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Vapor flows condensing at incidence onto a plane condensed phase in the presence of a noncondensable gas. II. Supersonic condensation

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This paper is the second part of the study of a steady flow of a vapor in a half space condensing onto a plane condensed phase of the vapor at incidence in the presence of a noncondensable gas near the condensed phase. The aim of the study is to clarify the behavior of the vapor and noncondensable gas on the basis of kinetic theory under the assumption that the molecules of the noncondensable gas are mechanically identical with those of the vapor. In the first part [S. Taguchi et al., Phys. Fluids 15, 689 (2003)], the case of subsonic condensation, where the Mach number corresponding to the flow-velocity component perpendicular to the condensed phase at infinity is less than unity, is considered. In the present second part, the case of supersonic condensation is investigated in detail on the same lines as the first part. © 2004 American Institute of Physics. [DOI: 10.1063/1.1630324]

I. INTRODUCTION

We consider a steady flow of a vapor in a half space condensing onto a plane condensed phase of the vapor at incidence in the case where another gas that does not condense (the noncondensable gas) is present near the condensed phase. We investigate the behavior of the vapor as well as the noncondensable gas on the basis of kinetic theory. Our main interest is to obtain the relation, among the parameters of the vapor at infinity (the pressure, temperature, and flow velocity of the vapor), those related to the condensed phase (the temperature of the condensed phase and the corresponding saturation pressure of the vapor), and the amount of the noncondensable gas contained in the system, that admits a steady solution.

This problem was investigated in detail in Ref. 1, where the case in which the vapor condenses perpendicularly to the condensed phase was considered. The essential point of Ref. 1 is a skillful analysis, based on the assumption that the noncondensable-gas molecules are mechanically identical with the vapor molecules, which clarifies the structure of the solution and reduces the necessary amount of computation dramatically. The same analysis was applied recently to the case where the vapor is condensing onto the condensed phase at incidence in Ref. 2, where the necessity of considering such a case is explained. In this reference, we restricted ourselves to the case where the magnitude of the flow-velocity component perpendicular to the condensed phase at infinity is less than the sonic speed there (subsonic condensation). In the present paper, we investigate the same problem in the case where it is equal to or greater than the sonic speed (supersonic condensation). Again, the analysis is a straightforward extension of that of Refs. 1 and 3 to the case of condensation at incidence.

The importance of the present problem comes from the following fact: the parameter relation mentioned in the first paragraph in the present section provides the boundary condition for the Euler set of equations on the condensing surface when a steady flow of a vapor around its condensed phases is considered in the continuum limit (the limit where the mean free path of the vapor molecules vanishes) in the presence of a trace of a noncondensable gas. The reader is referred to Ref. 4 for the details. Since the numerically constructed parameter relation plays the role of the numerical boundary condition for the Euler set of equations (see Ref. 4 for its application to a practical problem), we need to present a large amount of numerical data. This is the reason why we split the paper into two parts. In addition, as in Ref. 2, we will make use of the Electronic Physics Auxiliary Publication Service (EPAPS) to reduce the amount of the data contained in the paper.

II. PROBLEM AND ASSUMPTION

A. Problem

To begin with, we repeat the problem that is described in Ref. 2. Consider a vapor in a half space \( X_1 > 0 \) bounded by a stationary plane condensed phase of the vapor located at \( X_1 = 0 \), where \( X \) is a rectangular coordinate system. There is a uniform vapor flow at infinity toward the condensed phase with velocity \( (v_{x1}, v_{x2}, 0) \) \( (v_{x1} < 0, v_{x2} > 0) \), temperature \( T_\infty \), and pressure \( p_\infty \). The condensed phase is kept at a constant and uniform temperature \( T_w \). Steady condensation of the vapor is taking place on the condensed phase, and another gas neither condensing nor evaporating on the condensed phase, which we call the noncondensable gas, is confined near the condensed phase by the condensing vapor flow. (See Fig. 1.) We investigate the steady behavior of the vapor and the noncondensable gas on the basis of kinetic theory.

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Our basic assumptions are as follows: (i) the behavior of the vapor and the noncondensable gas is described by the Boltzmann equation for a binary mixture [the Garzó–Santos–Brey (GSB) model\(^2\) will be employed for numerical computation]; (ii) the vapor molecules leaving the condensed phase are distributed according to the corresponding part of the Maxwellian distribution describing the saturated equilibrium state at rest at temperature \(T_w\); (iii) the noncondensable-gas molecules leaving the condensed phase are distributed according to the corresponding part of the Maxwellian distribution with temperature \(T_w\) and flow velocity 0, and there is no net particle flow across the condensed phase (diffuse reflection); (iv) the molecules of the noncondensable gas are mechanically identical with those of the vapor.

\section*{B. Relevant parameters}

As in Ref. 2, we assign label \(A\) to the vapor (it will also be called \(A\)-component) and label \(B\) to the noncondensable gas (it will also be called \(B\)-component) throughout the paper.

The explicit form of the basic equation and boundary condition for the present problem as well as the definition of the macroscopic quantities is given in Ref. 2. To be more specific, the basic equation and its boundary condition are given by Eqs. (2)–(5b) in Ref. 2, and the macroscopic quantities are defined by Eqs. (10a)–(10f) there. Therefore, we omit them here for conciseness.

Now, let \(p_w\) be the saturation pressure of the vapor at temperature \(T_w\), and let \(M_{nA}\) and \(M_{vA}\) be the normal and tangential Mach numbers of the vapor at infinity, respectively:

\[ M_{nA} = -\frac{u_{\infty 1}}{\sqrt{5kT_w/3m_A}}, \quad M_{vA} = \frac{u_{\infty 2}}{\sqrt{5kT_w/3m_A}}, \]

where \(k\) is the Boltzmann constant and \(m_A\) is the mass of a molecule of the vapor, which is the same as that of the noncondensable gas because of assumption (iv). According to Ref. 2, the present (dimensionless) boundary-value problem is characterized by the following set of parameters:

\[ M_{nA}, \quad M_{vA}, \quad T_w/T_{\infty}, \quad p_{\infty}/p_w, \quad \Gamma, \]

where \(\Gamma\) is the dimensionless parameter corresponding to the amount of the noncondensable gas contained in the system and defined as follows:

\[ \Gamma = \frac{2}{\sqrt{\pi}} \frac{1}{n_{A0}} \int_0^\infty n_B dX_1, \]

