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THE RELATIONSHIP BETWEEN COGNITION AND AFFECT ON ONLINE MATHEMATICS AND THEIR INTERACTION OVER TIME

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The relationship between cognition and affect on online mathematics and their interaction over time

ABSTRACT

This paper presents a case study in order to explore the relationship between cognitive and affective factors in online mathematics learning. The study was carried out on a basic mathematics course which forms part of the engineering studies at the Open University of Catalonia (UOC), and focused not only in the relationship between these factors but also on their mode of interaction over one semester. The study looks at the relations between self-beliefs (specifically, self-confidence to do and learn mathematics), self-regulation processes (specifically, persistence in doing and learning mathematics), and performance (specifically, academic results).

KEYWORDS

Online mathematics, learning processes, cognition and affect, self-beliefs, behavioural engagement, self-regulation, persistence.

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INTRODUCTION

The literature suggests that "mathematics is an emotional thing", and that in learning it, affective factors play a role in cognitive guidance, facilitating or blocking the acquisition of knowledge (Gómez-Chacón, 2003).

In recent years, research has claimed that affective factors are key factors in understanding student behaviour in mathematics. Emotions and beliefs play a central role in students' success or failure in mathematics. In this sense, some of the most highlighted aspects by researchers are: the powerful impact they have on how students learn and use mathematics; the influence on the structure of self-conception as a learner of mathematics; the interactions produced with the cognitive system; and the obstacle they constitute for effective learning (Gómez-Chacón, 2000).

In spite of the growing increase over the last two decades of research focusing on the study of the role of affect in learning mathematics, there is still a need to consolidate a theoretical framework in order to increase the coherence between the different current approaches. In addition, further research is necessary that explores the dynamics of interaction between affect and cognition (Gómez-Chacón, 2010; Hannula, Evans, Philippou & Zan, 2004). We are particularly interested in tackling this issue in online mathematics in higher education. We expect that the method (e-learning) as well as the academic level (HE) will became significant in the discussion of our findings.

Student drop-out rates are higher for online mathematics courses than for other online disciplines. This is does not apply to face-to-face courses (Smith & Ferguson, 2005). The reasons suggested by researchers for this difference are: firstly, the profile of online students, who tend to be older, working full time, and who are often returning to study after a long break; and secondly the fact that mathematics is cumulative and makes them less tolerant of their gaps in knowledge. In addition, current pedagogical models of e-learning and characteristics of Learning Management Systems do not effectively solve the challenges of online mathematics. One such challenge we would like to highlight is that of learning abstract concepts and mathematical notation (Smith & Ferguson, 2005; John, Huertas, Steegmann, Córcoles & Serrat, 2008; Sancho & Pérez, 2009).

Because of such specific difficulties in online studies, the link between affect and cognition in the learning process may become decisive in students' academic paths. In-depth analysis of these factors can shed light on the definition of more flexible teaching strategies, adapted to their context so as to achieve lower drop-out rates and a higher level of satisfaction in the students who complete their studies.

In this study we not only focus on the relations between cognition and affect, but also on the mode of interaction between them over time in online mathematics on a basic mathematics course which forms part of the engineering studies course at the Open University of Catalonia (UOC).

This paper is organized in several sections as follows. Firstly, we provide a basic review of the literature related to affect and cognition in learning mathematics and present an appropriate model in which we work. Secondly, we give the purpose of the study and the research methodology. Thirdly, we present the results, which are then discussed. Finally, we suggest future lines of research.

THEORETICAL FRAMEWORK

McLeod (1992) identified three constructs in his research on affect in mathematics education: beliefs, attitudes and emotions. Since then, research into affect has included other constructs from which different theoretical frameworks have been developed. Hannula et al. (2004) presented four of the different approaches used in the research on affect in mathematics education, which have been treated by several authors and have been a useful guide in our work: affect as a representational system (Goldin, 2000), affect as one regulator of the dynamic self (Malmivuori, 2001), affect in a socio-constructivist framework (Op 't Eynde, 2004), and affect as embodied (Brown & Reid, 2006). Each approach presents a theoretical framework to explain the relationship between affect and cognition in learning mathematics, and each one is better suited to study specific research questions.

