The Observational Instruments at the Maragha Observatory after AD 1300*

S. Mohammad Mozaffari, Georg Zotti

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Abstract

The present paper introduces, investigates, analyses, and comments on an anonymous treatise in Persian named *al-Risāla al-Ghāzāniyya fī 'l-ālāt al-raṣādiyya*, "Ghāzān's (or Ghāzānid) treatise on the observational instruments", which describes the structure, construction, and functions of twelve "new" observational instruments in the medieval period that appear to have been proposed and invented during the reign of Ghāzān Khān, the seventh Ilkhan of the Ilkhanid dynasty of Iran (21 October 1295–17 May 1304). In the sections below we consider the treatise in the light of two issues: (1) the assumption that the primary historical sources may contain interesting notes and claims concerning Ghāzān Khān's astronomical

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activities and especially the new observatory that he founded in Tabriz, and (2) the fact that at present there are hardly any sound and historically reliable accounts of the activities of the Maragha Observatory from around 1280 onwards. It is thus essential to explore the issues that constitute the principal historical features of the research, i.e., Ghāzān, the Maragha Observatory, and astronomical activities, and to clarify the contextual relations between them. In what follows we present the key historical facts (derived from the primary sources) regarding Ghāzān and his connection to both astronomy and the Maragha Observatory. Second, we describe the Maragha Observatory in the period in question, giving further details about the observational programs conducted there and noting the substantial differences between them. These data cast new light on the activities of the observatory and, as we shall see below, may challenge the established history. We then examine the treatise, its contents, the manuscripts available, and the original approach applied to the design and construction of the instruments. In the final section, we examine the notes (and the possible misunderstandings as well) deduced from the treatise as regards the instruments, their physical construction, and their relation to Ghāzān and the Maragha Observatory. The section also contains two open discussions on the only possible archaeological evidence for the instruments and the authorship of the treatise. The most important evidence is, of course, provided by the instruments themselves and the new approach applied to their design and construction, which we discuss in the second part of the paper along with a classification of the different types of the instruments. We describe the configuration and functions of each instrument separately. These two sections are based on the text; a few changes in the order and arrangement of the materials are introduced to give a fuller account of each instrument in relation to the original text. These are followed by a separate section containing critical comments on the instrument with regard to either technical or historical considerations, including critical remarks such as probable mistakes or omissions in the treatise and some suggestions for corrections and completions, an analysis of our author's claim concerning the superiority of a new instrument over its precursors, the applicability of each instrument, the comparison of a new instrument with similar historical counterparts, and so on.

1. Introduction

1.1. Ghāzān Khān and astronomical activities

Ghāzān Khan, the seventh Ilkhan of the Ilkhanid dynasty of Iran (r. 21 October 1295–17 May 1304), was described by his Iranian Vizier, Rashīd al-Dīn Faḍl-Allāh (645 H / 1247 AD – 718 H / 1318 AD),[1] as being interested in theology, a prominent artisan (in gilding, blacksmithing, carpentry, painting, molding, and turnery), an alchemist, an expert in medicine and botany who invented a new antitoxin called *Tiryāq-i Ghāzānī*, Ghāzān's Antidote, and a mineralogist.[2] The primary historical sources[3] also contain references to his activities and skills in astronomy (especially in the field of observational instrumentation) and about the observatory he founded in Tabriz.[4]

Since the treatise studied here is directly related to Ghāzān Khān (it bears his name in the title, he is explicitly mentioned in the prologue, and we are told that the twelve new instruments appeared during his reign, etc.), we start by examining the historical materials connected to his astronomical activities.

The writings of Rashīd al-Dīn contain two interesting fragments regarding Ghāzān's astronomical interests (the Persian texts are in Appendix 1). They belong to two different contexts. In the first it is told that Ghāzān, after returning from the first war against the Mamlūk sultan of Egypt and Syria, stayed in Maragha from 15 Ramaḍān until some time before 24 Shawwāl

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[1] [Rashīd al-Dīn, *History*, Vol. 2, pp. 1331–41]; also see: [Sayılı 1960, p. 227].

[2] Furthermore, his attempts to introduce widespread political and social reforms in Iran, which had been devastated during seven decades of Mongol rule, establish him as a different kind of ruler. He was baptized a Christian. In his youth he was instructed by Buddhist Mongol monks (*Bagshī* in Mongolian language and *Bakhshī* in Persian). He converted to Islam after ascending the throne.

[3] Primary sources for this period are: (1) Rashīd al-Dīn Faḍl-Allāh al-Hamidānī, *Jāmiᶜ al-Tawārīkh (The Perfect[compendium] of Histories)* [1994, 2, pp. 1205ff]. Rashīd al-Dīn was Ghāzān Khān's vizier after 699 H/1300 AD. (2) *Banākitī's History* [1969] by Fakhr al-Dīn Abū Sulaymān Dāwūd b. Tāj al-Dīn Abu-al-Faḍl Muḥammad b. Dāwūd al-Bānāktī, Prince of Poets at Ghāzān Khān's court. (3) *Waṣṣāf's History* written by ᶜAbd-Allāh Waṣṣaf al-Ḥaḍrah Al-Nīshābūrī, who dedicated his work to Ghāzān on 3 March 1303 supported by two of his viziers (the abovementioned Rashīd al-Dīn and Saᶜd al-Dīn Mustawfī al-Sāwujī); for one of its editions see: [Waṣṣāf 1967]. For other important sources see: Ḥamd-Allāh Mustawfī's *Tarīkh-i Guzīdih* (written 730 H / 1329-30 AD) [1960], Mīrkhānd's *Tārīkh-i Rawẓa al-Ṣafā* (written 1434/4–1497/8 AD) [2002], and Khāndmīr, *Ḥabīb al-Sīyar* [1954].

[4] The data are compiled in [Sayılı 1960, pp. 224–232].

699 H (4 June–before 13 July 1300 AD).[5] According to Rashīd al-Dīn, Ghāzān arrived in Maragha on Saturday, June 4th, and
"On the next day [i.e., June 5th], he went to watch the observations; he looked at all the operations (*a'māl*) and instruments, studied them, and asked about their procedures, which he understood in spite of their difficulty. He gave orders for the construction of an observatory next to his tomb in Abwāb al-Birr [in the district] of al-Shām in Tabrīz[6] for several operations. He explained how to perform those operations with such clarity that local wise men marveled at his intelligence, because such work (*'amal*) had not been done in any era. Those wise men said that constructing it [the observatory] would be extremely difficult. He guided them, whereupon they commenced building it and they finished it following his instructions. Those wise men and all the engineers agreed that nobody had done such a thing before nor had imagined doing it."[7]

The second fragment is from the context in which Rashīd al-Dīn speaks of Ghāzān's skills and devotes a separate section to each one. In the paragraph on astronomy, we read:

"[1] On several occasions he [= Ghāzān] went to Marāgha, asked for an explanation of the instruments there, examined their configuration (*kayfiyya*) carefully, and studied them. He had a general idea of them. [2] As per his nature (*tab*'), everything having to do with the siting (*waḍ'*) and the building (*'imārat*) of the [Marāgha?] observatory he commanded to construct [3] And, as per his nature, he also erected a dome in order to investigate the Sun's motion and he spoke out with his astronomers about it. [4] All of them said that although we never have seen such an instrument, it is reasonable / sensible. [5] In the observatory next to Abwāb al-Birr in Tabrīz, a dome has

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[5] See: [Rashīd al-Dīn, *History*, Vol. 2, p. 1296]; Banākitī [*History*, p. 463] says nothing about his order to construct the Tabriz observatory; Khāndmīr [*Ḥabīb al-Sīyar*, 3, p. 154] says that Ghāzān lingered on in Maragha until Dhu-al-ḥijja 699/September 1300, but according to Rashīd al-Dīn, the king left Maragha for an Imperial council (*Kurultai*) on Tuesday 24 Shawwāl/13 July in Ujān.

[6] A rural area south of Tabriz where Ghāzān built a gigantic dodecahedral tomb and 12 social and scholarly institutes (including the observatory) around it during 16 Dhu-al-ḥijja 696–702 H (= 5 Oct 1297–1302/03 AD). See: [Rashīd al-Dīn, *History*, Vol. 2, pp. 1377–84]; [Waṣṣāf, *History*, pp. 229–231]; [Sayılı 1960, p. 226]. Ghāzān himself draws the plan of this complex [Rashīd al-Dīn, *History*, 2: 1376].

[7] [Rashīd al-Dīn, *History*, Vol. 2, p. 1296].

been constructed that contains these things ("*ma'ānī*" lit. purports), as can be seen."[8]

In the first quote it is reported that Ghāzān ordered unprecedented "operations" after visiting the Maragha Observatory and seeing the astronomical procedures performed there at the time. The new "operations" or "work" must have involved the new observatory that Ghāzān commissioned in Tabriz (1300 AD). We do not know exactly what these operations were, or whether or not they were related to Ghāzān's hemispherical instrument for solar observations, or how they differed from those carried out by his recent predecessors, especially the scholars who set up the Maragha observatory and worked in it for four decades before his visit. (In general, nothing is known about the activities of the Tabriz observatory and their results.) Even though the fragment is subject to the panegyric praise of a vizier for his sultan, two facts nonetheless emerge from it: first, the proposal to conduct new astronomical operations is attributed to Ghāzān (though no details are given); second, the Maragha observatory was active and alive at that time, and a number of astronomers and engineers were working there.[9] It is also understood that the operations carried out there were so extensive that an interested ruler was forced to devote a good deal of time to watching and/or learning them.

In the second quote, a tentative familiarity with the Persian language is enough to verify that the word "Observatory" in the second sentence, i.e., [2], refers back to "Maragha" in the first sentence, i.e., [1]. In addition, when, in

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[8] [Rashīd al-Dīn, *History*, Vol. 2, p. 1340]; cf. [Sayılı 1960, p. 228].

[9] One referee speculates that at that time the Ilkhanid astronomical activities were centered in Tabriz. Although, as we shall see below, a certain Shams al-Bukhārī (probably, Shams al-Dīn Muḥammad al-Wābkanawī al-Bukhārī) worked in Tabriz for a while in the 1290s (about him, see below), no observatory or the astronomical institute had yet been established in Tabriz. (The Tabriz Observatory, as the first quote above indicates, was founded in 1300 AD.) Shams al-Bukhārī performed some observations individually (the 1293–6 eclipses) or invited some foreign scholars (like Gregory Chioniades) to teach astronomy. After Ghāzān ascended the throne, by an imperial order (*yarlīq*), he instructed Wābkanawī, the most important astronomer of this period, who was officially an astrologer and connected to Ilkhān's court, to compile a new *zīj*. There is a table of parallax for the latitude of Tabriz (φ = 38°) in Wābkanawi's *zīj*, but the other tables of the *zīj* are based entirely on the latitude of Maragha (φ = 37;20°). Some observations mentioned in the *zīj* (e.g., the lunar observation in 1272 AD and the observation of the triple conjunction of Saturn and Jupiter in 1304–5) show that Wābkanawī was in Maragha before and after his presumably temporary settlement in Tabriz. In any case, it suffices to say that the data available do not allow us to conclude that astronomical activities during the period of Ghāzān's reign were essentially centered in Tabriz.

sentence [5] Rashīd al-Dīn wants to speak of the new observatory of Ghāzān in Tabriz, he refers to it with the complete name as "the observatory next to Abwāb al-Birr in Tabrīz". (In other words, if one assumes that the word "observatory" in the sentence [2] alludes to the "Tabriz Observatory" in the fifth sentence, then it is not clear why in the fifth sentence, Rashīd al-Dīn needed to mention the complete name of the Tabriz Observatory and indicate its location. In this quote, all that is said about the Tabriz Observatory is the construction of a dome (a hemispherical instrument) for the solar observations (sentences [3]–[5]). Now, based on our opinion that the word "observatory" in the second sentence refers to Maragha in the first sentence, on the strength of a straightforward reading of the full passage the phrase "everything having to do with the *siting* and the *building* of the [Marāgha] observatory he commanded to construct" becomes significant; it refers generally to the "*materials*" of the Maragha Observatory, which may include the new buildings and architectural structures, renovating the old ones that had been built some 40 years before and their superstructures, the instruments, and so on.

Therefore, as seen above, the primary historical sources establish a clear relationship between Ghāzān, the Maragha Observatory, and astronomical activities. From a historical perspective, there is no reason to doubt that Ghāzān paid attention to astronomical activities and, although he had founded a new observatory in his capital, he certainly did not neglect the Maragha Observatory. However, what the "operations" were or what Ghāzān constructed in the observatory is unclear. As we shall see below (Section 3), *Ghāzān's* or *the Ghāzānīd treatise* substantiates the historical claim to a large extent, by establishing a clear link between Ghāzān and observational instruments.

