Time Series Analysis Using Fractal Theory and Online Ensemble Classifiers with Application to Stock Portfolio Optimization

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A dissertation submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Science in Engineering.

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Declaration

I declare that this dissertation is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

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Dalton Lunga.

Abstract

Neural Network method is a technique that is heavily researched and used in applications within the engineering field for various purposes ranging from process control to biomedical applications. The success of Neural Networks (NN) in engineering applications, e.g. object tracking and face recognition has motivated its application to the finance industry. In the financial industry, time series data is used to model economic variables. As a result, finance researchers, portfolio managers and stockbrokers have taken interest in applying NN to model non-linear problems they face in their practice. NN facilitates the approach of predicting stocks due to its ability to accurately and intuitively learn complex patterns and characterizes these patterns as simple equations. In this research, a methodology that uses fractal theory and NN framework to model the stock market behavior is proposed and developed. The time series analysis is carried out using the proposed approach with application to modelling the Dow Jones Average Index's future directional movement. A methodology to establish self-similarity of time series and long memory effects that result in classifying the time series signal as persistent, random or non-persistent using the rescaled range analysis technique is developed. A linear regression technique is used for the estimation of the required parameters and an incremental online NN algorithm is implemented to predict the directional movement of the stock. An iterative fractal analysis technique is used to select the required signal intervals using the approximated parameters. The selected data is later combined to form a signal of interest and then pass it to the ensemble of classifiers. The classifiers are modelled using a neural network based algorithm. The performance of the final algorithm is measured based on accuracy of predicting the direction of movement and also on the algorithm's confidence in its decision-making. The improvement within the final algorithm is easily assessed by comparing results from two different models in which the first model is implemented without fractal analysis and the second model is implemented with the aid of a strong fractal analysis technique. The results of the first NN model were published in the Lecture Notes in Computer Science 2006 by Springer. The second NN model incorporated a fractal theory technique. The results from this model shows a great deal of improvement when classifying the next day's stock direction of movement. A summary of these results were submitted to the Australian Joint Conference on Artificial Intelligence 2006 for publishing. Limitations on the sample size, including problems encountered with the proposed approach are also outlined in the next sections. This document also outlines recommendations that can be implemented as further steps to advance and improve the proposed approach for future work.

I would like to dedicate this work to my loving wife Makhosazana Thomo, my mother, Jester Madade Hlabangana, my cousin, Khanyisile Hlabangana and other family members: thank you for your prayers and support throughout this demanding time.

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Contents

List of Figures

List of Tables

Nomenclature

Roman Symbols

F	transfer function
$\forall$	for all
$\ulcorner$	ulcorner

Greek Symbols

λ	lambda
ι	index
π	$\simeq 3.14\ldots$
ϵ	epsilon
δ	delta
β	beta
Σ	summation symbol

Superscripts

j	superscript index

Subscripts

0	subscript index

Acronyms

$AJCAI$2006	Australian Joint Conference on Artificial Intelligence 2006
$ICONIP$2006	International Conference on Neural Information Processing 2006
$LNAIS$2006	Lecture Notes in Artificial Intelligence Series 2006
$LNCS$2006	Lecture Notes in Computer Science 2006
$Eq.$	Equation
NNs	Neural Networks

Chapter 1

Introduction

Over the past two decades many important changes have taken place in the environment of financial time series markets. The development of powerful communication and trading facilities has enlarged the scope of selection for investors to improve their day to day practices. Traditional capital market theory has also changed and methods of financial analysis have improved. Forecasting stock market returns or a stock index is an important financial subject that has attracted researchers' attention for many years. It involves an assumption that fundamental information publicly available in the past has some predictive relationships to the future stock returns or indices.

The samples of such information include economic variables such as interest rates and exchange rates, industry specific information such as growth rates of industrial production and consumer price, and company specific information such as income statements and dividend yields. This is opposed to the general perception of market efficiency as proved by McNelis (2005). In fact, the efficient market hypothesis states that all available information affecting the current stock values is constituted by the market before the general public can make trades based on it, Skjeltorp (2000). Therefore, it is possible to forecast future returns since they already reflect all information currently known about the stocks. This is still an empirical issue because there is considerable evidence that markets are not fully efficient, and it is possible to predict the future stock returns or indices with results that are better than random as will be shown later in this investigation. Recently, researchers have provided evidence that stock market returns are predictable by means of publicly available information such as time-series data on financial and economic variables, especially those with an important business cycle component, Wong & Selvi (1998).

These studies identify various interest rates, monetary growth rates, changes in industrial production, and inflation rates as variables that are statistically important for predicting a portion of the stock returns. However, most of the conventional studies attempting to capture the relationship between the available information and the stock returns rely on simple linear regression assumptions. There is no evidence thus far to support the assumption that the relationship between the stock returns, the financial and economic variables is perfectly linear or to assume a random walk, which implies a normal distribution process, Leung *et al.* (2000). This is due to the fact that a significant residual variance of the actual stock returns exists from the prediction of the regression equation. Therefore, it is possible that nonlinear models are able to explain this residual variance and produce more reliable predictions of the stock price movements. Even though there exists a number of non-linear regression techniques, most of these techniques require that the non-linear model be specified before the estimation of parameters can be determined. One non-linear modelling technique that may overcome these problems is the neural networks .

Later we will see that the use of non-parametric techniques introduced by fractal theory proves to be very powerful in establishing the hidden trends and relationships within the time series data. Also, neural networks offer a novel technique that does not require a pre-specification during the modelling process because they independently learn the relationship inherent in the variables. This is especially useful in security investment and other financial areas where much is assumed and little is known about the nature of the processes determining asset prices. Neural networks also offer the flexibility of numerous architecture types, learning algorithms, and validation procedures. As a result, the discovery and use of non-linearity in financial market movements and analysis to produce better predictions of future stock returns or indices has been greatly emphasized by various researchers and financial analysts during the last few years. To this end, it has been found that stock trading driven by a certain forecast with a small forecasting error may not be as profitable as trading guided by an accurate prediction of the direction of stock return, McNelis (2005). Nonetheless, having an accurate prediction of a certain stock or stock index return still has numerous benefits that will be discussed later in the investigation.

In the traditional engineering thinking, machines, processes and complicated systems are easily understood by dividing the complete system into small subsystems. Focus is given to each small sub system to try and establish its behavior when subjected to certain conditions. An analysis is also done on each subsystem to try and understand how these small parts interact with each other to influence the overall decision that comes out of the main system. Robust scientific techniques such as chaos and fractal theory are emerging in these attempts, where researchers are seeking to study the complexity of systems as an interplay and self-organization of many small parts of the system that share some features in nature, Skjeltorp (2000). A distinct feature with complexity is that in many situations one has what is called sensitive dependence on initial conditions. These sensitivities limit our ability to predict the future accurately, but incorporate a kind of long-term memory effect in different processes, which is vastly and presently ignored in the basic theoretical and practical framework of time series analysis. As will be shown later, the non-linearity of time series data e.g. stock market may be studied using fractal theory concepts, which embody its own kind of simple laws.

Although we will get approximate results, these will prove the predictions with much confidence and accuracy more than the results obtained using conventional statistical averages. Through fractal geometry, we have access to tools and a new way of thinking that has been widely used within engineering and the physical sciences to describe complex systems and processes. In this study, we propose and implement a fractal theory technique that we also use to improve the performance of an existing incremental online NN algorithm. The problem that was identified from the existing models is: methods that are being used to find the coefficient for neural network or non-linear model are most likely to give a local optimum solution. That is the best forecast in the neighborhood of the initial guess, but not the coefficients for giving the best forecast if we look a bit further afield from the initial guesses for the coefficients. This makes it very difficult to determine the sign or direction of the stock/portfolio return value since a local minimum will rule out other solutions (global minimum) that are more valid in the decision-making process. Another shortfall that was discovered from the results of numerous researches in this field is that input data is passed onto the NN in batch form which results in the model having a requirement to go off-line every time new data is presented to the network so that architecture optimization is achieved for the new information. This means that the model will always be redesigned for all new data that is presented on its input. This is time consuming and uneconomical. A methodology is proposed for processing of input data before it is passed to the NN algorithm. This is done using non-parametric methods discovered from the study of fractal theory that are capable of eliminating noise random data points from the sample spaces.

Making use of fractals will result in classifying the sample parts of the signal as persistent, random signal or non-persistent. This approach has proven that the application of fractal theory to analyzing financial time series data is the better approach to providing robust solutions to this complex non-linear dynamic system. The proposed methodology offers to develop a model that takes input data in sequences and adapts (self-trains) to optimize its architecture without having to change the architecture design. The first model framework seeks to convert a weak learning algorithm implemented using the Multi-Layer Perceptron into a strong learning algorithm. The model has the ability to identify unknown data samples into correct categories and the results proves the incremental learning ability of the proposed algorithm. In other words the strong learner identifies the hard example and forces its capability to learn and adapt to these examples. The first model is observed to classify known data samples into correct labelled classes that give the predicted direction of movement of the future time series returns. A summary of these results were published in the Lecture Notes in Computer Science (LNCS) series by Springer 2006, Lunga & Marwala (2006a). The proposed second model framework applies the theory of fractal analysis. The model processes the input data in order to establish a form of a function that can identify features e.g self-similarity, data persistence, random walk pattern, non-persistent and the Hurst exponent. The relevance of these features in enabling predictions for future time series data returns is explained in the next chapter. A summary of fractal analysis framework results were submitted for publishing to the Australian Joint Conference on Artificial Intelligence in 2006, Lunga & Marwala (2006b).

Chapter 2

Time Series Analysis Using Fractal Theory and Online Ensemble Classifiers

(Presented at the Australian Joint Conference on Artificial Intelligence 2006)

2.1 Introduction

The financial markets are regarded as complex, evolutionary, and non-linear dynamical systems, MacKinlay (1988). Advanced neural techniques are required to model the non-linearity and complex behavior within the time series data, Mandelbrot (1997). In this paper we apply a non-parametric technique to select only those regions of the data that are observed to be persistent. The selected data intervals are used as inputs to the incremental algorithm that predicts the future behavior of the time series data. In the following sections we give a brief discussion on the proposed fractal technique and the online incremental Learn++ algorithm. The proposed framework and findings from this investigation are also discussed.

Fractal analysis is proposed as a concept to establish the degree of persistence and self-similarity within the stock market data. This concept is implemented using the rescaled range analysis (R/S) method. The R/S analysis outcome is applied to an online incremental algorithm (Learn++) that is built to classify the direction of movement of the stock market. The use of fractal geometry in this study provides a way of determining quantitatively the extent to which time series data can be predicted. In an extensive test, it is demonstrated that the R/S analysis provides a very sensitive method to reveal hidden long run and short run memory trends within the sample data. The time series data that is measured to be persistent is used in training the neural network. The results from Learn++ algorithm show a very high level of confidence of the neural network in classifying sample data accurately.

2.2 Fractal Analysis

The fractal dimension of an object indicates something about the extent to which the object fills space. On the other hand, the fractal dimension of a time series shows how turbulent the time series is and also measures the degree to which the time series is scale-invariant. The method used to estimate the fractal dimension using the Hurst exponent for a time series is called the rescaled range (R/S) analysis, which was invented by Hurst (1951) when studying the Nile River in order to describe the long-term dependence of the water level in rivers and reservoirs. The estimation of the fractal dimension given the approximated Hurst exponent will be explained in the following sections. The simulated fractal Brownian motion time series in Fig. 2.1 was done for different Hurst exponents. In Fig. 2.1 (1a) we have an anti-persistent signal with a Hurst exponent of $H = 0,3$ and the resulting fractal dimension is $D_f = 1,7$. Fig. 2.1 (1b) shows a random walk signal with $H = 0,5$ and the corresponding fractal dimension is $D_f = 1,5$. In Fig. 2.1 (1c) a persistent time series signal with $H = 0,7$ and a corresponding fractal dimension of $D_f = 1,5$ is also shown. From Fig. 2.1 we can easily note the degree to which the data contain jagged features that needs to be removed before the signal is passed over to the neural network model for further analysis in uncovering the required information.

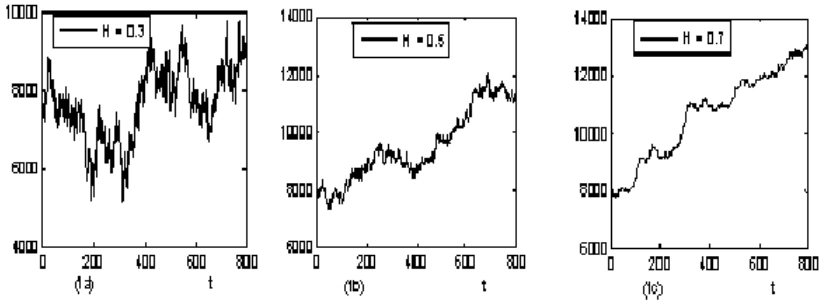


Figure 2.1: Fractal Brownian motion simulation-(1a) *anti-persistent signal*,(1b) *random walk signal*,(1c) *persistent signal*

2.3 Rescaled Range Analysis

In this section we describe a technique for estimating the quality of a time series signal to establish the intervals that are of importance to use in our Neural Network. The rescaled range (R/S) analysis is a technique that was developed by Hurst, a hydrologist, who worked on the problem of reservoir control on the Nile River dam project at around 1907. His problem was to determine the ideal design of a reservoir based upon the given record of observed river discharges. An ideal reservoir never empties or overflows. In constructing the model, it was common to assume that the uncontrollable process of the system, which at the time was the influx due to the rainfall, followed a random walk due to the many degrees of freedom in the weather. When Hurst examined this assumption he gave a new statistical measure, the Hurst exponent (H). His statistical method is very robust and has a few underlying assumptions. The Hurst statistical method can be used to classify time series signals into random and non-random series. The analysis is very robust with respect to the underlying distribution of the process. Using the R/S analysis, one also finds the average non-periodic cycle, if there is any, and the degree of persistence in trends due to long memory effects, Skjeltorp (2000).