Of the four models, the last one cannot be applied to an online environment, since it is based on the observation of somatic markers. Goldin's and Malmivuori's theoretical frameworks are highly compatible, and both are compatible with the socio-constructive approach of Op 't Eynde (Hannula et al., 2004). In addition, Malmivuori's model is consistent with the basis defined by Bandura in his social cognitive theory (Malmivuori, 2001). By the same token, Gómez-Chacón (2010), who is currently working in this field, cites Bandura and Malmivouri: "In particular, a point of interest has been the measurements of some dimensions of self-concept". Malmivuori's model also emphasizes the dynamic aspect of the interaction between cognition and affect which fits with the approach of our study, where we focus not only on the relationship between affective and cognitive factors, but also on their mode of interaction between them over time. For this reason we choose this model to focus and design our research.

Malmivuori (2001) presents a theoretical framework which incorporates the dynamic nature of this kind of element in a learning process (affect as one regulator of the dynamic self). She summarizes her ideas about this as follows:

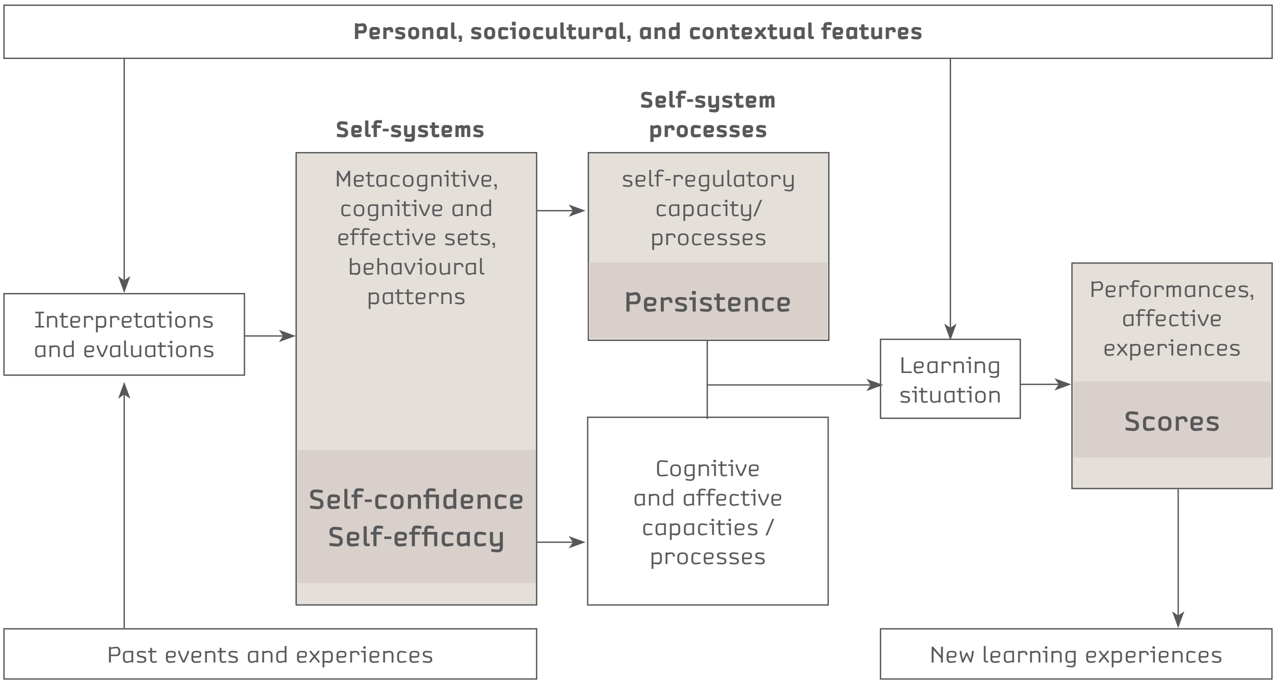
> *"In this perspective we stress the co-constructive and dynamic nature of affect and cognition, in which the functioning of self-appraisals and self-regulation ultimately determine the role of affect in students' mathematical learning or performance processes. This viewpoint also helps to deal with the complexity of affect-cognition interplay in learning situations." Malmivuori (2006)*

The main constructs in this model are self-system and self-system processes. Self-systems are stable internal structures involving mathematical knowledge, beliefs about mathematics, beliefs about the self in mathematics, affective schemata and habitual behavioural patterns in mathematical situations. Self-system processes include all self-regulatory processes: self-reflection, self-evaluations, self-directive constructions and personal agency, as well as self-regulation, self-direction and self-control activities.

Of the several constructs involved in this model, our goal was to measure three of them, two corresponding to the category of self-systems and the other one corresponding to the category of self-system processes. We measured self-confidence, self-efficacy, persistence and performance, and we placed them in Malmivuori's model (Fig 1).

