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Order of a holomorphic curve with maximal deficiency sum
for moving targets

by Seiki Mori

1. Introduction. Nevanlinna's defect relation remains valid for mutually distinct meromorphic target functions \( g_1, \ldots, g_q \) on \( \mathbb{C} \) which grow more slowly than a given meromorphic function \( f \) on \( \mathbb{C} \) (slow moving targets), that is, the Nevanlinna characteristic functions of those functions satisfy \( T_{g_j}(r) = o(T_f(r)) \) as \( r \to \infty \), \( (j=1, \ldots, q) \). (See N.Steinmetz [7]) On the other hand, in higher dimensional case, M.Ru - W.Stoll [4] [5] and Shirosaki [6] proved a defect relation with defect bound \( n+1 \) for slow moving targets to the case of nondegenerate holomorphic curve.

While, A.Edrei - W.H.J.Fuchs [1] proved that a finite order meromorphic function with \( \delta(\omega, f) = 1 \) is of positive integral order and regular growth if it has the maximal deficiency sum 2.

In this note, we investigate the order of holomorphic curve in some class with maximal deficiency sum for slow moving targets.

Let \( f : \mathbb{C} \to \mathbb{P}^n(\mathbb{C}) \) be a finite order and nondegenerate holomorphic curve and \( \bar{f} := (f_0, \ldots, f_n) \) a reduced representation of \( f \). Let \( g_j \) \( (j=0, \ldots, q) \) be slowly moving targets for \( f \) in general position,
We show that if there exists an \( f_{i_0} \) such that
\[
N_1(r, \frac{1}{f_{i_0}}) = o(T_f(r)) \quad \text{and} \quad T_1(r, \frac{1}{f_{i_j}}) = o(T_f(r)) \quad (j=0, \ldots, n-1)
\]
and
\[
\sum_{j=0}^{q} \delta(f, g^j) = n + 1,
\]
then \( f \) is of positive integral order and of regular growth. In the case \( n = 1 \), the theorem is sharp by F. Nevanlinna's example. But in the case \( n > 1 \), I could not find an example to show the sharpness of the theorem.

2. Preliminaries and statement of result.

Let \( f : \mathbb{C} \to \mathbb{P}^n(\mathbb{C}) \) be a holomorphic mapping of \( \mathbb{C} \) into \( \mathbb{P}^n(\mathbb{C}) \), and \( \bar{f} = (f_0, \ldots, f_n) : \mathbb{C} \to \mathbb{C}^{n+1} \) a reduced representation of \( f \). Set \( \|f(z)\|^2 := \sum_{i=1}^{n} |f_i(z)|^2 \).

We define the characteristic function \( T_f(r) \) of \( f \) by
\[
T_f(r) := \frac{1}{2\pi} \int_{0}^{2\pi} \log \|f(re^{i\theta})\| d\theta.
\]

We define the order \( \lambda_f \) and the lower order \( \mu_f \) of \( f \) as follows:
\[
\lambda_f := \limsup_{r \to \infty} \frac{\log T_f(r)}{\log r} \quad \text{and} \quad \mu_f := \liminf_{r \to \infty} \frac{\log T_f(r)}{\log r}.
\]
We say that \( f \) is of regular growth if \( \lambda_f = \mu_f \).

For a meromorphic function \( \phi(z) : \mathbb{C} \to \mathbb{C} \cup \{\infty\} \), its proximity function \( m_1(r, \phi) \), counting function \( N_1(r, \phi) \) and the characteristic function \( T_1(r, \phi) \) are defined by
\[
m_1(r, \phi) := \frac{1}{2\pi} \int_{0}^{2\pi} \log^+ |\phi(re^{i\theta})| d\theta, \quad N_1(r, \phi) := \int_{0}^{r} n_1(t, \phi) \frac{dt}{t}
\]
and
\[
T_1(r, \phi) := m_1(r, \phi) + N_1(r, \phi),
\]
respectively, where \( n_1(t, \phi) \) is the number of poles of \( \phi \) in \( |z| < t \).
counting multiplicities and \( \log^+ x := \max (\log x, 0) \).