2.3.1 The R/S Methodology

The main idea behind using the R/S analysis for our investigation is to establish the scaling behavior of the rescaled cumulative deviations from the mean, or the distance that the system travels as a function of time relative to the mean. The intervals that display a high degree of persistence are selected and grouped in rebuilding the signal to be used as an input to the NN model. A statistical correlation approach is used in recombining the intervals. The approach of recombining different data intervals is inherited from the discovery that was made by Hurst (1951) were it was observed that the distance covered by an independent system from its mean increases on average, by the square root of time e.g. t^2. If the system under investigation covers a larger distance than this from its mean, it cannot be independent by Hurst's definition; the changes must be influencing each other and therefore have to be correlated, Hutchinson & Poggio (1994). The following is the approach used in the investigation: the first requirement is to start with a time series in prices of length M. This time series is then converted into a time series of logarithmic ratios or returns of length $N = M - 1$ such that Eq.2.1 is

$$N_i = \log(\frac{M_{i+1}}{M_i}), i = 1, 2, ..., (M - 1) \qquad (2.1)$$

Divide this time period into T contiguous sub periods of length j, such that $T * j = N$. Each sub period is labelled I_t, with $t = 1, 2...T$. Then, each element in I_t is labelled $N_{k,t}$ such that $k = 1, 2, , j$. For each sub period I_t of length j the average is calculated as shown in Eq.2.2

$$e_t = \frac{1}{j}\sum_{k=1}^{j} N_{k,t} \qquad (2.2)$$

Thus, e_t is the average value of the N_i contained in sub-period I_t of length j. We then calculate the time series of accumulated departures $X_{k,t}$ from the mean for each sub period I_t, defined in Eq.2.3

$$X_{k,t} = \sum_{i=1}^{k}(N_{i,t} - e_t)k = 1, 2, ..., j \qquad (2.3)$$

Now, the range that the time series covers relative to the mean within each sub period is defined in Eq.2.4

$$R_{I_t} = max(X_{k,t}) - min(X_{k,t}), 1 < k < j \tag{2.4}$$

Next calculate the standard deviation of each sub-period as shown in Eq.2.5

$$X_{k,t} = \sqrt{\frac{1}{j}\sum_{i=1}^{k}(N_{i,t} - e_t)^2} \tag{2.5}$$

Then, the range of each sub period R_{I_t} is rescaled/normalized by the corresponding standard deviation S_{I_t}. This approach is done for all the T sub intervals we have for the series. As a result the average R/S value for length j is shown in Eq.2.6

$$e_t = \frac{1}{T}\sum_{t=1}^{T}(\frac{R_{I_t}}{S_{I_t}}) \tag{2.6}$$

Now, the calculations from the above equations were repeated for different time horizons. This is achieved by successively increasing j and repeating the calculations until we covered all j integers. After having calculated the R/S values for a large range of different time horizons j, we plot $log(R/S)_j$ against $log(n)$. By performing a least squares linear regression with $log(R/S)_j$ as the dependent variable and $log(n)$ as the independent one, we find the slope of the regression which is the estimate of the Hurst exponent (H). The relationship between the fractal dimension and the Hurst exponent is modelled in Eq.2.7

$$D_f = 2 - H \tag{2.7}$$

2.3.2 The Hurst Interpretation

If, $H \in (0,5;1]$ it implies that the time series is persistent which is characterized by long memory effects on all time scales, this is evident from a study done by Gammel (1998). This also implies that all hourly prices are correlated with all future hourly price changes; all daily price changes are correlated with all future daily prices changes, all weekly price changes are correlated with all future weekly price changes and so on. This is one of the key characteristics of fractal time series as discussed earlier. The persistence implies that if the series has been up or down in the last period then the chances are that it will continue to be up and down, respectively, in the next period. The strength of the trend reinforcing behavior, or persistence, increases as H approaches 1. This impact of the present on the future can be expressed as a correlation function G as shown in Eq.2.8

$$G = 2^{2H-1} - 1 \tag{2.8}$$

In the case of $H = 0,5$ the correlation $G = 0$, and the time series is uncorrelated. However, if $H = 1$ we see that that $G = 1$, indicating a perfect positive correlation. On the other hand, when $H \in [0; 0,5)$ we have an antipersistent time series signal (interval). This means that whenever the time series has been up in the last period, it is more likely to be down in the next period. Thus, an antipersistent time series will be more jagged than a pure random walk as shown in Fig. 2.1. The intervals that showed a positive correlation coefficient were selected as inputs to the Neural Network.

2.4 Incremental Online Learning Algorithm

An incremental learning algorithm is defined as an algorithm that learns new information from unseen data, without necessitating access to previously used data. The algorithm must also be able to learn new information from new data and still retain knowledge from the original data. Lastly, the algorithm must be able to teach new classes that may be introduced by new data. This type of learning algorithm is sometimes referred to as a 'memory less' on-line learning algorithm. Learning new information without requiring access to previously used data, however, raises 'stability-plasticity dilemma', this is evident from a study done by Carpenter *et al.* (1992). This dilemma indicates that a completely stable classifier maintains the knowledge from previously seen data, but fails to adjust in order to learn new information, while a completely plastic classifier is capable of learning new data but lose prior knowledge. The problem with the Multi-Layer Perceptron is that it is a stable classifier and is not able to learn new information after it has been trained. In this paper, we propose a fractal theory technique and online incremental learning algorithm with application to time series data. This proposed approach is implemented and tested on the classification of stock options movement direction with the Dow Jones data used as the sample set for the experiment. We make use of the Learn++ incremental algorithm in this study. Learn++ is an incremental learning algorithm that uses an ensemble of classifiers that are combined using weighted majority voting. Learn++ was developed by Polikar *et al.* (2002) and was inspired by a boosting algorithm called adaptive boosting (AdaBoost). Each classifier is trained using a training subset that is drawn according to a distribution. The classifiers are trained using a weakLearn algorithm. The requirement for the weakLearn algorithm is that it must be able to give a classification rate of at least 50% initially and then the capability of Learn++ is applied to improve the short fall of the weak MLP. For each database D_k that contains training sequence, S, where S contains learning examples and their corresponding classes, Learn++ starts by initializing the weights, w, according to the distribution D_T, where T is the number of hypothesis. Initially the weights are initialized to be uniform, which gives equal probability for all instances to be selected to the first training subset and the distribution is given by Eq.2.9

$$D = \frac{1}{m} \tag{2.9}$$

Where m represents the number of training examples in database S_k. The training data are then divided into training subset T_R and testing subset T_E to ensure weakLearn capability. The distribution is then used to select the training subset T_R and testing subset T_E from S_k. After the training and testing subset have been selected, the weakLearn algorithm is implemented. The weakLearner is trained using subset, T_R. A hypothesis, h_t obtained from weakLearner is tested using both the training and testing subsets to obtain an error,ϵ_t:

$$\epsilon_t = \sum_{t:h_t(x_i)\neq y_i} D_t(i) \tag{2.10}$$

The error is required to be less than $\frac{1}{2}$; a normalized error β_t is computed using:

$$\beta_t = \frac{\epsilon_t}{1 - \epsilon_t} \tag{2.11}$$

If the error is greater than $\frac{1}{2}$, the hypothesis is discarded and new training and testing subsets are selected according to D_T and another hypothesis is computed. All classifiers generated so far, are combined using weighted majority voting to obtain composite hypothesis, H_t

$$H_t = \arg\max_{y \in Y} \sum_{t:h_t(x)=y} \log \frac{1}{\beta_t} \tag{2.12}$$

Weighted majority voting gives higher voting weights to a hypothesis that performs well on its training and testing subsets. The error of the composite hypothesis is computed as in Eq.2.13 and is given by

$$E_t = \sum_{t:H_t(x_i) \neq y_i} D_t(i) \tag{2.13}$$

If the error is greater than $\frac{1}{2}$, the current composite hypothesis is discarded and the new training and testing data are selected according to the distribution D_T. Otherwise, if the error is less than $\frac{1}{2}$, the normalized error of the composite hypothesis is computed as:

$$B_t = \frac{E_t}{1 - E_t} \tag{2.14}$$

The error is used in the distribution update rule, where the weights of the correctly classified instances are reduced, consequently increasing the weights of the misclassified instances. This ensures that instances that were misclassified by the current hypothesis have a higher probability of being selected for the subsequent training set. The distribution update rule is given in Eq.2.15

$$w_{t+1} = w_t(i) \cdot B_t^{[|H_t(x_i) \neq y_i|]} \tag{2.15}$$

Once the T hypotheses are created for each database, the final hypothesis is computed by combining the composite hypothesis using weighted majority voting given in Eq.2.16

$$H_t = \arg\max_{y \in Y} \sum_{k=1}^{K} \sum_{t:H_t(x)=y} \log \frac{1}{\beta_t} \tag{2.16}$$

2.5 Confidence Measurement

An intimately relevant issue is the confidence of the classifier in its decision, with particular interest on whether the confidence of the algorithm improves as new data become available. The voting mechanism inherent in Learn++ hints to a practical approach for estimating confidence: decisions made with a vast majority of votes have better confidence than those made by a slight majority, Polikar *et al.* (2004). We have implemented weighted exponential voting based confidence metric by McIver & Friedl (2001) with Learn++ as

$$C_i(x) = P(y = i|x) = \frac{\exp^{F_i(x)}}{\sum_{k=1}^{N} \exp^{F_k(x)}}, 0 \leq C_i(x) \leq 1 \tag{2.17}$$

Where $C_i(x)$ is the confidence assigned to instance x when classified as class i , $F_i(x)$ is the total vote associated with the i^{th} class for the instance x and N is the number of classes. The total vote $F_i(x)$ class received for any given instances is computed as in Eq.2.18

$$F_i(x) = \sum_{t=1}^{N} \begin{pmatrix} \log \frac{1}{\beta_t},\ if\ h_t(x) = i \\ 0,\ otherwise \end{pmatrix} \tag{2.18}$$

The confidence of a winning class is then considered as the confidence of the algorithm in making the decision with respect to the winning class. Since $C_i(x)$ is between 0 and 1, the confidences can be translated into linguistic indicators as shown in Table 2.1. These indicators are adopted and used in interpreting our experimental results.

Table 2.1: Linguistic representation of confidence estimates

Confidence range (%)	Confidence level
$90 \leq C \leq 100$	Very High (VH)
$80 \leq C < 90$	High (H)
$70 \leq C < 80$	Medium (M)
$60 \leq C < 70$	Low (L)
$C < 60$	Very Low (VL)

Equations 2.17 and 2.18 allow the Fractal-Learn++ algorithm to determine its own confidence in any classification it makes. The desired outcome of the confidence analysis is to observe a high confidence on correctly classified instances, and a low confidence on misclassified instances, so that the low confidence can be used to flag those instances that are being misclassified by the algorithm. A second desired outcome is to observe improved confidences on correctly classified instances and reduced confidence on misclassified instances, as new data becomes available so that the incremental learning ability of the algorithm can be further confirmed.

2.6 Forecasting Framework

2.6.1 Experimental Data

In our empirical analysis, we set out to examine the daily changes of the Dow Jones Index. For a detailed discussion on the Dow Jones Indexes refer to a document by MacKinlay (1988). The database used consisted of 800 instances of the Dow Jones average consisting of the Open, High, Low and Close values during the period of January 2003 to December 2005; 400 instances were used for training and all the remaining instances were used for validation.

2.6.2 Model Input Selection

The function of choice is as discussed in R/S analysis section, the rescaled range analysis function. This technique was chosen because of its non-parametric characteristics, its ability to establish self-similarity within data points, long-memory effects on data, sensitivity to initial conditions as well as its ability to establish non-cyclic periods. From all these abilities the function makes it easy for determining regions of persistence and non-persistence within the time series using the R/S analysis method that estimates the Hurst exponent. The processed time series data from the R/S analysis function is combined with extra four data points to form a signal that is used as an input to the Neural Network model.This can be observed from figure Fig. 2.2. The proposed framework is to classify the eighth day's directional movement given the selected persistent signal from the previous seven days data. Within each of the seven previous days four data were collected in the form of the *Open*, *High*, *Low* and *Close* values of the stock on that day. In Fig. 2.3(a) a combination of all these data points resulted in a signal with twenty-eight points. Fig. 2.3(b) shows all highly correlated data intervals were combined to construct a signal that was later mixed with the previous day's raw data and the resulting signal was used as an input to the Learn++ algorithm.