1.2. The Maragha observatory and a new perspective on its periods of activity

The Marāgha observatory was built in 1259 by Hülegü (d. 1265), the founder of the Īlkhānid dynasty of Iran; during its fifty-eight years of operation, it represented the acme of Islamic astronomy. It appears that some observations in Maragha had begun before the construction of the observatory: In his treatise on the astrolabe (*fī kayfiyyat tasṭiḥ al-basiṭ al-kurī*), Ibn al-Ṣalāh al-Hamadhānī (d. 1153 AD) said that, in Maragha, he had found the magnitude of 23;35° for the Total Declination (*al-mayl al-kullī*; i.e., the obliquity of the ecliptic).[10]

Two *zīj*es were written during the first two decades of the observatory: al-Ṭūsī's *Zīj-i īlkhānī* in Persian and Muḥyī al-Dīn al-Maghribī's *Adwār al-'anwār* in Arabic. Al-Ṭūsī completed the *Īlkhānī Zīj* around 1270. We assume at present that it was the result of the observational program carried out by the main staff of the observatory in the 1270s (see below for further details). Al-Maghribī completed his *zīj* in Rajab 675 H (= December 1276 / January 1277).[11] It can be shown to have been based on the extensive observations carried out by Muḥyī al-Dīn himself. He later wrote *Talkhīṣ al-majisṭī*[12] in which he described the observations with the numerical data obtained from them and explained the procedures through which he recalculated Ptolemy's planetary parameters (e.g., eccentricity, the longitude of the apogee, the rate

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[10] See: [Lorch 2000, p. 401]. MS. Tehran, Majlis [Parliament] Library, No. 6412, fol. 62r: *wa huwa* ᶜ*alā mā wajadnāhu bi-'l-raṣad bi-Marāgha 23 juz'an wa 35 daqīqa.* Nevertheless, in some later copies of it (e.g., Tehran, Majlis Library, No. 602, pp. 33–52, written originally by Qāḍī-zādah al-Rūmī in Rajab 892 (= July 1487), and MS. Tehran, Majlis Library, No. 6329, pp. 24–35), the second part (*maqāla*) of the treatise is the "Projection of the Astrolabe" (*Tasṭīḥ al-asṭurlāb*) of Muḥyī al-Dīn al-Maghribī (d. 1283 AD), where that author stated his own figure for the magnitude for the total declination, 23;30° (*... bi-qadr al-mayl al-a*ᶜ*ẓam, huwa 23;30* ᶜ*alā mā wajadnāhu bi-al-raṣad ...* ; Edited text in the thesis of one of the authors: [Mozaffari 2007] (MS degree in History of Astronomy in Medieval Islam; Unpublished). In the later periods, there were few references to the 'new' value of Ibn Ṣalāh and his observations: e.g., in his *Zīj-i Ashrafī*, Al-Kamālī (see: [Kennedy 1956, no. 4]) wrote:

> "For observers, the amount of the extreme declination (*ghāyat al-mayl*) [i.e., the total declination] is, according to the Indians: 24;0°; Hipparchus and Ptolemy: 23;51°; Islamic astronomers: 23;35°; some modern scholars (*muḥaddithūn*?): 23;33°; and the most learned of the ancients and of [their] successors, Naṣīr al-Milla wa-'l-Dīn [Al-Ṭūsī], and the most learned of this era, Muḥyī al-Dīn al-Maghribī: 23;30°." [al-Kamālī, fol. 39r]

[11] A copy of it [Mashhad, No. 332] in the author's handwriting has survived, which bears the date Dhū al-qaᶜda 674 H (= April / May 1276) in the end of canons [fol. 55v] and the date Rajab 675 H (= December 1276 / January 1277) in the end of tables [fol. 124v].

[12] In the prologue of his last *zīj*, *Adwār al-anwār*, Muḥyī al-Dīn says that he wrote the *zīj* after completing a (now lost) treatise named *Manāzil al-'ajrām al-*ᶜ*ulwiyya* ("Mansions of the upper bodies"). *Talkhīṣ al-majisṭī* has been dedicated to Ṣadr al-Dīn Abū al-Ḥasan ᶜAlī b. Muḥammad b. Muḥammad b. al-Ḥasan al-Ṭūsī [*Talkhīṣ*, fol. 2r], a son of Naṣīr al-Dīn al-Ṭūsī, who was appointed director of the observatory on the death of his father [Sayılı 1960, p. 205]. So it seems that *Talkhīṣ* was written after the completion of the *Adwār al-anwār* and probably after the author's return to Maragha from Baghdad (see n. 14, below). The contents were presented and two parts of it were studied in [Saliba 1983, 1985, and 1986].

of mean motion, and so on) and derived new values for some of them.[13] According to the *Talkhīṣ*, the period of observations on which Muḥyī al-Din based his parameters was from 8 March 1262 (Lunar Eclipse) to 12 August 1274 (Jupiter).

Thus, the two observational programs proceeded more or less *simultaneously*. However, Muḥyī al-Dīn survived al-Ṭūṣī by nearly a decade, though there is no evidence that he performed other observations in this period (1274–1283), a part of which he had, of course, spent away from the observatory.[14] The two observations were also conducted *independently*: none of Muḥyī al-Dīn's new values for the parameters (except the obliquity of the ecliptic: 23;30°, derived from the observations performed on the three successive days after the two dates 12 June and 7 December of the year 1264[15]), have been used in the *Īlkhānī Zīj*.

A good time after the two *zīj*es appeared, the third was probably written in Maragha (see below): *al-Zīj al-muḥaqqaq al-sulṭānī* by Shams al-Dīn Muḥammad al-Khwāja Shams al-Munajjim al-Wābkanawī al-Bukhārī, completed between 1316 and 1324.[16] He states that he had written the *zīj* on receiving a royal order (*yarlīq*) from Ghāzān Khān and dedicated it to Sulṭān Abū Saᶜīd Bahādur (the ninth Ilkhān of the Ilkhanid dynasty, r. 1316–1335 AD) "*as an inheritance* (*bi ṭarīq-i 'irth*) *from his fathers*".[17] He praised Sulṭān Uljāytuw (the eighth Ilkhān of that dynasty, r. 1304–1316 AD), to whom the author had dedicated a compendium of the *zīj* before completing its final edition which, as Wābkanawī himself claims, pleased Uljāytuw so much that he ordered it to be copied and to be sent to several other cities.[18] Wābkanawī also mentioned Qutluq b. Zangī (Cotelesse in European sources),

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[13] The analysis of the lunar and planetary observations of Muḥyī al-Dīn will appear in two forthcoming papers by one of the authors.

[14] According to Ibn al-Fuwaṭī [vol. 5, p. 117], Muḥyī al-Dīn deserted the observatory and spent some days in the service of Al-Ṣāḥib Sharaf al-Dīn b. al-Ṣāḥib Shams al-Dīn in Baghdad. The date of the migration has not been given, but he probably left after he had finished writing the *zīj*, i.e., after 1276 AD.

[15] Note that the solstices were on 14 June and 13 December 1264, in the Julian calendar.

[16] It is preserved at least in three full copies, marked with (A), (B), and (C) in the bibliography (also, cf. [Mozaffari 2013a, pp. 241–242]). Two further partial copies of it are: (D) MS. Tehran Univ., No. 2452, pp. 122–128 (selected fragment for determination of hours) and (E) MS. Tehran University, Theology Faculty, No. 190 D, fol. 163v–175v. [King and Samsó 2001] only referred to (A).

[17] [Wābkanawī, A: fol. 4r, B: 6r].

[18] [Wābkanawī, A: fol. 4r, B: fol. 6r].

Amīr Čūpān,[19] and Tāj al-Dīn ʻAlīshāh of Guilan (d. 724 H / 1324 AD), the vizier of Uljāytuw and Abū Saᶜīd, for their support during his career. From what the author says it appears that Abū Saᶜīd, Amīr Čūpān, and Tāj al-Dīn ʻAlīshāh were alive at the time of the dedication of the *zīj*.

The information about Wābkanawī's career that can be drawn from his *zīj* shows that he probably worked in Maragha and Tabriz. The first observation documented in his *zīj* (see below) is reported to have been performed in Maragha, which shows that he had been there since at least 1272; the tables of the *zīj* are based on the latitude of Maragha (37;20º); and, as we shall see presently, he makes a number of remarks concerning the Maragha Observatory, the *zījes* written there, and the observational programs. This evidence allows us to conclude that he had connections with the Maragha Observatory. What is more, in Wābkanawī's *zīj*, besides the tables for parallax for the Seven Climates and Maragha, there is a separate table for Tabriz.[20] We also know that Wābkanawī was the astronomer royal of Ghāzān Khān and had been commissioned to prepare a new calendar, named *Khānī*.[21] This suggests that he probably spent some time in Tabriz, the Īlkhānīd capital of the day, and may have worked in the new observatory set up by Ghāzān. Also, since the preliminary version of his *zīj*, as mentioned above, was dedicated to Uljāytuw, it is probable that he would have worked in the observatory that Uljāytuw founded in Sulṭāniyya (see below). Nevertheless, since Wābkanawī says nothing about the Tabriz or Sulṭāniyya observatories, it seems that the Maragha Observatory was his main center of activity.

The period of Wābkanawī's observations, as he himself says, extended over 40 years. The first observation mentioned in his *zīj* is the measurement of the lunar altitude on 3 December 1272, which, as he explicitly mentioned, was performed in Maragha.[22] The last observation documented is that of the triple conjunction of the two superior planets, i.e., Jupiter and Saturn, in 1305–6 (we are not told the place of the observation).[23] Meanwhile, he

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[19] Both were generals in the Mongol Army. Qutluq was killed during the Mongol invasion of Guilan (one of the Northern provinces of Iran) in June 1307 AD. Čūpān was murdered by order of Abū Saᶜīd in 1328.
[20] [Wābkanawī, A: fol. 169r]. We know that a certain Shams al-Bukhārī was in Tabrīz and taught some mathematical astronomy to Gregory Chioniades. In the Greek writings, there are fragments concerning the observation and the calculation of the parameters of three eclipses (two solar, July 5, 1293 and October 28, 1296 and one lunar, May 30, 1295) in Tabriz (cf. [Pingree 1985, pp. 348, 352, 394]). "Shams al-Bukhārī" may be identified with Wābkanawī.
[21] [Wābkanawī, A: fol. 2v; B: fol. 3v; also cf. II, 6: A: fols. 28r–30r, B: fols. 49v–54r].
[22] [Wābkanawī, A: fol. 89v–90r, B: fol. 155r].
[23] [Wābkanawī, A: fol. 125r; B: fol. 235r].

mentioned the observation of the annular eclipse of 30 January 1283 in Mughān (see below, footnote 27) and that of the great conjunction of 1286 (again, the place of observation is not given).[24]

Based on the explanations given in the prologue of the *zīj*, these observations focused mainly on testing the data derived from the various *zīj*es at his disposal. It appears he paid considerable attention to testing the *Zīj-i Īlkhānī* (which was regarded as the main achievement of the observatory) and Muḥyī al-Dīn's *Adwār* against the observations.[25] He gives the numerical results concerning his comparative studies. He was finally convinced that the times of the occurrence of the astronomical phenomena such as conjunctions, oppositions, and eclipses as well as the planetary ecliptical coordinates calculated based on the *Zīj-i Īlkhānī* never coincide with the data derived from the observations, and added that, especially in the case of magnitudes and the instants of the eclipses' phases, *strong disagreements* and *evident differences* were observed.[26] In contrast, *Adwār al-anwār* gave the results in good agreement with the observations, which persuaded him to adopt all of Muḥyī al-Dīn's new values for the Ptolemaic parameters in his *zīj*: "we observed all of them [= the astronomical phenomena previously mentioned] based on the principles established in this *zīj* and found the calculated (*maḥsūb*) [position and/or time] in agreement with the observed (*mar'ī*) [position and/or time]".[27] Although the data presented in Wābkanawī's *zīj* still have to be checked with both the values derived from the *Zīj-i Īlkhānī* and *Adwār al-anwār* and the modern values, the fact that the author presented such quantitative conclusions is significant and merits further study. He was also highly critical of the *Zīj-i Īlkhānī* on the grounds that it was a mere copy of the earlier *zījes*, especially as regards the fundamental planetary

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[24] [Wābkanawī, A: fol. 3r, B: fol. 4v].

[25] In the case of the conjunctions of Jupiter and Saturn in 1286 and 1305/1306, as Wābkanawī said [Wābkanawī, A: fol. 3r, B: 4v], he checked all the earlier *zīj*es mentioned in the prologue of his *zīj* against observations, and the best agreement was obtained with Mūḥyī al-Dīn's.

[26] [Wābkanawī, A: fol. 2v, B: 3v]. Concerning the conjunctions, the differences that Wābkanawī found are:

Mars and Saturn: in the period of direct motion of Mars:	6 days
in the period of retrograde motion of Mars:	8 days
Mars and Jupiter: in the period of direct motion of Mars:	5 days

[27] [Wābkanawī, A: fol. 2v, B: 3v]. The most notable of the agreements observed between the data obtained based on Muḥyī al-Dīn's parameters and modern values is for the annular eclipse of 30 January 1283. Wābkanawī expounds step-by-step the procedure of obtaining this eclipse's parameters (including the iterative process of calculating the values of the luminaries' parallax) in the third book (Section 14) of his *zīj*; cf. [Mozaffari 2009; 2013a; 2013b].

parameters. (The opinion may be supported by al-Ṭūsī's own assertions in the prologue of the *Zīj-i Īlkhānī*.)[28] Wābkanawī stated that Muḥyī al-Dīn's *Adwār* was based on the *Raṣad-i jadīd-i Īlkhānī*, "the *New* Īlkhānīd *Observations*" (i.e., Muḥyī al-Dīn's own observations) in order to distinguish it from the *Zīj-i Īlkhānī* which was assumed to be obtained from the *Raṣad-i Īlkhānī*, the "Īlkhānīd Observations" (i.e. the observational plans supervised by Al-Ṭūsī and performed by his colleagues).[29] Since Wābkanawī contended that the *Zīj-i Īlkhānī* was based mainly on the earlier astronomical tables, rather than on independent observations, he referred only to Muḥyī al-Dīn's *Adwār* as the "Īlkhānīd Observations."[30] These terms, coined to differentiate between these observational activities in the observatory, appeared in Wābkanawī's work for the first time; however, the same terms, as we shall see now, may have been used in other works for different purposes.

The term *Raṣad-i jadīd* also appears in the works of an outstanding contemporary of Wābkanawī, Quṭb al-Dīn al-Shīrāzī, naming the observations that led to the writing of the *Zīj-i Īlkhānī*. For instance, in *Tuḥfa al-shāhiyya fī 'l-hay'a* ("Gift to the king on astronomy") and *Ikhtiyārāt-i muẓaffarī* ("Selections by Muẓaffar al-Dīn"; dedicated to Muẓaffar al-Dīn Bulāq Arsalān (d. 1305), a local ruler), he mentions that the solar eccentricity for the "recent observers" (*aṣḥāb al-arṣād min al-muta'akhkhirīn*) is 2;5,51 (when the radius of the geocentric eccentric orbit = 60 units) while the longitude of its apogee is time-based and so is different in their *zīj*es; but according to the "New Observation", *Raṣad-i jadīd*, it is 87;6,51° for the beginning of the year 650 Yazdigird (= 5 January 1281).[31] The value 2;5,51 for the solar eccentricity belongs to Ibn al-'A'lam (the corresponding maximum value for the equation of the centre is 2;0,10, which can be found in the table of solar equation attributed to Ibn al-A'lam in the *Ashrafī zīj*)[32]. However, in the *Zīj-i Īlkhānī*, the maximum value of the solar equation of center is 2;0,30 (the corresponding solar eccentricity = 2;6,10), which is the value applied to Ibn Yūnus' *al-Zīj al-kabīr al-ḥakimī*. Note that al-Shīrāzī here refers to his *recent* predecessors. The value he mentions for the longitude of solar apogee is nearly the same as the one tabulated in the *Zīj-i Īlkhānī* for

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28 [al-Ṭūsī, *zīj*, C: p. 7, T: 3r].
29 For example, [Wābkanawī, Book III, Section 3, Chapter 1: A: fol. 53r, B: fol. 96r; III, 9, 5: A: fol. 60r, B: fol. 108v; III, 13, 6: A: fol. 67r, B: 120v and many other places].
30 [Wābkanawī, A: fol. 3r, B: 4v].
31 [al-Shīrāzī, *Tuḥfa*, fol. 38v]; [al-Shīrāzī, *Ikhtiyārāt*, fol. 50v].
32 [al-Kamālī, fol. 236v].

the year 650 Yazdigird.[33] Despite the value adopted for the solar equation of center or eccentricity in the *Zīj-i Īlkhānī*, this value cannot be considered to have been borrowed from Ibn Yūnus' *zīj*, simply because Ibn Yūnus gives the value 86;10° for the longitude of the solar apogee in the year 372 Yazdigird[34]; since the value used for the precession rate in both *zīj*es is 1 degree per 70 Persian years, an approximate value of 90° will be produced for the longitude of the solar apogee in the year 650 Y, which deviates by almost three degrees from the value tabulated in the *Īlkhānī Zīj*.[35] When mentioning the solar apogee, al-Shīrāzī makes a bold difference and attributes its value to the "New Observations", a term that very likely means the ones performed at the Maragha Observatory. Thus, as we saw, the term *Raṣad-i jadīd* was used in al-Shīrāzī's work to highlight the difference between the observations at Maragha Observatory and those of his recent predecessors, Ibn al-A'lam and Ibn Yūnus. As mentioned earlier, Wābkanawī used the same term (in his first statement), but to distinguish al-Maghribī's observations from the ones conducted by the official staff of the observatory.

