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GENERALIZED PARENTHESES LANGUAGES AND
MINIMALIZATION OF THEIR PARENTHESES PARTS
(extended abstract)

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1. INTRODUCTION

The parenthesis grammar defined by McNaughton [2] is a context-free grammar \( G = (N,K,P,S) \) such that the terminal alphabet \( K \) contains a pair of parentheses, say \(< \) and \(>\), and the production rules are of form

\[ A \to <u> \]

where \( A \) is a nonterminal symbol, and \( u \) is a word not containing the parentheses \(<\) and \(>\). Then for parenthesis grammars the equivalence problem was proved to be decidable [2].

The generalized parenthesis language is defined [3] by extending the spirit of parenthesis languages so that it reflects the block structure prevalent in modern programming languages, while preserving the mathematical wealth.

Let \( K \) be an alphabet that includes a set

\[ \hat{I} = \{ a, \bar{a} \mid a \text{ is in } I \} \]

of parentheses, and \( G = (N,K,P,S) \) be a context-free grammar (cfg, for short) such that the production rules in \( P \) are of form

\[ A \to au\bar{a}B, \quad A \to bB, \text{ or } A \to e \]

where \( A \) and \( B \) are in the nonterminal alphabet \( N \), \( a \) is in
I, u is a word over \( \mathbb{N} \cup K \) not containing symbols in \( I \), and \( b \) is in \( J = K - \hat{I} \). (The e stands for the empty word.) Then we call \( G \) a generalized parenthesis grammar (gpg, for short), and the language generated thereby a generalized parenthesis language (gpl, for short) over \( K \) with the parenthesis part \( \hat{I} \) (or simply, over \( K[I] \)).

The class of gpl's so defined has been proved to have nice mathematical features; for example, the equivalence problem for gpg's over \( K[I] \) are decidable, and they enjoy various closure properties (under language-theoretic operations in relativized forms, with respect to the 'universal' gpl specified below) [3], [4]. On the other hand, the expressive power of gpl is sufficiently large; for example, it can describe the syntax of ALGOL 60 with five pairs of parentheses, \( (, ), [ , ] \), if, then, begin, end, and \( ' , ' \) [5].

In this paper, after a short preliminary in the rest of this section, in section 2 we study relations between regular sets and gpl's, and solve some decision problems affirmatively. In particular, we show that the regularity problem for gpl's is decidable, and that for a given regular set \( L \) over \( K \) and a set \( \hat{I} \) of parentheses in \( K \), one can decide whether \( L \) is a gpl over \( K[I] \) or not. In section 3 we apply these results to the study of parenthesis parts of gpl's, resulting in affirmative answers to more general problems. Among others we prove that for a given gpg \( G \) over \( K[I] \) and a subset \( I' \) of \( I \) it is decidable whether \( L(G) \) is a gpl over \( K[I'] \) or not. Thus we can minimalize the parenthesis part of a given gpl. (If the minimalized parenthesis part is empty then the gpl is regular.) In section 4, relations
between gpl's and context-free languages (cfl's, for short) are studied. We give a characterization of cfl's and that of gpl's, both in terms of universal gpl's, regular sets, and projections. We also give a negative answer to the decision problem to ask whether a given cfg generates a gpl or not.

Let $\hat{I} = \{ a, \bar{a} \mid a \text{ is in } I \} \subseteq K$, and $J = K - \hat{I}$ as above. Consider the gpg $G = (\{S\}, K, P, S)$ such that

$P = \{ S \rightarrow aSaS, S \rightarrow bS, S \rightarrow e \mid a \text{ is in } I \text{ and } b \text{ in } J \}.$

Any gpl over $K[I]$ is included in the gpl generated by $G$. We call the language $L(G)$ the universal gpl over $K[I]$, and denote it by $D_{I,J}$. In case of $J = \phi$, the language equals the Dyck set $D_I$ over $\hat{I}$. If $I = \phi$ then $D_{I,J} = J^*$. In general, $D_{I,J}$ is equal to $\text{Shuffle}(D_I, J^*)$, the shuffle product of $D_I$ and $J^*$.

For each element $w$ of $D_{I,J}$, the nonnegative integer $\text{depth}_I(w)$ is defined as follows:

$\text{depth}_I(e) = 0,$

$\text{depth}_I(auv) = \max\{ 1 + \text{depth}_I(u), \text{depth}_I(v) \},$

$\text{depth}_I(bu) = \text{depth}_I(u).$

where $a$ is in $I$, $b$ is in $J$, and $u$ and $v$ are in $D_{I,J}$.

For a language $L$ in $D_{I,J}$, we define

$\text{depth}_I(L) = \sup\{ \text{depth}_I(w) \mid w \text{ is in } L \},$

which may or may not be finite.