where \(n_{A0} = p_{\infty}/kT_{\infty}\) is the molecular number density of the vapor at infinity, \(n_B\) the molecular number density of the noncondensable gas, and \(l_{\infty}\) the mean free path of the vapor molecules in the equilibrium state at rest with temperature \(T_{\infty}\) and number density \(n_{A0}\) (or pressure \(p_{\infty}\)); e.g., \(l_{\infty} = (\sqrt{2\pi} (d^3)^2 n_{\infty})^{-1}\) for the hard-sphere molecules, where \(d_A\) is the diameter of a vapor molecule, and \(l_{\infty} = (2/\sqrt{\pi}) (2kT_{\infty}/m_A)^{1/2} K_{\infty A} n_{\infty}\) for the GSB model, where \(K_{\infty A}\) is a constant (see Appendix B of Ref. 2). The aim of the present study is to investigate the relation to be satisfied by the five parameters in Eq. (2) in the case of supersonic condensation.

\section*{III. MECHANICALLY IDENTICAL MOLECULES}

As described in Ref. 2, the assumption (iv) in Sec. II A that the molecules of the noncondensable gas are mechanically identical with those of the vapor simplifies the analysis dramatically. That is, the original problem of two coupled nonlinear equations is decomposed into two single boundary-value problems: a nonlinear problem for the total mixture and a linear homogeneous problem for the noncondensable gas. This approach was originally introduced in Ref. 1 for the case of \(M_{vA} = 0\). Since the approach plays an important role in the following analysis, we first summarize its outline (Sec. III A) and then investigate the relation among the parameters in the case \(M_{nA} \geq 1\) that was omitted in Ref. 2 (Sec. III B).

\section*{A. Outline of analysis}

Let \(F^A\) be the velocity distribution function of the vapor and \(F^B\) that of the noncondensable gas, and let \((\hat{F}^A, \hat{F}^B) = n_{\infty}^{-1} (2kT_{\infty}/m_A)^{3/2} (F^A, F^B)\) be their dimensionless counterparts. The original problem is a boundary-value problem of the simultaneous nonlinear Boltzmann equations for \((\hat{F}^A, \hat{F}^B)\). We now introduce the (dimensionless) velocity distribution function of the total mixture \(\hat{F} = \hat{F}^A + \hat{F}^B\) and transform the boundary-value problem for \((\hat{F}^A, \hat{F}^B)\) to the problem for \((\hat{F}, \hat{F}^B)\). The result of the transformation is given by Eqs. (14a)–(17b) in Ref. 2. This transformation essentially decomposes the problem into two separate problems, one for \(\hat{F}\) and the other for \(\hat{F}^B\), as described below.

The equation for \(\hat{F}\) [Eq. (14a) in Ref. 2] is the Boltzmann equation for a single-component gas, and the boundary condition for \(\hat{F}\) [Eq. (15a) in Ref. 2] contains a quantity denoted by \(n_0\) that depends on \(\hat{F}^B\). However, if we regard this \(n_0\) as the saturation number density of the vapor at temperature \(T_w\), or equivalently, \(p_0 = kT_w n_{A0}\) as the corresponding saturation pressure, then the equation and boundary condition mentioned above reduce to a closed problem for \(\hat{F}\), i.e., the half-space problem of condensation of a pure vapor (\(\hat{F}\) plays the role of the velocity distribution function of the pure vapor), which has been investigated by many authors (e.g., Refs. 6–17). The problem is characterized by the following four parameters:
\[\frac{p_\infty}{p_0} = F_s \left( M_\infty, M_{\infty}, T_\infty/T_w \right) \quad (M_\infty < 1), \]
\[\frac{p_\infty}{p_0} \geq F_b \left( 1, M_{\infty}, T_\infty/T_w \right) \quad (M_\infty = 1), \]
\[\frac{p_\infty}{p_0} > F_b \left( M_\infty, M_{\infty}, T_\infty/T_w \right) \quad (M_\infty > 1). \]

where \(p_0\) has been used rather than \(n_0\). According to Refs. 6–9 and 12–14, there is a solution only when these parameters satisfy a certain relation, which is expressed as follows:

Comprehensive numerical data for the functions \(F_s\) and \(F_b\) based on the Bhatnagar–Gross–Krook (BGK) model are obtained in Ref. 9. These data show the following properties:

\(M_\infty < 1\) (subsonic condensation), and all the four parameters satisfying the inequality (5b) or (5c) when \(M_\infty \geq 1\) (supersonic condensation).

Suppose that we have obtained the solution \(\hat{F}\) for a given value of \(n_0\). Then, the boundary-value problem for \(\hat{F}^B\) [Eqs. (14b), (15b), (16b), and (17b) in Ref. 2] reduces to a linear homogeneous boundary-value problem. Thus, a solution \(\hat{F}^B\) multiplied by an arbitrary constant is also a solution. The unique solution is determined by specifying the total amount of the noncondensable gas or, equivalently, the parameter \(\Gamma^*\) [Eq. (3)]. As in Ref. 2, we denote by \(\hat{F}^B\) the solution when \(\sigma_w^B\) in Eq. (15b) of Ref. 2 is equal to \(n_0\) (this corresponds to the case of \(p_\infty = 0\), i.e., the case where no vapor molecules are emitted from the condensed phase) and by \(\Gamma^*\) the corresponding \(\Gamma^*\). Then the solution \(\hat{F}^B\) for an arbitrary \(\Gamma^*\) is expressed as [Eq. (20) in Ref. 2]

\[\hat{F}^B = \left( \Gamma/\Gamma^* \right) \hat{F}^B, \]

and, therefore, the corresponding \(\hat{F}^A\) is given by

\[\hat{F}^A = \hat{F} - \left( \Gamma/\Gamma^* \right) \hat{F}^B. \]

The \(\hat{F}^A\) and \(\hat{F}^B\) thus obtained solve the original boundary-value problem with the saturation pressure \(p_\infty\) given by

\[p_\infty = (1 - \Gamma/\Gamma^*) p_0, \]

in terms of \(p_0\) (the virtual saturation pressure). Since \(p_\infty\) is non-negative physically, we obtain from Eq. (8) that

\[0 < \Gamma^* \leq \Gamma^*. \]

