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A note on picture insertion systems

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1 Introduction

Insertion and deletion systems are computing models based on the field of molecular biology. Several proposals have been made for generating two-dimensional languages based on insertion and deletion with replicative transposition operation.

In this paper, we focus on insertion operations and extend an insertion system from one dimension (1D) to 2D then introduce a picture insertion system to generate picture languages. The picture insertion operation introduced in this paper relates to the insertion operations in one dimensions of the form $(u, x, v)$ to produce a string $auv$ from a given string $uvw$ with context $wv$ by inserting a string $x$ [2]. We also present some examples and results concerning picture insertion systems.

2 Preliminaries

In this section, we introduce notation and basic definitions that are necessary for this paper. The basic notions and definitions in formal language theory are found in [4].

For an alphabet $T$, a picture $p$ is a two-dimensional rectangular array of elements of $T$. $T^{**}$ is the set of all pictures over $T$. A picture language over $T$ is a subset of $T^{**}$.

For a picture $p \in T^{**}$, let $\ell_{1}(p)$ (resp. $\ell_{2}(p)$) be the number of rows (resp. columns) of $p$. For a picture $p$ in $T^{**}$, $|p| = (m, n)$ denotes the size of the picture $p$ with $m = \ell_{1}(p)$ and $n = \ell_{2}(p)$.

The row and column concatenations are denoted $p \oplus q$ and $p \odot q$, respectively, and defined if $p$ and $q$ have the same number of columns (resp. rows). $p^{k \oplus}$ (resp. $p^{k \odot}$) is the vertical (horizontal) juxtaposition of $k$’s $p$.

A tiling system [3] is a tuple $T = (\Sigma, \Gamma, \theta, \pi)$, where $\Sigma$ and $\Gamma$ are alphabets, $\theta$ is a finite set of tiles over the alphabet $\Gamma$, and $\pi : \Gamma \rightarrow \Sigma$ is a projection. Let $TS$ be the class of picture languages generated by tiling systems.

3 Picture insertion systems

Definition 1 A picture insertion system is a tuple $\gamma = (T, P, A)$, where $T$ is an alphabet, $P$ is a finite set of picture insertion rules, and $A$ is a finite set of pictures over $T$. $P$ may contain the following three types of picture insertion rules:

- **R-type**: $(u, w, v)$, where $\ell_{1}(u) = \ell_{1}(v) = \ell_{2}(w)$.
- **C-type**: \( \begin{pmatrix} u \\ w \end{pmatrix} \), where $\ell_{2}(u) = \ell_{2}(v) = \ell_{2}(w)$.
- **RC-type**: \( \begin{pmatrix} u & w_{1} & v \\\ x & w_{2} & w_{3} & w_{4} \end{pmatrix} \), where $\ell_{1}(u) = \ell_{1}(w_{1}) = \ell_{1}(v)$, $\ell_{1}(w_{2}) = \ell_{1}(w_{3}) = \ell_{1}(w_{4})$, $\ell_{2}(u) = \ell_{2}(w_{2}) = \ell_{2}(x)$, $\ell_{2}(w_{1}) = \ell_{2}(w_{3}) = \ell_{2}(w_{4})$, $\ell_{2}(v) = \ell_{2}(w_{4}) = \ell_{2}(y)$, and $w_{3} \neq \lambda$. 

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Intuitively, R-type (resp. C-type) rule means an
insertion rule in row (resp. column), that is, the
picture \( w \) is inserted in between the pictures \( u \) and
\( v \). An RC-type rule is intend to insert the pictures
\( w_i \) (1 \( \leq i \leq 5 \)) into the picture consisting of
\( u, v, x, \) and \( y \). We break up the rectangle into sub-
pictures \( u, v, x, \) and \( y \) and secure the cross-shaped
space, then insert those pictures.

We show how to apply insertion rules in the fol-
lowing definition.