Let \( S \) be a finite set of holomorphic mappings \( g : C \rightarrow \mathbb{P}^n(C)^* \) with \( n+2 \leq q := \#S < \infty \). Here we say that \( g \) is a moving target.

Assume that

(A 1) \( S \) is in general position. (cf. [5])

This means that at least one point \( z_0 \in C \) exists, such that \( \#S(z_0) = q \) and \( S(z_0) \) is in general position, that is,

\[
\det (g^j_k)_{0 \leq k, l \leq n} \neq 0, \quad \text{where} \quad S = (g^1:C \rightarrow \mathbb{P}^n(C), \ (j=0,\ldots,q))
\]

and \((g^j_0,\ldots,g^j_n)\) a reduced representation of \( g^j \).

Let \((f_0,\ldots,f_n)\) and \((g_0,\ldots,g_n)\) be reduced representations of \( f \) and \( g \), respectively. Define \( N_{f,g}(r) := N_1(r, 1/h) \)

\[
m_{f,g}(r) := \frac{1}{2\pi} \int_0^{2\pi} \log \frac{\|f(re^{i\theta})\|\|g(re^{i\theta})\|}{|h(re^{i\theta})|} \, d\theta \geq 0,
\]

where \( h(z) := \sum_{i=0}^n f_i(z) g_i(z) \neq 0 \). Then it is known that

\[
T_f(r) + T_g(r) = N_{f,g}(r) + m_{f,g}(r) + O(1) \quad (r \rightarrow \infty).
\]

If \( f \) or \( g \) is nonconstant, then \( T_f(r) + T_g(r) \rightarrow \infty \) as \( r \rightarrow \infty \)

and the defect \( \delta(f,g) \) for the moving target \( g \) is defined by

\[
0 \leq \delta(f,g) := \liminf_{r \rightarrow \infty} \frac{m_{f,g}(r)}{T_f(r)+T_g(r)} = 1 - \limsup_{r \rightarrow \infty} \frac{N_{f,g}(r)}{T_f(r)+T_g(r)} \leq 1.
\]

Assume that

(A 2) \( T_g^j(r) = o(T_f(r)) \quad (r \rightarrow \infty), \) for all \( g^j \in S \).

Then the moving target \( g^j \) is said to grow more slowly than \( f \), and the defect \( \delta(f, g^j) \) is written as
\[ \delta(f, g^j) = \lim_{r \to \infty} \inf m \frac{r}{T_f(r)} = 1 - \lim_{r \to \infty} \sup N_{g_j^j} \frac{r}{T_f(r)}. \]

Let \( \mathbb{R}_G \) be the field generated by \( G \) over \( C \), that is, the field generated by elements of the form \( \xi_j^j = g_i^j / g_j^0, \) \( (i = 0, \ldots, n; j = 0, \ldots, q) \) over \( C \), where \( (g_0^j, \ldots, g_n^j) \) is a reduced representation of \( g^j \). By assumption (A 2), \( T_\psi(r) = o(T_f(r)) \) as \( r \to \infty \), for any \( \psi \in \mathbb{R}_G \). Assume that

(A 3) \( f \) is linearly nondegenerate over \( \mathbb{R}_G \), that is, \( f_0, \ldots, f_n \) are linearly independent over \( \mathbb{R}_G \). Then we have the following:

**Theorem.** Let \( f : C \to \mathbb{P}^n(C) \) be a finite order holomorphic curve and linearly nondegenerate over \( \mathbb{R}_G \), and \( (f_0, \ldots, f_n) \) a reduced representation of \( f \). Let \( G \) be a finite set of slowly growing moving targets as above. Assume that there exists an \( f_i^0 \equiv 0 \) such that

\[ N_1(r, 1/f_i^0) = o(T_f(r)) \quad \text{and} \quad T_1(r, f_i^j / f_i^0) = o(T_f(r)) \quad (r \to \infty), \]

\( (j = 0, \ldots, n-1) \). Then if \( \sum_{j=0}^{q} \delta(f, g^j) = n + 1, \) \( f \) is of positive integral order and of regular growth.

