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On Fano fourfolds with nef bundle $\Lambda^2T_X$ and $\rho(X) \geq 2$

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Abstract

In this poster, I explain about the structure of Fano fourfolds whose the second exterior power of tangent bundle $\Lambda^2T_X$ is nef and Picard number $\rho(X)$ is at least 2.

Definition of nef vector bundle

$X$: smooth proj. var. / $\mathbb{C}$, $E$: vector bundle on $X$,

$\pi: \mathbb{P}(E) \to X$: projectivization, $\xi$: tautol. div.

$(\emptyset) \in$ nef (ample) $\iff$ $\xi$: nef (ample)

Known results

One of a generalization of Mori’s Theorem (Hartshorne conjecture), K. Cho and E. Sato gave a characterization of smooth quadric as a variety with ample bundle $\Lambda^2T_X$.

Ample case. Cho-Sato [CS]

$X$ sm. proj. variety with ample vect. bundle $\Lambda^2T_X$

$\Rightarrow X \cong \mathbb{P}^n$ or $\mathbb{F}_4$ : smooth quadric hypersurface

As a further generalization of this theorem, F. Campana and T. Peternell classified threefolds with nef bundle $\Lambda^2T_X$.

Nef case in 3 dim. Campana-Peternell [CP1]

$X$ sm. proj. threefold with nef vect. bundle $\Lambda^2T_X$

$\Rightarrow$ Either $T_X$: nef, the blowing up of $\mathbb{P}^3$ at a point.

or del Pezzo threefold of degree $\geq 2$ with $\rho(X) = 1$

Problem

Classify smooth projective fourfolds with nef vector bundle $\Lambda^2T_X$.

Fourfolds with nef tangent bundle are already classified by F. Campana-T. Peternell, N. Mok and J.-M. Hwang. We review the classification in Fano case.

Fano fourfolds with $T_X$ nef, [CP2], [M] [H]

$X$: smooth Fano fourfold with nef tangent bundle $T_X$.

Then $X$ is one of the following:

- $\mathbb{P}^1$, $\mathbb{Q}_4$, $\mathbb{P}^3 \times \mathbb{P}^1$, $\mathbb{Q}_5 \times \mathbb{P}^1$, $\mathbb{P}^2 \times \mathbb{P}^2$;
- $\mathbb{P}^4 \times \mathbb{P}^3 \times \mathbb{P}^2$, $\mathbb{P}^2(\mathbb{P}^2 \times \mathbb{P}^2) \times \mathbb{P}^1$, $\mathbb{P}^4 \times \mathbb{P}^4 \times \mathbb{P}^1$, $\mathbb{P}^4(\mathbb{N})$ with null correlation bundle $\mathbb{N}$.

General results

In the case where $\kappa(X) = 0$, we can classify in all dimension.

Kodaira dimension $\kappa(X) = 0$. Theorem 1

$X$: smooth projective variety of $\kappa(X) = 0$.

Then the following conditions are equivalent:

1. $\Lambda^rT_X$ is nef for $1 \leq r \leq n - 1$;
2. $T_X$ is nef;
3. There is an étale covering $\nu: A \to X$ from Abelian variety.

Proof. Nefness of $\Lambda^rT_X$ implies that $X$ has the flat tangent bundle. The theorem follows from the result of Yau.

Next, we consider the case where $X$ is Fano and obtained by the blowing up of a smooth variety along a smooth subvariety. This proposition plays an important role in my study.

Blowing up. Proposition 2

$X$: blowing up of smooth variety $Y$ of dimension $n$ along smooth subvariety $Z$. If $X$ is Fano and $\Lambda^2T_X$ is nef

$\Rightarrow X$ is the blowing up of $\mathbb{P}^n$ at a point.

Main theorem

As a first step of classification of the case where $\kappa(X) = -\infty$, we consider Fano fourfolds with $\rho(X) \geq 2$.

Fano fourfolds with $\rho(X) \geq 2$. Theorem 3

$X$: smooth Fano fourfold with $\rho(X) \geq 2$. If $\Lambda^2T_X$ is nef and $T_X$ is not nef

$\Rightarrow X$ is the blowing up of $\mathbb{P}^4$ at a point.

Proof. Using results about extremal contractions on smooth fourfolds.

The proof of above theorem yields the following result.

Corollary

$X$: smooth Fano fourfold with $\rho(X) \geq 2$. If $\Lambda^2T_X$ is nef on every extremal rational curve in $X$

$\Rightarrow \Lambda^2T_X$ is nef.

References


[CP2] F. Campana and T. Peternell, 4-folds with numerically effective tangent bundles and second Betti numbers greater than one, Manuscripta Math. 79 (1993), 225–238.


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