If $uvw$ is a word in $D_{I,J}$ then we can write

$v = v_0\bar{a}_1v_1\bar{a}_2v_2...\bar{a}_n v_n a_{n+1}v_{n+1}...a_{n+m}v_{n+m}$

for some $a_1, a_2, ..., a_{n+m}$ in $I$, $v_0, v_1, ..., v_{n+m}$ in $D_{I,J}$ and $n,m \geq 0$. In this case we will write

$|v|_I = \bar{a}_1\bar{a}_2...\bar{a}_n a_{n+1}a_{n+2}...a_{n+m}.$

For a language $L$ in $D_{I,J}$, we define
subword\_(I)(L) = \{ v \text{ in } D\_{I,J} | uvw \text{ is in } L \text{ for some } u, w \}. 

For any word \( w \) in \( D\_{I,J} \), we define the word \( \text{surface}\_I(w) \) in \( J^* \) as follows: If

\[ w = u_0(a_1v_1\overline{a}_1)u_1(a_2v_2\overline{a}_2)u_2... (a_nv_n\overline{a}_n)u_n \]

for some \( n \geq 0 \), \( u_0, ..., u_n \) in \( J^* \), \( a_1, ..., a_n \) in \( I \), and \( v_1, ..., v_n \) in \( D\_{I,J} \), then

\[ \text{surface}_I(w) = u_0u_1...u_n. \]

For a language \( L \) in \( D\_{I,J} \), we define

\[ \text{surface}_I(L) = \{ \text{surface}_I(w) | w \text{ is in } L \}. \]

We may suppress the suffix \( I \) in these notations when it is clear from the context.

This paper is an extended abstract of [5], and we will omit the proofs of theorems.

2. REGULAR SETS AND GENERALIZED PARENTHESIS LANGUAGES

It has been proved [4] that the class of gpl's over \( K[I] \) is closed under intersection with regular sets, and therefore any regular set included in \( D\_{I,J} \) is a gpl over \( K[I] \). In this section we study properties of these regular sets, and give positive answers to some decision problems for gpl's.

Theorem 2.1 If \( L \) is a regular set included in \( D\_{I,J} \), then \( \text{depth}(L) \) is finite.

Theorem 2.2 If \( L \) is a gpl over \( K[I] \) and \( \text{depth}(L) \) is finite, then \( L \) is regular.

Corollary 2.3 For a language \( L \) in \( D\_{I,J} \) the following three
conditions are equivalent.

1. \( L \) is a regular set.
2. \( L \) is a gpl over \( K[I] \), and \( \text{depth}(L) \) is finite.
3. \( L \) is obtained from subsets of \( J \) by a finite number of applications of regular operations \( \cup, \cdot, * \), and bracketting by symbols in \( I \) (i.e., \( a\bar{X}a \) for \( X \), where \( a \) is in \( I \)).

Theorem 2.4 For a given regular expression \( E \) over \( K \) and a set of parentheses \( \hat{I} \) in \( K \), one can decide whether the regular set \( L \), denoted by \( E \), is a gpl over \( K[I] \). If this is the case, one can effectively obtain a gpg over \( K[I] \) to generate the set \( L \).

Note that any regular set in \( K^* \) is a gpl over \( K[\emptyset] \).

Therefore to specify the parenthesis part \( \hat{I} \) in theorem 2.4 is important. From the theorem, for a given regular set \( L \) in \( K^* \), we can effectively list up all the paired subalphabets \( \hat{I} \) of \( K \) such that \( L \) is a gpl with parenthesis part \( \hat{I} \).

Theorem 2.5 Whether a given gpg generates a regular set or not is decidable.

3. ON MINIMALIZATION OF THE PARENTHESIS PART

The regularity problem for gpg's (theorem 2.5) is nothing but to ask whether the parenthesis part of a given gpg can be reduced to the empty set. In this section we consider a more general problem to minimalize the parenthesis part of a given gpg. First
we note a property of the mapping \( \text{surface}: D_{I,J} \rightarrow J^* \).

Theorem 3.1 If \( L \) is a gpl over \( K[I] \), then \( \text{surface}(L) \) is a regular set over \( K{-}\hat{I} \).

As a consequence we know that if a gpl \( L \) over \( K[I] \) is also a gpl over \( K[I'] \) where \( I' \subseteq I \) then \( \text{surface}_{I'}(L) \) is a regular subset of \( D_{I-I',J} \). The converse of this statement is not true. However we can prove the following.

Theorem 3.2 Let \( G = (N,K,P,S) \) be a gpg over \( K[I] \), \( L_A = \{ w \text{ in } K^* \mid A \not\rightarrow^* w \text{ in } G \} \) for each \( A \) in \( N \), and \( I' \subseteq I \). If \( \text{surface}_{I'}(L_A) \) is regular for each \( A \), then each \( L_A \) is a gpl over \( K[I'] \).