Self-confidence and self-efficacy refer to self-perception (self-beliefs) about the ability of self. To define these constructs, we have referred to Cretchley's (2008) definitions in her study. Self-confidence refers to self-beliefs about abilities in doing and learning mathematics in a particular context, not necessarily in general. Self-efficacy refers

Figure 1. Learning model (Malmivuori, 2001).



to self-beliefs about the capabilities to perform specific tasks. This definition is in line with Bandura's conception: "the efficacy belief system is not a global trait but a differentiated set of self-beliefs linked to distinct realms of functioning" (Bandura, 2006). In addition, according to Bandura's "Social Cognitive Theory", self-efficacy heavily influences the choices a person makes, the effort and perseverance they make in the face of adversity and the degree of anxiety experienced. Self-efficacy can be conceptualized as students' task-related self-confidence with mathematics (Malmivuori, 2001).

Other studies also mention the key role of these constructs:

> *"Both datasets indicated the significant role of self-confidence and affective responses in self-regulation of mathematics learning or problem solving." (Malmivuori, 2006).*

> *"Among the mechanisms of agency, none is more central or pervasive than people's beliefs about their capabilities to exercise control over their own level of functioning and over events that affect their lives. Efficacy beliefs influence how people feel, think, motivate themselves, and behave". (Bandura, 1993)*

> *"Also research indicates that self-efficacy has significant influences on self-management behaviours and self-regulated learning process, such as self-observation, self-judgement and self-reaction." (Wang & Wu, 2008)*

Persistence in learning and doing mathematics was chosen as one of the variables in self-regulatory behavioural patterns under the self-system processes category in Malmivuori's (2001) framework.

Persistence is closely linked to the self-perception of self-efficacy and academic performance:

"Efficacy expectations are presumed to influence level of performance, by enhancing intensity and persistence of effort."
(Bandura, 1977)

PURPOSE OF THE STUDY

The aim of this study is to explore the relationship between cognitive and affective factors in online mathematics in a basic mathematics course which forms part of the engineering studies course at the Open University of Catalonia (UOC). It specifically aims to explore the relationship between students' self-beliefs in mathematics, students' behavioural engagement in learning mathematics and students' performance, as well as to analyze the mode of interaction between these factors over time. In fact, we will provide answers to the following questions:

1. To what extent are self-confidence and performance related?
2. To what extent are self-confidence and persistence in learning and doing mathematics related?
3. What is the mode of interaction between these factors over time?

METHODOLOGY

PARTICIPANTS AND TEACHING PLAN

Participants in this study are students on a basic course in mathematics for engineering students at the Open University of Catalonia. The teaching methodology on this course is based on formative assessment through practice exercises with immediate and automatic feedback, thereby promoting students self-regulation in their learning process. The course is structured into two blocs: algebra with five teaching units (numbers, equations, systems of equations, polynomials, matrices), and analysis, with six units (polynomial functions, trigonometric functions, exponential and logarithmic functions, continuity, derivation and integration). Each unit takes a week, except for derivation and integration, which are studied in two weeks. Each week the students have to complete an assessment exercise and a synthesis assessment exercise for each bloc. The assessment exercise must be completed at the end of the study period. In addition, the students have the opportunity to complete as many practice exercises as they need before the assessment exercise, with immediate and automatic feedback explaining the reasons why their answer was right or wrong.

SAMPLE AND DATA COLLECTION

Data collection was carried out during the first term of the 2011-12 academic year. A total of 19 students enrolled on the course, 18 males and 1 female. After the third unit of analysis (week nine of the course), at the beginning of each week, all the students were asked to complete the self-appraisal questionnaire in order to measure self-efficacy, self-confidence and persistence in relation to the content to be studied. Obviously, the confidentiality of students' responses was ensured.

Table 1 summarizes the number of students completing the assessment exercise and the self-appraisals questionnaire.

The students selected for the case study were those who had completed at least one of the self-appraisal questionnaires sent each week. A total of seven male students completed all self-appraisal questionnaires (or all except one), correctly and on time. It should be noted that this sample is not representative of the group. In fact all selected students passed the course, and in the sample there are no students who failed or dropped out.

Table 1. Number of students completing assessment exercise and self-appraisal questionnaire.