2.6.3 Experimental Results

The two binary classes were 1 (to indicate an upward movement direction of returns in Dow Jones stocks) and -1 (to indicate a predicted fall/downward direction of returns in the Dow Jones stocks). The training dataset of 400 instances were divided into three subsets S_1, S_2 and S_3, each with 100 instances containing both classes to be used in four training sessions. In each training session, only one of these datasets was used. For each training session $k, (k = 1, 2, 3)$ three weak hypotheses were generated by Learn ++. Each hypothesis h_1, h_2 and h_3 of the k^{th} training session was generated using a training subset T_{R_t} and a testing subset T_{E_t}. The results show an improvement from coupling the MLP with a fractal R/S analysis algorithm; MLP hypothesis (weakLearner) performed over 59% compared to the 50% without fractal analysis. We also observed a major

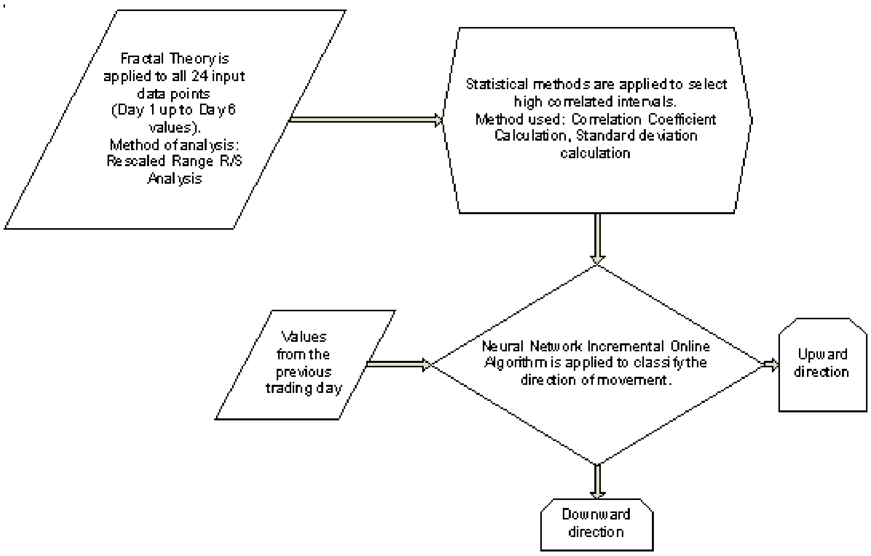

Fractal Theory is applied to all 24 input data points (Day 1 up to Day 6 values). Method of analysis: Rescaled Range R/S Analysis
Statistical methods are applied to select high correlated intervals. Method used: Correlation Coefficient Calculation, Standard deviation calculation
Values from the previous trading day
Neural Network Incremental Online Algorithm is applied to classify the direction of movement.
Upward direction
Downward direction

Figure 2.2: Model framework for fractal *R/S* technique and online neural network time series analysis

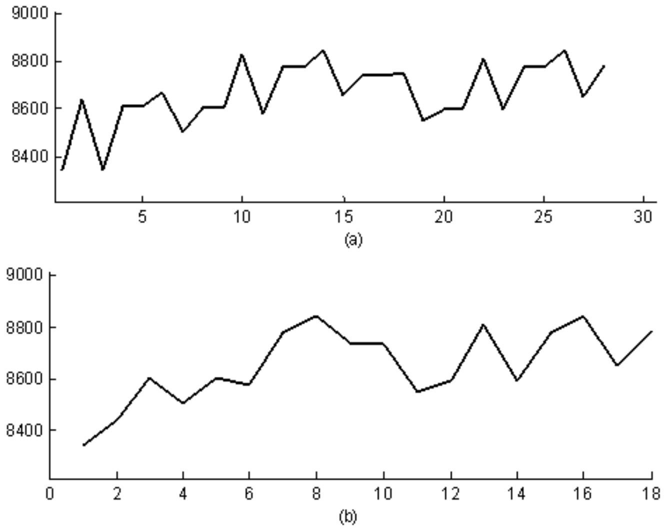


Figure 2.3: A simulated fractal analysis for 7-day time series *H=0.563* (a) represent the all data points for *7-day* time series (b)represent the reconstructed signal from the optimal correlated intervals that are selected using fractal theory

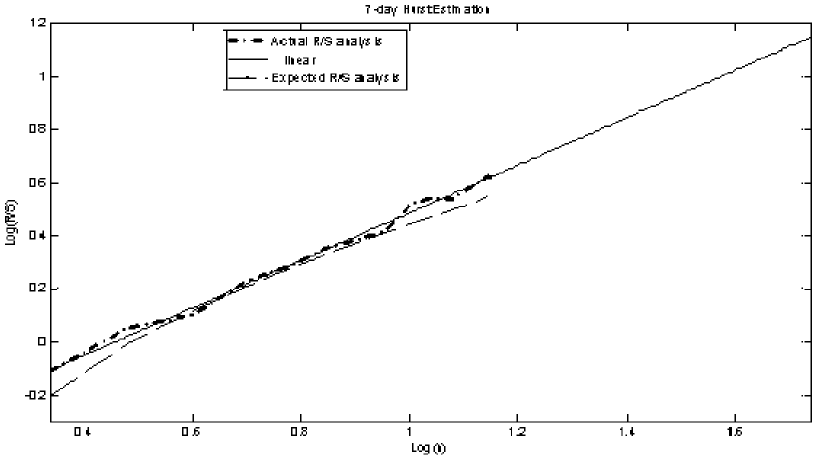


Figure 2.4: Log *(R/S)* as a function of log n for the *7-day* data fractal analysis The solid line for $n > 3$ is a linear fit to Actual R/S signal using $R/S = an^H$ with *h=0.557* and intercept a *=-0.4077*

classification accuracy improvement from 73% (without fractal R/S algorithm) to over 83% (with fractal R/S algorithm implemented) on the testing data. The hypothesis were combined by making use of weighted majority voting that was introduced by Littlestone & Warmuth (1994).

The results demonstrate a performance improvement property of the Fractal-Learn++ on a given sample space. This can be traced back to the persistent data that was used as input to the network. The last row of Table 2.2 shows the classification performance on the validation dataset, which gradually improved indicating that the confidence of framework increases as hard examples are introduced. Making use of the fractal techniques is observed to have boosted the incremental learning capability of Learn++. This improvement is due to the long memory relationships and dependencies on the data that are established by the fractal R/S algorithm.

The performance shown in Table 2.2 indicates that the algorithm is improving its generalization capacity as new data become available. The improvement is modest due to the fact that the majority of all new information is already learned in the first training session. Table 2.3 indicates that the vast majority of correctly classified instances tend to have very high confidences, with continually improved confidences at consecutive training sessions. While a considerable portion of misclassified instances also had high confidence for this database, the general desired trends of increased confidence on correctly classified instances and decreasing confidence on misclassified ones were notable and dominant, as shown in Table 2.4.

Table 2.2: Training and generalization performance of Fractal-Learn++ model

Database	Class(1)	Class(-1)	Test Performance (%)
S_1	68	32	74
S_2	55	45	77
S_3	62	48	82
Validate	58	52	–

Table 2.3: Algorithm confidence results on the testing data subset

		VH	H	M	VL	L
Correctly classified	S_1	26	24	25	15	9
	S_2	60	7	22	8	5
	S_3	55	11	21	3	11
Incorrectly classified	S_1	23	7	13	3	8
	S_2	27	0	1	3	4
	S_3	21	1	2	4	2

Table 2.4: Confidence trends for the time series: testing data subset

	Increasing Steady	Decreasing
Correctly classified	123	3
Misclassified	9	31

2.7 Results Summary

In this chapter, we propose the use of fractal analysis techniques in conjunction with a neural network incremental algorithm to predict financial markets movement direction. As demonstrated in our empirical analysis, fractal re-scaled range (R/S) analysis algorithm is observed to establish clear evidence of persistence and long memory effects on the time series data. For any 7-day interval analysis we find the Hurst exponent of $H = 0,563$ for the range $7 < n < 28$ (where n is the number of data points). A high degree of self-similarity is noted in the daily time series data. These long-term memory effects observed in the previous section are due to the rate at which information is shared amongst the investors. Thus, the Hurst exponent can be said to be a measure of the impact of market sentiment, generated by past events, upon future returns in the stock market. The importance of using fractal R/S algorithm is noted in the improved accuracy and confidence measures of the Learn++ algorithm. This is a very comforting outcome, which further indicates that the proposed methodology does not only process and analyze time series data but it can also incrementally acquire new and novel information from additional data. As a recommendation, future work should be conducted to implement techniques that are able to provide stable estimates of the Hurst exponent for small data sets as this was noted to be one of the major challenges posed by time series data.

Chapter 3

Main Conclusion

In this document, a study is conducted to explore new ways in which time series analysis can be carried out using emerging computational intelligence techniques and physical sciences techniques. One of the observations that also emerged from this investigation is an improvement on accurate performance of an existing neural network model after the introduction of a fractal analysis technique. In this study it is proven that the computational intelligence tools are essential for describing and characterizing the dynamics of non-linear processes without any intrinsic time scale.

The observations from this investigation are presented in the form of a proceedings paper in which the technique of converting a weak learning algorithm into a strong classifier is introduced. This is done through the use of generating an ensemble of hypotheses and later regrouping them using the weighted majority voting technique. This study also introduces the concept of incremental learning whereby the classifier is able to continuously identify data that it has not seen before and group it into the respective categories thereby enforcing the ability of adapting to learning new information without the need for retraining the model with a new data set. A summary of the initial results were published in the Lecture Notes in Computer Science 2006 by Springer, Lunga & Marwala (2006a), of which a copy of the published paper is attached at the end of this document.

Although the results from the first proceedings paper indicated a success in converting a weakLearner into a strong learning classifier, the initial proposed framework faced some challenges. One of the major challenges that is observed from the first model is the presents of noise samples that continuously biases the initial guessing of the weakLearner hypothesis. The weak learning algorithm is noted to constantly initialize its hypothesis to an average correct labelling (classification) of 50%, which is a result of random guessing. This result indicates the requirement for boosting the initial guessing of the weakLearner. A method proposed in this study provides a very powerful approach in eliminating noise samples from the data base.

In this section we propose the use of advanced fractal techniques to analyze time series data. The objective of this new proposal is to establish regions of persistent data, random data, and non-persistent data within the input signal and later choose only a persistent portion of the signal to train the weakLearner. The outcome has resulted in a improved algorithm that took less time in training and also showed an increase in the algorithm's confidence in decision making. These findings imply that there are patterns and trends in the time series stock returns that persist over time and different time scales. This provides a theoretical platform supporting the use of technical analysis and active trading rules to produce above average returns. The findings prove to be also useful in improving the current models or to explore new models that implement the use of fractal scaling. A summary of the results as presented in chapter 2 were submitted for publishing to the Australian Joint Conference on Artificial Intelligence.

A continuation of this work would be to use multifractals, which is a kind of generalized fractal analysis, and carries the analysis of dynamic non-linear behavior even to greater analysis. The concept of multi-fractals has been applied successfully in analyzing non-linear systems from a synthesis perspective. The approach would improve on the challenges that we faced with our proposed model, which are: the sensitivity of the proposed re-scaled range (R/S) analysis method to short-term dependencies, which constantly biased our estimate of the Hurst exponent H, which is in agreement to findings by other researchers. Another shortfall that can be addressed by multifractals is the handling of very small datasets ($n < 8$), where n is the number of data points, and very large datasets. It is observed that for a very small n the Hurst exponent turns to be unstable.

Thus for the small sample sizes the approximation of the Hurst exponent are not accurate. And also robust ways in which the lower bound sample size can be defined are necessary to bring about an efficient algorithm.

Appendix A

Neural Networks

A.1 Introduction

This section covers the models that have been implemented in modelling the financial markets. Neural Networks (NNs) are currently being applied to nearly every field in the engineering and financial industries e.g. bio-medical equipment and risk analysis, Zhang *et al.* (1998). NNs are used in the banking sector to predict the issuing of bonds as well as the risk of issuing a loan to a new customer. NNs are also used in the finance market to predict share prices which helps in portfolio management. Furthermore, NNs are used in industry for predicting, the life processes of products, optimization of business process, conflict management, and the loss of information in databases as well as in most radar applications for object identification, Bishop (1995).

Most of the conventional financial markets forecasting methods use time series data to determine the future behavior of indicators. Wong & Selvi (1998) conducted a survey which indicated that NNs are more appropriate for time-series data rather than conventional regression methods. They also discovered that the integration of neural networks with other technologies, such as decision support systems ($DSSs$), expert systems, fuzzy logics, genetic algorithms, or robotics can improve the applicability of neural networks in addressing various types of finance problems.

Quah & Srinivasan (1999) developed a methodology for improving returns on stock investment through neural network selection. Their findings displayed the generalization ability of the neural network in its application to financial markets. This was evident through the ability to single out performing stock counters and having excess returns in the basic stock selection system overtime. Moreover, neural networks also showed its ability in deriving relationships in a constrained environment in the moving window stock selection system thus making it even more attractive for applications in the field of finance, Leke (2005).

Kuo *et al.* (2002) proposed a methodology that uses artificial neural networks and fuzzy neural networks with fuzzy weight elimination for prediction of share prices. Previously, statistical methods, which include regression methods and moving average methods, were used for such predictions. These methods are limited in that they are efficient only for seasonal or cyclical data. Results obtained by Kuo *et al.* (2002) proved to be more accurate than conventional statistical methods. In the following section an investigation is conducted to try and explain what a neural network is. The study goes on to investigate the back-propagation neural network-training algorithm. The study also introduces the Multi-Layer Perceptron neural network architecture

A.2 What is a Neural Network

The area of NNs is the intersection between the Artificial Intelligence and Approximation Algorithms. One could think of it as algorithms for ‘smart approximation’. These algorithms are used in (to name a few) universal approximation (mapping input to the output), development of tools capable of learning hidden patterns from their environment, tools for finding non-evident dependencies between data and so on.

The NNs structure models the human brain in how it processes information. Neurolobiologists believe that the brain is similar to a massively parallel analog computer, containing about 10^{10} simple processors which each require a few milliseconds to respond to input. The brain is a multi layer structure that works as a parallel computer capable of learning from the 'feedback' it receives from the world and changing its design by growing new neural links between neurons or altering activities of existing ones. The brain is composed of neurons, which are interconnected. With NNs technology, we can use parallel processing methods to solve some real-world problems where it is difficult to define a conventional algorithm. NNs possess the property of adaptive learning which is the ability to learn how to do tasks based on the data given for training or initial experience and this was shown in an investigation that was conducted by Vehtari & Lampinen (2000). In Fig. A.1 and Fig. A.2, Bishop (1995), the network functions as follows: each neuron receives a signal from the neurons in the previous layer, and each of those signals is multiplied by a separate weight value. The weighted inputs are summed, and passed through a limiting function, which scales the output to a fixed range of values. The output of the limiter is then broadcast to all of the neurons in the next layer. So, to use the network to solve a problem, we apply the input values to the inputs of the first layer, allow the signals to propagate through the network, and read the output values. In Fig. A.1: stimulation is applied to the inputs of the first layer, and signals propagate through the middle (hidden) layer(s) to the output layer. Each link between neurons has a unique weighting value.

