 9qyq	9a) $7a^2b$ / $8a^2b$ 9b) $2a^2 + 5b$ 9c) Other
3	✔	**6+p**								✔
4	✔	**6p**								9a) 6xp 9b) Other
5.1	✔	**48**; 4x12 (ignore insertion of $units^2$ or $numbers^2$)								9a) 16; 4+12 9b) Other
5.2	✔	1a) **mn, mxn** (ignore insertion of $units^2$) 1b) **m+n**; if answer to 5.1 was 16								9a) m+n (if 5.1 was not 16) 9b) 2(m+n) / 2m+2n / m+m+n+n 9c) Other

Ques	**Code 0** Missing	**Code 1** Correct	**Code 2** Ambiguous	**Code 3**	**Code 4**	**Code 5**	**Code 6**	**Code 7** Letter not used	**Code 8** Premature Closure	**Code 9** Wrong
				Letter Evaluated		Letter as Object				
6	✔	**<u>20</u>**; 18; **<u>19; 21</u>**								✔
7.1	✔	1a) **<u>3k</u>** 1b) **<u>k+k+k</u>**	3g	9, 12, 15 … (any number added 3 times)						✔
7.2	✔	1a) **<u>4c+d</u>** 1b) **<u>4c+1d</u>** 1c) **<u>c+c+c+c+d</u>**	4c, d operation missing						8a) **<u>4cd</u>** / 4c1d 8b) ccccd	9a) 5cd 9b) Other
7.3	✔	1a) **<u>2e+23</u>** 1b) **<u>2.e+2.7+1.9</u>** *or* 2e+2(7)+9 1c) 2e+14+9 *or* 2e+7+7+9	2a) 2e,23 2b) 49 2e 23						8a) **<u>2e23</u>** 8b) **<u>ee779</u>** 8c) 25e	✔
8	✔	**<u>h = 14</u>** / 6+8							**<u>2</u>**	9a) 6 9b) Other
9	✔	**<u>f = 42</u>** / 5(8)+2								9a) 15 (8+5+2) 9b) ±1 9c) 10 9d) Other