A more important note here is that, in spite of Wābkanawī's second claim (and the general idea propounded in modern research as well)[36], the *Zīj-i Īlkhānī* is not based completely on the earlier *zīj*es and *at least* some independent observations were made in the observatory to measure the longitude of the solar apogee. More definite evidence of the observations made by the main staff of the observatory is *Zīj-i Īlkhānī*'s second star table[37] which tabulates the ecliptical coordinates of eighteen stars, observed in the observatory, accompanied by the coordinates measured by Ptolemy, Ibn al-A'lam,[38] and Ibn Yūnus. The star table is also independent of Muḥyī al-Dīn's observations.[39] Also, *Īlkhānī zīj* uses the value 40;18 for the radius of the

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[33] 87;6,21°.[al-Ṭūsī, *Zīj*, C: p. 56].
[34] [Ibn Yūnus, p. 120].
[35] The value is also independent of Muḥyī al-Dīn's observations. He gives the value 88;20,47 for the longitude of the solar apogee in the year 600 Y [al-Maghribī, *Talkhīṣ*, fol. 64v] and so, according to him, the solar apogee in the year 650 would have a longitude greater than the value adopted in the *Zīj-i Īlkhānī*.
[36] E.g., cf. [King 2000, p. 604].
[37] [al-Ṭūsī, *Zīj*, C: p. 195, T: fol. 100r]. The table may be found in [Kennedy 1956, Chapter 17].
[38] Note that the coordinates for the 18 stars which *Īlkhānī zīj* attributed to Ibn al-A'lam are, in fact, derived from the star table of the *Mumtaḥan zīj*; cf. [Dalen 2004, pp. 27–28].
[39] Muḥyī al-Dīn reported the measurement of the ecliptical coordinates of eight stars from observing and measuring their meridian altitude and the time elapsed from the true noon to their meridian transit. For some unknown reason, he did not use the armillary sphere of the observatory; rather he used the central quadrant erected in the southern half of the

epicycle of Mars (the radius of the deferent = 60), which was not known in any text prior to the *Īlkhānī zīj*.[40] A detailed numerical study is needed to estimate what other differences may exist between the *Zīj-i Īlkhānī* and its near or contemporary counterparts. Nevertheless, the points made above make it clear that: (1) certainly some observations other than Muḥyī al-Dīn's had been conducted at the Maragha Observatory, and that the results may be found in the *Zīj-i Īlkhānī*. (2) Wābkanawī differentiated between the observations conducted by the main staff of the observatory and the individual observations by Muḥyī al-Dīn, and there is no doubt that they were independent of each other. However, his latter claim that no observations other than those of Muḥyī al-Dīn had been performed in the observatory is exaggerated and may be interpreted as indicating the inferiority of the results of the observational activities of the main staff of the observatory in comparison with Muḥyī al-Dīn's vast corpus of observations and measurements.

Over a century later, another account of the observational programs in the Maragha Observatory appeared in the prologue of Rukn al-Dīn al-Āmulī's *Zīj-i jāmi'-i Būsa'īdī* (written around 842 H / 1438 AD). The text is edited in Appendix 2, based on the two manuscripts listed in the Bibliography, one of which, MS. T, is in the author's handwriting. It was considered a key text for making inferences and conclusions about the Maragha Observatory in Sayılı's work (see below).[41] Āmulī states that his friends approached him because it was necessary to compose a *zīj* whose results were to be in agreement with the *Raṣad-i Īlkhānī*, "Īlkhānīd Observations" – not with the *Īlkhānī zīj,* because al-Ṭūsī had made some errors when he wrote it. The errors in the *Īlkhānī zīj*, as Āmulī explicitly says, were *well known* (*mashhūr*)

observatory's main building. As he noted in *Talkhīṣ*, "It is not possible for us to observe either Vega (α Lyrae), or Capella (α Aurigae), both of which transit the circle of meridian in its northern direction, because there is no northern quadrant [established] on the meridian line by this auspicious, blessed observation [i.e. in the Maragha observatory]" [Muḥyī al-Dīn, *Talkhīṣ*, fol. 114v]. The declinations of Vega and Capella were around 44° 51.5′ and 38° 17.5′, respectively, in those days, and so both transited Maragha's meridian (φ=37;23,46°) in its northern direction. It is interesting that the *Īlkhānī Zīj*'s star table includes the ecliptical coordinates of both Vega and Capella. A detailed study of the observations in Maragha by one of the authors is being prepared.

40 The table for the epicyclic equation of Mars for the adjusted anomaly is symmetric with the maximum value 42;12° in the mean distance (i.e., when the distance between the centre of the planet's epicycle and the centre of the Earth is equal to the radius of the deferent, which is taken $R = 60$); al-Ṭūsī, *Zīj*, C: p. 116, P: fols. 38v–39r, M: fols. 70v–71v. Also, see Al-Kāshī, *Zīj*, IO: fols. 99r, 112r.

41 Sayılı [1960, pp. 214–215] gives a translation of selected Sections.

in his day. In order to explain what *Raṣad-i Īlkhānī* is, Āmulī tells a historical anecdote: al-Ṭūsī wished that his son, Aṣīl al-Dīn Ḥasan, should correct the tables of the *Īlkhānī zīj* with the aid of Quṭb al-Dīn al-Shīrāzī. But Quṭb al-Dīn rejected the request and only submitted *some words* in the *margins of the zīj*. According to Āmulī, the proposed corrections by al-Shīrāzī were:

Moon:	the tabulated mean longitude	+0;30°
Saturn:	Centrum	+0; 7
Jupiter:	Centrum	–1;36
	Anomaly	+1;21
Mars:	Centrum	+1;30
Venus:	Centrum	–1;30

The correction values, as Āmulī says (see Appendix 2), should be added to or subtracted from the accumulated values for the mean anomaly or mean centrum extracted from the tables of the *Īlkhānī zīj*. Namely, the accumulated values of the mean motions for a specific date extracted from the columns and tables for the mean motion in years, months, days, and hours must be corrected with the above values and then the ecliptical coordinates must be computed. (However, it would be easier to add or subtract these values, once and for all, to or from the epoch mean positions of the *Īlkhānī zīj*.)

Āmulī continues his anecdote by saying that after the death of al-Ṭūsī, some of the observational astronomers (he mentioned four persons including al-Maghribī and Najm al-Dīn Dabīrān Qazwīnī) made observations during a 30-year period, with the result that in addition to the above corrections, three arc-minutes must be subtracted from the solar Centrum so that the results calculated (*maḥsūb*) for eclipses, planetary conjunctions (*qirānāt*) and solar conjunctions (*iḥtirāqāt)* are in agreement with the observed (*mar'ī*) data.

Raṣad-i Īlkhānī is again used in a historical account but to name a different observational activity, performed after the death of al-Ṭūsī. This, however, is purely a *fable* that is at odds with some definite historical facts: e.g., al-Maghribī lived around *nine* years after al-Ṭūsī, during which time, as we saw earlier, he was not continually present in the observatory, and in addition there is no evidence that he made any other observations. Qazwīnī also died on 4 Rabī' II 675 (= 23 September 1276), i.e., only *two* years after al-Ṭūsī's death.

Āmulī's comment that the *Īlkhānī zīj* is different from the *Īlkhānī observations* also appears to be a misinterpretation of Wābkanawī's statements. Support comes from the manuscript T, which is in the author's handwriting: it shows that the author first wrote *Raṣad-i jadīd-i Īlkhānī* but had crossed out the word *jadīd* (Figure 1) so that it does not appear in the other manuscript used (P), which was copied in Ṣafar 889 (= March 1484).

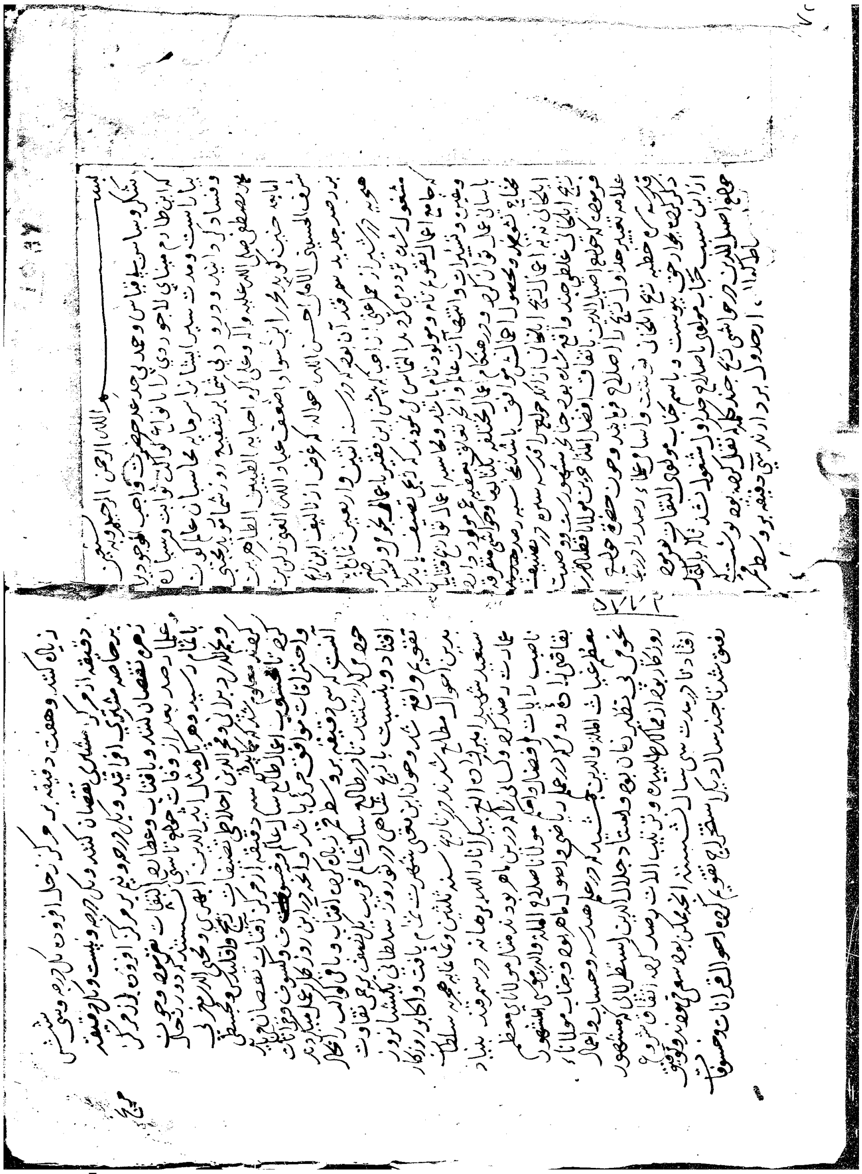

Figure 1: Āmulī's *Zīj-i jāmi'-i Būsa'īdī*, MS. T: University of Tehran, no. 2558, fols. 1v–2r.

The corrections attributed to al-Shīrāzī merit our attention. As a matter of fact the *Zīj-i Īlkhānī* was updated on several occasions: the tables of planetary mean motions in the surviving copies are from different periods.[42] Also, a tentative inspection of the manuscripts shows that there are two editions of some other tables. Referring to the manuscripts used in the present study (mentioned in the Bibliography at the end of the paper), some variants appear. In the tables of Mss. C and P, the equations of the luminaries are given in steps of 0;6º, but those of the planets for each integer degree, while in MS. T, all the tables of equations are in steps of 0;6º. MS. T bears extensive marginal notes. At the beginning of the tables of the planetary equations (the first equation of Saturn; T: fol. 50r), an unknown commentator has submitted that "in the original version (*dar aṣl*), they [= the compilers of the *zīj*] put the tables for each one degree. *We* have expanded them in order to be consistent with the tables for [the equations of] the luminaries. They can also be written in the same way as they are submitted in the original version." (Therefore, Mss. C and P are original and MS. T is the later edition.) MS. T is assumed to belong to the eighth or ninth centuries after Hijra (= 14th and 15th centuries AD). Nevertheless, there are good reasons for believing that MS. T, or the earlier manuscript of which MS. T is a copy, belong, at least, to the early 14th century.[43] Who rearranged and modified the tables is not known. The marginal explanations remind us of Āmulī saying that al-Shīrāzī had submitted *some words* in the *margins of the zīj* while correcting it. Nevertheless, whether or not they belong to al-Shīrāzī cannot be established with the evidence available.

With regard to the later updates and modifications in the *Zīj-i Īlkhānī*, further study is needed to establish whether, as one may expect, the corrections given by Āmulī had been applied to its tables for the mean motions. Also, testing the coordinates obtained by adopting these corrections may show to what extent they were either useful in improving the quantities derived from the *Zīj-i Īlkhānī*, or even factual. At present, it can only be said that the source of corrections is unknown; moreover, there is no reliable

[42] For example, Mss. P and C contain the original tables for the years 600–700 Yazdigird, MS. T for 700–800 Y, and MS. B contains the tables for 861–90 Y in addition to the original ones [Kennedy 1956, Chapter 13].