From Eqs. (5a)–(5c) and (8) and the structure of the solution, we can derive the fundamental property of the relation among the parameters that we are seeking. In the case of subsonic condensation \((M_\infty < 1)\) which is considered in Ref. 2 (see Sec. III D of the same reference), the \(\Gamma^*\) turns out to be a function of the three parameters \(M_\infty, M_{\infty}, T_\infty/T_w\), and the relation among the parameters takes the following form:

\[\frac{p_\infty}{p_0} = F_s(M_\infty, M_{\infty}, T_\infty/T_w, \Gamma), \]

where

\[F_s(M_\infty, M_{\infty}, T_\infty/T_w, \Gamma) = \left( 1 - \frac{\Gamma}{\Gamma^*(M_\infty, M_{\infty}, T_\infty/T_w)} \right)^{-1} \times F_b(M_\infty, M_{\infty}, T_\infty/T_w). \]

Comprehensive numerical data, based on the BGK model, for the function \(F_s\) are available in Ref. 9 (see also Ref. 2), while those for the function \(\Gamma^*\) are constructed in Ref. 2 using the GSB model that is compatible with the BGK model. Therefore, we now have the numerical data for the function \(F_s\). It should be noted that the dependence of \(F_s\) on \(\Gamma^*\) is explicit.

**B. Existence range of a solution: Supersonic condensation**

Next, we investigate the relation among the parameters \(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0\), and \(\Gamma^*\) that allows a solution in the case of supersonic condensation \((M_\infty \geq 1)\). The description below is essentially the same as that in Ref. 1. For the problem of \(\hat{F}\), we can freely choose the parameters \(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0\) satisfying the relation (5b) or (5c), and hence the solution \(\hat{F}\) depends on these four parameters. On the other hand, the problem for \(\hat{F}^B\) contains the parameters \(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0\) through \(\hat{F}\). Therefore, \(\hat{F}^B\) and \(\hat{F}^A\), which is obtained from \(\hat{F}^B\), are the functions of \(M_\infty, M_{\infty}, T_\infty/T_w, \) and \(p_\infty/p_0\). Thus we may write Eq. (8) explicitly as

\[p_\infty/p_0 = \left( 1 - \frac{\Gamma}{\Gamma^*(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0)} \right)^{-1} \times p_0, \]

and Eq. (9) as

\[0 \leq \Gamma^* \leq \Gamma^*. \]

Now, let us suppose that \(\Gamma^*\) is a decreasing function of \(p_\infty/p_0\) (It is numerically confirmed in Ref. 1 that \(\Gamma^*\) is a monotonically decreasing function of \(p_\infty/p_0\) when \(M_{\infty} = 0\)). This hypothesis will be confirmed numerically in Sec. IV A below. Then, for fixed \(M_\infty, M_{\infty}, T_\infty/T_w, \) and \(\Gamma^*\), the range of \(p_\infty/p_0\) is from \(F_b\) [Eq. (5c)] to the value of \(p_\infty/p_0\) such that \(\Gamma^*(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0) = \Gamma^*\) holds, since \(\Gamma^*\) cannot be less than \(\Gamma^*\) by Eq. (13). When \(p_\infty/p_0\) ranges in this interval, \(p_\infty/p_0\) ranges from \([1 - \Gamma/\Gamma^*(M_\infty, M_{\infty}, T_\infty/T_w, p_\infty/p_0)]^{-1} F_b\) to infinity because the right-hand side of Eq. (12) is an increasing function of \(p_\infty/p_0\).

To summarize, in the case of supersonic condensation, there exists a solution only when the parameters satisfy the following relation:
where

\begin{align}
\mathcal{F}_b(M_{\infty}, M_{\infty}, T_{\infty}/T_w, \Gamma) &= \left(1 - \frac{\Gamma}{\Gamma_b(M_{\infty}, M_{\infty}, T_{\infty}/T_w)}\right)^{-1} \\
& \times F_b(M_{\infty}, M_{\infty}, T_{\infty}/T_w), \tag{15a}
\end{align}

\begin{align}
\Gamma_b(M_{\infty}, M_{\infty}, T_{\infty}/T_w) &= \Gamma_a(M_{\infty}, M_{\infty}, T_{\infty}/T_w, p_{\infty}/p_0 \rightarrow F_b). \tag{15b}
\end{align}

As in the case of subsonic condensation, if we exploit the comprehensive numerical data for \( F_b \) based on the BGK model given in Ref. 9, we just need to compute \( \Gamma_b \) for various values of the set \((M_{\infty}, M_{\infty}, T_{\infty}/T_w)\), making use of a model Boltzmann equation compatible with the BGK model. In this way, we can construct the function \( \mathcal{F}_b \). It should be stressed that, since the \( \Gamma \)-dependence of \( \mathcal{F}_b \) is explicit, we are able to construct \( \mathcal{F}_b \) of four variables by obtaining the function \( \Gamma_b \) of three variables. This reduces the amount of necessary computation dramatically. We will carry out the actual numerical computation to obtain \( \Gamma_b \) in the next section.

IV. NUMERICAL ANALYSIS AND RESULTS

In this section, we carry out actual numerical analysis to obtain \( \Gamma_b \). As in Refs. 1–3, we employ the GSB model of the Boltzmann equation, which is summarized in Appendix B of Ref. 2. We solve the problem by means of a finite-difference method. Since the solution method is essentially the same as that given in Ref. 9 for the case of a single-component system, we omit it here. See also Sec. IV A of Ref. 2 for some remarks on numerical analysis. Information about the accuracy of the present computation is given in the Appendix.

A. Existence range of a solution

In this section, we show some numerical results for the existence range of a solution discussed in Sec. III B. First, we confirm the assumption we made in Sec. III B, namely, \( \Gamma_a(M_{\infty}, M_{\infty}, T_{\infty}/T_w, p_{\infty}/p_0) \) is a decreasing function in \( p_{\infty}/p_0 \) (this has already been confirmed numerically in Ref. 1 for the case \( M_{\infty}=0 \)). The \( \Gamma_a \) versus \( p_{\infty}/p_0 \) for various values of \( M_{\infty} \) and \( M_{\infty} \) in the case of \( T_{\infty}/T_w=1 \) are shown in Fig. 2. Clearly, the function \( \Gamma_a \) is decreasing in \( p_{\infty}/p_0 \). It is true also for other values of \( T_{\infty}/T_w \). Hence the discussion in Sec. III B is valid, and the existence range of a solution is given by Eqs. (14a)–(15b).

