THREE CASTILIAN MANUSCRIPTS ON MERCANTILE ARITHMETIC AND THEIR PROBLEMS OF ALLOYS

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Abstract

The aim of this work is to throw light on the existence of three Castilian manuscripts on mercantile arithmetic, all from the fourteenth century. The first, which is moreover the most general and most complete of the three, is a manuscript entitled *Libro de Arismética*, which is kept in the Royal Collegiate Church of San Isidoro in León (MS.46). It can be considered as the first book on mercantile arithmetic written in Castilian and it takes back, by some 100 years, the date which had been used up until then when speaking of treatises on mercantile arithmetic in the Peninsula. It contains an ample collection of practical examples, 192 in all. Of these, 23 (12%) are on alloys.

A second manuscript is kept in the National Library in Madrid, in the Rare Manuscripts section. It is incomplete and does not have a title and it offers us a total of 76 problems. 13 (16%) of these are about alloys.

The third example analysed, *De Arismetica*, is preserved in The Spanish Academy, Ss.155, bound in a miscellaneous work entitled *Escritos Diversos*. We believe that it is incomplete, although it forms a coherent and perfectly intelligible whole. It omits all general aspects and, after advising us that its aim is to illustrate how to work with fractions, it begins directly with a collection of problems —48— of which only two (4%) are on alloys.

Key words

Arithmetic for merchants, Development of trade, Learning mercantile techniques, Manual from the lower Middle Ages and Renaissance.

Capitalia Verba

Mercatorum arithmetica, Rerum incrementum, De re mercatoria, Enchiridion ad Medium Aeuum inferius ac Humanitatum studium tractati.

For the purpose of this study, we have concentrated on three Castilian manuscripts concerning commercial arithmetic[1]. All three of them were written in the XIV century. This early date and the language in which they are written —Castilian — mean that they are valuable and also reliable. Thus we can discard the idea that in the late Middle Ages there was no technical literature in the Spanish kingdoms about applying the new arithmetic (algorithm) to the field of commerce; it had spread through Europe from the beginnings of the thirteenth century as a consequence of the contact with the Moslem world and Latin Europe; it spread across the extensive southern European frontier, via the Mediterranean[2], that impressive pathway of communication. These manuscripts also demonstrate how technical activity was put to the service of commercial activity under the Castilian crown.

The first manuscript, considered to be the most important, is the *Libro de Arismética*, which is kept amongst the manuscripts in the Royal Collegiate Church of San Isidoro in León (MS 46). A study of this manuscript, published in the year 2000[3], allowed us to move back, by over a hundred years, the date which, up until then, had been used in the context of treatises on mercantile arithmetic in the Peninsula[4]; this study meant that we could begin to cover a considerable historiographic void. The sample conserved and analysed is a manuscript copy from the sixteenth century of a written work dated 1393; its content includes explanations from previous manuscripts, now lost, which date from the beginnings of the fourteenth century.

Knowledge of the existence of this manuscript, and its study has also obliged us to seriously wonder whether there might not in fact be more of this type of study buried in oblivion in the different Castilian archives, to which the historian has not yet paid sufficient attention, because he is resigned to the idea that the first commercial arithmetic found in the Spanish kingdoms was that of Francesc Santcliment, a publication from the first Catalan printing house (1482)[5]. After studying MS 46, we can more clearly reiterate the classical question of how it had been possible that the intense commercial activity detected in the Hispanic kingdoms did not have its own technical literature, and/or translated literature, when contact with the Italian world and the existence of a rich Andalusian tradition lead to the

1. This work has been elaborated within the framework of the investigation project HUM 2007- 63856, *La transmisión del saber técnico y profesional: literatura técnica en la España Medieval*, subsidized by the Ministry of Science and Technology.

2. None of the monographs in existence at the time of the publication of the work, *El Arte del Alguarismo: un libro de aritmética commercial y de ensayo de moneda del siglo XIV*, eds. Betsabé Caunedo del Potro, Ricardo Córdoba de la Llave. Salamanca: Consejería de Educación y Cultura, Junta de Castilla y León-Caja Duero, 2000, included the existence of this technical literature in the Hispanic kingdoms in the late Middle Ages —or under the crowns of Castile or Aragon.

3. Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El Arte del Alguarismo. Un libro castellano de aritmética comercial y ensayo de moneda del siglo XIV. Estudio, edición, glosario e índices*. Salamanca: Junta de Castilla y León, 2000.

4. Supremacy, up until then, belonged to the *Summa de l'árt d'Aritmètica* by Francesc Santcliment, written in Catalan in 1482, and this is still the first printed text on this discipline. See a critical edition of this in Santcliment, Francesc. *Summa de l'art d'aritmètica*, ed. Antoni Malet. Vic: Eumo, 1998.

5. See previous note.

belief in the existence and circulation of this type of literature. This same question encouraged us to search for a document of this nature and our work has not been fruitless. In scarcely three years we have located another two manuscripts on this discipline, which we believe to have originated in the fourteenth century. This means that the *Libro de Arismética* is not only a magnificent copy, but is also rare, curious and therefore exceptional[6]. These three essays are proof of an authentic technical literature and of the existence of didactic activity in Castile at the service of the active, flourishing commerce typical of late medieval Castile.

The two other manuscripts which we intend to present are located in Madrid. The first of these, entitled *De Arismetica*, is preserved in very good condition in the *Real Academia Española*, MS.155. It is bound in a miscellaneous work entitled *Escritos Diversos: Dichos de sabios y filósofos; Libro del regimiento de la salud; Regimiento para conservar la salud de los omes; Coplas de Mingo Revulgo...* amongst these are some brief insertions: *notas sobre las estaciones, recetas médicas, Sentencias de Salomón, notas sobre el componente de oro y plata en diferentes monedas y sobre algunos signos del zodíaco y sus características*[7]. The third manuscript, which is untitled, is housed in the Biblioteca Nacional de Madrid, in the section dedicated to Rare Manuscripts, MS.10,106[8]. The work of José María Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca catedral de Toledo*[9]), refers to it, explaining that it is bound together with a *Libro de Agricultura*, which title figures on its binding.

These three manuscripts fulfilled the obvious purpose of providing Castilian merchants with suitable arithmetical training, urgently demanded by the commercial techniques which were becoming richer, more sophisticated and also more complicated. Moreover, as participants in the so-called "commercial revolution" promoted by the Italians, they stimulated and urged for a specific technical development,

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6. We made reference to this in our work: Caunedo del Potro, Betsabé. "Usos y prácticas mercantiles a fines de la Edad Media", *La Península Ibérica entre el Mediterráneo y el Atlántico, siglos XIII-XV. Jornadas celebradas en Cádiz, 1-4 de abril de 2003*, Manuel González Jiménez, Isabel Montes Romero Camacho, eds. Sevilla – Cádiz: Sociedad Española de Estudios Medievales – Diputación de Cádiz, 2006: 35-55.