[43] Below the fixed stars table [T: fol. 100r] is written "Until the year 688 Y (= 1318 AD, i.e., 87 years since the epoch of the *zīj*), (the longitudes of) the fixed stars have increased by the amount of 1;18,18°." This is, however, not in agreement with the increment of the precession motion derived from the precession rate ψ = 1°/70y adopted in the *Īlkhānī zīj*; error = +0;3,44°.

historical evidence to support them, and no evidence that they are al-Shīrāzī's. Note that when writing the astronomical part of his encyclopedia, *Durrat al-tāj li qurrat al-Dibāj*, dated 24 Rabīᶜ I 674 H (= 17 September 1275 AD), i.e., one year after the death of al-Ṭūsī, he was in Shiraz. In the following years, he spent a great deal of time either as a political envoy in Egypt or living in Anatolia. Moreover, nowhere in his preserved treatises does he refer to these or any similar corrections, even when he explicitly speaks of the "New Observations."

As mentioned earlier, Āmulī was Sayılı's main source for contemplating the activities in the Maragha Observatory in its latter period. He adds the 30-year period of observations after al-Ṭūsī to the date of his death (1274), arriving at 1304, and then concluded that "the work at the observatory continued up to the year 1304."[44] Based on this he continues his line of reasoning that the life of the observatory was, at least, 45 years.[45] He first rightly deduced from Wābkanawī's statements that "the *zīj* of Muḥyī al-Dīn al-Maghribī is more truly representative of the work done at Maragha"[46] but he wrongly inferred that "al-Wābkanawī seems to contradict here his statements previously referred to the effect that none of the astronomical tables existing in his time was complete as none had been based upon observations lasting for thirty years". Wābkanawī does not appear to have said this. At the beginning of the prologue to his *zīj*, Wābkanawī mentions the prominent *zīj*es written up to his day, i.e., before the establishment of the Maragha Observatory. He briefly considers the relations between them and the statements of certain authors regarding others, and then he mentions that he tested these *zīj*es against his observations. Wābkanawī *quotes* al-Fahhād's criticism of the works of his earlier Islamic predecessors[47], that all of them are in error because the parameters calculated based on these works were never in agreement with the observed data.[48] Although Wābkanawī says that "[in the case of] those great men who constructed those Tables, despite their perfect knowledge and their copious funding, and the order of the king, they died before completing those important affairs"[49], this is before his discussions of the earlier *zīj*es; secondly, a contextual consideration of the text shows that he

44 [Sayılı 1960, p. 212].
45 [Sayılı 1960, p. 213].
46 [Sayılı 1960, p. 214].
47 Al-Fahhād al-Dīn Abu-'l-Ḥasan ᶜAlī b. ᶜAbd-al-Karim Al-Fahhād of Shīrwān; About him, see: [Kennedy 1956, no. 84]; [King and Samsó 2001, p. 45]; [Pingree 1985, pp. 7–8].
48 [Wābkanawī, A: fol. 3r, B: 4v].
49 [Wābkanawī, A: fol. 2r, B: 2v].

is referring here to the writers of the *Īlkhānī zīj* because the statements are immediately exemplified by the deviations found in the time of occurrence of the Saturn-Jupiter conjunctions of 1286 and 1305-06 calculated based on the *Īlkhānī zīj* vis-à-vis the observed data.

In any case, putting his interpretation of the two statements by Wābkanawī together, Sayılı is wrong to state that "we may thus attach greater credence to the words of Rukn al-Dīn al-Āmulī, according to whom [...] the astronomers of the Maragha Observatory *actually* did complete an observation program of thirty years after the death of Naṣīr al-Dīn al-Ṭūsī. Rukn al-Dīn gives some details concerning this activity of observation" (our emphasis). As seen here, if we pay the necessary attention to the date of death of Mūḥyī al-Dīn and Najm al-Dīn al-Qazwīnī, it will be obvious to what extent Āmulī's account may be wrong, untrustworthy, and pseudo-fable. In addition, because in relation to the history of the Maragha Observatory Wābkanawī is a primary source while Āmulī is a secondary one (the two works are separated from each other by around a century and a half), preference should be given to Wābkanawī, who had been present in the observatory since at least 1272, had inside first-hand knowledge about the observatory after the death of its first director, and was thoroughly familiar with its activities and members. Therefore, his account should not be placed in doubt by the comments of an astronomer who lived a long time after it.

We would also challenge some other points of Sayılı's description of the later period of the Maragha Observatory. He asserts that "Wābkanawī also states that Muḥyī al-Dīn was busy observing at Maragha *after* Naṣīr al-Dīn al-Ṭūsī's death"[50] (our emphasis); but this is neither deducible from Wābkanawī's *zīj*, nor is it historically true, as shown above. Nor do we know why Sayılı says that "al-Wābkanawī [...] speaks of the Maragha Observatory as a thing of the past".[51] We have not been able to find a statement of this in the place to which Sayılı refers to. Sayılı also claims that "he [= al-Wābkanawī] started writing his *zīj* during the reign of Uljāytuw" while, as noted earlier, it is certain that he started writing it during Ghāzān's reign and dedicated some parts of it (a preliminary draft, maybe) to Uljāytuw before the whole *zīj* was completed.

At the end of this section, we should mention the third Īlkhānīd observatory, of which little (if anything) has been said in the secondary literature: the observatory that Uljāytuw established in Sulṭāniyya (Iran,

[50] [Sayılı 1960, p. 214].
[51] [Sayılı 1960, p. 212].

Zanjān). Waṣṣāf was present in the observatory when the Ilkhān visited on Thursday, 24 Muḥarram 712 (10 June 1312), and read a poem (written in the year 710 H /1310-11 AD, as Waṣṣāf himself says) praising the Ilkhān and his new observatory. In that time, Aṣīl al-Dīn Ḥasan, a son of Naṣīr al-Dīn, was present in the congregation and explained the meaning of an astronomy-related verse of the poem for Uljāytuw.[52] Based on the above, it is conspicuous that the observatory was built before 1310 and that it was in use. However, like Ghāzān's observatory, nothing more can be said about it at the present time.

As a conclusion, the historical facts presented in these previous pages lead to these results: there were probably three observational programs at the Maragha Observatory of different durations and with different goals and values: first, the one presumably conducted under supervision of al-Ṭūsī and carried out by the main staff of the observatory; second, the one performed by al-Maghribī. The two were nearly simultaneous but objectively independently of each other, and they cover around two decades of the lifetime of the observatory, i.e. 1260–1283. We suggest here the existence of a third program consisting of a relatively long period of observations by Wābkanawī to test and examine the results of the two previous programs. Based on his observational data, at least a 30-year period of observations (from the lunar observation in 1272 to the observation of the great conjunction in 1305-6) can be safely assumed. Although, as mentioned above, there is sufficient evidence to show that he was at the Maragha Observatory, we do not know whether he was continually present there; so he may also have worked in the other Īlkhānīd observatories (especially Tabriz). Nevertheless, as argued above, the Maragha Observatory was probably the main center of his activities. Besides, when he completed his *zīj* around 1320 AD, he spoke in such detail of the Maragha Observatory, its history and the nature of the observational programs conducted there (while saying nothing about the other contemporary observatories) that we could argue that around five decades after the death of its first director the Maragha Observatory still retained its position of prominence in the astronomical activities. Therefore, since Wābkanawī's observational activities were, in any case, connected to the Maragha Observatory and the observatory has a colourful presence in his work, it is safe to consider them the third observational program of the Maragha Observatory.

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52 [Waṣṣāf, p. 285].

The errors and defects of the *Īlkhānī zīj* were known soon after it was completed. In fact they seem to have been so well known that not even the astrological predictions made based on the *Īlkhānī zīj* were accepted.[53] Wābkanawī's *zīj* is the first text in which criticisms appear. In the following decades the scholars and astronomers were well aware of the errors and some (especially al-Kāshī) tried to correct or remove them.

Traditionally, the activities of the Maragha Observatory have been divided into two periods, the one marked by al-Ṭūsī and the *Īlkhānī zīj* and the other by Muḥyī al-Dīn. However, all the historical facts presented here show that this division is anachronistic and unjustified, simply because the two observational programs were performed simultaneously. In fact, since Muḥyī al-Dīn's results were of importance to the later astronomers working in the observatory, and since his *zīj* was considered the great achievement of the Maragha Observatory, the date of his death can be regarded as a watershed and the activities can be divided into two distinct periods, before and after Muḥyī al-Dīn's death. The first period is marked by the observations conducted by the main staff of the observatory and individually by Muḥyī al-Dīn, and the later period by the investigation, testing, and critical study of the results. Of course, although the classification is *conventional*, it provides us with a clearer view of the activities conducted at the Maragha Observatory in the light of the facts mentioned above.

The second period lasted approximately twice as long as the first, but nothing worthy of comment has previously been mentioned; Sayılı's concluding remarks are based on Āmulī's pseudo-fable anecdote and so remain at best a *story*. Wābkanawī's *zīj* appears to be the last achievement of the observatory in the field of mathematical and observational astronomy, so the observational reports and the comparative studies embedded in it constitute the main characteristics of the second period. As we saw earlier, the

[53] According to a historical anecdote told by Rashīd al-Dīn Faḍl-Allah, the astrological predictions made by Aṣīl al-Dīn and Ṣadr al-Dīn, the two sons of Naṣīr al-Dīn al-Ṭūsī, based on the *Ilkhānī zīj* were not acceptable to the third Ilkhān, Aḥmad Tekudār (r. 1282–1284) [Rashīd al-Dīn, *History*, Vol. 2, pp. 1138–39]. In an astrological text entitled *Laṭā'if al-kalām fī aḥkām al-a'wām* written by Sayyid Muḥammad the Astrologer on 7 Rabī^c I 824 H/21 March 1421 AD in Lāhījān (north of Iran), the author says that the calculations based on the *Ilkhānī zīj* never coincide with the real observations. The clear differences in the ecliptical latitudes and longitudes were so great that even astrological predictions based on its quantities were mostly in error! As he says, in that era, the astronomers relied mainly on Wābkanawī's *Zīj* [Siyyid Muḥammad, pp. 322–324]. Cf. the already-mentioned statement by Āmulī, a younger contemporary of Siyyid Muḥammad, concerning the reputation of errors in the *Īlkhānī zīj*.

Maragha Observatory was active and alive around 1300 (Section 1). Nevertheless, the activities at the observatory after 1300 may have been influenced by the two other Ilkhanid observatories established in Tabrīz and Sulṭaniyya, but, as we stress, Wābkanawī used the longitude of Maragha as the basic longitude of his *zīj* completed between 1316 and 1324, showing that the observatory maintained its leading position around five decades after the death of its first director. Therefore, any other evidence that may shed more light on this period should be seriously considered. After this long, but necessary, explanation of the nature and periods of the observational programs in the observatory, the rest of the paper will focus on one of them.

Significantly, during this period, besides Wābkanawī, there was an outstanding astronomical writer, Niẓām al-Dīn al-Nīshabūrī,[54] who wrote detailed commentaries on al-Ṭūsī's works and the *Īlkhānī zīj*. Wābkanawī severely criticized and sometimes sneered at al-Nīshabūrī's gigantic commentary on the *Īlkhānī zīj* named *Kashf al-ḥaqā'iq* on account of its unnecessarily long explanations and, more importantly, because it said nothing about the conspicuous deficiencies of the *Īlkhānī zīj* in spite of its bombastic wording.[55] Later historical sources also refer to a mathematician named Shams al-ᶜUbaydī in the period of Ghāzān Khān.[56]

In this period, Gregory Chioniades was in Tabriz, where he translated Al-Khāzinī's *Zīj Al-Sanjarī*, al-Fahhād's *Zīj Al-ᶜAlā'ī*, and a text on the *ʿIlm al-hay'a*[57] into Greek.[58] Chioniades says that he used the oral instructions of a person named Σάμψ Πουχαρής born at Bukhārā on 11 June 1254,[59] who is probably the same Shams al-Dīn al-Wābkanawī (cf. above, n. 20).[60] This obviously makes the second period highly significant as the last phase in the transfer of the astronomical heritage and ideas from the Islamic world to Europe.[61] In his *al-As'ila wa l-ajwiba* ("The Questions and the Answers"),

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[54] About him, cf. [Morrison 2007].

[55] Although the objections of Wābkanawī are valid, the *Kashf* contains a detailed explanation of mathematical astronomy; cf. [Al-Nīshābūrī].

[56] [Khāndmīr, vol. 3, p. 191].

[57] Cf. [Paschos and Sotiroudis 1998].

[58] We know for sure that he spent several years between 1295–1297 AD and 1310–1314 AD in Tabriz; cf. [Westerink 1980]. Pingree [1985, p. 22] noticed that he was in Constantinople in 1302. Ghāzān Khān received envoys of from Emperor Andronicus II (1282–1328 AD) in September 1302, so probably Chioniades was among them.

[59] [Pingree 1985, pp. 16–17].

[60] Cf. [Dalen 2007]; [Mercier 2007].

[61] While the teaching and translating were based in Tabriz, this appears to be an unavoidable consequence of the fact that Tabriz had become the Ilkhanid capital before the foundation of

Rashīd al-Dīn answered the theological questions of a "wise Frank" (*ḥakīm-i farang*),[62] who is probably Chioniades.

1.3. The Treatise

The anonymous Persian treatise investigated here is, as already mentioned, titled *Risāla al-Ghāzāniyya fī 'l-ālāt al-Raṣadiyya* ("Ghāzān's" or "Ghāzānid treatise on the observational instruments").[63] As we shall see presently, it was written during the reign of Ghāzān and contains a full description of twelve observational instruments which were proposed and constructed in that period.

Three copies of this treatise are preserved in libraries in Iran: one in the Sipahsālār Library (No. 555D, fols. 15v–49v, henceforth referred to as **S**), another in the Majlis [Parliament] Library (No. 791, pp. 29–97, **P**) and a last – incomplete – copy in the Malik National Library (MS. No. 3536, pp. 41–56, henceforth **M**).[64] MSS **S** and **P** are identical; they are in the same handwriting, and they contain the same figures, scribal errors, repetitions, and so on (see Figure 2). They were copied by a scribe named Ra'īs al-Kuttāb ("Head of the scribes") on Thursday 23 Jumādā II 1294 H / 5 July 1877. In both, the date is inaccurately written as 23 Jumādā II *194* (**P**: p. 96; **S**: fol. 49r); on the opening page of MS **P** (before page 1), the scribe explicitly mentioned his name and the date of copy as "1294", and set his seal below it

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an observatory there. The Byzantine scholar would have traveled from Constantinople to the Ilkhanid realm because of the activities in the Maragha Observatory.