3. Proof of the theorem.

We may assume that \( i_j = j, \) \( (j = 0, \ldots, n) \). We may assume that \( g_n^j \equiv 0 \) \( (j = 0, \ldots, n-1) \), by adding some constant targets in general position, if necessary, and also may assume that \( g_0^j \equiv 0 \) \( (j = 0, \ldots, q) \), by a unitary transformation of \( \mathbb{P}^n(C) \). Set

\[ \xi_{jk} = g_k^j / g_0^j, \quad (j = 0, \ldots, q; k = 0, \ldots, n), \] so \( \xi_{j0} = 1 \) \( (j = 0, \ldots, q) \), and
\[ h_j = g_0^j f_0 + \cdots + g_n^j f_n, \quad (j=0, \ldots, q). \]

Then the assumption (A.3) yields \( h_j(z) \neq 0 \). Let \( \mathcal{E}(p) \) be the vector space over \( \mathbb{C} \) spanned by the set

\[ \{ \prod_{i=0}^{n} \xi_{j_i}^{p_{j_i}} | p_{j_i} \text{ are non-negative integers with } \sum_{0 \leq j \leq q} p_{j_i} = p \} \]

and \( (b_1, \ldots, b_t) \) be a basis of \( \mathcal{E}(p+1) \) such that \( (b_1, \ldots, b_s) \) a basis of \( \mathcal{E}(p) \) \((s \leq t)\). Then \( \mathcal{E}(p) \subset \mathcal{E}(p+1) \) and \( (b_j f_{\alpha_k}) \)

\((j=1, \ldots, t; \ k=0, \ldots, n)\) are linearly independent over \( \mathbb{C} \). Let

\[ F_j = h_j / g_0^j = \sum_{i=0}^{n} \xi_{j_i} f_i, \quad (j=0, \ldots, q). \]

Then \( b_j f_{\alpha_k} \)

\((j=1, \ldots, s; \ k=0, \ldots, n)\) are linearly independent over \( \mathbb{C} \). Since \( b_j f_k \)

\((j=1, \ldots, s; \ k=0, \ldots, n)\) are written as linear combination of \( b_j f_k \)

\((j=1, \ldots, t; \ k=0, \ldots, n)\) over \( \mathbb{C} \), there exist \( \delta_{mj}^{kl} \in \mathbb{C} \) and \( C \in \text{GL}(n+1) \cdot t, \mathbb{C} \) such that

\[
\left( b_j f_k \right) \left( 1 \leq j \leq s, \ 0 \leq k \leq n \right); \quad h_m \left( s+1 \leq j \leq t, \ 0 \leq m \leq n \right) = \left( b_j f_k \right) \left( 1 \leq j \leq t, \ 0 \leq k \leq n \right) \cdot C,
\]

where \( h_m = \sum_{1 \leq k \leq t} \delta_{mj}^{kl} b_k f_l \) \((j=s+1, \ldots, t; \ m=0, \ldots, n)\). Then we have

\[
W \left( b_j f_k \left( 1 \leq j \leq s, \ 0 \leq k \leq n \right); \quad h_m \left( s+1 \leq j \leq t, \ 0 \leq m \leq n \right) \right)
\]

\[ = W \left( b_j f_k \left( 1 \leq j \leq t, \ 0 \leq k \leq n \right) \right) \cdot \det C. \]

Let \( \alpha = (\alpha_0, \ldots, \alpha_n) \) \((\alpha_k \in (0, \ldots, q))\) be multi-indices. Put

\[ W_\alpha := W \left( b_j f_{\alpha_k} \left( 1 \leq j \leq s, \ 0 \leq k \leq n \right); \quad h_m^\alpha \left( s+1 \leq j \leq t, \ 0 \leq m \leq n \right) \right) \]

and

\[ W := W \left( b_j f_k \left( 1 \leq j \leq t, \ 0 \leq k \leq n \right) \right) \neq 0. \]
Then from a similar argument as above by using $F_{\alpha_0}, \ldots, F_{\alpha_n}$ instead of $F_0, \ldots, F_n$, we have $W_\alpha = C_\alpha W$, where $C_\alpha \in GL((n+1) \cdot t, \mathbb{C})$.