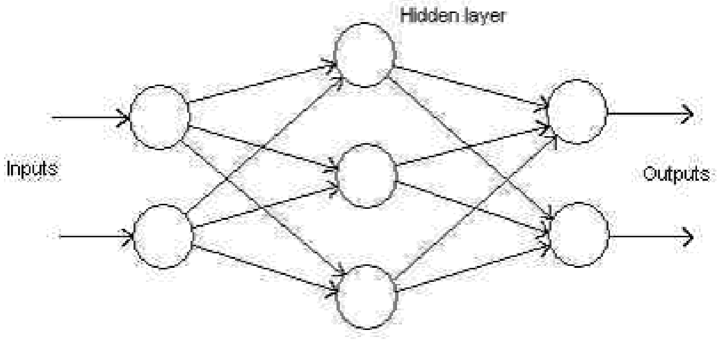


Figure A.1: An example of a neural network model

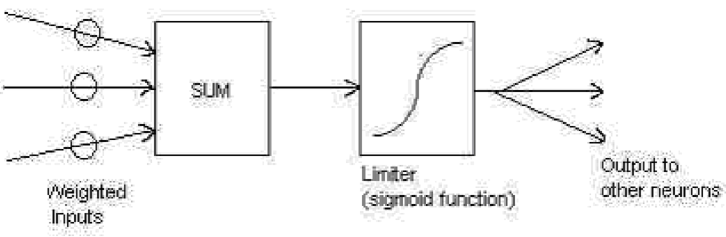


Figure A.2: The Structure of a Neuron

Inputs from one or more previous neurons are individually weighted, then summed. The result is non-linearly scaled between 0 and +1, and the output value is passed on to the neurons in the next layer. Since the real uniqueness or 'intelligence' of the network exists in the values of the weights between neurons, we need a method of adjusting the weights to solve a particular problem. For this type of network, the most common learning algorithm is called Back Propagation (BP). A BP network learns by example, that is, we must provide a learning set that consists of some input examples and the known-correct output for each case. So, we use these input-output examples to show the network what type of behavior is expected, and the BP algorithm allows the network to adapt.

A.3 Back Propagation Algorithm

The BP learning process works in small iterative steps: one of the example cases is applied to the network, and the network produces some output based on the current state of its synaptic weights (initially, the output will be random). This output is compared to the known-good output, and a mean-squared error signal is calculated. The error value is then propagated backwards through the network, and small changes are made to the weights in each layer. The weight changes are calculated to reduce the error signal for the case in question. The whole process is repeated for each of the example cases, then back to the first case again, and so on. The cycle is repeated until the overall error value drops below some pre-determined threshold. At this point we say that the network has learned the problem ‘well enough’ - the network will not exactly learn the ideal complex function, but rather it will asymptotically approach the ideal function.

A.3.1 Advantages of the Back Propagation Algorithm

In the literature review it is found that NNs architectures, which were implemented using the BP algorithm, displayed the algorithm’s strong generalization ability to predict the performance of securities. This is evident through the ability to single out performing stock and having excess returns in the basic stock selection system overtime from the results that were presented by Quah & Srinivasan (1999).

A.3.2 Disadvantages of the Back Propagation Algorithm

The selection of the learning rate and momentum is very important when using this type of algorithm. Failure to choose value for these to the required tolerance will result in the output error oscillating and thus the system will become unstable because it will not converge to a value (system will diverge). In the following section a brief discussion is given on how to select the required optimal parameters for this algorithm.

A.4 Neural Network Architectures

Most studies use the straightforward Multi-Layer Perceptron (MLP) networks while others employ some variants of the MLP, Vehtari & Lampinen (2000). The real uniqueness or intelligence of the MLP network exists in the values of the weights between the neurons. Thus the best method to adjust these weights in our study is the Back Propagation as discussed in the previous sections. There is a criterion required to assist us in choosing the appropriate network structure as proposed by Quah & Srinivasan (1999). There are four major issues in the selection of the appropriate network:

- Appropriate training algorithm.
- Architecture of the ANN.
- The Learning Rule.
- The Appropriate Learning Rates and Momentum.

A.4.1 Selection of the Appropriate Training Algorithm

Since the sole purpose of this project is to predict the performance of the chosen stock portfolio's movement direction, the historical data that is used for the training process will have a known outcome (whether it is considered moving up or moving downwards). Among the available algorithms, the BP algorithm designed by Rumelhart & McClelland (1986) is one of the most suitable methodologies to be implemented as it is being intensively tested in finance. Moreover, it is recognized as a good algorithm for generalization purposes.

A.4.2 Selection of the System Architecture

Architecture, in this context, refers to the entire structural design of the NNs (Input Layer, Hidden Layer and Output Layer). It involves determining the appropriate number of neurons required for each layer and also the appropriate number of layers within the Hidden Layer. The logic of the Back Propagation method is the hidden layer. The hidden layer can be considered as the crux of the Back Propagation method. This is because hidden layer can extract higher-level features and facilitate generalization, if the input vectors have low-level features of a problem domain or if the $\frac{output}{input}$ relationship is complex. The fewer the hidden units, the better is the NNs is able to generalize. It is important not to over-fit the NNs with large number of hidden units than required until it can memorize the data. This is because the nature of the hidden units is like a storage device. It learns noise present in the training set, as well as the key structures. No generalization ability can be expected in these. This is undesirable, as it does not have much explanatory power in an unseen environments.

A.4.3 Selection of the Learning Rule

The learning rule is the rule that the network will follow in its error reducing process. This is to facilitate the derivation of the relationships between the input(s) and output(s). The generalized delta rule developed by McClelland & Rumelhart (1988) is mostly used in the calculations of weights. This particular rule is selected in most researches as it proves to be quite effective in adjusting the network weights.

A.4.4 Selection of the Appropriate Learning Rates and Momentum

The Learning Rates and Momentum are parameters in the learning rule that aid the convergence of error, so as to arrive at the appropriate weights that are representative of the existing relationships between the input(s) and the output(s). As for the appropriate learning rate and momentum to use, some neural network software has a feature that can determine appropriate learning rate and momentum for the network to start training with. This function is known as 'Smart Start'. Once this function is activated, the network will be tested using different values of learning rates and momentum to find a combination that yields the lowest average error after a single learning cycle. These are the optimum starting values as using these rates improve error converging process, thus require less processing time.

This function automatically adjusts the learning rates and momentum to enable a faster and more accurate convergence. In this function, the software will sample the average error periodically, and if it is higher than the previous sample then the learning rate is reduced by 1%. The momentum is 'decayed' using the same method but the sampling rate is half of that used for the learning rate. If both the learning rate and momentum decay are enabled then the momentum will decay slower than the learning rate. In general cases, where these features are not available, a high learning rate and momentum (e.g. 0.9 for both the Learning Rates and Momentum) are recommended, as the network will converge at a faster rate than when lower figures are used. However, too high a learning rate and momentum will cause the error to oscillate and thus prevent the converging process. Therefore, the choice of learning rate and momentum are dependent on the structure of the data and the objective of using the NNs. The following section introduces the Multi-Layer Perceptron neural network architecture that is implemented in this study.

A.5 Multi-Layer Perceptron

A Multi-Layer Perceptron (MLP) is made up of several layers of neurons, each layer is fully connected to the next one. Moreover, each neuron receives an additional bias input as shown in A.3. It is both simple and based on mathematical grounds. Input quantities are processed through successive layers of "neurons". There is always an input layer, with a number of neurons equal to the number of variables of the problem, and an output layer, where the perceptron response is made available, with a number of neurons equal to the number of quantities computed from the inputs. The layers in between are called hidden layers. With no hidden layer the perceptron can only perform linear tasks. Each neuron of a layer other than the input layer computes a linear combination of the outputs of the previous layer, plus the bias. The coefficients of the linear combinations plus the biases are called the weights. Neurons in the hidden layer then compute a non-linear function of their input. The non-linear function is the sigmoid function $y(x) = \frac{1}{1-\exp(-x)}$. $MLPs$ are probably the most widely used architecture for practical applications.

- W_{ij}^k = weight from unit i (in layer k) to unit j (in layer $k+1$)

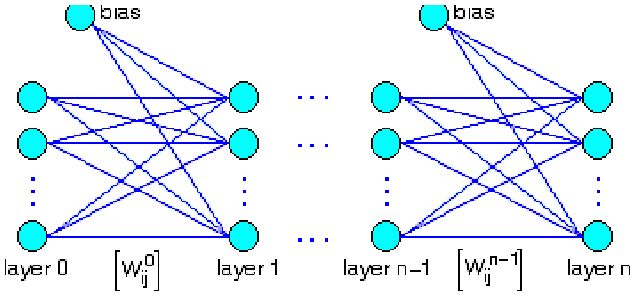


Figure A.3: A fully interconnected, biased, n-layered back-propagation network

- I_p = input vector (pattern p) = $(I_1^p, I_2^p, ..., I_b^p)$ where b is the dimensionality of the input vector.
- A_p = Actual output vector (pattern p) = $(A_1^p, A_2^p, ..., A_c^p)$ where c is the dimensionality of the output vector.

The network can be described as in Eq.A.1:

$$y_k = f_{outer}(\sum_{j=1}^{M} w_{kj}^2 \times f_{inner}(\sum_{j=1}^{d} w_{ji}^1 + w_{j0}^1) + w_{k0}^2) \tag{A.1}$$

Where y_k represents the k^{th} output, f_{outer} represents the output layer transfer function, f_{inner} represents the input layer transfer function, w represents the weights and biases, (i) represent the i^{th} layer. For this project the linear combiner (activation) function will be used for each output and the hyperbolic tangent or the sigmoid function will be used in the hidden layers.

A.6 Conclusion

In this appendix we give a brief overview of neural networks. A literature background was conducted so that one could understand the existing applications of neural networks as well as its relevance in solving the problem within this study. The back propagation training algorithm is also discussed. A procedure on how to choose the required parameters for the back propagation algorithm is also explained in detail. An introduction of the neural network architecture that is to be implemented in the study is given. The appendix ends by showing a mathematical model of the multi-layer perceptron.

Appendix B

Learn++

B.1 Ensemble of Classifiers

Learn++ was developed by Polikar *et al.* (2002) from an inspiration by Freund & Schapire (1997)'s 'adaptive boosting' (AdaBoost) algorithm, originally proposed for improving the accuracy of weak learning algorithms. In 'Strength of weak learning', Freund & Schapire (1997) showed that for a two class problem, a *weakLearner* that almost always achieves high errors can be converted into a strong learner that almost always achieves arbitrarily low errors using a procedure called boosting. Both Learn++ and AdaBoost are based on generating an ensemble of weak classifiers, which are trained using various distributions of the training data and then combining the outputs (classification rules) of these classifiers through a majority voting scheme. In the context of machine learning, a classification rule generated by a classifier is referred to as a hypothesis. The voting algorithm "weighted majority algorithm" was developed by Littlestone & Warmuth (1994), this algorithm assigns weights to different hypotheses based on an error criterion. Weighted hypotheses are then used to construct a compound hypothesis, which is proved to perform better than any of the individual hypotheses.

Littlestone & Warmuth (1994), also showed that the error of the compound hypothesis is closely linked to the error bound of the best hypothesis. Freund & Schapire (1996) later developed AdaBoost extending boosting to multi-class, learning problems and regression type problems. They continually improved their work on boosting with statistical theoretical analysis of the effectiveness of voting methods. Recently, they have introduced an improved boosting algorithm that assigns confidences to predictions of decision tree algorithms. Their new boosting algorithm can also handle multi-class databases, where each instance may belong to more than one class.

Independent of Schapire and Freund, Breiman (1996) developed an algorithm very similar to boosting in nature. Breiman's *bagging*, short for 'bootstrap aggregating', is based on constructing ensembles of classifiers through continually retraining a base classifiers with bootstrap replicates of the training database. In other words, given a training dataset S of m samples, a new training dataset S is obtained by uniformly drawing m samples with replacement from S. This is in contrast to AdaBoost where each training sample is given a weight based on the classification performance of the previous classifier. Both boosting and bagging require weak classifiers as their base classification algorithm because both procedures take advantage of the so-called *instability* of the weak classifier.

This instability causes the classifiers to construct sufficiently different decision surfaces for minor modifications in their training datasets , this is observed from the results presented by Schapire *et al.* (1998). Both bagging and boosting have been used for construction of strong classifiers from weak classifiers, and they have been compared and tested against each other by several authors. The idea of generating an ensemble of classifiers is not new. A number of other researchers have also investigated the properties of combined classifiers. In fact, it was Wolpert (1992) who introduced the idea of combining hierarchical levels of classifiers, using a procedure called 'stacked generalization' and other authors analyzed error sensitivities of various voting and combination schemes whereas other researchers concentrated on the capacity of voting systems. Ji & Ma (1997) proposed an alternative approach to AdaBoost for combining classifiers. Their approach generates simple perceptrons of random parameters and then combines the perception outputs using majority voting. Ji & Ma (1997) also gave an excellent review of various methods for combining classifiers.