7. Real Academia Española, Ms. 155. The treatise *De Arismetica*, like all other writings, is numbered with modern Arabic numerals in the top right-hand margin. It occupies folios 144r – 164r We also offer the pagination of the other writings: pages 1 – 86v, *Dichos de sabios y filósofos*; pages 92r – 119r, *Regimiento para conservar la salud de los omes*, written, as mentioned in the work itself on page 109, by the Sevillian doctor Estéfano de Sevilla; pages. 121r- 143r *Glosas sobre el tratado de Domingo con las respuestas dirigidas al muy magnifico señor D. Diego Furtado de Mendoça, marqués de Santillana, conde del Real, acabado por metro y prosa*. This is followed by the treatise *De Arismética*. The middle, page 86v, is occupied by *Apuntes sobre los nacimientos de Pedro (1947) y Diego de Molina (1451) hechos por su padre*; pages 87r – 88r *Notas sobre estaciones*; pages. 88r – 89v *Regimiento de salud*; pages. 89v – 90r *Varias recetas médicas*; pages. 90v – 91r *Sentencias de Salomón*; page 91v *notas sobre el componente de oro y plata de diferentes monedas*; and pages. 120r-v *notas sobre los signos del zodiaco*. It was Bartolomé José Gallardo in his *Ensayos de una biblioteca de libros raros y curiosos* who first informed of the existence of this interesting document. Gallardo, Bartolomé José. *Ensayos de una biblioteca de libros raros y curiosos*. Madrid: Imprenta y Estereotipia de M. Rivadeneyra, 1863: I, doc. nº 758. I have done a short study on the same subject, Caunedo del Potro, Betsabé. "'De Arismética'. Un manual de aritmética para mercaderes". *Cuadernos de Historia de España*, 78 (2003-2004): 35-46.

8. Biblioteca Nacional (BN), Raros, Ms. 10,106.

9. Millás Vallicrosa, José María. *Las traducciones orientales en los manuscritos de la Biblioteca Nacional de Toledo*. Madrid: Consejo Superior de Investigaciones Científicas, 1942: 91.

because the smooth running of business evidently came to require improved preparation and training. The new businessmen needed an efficient, rapid apprenticeship and, for this purpose, they began to elaborate an authentic "mercantile and financial culture"; this surpassed the elementary level of oral tradition or empirical practices —which they were never to do away with— and this financial culture was set out in writing. The *Manuales de Mercaduria* and the *Aritmética Mercantil* are the best exponent of these new requirements for education, and they were put to the service of a marketing technique which grew in refinement. With them, future merchants learned the theoretical rudiments of their profession, which they were later to perfect in practice in the shop, the workshop or the factory. A good combination of both ingredients, i.e. theory and practice in their training, could provide them with the professional success they desired. Solid teaching was a guarantee for this, because in accordance with the beliefs of the era, there were three essential conditions for anyone wishing to become a merchant: money or inheritance, aptitude in mercantile calculation and knowledge of accounting[10]. The two latter clearly allude to training. The Arithmetic Manuals offered this training or, at least, helped to achieve it. And so the merchants were the principal users, yet they were not the only ones; there were also many others in the city with an ever-pressing need for accurate reckoning in order to be able to carry out their activities[11]. For example, tax collectors, bankers, craftsmen... and particularly those who worked in jobs related to the minting of coinage, as they were obliged to be familiar with metals, with methods of alloying them and also with precise calculations for obtaining the suitable proportions established in the prevailing rules. These people also made a specific demand on the mathematics of the era and they developed certain themes of a practical nature. In almost all of what were known as manuals of mercantile arithmetic, and even in some of the so-called "marketing manuals"[12], amongst their exercises we find what are referred to as "problems of alloys"; the objective of which was to teach such concepts. This occurs in the three Castilian manuscripts on which we are going to concentrate. One of them, *El arte del alguarismo*, even goes a step further, because, by including most of the practical exercises on alloys, it offers a technical manual, the *Libro que enseña ensayar qualquier moneda*, concerning the refining of silver and the manufacture of coins. In the opinion of Ricardo Córdoba, this treatise constitutes an authentic *Manual de ensayadores y maestros de moneda*[13].

10. These are presented by Luca Paccioli in his *Summa Aritmética geometria proportioni et proportionalità*, published in Venice in 1494, which caused a sensation in Italian intellectual circles. See Hernández Esteve, Esteban. *De las Cuentas y las escrituras*. Madrid: AECA e Ilustre Colegio Oficial de Titulados Mercantiles y Empresariales de Madrid, 1994: 172.

11. Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. "Oficios urbanos y desarrollo de la ciencia y de la técnica en la Baja Edad Media: la Corona de Castilla". *Norba, Revista de Historia*, 17 (2004): 41-68. In this study we deal with how late medieval urban society, with its particular economic requirements, demanded and propitiated new technical challenges.

12. Ricardo Córdoba takes a look at some of the best known in: Córdoba de la Llave, Ricardo. "Cálculo, Técnica y Moneda", *El arte del Alguarismo...*: 86-88.

13. This manual is extremely interesting because it is rare that we find manuals from medieval times which "teach" how to do a job; it was more normal to transmit knowledge by word of mouth. Ricardo

1. First steps

Where should we look for the origins of these good examples of technical literature put to the service of commercial activity and, in general, of urban expansion? A first answer, which is straightforward and easy, and not too open to error, points to the Moslem domination in the Peninsula and to the cultural prosperity of *Andalusí*. Nobody doubts the immense contribution of Arab erudition and its significant influence in the West, due to its vast flow of knowledge, some of which was classical, and which was certainly far richer than what had been preserved here, and consequently completely overshadowed the foundations of Western knowledge. The term *"alguarismo"*, the numbering system, and the style of the essays —the treatise of mercantile arithmetic, *Al-Muawalat*, all demonstrate this. Neither is there any doubt about the Jewish contribution, at least about the outstanding role of the Jews as good disseminators of culture and of the discipline which concerns us here. However, we present these manuscripts as the product of a prolific, double-sided tradition: the Latin tradition and the Arabic tradition, both elaborated within an insuperable framework for that purpose —the Iberian Peninsula. There, two cultures —Latin and Arab— as well as the participation of Christians (Mozarabs) and Jews —intermingled, developed and were of mutual enrichment. We would also like to place emphasis on the humblest link in the chain of transmission, which was influenced by priceless Arab science. This link was the Latin-Christian element, visible at least in the first of the texts in question here. Two names are of exceptional significance: Boethius and the Venerable Bede. With Boethius (435-480) a first classical tradition arrived on the scene of mathematics. Using Greek sources, he compiled Latin selections of elemental studies on arithmetic, geometry and astronomy. He wrote the *Institutio Aritmética*[14], a translation/summary of the *Introductio Aritmética* by Nicómaco. His low level in mathematics was lowered even further in later compilations by Casiodoro (475-570)[15] and Isidoro of Seville (560-636), whom we have a particular interest in mentioning because his work *Las Etimologías*, dedicates Book III to the study of the four mathematical sciences. After a brief definition of these, he mentions the "investigators" of mathematics and goes on to focus on the definition of the number, on its importance and its types. He concludes with different geometrical annotations. He adopts the position of the

Córdoba has studied this in: Córdoba de la Llave, Ricardo. "Cálculo, Técnica y Moneda", *El arte del Alguarismo* ...: 86-88.

14. Boethius, Anicio Maulio Torcuato Severino. *Institutio Arithmetica*, ed. Jean Yves Guillaumin. Paris: Les Belles lettres, 1995, in the introduction of which the value of the number for Boethius and Nicómaco is analysed, and the consideration of the study of arithmetic as a phase which should precede other higher studies.

15. Boethius, Anicio Maulio Torcuato Severino. *Institutio Arithmetica*... Jean Yves Guillaumin also offers us a first edition of the text, converted into a manual for the monks of Vivarium and later for the Bobbio monks. Its presence in these libraries and in the Lateran Library would have been of significance for its circulation throughout Europe.

ancient philosophers in his explanation of the doctrine of numbers studied as a manifestation of the Creator's plan[16].