Once we have the data for the functions \( F_b \) and \( \Gamma_b \), Eq. (15a) gives the function \( \mathcal{F}_b \) immediately. In principle, \( F_b \) can be constructed by trying to obtain the solution \( \tilde{F} \) for many sets of values of the parameters \((M_{\infty}, M_{\infty}, T_{\infty}/T_w, p_{\infty}/p_0, p_0)\). However, since such an approach is not practical, another indirect method was used in Ref. 9 to obtain numerical values of \( F_b \), which will be explained below. Now let us suppose that \( F_b \) is known and recall that \( \Gamma_b \) is the limiting value of \( \Gamma_a(M_{\infty}, M_{\infty}, T_{\infty}/T_w, p_{\infty}/p_0) \) as \( p_{\infty}/p_0 \rightarrow F_b(M_{\infty}, M_{\infty}, T_{\infty}/T_w) \) [see Eq. (15b)]. The straightfor-
ward way to compute this limit is to obtain the solution \( \tilde{F}_b \) for several values of \( p_\infty/p_0 \) close enough to \( F_b \) and deduce the limit of \( \Gamma_b \) by extrapolation. However, as \( p_\infty/p_0 \) approaches \( F_b \), the computation for obtaining \( \tilde{F}_b \) becomes increasingly difficult (note that there is no solution at \( p_\infty/p_0 = F_b \)). Therefore, such a method is not practical. This situation is the same as that in Ref. 1 for the case \( M_\infty = 0 \), where a different (indirect) method to obtain \( \Gamma_b \) is proposed. We make use of the same method, which will be described, together with the method for obtaining \( F_b \), in the following.

According to Refs. 7–9, the behavior of \( \tilde{F}_b \) for \( (M_\infty, M_\infty, \Gamma_b) \) is summarized as follows.

(i) When \( p_\infty/p_0 \) is sufficiently close to \( F_b(M_\infty, M_\infty, \Gamma_b) \), the solution \( \tilde{F}_b \) is described as follows: the part near the condensed phase is almost a subsonic solution (a solution with \( M_\infty < 1 \)), and this part is followed by an almost entire profile of a standing shock, parallel to the condensed phase, whose upstream state is the state at infinity \( (p_\infty, T_\infty, M_\infty, M_\infty) \). As \( p_\infty/p_0 \) approaches \( F_b \), the position of the standing shock moves upstream, and the separation between the subsonic-solution part and the standing-shock part becomes clearer.

(ii) In the limit \( p_\infty/p_0 \to F_b \), the position of the standing shock moves to upstream infinity, and thus the separation becomes complete. This means that the limiting solution is a subsonic solution with a standing shock at infinity and thus is not a true supersonic solution. However, we can interpret it as the marginal supersonic solution. Therefore the limiting solution is the subsonic solution with the upstream parameters \( M'_\infty, M'_\infty, \) and \( T'_\infty \) [thus \( p'_\infty = p_0 F_s(M'_\infty, M'_\infty, T'_\infty/T_w) \)], where \( M'_\infty, M'_\infty, \) and \( T'_\infty \) are given by the standing shock relation (Rankine–Hugoniot relation) from \( M_\infty, M_\infty, \) and \( T_\infty \) as follows:

\[
M'_\infty = (M_\infty^2 + 3)^{1/2}(5M_\infty^2 - 1)^{-1/2},
\]

\[
M'_\infty = \left( \frac{T_\infty}{T_w} \right)^{1/2}\left( \frac{T'_\infty}{T_w} \right)^{-1/2} M'\infty,
\]

\[
T'_\infty = \frac{M'_\infty^2 + 3(5M_\infty^2 - 1)}{16M_\infty^2} T_\infty.
\]

Equation (16b) indicates the continuity of the tangential velocity component across the shock. Since the shock relation also gives

\[
p'_\infty = 5M_\infty^2 - 1 \frac{p_\infty}{p_0},
\]

the \( F_b \), which corresponds to \( p_\infty/p_0 \), is obtained as

\[
F_b(M_\infty, M_\infty, T_\infty/T_w) = \frac{4}{5M_\infty^2 - 1} F_s(M'_\infty, M'_\infty, T'_\infty/T_w).
\]

The behavior described above is based on physical consideration with some numerical evidence.\(^9\) For \( M_\infty \) close to unity, however, it has been justified analytically in Ref. 12. Although \( M_\infty = 0 \) is assumed in this reference, it is not essential for the analysis there.

Since \( F_b^B \) vanishes at infinity in the subsonic solution, the limiting value of \( f^B dX \) as \( p_\infty/p_0 \to F_b^B \) is equal to the value calculated from the corresponding subsonic solution, i.e., the solution with the parameters \( M'_\infty, M'_\infty, \) and \( T'_\infty \) (thus \( p'_\infty \)) at infinity. Taking account of this fact, we obtain from Eq. (3) that \( n_{s,b} \) in \( \Gamma_b \) is equal to the mean free path of the vapor in the equilibrium state at rest with temperature \( T'_\infty \) and number density \( n_{s,b}^- \). Since \( n_{s,b}^- = p'_\infty/kT'_\infty \), the limiting value of \( \Gamma_b \) for subsonic solution \( T'_\infty/\Gamma_b \) holds for the GSB model (see the last paragraph of Sec. II B), we obtain the formula

\[
\Gamma_b(M_\infty, M_\infty, T_\infty/T_w) = \frac{T'_\infty}{T_w} \left( \frac{T'_\infty}{T_w} \right)^{-1} \Gamma_b(M'_\infty, M'_\infty, T'_\infty/T_w).
\]

From Eqs. (16a)–(16c) and (18), we can obtain \( F_b \) immediately for any \( M_\infty, M_\infty, \) and \( T_\infty/T_w \) by using the numerical data of \( F_b \) tabulated in Ref. 9 (see also Ref. 2) and interpolation. In Ref. 9, however, for the purpose of presenting accurate numerical values of \( F_b \), in a well-arranged way, the subsonic solution \( \tilde{F}_b \) for each set of \( (M'_\infty, M'_\infty, T'_\infty/T_w) \) \( (M'_\infty < 1) \) given by Eqs. (16a)–(16c) was recomputed to obtain \( F_b' \) from which \( F_b \) was obtained by Eq. (18) without the help of interpolation. More specifically, Fig. 7 and Tables V–VIII in Ref. 9 show the numerical data of \( F_b(M_\infty, M_\infty, T_\infty/T_w) \) for \( T_\infty/T_w = 0.5, 1, 1.5, \) and 2 obtained in this way. In the present study, we have repeated the same computation with higher accuracy and confirmed the accuracy of the data given in Ref. 9. The results are as follows: in Tables VI–VIII in Ref. 9, the last figure should be changed by one in several data, and in Table V there, the last figure should be changed by one in five data for \( M_\infty = 1.2 \) and by at most six in the data for \( M_\infty = 1.1 \) and 1.01. We also made additional computations to supplement these data. Some of the results are shown in Table I, the more comprehensive data being given in Tables I–IV in Ref. 20.