B.2 Strong and Weak Learning

Consider an instance space X, a concept class $C = c : X \rightarrow \{0, 1\}$, a hypothesis space $H = h : X \rightarrow \{0, 1\}$, and an arbitrary probability distribution D over the instance space X. In this setup, c is the true concept that we wish to learn, h is the approximation of the learner to the true concept c. Although the following definitions are for a two-class concept, they can be naturally generalized to the n-class concept as was demonstrated by Polikar (2000) when he investigated the application of Learn++ to the gas sensing problem . We assume that we have access to an oracle, which obtains a sample $x \in X$, according to the distribution D, labels it according to c, and outputs $< x, c(x) >$.

Definition *(Strong Learning)*: A concept class C defined over an instance space X is said to be potentially Probably Approximately Correct (PAC) learnable using hypothesis class H if for all target concepts $c \in C$, a consistent learner L is guaranteed to output a hypothesis $h \in H$ with error less than $\frac{1}{2}$ observing ϵ to be $\epsilon > 0$ and probability of at least $1 - \delta$, $\delta > 0$ after processing a finite number of examples, obtained according to D. The learner L is then called a PAC learning algorithm, or a *stronglearner* of the sample space. Note that PAC learning imposes very stringent requirements on the learner L, since L is required to learn all concepts within a concept class with arbitrarily low error (approximately correct) and with an arbitrarily high probability (probably correct). A learner that satisfies these requirements may not be easy to implement practically, hence such a learner is only a potentially PAC learning algorithm. However, finding a learner L_0 that can learn with fixed values of ϵ, (say ϵ_0) and δ , (say δ_0) might be quite conceivable.

Definition *(Weak Learning)*: A concept class C defined over an instance space X is weakly learning using the hypothesis class H, if there exists a learning algorithm L_0 and constants $\epsilon_0 > \frac{1}{2}$ and $\delta_0 < 1$ such that for every concept $c \in C$

and for every distribution D on the instances space X, the algorithm L_0, given access to an example set drawn from (c, D), returns a hypothesis $h \in H$ with probability of at least $1 - \delta_0$ and $Error_c, D(h)$. Note that unlike strong learning, weak learning imposes the least possible stringent conditions, since it is required to perform only slightly better than a random guess for a two class problem Polikar (2000).

B.3 Boosting the Accuracy of a Weak Learner

Boosting is based on running the weak learning algorithm for a number of times to obtain many weak hypotheses, and using a majority vote to determine the final hypothesis whose error is less than any one of the individual weak hypotheses. For the generation of each additional hypothesis, the learner is presented with a different distribution of the training data, and it is forced to learn increasingly misclassified examples.

B.4 Boosting for Two-class Problems

Let c be a boolean target concept and $D_1 = D$, where D is the original distribution of the training data. The weak learning algorithm L_0 is ran with training examples from D_1 and weak hypothesis h_1 is obtained such that $Error_c$, $D_1(h_1) = \epsilon_1 < \epsilon < \frac{1}{2}$. Attention is then focused on examples misclassified by h_1. A new set of training examples is obtained from a new distribution D_2 as follows: An oracle flips a fair coin; on heads, it returns an example $< x, c(x) >$ such that $h_1 = c(x)$. On tails the oracle returns an example $< x, c(x) >$ such that $h_1 = c(x)$. Therefore, the new distribution D_2 picks up correctly classified examples with a probability of $\frac{1}{2}$ and picks up misclassified examples with probability of $\frac{1}{2}$. Now let h_2 be the new hypothesis returned by the learner L_0 on D_2. This hypothesis would also have an $Error_c$, $D_2(h_2) = \epsilon_2 < \epsilon_1 < \frac{1}{2}$. The next hypothesis h_3 is then returned by L_0 in which h_1 and h_2 disagree. The hypothesis h_3 will then have

$Error_c$, $D_3(h_3) = \epsilon_3 < \epsilon_1 < \frac{1}{2}$. Then the final hypothesis h is chosen from the majority voting of the three hypotheses. Freund & Schapire (1997) showed that $Error_c$, $D(h)$ is bounded by $3 \times \epsilon^2 - 2 \times \epsilon^3$, which is less than ϵ. Thus with each iteration, the error of the final hypothesis decreases and can potentially converge to an arbitrarily low value of error. Furthermore, it is proven that the error only takes polynomial time to reach arbitrarily low values. Byorick & Polikar (2003) gave an upper bound on the number of training examples required to reach these low error levels.

B.5 Boosting for Multi-class Problems

AdaBoost algorithm is based on the belief that a large number of solvers, each solving a simple problem, can be used to solve a very complicated problem when the solutions to simple problems are combined in an appropriate form. Learn++ is an extension to the original boosting algorithm, which was developed to boost the performance of multi-class weak learning classifiers by generating various weak classification hypotheses and combining them through weighted majority voting for the classes predicted by the individual hypotheses. These hypotheses are obtained by retraining the classifier using a different subset of the training dataset, chosen strategically based on the performance of the previous hypotheses. In general terms, each instance in the training database is assigned a weight, and these weights are updated based on the performance of the previous hypothesis. Misclassified instances are assigned larger weight, whereas correctly classified instances are smaller weights. The training dataset for the next hypothesis is then chosen based on the current weights of the instances. Instances with higher weights have higher chances of being selected into the next training set. Furthermore, the weights are normalized to satisfy the conditions to form a probability distribution function, referred to as the distribution D of the training dataset. This is observed in a study that was conducted by Carpenter *et al.* (1992). Consistently misclassified instances are considered as hard examples of the dataset, and the algorithm is designed to train subsequent classifiers with increasingly harder instances of the dataset. Inputs to the AdaBoost algorithm are:

- Sequence of labelled examples (training data,S) drawn randomly from an unknown distribution D,
- Weak learning algorithm, **weakLearner**
- An integer T that specifies the number of hypotheses (iterations) to be generated by **weakLearner**.

The algorithm AdaBoost, which is given in the next section, proceeds as follows: In iteration $t = 1, 2, ..., T$, AdaBoost provides the weak learning algorithm, weakLearner, with a training subset data drawn according to distribution D_t from the original training data $S = [(x_1, y_1), (x_2, y_2), ..., (x_m, y_m)]$ where x_1 are training data instances and y_1 are the corresponding correct labels. weakLearner then computes a hypothesis (classifier) $h_t : X \rightarrow Y$, which correctly classifies a percentage of the training set. That is, weakLearner's goal is to find a hypothesis h_t, which minimizes the training error as in Eq.B.1

$$\epsilon_t = \sum_{t:h_t(x_t)\neq y_t} D_t(i) \tag{B.1}$$

The initial distribution D_t is typically chosen to be uniform over S, unless there is prior knowledge to choose otherwise, that is, $D_1(i)= \frac{1}{m} \ \forall_i$. This gives equal probability to all instances in S to be drawn into the initial data subset. The distribution is updated using Eq.B.2

$$D_{t+1} = \frac{D_t(i)}{Z_t} \tag{B.2}$$

Where $Z_t = \sum_i D_t(i)$ is a normalized constant chosen to ensure that D_{t+1} will be a distribution, and $\beta_t = \frac{\epsilon_t}{1-\epsilon_t}$. The parameter β can be thought of as a normalized error term, since for

$0 < \epsilon_t < \frac{1}{2}$,$0 < \beta < 1$. In fact, Schapire *et al.* (1998) showed that Eq.B.3 is the optimum choice for the parameter

$$\beta_t = \frac{\epsilon_t}{1 - \epsilon_t} \tag{B.3}$$

The distribution update rule in Eq.B.2 ensures that weights for misclassified instances are increased, whereas weights for correctly classified instances are reduced. Thus, AdaBoost focuses on examples that seem to be hardest for weakLearner to learn. At the end of T iterations, AdaBoost combines the weak hypotheses $h_1, ..., h_T$ into a single final hypothesis h_{final} by computing the weighted majority of the weak hypothesis as in Eq.B.4

$$h_{final} = argmax_{y \in Y} \times \sum_{t:h_t(x)=y} \log \frac{1}{\beta_t} \tag{B.4}$$

Where the weight of hypothesis h_t is defined to be $\log{(\frac{1}{\beta})}$ so that the greater weight is given to a hypothesis with lower error. For a given instance x, Eq.**??** outputs the label y, that maximizes the sum of the weights of the weak hypothesis predicting that label. It should be noted that AdaBoost requires ϵ_t, error of each hypothesis h_t, to be less than $\frac{1}{2}$. For a binary class problem, this is the least restrictive requirement one could have, since an error of $\frac{1}{2}$ for a binary class problem is equivalent to random guessing.

Algorithm AdaBoost **Input**:

- Sequence of m examples $S = [(x_1, y_1), (x_2, y_2), ..., (x_m, y_m)]$ with labels $y_i \in Y = 1, ..., C$ drawn from a distribution $\mathbf{D}$
- Weak learning algorithm **weakLearner**,
- Integer T specifying number of iterations.

Initialize $D_1(i) = \frac{1}{m}, \forall_i$

Do for $t = 1, 2, ..., T$

1. Call **weakLearner**, providing it with the distribution D_t
2. Get back a hypothesis $h_t : X \rightarrow Y$
3. Calculate the error of h_t: $\epsilon_t = \sum_{i:h_t(x_i) \neq y_i} D_t(i)$ if $\epsilon_t > \frac{1}{2}$, Then set $T = t - 1$ and abort loop.

4. Set $\beta_t = \frac{\epsilon_t}{1-\epsilon_t}$

5. Update distribution D_t: $D_{t+1}(i) = \frac{D_t(i)}{Z_t} \times B_t$ if $h_t(x_i) = y_i$ **Otherwise** $D_{t+1}(i) = \frac{D_t(i)}{Z_t}$

Where $Z_t = \sum_i D_t(i)$ is a normalized constant chosen to ensure that D_{t+1} will be a distribution function. The final hypothesis is

$$h_{final}(x) = argmax_{y \in Y} \sum_{t:h_t(x)=y} \log(\frac{1}{\beta_t}) \tag{B.5}$$

Note that any hypothesis with an error larger than $\frac{1}{2}$ can be negated to obtain an alternative hypothesis with an error less than $\frac{1}{2}$. However, obtaining a maximum error of $\frac{1}{2}$ becomes increasingly difficult as the number of classes increases, since for a k class problem the error for random guessing is proven to be $(k-1)$ k by Freund & Schapire (1997). Therefore, the choice of a weak learning algorithm with a classification performance of at least 50% may not be very easy. Any classification algorithm can be substituted as a weak learner by modifying appropriate parameters. For example, an MLP with a larger number of nodes/layers and a smaller error goal is, in general, a stronger learner than the one with smaller number of nodes and a higher error goal.

It should be noted that the use of strong learners that achieve high classification performance on a particular training data are not recommended for use with boosting since there is little to be gained from their combination, or they may lead to over fitting of the data. One of the nice properties of the AdaBoost algorithm is that it is less likely to encounter over fitting problems since only a portion of the instances space is learned by individual hypothesis. In addition, an ensemble of weak learners performs at least as well as a strong learner, but in considerably less time, since strong learners spend most of the training time during fine-tuning at lower error rates.

B.6 Connection to Incremental Learning

In order to achieve incremental learning capability we assume that the new dataset S_{new} belongs to a slightly or significantly different portion of the original data distribution (data space) D. In boosting, classifiers are added to learn regions of the pattern space that include increasingly 'difficult' instances. Since, S_{new} is likely to be misclassified by the learner, instances of S_{new} can be considered to come from a 'misclassified' region of the data distribution D.

Fig. B.1 conceptually illustrates the procedure of combining simpler classifiers, similar to Fig. B.2, but this time in the context of incremental learning.

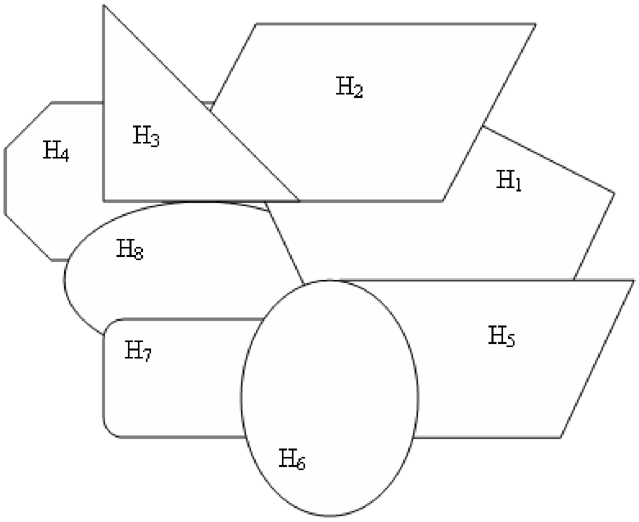


Figure B.1: Conceptual representation of combining classifiers

The dark curve in Fig. B.2 is the decision boundary to be learned and the two sides of the dashed line represent the two training data S_1 and S_2. Individual classifiers (hypotheses) are illustrated with simple geometric figures, where h_1 through h_4 are generated due to training with S_1 and h_5 through h_8 generated due to training with S_2. Each hypothesis decides whether a data point is within or outside the decision boundary, where simple shapes represent the region learned by a weak learner. Note that this setup is identical to that used by AdaBoost, and hence AdaBoost can be used for incremental learning of new data, with the understanding that new data corresponds to *harder* examples of the distribution.