This work spread quickly and widely prior to the ninth century and it was to be found in almost all medieval monasteries[17]. This significant distribution ensured that the inheritance which was included, conserved and explained in the work was widely transmitted. Moreover, in many of these monasteries, they did not only keep such works, but they also studied them conscientiously, although for different purposes to those of the thirteenth and fourteenth centuries. One of these, which encouraged the culture of arithmetic, was the determination of the liturgical calendar, as most of these religious feasts which were moveable, were to be established according to the celebration of Easter, as had been decided by the Council of Nicea[18]. Once the religious-dogmatic controversy had been overcome, it only required a certain arithmetical knowledge to bring it into being. Some monks concentrated on acquiring and transmitting this knowledge. This was the case of the Venerable Bede (673-735), known as the historian monk of Jarrow[19], who wrote six studies on chronology. In one of them, *De Temporum ratione*, written in the year 725, he calculated the Easter calendar for the period between the years 532 and 1063 and also tried, for the first time, to prepare a world chronology up to the reign of the Byzantine Emperor of his era, Leo the Isaurian[20]. Yet we are not going to concentrate on these aspects, which are already familiar, nor on how the calculations, the *Computus*, which meant that the liturgical year could be firmly established, continued to be cultivated in the West. But we wish to highlight a small work of arithmetical from Bede, *De Arithmeticis Propositionibus*[21], in which he attempts to solve real issues which could be found in daily life or in the work of

16. Sevilla, Isidoro de. *Las Etimologias*, eds. José Oroz Reta, Manuel Marcos Casquero. Madrid: Biblioteca de Autores Cristianos, 1982, with a general introduction by Manuel C. Díaz y Díaz. Apart from this, Book III of *Las Etimologias, Acerca de las Matemáticas, Libro de los Números* has also been attributed to Saint Isidoro; this is a treatise in which he returns to the study of the Bible to apply mystical interpretations to all the numerical mentions in the sacred books (Sevilla, Isidoro de. *Las Etimologías...*: 129-130.

17. De Sevilla, Isidoro. *Las Etimologias...*: 200-222 offers us a complete schema of the circulation of the work prior to the ninth century.

18. Muñoz Box, Fernando. "El tiempo y la medida del tiempo", *Historia de la Ciencia y de la Técnica en la Corona de Castilla, II. Edad Media*, Luis García Ballester. dir. Salamanca: Junta de Castilla y León, 2002: 539-550, once again insists that it was the monks who were responsible for the progress in the sciences of chronology and horology.

19. Bede's seminal work is perhaps his *Historia Eclesiástica gentis anglorum*, the principal source of the history of the church in that country and the one which has given the author the name of the historian Monk, Bede. *The Ecclesiastical History of the English Nation*, eds. John Allen Giles, John Stevens, Vida Dutton Scudder. London: Denton, 1935.

20. Whitrow, Gerald James. *El tiempo en la historia. La evolución de nuestro sentido del tiempo y de la perspectiva temporal*. Barcelona: Crítica, 1990: 102.

21. Menso Folkerts stresses the mathematical value of this volume in his comprehensive work: Folkerts, Menso. " *De Arithmeticis Propositionibus*. A Mathematical Treatise Ascribed to the Venerable Beda", *Essays on Early Medieval Mathematics. The Latin Tradition*. Aldershot: Ashgate Variorum, 2003: 12-30.

Alcuin of York, *Prepositiones ad acuendos juvenes*[22]. This volume includes an exercise which might be the starting point for those considered here. It contains a small collection of arithmetical and geometrical problems preceded by some notes on the numbers, and it intends, according to the author, to "develop the talent of the young"[23]. It should be noted that several of its exercises are repeated with the same text or with minor modifications —adaptations of time and location— in subsequent works of arithmetics, specifically in many of those lumped together under the appellative of "recreational arithmetics" integrated in turn in the medieval and Renaissance commercial arithmetics. Possibly those exercises were already traditional in Alcuin´s time, given the Greek, Indian and Chinese origins of some of them[24], and it is conceivable that Alcuin, his contacts with the Moslem world aside, was directly inspired by Bede, whose work was quite familiar to him. Besides the potential connections and setting, brilliantly examined by Menso Folkers[25], we wish to highlight, as mentioned, one of the exercises of the collection, which we can considerer the model in the West of the "problems of alloys". At least the ties are clear. Alcuin presents it as follows:

> *A metal disc weighs 30 pounds and is worth 600* solidi. *The disc is made of a mixture of gold, silver, copper and tin. For each part of gold there are three of silver. For each part of silver, there are three of copper. For each part of copper there are three of tin. Let he who can, say: How much is there of each metal?*[26].

22. Folkerts, Menso."*The Prepositiones ad acuendos iuvenes* Ascribed to Alcuin", *Essays on Early Medieval Mathematics...*: 31–76.

23. Alcuin stated 55 problems, solved 32 and thus left 23 unsolved. These were those to develop the talent: "the other solutions are desirable, however anyone can solve these propositions by using arithmetics, so that the non-solved are good to put talent into practice".

24. Singmaster, David. "Some early sources in recreational mathematics", *Mathematics from Manuscript to Print, (1300-1600)*, Cynthia Hay, dir. Oxford: Clarendon Press, 1988: 95-208, points to certain Indo and Chinese origins for many of these problems. Some of them appear in collections of Mohavira (850) and Abu Kamil (900) and he insists on the fact that there must have been previous Arab studies which introduced them.

The amusing problem of a hare which fled chased by a dog which appears in Bede's collection, had already been done in what can be considered as one of the most ancient mathematical texts in the world (the early years of our era) and the most influential of all Chinese mathematical texts, the *Chin Chang Suan Shu or los Nueve Capítulos sobre las artes matemáticas*. In this work, composed of 246 problems distributed throughout nine sections or chapters, he incorporates into Chapter 6, *"Justos impuestos"*, where there are exercises dealing with the distribution of taxation between different sectors of the population and others on the time needed for transporting grain (tax) from different villages to the capital. Here he includes those of the "persecution type" which we can prove reached Europe before the reiterated Arabic influence. The problem reads: *"una liebre lleva a un galgo 50 pu (pasos) de ventaja. El perro persigue a la liebre 150 pu, pero la liebre aún está 30 pu por delante. ¿Al cabo de cuántos pu el perro alcanzará a la liebre?"* (Joseph, George. *La cresta del pavo real. Las matemáticas y sus raíces no europeas*. Madrid: Pirámide, 1996: 236).

25. In his works referred in notes 20-21, Menso Folkerts provides a superb account of the various copies still existing, their analogies and their differences.

26. Note our translation of Folkerts, Menso."*The Prepositiones...*": 48-9. (7) PROPOSITIO DE DISCO PENSANTE LIBRAS XXX.

The solution which Alcuin offers us is as follows:

Metal	In weight	In coins
Gold	9 ounces	15 *solidi*
Silver	2 pounds, 3 ounces	45 *solidi*
Copper	6 pounds, 9 ounces	135 *solidi*
Tin	20 pounds, 3 ounces	405 *solidi*
Total	30 pounds	600 *solidi*

2. The characteristics of the Castilian texts

The most complete text of those which we are going to discuss is, as we have already said, the one entitled *El Arte del Alguarismo*, manuscript 46 in the archives of the Royal Collegiate Church of San Isidoro in León. In its general layout it follows a line common to all the arithmetical literature of the era. After an index or summary, setting out the most general, basic aspects of the different operations, there follows a description and brief explanation of each one, and a fairly long collection of practical examples. Its layout corresponds to the practical nature of this type of work, and this is evident from its brevity and apparent simplicity. Quotes and any digressions are eliminated, to concentrate on what is really believed to be of interest to the merchant, to the businessman and to anyone who wishes to learn the noble art of arithmetic.