**Definition 2** For pictures \( p_1, p_2 \) in \( T^{**} \), we say
that \( p_1 \) derives \( p_2 \) in one step if

- there is an R-type rule \((u, w, v)\) with \( u, v, w \in
  T^{m*} \) for \( m \geq 1 \) such that \( p_1 = \alpha \odot u \odot v \odot \beta \)
  and \( p_2 = \alpha \odot u \odot v \odot \beta \) with \( \alpha, \beta \in T^{m*} \).
  We write \( p_1 \rightarrow_R p_2 \). In a graphical represent-
  ation, it means

\[
\begin{array}{ccc}
\alpha & u & v \\
\beta & \rightarrow_R & \alpha & u & w & v & \beta
\end{array}
\]

- there is a C-Type rule \((u, w)\) with \( u, v, w \in
  T^{m*} \) for \( n \geq 1 \) such that \( p_1 = \alpha \odot u \odot v \odot \beta \)
  and \( p_2 = \alpha \odot u \odot w \odot v \odot \beta \) with \( \alpha, \beta \in T^{m*} \).
  We write \( p_1 \rightarrow_C p_2 \). In a graphical representa-
  tion, it means

\[
\begin{array}{ccc}
\alpha & u & v \\
\beta & \rightarrow_C & \alpha & u & w
\end{array}
\]

- there is an RC-Type rule \((u, w_1, v)\)
  such that \( p_1 = (u \odot v) \odot (x \odot y) \) and \( p_2 =
  (u \odot w_1 \odot v) \odot (w_2 \odot w_3 \odot w_4) \odot (x \odot w_5 \odot y).
  
We write \( p_1 \rightarrow_{RC} p_2 \). In a graphical represen-
  tation, it means

\[
\begin{array}{ccc}
u & v & \Rightarrow & u & w_1 & v \\
x & y & & w_2 & w_3 & w_4 \\
& & & x & w_5 & y
\end{array}
\]

If there is no confusion, we write \( \rightarrow \) instead of
\( \rightarrow_R, \rightarrow_C, \) and \( \rightarrow_{RC} \). The reflexive and transitive
closure of \( \rightarrow \) (resp. \( \rightarrow_R, \rightarrow_C \)) is defined as \( \rightarrow^* \)
(resp. \( \rightarrow^*_R, \rightarrow^*_C \)). The transitive closure of \( \rightarrow \) (resp.
\( \rightarrow_R, \rightarrow_C \)) is denoted by \( \Rightarrow \) in the following de-
dinition.

**Definition 3** For pictures \( p_1, p_2 \) and \( p_1 \Rightarrow p_2 \) is
defined in the following three cases:

1. [Using R-type rules]

- pictures \( p_1 \) and \( p_2 \) satisfy
  \( p_1 = (\alpha_1 \ominus \beta_1 \ominus \cdots \ominus (\alpha_n \ominus \beta_n)) \) and
  \( p_2 = (\alpha_1 \ominus w_1 \ominus \beta_1 \ominus \cdots \ominus (\alpha_n \ominus w_n \ominus \beta_n)) \), where
  for each \( 1 \leq i \leq n \),
  - there is a derivation \( \alpha_i \ominus \beta_i \rightarrow^*_R \alpha_i \ominus w_i \ominus \beta_i \),
  - there are \( l_0, l_1, l_w \geq 0 \) such that \( l_2(\alpha_i) = l_0, l_2(\beta_i) = l_0, l_2(w_i) = l_w \),
  - there is no picture \( p' \) in \( T^{**} \) such that \( p_1 \rightarrow^*_R p' \),

In a graphical representation, it means

\[
\begin{array}{cc}
\alpha_1 & \beta_1 \\
\cdots & \cdots \\
\alpha_n & \beta_n
\end{array}
\]

\[
\begin{array}{ccc}
\alpha_1 & w_1 & \beta_1 \\
\cdots & \cdots & \cdots \\
\alpha_n & w_n & \beta_n
\end{array}
\]

2. [Using C-type rules]

- pictures \( p_1 \) and \( p_2 \) satisfy
  \( p_1 = (\alpha_1 \ominus \beta_1 \ominus \cdots \ominus (\alpha_n \ominus \beta_n)) \) and
  \( p_2 = (\alpha_1 \ominus w_1 \ominus \beta_1 \ominus \cdots \ominus (\alpha_n \ominus w_n \ominus \beta_n)), \)
  where for each \( 1 \leq i \leq n \),
- There is a derivation \( \alpha_i \ominus \beta_i \rightarrow^* \alpha_i \ominus w_i \ominus \beta_i \).