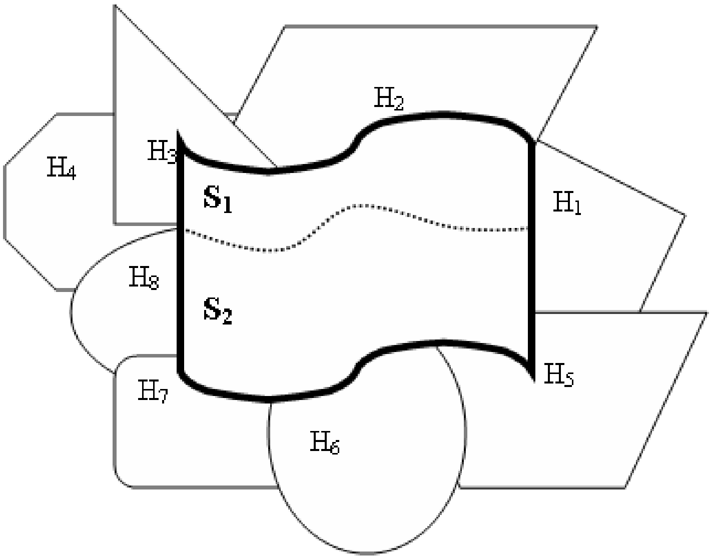


Figure B.2: Conceptual representation of combining classifiers

However, the distribution update rule given in the AdaBoost algorithm in the previous section does not allow efficient incremental learning, particularly when new data include new classes. This is because the distribution update rule for D_{t+1} depends on the classification performance of h_t, a single hypothesis. To understand the shortcoming of this distribution update scheme with respect to incremental learning, consider T hypotheses $h_1, h_2, ..., h_T$ generated with training datasets $S_1, S_2, ..., S_T$, all drawn from the same distribution D_1, consisting of C classes. Assume that a new database of distribution D_2 become available which includes instances from an additional $(C+1)^{st}$ class. AdaBoost will select the next training set S_{T+1} from D_2 based on the classification performance of h_T, which was generated from a database that did not include the $(C+1)$ class.

Furthermore, note that once the training set is selected, each classifier is independent and is likely to perform equally well (or poorly) on all classes (unless one class is particularly more difficult than others). In other words, hypothesis

h_{T+1} will perform equally well on instances coming from $(C+1)^{st}$ class. Therefore, patterns from the $(C+1)^{st}$ class will not necessarily be selected into S_{T+1}, and they will have no advantage of being selected into the training dataset in the next little iteration. Consequently, learners will not be forced to focus on the patterns of the new class. The final weighted majority will then fail to recognize samples from the new class for much iteration to come, increasing the time and space complexity of the algorithm.

The distribution update rule can, however, be forced to focus on instances of the new class, if the updated rule is based on the combined performance of all t hypothesis generated during the previous t iterations. Let us call the weighted majority voting of the previous t hypotheses the composite hypothesis H_t. Note that when instances from a new class become available, they will be misclassified by H_t, since none of the previous t training sessions have seen instances from the new class. Therefore, updating the training dataset distribution based on the classification results of H_t will ensure that the selection of instances from the new class is favored. The Learn++ algorithm incorporates these ideas into a smarter distribution update rules, this algorithm is described in the following section. As will be shown in the following sections, Learn++ does not only allow the weak learning algorithm to learn incrementally from new data, but at the same time it converts the weak learning algorithm into a very powerful classifier.

B.7 Learn++: An Incremental Learning Algorithm

Learn++ is an algorithm that allows any classifier to learn incrementally from additional data, without forgetting what is previously learned, even when the new data includes a new class. To achieve this rather ambitious task, Learn++ introduces a number of modifications to the basic ideas of AdaBoost. First, the *training error* is redefined. In AdaBoost, the error ϵ_t is the training error of the weak learner, calculated using the training patterns misclassified by h_t. This constitutes a problem, when using neural network type classifiers such as $MLPs$ as base classifiers, since a converged neural network almost always performs close to 100% on its training data for any nontrivial error goal. This is particularly true for Radial Basis Functions, $ARTMAP$ type algorithms, since these algorithms guarantee 100% correct classification on their training database Byorick & Polikar (2003).

In order to ensure weak learning and a nonzero ϵ_t, Learn++ first divides the selected training dataset T_t into subsets TR_t and TE_t where TR_t is the training subset and TE_t is the testing subset for the current training dataset T_t. During the t^{th} iteration, the weak learner is trained on TR_t and tested on the entire set $T = TR_t + TE_t$. For each iteration, different training and testing subsets are selected based on previous performance. The error of t^{th} hypothesis on the combined $TR_t + TE_t$ set is defined as ϵ_t. Eventually (almost) all patterns in the original training dataset will be established by the weakLearner, and hence using this definition of training error is justified.

Initially all instances have equal likelihood to be selected into the first training dataset (unless there is prior knowledge to choose otherwise). In the following discussion, new *databases* that become available for incremental learning are denoted with the subscript k and the (unknown) distribution from which the k^{th} database is drawn will be denoted by the script D_k, whereas the distribution of the current training dataset at t^{th} iteration is denoted by D_t. In each iteration t, the distribution D_t. is obtained by normalizing the current weights of the instances in step 1. In step 2, training (TR_t) and testing (TE_t) subsets are randomly generated from the current database D_k, according to the distribution D_t. These subsets are used as inputs to $WeakLearn$ in step 3, which return the hypothesis h_t in step 4. The error, ϵ_t, is then computed from the misclassified patterns $TR_t + TE_t$. If $\epsilon_t > \frac{1}{2}$, h_t is discarded, and new TR_t and TE_t are generated. Instead of updating the distribution based on instances misclassified by h_t, Learn++ then calls the weighted majority voting in step 5 to compute the composite hypothesis, H_t.

B.7.1 Learn++ Algorithm

Input: For each database drawn from D_k, $k = 1, 2, ..., K$

- Sequence of m examples $S = [(x_1, y_1), (x_2, y_2), ..., (x_m, y_m)]$ with labels $y_i \in Y = 1, ..., C$
- Weak learning algorithm **weakLearner**,
- Integer T specifying number of iterations.

Do for $t = 1, 2..., T$ **Initialize** ${w_1}^i = D(i) = \frac{1}{m}\forall_i$, unless there is prior knowledge to select otherwise. **Do for** $k = 1, 2, ...K$

1. Set $D_t {=} \frac{w_t}{\sum_{i=1}^{m} w_t(i)}$ so that D_t is a distribution
2. Randomly choose training TR_t and testing TE_t subsets according to D_t
3. Call weakLearner, providing it with TR_t
4. Get back a hypothesis $h_t : X \rightarrow Y$, and. Calculate the error of $h_t{:}\epsilon_t = \sum_{i:h_t(x_i)\neq y_i} D_t(i)$ on $T = TR_t + TE_t$ if $\epsilon_t > \frac{1}{2}$, set $t = t - 1$, discard h_t and go to step 2. Otherwise, compute normalized error as $\beta_t {=} \frac{\epsilon_t}{1-\epsilon_t}$
5. Call weighted majority, obtain the overall hypothesis $h_t = argmax_{y\in Y} \sum_{t:h_t(x)=y} (\log \beta_t)$ and compute the overall error $E_t = \sum_{i:H_t(X_i)\neq y_i} D_t(i) = \sum_{i=1}^{m} D_t(i) \lceil |H_t(x_i \neq y_i|) \rceil$ if $E_t > \frac{1}{2}$, set $t = t - 1$, discard H_t and go to step 2.
6. Set $\beta_t {=} \frac{\epsilon_t}{1-\epsilon_t}$ and update the weights of the instances: $w_{t+1} = w_t(i) \times B_t^{[|H_t(x_i)\neq y_i|]}$

Call weighted majority on combined hypotheses H_t and **Output** the final hypothesis $H_{final} = argmax_{y\in Y} \sum_{k=1}^{K} \sum_{t:H_t(x)=y} (\log \beta_t)$ Note that the composite hypothesis H_t is computed similar to the final hypothesis h_{final} in AdaBoost, that is Eq.B.6

$$h_t = argmax_{y\in Y} \sum_{t:h_t(x)=y} (\log \beta_t) \tag{B.6}$$

which makes the training error given in Eq.B.7 to be :

$$E_t = \sum_{i:H_t(X_i)\neq y_i} D_t(i) = \sum_{i=1}^{m} D_t(i) \left\lceil |H_t(x_i \neq y_i|) \right\rceil | \tag{B.7}$$

on misclassified instances, where $\lceil\cdot\rceil$ is 1 if the predicate holds true, and 0 otherwise. From this error, we compute the normalized error as in Eq.B.8

$$\beta_t = \frac{\epsilon_t}{1-\epsilon_t} \tag{B.8}$$

In step 6, the composite hypothesis, H_t , and its normalized error, B_t are then used to update the distribution of the distribution of the instances to be used in the next training session. The update rule is given by Eq.B.9

$$w_{t+1} = w_t(i) \times B_t^{[|H_t(x_i)\neq y_i|]} \tag{B.9}$$

where $w_t(i)$ is simply the weights of the i^{th} instance for the t^{th} training session. At the end of T iterations (for each database D_k), the final hypothesis is obtained by combining the composite hypothesis H_t as in Eq.B.10

$$h_t = argmax_{y\in Y} \sum_{t:h_t(x)=y} (\log \beta_t) \tag{B.10}$$

In this algorithm, when a new dataset contains new classes, the composite hypothesis H_t will misclassify instances from the new class, and the algorithm will be forced to learn these instances. A disadvantage of this approach is the large storage capacity required to store all hypotheses generated to learn the additional class. Addition of data with new classes results in generating a large number of hypotheses in order to remove the bias of the combined classifier towards the previous classes. This bias can be reduced significantly by changing the final classification rule to Eq.B.11

$$H_{final} = argmax_{y\in Y} \sum_{k=1}^{K} \sum_{t:H_t(x)=y} (\log \beta_t) \tag{B.11}$$

Which combines the original weak hypothesis h_t It should be noted however that subsequent hypotheses are still generated with training data selected according to a distribution based on the performance of the composite hypotheses H_t, which, along with other modifications, distinguishes Learn++ from AdaBoost. Experimental results that are summarized in Chapter 2 and in Appendix D demonstrate that both final classification rules given by equations Eq.B.10 and Eq.B.11 achieve the same performance level in incremental learning problems including new classes.

B.8 Conclusion

This chapter introduced the incremental algorithm, Learn++, in detail focusing more on its development from the AdaBoost algorithm. As shown in this study, Learn++ displays its ability to learn from new data even when the data introduces new classes. Learn++ makes no assumptions as to what kind of weak learning algorithm is to be used. Any weak learning algorithm can serve as the base classifier of Learn++, though the algorithm is optimized for supervised neural network-type classifiers, whose weakness can be easily controlled via network size and error goal. Learn++ is also intuitively simple, easy to implement,and converges much faster that strong learning algorithms. This is because using weak learners eliminates the problem of fine-tuning and over fitting, since each learner only roughly approximates the decision boundary of the sample data. Initial results using this algorithm looked promising, but because of the significant room for improvement we proposed the use of fractal theory to eliminate non-persistent data that introduced a huge margin of randomness in our initial results, Lunga & Marwala (2006a). This results of this improvement are summarized in Lunga & Marwala (2006b).

Appendix C

Fractals Theory

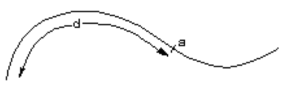
C.1 An Introduction to Fractals

C.1.1 What is a fractal ?

Definition: A rough or fragmented geometric shape that can be subdivided into parts, each of which is (at least approximately) a reduced/size copy of the whole. Mathematically: this is a set of points whose fractal dimension exceeds its topological dimension.

C.1.2 Fractal Geometry

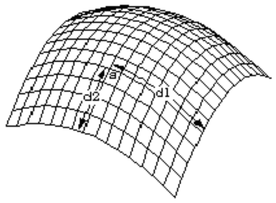
Almost all geometric forms used for building man made objects belong to Euclidean geometry; they are comprised of lines, planes, rectangular volumes, arcs, cylinders, spheres, etc. These elements can be classified as belonging to an integer dimension 1, 2 or 3. This concept of dimensioning can be described both intuitively and mathematically. Intuitively we say that a line is one-dimensional because it only takes one number to uniquely define a point on it. That one number could be the distance from the start of the line. This applies equally well to the circumference of a circle, a curve, or the boundary of any object.



Any point "a" on a one dimensional curve can be represented by one number, the distance d from the start point.

Figure C.1: Line dimension representation

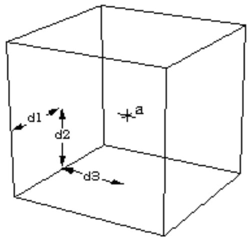
A plane is two-dimensional since in order to uniquely define any point on its surface we require two numbers. There are many ways to arrange the definition of these two numbers but we normally create an orthogonal coordinate system. Other examples of two-dimensional objects are the surface of a sphere or an arbitrary twisted plane.



Any point "a" on a two dimensional surface can be uniquely represented by two numbers. One of the many possible methods is to grid the surface and measure two distances along the grid lines.

Figure C.2: Two dimension representation

The volume of some solid object is 3 dimensional on the same basis as above; it takes three numbers to uniquely define any point within the object.



Any point "a" in three dimensions can be uniquely represented by three numbers. Typically these three numbers are the coordinates of the point using an orthogonal coordinate system.

Figure C.3: Three dimension representation

A more mathematical description of dimension is based on how the 'size' of an object behaves as the linear dimension increases. In one dimension consider a line segment. If the linear dimension of the line segment is doubled then obviously the length (characteristic size) of the line has doubled. In two dimensions, if the linear dimension of a rectangle for example is doubled then the characteristic size, the area, increases by a factor of 4. In three dimension, if the linear dimension of a box are doubled then the volume increases by a factor of 8. This relationship between dimension D, linear scaling L and the resulting increase in size S can be generalized and written as

$$S = L^D \tag{C.1}$$

This is just telling us mathematically what we know from everyday experience. If we scale a two-dimensional object for example, then the area increases by the square of the scaling. If we scale a three dimensional object the volume increases by the cube of the scale factor. Rearranging the above gives an expression for dimension depending on how the size changes as a function of linear scaling, namely

$$D = \frac{log(S)}{log(L)} \tag{C.2}$$

In the examples above the value of D is an integer 1, 2 or 3 depending on the dimension of the geometry, Feder (1988). This relationship holds for all Euclidean shapes. There are, however, many shapes which do not conform to the integer based idea of dimension given above in both the intuitive and mathematical descriptions. That is, there are objects, which appear to be curves for example (e.g. time series signals), but which a point on the curve cannot be uniquely described with just one number. If the earlier scaling formulation for dimension is applied the formula does not yield an integer. There are shapes that lie in a plane but if they are linearly scaled by a factor L, the area does not increase by L squared but by some non-integer amount. These geometries are called fractals. One of the simpler fractal shapes is the von Koch snowflake in Fig. C.4. The method of creating this shape is to repeatedly replace each line segment with the following 4 line segments. The process starts with a single line segment and continues forever. The first few iterations of this procedure are shown in Figure C.5.