[62] [Rashīd al-Dīn, *As'ila wa-'l-Ajwiba*, Vol. 1, pp. 28–50].

[63] One referee mentioned that the translation given is a deliberate mistake that the authors make in order to gain a desired result. The term "*Ghāzāniyya*" denotes that the treatise is related or connected to Ghāzān. Of course, he is not its author, nor is the treatise dedicated to him (see below).

[64] Two other copies appear to be available in the Library of Āṣafīya, Hyderabad, India and in the Egyptian National Library, Cairo, Egypt, but we have not had access to them: [Storey 1958, Vol. 2, Part 1, p. 64]; [King 1986, p. 166].

In **M**, the prologue of the treatise has been wrongly attributed to Al-Kāshī (d. 1436) probably due to its similarity with Al-Kāshī's "Description of Observational Instruments" (completed in Dhu al-Qa^{c}da 818 H / January 1416),. His two treatises are also available in the following MSS: *Risāla fī 'istikhrāj jayb daraja wāḥida* ("Treatise on calculating the Sine of one degree"; **S**: fol. 1v–8v; **P**: pp. 1–15, **M**: pp. 31–39) and *Sharḥ-i Ālāt-i Raṣad* ("Description of the Observational Instruments"; **S**: fol. 9v–14v, **P**: pp. 17–27, **M**: pp. 31–39, edited in [Kennedy 1961]. On Al-Kāshī's *Zīj*, see: [Kennedy 1956, no. 12].

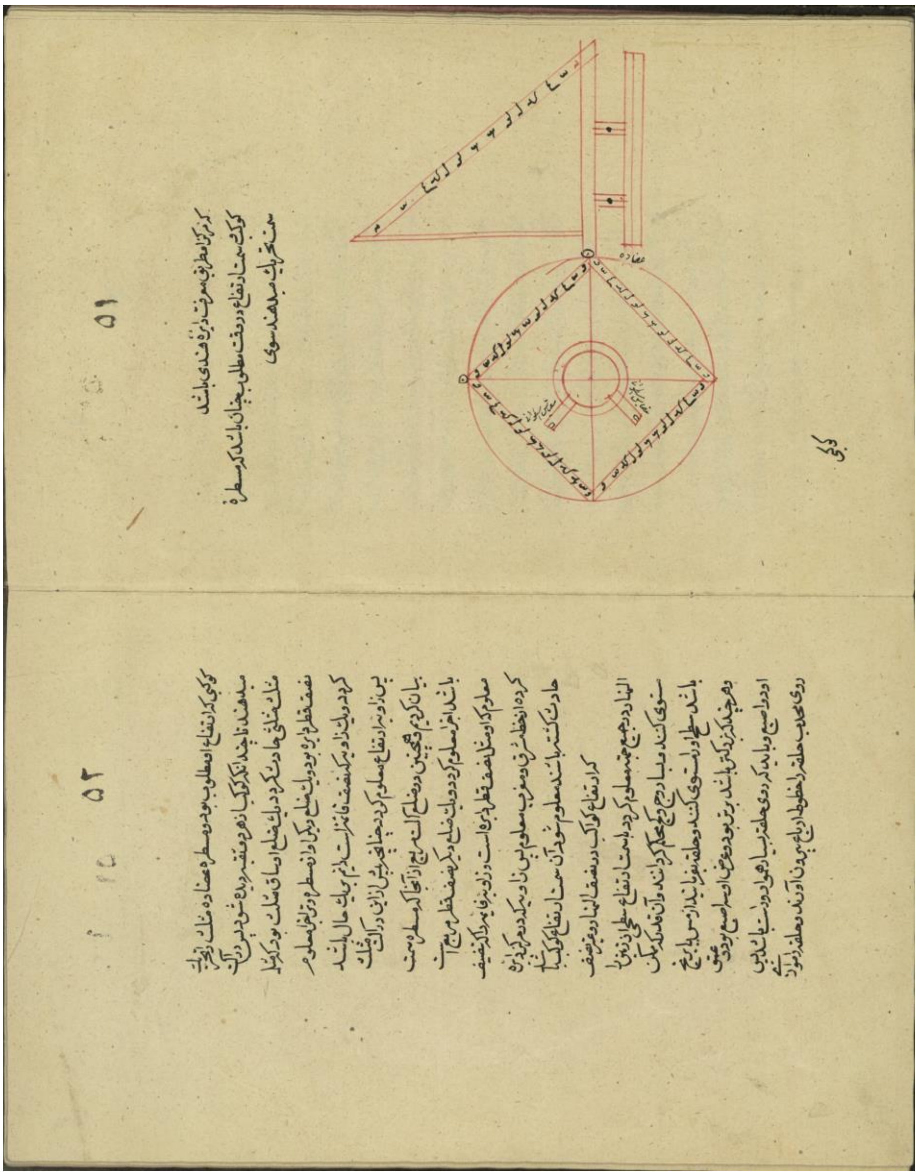

Figure 2: (A) MS. **P**, pages 51–2

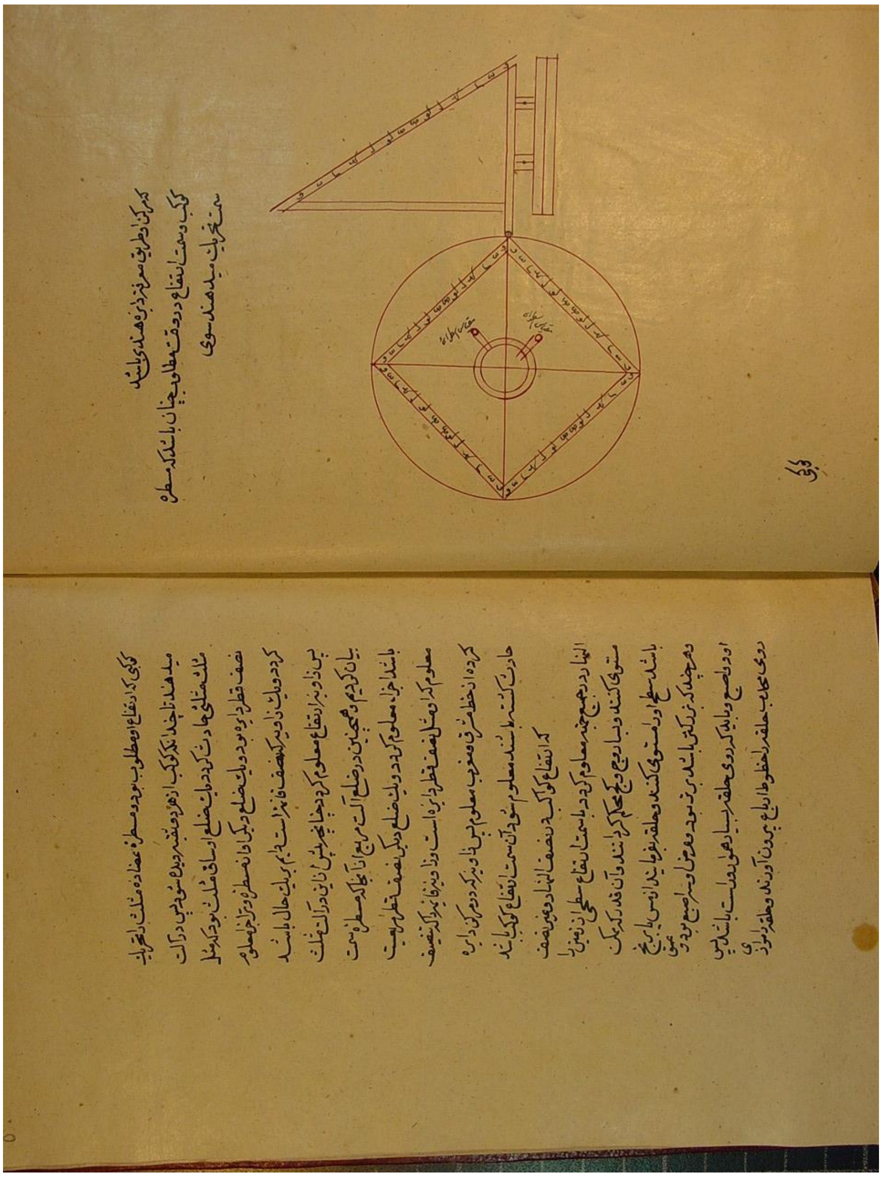

Figure 2: (B) MS. **S**, fols. 26v–27r.

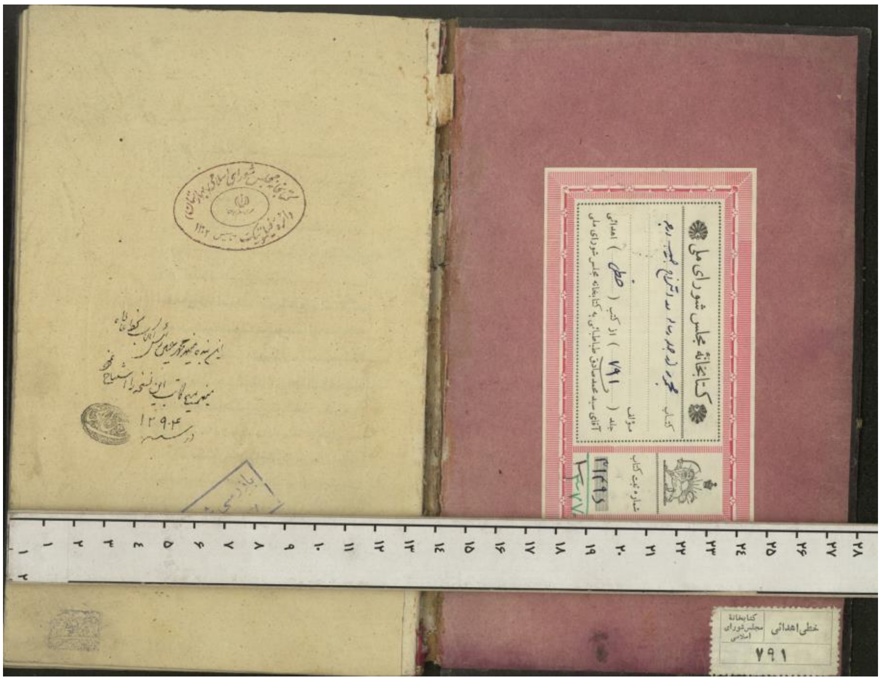

Figure 3: The opening page of MS. **P** on which the name of scribe and his seal can clearly be seen.

In the following, the prologue and the contents of the treatise are introduced.

Classic instruments and our author's critical notes

Our treatise begins with the description of the five classical instruments mentioned in Ptolemy's *Almagest*:

1. *al-Ḥalqa al-nuḥāsiyya* (Two Rings, I, 12);
2. *Lubna* (Quadrant, I, 12);
3. *al-Ḥalqa al-nuḥāsiyya* [sic!] (Equinoctial Ring, III, 1);
4. *Dhāt al-ḥalaq* (Armillary Sphere, V, 1);
5. *Dhāt al-shu*^{c}*batayn* (Parallactic Instrument, V,12).

In **S** and **P**, the name of the classic instrument no. 1 is omitted, but in **M** it is called "*al-ḥalqa al-nuḥāsiyya*". In accordance with the author's description, no. 3, which has been wrongly given the repeated name of no. 1 in the list, should be referred to by its usual name *ḥalqat al-'i*^{c}*tidāl*. Although both instruments were mostly made of copper, only the "Two Rings" were customarily called *al-ḥalqa al-nuḥāsiyya* (lit. "the ring of copper").[65]

Our author reviews the classical instruments and in general is critical of them.

In the case of the equinoctial ring, he repeats (**S**: fols. 17r–17v, **M**: p. 44) the difficulty which was encountered by Ptolemy in the *Almagest* (III, 1), namely that the weight of the ring causes it to divert from its true position of angle $90^{\circ} - \varphi$ with respect to the horizon. Al-ᶜUrḍī has mentioned this problem, and has introduced a solution for preventing it by fixing the ring into a larger ring erected in the meridian plane.[66]

In the case of the armillary sphere, to measure the longitude of any star we must first have the coordinates of a reference star. In the *Almagest* (V, 1), Ptolemy describes a method by which one considers the position of the sun determined beforehand (from the tables based on the solar theory) as the *primary reference*, and goes on to measure the unknown coordinate of a given star with the moon's position as an *intermediate reference.*[67] Thus, the measurements will be approximate, not certain, or will not derive completely from the experiments. The sun's position may be obtained by the armillary sphere, of which, however, Ptolemy does not speak. To this end, the instrument is set in its correct position (regarding the geographical latitude, the zenith [vertical orientation], and placement of the meridian ring of the instrument in the meridian plane). The ecliptic ring of the instrument is to be placed in such a position that its upper limb obscures its lower limb so that the ecliptic ring's inner surface will be in shadow. In this case, the instrumental ecliptic will be in the plane of the true ecliptic. Now, the sun's position may be obtained by placing the outer latitudinal ring in line with the sun. This method, of course, has some practical difficulties: evidently, with this method, only an object on

[65] In his treatise, Al-Kāshī referred to the *Ḥalqat al-iᶜtidāl* as *Ḥalqat 'Iskandariyya* ("Alexandria Circle"), which is connected with Ptolemy's famous observation (*Almagest* III, 1) that our author has mentioned: "…as happened to the author of the *Almagest* [*i.e.*, Ptolemy] in Alexandria's "Gymnasium" or "Palaestra" *παλαίστρά*, *al-riwāq al-malᶜab al-'Iskandariyya*, the [Sun's] light appeared in the [Equinoctial] circle two times in one equinox." (**S**: fol. 17r–17v; **M**: p. 44). (However, it must be noted that "Palaestra" was not a Stoa, *riwāq*, as our author says) Some lines previously our author speaks of *al-riwāq al-murabbaᶜ* which is identical to "Square, *τετραγών*, Stoa" in *Almagest* [Toomer 1998, p. 133, esp. n. 7 and p. 134]. Ptolemy, as mentioned in *Almagest* III, 1, had "one" equinoctial circle in Square Stoa and some in Palaestra. The terminology of our treatise here is adopted from an Arabic translation of the *Almagest* [Arabic *Almagest*, fols. 28r, 28v].