Bloc	Unit	week	Assessment exercises	Self-appraisal questionnaire
Algebra	Numbers	1	16	-
	Equations	2	16	-
	Systems of equations	3	16	-
	Polynomials	4	16	-
	Matrices	5	14	-
	Synthesis Algebra	6	15	-
Analysis	Polynomial functions	7	15	-
	Trigonometric functions	8	14	-
	Exponential and logarithmic functions (ExpLog)	9	15	12
	Continuity (Cont)	10	14	9
	Derivation 1 (Der1)	11	14	11
	Derivation 2 (Der2)	12	14	11
	Integration 1 (Int1)	13	13	10
	Integration 2 (Int2)	14	13	9
	Synthesis Analysis (SA)	15	13	11

INSTRUMENTS

A questionnaire developed from the ideas expressed by Cretchley (2008), and Pierce, Stacey and Barkatsas (2007) was used to measure self-confidence, self-efficacy and persistence.

The questionnaire includes eight items - three measure self-confidence, three measure persistence, and two measure self-efficacy. The students answered one questionnaire every week for seven weeks. Table 2 shows the indicators and values for each variable.

Performance is measured by the score achieved in the assessment exercise. Regarding the capability to use correct notation and to reason and justify, the score and the development of the last question of the assessment exercise has also been taken into account. This is an open question where the student has to reason and justify all the processes used. Table 3 shows the indicators and its values.

In addition to the data obtained from these questionnaires and assessment scores, the following information has been taken into account: the voluntary initial presentation of each student to the forum, all messages with mathematical content sent by the students to the forum, and all practice exercises submitted by the students.

It is important to stress that all variables under study have been measured over time with a set of data regularly collected for seven weeks and complemented with further information such as the voluntary presentation to the forum. This type of almost-continuous data allows us to

Table 2. **Indicators and values for Self-confidence, Self efficacy and Persistence measured using a questionnaire.**

VARIABLE	INDICATOR	VALUES
Self-confidence	Outcome expectations	1 (strongly disagree), ..., 5 (strongly agree)
	Overcoming difficulties	1, 2, 3, 4, 5
	Ease of learning maths	1, 2, 3, 4, 5
Self-efficacy	Capability to use the correct mathematical notation	1 (never), 2, 3, 4, 5 (always)
	Capability to reason and justify all mathematical procedures used	1, 2, 3, 4, 5
Persistence	Correction of error	1 (never), 2, 3, 4, 5 (always)
	Self assessment	1, 2, 3, 4, 5
	Continued search for solutions	1, 2, 3, 4, 5

Table 3. **Indicators and values for Perseverance measured using assessment exercise score.**

VARIABLE	INDICATOR	VALUES
Performance	Assessment exercise score	0,....,10
	Reasoning question score (scaled to 10)	0,....,10

analyze the progress of these variables and the relationship between them.

ANALYSIS AND RESULTS

All available data for each student was represented in a graph so as to have an overall view of each student and to look for any relationships between the variables measured. The average week-to-week progress for self-confidence, self-efficacy, persistence and performance, are shown in figures 1, 2, 3 and 4 respectively.

The distribution of both variables (Self-confidence and Self-efficacy) is similar, although the latter is smoother. In both cases the curves are almost steady with two slight but significant declines (Continuity and Integration). Thus, self-confidence and self-efficacy depend on the content studied by the students at the moment of answering the self-appraisal questionnaire (see Figures 1 and 2).

We can also see a slight increase in relation to the averages for self-confidence and self-efficacy in the last week of the course.

The curve in Fig 3 is steady at around 4.5.

The progress of scores in reasoning question is more irregular due to two factors. Firstly, the small sample size which makes it more sensitive to extreme variations; secondly, the curve shows the scores (scaled to 10) for only one question, whereas the other curve (assessment exercise scores) shows the sum of six.

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Figure 1. Perceived Self-Confidence.