Theorem 3.3 Let \( L \) be a gpl over \( K[I] \), and \( I' \subseteq I \). Then \( L \) is a gpl over \( K[I'] \) if and only if \( \text{surface}_{I'}(\text{subword}_I(L)) \) is regular (i.e., depth\( _{I-I'}(\text{surface}_{I'}(\text{subword}_I(L))) \) is finite).

Corollary 3.4 Let \( G \) be a gpg over \( K[I] \), and \( I' \subseteq I \). Then it is decidable whether \( L(G) \) is a gpl over \( K[I'] \) or not. If the answer is affirmative, one can effectively obtain a gpg \( G' \) over \( K[I'] \) to generate the language \( L(G) \).

As for the expansion of the parenthesis parts of gpl's, we can extend theorem 2.4 as follows.

Theorem 3.5 Let \( L \) be a gpl over \( K[I] \), \( I \subseteq I' \), and \( \hat{I}' \subseteq K \).
Then $L$ is a gpl over $K[I']$ if and only if $L \subseteq D_{I', j'}$, where $J' = K - \hat{J}$. For a given gpg $G$ over $K[I]$ and an expansion $I'$ of $I$, one can decide whether the condition is satisfied or not. If this is the case one can effectively obtain a gpg $G'$ over $K[I']$ to generate the language $L(G)$.

By corollary 3.4 and theorem 3.5, for a given gpg $G$ over $K[I]$ we can list up all restrictions and expansions $I'$ of $I$ such that $L(G)$ is a gpl over $K[I']$. In particular, we can obtain all minimal parenthesis parts for a given gpg $G$ over $K[I]$, i.e., all minimal subsets $I'$ of $I$ such that $L(G)$ is a gpl over $K[I']$.

It is interesting to note that a gpl may have no 'minimum' nor 'maximum' parenthesis part. For instance, consider

$$L = \{ (ab)^i (cd)^i \mid i=0,1,2,\ldots \}.$$ 

The language $L$ is a gpl in various ways; it is a gpl with $\hat{I}_1 \sim \hat{I}_5$ below as the parenthesis part.

$$\hat{I}_1 = \{ a, d \},$$
$$\hat{I}_2 = \{ a, c \},$$
$$\hat{I}_3 = \{ b, c \},$$
$$\hat{I}_4 = \{ b, d \},$$
$$\hat{I}_5 = \{ a, d; b, c \}.$$ 

Among these, $\hat{I}_1 \sim \hat{I}_4$ are minimal, while $\hat{I}_2$, $\hat{I}_4$ and $\hat{I}_5$ are maximal. But none of them is the minimum, nor the maximum. At present we do not know any algorithm to get all possible parenthesis parts of a given gpl, or whether one can expect it at all or not.
4. CONTEXT-FREE LANGUAGES AND GENERALIZED PARENTHESIS LANGUAGES

First we note that any gpl is a deterministic cfl, indeed Greibach and Friedman [1] have shown a stronger result that any gpl is a superdeterministic language.

We prove that a language \( L \) is a cfl (or gpl, respectively) if and only if \( L = h(D_{I,J} \cap R) \) for a regular set \( R \) and a projection (or pair preserving projection) \( h \). Finally we prove the undecidability of whether a given cfl is a gpl or not.

Let \( K \) and \( K' \) be alphabets. A homomorphism \( h : K^* \rightarrow K'^* \) is said to be a projection if \( h(K) \subseteq K' \). A projection \( h : (\hat{I} \cup J)^* \rightarrow (\hat{I'} \cup J')^* \) is said to be pair preserving if \( h(I) \subseteq I' \), \( h(J) \subseteq J' \) and \( h(\bar{a}) = \overline{h(a)} \) for any \( a \) in \( I \).

Theorem 4.1 A language \( L \) is a gpl over \( K[I] \) if and only if \( L = h(D_{I',J'} \cap R) \) for some alphabets \( I' \) and \( J' \), a pair preserving projection \( h \), and a regular set \( R \) over \( \hat{I'} \cup J' \).

Theorem 4.2 A language \( L \) is a cfl if and only if \( L = h(L') \) for a gpl \( L' \) over some alphabet \( K[I] \), and a projection \( h \).

Corollary 4.3 A language \( L \) is a cfl if and only if \( L = h(D_{I,J} \cap R) \) for a projection \( h \) and a regular set \( R \) over some alphabet \( K = \hat{I} \cup J \).

Theorem 4.4 For a given cfg whether it generates a gpl or not is undecidable.
REFERENCES