Similarly, from Eqs. (16a)–(16c) and (19), we can obtain \( \Gamma_b \) using the numerical values of \( \Gamma_b \) for subsonic solutions tabulated in Ref. 2 with the help of interpolation. But, in order to give accurate numerical data in a well-arranged way, we recompute \( \tilde{F}_b^B \) that corresponds to the subsonic solution \( \tilde{F}_b \) obtained above [i.e., the subsonic solution for \( (M'_\infty, M'_\infty, T'_\infty/T_w) \) given by Eqs. (16a)–(16c)] and obtain \( \Gamma_b \) for this \( \tilde{F}_b^B \). Thus, we obtain accurate numerical values of \( \Gamma_b \) by Eq. (19) without using interpolation. Some of the results for \( \Gamma_b \) obtained in this way are shown in Table II, the more comprehensive data being given in Tables V–VIII in Ref. 20.

The function \( F_b \) obtained with the help of the numerical data for \( F_b \) and \( \Gamma_b \) in the case \( T_\infty/T_w = 1 \) is shown in Fig. 3. In the figure, \( F_b \) versus \( M_\infty \) is shown for various \( \Gamma \) at four values of \( M_\infty \), i.e., \( M_\infty = 0, 1, 2, \) and 3. Similar figures for \( T_\infty/T_w = 0.5, 1, 1.5, \) and 2 are given in Figs. 1, 3, and 4 in Ref. 20 (Fig. 2 in Ref. 20 is the same as Fig. 3 here). The \( F_b \) is a decreasing function of \( M_\infty \), and its curve moves upward as
Therefore the features of $G$ increase. The $\Gamma_c$ in the figures, which depends on $M_{\infty}$ and $T_c/T_w$, is a critical value of $\Gamma$, that is, when $\Gamma < \Gamma_c$, $F_b$ takes a finite value at $M_{\infty} = 1$, whereas when $\Gamma > \Gamma_c$, $F_b$ becomes infinitely large as $M_{\infty}$ approaches a certain value of $M_{\infty}$ depending on $T_c/T_w$, $\Gamma_c$, and $M_t$. We denote this value by $\tilde{M}_c$. When $\Gamma = \Gamma_c$, $F_b$ goes to infinity at $M_{\infty} = 1$ (hence $\tilde{M}_c = 1$). In other words, $M_{\infty} = M_c$ is the asymptote of the curve. Consequently, there is no solution in the interval $1 < M_{\infty} \leq \tilde{M}_c$ when $\Gamma > \Gamma_c$. Further properties of $F_b$ will be discussed in the next paragraph, where more detailed information about $\Gamma_c$ and $\tilde{M}_c$ will also be given.

Because the dependence of $F_b$ and $\Gamma_b$ on $M_{\infty}$ is not strong, the function $F_b$ does not depend much on $M_{\infty}$. Therefore the features of $F_b$ are essentially the same as those described in Refs. 1 and 3 for $M_{\infty} = 0$. In particular, for $M_{\infty}$ smaller than around 1, $F_b$ is almost independent of $M_{\infty}$. The dependence of $F_b$ on $T_c/T_w$ is also weak (see Figs. 1–4 in Ref. 20). The $F_b$ is a decreasing function of $M_{\infty}$, whereas $\Gamma_b$ is its increasing function. Therefore, as is seen from Eq. (15a), $F_b$ is a decreasing function of $M_{\infty}$. It follows from Eqs. (16a)–(16c), (18), and (19) that at $M_{\infty} = 1$, the following relations hold:

\begin{align}
F_b(1, M_{\infty}, T_c/T_w) &= F_b(1 - M_{\infty}, T_c/T_w), \\
\Gamma_b(1, M_{\infty}, T_c/T_w) &= \Gamma_b(1 - M_{\infty}, T_c/T_w)
\end{align}

(20a)

(20b)

where $\Gamma_b$ is the same as that used in Ref. 2. Analytical evidence for the relation (20a) is found in Ref. 12. Then, the properties of $F_b$ described in the preceding paragraph follow immediately from Eqs. (15a) and (20b) and from the fact that $\Gamma_b$ is an increasing function of $M_{\infty}$. That is, when $\Gamma < \Gamma_c$, $\Gamma_c/T_w$ in Eq. (15a) is less than unity and thus $F_b$ remains finite in the whole range of $M_{\infty} = 1$. From Eqs. (11), (15a), (20a), and (20b), we have

\begin{align}
\Gamma_b(1, M_{\infty}, T_c/T_w, \Gamma) &= \Gamma_b(1 - M_{\infty}, T_c/T_w, \Gamma)
\end{align}

(21)

in this case. When $\Gamma > \Gamma_c$ (or $\Gamma_c$), the $\Gamma_c/T_w$ in Eq. (15a) becomes unity at $M_{\infty} = 1$ (or at $M_{\infty} = 1$. If we denote this value of $M_{\infty}$ by $\tilde{M}_c$ ($\tilde{M}_c = 1$ for $\Gamma = \Gamma_c$), $F_b$ increases indefinitely as $M_{\infty}$ approaches $\tilde{M}_c$. The $\Gamma_c$, which is a function of $M_{\infty}$ and $T_c/T_w$, is shown in Fig. 3 of Ref. 2. The $\tilde{M}_c(M_{\infty}, T_c/T_w, \Gamma)$, which is the solution of
is shown in Fig. 4, where \( \tilde{M}_c \) versus \( M_{\alpha} \) at \( T_{w}/T_w = 0.5, 1, 1.5 \), and 2 is plotted for various values of \( \Gamma \); \( \tilde{M}_c \) is taken as the abscissa for easy comparison with Fig. 3 (and Figs. 1–4 in Ref. 20).