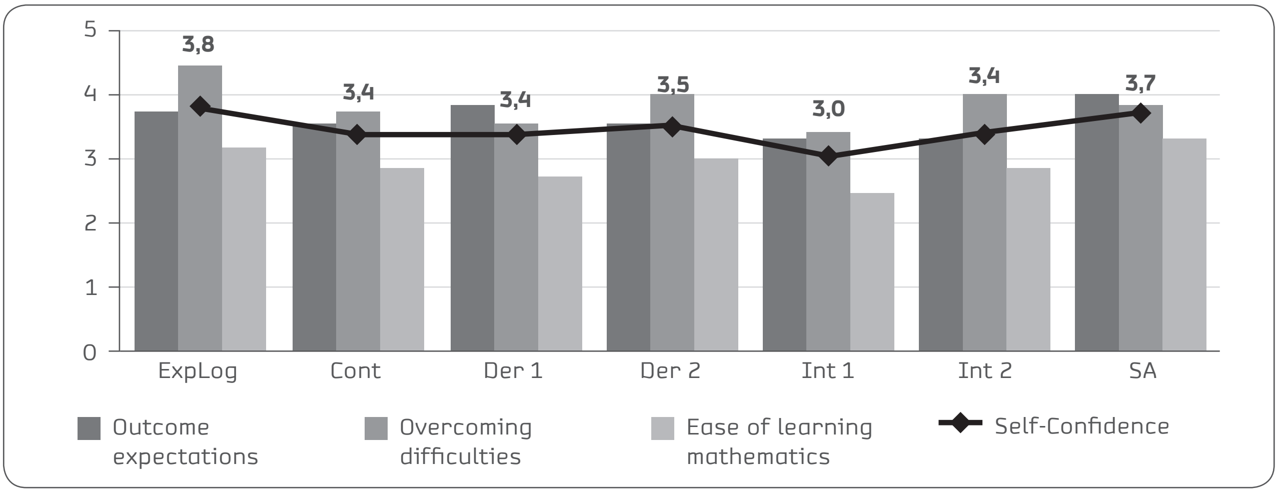


Figure 2. Perceived Self-efficacy.

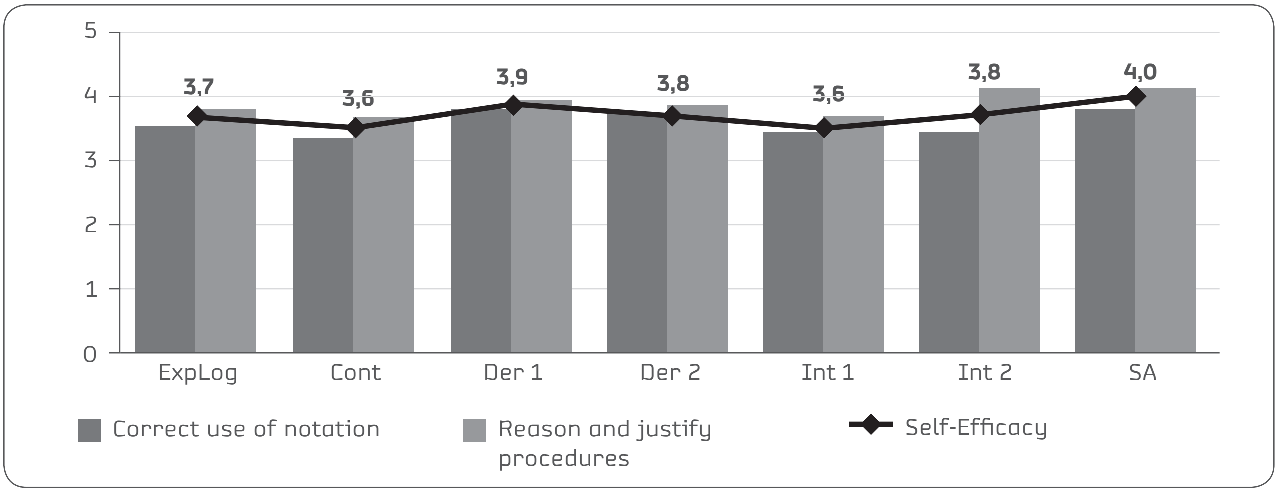


Figure 3. Declared Persistence.