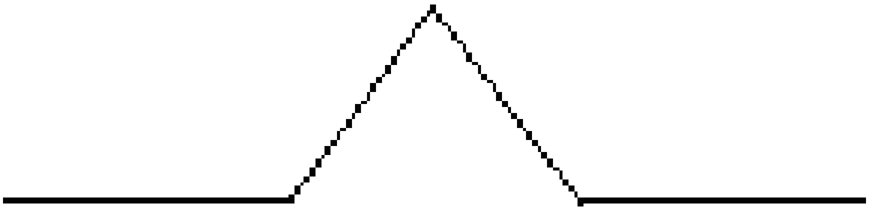


Figure C.4: Van Koch snowflake

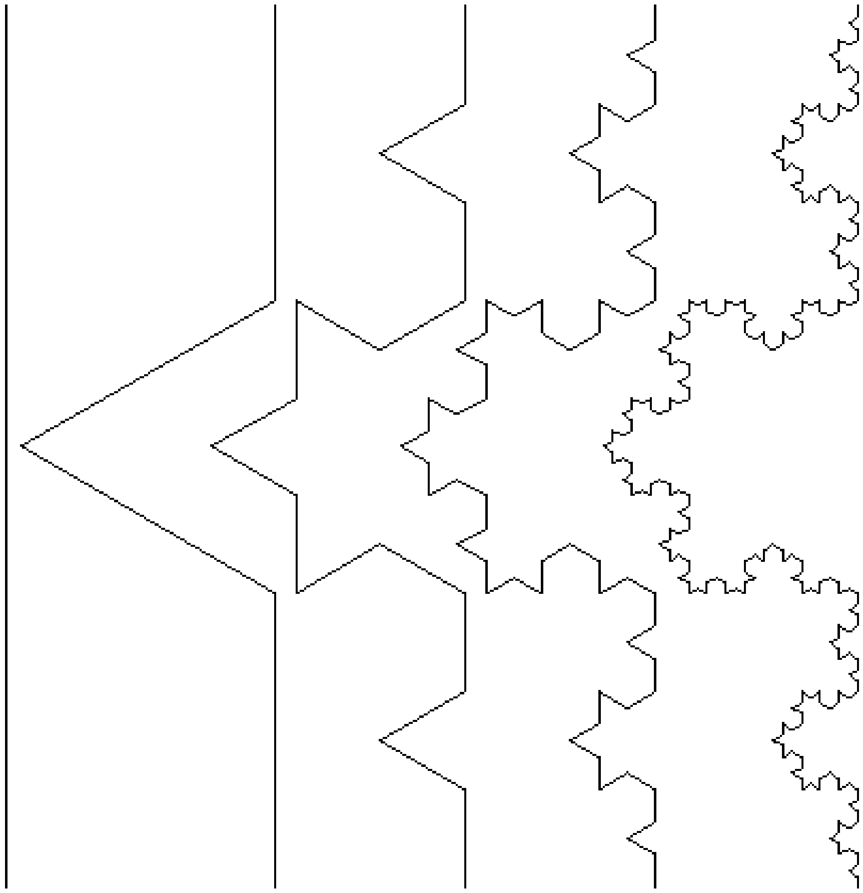


Figure C.5: Multi Van Koch Snowflake shapes combined, Goldberger(2006)

This demonstrates how a very simple generation rule for this shape can generate some unusual (fractal) properties. Unlike Euclidean shapes this object has detail at all levels. If one magnifies an Euclidean shape such as the circumference of a circle it becomes a different shape, namely a straight line. If we magnify this fractal more and more detail is uncovered, the detail is self-similar or rather, it is exactly self-similar. Put in another way, any magnified portion is identical to any other magnified portion. Note also that the curve on the right is not a fractal but only an approximation of one. This is no different from when one draws a circle; it is only an approximation to a perfect circle. At each iteration, the length of the curve increases buy a factor of $\frac{4}{3}$. Thus the limiting curve is of infinite length and indeed the length between any two points of the curve is infinite. This curve manages to compress an infinite length into a finite area of the plane without intersecting itself. Considering the intuitive notion of 1 dimensional shape, although this object appears to be a curve with one starting point and one end point, it is not possible to uniquely specify any position along the curve with one number as we expect to be able to do with Euclidean curves, which are one dimensional. Although the method of creating this curve is straightforward, there is no algebraic formula that describes the points on the curve. Some of the major differences between Fractal and Euclidean geometry are outlined in the following in Table C.1.

Table C.1: Fractal vs. Euclidean geometries

Fractal	Euclidean
Modern invention	Traditional
No specific size or scale	Based on characteristic size or scale
Appropriate geometry in nature	Suits description of man made objects
Described by an algorithm	Described by a usually simple formula

Firstly the recognition of fractal is very modern; they have only formally been studied intensively in the last 30 years compared to Euclidean geometry which goes back over 2000 years, Bunde & Havlin (1994). Secondly whereas Euclidean shapes normally have a few characteristic sizes or length scales (eg: the radius of a circle or the length of of a side of a cube) fractals have so many characteristic sizes. Fractal shapes are self-similar and independent of size or scaling. Third, Euclidean geometry provides a good description of man-made objects whereas fractals are required for a representation of naturally occurring geometries. It is likely that this limitation of our traditional language of shape is responsible for the sticking difference between mass produced objects and natural shapes. Finally, Euclidean geometries are defined by algebraic formulae, as in equation:

$$x^2 + y^2 + z^2 = r^2 \tag{C.3}$$

Equation C.3 defines a sphere. Fractals are normally the result of an iterative or recursive construction or algorithm.

C.2 Fractal Objects and Self-Similar Processes

Before describing the metrics that are used to quantitatively characterize the fractal properties time series dynamics, we first review the meaning of the term *fractal*.

The concept of a fractal is most often associated with geometrical objects satisfying two criteria: *self-similarity* and *fractional dimensionality.* Self-similarity means that an object is composed of sub-units and sub-sub-units on multiple levels that (statistically) resemble the structure of the whole object, Iannaconne & Khokha (1996). Mathematically, this property should hold on all scales. However, in the real world, there are necessarily lower and upper bounds over which such self-similar behavior applies.

The second criterion for a fractal object is that it has a fractional dimension. This requirement distinguishes fractals from Euclidean objects, which have integer dimensions. As a simple example, a solid cube is self-similar since it can be divided into sub-units of 8 smaller solid cubes that resemble the large cube, and so on. However, the cube (despite its self-similarity) is not a fractal because it has an *(=3)* dimension. The concept of a fractal structure, which lacks a characteristic length scale, can be extended to the analysis of complex temporal processes. However, a challenge in detecting and quantifying self-similar scaling in complex time series is the following: Although time series are usually plotted on a 2-dimensional surface, a time series actually involves two different physical variables.

For example, let us consider the study of time series analysis when applied to heart rate and gait dynamics which is shown in Fig. C.6, the horizontal axis represents 'time', while the vertical axis represents the value of the variable that changes over time (in this case, heart rate). These two axes have independent physical units, minutes and beats/minute, respectively. (Even in cases where the two axes of a time series have the same units, their intrinsic physical meaning is still different.) This situation is different from that of geometrical curves (such as coastlines and mountain ranges) embedded in a 2-dimensional plane, where both axes represent the same physical variable.

To determine if a 2-dimensional curve is self-similar, we can do the following test: (i) take a subset of the object and rescale it to the same size of the original object, using the same magnification factor for both its width and height; and then (ii) compare the statistical properties of the rescaled object with the original object. In contrast, to properly compare a subset of a time series with the original data set, we need two magnification factors (along the horizontal and vertical axes), since these two axes represent different physical variables. Fig. C.6 shows a heart rate *normal sinus rhythm* time series of 30 min from (a) a healthy subject at sea level,(b) a subject with congestive heart failure,(c) a subject with obstructive sleep apnea, and (d) a sudden cardiac death subject who sustained a cardiac arrest with ventricular fibrillation (VF).

Note the highly non-stationary and "noisy" appearance of the healthy variability which is related in part to fractal (scale-free) dynamics. In contrast, pathologic states may be associated with the emergence of periodic oscillations, indicating the emergence of a characteristic time scale.

To put the above discussion into mathematical terms: A time-dependent pro-

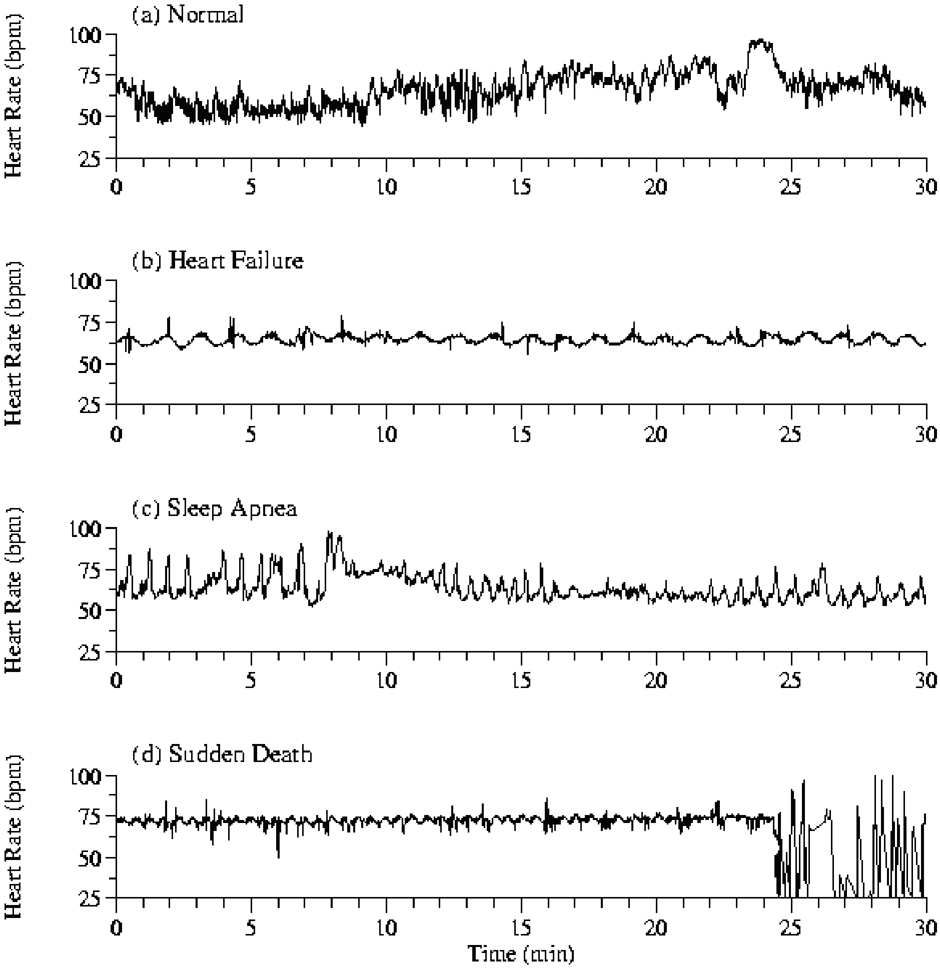


Figure C.6: Representative complex physiological fluctuations, Goldberger(2006)

cess (or time series) is self-similar if

$$y(t) \equiv a^{\alpha} \times y \times (\frac{t}{a}) \tag{C.4}$$

where $\equiv$ means that the statistical properties of both sides of the equation are identical. In other words, a self-similar process, $y(t)$, with a parameter a has the identical probability distribution as a properly rescaled process, $a^{\alpha} \times y \times (\frac{t}{a})$, i.e. a time series which has been rescaled on the $x - axis$ by a factor $a(t \rightarrow \frac{t}{a})$ and on the $y - axis$ by a factor of $(a^{\alpha}(y \rightarrow a^{\alpha} \times y))$. The exponent α is called the self-similarity parameter. In practice, however, it is impossible to determine whether two processes are statistically identical, because this strict criterion requires the same processes to have identical distribution functions (including not just the mean and variance, but all higher moments as well). Therefore, one usually approximates this equality with a weaker criterion by examining only the means and variances (first and second moments) of the distribution functions for both sides of Eq.C.4 can be calculated by a simple relation as in C.5

$$\alpha = \frac{\ln M_y}{\ln M_x} \tag{C.5}$$

Where M_y and M_x are the appropriate magnification factors along the horizontal and vertical direction, respectively. In practice, we usually do not know the value of the α exponent in advance. Instead, we face the challenge of extracting this scaling exponent (if one does exist) from a given time series, Beran (1994). To this end it is necessary to study the time series on observation windows with different sizes and adopt the weak criterion of self-similarity defined above to calculate the exponent α .