Our text actually begins with a doctrinal prayer and, after an allusion to the mystery of the Trinity and the greatness of the Creator who gave us the *understanding* to be able to learn the sciences, it comments on the seven liberal arts and praises the value of arithmetic. It then goes on to describe the Indo-Arabic numbering system, and afterwards details, one by one, the seven arithmetical operations which were considered fundamental: addition, subtraction, multiplication, division, pro-

Est discus qui pensat libras XXX, sive solidus DC, habens in se aurum, argentum, auricalcum et stagnum. Quantum habet auri, ter tantum, habet argenti; quantum habet argenti, ter tantum auricalci; quantum auricalci, ter tantum stagni. Dicat, qui potest, quantum unaquaeque species penset.

SOLUTIO DE DISCO

Aurum pensat uncias novem. Argentum pensat ter VIIII uncias, id est libras duas et tres uncias. Auricalcum pensat ter libras duas et ter III uncias, id est libras VI et uncias VIIII. Stagnum pensat ter librasVI et ter VIIII uncias, hoc est libras XX et III uncias. VIIII unciae et II librae cum III unciis, et VI librae cum VIIII unciis, et XX librae cum III unciis adunatae XXX libras efficiunt.

Item aliter ad solidos. Aurum pensat solidos argenteos XV. Argentum ter XV, id est XLV. Auriaclcum ter XLV, id est CXXXV. Stannum ter CXXXV, hoc est CCCCV. Iunge CCCCV et CXXXV et XLV et XV, et iuvenies solidos DC, qui sunt librae XXX.

portional distribution, the rule of three and fractions, which the author denominates *especias*. The numbering system used in the manuscript is, as we have already mentioned, the Indo-Arabic, denominating the symbols used as *letters of the algorism* —today, the figures 1,2,3,4,5,6,7,8,9. This is not the case with the 0, which receives the category of *figure*. The value of each one of the figures themselves, which depends on their position, is referred to in the manuscript as *presçio* of the value of the *letras del alguarismo*.

The text also includes an ample collection of problems, 192 in total, which make up the lion's share of the manuscript[27]; they are correctly solved, both in their mathematical procedure and in their results[28]. There is a great variety of problems in such a wide collection: calculation, mental arithmetic, solving situations with associated numbers, the prices of products, the time elapsed between the occurrence of events, the distribution of money or products, interest rates, simple geometry... and alloys, which are what particularly interests us here. There are 23 alloys, 12% of the whole collection, and they are all solved using basic operations[29]. These 23, which are categorised from minor to major difficulty, might be classified in the following types. It should be emphasised that they refer exclusively to silver, and not to gold:

- calculations to debase the legal standard, to obtain an inferior quality silver, with a higher copper content.
- calculations to obtain silver of an intermediate standard, from two silvers of different standards.
- calculations to determine the standard which results from the mixture of three or more silvers of different standards.

These 23 problems concerning alloys are all correctly solved[30], using, as we have already pointed out, elemental operations. It seems curious that the author

27. They are set out from folio 22r. where the explanation of the *especias* finishes, to the end of the work, *with no other interruption apart from three short cuts*. The author takes advantage of the first one to show us how fractions are simplified (f. 59v–61r), the second how roots are calculated (f. 106v–107r), and the third, the longest, teaches us how to melt metals (f.118v–137r) Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*

28. However, there are errors, logical in a copy, which can mostly be obviated, because simply by extracting the mathematical operations done by the author, the correct data can be obtained. See Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*: 76.

29. If we summarise the classification of problems according to the mathematical procedure used to solve them, we find: elemental operations: 52 (27%); rule of three: 28 (14%); proportions: 16 (8%); fractions 67 (35%); alloys: 2 (12%); square root: 5 (3%) and mental arithmetic 1 (1%). See Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*: 76.

30. Certain elements which have not been corrected on the copy may appear at first sight as errors, but they are not; they are merely out of order. The problems of alloys are described on folios 112r–118v and 137v–141v, whilst folios 119v, 120r-v, show two problems of alloys which connect with those presented earlier on. Therefore the problem on folio 119v, in which the procedure is explained for debasing the legal standard of an alloy, which appears to have neither beginning nor end, actually joins on to the one on folio 116 which is interrupted, and offers the solution to this one. Something similar occurs with the problem on folios 120r and v, a way to make an alloy taking silver of two different standards, one of more value and the other of less value than that required. It does not seem to have any conclusion, but this is included in the problem set out on folio 118r., Córdoba de la Llave, Ricardo. "Cálculo, Técnica y Moneda"...: 191.

has systematised his solution with specific procedures, subtly explained in two of them[31]; this is completed by a graphic which easily and immediately informs us of how to go about obtaining an alloy of the standard we require. A drawing clarifies the text and offers a rule for its solution.

> *si te dixeren, el rey manda labrar a 7 dineros de ley e tenemos aquí dos platas, que es la una de ley de 11 dineros e la otra de ley de 2 dineros, ¿qué tomaremos de cada una destas platas para que nos venga aleado a ley de 7 dineros?. Primeramente faz tu fegura segund aquí está e di, de 7 sacando 2, quedan estos 5, ponlos de yuso del 11 y estos 5 marcos as de tomar de la plata que es a ley de 11 dineros e di otra vez, de 11 sacar 7 quedan 4, los quales 4 se an de poner de yuso del 2 y estos 4 marcos as de tomar de la plata que es a ley de 2 dineros, asy que de la una tomaríamos 5 marcos de la que es a ley de 11 dineros e de la que es a ley de 2 dineros tomaríamos 4 marcos, asy que 4 y 5 son 9 marcos, estos 9 marcos son aleados a ley de 7 dineros e para fazer la prueva, multiplica el 7, que está ençima del 9 con el mismo e serán 63 e multiplica agora el 2, que está ençima del 4 con el mismo e son 8 e multiplica agora eso mismo, el 5 con el 11 que está ençima e son 55 e añádeles los 8 e serán 63, que es tanto lo uno como lo otro, asý que esta cuenta es bien fecha e provada*[32]

If we subtract 2 from 7 (the order of the alloy), 5 remain, which he places beneath 11 (which is one of the silvers available); on the other hand, if we subtract 7 from 11, we are left with 4 which we place beneath 2. As 5 + 4 = 9 the answer is, that to manufacture 9 marks of standard silver from 7 coins, we have to take 5 standard marks of 11 coins and 4 standard marks of 2 coins.

There is a list on the last pages of the manuscript which contains the different coins: *real, tornés, barcelonés...*; it concludes with some definitions referring to musical notions and staves with notes.[33]

We believe that the treatise entitled *De Arismetica* is incomplete, even though it forms a coherent whole and is perfectly intelligible. It does not begin, as is usual, with a prayer. Neither is there an introduction to the work, nor are its usage, value and utility specified. It should have a brief index or summary and another section, most important in these manuals, namely a general explanation of the Indo-Arabic numbering system (which is, of course, the one which is used in the work), of the value of position, and a brief explanation on each one of the basic operations. It thus omits all these general aspects and starts straightaway with a collection of problems. Although there are not many problems —only 48— we can, however, affirm that the book is extraordinarily valuable in as far as content and solution of the problems presented. At the start of the book, we are told: *Este libro es muy*

31. Problems number 160 and 164, f. 112 r. v. and f. 114-115r.; Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*: 209-10.

32. Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*: 210.

33. The definition *mutança es mandamiento de dos bozes* is included on f. 159v, after an allusion that musical notes are familiar *después que señor supisteis los signos aveys señor de saber e venir a las mutanzas e saber qué cosa es mutança*. There are some staves with notes on f. 160-161v and 173v are missing and there are more staves with notes on f. 174r and 175v Caunedo del Potro, Betsabé; Córdoba de la Llave, Ricardo. *El arte del Alguarismo...*: 78.

bueno y muy provechoso para saber partir e multiplicar enteros e rotos.[34]..., leaving us in no doubt that its aim is to provide the reader with suitable knowledge of problems with fractions, for their immediate use in exchange and marketing operations, insisting on their benefit for merchants.

The aim of the work to concisely transmit knowledge of operations with fractions, is obviously fulfilled. Of the 48 problems in the collection, two (4%), are solved using basic operations (addition, subtraction, multiplication and division), four (8%) by proportions, and 42 (88%) with fractions. The author shows considerable mastery also in these, presenting the problems with short, direct, appropriate procedures, which allow him to arrive rapidly at the solutions. He also assists the reader by adding the "proof", or verification, including this in the more complex problems where there might otherwise be doubt about the accuracy of the result.

Thus, we find two problems of alloys in the collection, which reflect one of the most typical applications of operations with fractions. Although he does not enter in depth into any complex problems of alloys, which require specific knowledge to determine the purity of metals, the author encourages us to investigate the difficulties surrounding them by presenting us with cases of simple alloys which can be easily solved with a knowledge of fractions. A correct approach and results, such as we are already accustomed to with this author.

> *E si te dixeren, yo tengo de tres suertes plata, la una suerte es de 1 marco, 7 onças 1/3 de onça de plata fina e la otra suerte de 6 onças e ¼ de plata fina e la otra de 7 onças ½ de plata fina, e de todas estas 3 suertes tengo 348 marcos tanto de uno como de otro, quiero yo afinar esta plata para saber quanto ha en ella de plata fina, primeramente farás en esta manera, ayuntarás las 3 suertes que dichas son que tienen de plata fina la 1 7 1/3 e la otra 6 1/4, la otra 7 1/2 que fasen por todos ayuntados los 21 /// 1/12, estos 21 1/12 partirás por 3 por quanto son 3 suertes de plata e salen a la parte 7 1/36 de marco por 348, que es todo*[35].

In the second exercise, which is very similar to the one we have described, as it also deals with determining the standard resulting from the mixture of three silvers of different standards, data are missing, precisely the data concerning the types of silver; however, he also solves the problem[36].

The manuscript concludes with some multiplication tables. The purpose of these is to facilitate their learning and memorisation. The author differentiates between

34. Real Academia Española, Ms. 155, f. 145r.

35. Real Academia Española, Ms. 155, f. 151r-v.

36. Real Academia Española, Ms. 155, f. 150v-151r. *Regla para saber allegar la plata. Pongamos que tengas de tres suertes de plata, la primera plata que sea de 15 marcos, 7 onças, de ley /// de 11 dineros, 8 granos e la segunda (blanco en el doc.), e la terçera (blanco en el doc.) e quieres fundir toda esta plata en uno e saber de qué ley será. Primeramente ayuntarás los marcos e las honças de la dicha plata en uno quel ??? 49 marcos e 1 onça, que son 49 1/8 por quanto 8 onças fasen el marco, estos 49 1/8 es el tu partidor, después tomarás la ley de la primera plata que es 11 dineros,8 granos, que son 11 ¼, e este 11 ¼ marcarás por el peso de los marcos que pesava la dicha plata, que son 12 marcos, 4 onças, que es 12 marcos e ½, por quanto 8 onças es un marco como dicho es, agora multiplica 11 ¼ por 12 ½ e salen 140 5/8 la plata e sálente 2448 4/9, éstas son onças, agora pártelos por 8 e lo que saliere serán marcos e fallarás que estos 2448 4/9 partidos por 8, como dicho es, que salen 306 1/8 e tantos marcos de plata fina serán en toda esta plata.*

the *Minor table and major tables*[37]. The *minor table* consists of a simple list of the tables from 1 to 9; whilst the *major tables*, apart from including the previous ones, present us with the higher numbers: 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 29, 31, 33, 37, 41, 43, 47, 51, 53, 57, 59, 61 where there are also results of multiplying numbers over 10, likewise for memorisation.

The third of the manuscripts is preserved in good condition in the National Library of Madrid, in the Rare Manuscripts section[38]. We know about it from the work of José María Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo*; from this, it is evident that it is bound together with a *Libro de Agricultura*, the title of which can be seen on its binding. It is therefore in book format (275 x 207 mm) and consists of 81 unnumbered folios and 65 with Arabic numbering in pencil in the upper right-hand margin. It is written on paper in black ink, except for the initial letters in red and purple, and the initials of the chapters, some of the epigraphs and the paragraph signs in red. Bound in wood and leather-lined, it fastens with two small clasps.

Throughout all 81 folios certain alterations have obviously been made and other folios are missing; the beginning of the text is incomplete and other pages are out of place, in their current order. Thus, for example, folio 13v. is written the wrong way round and, at the end of folio 16v., two lines are also written the wrong way round —the first problem in the collection. From folio 17r. onwards according to Millás, there is what ought to be another treatise, written in different handwriting to the previous one. It is fairly evident that there are two different handwritings —the copy we are working on is written in processed courtesan script— but we cannot affirm with any certainty whether they are two different treatises, or whether the first one simply continues with a new problem after a blank space.

This third text is incomplete —we have already pointed out that the beginning is incomplete— which means that it does not wholly respect the schema typical of this type of manuscript, although, in its original format, it must have done so, judging by what has been preserved. It begins with a very superficial explanation of the Indo-Arabic numbering system and the value of position, explaining how the new characters, *fegueras*, should be read. It makes no comment on the value of 0, but when indicating the value of the position of the *feguras*, it shows the equivalence with Roman numbering, distinguishing the ease and operative advantages of the new system[39]. These clarifications are extremely brief and succinct, as the author refers to having made them before[40].

37. Real Academia Española, Ms. 155, f. 161r–164r.

38. B. N., Raros, Ms. 10.106. We know of it from the work of Millás Vallicrosa, José María. *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo*. Madrid: Consejo Superior de Investigaciones Científicas- Instituto Arias Montano, 1942: 91.

39. The beginning of the arithmetical text is *" segunt que ya avemos dicho una figura sola sinifica unidat, así como 1 senefica uno, e dos feguras en uno así puestas sinifica veynte e uno"*...From one hundred onwards, the quantities are noted in Roman numbering: *"tres figuras asy puestas sinifican 321,III XXI, otrosí quatro feguras asy fechas 4321 sinifican IIII M CCC XXI, otrosí çinco figuras así puestas 54321 sinifican L IIII M CCC XXI..., otrosí nueve feguras e tales 987654321 sinifican IX LXXX VII MM DC IIII M XXI...* (B. N., Raros, Ms. 10106).