- There are \( l_a, l_b, l_w \geq 0 \) such that \( \ell_1(\alpha_i) = l_a, \ell_1(\beta_i) = l_b, \ell_1(w_i) = l_w \).

* There is no picture \( p' \) in \( T^* \) such that \( p_1 \rightarrow^+ p' \).

In a graphical representation, it means

\[
\begin{array}{ll}
\alpha_1 \ldots \alpha_n & \Rightarrow \alpha_1 \ldots \alpha_n \\
\beta_1 \ldots \beta_n & \Rightarrow w_1 \ldots w_n
\end{array}
\]

3. [Using an RC-type rule]

- There is an RC-type rule \( (u, w_1, v, w_2, w_3, w_4, x, w_5, y) \).

- Pictures \( p_1 \) and \( p_2 \) satisfy \( p_1 = (q_1 \ominus q_2) \ominus (q_3 \ominus q_4) \) and \( p_2 = (q_1 \ominus (z_1 \ominus w_1) \ominus q_2) \ominus (z_2 \ominus z_3) \ominus (q_3 \ominus (w_5 \ominus z_4) \ominus q_4) \).

- The lower right corner (resp. lower left, upper right, upper left) of \( q_1 \) (resp. \( q_2, q_3, q_4 \)) is \( u \) (resp. \( v, x, y \)).

- \( z_1 \) (resp. \( z_2, z_3, z_4 \)) is inserted by R-type (resp. C-type, C-type, R-type) rules.

In a graphical representation, it means

\[
\begin{array}{ll}
q_1 & q_2 \\
\vdots u \vdots v \vdots & \vdots z_1 \vdots q_2 \\
x \vdots y \vdots & x \vdots \vdots w_3 \vdots w_4 \\
q_3 & q_4 \\
\vdots \vdots \vdots \vdots & \vdots \vdots \vdots \vdots \vdots \\
& \vdots \vdots \vdots \vdots \vdots
\end{array}
\]

Intuitively, the standard derivation \( \Rightarrow \) is the smallest unit to apply to a picture by applying picture insertion rules. The reflexive and transitive closure of \( \Rightarrow \) is defined as \( \Rightarrow^* \).

A picture insertion system \( \gamma = (T, P, A) \) is said to be of weight \( (i, j; k, l) \) if the number of rows (resp. columns) for context checking picture is not more than \( i \) (resp. \( j \)), and the number of rows (resp. columns) for inserted picture is not more than \( k \) (resp. \( l \)).

For \( i, j, k, l \geq 0 \), let \( INS_{i,j}^{k,l} \) be the class of picture languages generated by picture insertion systems of weight \( (i', j'; k', l') \) with \( i' \leq i, j' \leq j, k' \leq k, \) and \( l' \leq l \). If some of the parameters \( i, j, k, l \) are not bounded, we use \( \star \) in place of the symbols for those parameters.

**Example 1** Consider a picture insertion system \( \gamma = (T, P, A) \), where \( T = \{a, b\}, P = \{\lambda, ab, \lambda\} \), \( A = \{\lambda\} \). The picture language generated by \( \gamma \) is viewed as a Dyck's string language.

As shown in Example 1, picture insertion systems are 2D generalizations of insertion systems in 1D cases. We slightly note that Dyck language is not regular (in 1D sense).