For any fixed $z \in \mathbb{C}$, we arrange $F_{j_k}$'s in order that

$$|F_{j_1}(z)| \leq |F_{j_2}(z)| \leq \cdots \leq |F_{j_n}(z)| \leq \cdots \leq |F_{j_{q+1}}(z)| \leq \infty.$$ 

Then we have

$$\|f(z)\| \leq A_1(z) \cdot \prod_{j_k} |F_{j_k}(z)|, \quad (k=n+1, \ldots, q+1),$$

where $\int_0^{2\pi} \log^+ A_1(re^{i\theta}) \, d\theta = o(T_f(r)) \quad (r \to \infty)$, since $A_1$ can be represented by a combination of $\xi_{j_k}$'s. Hence we have

$$q \prod_{j=0}^{q} \left( \frac{\|f(z)\|}{\|F_{j_k}(z)\|} \right) \leq A_1(z) q^{-1} \sum_{k=1}^{n} \left( \frac{\|f(z)\|}{\|F_{j_k}(z)\|} \right).$$

Thus we obtain that for any $z \in \mathbb{C}$,

$$q \prod_{j=0}^{q} \left( \frac{\|f(z)\|^s}{\|F_{j_k}(z)\|^s} \right) \leq A_2(z,s) \cdot \sum_{j=0}^{n-1} \frac{\|f(z)\|^n s}{\|F_{j_k}(z)\|^s},$$

where $\int_0^{2\pi} \log^+ A_2(re^{i\theta}) \, d\theta = o(T_f(r)) \quad (r \to \infty)$. Therefore we have

$$q \prod_{j=0}^{q} \left( \frac{\|f\|^s}{\|F_{j_k}(z)\|^s} \right) \leq A_2(z,s) \cdot \|f\|^n s \cdot \left( 1 + \sum_{i=0}^{n-1} \frac{\|F_{i}^s / \|F_{j_k}^s\|}{\|F_{j_k}^s\|} \right) \left( 1 / \prod_{i=0}^{n-1} \|F_{i}^s\| \right)$$

Here the summation $\sum_{(j_1, \ldots, j_n)}$ and $\sum_{(j_{j_k})}$ is taken over all combinations of

$(j_1, \ldots, j_n)$ and $\sum_{(j_{j_k})}$ is taken over all combinations without

$(0, \ldots, n-1)$. Hence we have

$$\log q \prod_{j=0}^{q} \left( \frac{\|f\|^s}{\|F_{j_k}^s\|} \right) \leq \log^+ \sum_{i=0}^{n-1} \frac{\|F_{i}^s / \|F_{j_k}^s\|}{\|F_{j_k}^s\|} + \log \left( \frac{\|f\|^n s}{\|F_{j_k}^s\|} \right) + \log A_2(z,s) + O(1).$$
and

\[
\log^+ \sum_{i=0}^{n-1} \left( \prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \right) \leq \log^+ \sum_{i=0}^{n-1} \left( |W_{\alpha}| \prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\|(n+1)(t-s) \right)
\]

\[
+ \log^+ \left( \prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\|(n+1)(t-s) \right) / |W_{\alpha}|
\]

\[
= \sum_{i=0}^{n-1} \log^+ D_{\alpha} + \log^+ \left( \prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\|(n+1)(t-s) / |W| \cdot |C_{\alpha}| \right)
\]

\[
\leq \sum_{i=0}^{n-1} \log^+ D_{\alpha} + \log^+ \frac{1}{|\tilde{W}|} + \log^+ \|f\|(n+1)(t-s)
\]

\[
+ \log^+ \left( \prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\|(n+1)(t-s) \right) / |C|.
\]

where \( D_{\alpha} = \frac{|W_{\alpha}|}{\prod_{k=1}^{n} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\|(n+1)(t-s)} \) and