40. The beginning of the arithmetical text is: *" segunt que ya avemos dicho...*

Immediately afterwards, he introduces a *fourth rule*, which is operating with fractions[41]; this is a clear example that the first pages of the manuscript have been lost. He then sets out different divisions of fractions: fractions between fractions or with whole numbers, and also mixed. After 27 exercises of this type, he includes two which could be assigned to what we know today as arithmetical progressions. In them, perfectly valid mathematical procedures are indicated, to calculate the sum of the terms of the arithmetical progressions formed by even numbers in one exercise and by odd numbers in the other[42].

At the end, as is usually the case, he incorporates different statements of problems, to make up the total of the 76 in the collection, with exercises where he generally uses the rule of three and proportions. His presentations of the problems reflect the obvious mercantile slant of the document more noticeably than in the case of the two previous manuscripts. It is also meant for use by other sectors of urban society, interested in the practice of arithmetic, which useful manuals, written in clear, colloquial language, could provide them with. If we concentrate on the presentation and content of its 76 problems, we see that none of them can be qualified with the common label of recreational arithmetic. Not one of them enters into the category of the so-called "bird problems", or "reservoir problems" or the distribution of wills. Neither is there any simple geometry or calculation of ages... nor any which can be classified as mental arithmetic because they do not use any type of operation for their solution. However, the manuscript concentrates on problems of a commercial nature and solves them; problems involving the conversion of coins, the purchase and sale of merchandise using units of weight and a variety of measures valid in the different markets frequented, the distribution of profits... The presentation of the problems is typically Mediterranean, and clearer than in the two previous manuscripts. The cities mentioned are Mediterranean, as well as the coins and the merchandise. Venice, Acre, Pisa, Marseille and Valencia are the cities which are most frequently repeated; most of the merchandise mentioned was transported from Acre to Venice and from Sicily to Venice: silks, Pisa cloth, Marseille cloth, oil, pepper and cloves... all paid for with numerous coins, amongst which, together

41. *Capitulo de la quarta de la postrimera regla abba, eso es partir por rotos e por enteros segunt que luego se sigue...* (B. N., Raros, Ms. 10106).

42. We might say that they are arithmetical progressions of ratio 2, which means that each number of the progression is obtained by adding two units to the previous one. *"Otrosi sy quisieres asumar todos los nombres pares de 1 fasta en 20 asy como 2 junto con 4 et 6 et con 8, asi por esta via fasta en 2/ que non metas y ningun dispar, aquesta es la regla, toma la meytad de 20 que es 10 et /// por1 mas de 10 eso es por 11, agora multiplica 10 por 11 fasen 110 tantos son los nonbres pares de 1 fasta en 20 et si el nonbre mayor era de 1 fasta 23 et quisieres asumar todas las partes que son de 1 fasta en 23 toma la mayor meytad entrega que es 11, multiplicala por 1 mas eso es por 12, ende multiplica 11 con 12 fasen 132 tantos son los pares que son de 1 fasta, 23 et por aquesta regla faras de quantas tu quieras".*
Otrosi si quisieres asumar todos los nonbres dispares que son de 1 fasta en 20 asi como 1 junto con 3 e con 7 e con 9, que noy metas ningun nonbre par, aquesta es la regla, toma la meytad de 20 que es 10 e multiplícala en sy mesma, ende dy 10 con 10 fasen 100 e a tantos son los dispares de 1 fasta en 20 et sy el nonbre mayor era dispar, asi como 23, et quisieres asumar todos los dispares, toma la mayor parte eso es la mayor parte entrega ques 12 e multiplícalo ensy mesmo,12 con 12 fasen 144 et a tantos son ayuntados todos los dispares que son de 1 fasta en 23, e por esta regla lo puedes faser de tantos como tu quieres", (B. N., Raros, Ms. 10106, f. 4 r-v).

with the normal ones in the Castilian markets of the era, we should not omit the *besantes, quirates, tarines, torneses* or *barceloneses*.

This third manuscript with its new, rather special collection of problems, also offers us some very interesting information about problems of alloys; it is, in fact, one of the clearest and most concise repertories known, at least as far as gold alloys are concerned. Despite the fact that there are not many of them —10 problems out of a total of 76— we should emphasise that the author has been extremely careful to avoid repeating the statements presented, by offering a high quality, didactic selection. Apart from solving all the problems of alloys perfectly[43], he does the mathematical operations apparently with great ease, describing his calculations in minute detail; this is typical only of authors who dominate the material they deal with. He was probably also aware of the difficulties which the pupil or reader could have in understanding this type of problem —let us not forget that this is arithmetic for merchants— and for this reason he takes great care to facilitate their understanding and learning, avoiding numerous useless, tedious repetitions which, in many cases, would render them less beneficial. His selection truly amazes us, given that it is not easy to summarise problems of alloys so exhaustively and so clearly, and particularly difficult for merchants.

We could classify this interesting collection, bearing in mind the problem posed in each one of them, as follows:

- In three problems the necessary calculations are made to refine gold or to debase its purity. In two of them, with a piece of 17 carat gold, the idea is to refine it to obtain gold of 22 carats in one case and of 24 in another. In the third, the aim is to debase the purity of 24 carat gold to 17 carats, working out the corresponding calculation of ounces to be added [44].
- In two problems, five and three types of gold are mixed and the resultant carats are calculated, whilst in another two, two types of gold or silver are mixed with a third of unknown quantity and it is calculated how to obtain gold of a determined price[45].
- One problem solves how to mix three types of gold to obtain gold of a certain price[46], whilst another one mixes two types of gold, calculating the necessary quantities of each one of them to obtain gold of a specific purity[47]. Lastly, we would point out that, in one case, the problem of finding the proportion of each metal in a three-metal alloy has also been solved.[48]

With regards to the operations carried out, we can conclude that fractions are widely used, as well as the basic operations of addition, subtraction, multiplication and division.

43. The very few errors found in them can be attributed to the copy.

44. We have numbered them with Arabic numbering in brackets. Numbers (1), (2), (3).

45. Problem nums. (4), (5), (6) and (7).

46. Problem num. (8).

47. Problem num.(9).

48. Problem num. (10).

(1) *Un ome a oro de 17 quilates et quierélo meter en çemitre, eso es al fuego, et quiérelo afinar fasta que sea de 24 quilates et es a saber ¿quánto se querrá aminuar el marco que es 8 onças?. Aquesta es la regla, tu deves guardar a los quilates postrimeros, eso es a saber, aquellos que querras tornar el oro, el qual es 24, et dirás si 24 quilates valen 8 onças, eso es, 1 marco, ¿ quánto valdran 17 quilates?, multiplica 8 onças con 17, parte por 24, et viénele 5 2/3 de 1 onça 2 tornará el marco a 5 onças 2/3 de 1 onça et será aminuado 2 honças 1/3 de honça*[49].

(2) *Otrosí, 1 ome a 10 marcos, 5 honças ½ de oro, que es de liga de 17 quilates 1 grano ½ et quiérelo meter en çemitre para afinar, a tanto que sea de liga de 22 quilates 1 grano ½, di ¿quánto le aminuará todo el sobre dicho?. Aquesta es la regla, tu pornás 10 marcos 51/82, otrosy pornás 17 quilates 3/8, otrosy pornás 22 3/8 et pues guarda a los quilates a que quieres tornar el oro, eso es 22 quilates 3/8 et dirás, si 22 3/8 valen 10 marcos 51/82 ¿qué valdrán 17 3/8? et por tal que partirás por entrego 8, redresa 22 3/8 con 10 51/82 et multiplica 22 quilates por 8, junta 3 et la suma por la segunda verga, et eso es por 8 et la suma por 2 fasen 2864 quilates, otrosí multiplica 10 marcos por 8, junta 5 e la suma por 2, junta 1 e la suma por la primera verga eso es por 8 et fasen 1368 marcos et así as que 2684 quilates valen 1368 marcos, aminúa eso que podrás amos a dos los nombres et fallarás finalmente que 358 quilates valdrán 171 marcos, ende que valdrán los 17 quilates 3/8 con los 171 marcos et fasen 2907 marcos, ponlos aparte pues los 3/8 de 171 marcos que son 64 et 1 onça, júntalo con 2907 et será 2971 marcos et 1 onça, pártelos por los 358 et viénele 8 marcos 2 onças 140/358 de honça e tanto tornarán los 10 marcos e 5 onças ½ et así será aminuado todo 2 marcos, 3 onças, 38/358 de onça*[50].