[66] [Seemann 1929, pp. 57ff, Instrument IV].

[67] [Toomer 1998, p. 219].

the ecliptic can be used as the primary reference. But only the sun is both on the ecliptic and so luminous that it can place the inner surface of the instrumental ecliptic in shadow, thus determining whether or not the ecliptic ring is exactly in its true position. (Accordingly, if we do not want to use the sun as the reference object, then we again need a source giving us a previously determined position of such an object.) The fact that the reference star has to be an object on the ecliptic (and this should be solely the sun) limits the possible uses of the instrument. Our author criticizes this point (**S**: fols. 18r–18v, **M**: p. 45). In his view, this procedure will make the measurements *approximate* (*bi taqrīb*), not *certain* (*bi taḥqīq*).[68]

In the case of the parallactic instrument, he says that

> With this instrument, the highest altitudes of the stars [when they are placed] on the meridian circle that are not in excess of 30° will [only] be known approximately; because this instrument's third rule, by which the chord of the [zenith] angle is found, truly does not show the chord of [this large] angle. (**S**: fol. 18v., **M**: p. 46)

This critique is actually similar to al-ᶜUrḍī's views of Ptolemy's Parallactic Rule given at the end of his treatise.[69] With a Chord Rule of only 60^p, altitudes below 30° could not be measured, only estimated.[70]

A new approach to constructing the observational instruments

Once our author had finished his critical survey of the classic instruments, he immediately proposed a new approach to the design and construction of the observational instruments (**S**: fols. 18v–19r/**M**: pp. 47–48). The section also contains historical information about the instruments, i.e. when the new approach was presented and/or the instruments were designed and constructed:

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[68] See also: [Wlodarczyk 1987, pp. 177 and 182]. In his *Talkhīṣ al-Majisṭī*, Muḥyī al-Dīn al-Maghribī introduced a rather complicated method for determining the longitudes of fixed stars involving observation of their and the Sun's culminations, the determination of the oblique ascension of the ascendant, and calculation of the objects' oblique ascensions, rather than using the armillary sphere; see: [Saliba 1983, pp. 398–399].

[69] [Seemann 1929, p. 107].

[70] However, as Ptolemy used this instrument to measure lunar altitudes in Alexandria, this limitation did not affect him.

Over the years [I][71] have been praying for the imperial government of the king of the world, the Great Īlkhān, the King of Kings on Earth, the sultan Ghāzān Khān—may God perpetuate his kingdom and spread his shadow over all the inhabitants of the world.

For a time, I thought about and searched for observational instruments by which observations can be produced precisely and certainly without suffering and trouble, until in the government of the world's king [i.e., Ghāzān Khān], twelve kinds of observational instruments appeared (*rūy numūd*) that had not appeared with any of his antecedents and their descendants, and had not been possible for them [to conceive]. By them [i.e., these instruments], all observations can be exactly and certainly known with the least cost and effort, because these instruments consist entirely of rules and straight lines. Although they are long, constructing them fully straight and dividing them into minutes and seconds is possible and by them those [observational] matters can be found exactly, whereas finding them by the five classical instruments is not possible, as we will describe.

The panegyrical introduction in the above quote obviously shows that the description that follows, as well as the twelve new instruments whose construction and application are explained in the treatise, date from the reign of Ghāzān and it implies that the treatise itself was also written in this period, or, more exactly, between 1300 and 1304.

Then, our author announces two parts (*qism*) of the treatise (**S**: fol. 19r; **M**: p. 48):

1. Description of observational instruments and their applications, and
2. Calculating the stars' positions in ecliptical longitude and latitude.

The second part, however, is not found in the extant copies.

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[71] Our author introduces himself as *bandih-i kamīnih kamāl*, "a pupil, having minor perfection", to indicate his humility, a common practice in the Islamic medieval period. One referee assumed that the name of our author is "Kamāl". Even if we misread the above phrase as "a minor pupil, *Kamāl*", the opinion cannot be verified for two reasons: first, authors did not usually introduce themselves by only their first name, without a title or their father's name, even if they copied the treatise in their handwriting. In such cases the phrases of humility were often used instead of honorific titles, but the full name was given. The examples are numerous: e.g., in the only copy survived from Muḥyī al-Dīn's *Talkhīṣ al-majisṭī* (MS. Leiden, Or. 110), which is in the author's handwriting, he calls himself "*al-ʿabd al-faqīr ila Allāh …*" ([fol. 1v]; see also [Saliba 1983, pp. 389–391]) and immediately mentions his title and father's name. Second, if the scholars or scribes of the previous centuries connected to the treatise had read the phrase as the referee assumed, they could not have misattributed the treatise to al-Kāshī, as is clear in all three copies inspected (*cf.* note 64). D. A. King, *Survey* (*cf.* note 64) and A. Monzavī [2003, vol. 4, p. 2999] have also mentioned that the treatise is anonymous.

On the superiority of this construction over old instruments (i.e., that the new ones are based on long straight beams and avoid the building of circular structures) he later adds (**S**: fol. 23r–23v; **M**: pp. 54–56):

This instrument [i.e., #2] is preferable and superior to all [observational] instruments for four reasons:

1. Each [older] instrument, which is well known and in common use, is dedicated to an important [application]—as we said earlier—while with this [new] instrument the determination of all quantities that can be found by those [previous] instruments is possible.

2. The expenditure, cost, effort, and occupation of [constructing] this instrument are less than for the preceding instruments, as a whole.

3. What was observed was not revealed with certainty and exactitude by the [previous] instruments, because all those instruments are made up of arcs and circles, so that if they are small, it is not possible to divide them into minutes and seconds, and the results are approximate. If they are large, it is not possible to make them completely circular, as they ought to be, and then their defect (*fasād*) and disorder (*khalal*) are more than their benefit, whereas these [new] instruments are made up of straight rules (*misṭara-hāy-i mustaqīm*) and straight lines (*khuṭūṭ-i mustaqīm*), [so that] however long, to construct them being straight, without disorder and trouble, is possible.

4. In those [old] instruments, the arcs are determined [directly], whereas in these [new] instruments, the parts [functions of arcs; i.e., sin, tan, etc.] whose corresponding arc (*Ḥiṣṣa-yi qaws*) is often smaller are determined; therefore the arcs are determined [more] exactly and with certainty.

A simple comparison of this treatise with the treatises written before and after this era confirms our author's claims.[72]

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[72] To validate our author's claim, we considered several treatises on astronomical instrumentation before this period. From the Buwayhid period, we find only one important instrument named *al-ḥalqa al-ᶜaḍudiyya* which al-Ṣufī mentioned in his *Book of Fixed Stars*. The instrument was more likely a solstice ring (and probably a large version of Ptolemy's "Two Circles"). See [Charette 2006] for a discussion of the instruments of the early Islamic period. Charette mentions [p. 133] that *al-ḥalqa al-ᶜaḍudiyya* was an equinoctial ring, while al-Ṣufī clearly says that he used it to measure the latitude of Shīrāz, which is not the assumed application of an equinoctial ring – it is not for measuring the celestial arcs but for finding the instant or day when an equinox occurs. From the Seljuk Period (the scientific circle of Seljuk Sultan Malikshāh I), we have a few treatises by Al-Khāzinī (on Al-Khāzinī and his treatises, see: [Lorch 1995, Papers XI, XIV] and [Sayılı 1956]). More important than these two is Muʾayyad al-Dīn al-ᶜUrḍī's *Fī kayfiyyat al-arṣād* [Seemann 1929], from the first period of the Maragha Observatory. Al-ᶜUrḍī described the instruments built in Maragha for al-Ṭusī: a great mural quadrant; armillary sphere (with some improvements over Ptolemy's); solstitial and equinoctial armillae; Hipparchus's dioptra (with improvements to observe eclipsed diameters

In all copies, the places for the names of the instruments are left blank. We find only the names of instruments #1 and #2 in the description of the latter, where they are called, respectively, the "Triangle Instrument" (*ālat-i muthallath*) and "Perfect Instrument" (*ālat-i kāmila*).[73] Instrument #12 is introduced with a rather long descriptive sentence.

In what follows, our author adds that, up to this time, the best observational instrument was the "Azimuth Instrument" (*ālat-i samtiyya*) invented by Abū al-ᶜAbbās al-Lawkarī, which however also suffers from the three previously mentioned flaws).[74]

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of Sun or Moon); a double quadrant made of copper inside a circular wall, capable of measuring, at the same time, azimuths and altitudes of two objects; an improved version of Ptolemy's parallactic instrument; an instrument to determine sine (of zenith distance) and azimuth using a wooden bar rotating on an iron axis inside another circular wall, on which one end of the alidade can slide, the other end sliding up a vertical central pillar; another similar instrument to determine Sine and Versine; and a "Perfect Instrument" consisting of a rotating parallactic rule inside a circular wall. We can see parallels between those instruments and the ones described here, and will note them where appropriate. From the time after our treatise, we have, e.g., from Ulugh Beg's period, al-Kāshī's *Description of Observational Instruments* (See above, n. 64) From Istanbul Observatory, e.g., Taqī al-Dīn Muḥammad Al-Maᶜrūf's works. The approach proposed in our treatise cannot be found in any of these treatises.

[73] Note also that al-ᶜUrḍī names one of his instruments, a rotating parallactic rule for azimuth and altitude measurements, the *perfect instrument* [Seemann 1929, pp. 96–104]. He had built this instrument for Malik Manṣūr, the ruler of Ḥimṣ (the ancient Emesa, now Homs), in 650 H/1252–3 AD, in presence of Manṣūr's vizier Najm al-Dīn al-Lubūdhī. Clearly, this is not the same as our *perfect instrument*.

[74] Abu al-ᶜAbbās al-Lawkarī (d. 464 H/1071–2 AD), one of the leading figures of Islamic philosophy, was a contemporary of Khayyām (d. 515H/1121 AD). He is one of the intermediaries of a famous chain of Islamic peripatetic philosophers: Ibn Sīna – Bahmanyār – Abū al-ᶜAbbās al-Lawkarī – … – Naṣīr al-Dīn al-Ṭūsī. The ancient sources provide little information about him. For example, see: [al-Bayhaghī 1994, pp. 110–111] and [al-Shahrzūrī 1976, vol. 2, p. 54]. We know nothing of his role in the scientific circle of Malikshāh Saljūqī (447 H/1055 AD–485 H/1092 AD, r. from 465H/1072AD) led by Khayyām, which is our only source of information on the scientific/astronomical activities of the period. (On Malikshāh's command, the task of purifying the solar (Iranian) calendar (with its origin at the spring equinox of 1079 AD) commenced in 467 H / 1074 AD and possibly continued until the king's death. In his History, Ibn al-'Athīr named Khayyām's collaborators, but did not mention al-Lawkarī. See: [Ibn al 'Athīr 1966, p. 98]; [Kennedy 1968, pp. 671–2]. Lawkarī, though, wrote an encyclopedia entitled *Bayān al-ḥaqq fī ḍimān al-ṣidq* including an epitome of Ptolemy's *Almagest*, which must have been well known at the time since Quṭb al-Dīn al-Shīrāzī referred to it in the prologue on astronomy of his own Persian encyclopedia (*Durrat al-tāj li qurrat al-Dibāj*) dated 24 Rabīᶜ I 674 H/17 Sep. 1275 AD in Shiraz [al-Shīrāzī 1944, Vol. 2, 1]. On the other hand, an instrument only used for determining azimuths dates back to

The Tables of the Treatise

The pages for the tables are shown in the manuscripts **S** and **P**.

1. *Ẓill-i mustawī* (Cotangent, or Umbra Recta) (**S**: fol. 44v);
2. *Mayl-i awwal* and *mayl-i thānī*, First and Second Declination (**S**: fol. 45r);
3. *Ẓill-i maᶜkūs* (Tangent, or Umbra Versa; **S**: fol. 45v);
4. *Jayb*, Sine (**S**: fol. 46r);
5. *Sahm-i qaws-i niṣf-i dawr*, Versine for the arcs 0, …, 180º (**S**: fol. 46v);
6. *Maṭālᶜ-i mustaqīm wa mā'il*, Right and Oblique Ascension of the parts of the ecliptic for the latitude of Maragha (**S**: fol. 47r);
7. *Hādhā al-jadwal* [correct: *jadwal*] *al-samt alladhī li-hādhihi al-ᶜuruḍ li-kul khums rubᶜ rubᶜ dā'irat a-'ufuq*, an uncommon table titled "The table of azimuths of these [geographical] latitudes for each quadrant of the horizon circle per five degrees".

Our author says that he copied these tables from the *Īlkhānī Zīj*, although we have not found table no. 7. Tables 1–5 have been left empty. From the text it can be deduced that the treatise originally contained other tables for the equation of days and daylight hours for the equator line[75] and for Maragha, which are also lacking in these copies.

1.4. Discussion and Conclusions

(A) The first quote from our author (above, Section 3) suggests that these new instruments were indeed constructed. There is no reason to doubt his statement that "the … instruments appeared …", even though there is no support from the material record, as in the case of al-'Urḍī's instruments.[76] The general atmosphere regarding the treatise also reinforces the idea that it is a "manual" for building and operating the new instruments rather than a "proposal" for constructing them. For instance, when introducing the tables (which can only be applied while using the instruments) our author used the title "some tables from the *Īlkhānī zīj* in order to correct (*taṣḥīḥ*) what is

the epoch of Ibn Sīnā, who built a model of it in Isfahan (between 415 H / 1024 AD – 428 H / 1037 AD); see: [Wiedemann and Juynboll 1926].

[75] Of course, this is an erratum, because there are always 12 hours of daylight in the equator line and so the equation of days for the places located on it does not exist.

[76] Note that neither al-'Urḍī's instruments nor those belonging to Ghāzān's period have survived and thus there is no support for them from the material record. We have roughly the same evidence for both: some references scattered throughout the historical sources and a treatise describing them.

mentioned in this treatise so that, whenever they [the tables] are needed [when measuring and observing by the instruments], it is not referred [i.e., it become unnecessary to refer] to the *zīj*es" (**S**: fol. 48v). Also, suggesting alternatives for the configuration of the instruments in order to facilitate (*āsānī*) or to simplify (*ikhtiṣār*) fabricating them may readily be interpreted as results from experiments due to engaging in the physical construction and to avoid difficulties with it. Thus, until we find other evidence to disprove what our author clearly says (the first quote in Section 3), we may safely *assume* that the instruments were indeed constructed[77].