we write \( \tilde{u}(z) = u(z) / f_0^m \) for a function \( u(z) \) with homogeneous form of degree \( m \) in \( f_0, \ldots, f_n \). Thus we obtain that

\[
(1) \quad \log^+ \prod_{j=0}^{q} \left( \frac{\|f_j\|^S}{|F_j|^S} \right) \leq \sum_{i=0}^{n-1} \log^+ D_{\alpha} + \log^+ \frac{1}{|\tilde{W}|} + \log^+ \|f\|(n+1)(t-s)
\]

\[
+ \log^+ \prod_{i=0}^{n-1} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) + \log^+ \|f\| / |C_{\alpha}|
\]

\[
+ \log^+ \left( \prod_{i=0}^{n-1} \frac{|F_i|^S}{|F_k|^S} \cdot \left( |F_0|^S + \cdots + |F_{n-1}|^S \right) \cdot \|f\| / |C| \right) + \log^+ A_2(z,s) + O(1).
\]

By integrating both sides of (1) on a circle \( |z| = r \), we obtain

\[
s \cdot \sum_{j=0}^{q} \frac{1}{2\pi} \int_0^{2\pi} \log^+ \frac{\|f\|}{|F_j|^S} \ d\theta \leq o(T_f(r)) + \frac{1}{2\pi} \int_0^{2\pi} \log^+ 1 / |\tilde{W}| \ d\theta
\]

\[
+ (n+1)(t-s) \cdot T_f(r) + 2s \cdot \sum_{i=0}^{n-1} \frac{1}{2\pi} \int_0^{2\pi} \log^+ |\tilde{F}_i| \ d\theta
\]
\[ + s \cdot \sum_{i=0}^{n-1} \frac{1}{2\pi} \int_{0}^{2\pi} \log \frac{\|f\|}{\|f_{i}\|} \, d\theta + \int_{0}^{2\pi} \log A_{2}(z,s) \, d\theta + O(1) \quad (r \to \infty), \]

by the lemma on logarithmic derivatives and the assumption
\[ T_{1}(r, f_{j}/f_{0}) = o(T_{f}(r)) \quad \text{as} \quad r \to \infty, \quad (j=0,\ldots,n-1). \]
Hence we have
\[ (2) \quad \frac{q}{j=0} \frac{1}{2\pi} \int_{0}^{2\pi} \log \frac{\|f\|}{\|f_{j}\|} \, d\theta \leq (n + (n+1)\left(\frac{t}{s} - 1\right) + o(1)) \cdot T_{f}(r) \]
\[ + \frac{1}{2\pi s} \int_{0}^{2\pi} \log^{+} \frac{1}{|\hat{\omega}|} \, d\theta, \quad (r \to \infty). \]

We note that
\[ m_{f,g^{j}}(r) = \frac{1}{2\pi} \int_{0}^{2\pi} \log \frac{\|f\|}{\|g^{j}\|} \, d\theta = \frac{1}{2\pi} \int_{0}^{2\pi} \log \frac{\|f\|}{\|f_{j}\|} \, d\theta + o(T_{f}(r)) \quad (r \to \infty), \quad (j=0,\ldots,q). \]
Therefore dividing both sides of (2) by
\[ T_{f}(r) \quad \text{and taking a limit infimum as} \quad r \to \infty, \quad \text{we obtain} \]
\[ \sum_{j=0}^{q} \frac{\delta(f,g^{j})}{s} \leq n + (n+1)\left(\frac{t}{s} - 1\right) + \liminf_{r \to \infty} \left(\frac{1}{2\pi} \int_{0}^{2\pi} \log^{+} \frac{1}{|\hat{\omega}|} \, d\theta\right)/s \cdot T_{f}(r). \]

From Steinmetz' lemma [7, p.138], we see \( \inf \frac{t}{s} = 1 \), so we obtain \( p \to \infty \)
that for any small \( \varepsilon > 0 \) there exists \( p \) such that \( t_{\varepsilon}/s_{\varepsilon} < (1 + \frac{\varepsilon}{n+1}) \).
Hence we have
\[ \sum_{j=0}^{q} \delta(f,g^{j}) \leq (n + \varepsilon) + \liminf_{r \to \infty} \left(\frac{1}{2\pi} \int_{0}^{2\pi} \log^{+} \frac{1}{|\hat{\omega}|} \, d\theta\right)/s_{\varepsilon} \cdot T_{f}(r), \]