### B. Macroscopic quantities

In this section, we show the behavior of the macroscopic quantities. In the case of supersonic condensation (\( M_{\alpha} \approx 1 \)), we can freely choose the parameters \( (M_{\alpha}, M_{\gamma}, T_{w}/T_w,p_{\alpha}/p_{\gamma}, \Gamma) \) in the region (14a) or (14b). However, it is more convenient to arrange the results using the parameters \( (M_{\alpha}, M_{\gamma}, T_{w}/T_w,p_{\alpha}/p_{\gamma}, \Gamma) \) rather than the original parameters \( (M_{\alpha}, M_{\gamma}, T_{w}/T_w,p_{\alpha}/p_{\gamma}, \Gamma) \), since the basic quantities \( (\tilde{F}, \tilde{F}_B^{\gamma}) \) and \( \Gamma \), which give the solutions \( (F^A,F^B) \) for arbitrary \( \Gamma \), are determined by \( M_{\alpha}, M_{\gamma}, T_{w}/T_w, p_{\alpha}/p_{\gamma}, \Gamma \), and so on. We use the same notations for the macroscopic quantities as in Ref. 2 (cf. Sec. II B of Ref. 2): \( n^\alpha \) denotes the molecular number density, \( v^\alpha \) the flow velocity, \( T^\alpha \) the temperature, and \( p^\alpha (=kT) \) the pressure of the total mixture. \([n^B\) has already been introduced in Sec. II in the sentence following Eq. (3).]

The typical profiles of the macroscopic quantities for \( T_{w}/T_w = 1 \) are shown in Figs. 5–7, i.e., Fig. 5 for \( M_{\alpha} = 1.5 \) and \( p_{\alpha}/p_{\gamma} = 2.593 \), Fig. 6 for \( M_{\alpha} = 2 \) and \( p_{\alpha}/p_{\gamma} = 4 \), and Fig. 7 for \( M_{\alpha} = 1.05 \) and \( p_{\alpha}/p_{\gamma} = 22 \). In each figure, the result for \( M_{\alpha} = 1 \) is shown in (a) and that for \( M_{\alpha} = 2 \) in (b), and the notation \( a_{\alpha} = (5kT_{w}/3m\alpha)^{1/2} \) has been introduced. It should be noted that \( v_{\alpha} = 0 \) in the whole region of \( X_{\alpha} > 0 \), and the quantities \( (\Gamma_{\alpha}/\Gamma)^{n_{\alpha}}, v_{\alpha}^{B}, T_{\alpha}^{B}(=p_{\alpha}/kT_{\alpha}), (\Gamma_{\alpha}/\Gamma)^{P_{\alpha}} \) are independent of \( \Gamma \) (see Secs. II C and IV C of Ref. 2). The \( n, v_{\gamma}, v_{\alpha}, T, \) and \( p \) for the total mixture are the same as \( n^A, v^A, v_{\gamma}^{B}, T_{\gamma}^{A}, \) and \( p^A \) for \( \Gamma = 0 \), respectively.

Figure 5 demonstrates the profiles for the parameters close to the boundary of the existence range \( (5c) \) [or Eq. (14a)] (note that \( F_{\beta} = 2.1801 \) for \( M_{\gamma} = 1 \) and \( F_{\beta} = 2.5929 \) for \( M_{\gamma} = 2 \) in the case \( M_{\alpha} = 1.5 \) and \( T_{w}/T_w = 1 \). Since the parameters for Fig. 5(b) are very close to the boundary of the existence range, the profile exhibits the features described in the fourth paragraph in Sec. IV A, that is, the profile is a combination of a subsonic solution and a standing shock that are well separated from each other. The noncondensable gas

<table>
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<th>( M_{\alpha}/M_{\gamma} )</th>
<th>( T_{w}/T_w = 0.5 )</th>
<th>( T_{w}/T_w = 1 )</th>
<th>( T_{w}/T_w = 1.5 )</th>
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<th>( T_{w}/T_w = 2 )</th>
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<td>1.620 5</td>
<td>1.585 1</td>
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<td>1.360 8</td>
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</tbody>
</table>

\[ \Gamma_{\beta}(\tilde{M}_c,M_{\gamma},T_{w}/T_w) = \Gamma_{\beta} \]
is confined only in the subsonic-solution part. Figure 6 shows the profiles for the parameters well inside the existence range (5c) [or Eq. (14a)] (note that \( F_b = 0.87560 \) for \( M_{\infty} = 1 \) and \( F_b = 1.00691 \) for \( M_{\infty} = 2 \) in the case \( M_{\infty} = 2 \) and \( T_e/T_w = 1 \)). These profiles are of the same type as Fig. 11 of Ref. 9. Figure 7, which corresponds to Fig. 12 of Ref. 9, shows the profiles for large \( p_\infty/p_0 \) and \( M_{\infty} \) close to 1 (note that \( F_b = 10.825 \) for \( M_{\infty} = 1 \) and \( F_b = 13.300 \) for \( M_{\infty} = 2 \) in the case \( M_{\infty} = 1.059 \) and \( T_e/T_w = 1 \)). Since the features of the macroscopic quantities demonstrated in Figs. 5–7 are essentially the same as those already discussed in Ref. 9 for the pure-vapor case \((\Gamma = 0)\) and Refs. 1 and 3 in the case of \( M_{\infty} = 0 \), we omit the repetition of the explanations here.

As discussed in Ref. 1, \( v_2^B \) and \( T_B \) do not approach \( v_\infty^2 \) and \( T_\infty \) and may also have gradients at infinity. In any case, \( v_2^B \) and \( T_B \) are not meaningful in the far field where \( n_B \) becomes practically zero.

