LEARNERS’ MATHEMATICAL REASONING WHEN GENERALIZING FROM NUMBER PATTERNS IN THE GENERAL EDUCATION AND TRAINING PHASE

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ABSTRACT

This study aims to explore GET learners’ mathematical (algebraic) reasoning when generalizing from number patterns. Data was collected in a former model C school in greater Johannesburg area by means of a questionnaire based task involving number patterns. The mathematical reasoning of the grade 9 participants when generalizing from number patterns was examined within a commognitive framework. According to this perspective, thinking is a special activity of communication in which a participant of a discourse engages. The participants’ responses to questions in the questionnaire based task were classified according to particular aspects of the discourse they used, specifically routines (strategies) and visual mediators. The participants’ generalization routines were further classified into one of the three main categories; numeric, figural and pragmatic generalizations. The analysis focused on how the learners’ derived rules for the \( n \)th term and their justifications for their responses.

The results of this study strongly support the notion that students’ algebraic reasoning when generalizing in number patterns is intertwined with their choices of routines and mediators. Most learners used recursive routines while a few used explicit routines (classified and categorized as numeric routines) and number-mediators. Also, most participants found it easier to informally verbalize their generalizations. However participants’ spoken justifications of their written and spoken responses often did not match their use of routines and visual mediators. As such, an awareness and appreciation (by teachers) of students’ diverse use of routines and mediators when generalizing from number patterns could have direct pedagogical implications in the mathematics classrooms.

KEYWORDS

Algebra, Generalization, Commognition, Thinking, Communication and Reasoning
DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg South Africa. It has not been submitted before for any degree or examination at any other University.

________________________________________
(Signature of Candidate)

7th day of June in the year 2011
DEDICATION

In loving memory of my late Parents: Enala & Chapasuka
ACKNOWLEDGEMENTS

First and foremost I thank the Almighty GOD for giving me life and a good health. And also, I would like to thank the following people for their contributions towards my work:

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ABBREVIATIONS

GET  Education and Training
GD  Gauteng Department of Education
FET  Further Education and Training
OBE  Outcome Based Education
RNCS  Revised National Curriculum Statement
C2005  Curriculum 2005
NCS  National Curriculum Statement
DoE  Department of Education
LR  Learners’ Routines
LVM (LM)  Learners’ Visual Mediators
QRASS  Question Response and Summary Sheet
LD  Level Descriptors
TLA  Task Level of Attainment
MALATI  Mathematics Learning and Teaching Initiative
SO  Specific Outcomes
AS  Assessment Standards
LO  Learning Outcomes
AF  Analytical Framework
NCTM  National Council for Teachers Mathematics