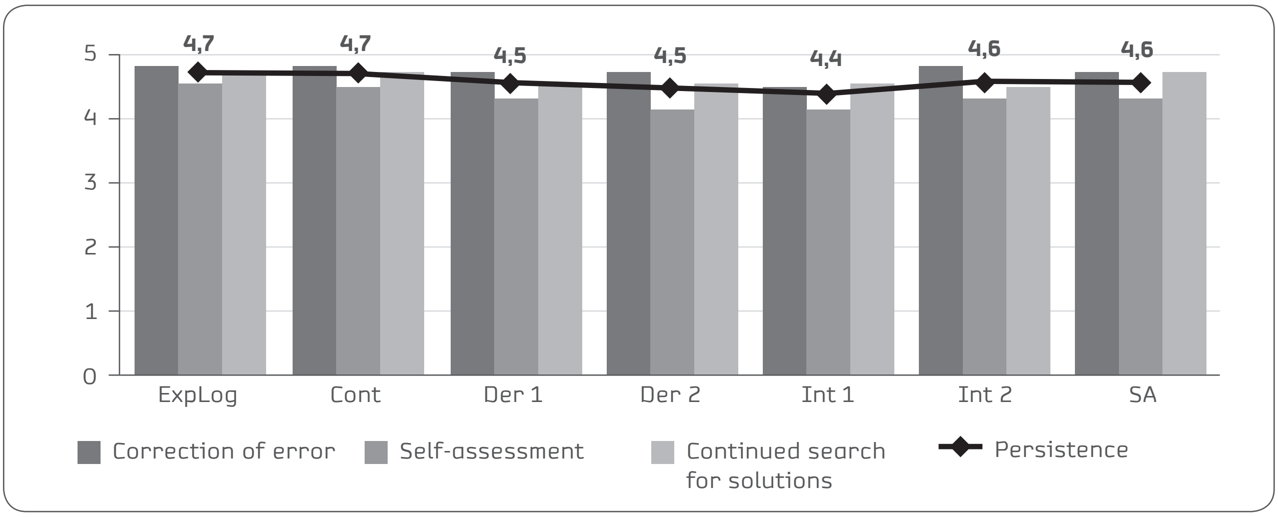
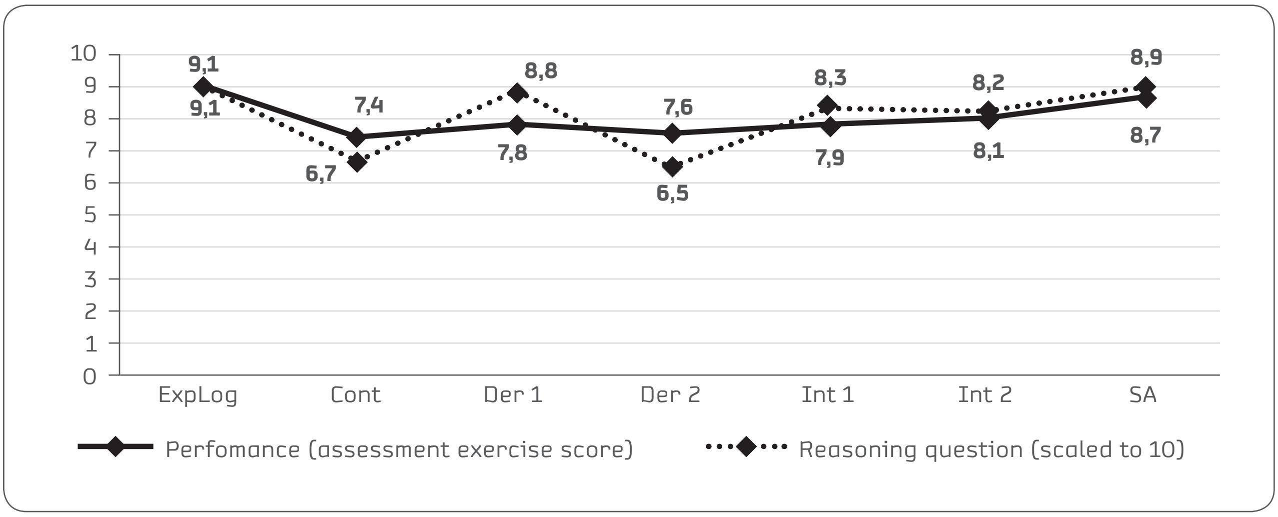


Figure 4. Evolution of practice and assessment exercises.



Previous figures show the average student profile. After a case-by-case analysis, the students were grouped into different profiles taking into account several things: a) all quantitative data mentioned before and b) some qualitative data as the initial presentation of each student to the forum and all messages with mathematical content sent by each student to the forum. In respect with self-confidence and self-efficacy, the answers of students to the questionnaire are validated by monitoring their actions throughout the semester. After this analysis, we could define three profiles of students that help us understand several behaviours into the group.

PROFILE 1

The students within this profile stated the highest level of persistence in the study over the course regardless of whether self-confidence was higher or lower. They have a high level of capability to use correct mathematical notation, and capability to reason and justify all mathematical procedures used.

PROFILE 2

The students within this profile stated a high level of persistence in the study over the course regardless of whether self-confidence is higher or lower. They have a medium-low level of capability to use correct mathematical notation, and capability to reason and justify all mathematical procedures used. They do not always achieve good results, but never less than 4 (out of 10), except for one unit.