Thus if \( \sum_{j=0}^{q} \delta(f,g^{j}) = n + 1 \), we have
\[ 1 - \varepsilon \leq \liminf_{r \to \infty} \left(\frac{1}{2\pi} \int_{0}^{2\pi} \log^{+} \frac{1}{|\hat{\omega}|} \, d\theta\right)/s_{\varepsilon} \cdot T_{f}(r) \leq \liminf_{r \to \infty} T_{1}(r,\tilde{\omega})/s_{\varepsilon} \cdot T_{f}(r), \]
where \( \tilde{\omega} = \tilde{\omega}(z,s_{\varepsilon}) \) depends on \( s_{\varepsilon} \). Also, we have
\[ T_{1}(r,\tilde{\omega}) \leq \frac{1}{2\pi} \int_{0}^{2\pi} \log^{+} \left(\tilde{\omega}(b_{1}\tilde{f}_{0},\ldots,b_{1}\tilde{f}_{n},\ldots,b_{n}\tilde{f}_{n})/\prod_{k=0}^{n} |\tilde{f}_{k}|^{t_{\varepsilon}}\right) \, d\theta \]
\[ + \frac{1}{2\pi} \int_{0}^{2\pi} \log^{+} \prod_{k=0}^{n} |\tilde{f}_{k}|^{t_{\varepsilon}} \, d\theta + m \cdot N_{1}(r,0,f_{0}) \]
\[ = o(T_f(r)) + t_{E_k} \sum_{k=0}^{n} T_1(r, \tilde{r}_k) = (t_{E_k} + o(1)) T_f(r) \quad (r \to \infty). \]

Thus we have

\[ 1 - \varepsilon \leq \liminf_{r \to \infty} \frac{T_1(r, \tilde{w})}{s_{E_k} T_f(r)} \leq \limsup_{r \to \infty} \frac{T_1(r, \tilde{w})}{s_{E_k} T_f(r)} \leq t_{E_k}/s_{E_k}. \]

This yields

\[ 1 - \varepsilon \leq \left( \frac{t_{E_k}}{s_{E_k}} \right) \liminf_{r \to \infty} \frac{1}{2\pi} \int_{0}^{2\pi} \log \frac{1}{|\tilde{w}|} d\theta / T_1(r, \tilde{w}). \]

Therefore we deduce that

\[ 0 \leq \limsup_{r \to \infty} \frac{(N_1(r, \tilde{w}) + N_1(r, 1/\tilde{w}))/T_1(r, \tilde{w})}{2\varepsilon}. \]

From Edrei-Fuchs' theorem [1, p.298], if

\[ \kappa := \limsup_{r \to \infty} \frac{(N_1(r, \tilde{w}) + N_1(r, 1/\tilde{w}))/T_1(r, \tilde{w})}{2\varepsilon}, \]

then there is an integer \( \gamma \) such that

\[ \gamma - 10e(\gamma + 1)\varepsilon \leq \mu_{\tilde{w}} \leq \lambda_{\tilde{w}} < \gamma + e(\gamma + 1)\varepsilon. \]

Thus, we deduce that \( \tilde{w} = \tilde{w}(z, s_{E_k}) \) is a meromorphic function of order \( \lambda_{\tilde{w}} \)

and of lower order \( \mu_{\tilde{w}} \), satisfying (3). On the other hand, since

\[ (1 - \varepsilon)s_{E_k} T_f(r) \leq T_1(r, \tilde{w}) \leq (1 + \varepsilon)s_{E_k} T_f(r). \]

Hence we obtain that the order and lower order of \( f \) are equal to the order and lower order of \( \tilde{w} \), respectively. Now taking \( p \to \infty \)

and \( \varepsilon \to 0 \), we obtain that \( f \) is of positive integral order and

regular growth. This completes the proof of the theorem.

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