### C. Particle flux of the noncondensable gas along the condensed phase

As is seen from Figs. 5–7, there is a macroscopic motion of the noncondensable gas along the condensed phase (i.e., in the \( X_2 \) direction) when the vapor flow at infinity has a transversal component \((M_{\infty} \neq 0)\). As in Ref. 2, we introduce the following dimensionless quantity corresponding to the total particle flux of the noncondensable gas:

\[
\tilde{N_f} = \langle 2/\sqrt{\pi} \rangle [n_\infty l_s (2kT_\infty/m^4)^{1/2}]^{-1} \int_0^\infty n_i^B v_2^B dX_1.
\]

(23)

Actually, \((\sqrt{\pi}/2)n_\infty l_s (2kT_\infty/m^4)^{1/2}\tilde{N_f}\) indicates the total particle flux of the noncondensable gas in the direction of \(X_2\) per unit width in \(X_3\) and per unit time. If we use the definition of the macroscopic quantities and the relation between dimensional and dimensionless quantities given in Ref. 2, Eq. (23) is written in terms of \( \hat{F}^B \) as

\[
\tilde{N_f} = \int_0^\infty \langle \int \zeta_2 \hat{F}^B d\zeta \rangle dX_1,
\]

(24)
where $x_1 = (2/\sqrt{\pi})l_{\infty}^{-1}X_1$ is the dimensionless space coordinate, $\xi_i$ is the dimensionless molecular velocity nondimensionalized by $(2 k T_\infty/m A)^{1/2}$, and $d^3\xi = d\xi_1 d\xi_2 d\xi_3$; here and in what follows, the domain of integration with respect to $\xi_i$ is the whole space of $\xi_i$.

Since $F^B$ is determined by $M_{n\infty}$, $M_{v\infty}$, $T_\infty/T_w$, $p_{\infty}/p_w$, and $\Gamma$ in the case of supersonic condensation, $\tilde{N}_f$ is written in the following form:

$$\tilde{N}_f = \tilde{N}_f(M_{n\infty}, M_{v\infty}, \frac{T_\infty}{T_w}, \frac{p_{\infty}}{p_w}, \Gamma), \quad (M_{n\infty} \geq 1).$$

FIG. 5. Profiles of the macroscopic quantities for $M_{n\infty} = 1.5$, $T_\infty/T_w = 1$, and $p_{\infty}/p_w = 2.593$. (a) $M_{v\infty} = 1$ and (b) $M_{v\infty} = 2$. Here, $a_{\infty} = (5 k T_\infty/m A)^{1/2}$ is the sound speed at temperature $T_\infty$. The macroscopic quantities of the total mixture are given by those of the vapor for $\Gamma = 0$. The profiles of $(n/\Gamma)(n^p/\Gamma)(n^p/\Gamma)$ are independent of $\Gamma$. 

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According to Ref. 4, the function \( \hat{N}_f \), along with the continuity equation in the Knudsen layer, is required as a part of the boundary condition for the Euler set of equations in the continuum limit. For this reason, we give some of the numerical data for \( \hat{N}_f \). Table III shows the data for \( \hat{N}_f \) in the case \( T' / T_w = 5 \). The data for \( T' / T_w = 0.5, 1.5, \) and 2 are given in Tables IX, XI, and XII in Ref. 20, respectively.

As is seen from Table III, the \( \hat{N}_f \) depends weakly on \( M_{\infty} \) and \( T' / T_w \) (see also Tables IX–XII in Ref. 20). Figure 8 shows \( \hat{N}_f \) versus \( M_{\infty} \) for various \( \Gamma \) and that versus \( \Gamma \) for various \( M_{\infty} \). Figure 8 shows \( \hat{N}_f \) in the case of \( M_{\infty} = 1.2 \), \( T' / T_w = 1 \), and \( p' / p_\infty = 20 \) [(a)] and of \( M_{\infty} = 2 \), \( T' / T_w = 2 \), and \( p' / p_\infty = 10 \) [(b)]. As is seen from Table III, the \( \hat{N}_f \) depends weakly on \( M_{\infty} \) and \( p' / p_\infty \) (see also Tables IX–XII in Ref. 20) and increases with \( M_{\infty} \) and \( \Gamma \) (see Fig. 8). The dependence on...
$T_w/T_w$ is also weak (see Tables IX–XII in Ref. 20).

If we denote by $\hat{N}_{f*}$ the $\hat{N}_f$ corresponding to $F_{\hat{g}*}$, we have [see Eq. (31) in Ref. 2]

$$\hat{N}_{f*} = (\Gamma/\Gamma_*)\hat{N}_{f*}.$$  \hspace{1cm} (26)

Since $\hat{F}_{\hat{g}*}$ depends on $M_{n*}$, $M_{t*}$, $T_w/T_w$, and $p_*/p_0$, the quantity $\hat{N}_{f*}$, as well as $\Gamma_*$, is a function of these four parameters. The data in Table III have been computed in the following way. We first solve $p_*/p_0$ corresponding to the given set ($M_{n*}$, $M_{t*}$, $T_w/T_w$, $p_*/p_0$, $\Gamma$) from Eq. (12) with the help of interpolation based on the numerical data of $\hat{F}_* (M_{n*}, M_{t*}, T_w/T_w, p_*/p_0)$. Next, we solve the half-space problem for the total mixture numerically to obtain $\hat{F}$ for the original values of $M_{n*}$, $M_{t*}$, and $T_w/T_w$ and the

FIG. 7. Profiles of the macroscopic quantities for $M_{n*} = 1.05$, $T_w/T_w = 1$, and $p_*/p_0 = 22$. (a) $M_{n*} = 1$ and (b) $M_{n*} = 2$. See the caption of Fig. 5.
and computed $\hat{N}_f(M_{\infty}, M_0, T_\infty/T_w, p_\infty/p_w, \Gamma)$ as a function of $M_{\infty}$, $M_0$, $p_\infty/p_w$, and $\Gamma$ ($T_\infty/T_w = 1$). There is no solution for $\Gamma = 0.1$ in the case $M_{\infty} = 1.2$ and $p_\infty/p_w = 10$.

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obtained value of $p_\infty/p_0$. Then, we solve the linear problem for the noncondensable gas numerically to obtain $\hat{F}^B_*$, from which $\hat{N}_{f*}$ is computed. The $\hat{N}_f$ is obtained from Eq. (26). However, once we know the existence range (14a) and (14b), there is no merit to use the above indirect procedure. Therefore, in order to obtain the data given in Tables IX, XI, and XII in Ref. 20, we made use of a direct method, namely, we numerically solved the original boundary-value problem for $(\hat{F}_A^*, \hat{F}^B_*)$, rather than the problem for $(\hat{F}_*, \hat{F}^B_*)$, specifying the original parameters ($M_{\infty}$, $M_0$, $T_\infty/T_w$, $p_\infty/p_w$, $\Gamma$) and computed $\hat{N}_f$ directly.