(B) Assuming this to be the case, the instruments were probably located in the Maragha and Tabriz observatories. Let us consider the material at our disposal to consider this possibility.

The tables of the treatise that contain the latitude-based functions (for instance, the hours of daylight), have been extracted, as our author says, from the *Īlkhānī zīj*, which is based on the latitude of Maragha. This suggests that Maragha was probably the place of the instruments, because it initially seems unlikely that the instruments would have been installed in (or even "proposed" for) a specific site and the auxiliary tables given for a different location. However, it does not completely exclude the Tabriz Observatory from being the location of the instruments; it is possible that the instruments had been in the Tabriz Observatory but for simplicity's sake and because of the small difference in latitude between Maragha and Tabriz, our author would have copied the tables from the *Īlkhānī zīj*. But we cannot be sure of this; as we saw earlier in the case of Wābkanawī, he had a table of parallax for the latitude of Tabriz which is probably one of the places where he worked. Similarly, if the instruments were in fact constructed and were installed in the Tabriz Observatory, it is only natural that the astronomers would arrange the tables mentioned in the treatise for the latitude of Tabriz (this would not have been a difficult task). In addition the latter possibility conflicts with the explicit phrase "our city of observation which is Maragha" which the author of the treatise used in association with tables (S: fol. 48v).

We saw in Section 1 that Rashid al-Dīn spoke *only* of a hemispherical instrument for the solar observations in the Tabriz Observatory. No other instruments or implements from the Tabriz Observatory are indicated throughout Rashīd al-Dīn's *history*. Nor do we know whether the Tabriz

[77] Only for instrument #10 may we have doubts about the physical implementation. The description lacks dimensions and instructions for its assembly.

Observatory, an institute founded on one of the sides of Ghāzān's dodecahedral tomb (see above, footnote 6) was large enough to hold the observational instruments other than a hemispherical one. We have already seen (the first quote in Section 1) that the Maragha observatory was active at that time. Thus, the contention that the instruments were used there does not *ipso facto* present any historical difficulties. And as we shall see below, archaeological remains from the site of the Maragha Observatory are very similar to the foundations proposed for instrument no. 11 of Ghāzān's treatise.

After all, since the material at our disposal does not appear sufficient to identify either Maragha or Tabriz as the place of the instruments (supposing they were indeed constructed), we prefer to leave the question open. However, on the basis of the evidence available it is much more likely that they were installed in the Maragha observatory than in Tabriz.

(C) Archaeological Evidence: In Maragha, with an average rainfall of 300–1000 mm during spring and long periods of freezing weather (114 days), after four decades the mainly wooden instruments of Al-ᶜUrḍī, and especially their fine graduations required for observations, would have been destroyed. Al-ᶜUrḍī (d. 1266 AD) died eight years before Al-Ṭūsī (d. 1274 AD); obviously, then, he could not reconstruct his own inventions, but we know that the activities in the observatory continued for, at least, four decades after the 1270s.

In addition, there are five circular traces to the south, southeast and north of the central building of the observatory in Maragha. These traces appear to be the remnants of the instruments' foundations. Al-ᶜUrḍī described only four large instruments that required circular foundations. The fifth is a Double Circular Trace (Figure 4a). It is strikingly similar to the foundation of the instrument no. 11 in the treatise (Figure 4b), which described two concentric circles of radii 2 and 3 cubits (≈ 133 and 199.5 cm) (*cf.* below, Section 2.11.3). The radii of the available circles in the trace are, however, appreciably larger (the inner radius of the lesser circle is around 288 cm and the outer radius of the bigger circle is around 2·288 cm). Nevertheless, the average radii of the two circles (see Figure 4a) maintain a proportion of roughly 3/2. Before the summer of 2011, the trace was marked only with separate stones and so it was exposed to changes of all kinds; it was then re-constructed, but imprecisely (as evident from Figure 4a) so it is difficult to fix a reliable measure for the radii of the circles and for the gap between the two.

Nevertheless, Ghāzān's treatise is evidently the only document that refers to a configuration of this kind for an instrument of the Maragha Observatory. [78]

Figure 4: (a) Double Circular Trace, one of the five circular traces around the central building of the Maragha Observatory, located to the southwest of it.

[78] According to one referee, the archaeological excavations suggest that all the remnants of the observatory were constructed in the same "historical period"; he therefore doubted that these instruments were built in the Maragha observatory. Even if correct, these findings are clearly irrelevant to the question of whether the instruments were really constructed there; the remnants are merely the foundation of the central building of the observatory and its peripheral architectural structures, while the installation and/or replacement of the instruments in particular or any other change in the superstructures in general would not have modified the foundations or the main architectural structure of the observatory in any way that might be detected by archaeological inspection. Obviously, any changes in the instruments are unrelated to the renovation of the buildings. In addition, when one uses the term "period" in a historical sense, one should keep in mind that it means "a stage in the history of a culture or of a civilization having a definable place in space and time" and may last from several centuries (e.g., Ancient Rome: 753 BC–476 AD) to several millennia (e.g., Mesopotamia: BC 6000–1100 BC). In our case, it clearly denotes the Ilkhanid Period, which covers the whole lifetime of the Maragha Observatory (AD 1260–1324). Simply, *everything* concerning Maragha Observatory belongs to "one historical period". It is not clear how even archaeological inspections can distinguish any difference in the stone foundations whose destroyed buildings belong to the same architectural and historical period and are separated in time by only four decades or so. It is worth emphasizing that we are dealing with instruments whose original models have not survived. For this reason, we do not draw any conclusions from the presence in our treatise of a documented trace corresponding to the double circular remnant (Figures 4). Since the purpose of the two circles which are clearly depicted in the figure related to instrument 11 in the treatise (Figure 4b), and whose sizes are also given, is not clear to us at present (see below, instrument no. 11 in Section 2.11.3), further study is needed to demonstrate whether or not there exists a solid relation between the two, or even between the remains and instrument #5.

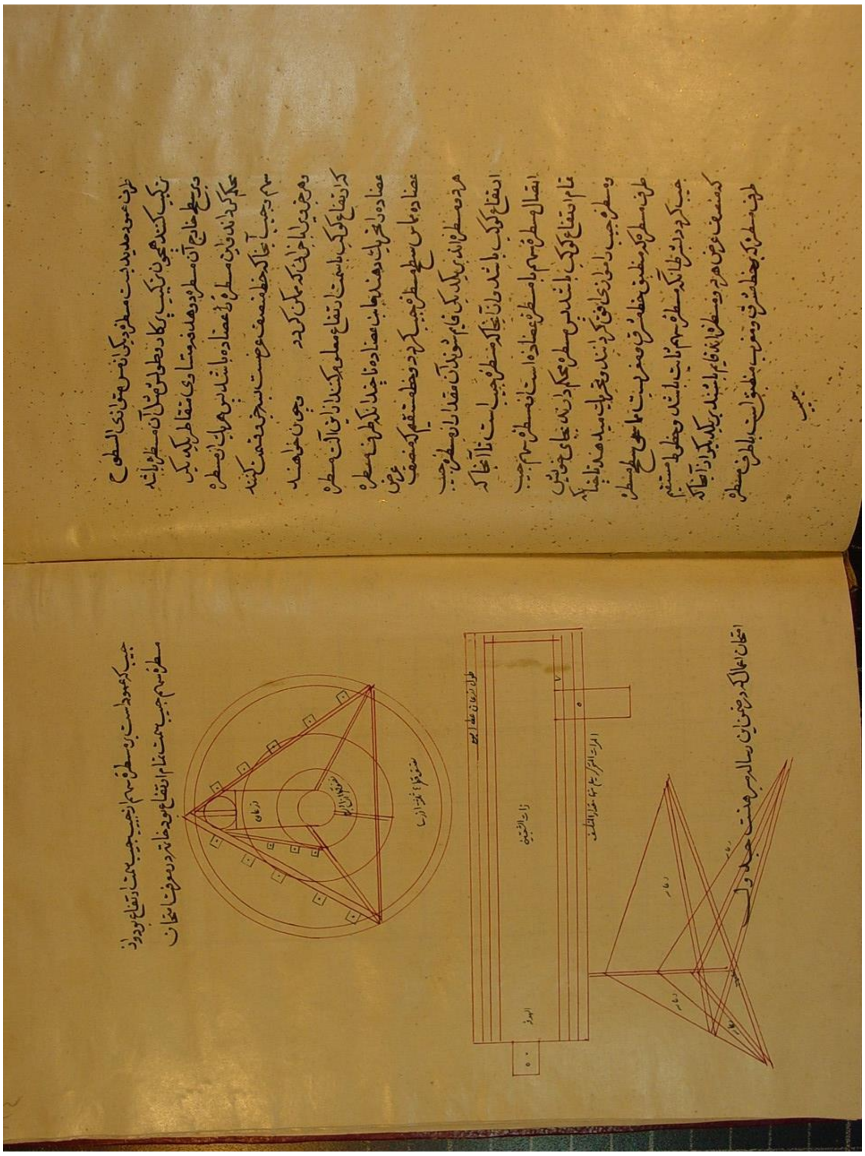

Figure 4: (b) the circles around the foundation of the instrument no. 11 as depicted in MS. S, fol. 44r.

(D) Authorship: Two centuries after the fall of the Buwayhids, Persian scholars again began writing in their native language, with result that the great majority of mathematical and astronomical treatises written after the mid-thirteenth century are in Persian. However, the language of these treatises is not as pure as that of their tenth-century counterparts; Arabic and Mongolian are used liberally (although the latter is less frequent in astronomical treatises, except in descriptions of the Chinese-Uighur calendar). This has a corrupting influence on phraseology, and since mathematical and astronomical treatises, whether in Arabic or Persian, also usually adopted a simple unitary style, the combination of the two makes the identification of a distinctive literary style very difficult. It is clear that our author is a professional astronomer with ample knowledge of observational astronomy and instrumentation and capable of comparing and drawing critical conclusions. We can therefore propose al-Wābkanawī and al-Nīsābūrī (though less confidently), as the most likely authors. In some respects—particularly in terms of the type and order of explanation, the frequency of technical terminology, the structure of sentences, and the remarkably sparse descriptive material—our treatise is so similar to al-Wābkanawī's *zīj* that in fact the authorial voice seems to be identical; it is less similar to al-Nīsābūrī's treatises, with their typical excess of explanation. The style of Wābkanawī's *zīj* and the *Ghāzānīd Treatise* seems to be practically indistinguishable. All this, along with the fact that al-Wābkanawī was Ghāzān's astronomer-royal and had received an order (*yarlīgh*) to complete a *zīj* in that period, indicates that the author was indeed Wābkanawī. Nevertheless, no further evidence is available at present to establish this identification, or to pose an alternative for the authorship of the *Ghāzānīd Treatise*.

(E) Ghāzān and the *Ghāzānīd Treatise*: In Section 1.2 we discussed the straightforward historical notes concerning Ghāzān's relation to observational activities and to the Maragha Observatory. As mentioned above, new operations in this field have been attributed to him. Although we are not told what these operations were, or what he ordered to be constructed there, the fact that he is associated with the construction of a hemispherical instrument for the solar observations at Tabriz (about which, however, we know nothing), and the involvement of engineers in the operations suggest that they may have included instruments. Now, a treatise at our disposal describes a new set of observational instruments inspired by a new approach and constructed during the reign of Ghāzān. We will not assess whether the sentences in the first quote from our author (Section 1.3) may be taken to indicate that Ghāzān was the inventor of the instruments. However, the author's clear insistence that the instruments appeared during his reign and his emphasis on the fact that they were unparalleled echoes the language used in the historical sources to describe Ghāzān's astronomical activities. This raises the question of whether the *Ghāzānīd treatise* documents Ghāzān's observational operations (possibly with some improvements or changes made by his engineers or astronomers working at the observatory, in view of the role attributed to them in the historical sources). In our view, this may be one of the instances in which the historical sources and a treatise on a specific topic are intimately coupled with each other to support a historical claim (i.e., the second quote in Section 1.1). The claim now seems to be justified, yet we prefer to still keep the question open. Nevertheless, there is no reason now to deny or to cast doubt on the evidently established link between the three main elements mentioned at the beginning of the paper: that is, Ghāzān, observational activities, and the Maragha Observatory. What is obvious, indisputable, and important is that the instruments are based on a new approach and are also connected, in some way, to Ghāzān, the period of his reign, and the Maragha (or maybe Tabriz) Observatory.

Regardless of who invented the instruments, whether they were actually constructed, where they were installed, what makes the research on this topic truly significant is the fact that we have fairly complete information on the structure, configuration, and the applications of the new twelve observational instruments in the medieval period, with the aid of which we can perform a critical study of the technical aspects. This study is the subject of the following section.

2. The Instruments

The instruments are described but not named (except for instruments #1, #2 and #12), so we will refer to them by their sequence number. We describe each instrument with subsections "configuration" and "application" closely following the treatise, plus a "comments" section, in which we discuss difficulties with the translation and some practical considerations found during the creation of the virtual reconstructions made for illustrative purposes. #12 will be presented in more detail and a full translation will be provided.

The treatise gives numerical values for the dimensions of the instruments and their parts. Our author, though, has not explicitly specified which of the definitions of *dhirāᶜ* (cubit) used at his time he is adopting.[79] The definition we used in the reconstruction is the one found in al-ᶜUrḍī's treatise, which applies the Old Royal Iranian Cubit (*dhirāᶜ al-malik* or *dhirāᶜ al-hāshimī*) of 665 mm and its fractions as shown in Table 1. Frequently, however, not all dimensions are given, and for the reconstruction, we had to estimate useful dimensions ourselves. In addition, some numbers are obviously copying errors and do not make sense when used as given literally. We will mention occurrences of this kind in the comments.