PROFILE 3

The students with this profile stated a level of persistence which is not constant over time; it is high in the first units, but decreases significantly from the second unit of derivation on. The level of perceived self-efficacy and self-confidence is not constant over time and their evolution is similar. Their level of capability is higher than that stated. They achieved good results although their self-appraisals are below real results.

In order to tackle the research questions posed, we analyse all previous average variables and their evolution over time. All

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variables were rated over five to be comparable. The next figure (Figure 5) uses average figures to show this evolution.

In order to visualize the extent to which these variables are related, individual data points are represented by point markers in two-dimensional space, combining three variables in pairs.

After analysing all students, and variables on average, one of the most evident results according to the data we have, is that persistence remains constant over time irrespective of self-confidence (see Figure 6), and student's scores (see Figure 8). We can say that persistence is a strong variable constant over time as we see also in Figure 3. However, after a first overview, we did not observe strong evidence of a relationship between scores and persistence (see Figure 7).

Figure 5. Evolution of all variables on average.

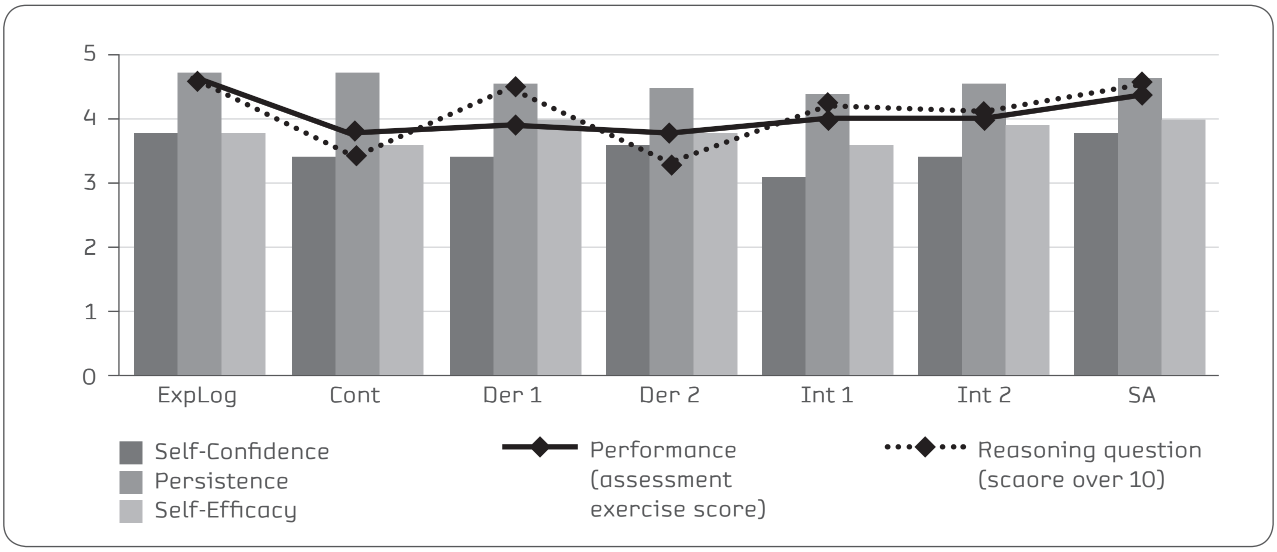


Figure 6.

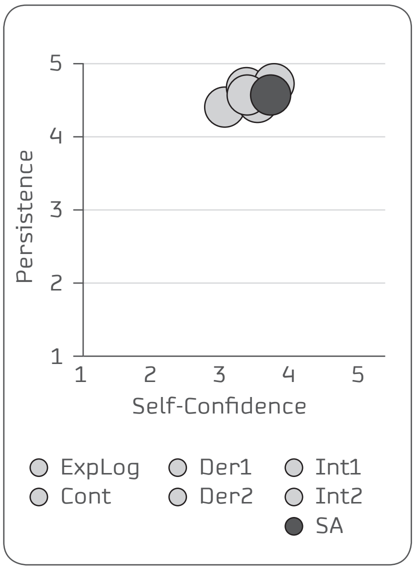


Figure 7.