V. CONCLUDING REMARKS

The present paper is the second part of the study of a flow of a vapor condensing onto a plane condensed phase at incidence in the case where a noncondensable gas is present near the condensed phase. The case of subsonic condensation, i.e., the case where the component of the flow velocity of the vapor perpendicular to the condensed phase at infinity is subsonic, is studied in the first part,\textsuperscript{2} whereas the case of supersonic (and sonic) condensation is considered in the present paper. As in the first part\textsuperscript{2} and in the earlier works,\textsuperscript{1,3} we restricted ourselves to the case where the molecules of the vapor and those of the noncondensable gas are mechanically identical. Making use of the general features of the solution discussed in Ref. 2, we derived essential properties of the parameter range that admits a steady solution (Sec. III). Then, with the help of the property of the boundary of the parameter range discussed in Ref. 1 and extensive numerical computation based on the GSB model, the parameter range was constructed numerically (Sec. IV A). Finally, the behavior of the macroscopic quantities was clarified (Secs. IV B and IV C).

The present result for the parameter range that admits a solution, together with the corresponding result for subsonic condensation in Ref. 2, completes the boundary condition for the compressible Euler set of equations that describes the steady flows of the vapor around arbitrarily shaped condensed phases in the continuum limit in the presence of a tiny amount of the noncondensable gas.\textsuperscript{4} The boundary condition, however, is still subject to the limitation that the vapor molecules are mechanically the same as the noncondensable-gas molecules and that the numerical results are obtained on the basis of the GSB model. The relaxation of these limitations would be an important future work.

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The values of was performed for several cases for $M_t$. The lattice systems for 1, 2, 3, and 4 are spatially uniform and are expressed in terms of the quantities at infinity as

$$I_1 = I_{1\infty} = -(5/6)^{I2}M_{\infty}$$

$$I_2 = I_{2\infty} = (5/3M_{\infty}^2 + 1)/2$$

$$I_3 = I_{3\infty} = -(5/6)^{I2}M_{\infty}$$

$$I_4 = I_{4\infty} = (5/6)^{I2}M_{\infty}(M_{\infty}^2 + 2 + M_{\infty}^2 + 3).$$

Because of numerical error, this uniformity is not satisfied exactly, and $I_4^B$ does not vanish exactly. The deviations of the numerical values of $I_{m\infty} - I_{m\infty}$ and $I_{m\infty}^B$ from zero, where $I_{m\infty}^B$ is the $I_1$ with $F^B = F^B_w$ (see the third paragraph in Sec. III A), are estimated as follows:

$$\left| \frac{(I_m - I_{m\infty})}{I_{m\infty}} \right| < 0.89 \times 10^{-7},$$

for all $M_{\infty}$, $M_{\infty}$, and $T_w/T_w$ included in Tables I–IV in Ref. 20 ($M_{\infty} = 0$ is excluded for $m = 3$ because $I_3 = I_{3\infty} = 0$ in this case).

APPENDIX: DATA ON NUMERICAL COMPUTATION

In this appendix, we give some information on the accuracy of the present numerical analysis. In the present computation based on the GSB model collision term, we only need to handle two independent variables, $x_1 = \sqrt{2/\pi}l_x^{-1}X_1$ and $\xi_1$, because the transversal components $\xi_2$ and $\xi_3$ of the molecular velocity can be eliminated (see Sec. IV A of Ref. 2). The lattice systems for $x_1$ and $\xi_1$ used here are essentially the same as those used in Ref. 9 (see Appendix A of Ref. 9). In the present computation, however, the higher accuracy was attained by using wider computational regions, more lattice points, and smaller lattice intervals. The details of the lattice systems are omitted here.

We checked the accuracy of the computation in various ways. For many cases included in Tables I–IV in Ref. 20, we carried out computation with finer lattice systems with double lattice points either in $x_1$ or in $\xi_1$ and confirmed that the values of $F_{\theta}$ and $\Gamma_{\theta}$ in Tables I and II (and Tables I–VIII in Ref. 20) did not change. More specifically, concerning the $x_1$ lattices, this check was performed for all $M_{\infty}$ and for $M_{\infty} = 0$ and 3 in the cases included in Tables I–IV in Ref. 20. The same check was also performed for many other cases in Tables I–IV in Ref. 20. As for the $\xi_1$ lattices, the check was performed for several cases for $M_{\infty} = 3$ of Tables I–VI in Ref. 20. In general, accurate computation becomes more difficult as $M_{\infty}$ increases.

The conservation laws were also used for checking the accuracy. As in Ref. 2, let us introduce the following quantities:

$$I_1 = \int \xi_1(1, \xi_1, \xi_2, \xi_3) F_{\theta} d^3 \xi,$$

$$I_2^B = \int \xi_1 F_{\theta} d^3 \xi.$$  

(A1)

(A2)

The $n_{n}(2kT_{n}/m^A)^{1/2}I_1$, $2p_{s}I_2$, $2p_{s}I_3$, and $p_{s}(2kT_{s}/m^A)^{1/2}I_4$ indicate, respectively, the number of molecules, the $X_1$ component of the momentum, its $X_2$ component, and the energy of the total mixture transported in the positive $X_1$ direction across a unit area of the plane $X_1 = \text{const}$ per unit time; $n_{n}(2kT_{n}/m^A)^{1/2}I_1$ is the molecular flux of the noncondensable gas corresponding to $n_{n}(2kT_{n}/m^A)^{1/2}I_1$. It is shown in Appendix C of Ref. 2 that $I_5^B = 0$ and that $I_m (m = 1, 2, 3, 4)$ are spatially uniform and are expressed in terms of the quantities at infinity as

$$I_1 = I_{1\infty} = -(5/6)^{I2}M_{\infty}$$

$$I_2 = I_{2\infty} = [ (5/3M_{\infty}^2 + 1)/2 ]$$

$$I_3 = I_{3\infty} = -(5/6)^{I2}M_{\infty}$$

$$I_4 = I_{4\infty} = (5/6)^{I2}M_{\infty}(M_{\infty}^2 + 2 + M_{\infty}^2 + 3).$$

(A3)


Y. Sone, Kinetic Theory and Fluid Dynamics, Modeling and Simulation in Science, Engineering and Technology (Birkhäuser, Boston, 2002).


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