(3) *Otrosí sy quisieres desir que oro que sea de 24 quilates ¿quánto querrás meter que torne a 17 quilates?, tu dirás sy 17 valen 8 onças ¿quánto valdrán 24?, multiplica 8 con 24, son 192, parte por 17 et viénelen 11 5/17 de onça et así sería çrecido el marco 3 onças 5/17 de honça et tanto querrás ayuntar al marco et es fecha la rasón*[51].

(4) *Otrosí un ome a oro de 5 maneras, primeramente ha 7 marcos de 22 quilates, otrosí 6 marcos de 20 quilates, otrosí 9 marcos de 18 quilates, otrosy 7 marcos de 15 quilates, otrosy 5 marcos de 13 quilates et quiere todo este oro meter en un troçel et fundir en uno, di ¿quándo todo este oro fuere mesclado de quántos quilates saldrá?, aquesta es la regla, por la qual /// regla lo podrás faser de quantas maneras de oro como tu quieras, tu deves multiplicar todos los marcos por los sus quilates cada uno e ayuntar la suma de todos en uno e partirlos as por la suma de los marcos, que es 34 marcos, et verná la rasón fecha, ende tu deves multiplicar 7 con 22 quilates que fasen 254, otrosí multiplica 6 con 20 quilates e fasen 120, otrosy multiplica 9 por 18 quilates que fasen 162, otrosy multiplica 7 por 15 et fasen 105 quilates, otrosí multiplica 5 marcos con 13 e fasen 65 quilates, agora ayunta en uno todas las sumas de los quilates, eso es 154 et 120 e 162 e 105 et 65 et fasen 606 quilates, agora toma la suma de los marcos, eso es 7, e 6 e 9 e 7,5 marcos que son 34, et parte la suma de los quilates por los marcos et viénele 17 quilates e 3 granos et 5/17 de 1 grano et de tantos quilates saldrá todo el dicho oro mezclado. Por aquesta regla farás todas las semejantes rasones de quantas maneras de leyes de oro fuese.*

7 marcos de 22 quilates 154 quilates
6 marcos de 20 quilates 120 quilates
9 marcos de 18 quilates 162 quilates
7 marcos de 15 quilates 105 quilates

49. B. N., Raros, Ms. 10106, f. 14r.-v.
50. B. N., Raros, Ms. 10106, f. 13v.
51. B. N., Raros, Ms. 10106, f. 13v.

5 marcos de 13 quilates 65 quilates

mayor 606 quilates

parte 606 quilates por 34

et viénele 17 quilates 3 granos 5/17 de grano[52].

(5) *Otrosí un ome ha 3 maneras de oro, primeramente a 3 marcos 3 onças ½ de oro de 21 quilates, 1 grano ½, otrosí 5 marcos 1 onça de oro de 17 quilates 2 granos, otrosí 7 marcos 5 onças de ley de 17 quilates ½ et quiere todo aqueste oro fondir e mesclar en uno, dy ¿de quántos quilates saldrá aqueste oro quando sea mesclado en uno?. Aquesta es la regla semejante a la sobre dicha, tu pornás 7 marcos 5/8 de 17 quilates ½ et multiplica los marcos con sus quilates, eso es, a saber, 3 31/82 con 21 quilates 11/42 et fasen 73 quilates 61/128 de quilate /// et ponlo aparte, otrosí multiplica 5 1/8 con 19 quilates ½ et fasen 89 quilates 11/16, ponlos con la otra suma de los quilates, otrosí multiplica 7 quilates 5/8 con 17 quilates ½ e fasen 133 quilates 7/16 et ponlo con las otras sumas de los quilates, agora ayunta en uno las 3 sumas de los quilates, fasen 296 445/428, pues ayunta todos los marcos en uno, eso es 3 7/16 e 5 marcos 1/8 et 7 marcos 5/8 et fasen 16 3/16, agora parte los 296 quilates por la suma de los marcos, eso es por 16 3/16 et faslo por la regla de partir sanos por sanos e rotos et venirle a 17 quilates 45/67 de quilate, o si quieres puedes desir que le viene 17 quilates et 2 granos et 46/67 de 1 grano de tantos quilates será el oro todo quando fuese mesclado.*

3 marcos 7/16 de 21 quilates 3/8 73 61/128

5 marcos 1/8 de 17 quilates ½ 89 11/16

7 marcos 5/8 de 17 quilates ½ 133 7/16

mayor 296

parte 296 por 16 3/16, viénele 17 quilates

2 granos 46/67 de 1 grano[53].

(6) *Otrosí un ome a oro o plata de 3 maneras, a 10 marcos ½ que vale el marco 10 libras, otrosí a 6 marcos que vale el marco 8 libras, otrosí de otro que non digo quánto es de que vale el marco, 3 libras, e de aqueste que no vale sino 3 libras el marco, quiere mesclar con todo el oro de los 2 preçios sobre dichos, que quando sea mesclado no venga a costar el marco sino 7 libras uno con otro. Agora demando ¿quánto querra de aquel de 3 libras el marco, en guisa que todo lo otro de los otros preçios non cueste con aqueste en uno sino 7 libras el marco?. Aquesta es la regla, tu deves guardar la deferençia que es de 7 fasta en 10 et es la deferençia 3 et aqueste 3 multiplícalo con 10 marcos ½, fasen 31 marcos ½, otrosí toma la deferençia que es de 7 fasta e 8, que es 1, et multiplica 1 por 6 et fasen 6 marcos, ayunta en uno los marcos, eso es 31 ½ 6 marcos, et serán 37 marcos ½, aquestos 37 marcos ½ se quieren partir por la deferencia, que es de 7 fasta en 3, el qual es 4, agora parte 31 marcos ½ por 4 et viénele 9 marcos et 3/8 de 1 marco, que son 9 marcos e 3 onças et tantos marcos et tantas honças de aquel oro de 3 libras el marco querrá mesclar con todo el oro de los 2 preçios et verná mesclado et costará 7 libras el marco apunto. E por aquesta regla puedes faser las semejantes rasones de quantos preçios como tu quieras*[54].