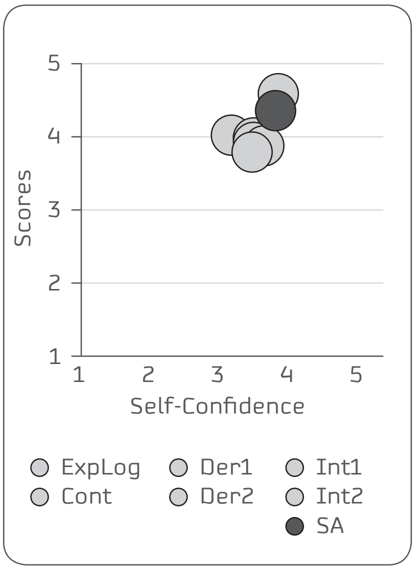
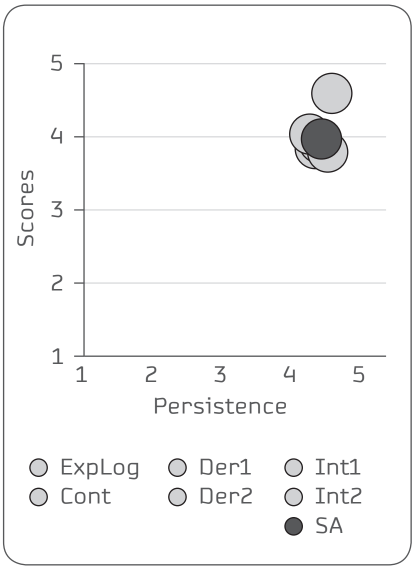


Figure 8.



DISCUSSION AND CONCLUSIONS

The aim of this study was to explore and analyse the evolution of self-confidence, self-efficacy, declared persistence and performance over time in the different student profiles.

The methodological approach to observe the mode of interaction over time was to measure the students' self-appraisals through weekly self-appraisal questionnaires and to collect data on their progress in learning over the same period. In this way we have fairly continuous data, which allow us to evaluate their evolution over time. Although the methodology should be refined, the conclusions we reach are discussed below.

For the relationship between self-confidence, self-efficacy and persistence:

Self-confidence and Self-efficacy are content-related, that is, the evolution over time of both variables depends on the content the students are facing at the moment of answering the questionnaire. This is aligned with the definition of the variables we used for this study. "… a learner may be confident within one area of mathematics, but perhaps not in another […] a student may have high level of self-efficacy for factorizing a quadratic polynomial, but a low level for a cubic" (Cretchley, 2008).

Although other studies show evidence of a clear correlation between self-confidence and student persistence and preference for challenge (Malmivuori, 2006), the results in this study show that persistence becomes constant over time, irrespective of self-confidence and self-efficacy. We could justify this result taking into account two key factors: the teaching plan, where the students must complete a weekly assessment, and the particular profile of online students (adults with little availability and high motivation).

For the relationship between self-confidence, self-efficacy and performance:

Although other studies conclude that self-confidence play a significant role in maths performance (Malmivuori, 2006), we did not find strong evidence of a relationship between them. When analysing this result it has to be taken into account that most of the students who answered the self-appraisal questionnaires achieved from medium to good results, and we did not have information about self-confidence, self-efficacy and persistence of the students who dropped-out or did not pass.

There was clear evidence of this relationship in one unit in one student of profile 1. The student's personal circumstances did not allow him study during the week. Before the assessment exercise he showed his fear of failing the exam in a message sent to the forum. His low self-confidence, together with the fact that he could fail one assessment with no consequence (as he passed all the previous ones and the lowest mark does not compute in the final mark), led him to lower persistence (he did not complete any practice exercise), and to lower performance. This is evidence that external factors are also a key factor to take into account.

For the dynamic of interaction between self-confidence and performance over time:

In derivation and integration units, it is observed that in some students, the good results obtained in the first week may have influenced in higher perceived self-confidence at the beginning of the second week. The opposite was also observed: bad results lead to lower perceived self-confidence.

Finally, we observe a slight increase in relation to the average of self-confidence and self-efficacy in the last week, at the end of the course, indicating that through work and continuous practice, students develop higher perceived self-confidence and self-efficacy in related fields of work over time. As Bandura (1994) states, “the most effective way of developing a strong sense of efficacy is through mastery experiences [...] A resilient sense of efficacy requires experience in overcoming obstacles through perseverant effort”.

FUTURE LINES

With a view to future research, it would be interesting, firstly, to continue refining the variables used in this study and to see how to measure them using more appropriate instruments. Secondly, it is necessary to carry out further research into the relationship between affect and cognition on online mathematics including other variables defined by other authors in this research line.

It is also important to keep working on the understanding of the dynamic of interaction between affect and cognition. In this study the evolution over time was analyzed throughout the development of a subject in a semester, but it would also be interesting to explore this evolution, for example, throughout all mathematical subjects of a degree.

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