**Example 2** Consider a picture insertion system \( \gamma = (T, P, A) \), where \( T = \{a, b\}, P = \{\lambda, \gamma_{ba}, \lambda\} \), \( A = \{\lambda\} \). The followings are some of the pictures generated by \( \gamma \):

\[
\begin{array}{l}
\lambda \rightarrow \gamma_{ba} \rightarrow abbaa, \quad qabbbba \rightarrow bbaa \\
\lambda \rightarrow \gamma_{ba} \rightarrow abbaa, \quad qabbbba \rightarrow bbaa \\
\end{array}
\]

For example, the picture \( abbbab \) is derived in two ways as follows:

\[
\begin{array}{l}
\lambda \rightarrow abba \rightarrow abbbab \\
\lambda \rightarrow abba \rightarrow abbbab \\
\end{array}
\]

**Example 3** Consider a picture insertion system \( \gamma = (T, P, A) \), where \( T = \{a, b\}, P = \{b, b, \lambda\} \),
A derivation in $\gamma$ proceeds as follows:

\[
\frac{ab}{ab} \Rightarrow \frac{abb}{bbb} \Rightarrow \frac{abbaab}{bbaab} \Rightarrow \cdots.
\]

The derivation proceeds deterministically using the RC-type rule. A language generated by $\gamma$ is a set of squares whose positions in the main diagonal are covered by $a$ and the remaining ones are covered by $b$.

**Lemma 1** There is a picture language which cannot be generated by any picture insertion systems.

**Proof** Consider a picture language defined by $(a^{2n+1})^\theta \circ (ab)^n \circ (a^{2n+1})^\theta$ for $n \geq 1$.

The claim can be proved by contradiction. □

**Lemma 2** There is a picture insertion system $\gamma$ such that $L(\gamma)$ is not generated by a tiling system.

**Proof** Consider a picture insertion system $\gamma = (T, \{(a, \frac{ab}{ba}, \lambda), (\lambda)\})$ with $T = \{a, b\}$. From the definition of $\gamma$, a picture $p$ in $L(\gamma)$ satisfies that the number of $a$ in $p$ is equivalent to that of $b$.

Suppose that there is a tiling system $\mathcal{T} = (T, \Gamma, \theta, \pi)$ such that $L(\gamma) = L(\mathcal{T})$, where $\Gamma$ is a finite alphabet, $\theta$ is a finite set of tiles over $\Gamma$, and $\pi : \Gamma \to T$ is a projection. Then we can generate a contradiction. □

**Lemma 3** For any $i, j \geq 0$, $INS_{i,j}$ is incomparable with $TS$.

**Proof** As an example, for the class of picture insertion systems, we consider $INS^{0,0}_{i,j}$.

From Lemma 2, we can prove that there is a picture language $L(\gamma)$ in $INS^{0,0}_{i,j}$ but not in $TS$.

Consider a tiling system $\mathcal{T} = (\{a, b\}, \{a, b\}, \theta, \pi)$, where $\theta = \{ \frac{ab}{ba}, \frac{ba}{ab} \}$, $\pi : \{a, b\} \to \{a, b\}$ is an identity projection such that $\pi(x) = x$ with $x \in \{a, b\}$. The followings are some examples of pictures in $L(\mathcal{T})$: $\frac{ab}{ba}, \frac{abq}{ba}, \frac{ab}{ab}, \frac{ab}{aba}, \cdots$.

Suppose that there is a picture insertion system $\gamma$ such that $L(\gamma) = L(\mathcal{T})$, then we can generate a contradiction.

Similarly, for the case of $INS_{i,j}$ with $i, j \geq 0$, the claim can be proved. □

## 4 Concluding Remarks

In this paper, we introduced picture insertion systems which generate two-dimensional languages.

As considered in 1D case, picture insertion-deletion systems can be defined in which we can use not only picture insertion operations but also deletion operations.

Using insertion systems together with some morphisms, representation theorems are shown in 1D case [1]. Those representation might be possible in 2D case. Furthermore, to compare with cellular automaton is also our future work.

## References