(7) *Otrosí es un ome que a plata de 3 preçios, a 5 marcos que cuestan a rasón de 12 libras el marco, otrosí a 8 marcos que le cuestan a rasón de 3 libras el marco, otrosí a de otra plata e no digo quántos marcos, que le cuesta a rasón de 2 libras el marco. Agora demando ¿quánta plata de aquesta de 2 libras quieres mesclar con todo lo otro sobre dicho de los 2 preçios en guisa quel marco no le venga a costar sino 5 libras?. Aquesta es la regla, toma la deferençia que es de 5 libras a que lo quieres tornar fasta en 12, e es 7, multiplícalos por los marcos del*

52. B. N., Raros, Ms. 10106, f. 13v-14r.

53. B. N., Raros, Ms. 10106, f. 14v-15r.

54. B. N., Raros, Ms. 10106, f. 15r.

preçio de 12 libras, eso es por 5, fasen 35 marcos, otrosy toma la deferençia que es de 5 a 3, que es 2 e multiplica 2 con 8, fasen 16 marcos, et por eso como 5 libras aquel quieres tornar en mayor preçio 93 libras que es preçio del primero, tu deves abatir aquestos 16 marcos de 35 marcos que avias puesto et fincaran 19 marcos, aquestos 19 marcos se deven partir por la deferençia que es de 5 a 2, que es 3, onde parte 19 marcos por 3 et viénele 6 marcos et 1/3 et tantos marcos del preçio de 2 libras querrás mesclar con todo lo otro. Et por esta regla podrás faser las semejantes rasones de tantos preçios como tu quieras.

5 marcos a rasón de 12 libras

8 marcos a rasón de 3 libras

otra plata a rasón de 2 libras

et quierese tornar a rasón de libras

e deferençia de 5 a 12, es 7

e multiplica 7 por 5, fasen 35

e deferençia de 5 a 3, es 2

e multiplica 2 con 8, fasen 16

abate 16 de 35, fincan 19

e quiérese partir por deferençia de 5 a 2, que es 3

et viénele 6 marcos 1/3 de marco[55].

(8) *Otrosí un ome ha oro de 3 preçios, a 5 marcos que le cuestan a rasón de 50 libras el marco, otrosí a 7 marcos que le cuestan a rasón de 47 libras el marco, otrosy ha 9 marcos que le cuestan a rasón de 43 libras el marco, et aqueste ome quiere fondir todo aqueste oro en uno e quiérele faser tanta mescla o sea de cobre o de plata quel marco no le venga a costar syno a rason de 35 libras el marco, dime ¿quánta mescla se querrá faser en todo aqueste oro?. Aquesta es la regla, tu deves asumar todos los marcos, eso es, 5 e 7 e 9 et son 21 marcos et guarda quanto costaron todos los marcos de la primera compra et multiplica cada uno por el su preçio, eso es, 5 por 50, fasen 250 et ponlo aparte. Otrosí, multiplica 7 con 47, fasen 329, ponlo la otra suma. Otrosí, multiplica 9 por 43 et fasen 387 et ponlo con las otras sumas, agora ayunta en uno 250 et 329 e 387 et fasen todos 966 libras, así as que 21 marcos costaron 966 /// libras, agora guarda 21 marcos a rasón de 35 libras el marco montaran et multiplica 21 con 35 libras, fasen 735 onde (...)*[56] *el señor del oro quiere que et (...) son que le costava 966 libras que le tornen a 735 libras por que guarda la deferencia, que es de 735 fasta en 966 libras et ay 231 libras porque tomarás (...) otros tantos marcos como es la deferençia dellos, eso es 231 marcos et parte por el preçio a que quieres tornar el 1 marco, eso es por 35 e viénele 6 marcos, 4 onças 4/7 de onça et tanto querrán juntar omes con lo (...) en todos los 21 marcos. Et por aquesta regla lo podrás faser de a tantos preçios et de a tantos marcos como tu quieras*[57].

(9) *Otrosí, un ome a 2 maneras de oro, eso es, de 22 quilates et de 13 quilates et quiere tomar de aquestas 2 maneras de oro 1 marco que sea de 16 quilates quando sea mesclado, dy ¿quánto quieres tomar del 1 oro e quánto del otro?. Aquesta es la regla, tu deves guardar la deferençia que es de 16 a 22 et es 6 et pues /// toma la deferençia que es de 13 a 16 que es 3, pues ayunta en uno las deferencias, eso es, 6 e 3 son 9, aqueste 9 será colonia, pues toma la deferençia de los 22 quilates, que es 6, et dirás si 9 es venido de 6 ¿onde verná 1 marco?. Multiplica 6 con 1 marco, fasen 6 marcos, et parte por 9, viénele 5 honças 1/3 de 1 onça et tanto quiere del oro de 13 quilates, et pues tomarás la otra deferençia que es de 3 e dyrás sy 9 es venido de 3 ¿ende verná 1 marco?. Multiplica 3 por 1 marco e fasen 3 marcos et pártelos por 9 et viénele 2 honças 2/3 de 1 onça et tanto quieres del oro de 22 quilates, et así quiere*

55. B. N., Raros, Ms. 10106, f. 16r.

56. It was impossible to read the word.

57. B. N., Raros, Ms. 10106, f. 16r.

del oro de 13 quilates 5 onças 1/3 et del oro de 22 quilates 2 onças 2/3 et sy quisieres di si 8 honças, eso es, 1 marco tienen 22 quilates a 2 onças 2/3, multiplica 2 2/3 con 22 quilates e la suma pártela por 8, viénele 7 1/3 e tantos quilates las 2 onças 2/3. Otrosy 8 honças tienen 13 quilates ¿quánto tienen las 5 onças 1/3? multiplica 5 1/3 con 13 quilates e la suma parte por 8, viénele 8 2/3 et tantos quilates tienen las 5 onças 1/3 et asy las honças son bien 8 et los quilates son 16 et asy podras faser las semejantes rasones[58].

(10) *Es una copa que es de 3 metales e pesa toda 14 onças, de que son de plata 4 honças et 3 onças de cobre et 7 onças de oro fino et de aquesta copa se quebró 1 pieça que pesó 6 honças, dy ¿quánta plata e quánto cobre e quaáto oro avrá en la pieça quebrada?. Aquesta es la regla, tu deves tomar la suma deso que pesa toda la copa, que son 14 onças et aquestos 14 serán colonia et pues guarda quanta era toda la plata que era en la copa, que era 4 onças, et multiplícalo con el pedaço quebrado, eso es por 6, et fasen 24, parte por 14, viénele 1 5/7 e tanta plata avra en la pieça quebrada. Otrosí toma el peso del cobre, que son 3 honças, multiplica por 6 fasen 18, parte por 14 e viénele 1 2/7 et tanto cobre avrá. Otrosí, toma el preçio del oro que son 7 onças e multiplica por 6, fasen 42, parte por 14 et viénele 3 onças et tanto oro avrá et así avía de plata 1 onça 5/7 et de cobre 1 2/7 et de oro 3 onças*[59].

The practical purpose of the medieval manuals on mercantile arithmetic is obvious. The challenge of the significant development of trade had been extraordinary; the response would be extraordinary too: medieval society and the men of commerce did develop a complete set of procedures and resources which helped them to carry out their activity successfully. A basic training in arithmetic was essential. The manuals of mercantile arithmetic were produced for that purpose, and the merchants were their main but not their only users. Many other urban activities required them. Various craftsmen, money-changers, tax collectors etc. likewise needed further training in arithmetic to perform their tasks. We should mention especially the men involved in minting and money-change, as the "problems of alloys" which we present here were devoted mainly to them. These texts explain clearly the calculations to obtain the adequate proportion of silver-copper or gold-silver in the alloys for minting coins, and those necessary to prepare the artefacts with which to control their weight and size. These calculations were as important in "their" jobs as in commerce. Therefore they were a necessary part of their extensive and specialized training.

58. B. N., Raros, Ms. 10106, f. 16r-v.

59. B. N., Raros, Ms. 10106, f. 16v–17r.