The basic idea is illustrated in Fig. C.7. Two observation windows Fig. C.7(a), window 1 with horizontal size n_1 and window 2 with horizontal size n_2, were arbitrarily selected to demonstrate the procedure. The goal is to find the correct magnification factors such that we can rescale window 1 to resemble window 2. It is straightforward to determine the magnification factor along the horizontal direction, $M_x = \frac{n_1}{n_2}$. But for the magnification factor along the vertical direction, M_y, we need to determine the vertical characteristic scales of windows

1 and window 2. One way to do this is by examining the probability distributions *histograms* of the variable y for these two observation windows Fig. C.7.

A reasonable estimate of the characteristic scales for the vertical heights, i.e., the typical fluctuations of y, can be defined by using the standard deviations of these two histograms, denoted as s_1 and s_2, respectively. Thus, we have $M_y = \frac{s_1}{s_2}$. Substituting M_x and M_y into Eq.C.5, we obtain

$$\alpha = \frac{\ln M_y}{\ln M_x} = \frac{\ln s_2 - \ln s_1}{\ln n_2 - \ln n_1} \tag{C.6}$$

This relation is simply the slope of the line that joins these two points, (n_1,s_1) and (n_2,n_2), on a log-log plot Fig. C.7(a) Two observation windows, with time scales n_1 and n_2 are shown for a self-similar time series *y(t).* In Fig. C.7(b)) Magnification of the smaller window with time scale n_1. Note that the fluctuations in Fig. C.7(a) and Fig. C.7(b) look similar provided that two different magnification factors, M_x and M_y, are applied on the horizontal and vertical scales, respectively. Fig. C.7(c) The probability distribution, $P(y)$, of the variable y for the two windows in Fig. C.7(a), where s_1 and s_2 indicate the standard deviations for these two distribution functions. Fig. C.7(d) *Log-log* plot of the characteristic scales of fluctuations, s, versus the window sizes, n.

In analyzing 'real-world' time series, we perform the above calculations using the following procedures: (1) For any given size of observation window, the time series is divided into subsets of independent windows of the same size. To obtain a more reliable estimation of the characteristic fluctuation at this window size, we average over all individual values of s obtained from these subsets. (2) We then repeat these calculations, not just for two window sizes (as illustrated above), but for many different window sizes. The exponent α is estimated by fitting a line on the log-log plot *linear regression* of s versus n across the relevant range of scales. A more profound non-parametric technique(re-scaled range analysis) that combines all the steps presented above will be introduced in the next sections.

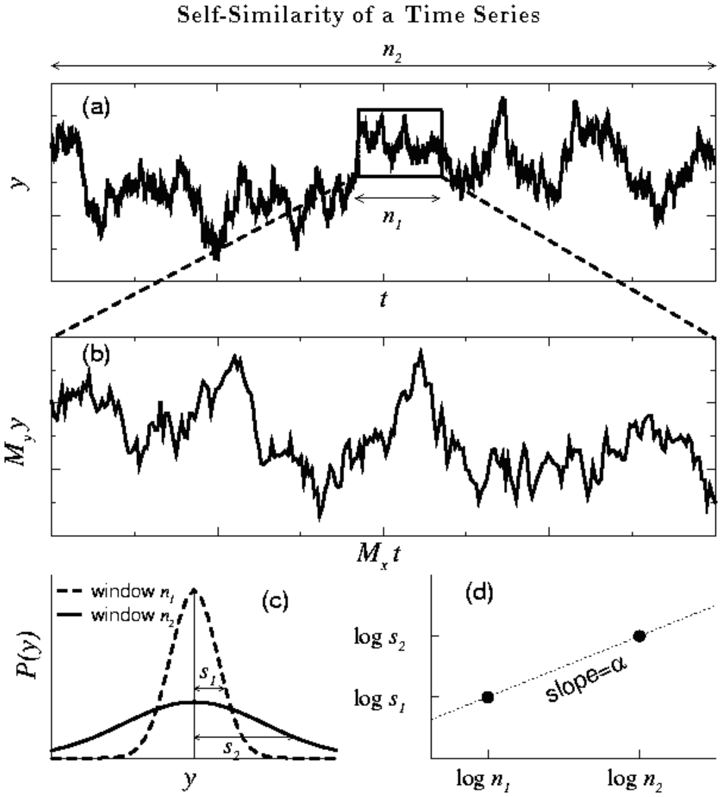


Figure C.7: Illustration of the concept of self-similarity for a simulated random walk

C.3 Mapping Real-World Time Series to Self-Similar Processes

For a self-similar process with $\alpha > 0$, the fluctuations grow with the window size in a power-law way. Therefore, the fluctuations on large observation windows are exponentially larger than those of smaller windows. As a result, the time series is *unbounded.* However, for most time series of interest, such as heart rate and financial time series, are *bounded*–they cannot have arbitrarily large amplitudes no matter how long the data set is, this is evident from a study conducted by Goldberger (2006). This practical restriction causes further complications for our analysis. Consider the case of the time series shown in Fig. C.8(A). If we zoom in on a subset of the time series, we notice an apparently self-similar pattern. To visualize this self-similarity, we do not need to rescale the y-axis ($M_y = 0$), only rescaling the $x - axis$ is needed. Therefore, according to Eq.C.6, the self-similarity parameter is 0–not an informative result. Consider another example where we randomize the sequential order of the original heart rate time series generating a completely uncorrelated *control* time series Fig. C.8(B)-white noise. The white noise data set also has a self-similarity parameter of 0. However, it is obvious that the patterns in Fig. C.8(A) and (B) are quite different. An immediate problem, therefore, is how to distinguish the trivial parameter 0 in the latter case of uncorrelated noise, from the non-trivial parameter 0 computed for the original data. Successive magnifications of the subsets show that both time series are self-similar with a trivial exponent $\alpha = 0$ (i.e.,$M_y = 1$), albeit the patterns are very different in Fig. C.8(A) and (B)

Physicists and mathematicians have developed an innovative solution for this central problem in time series analysis, Hurst (1951). The "trick" is to study the fractal properties of the accumulated (integrated) time series, rather than those of the original signals, Beran (1994). One well-known physical example with relevance to biological time series is the dynamics of Brownian motion. In this case, the random force (noise) acting on particles is bounded, similar to physiologic time series. However, the trajectory (an integration of all previous forces) of the Brownian particle is not bounded and exhibits fractal properties

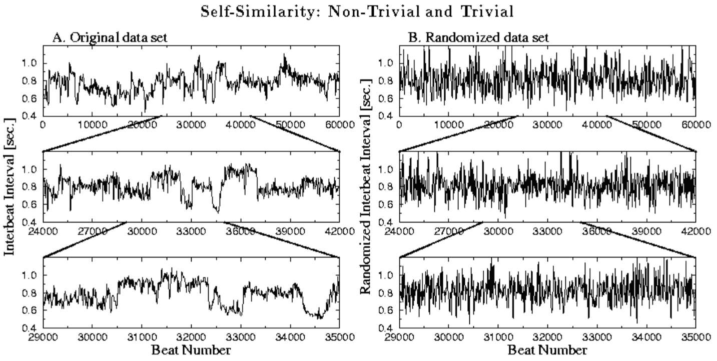


Figure C.8: A cardiac inter-heartbeat interval (inverse of heart rate) time series is shown in (A) and a randomized control is shown in (B)

that can be quantified by a self-similarity parameter. When we apply fractal scaling analysis to the integrated time series of Fig. C.8(a) and (b), the self-similarity parameters are indeed different in these two cases, providing meaningful distinctions between the original and the randomized control data sets. The details of this analysis will be discussed in the next section.

In summary, mapping the original bounded time series to an integrated signal is a crucial step in fractal time series analysis. In the rest of this chapter, therefore, we apply fractal analysis techniques after integration of the original time series. Such techniques need be non-parametric techniques as will be discussed in the following sections. A technique to be discussed in the next section was developed by Hurst (1951), a hydrologist who worked on a problem of estimating the behavior of the water flow on the Nile River in 1907.

C.4 Re-Scaled Range Analysis

In this section we describe a technique for estimating the quality of a time series signal to establish the intervals that are of importance to use in our Neural Network. The rescaled range (R/S) analysis is a technique that was developed by Hurst, a hydrologist, who worked on the problem of reservoir control on the Nile

River dam project at around 1907. His problem was to determine the ideal design of a reservoir based upon the given record of observed river discharges. An ideal reservoir never empties or overflows. In constructing the model, it was common to assume that the uncontrollable process of the system, which at the time was the influx due to the rainfall, followed a random walk due to the many degrees of freedom in the weather. When Hurst examined this assumption he gave a new statistical measure, the Hurst exponent (H). His statistical method is very robust and has a few underlying assumptions. The Hurst statistical method can be used to classify time series signals into random and non-random series. The analysis is very robust with respect to the underlying distribution of the process. Using the R/S analysis, one also finds the average non-periodic cycle, if there is any, and the degree of persistence in trends due to long memory effects, Skjeltorp (2000).

C.4.1 The R/S Methodology

The main idea behind using the (R/S) analysis for our investigation is that we establish the scaling behavior of the rescaled cumulative deviations from the mean, or the distance that the system travels as a function of time relative to the mean. The intervals that display a high degree of persistence are selected and grouped in rebuilding the signal to be used as an input to the Neural Network model. A statistical correlation approach is used in recombining the intervals. From the discovery that was made by Hurst, the distance covered an independent system increases, on average, by the square root of time. If the system covers a larger distance than this, it cannot be independent by this definition; the changes must be influencing each other and therefore have to be correlated, Hutchinson & Poggio (1994). The following is the approach we used in our investigation: we first start with a time series in prices of length M. This time series is then converted into a time series of logarithmic ratios or returns of length $N = M - 1$ such that

$$N_i = \log(\frac{M_{i+1}}{M_i}), i = 1, 2, ..., (M - 1) \tag{C.7}$$

We divide this time period into T contiguous sub periods of length j, such that $T * j = N$. Each sub period is labelled I_t, with $t = 1, 2...T$. Then, each element in I_t is labelled $N_{k,t}$ such that $k = 1, 2, , j$. For each sub period I_t of length j the average is calculated as

$$e_t = \frac{1}{j} \sum_{k=1}^{j} N_{k,t} \tag{C.8}$$

Thus, e_t is the average value of the N_i contained in sub-period I_t of length j. We then calculate the time series of accumulated departures $X_{k,t}$ from the mean for each sub period I_t, defined as

$$X_{k,t} = \sum_{i=1}^{k} N_{i,t} - e_t k = 1, 2, ..., j \tag{C.9}$$

Now, the range that the time series covers relative to the mean within each sub period is defined as

$$R_{I_t} = max(X\_k, t) - min(X\_k, t, 1 < k < j \tag{C.10}$$

Next we calculate the standard deviation of each sub-period as

$$X_{k,t} = \sqrt{\frac{1}{j} \sum_{i=1}^{k} (N_{i,t} - e_t)^2} \tag{C.11}$$

Then, the range of each sub period R_{I_t} is rescaled/normalized by the corresponding standard deviation S_{I_t}. This approach is done for all the T sub intervals we have for the series. As a result the average R/S value for length j is

$$e_t = \frac{1}{T} \sum_{t=1}^{T} (\frac{R_{I_t}}{S_{I_t}}) \tag{C.12}$$

Now, the calculations from the above equations were repeated for different time horizons. This is achieved by successively increasing j and repeating the calculations until we covered all j integers. After having calculated the R/S values for a large range of different time horizons j, we plot $log(R/S)_j$ against

$log(n)$. By performing a least squares linear regression with $log(R/S)_j$ as the dependent variable and $log(n)$ as the independent one, we find the slope of the regression which is the estimate of the Hurst exponent (H). The relationship between the fractal dimension and the Hurst exponent is modelled as

$$D_f = 2 - H \tag{C.13}$$

C.4.2 The Hurst Interpretation

If, $H \in (0,5;1]$ it implies that the time series is persistent which is characterized by long memory effects on all time scales, Gammel (1998). This also implies that all hourly prices are correlated with all future hourly price changes; all daily price changes are correlated with all future daily prices changes, all weekly price changes are correlated with all future weekly price changes and so on. This is one of the key characteristics of fractal time series as discussed earlier. The persistence implies that if the series has been up or down in the last period then the chances are that it will continue to be up and down, respectively, in the next period. The strength of the trend reinforcing behavior, or persistence, increases as H approaches 1. This impact of the present on the future can be expressed as a correlation function G as in

$$G = 2^{2H-1} - 1 \tag{C.14}$$

In the case of $H = 0,5$ the correlation $G = 0$, and the time series is uncorrelated. However, if $H = 1$ we see that that $G = 1$, indicating a perfect positive correlation. On the other hand, when $H \in [0;0,5)$ we have an antiperspirant time series signal (interval). This means that whenever the time series has been up in the last period, it is more likely to be down in the next period. Thus, an nonpersistent time series will be more jagged than a pure random walk as shown in Fig. C.8(b). The intervals that showed a positive correlation coefficient were selected as inputs to the Neural Network.

C.5 Conclusion

In this section we introduce fractal analysis and its applications to time series data. An example where fractal theory has been successfully used to extract required features within data is also given. The example we introduce in this chapter is that of estimating the *inter-beat heartbeat interval* which has been proven by Goldberger (2006) to contain the same characteristics as financial time series data. The reason for using this example in this study was due to the availability of already successfully proven applications of fractal theory to time series by, Hutchinson & Poggio (1994). The other reason was that this example seeks to establish the self-similarity characteristics between the *inter-beat heartbeat interval* of which these are same characteristic we seek to establish within the financial time series data. Another example where fractal theory has been applied successfully was that given by, Hurst (1951) in his study of the Nile River in 1907. In the implementation of the proposed methodology the re-scaled range analysis R/S is adapted.

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Appendix D

Published Paper

A summary of the paper has been accepted for publishing and it will appear in the Lecture Notes in Computer Science by Springer 2006. The full reference of the paper in this document is Lunga & Marwala (2006a). A copy of the published paper is attached with this document.