Symbol	Unit	Arab	Persian	Relation		Length (mm)
cb	cubit	*dhirāᶜ*	*gaz*		24 fg	665
sh	handspan	*shibr*	*wajab*	1/3 cb	8 fg	221.67
hd	handbreadth	*qabḍa*	—	1/6 cb	4 fg	110.83
fg	finger	*'aṣbaᶜ*	*angusht*	1/24 cb	1 fg	27.71

Table 1: Units of measurement used in the treatise

Instruments #1–#4 are based on the concept of rigid right isosceles triangles, or moving triangle legs with a Chord Rule to measure the chord of the angle, crd $\alpha = 2\sin(\alpha/2)$. In accordance with common practice, lengths and angles are defined so that the base length of the triangle legs is divided into 60 parts which are each divided into 60 "minutes", and the hypotenuse/chord of the triangle shows a graduation in the same units; so we will write $\mathrm{Crd}\,\alpha = 60\,\mathrm{crd}\,\alpha$, and similarly, $\mathrm{Sin}\,\alpha = 60\sin\alpha$, *etc.* The hypotenuse of the right triangle will then be of length

$$\mathrm{Crd}\,90° = \sqrt{2}.60^{\mathrm{p}} = 84^{\mathrm{p}}51'10''.$$

Most instrument descriptions with Chord Rules mention regular graduations up to 85^{p}, which slightly exceeds the required length, but was obviously done for practical purposes.

In the course of presenting the first instrument, we describe some mathematical principles of working with these triangle-based instruments which are then also used in later instruments.

In this treatise, the azimuth is counted from the east–west line; in the Islamic period, there were three systems (*naẓm*) for the azimuth: 180° west or east from north, 90° from the east–west line to north or south, and 90° from the meridian to the east–west line.

General Scope of the Instruments

We can classify the instruments into seven types, as shown in Table 2. In types A, D and E, we see that the latter instrument in each type is an

[79] For the numeral values of kinds of *dhirāᶜ*, see W. Hinz, "dhirāᶜ" in *EI²* (B. Lewis *et al.* 1965).

improved and more sophisticated version of the first. We can compare #9 and #4 in type F in a similar way. In other words, the treatise approximately follows an evolutionary model for describing the instruments. The instruments of type C form a related couple, in which one instrument is better for high elevations than the other.

An explicit distinction between al-ᶜUrḍī's instruments and the ones described here is that al-ᶜUrḍī mostly had made his instruments in teak brought from India,[80] while in the latter metals play a more important role.[81]

Type	*Functions involved*	*Nos.*
A (triangle-type instruments)	A special function described in the treatise	#1 & #2
B	Chord arm on circle	#3
C (wire instruments)	Sine, Chord and Tangent	#5 & #6
D (square-type instruments)	Tangent	#7 & #8
E	Sine and Versine	#1& #11
F Improved Ptolemy's Parallactic Instrument	Chord	#9 & #4
G	Pinhole Device	#12

Table 2: The types of the 12 instruments

The Instruments and similar examples from Europe

Some of the instruments bear a great similarity to instruments constructed in Europe in the following centuries.

The most notable similarity, clearly, is between our #7 and #8 and Georg von Peuerbach's *Quadratum Geometricum* (described, *e.g.*, in *Canones Gnomonis*, MS. Vienna No. 5292, fol. 86v–93r, printed in Nuremberg, 1516)[82] (See Figure 15).[83]

Instrument #12 is a pinhole image device, and the considerations applied in its construction are quite similar to those described at least two decades later by Levi ben Gerson (1288–1344 AD). As we will see in Section 2.12.3,

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[80] [Seemann 1929, p. 29].

[81] Iron and pure tin (*Arzīz*) were brought from Minor Asia (Waṣṣāf, *History*, 229). There is also a seventeenth- century treatise on the construction of the astrolabe stating that brass was brought from Hashtarkhān (today, Astrakhan) in the North of Azerbaijan. [al-Yazdī, fol. 13v].

[82] [Hellman and Swerdlow 1974, pp. 477–8].

[83] One referee assumed that instrument no. 7 is simply a quadrant (*rub'*), the models of which were usually built and widely used in the medieval period, and so doubted that its similarity with Peuerbach's Quadratum Geometricum was significant. It should be noted that instrument no. 7 is a square (*murabba'*) not a quadrant (*rub'*), simply because the quadrant is the instrument consisting of one-fourth of a *circle* while the instrument no. 7 has a *square* frame. The application of the two instruments is also different: in the quadrant, the operator directly reads the altitude off its graduated scale while in the square, the tangent of the angle is read. As we see in Section 2.7.3, squares were not widely used for *observational* purposes.

instrument #12 can be seen as a link between the ancient Dioptra and the instruments that were used in particular as Camera Obscura.

The basic knowledge needed to create both instruments was readily accessible to the astronomers of both cultures, so it is probable that the instruments developed independently. However, this period saw strong politico-cultural relations between Iran and Europe.[84]

2.1. Instrument #1

The applications of this instrument are: establishing the altitude and zenith distance of a [culminating] star, determination of the obliquity of the ecliptic, finding local latitude.

2.1.1. Configuration

The instrument (Figure 5) consists of:

Three rules (*misṭara*) of copper, iron or wood, namely *AC*, *AF* and *CF*, so that $\overline{CF} = \sqrt{2}\,\overline{AC} = \sqrt{2}\,\overline{AF}$. The rule *CF* is divided lengthwise into two halves by a line which is erased after construction of the instrument.

Alidade: Its length is identical to $\overline{CB}$ and its width and thickness are slightly less than the rules *AC* and *AF*.

There is a hole (*thuqba*) in point *B* (at the midpoint of the line that divides *CF* into two halves), in which the alidade is joined to the instrument by a ring (*ḥalqa*), washer (*fals*) and axis (*quṭb*). The alidade is bisected lengthwise like the rule *CF* by a virtual line (*khaṭṭ-i muntaṣif-i*^{c}*arḍ*). One half of its outermost 1/3 length, defined by this line, is removed.[85]

Two vanes of copper or brass or wood are placed as in an astrolabe, with two holes (sights). Note that the vanes are perpendicular on the alidade's outer surface.

[84] Cf. [Comes 2004]. On a possible connection of Peuerbach and Maragha astronomy not concerning instruments, see: [Dobrzycki and Kremer 1996].

[85] In other words, 1/6 of the alidade is removed, so that the centerline now forms a fiducial edge.

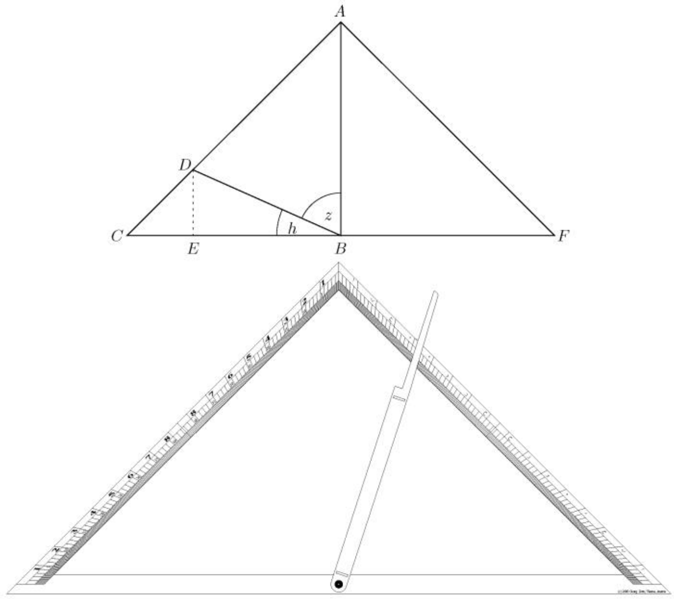


Figure 5: Instrument #1: (top) Mathematical principle. – (bottom) Instrument reconstruction.

Both legs AC and AF are bisected in the middle of their lengths so that the upper and lower parts are used in order to measure zenith distance z or altitude h of a given star respectively ($z = 0°$ in A and 45° in the midway of AC/AF, and $h = 0°$ in C/F and 45° in midway of AC/AF).

In addition, the legs are divided lengthwise into three bands, on which there are several types of graduations of the rules [counted from both ends]:

The **outer band** is divided into 1/12-parts of the length $\overline{BC}$ (in total, eight parts in each half of the rule, plus some excess).

The middle band is divided into 1/60-parts of the length $\overline{BC}$. (≈ 45° according to our author; the exact magnitude is $\text{Sin}\,45° = 42^{p}25'35''$, see below.)

The inner band is divided into the minutes of aforementioned parts, namely each part of the middle band is divided into 60 parts. In total, we have $2 \times (42 \times 60 + 25) = 2 \times 2545$ divisions on the inner band.

Finally, the instrument is fixed into a wooden framework and is placed on a wall parallel to the meridian line.

2.1.2. Application

The problem that this instrument is used to solve is the determination of an altitude *h* (or zenith distance *z*) from a value $x = \overline{CD}$, where $\overline{BC} = 60$. Our author describes several methods which are, in modern formula language:

Method 1

In the triangle CDE we have:

$$\frac{\text{Sin}\angle DCE}{\overline{DE}} = \frac{\text{Sin}45°}{\overline{DE}} = \frac{\text{Sin}90°}{\overline{CD}}$$

from which we can deduce $\overline{DE}$, and $\overline{CE} = \sqrt{\overline{CD}^2 - \overline{DE}^2}$. (However, it is obvious that $\overline{DE} = \overline{CE}$.) With $\overline{BC} = 60$, $\overline{BE} = \overline{BC} - \overline{CE}$, and $\overline{BD}^2 = \overline{DE}^2 + \overline{BE}^2$.

The altitude of a star, $h = \angle DBC$ is now known from

$$\text{Sin}\angle DBC = \frac{\overline{DE}}{\overline{BD}}$$

Also, $\angle BDC = 180° - (\angle DCB[= 45°] + \angle DBC)$. In ΔBCD,

$$\frac{\text{Sin}\angle CBD}{\overline{CD}} = \frac{\text{Sin}\angle BDC}{\overline{BC}}$$

which gives $\angle BDC$ as well.

Method 2

If we divide the alidade into 60 parts, we can read $\overline{BD}$ directly, so that if we know the zenith distance $z = \angle ABD$, we have in ΔCBD

$$\frac{Sin\angle DCB[= 45°]}{\overline{BD}} = \frac{Sin\angle BDC}{\overline{BC}}$$

Then, $\angle ADB = 180° - \angle BDC$, and therefore

$$\angle ABD = 180° - (\angle ADB + \angle BAD[= 45°]).$$

Our author says that this method does not give accurate results, which is clear given that $\overline{BD}$ can only be in the range $42^{p}25'$, …, 60^{p} and changes only slowly with the measured angle.

Method 3
We calculate the Sine of any angle between 0, …, 45° from the base up to half of $\overline{AC}/\overline{AF}$ (= 45°) in accordance with Method 1. In this case, by reading a distance such as $\overline{CD}$ on the rule CA, we can directly obtain the angle $h = \angle DBC$ by looking up in the table.

In modern formula language,

$$h(\overline{CD}) = \arcsin\left(\frac{\overline{CD}}{\sqrt{2}.\sqrt{\overline{CD}^2 + \overline{BC}^2 - \sqrt{2}.\overline{CD}.\overline{BC}}}\right)$$

The results of these calculations were apparently listed in the manuscript under the title "The parts of chords of one eight (1/8) of a circle". Our manuscript copies leave empty space for the table, of which we have calculated a short version (Tab. 3 (b)).

2.1.3. Comments
The author presents triangle-based instruments instead of other instruments that were more common in his time, such as quadrants and armillary spheres which are based on circular arcs as described, *e.g.*, by al-ᶜUrḍī, and declares that his instruments are superior in accuracy and usability. Now, can these claims be supported? Clearly, producing straight rules and graduating them uniformly is much easier than producing accurate circles. On the other hand, to use them for angle measurements, a lookup table for either chord values or a specially derived length (Table 3(b)) was required. Another option would have been to precompute the lengths $\overline{CD}$ for all angles and graduate the scale in unequal units according to the measured angles, as al-ᶜUrḍī described,[86] but apparently this was not done in our case.

Table 3(a) shows both the chord function (dashed) and the function $\overline{CD}(h)$ (solid curve). Obviously, they are equal at $h = 0°$ and $h = 90°$. Note also the simple forward computation:

$$\overline{CD}(h) = \frac{\overline{BC}.\sin(h)}{\sin(135° - h)}$$

The chord function is a very smooth curve, giving a good correlation between h and Crd(h). Our new function shows higher variability. Given the steeper ascension of the solid curve in the first 13 degrees, it can be argued that for small angles, reading a length $\overline{CD}$ on our triangular instrument allows

86 [Seemann 1929, p. 85].

a more accurate determination of these angles. For angles of 13º, …, 70º, a chord instrument appears to be more accurate. The graduation step width, for a "minute" graduation of 60′/p, where $\overline{BC} = 60^p$, results in an angular step width of 40.5″ at $h = 0$, and 1′ 21″ at 45º. For the chord function, the step width per "minute" of graduation is 57″ at 0º, and 1′ 21″ at 90º. The instrument graduation is counted from both outer edges towards $h = z = 45º$, so the more accurate regions are around $h = 0, …, 13º$ and $z = 0, …, 13º$ or $h = 77, …, 90º$. However, these differences are small, and for practical considerations, accurate partition of the scales, the absolute size of the instruments, and issues of flexion of the long rules involved were almost certainly more important than these differences in the curves.

2.2. Instrument #2

The second instrument adds the capability of measuring altitude in a (presumably) preselected azimuth to instrument #1.

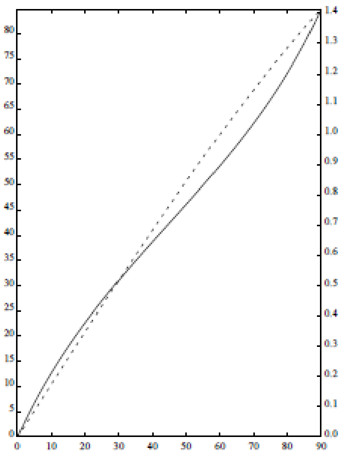
2.2.1. Configuration

A. Base of the instrument

Iron Shaft: A round cylindrical shaft (*miqyās*, scale) of iron with dimensions 3 fg (diameter) × 1 cb (height) is placed into a cavity with dimensions 3 fg (diameter) × 1/2 cb (height) and is strengthened here by tin and lead. It is clear that half of the shaft rises up from the ground.

Diagonal Rules: Two intersecting long rules called *diagonal rules* (*misṭarah-i quṭr*), with dimensions 15 cb (length) × 4 fg (width) × 2 fg (thickness) if made from copper, or 15 cb (length) × 4 fg (width) × 1/4 cb (thickness) if made from teak wood, perpendicular to each other in their mid-lengths, where there is a hole 3 fg in diameter so that the iron shaft passes through it. If copper rules are used, two other wooden rules with the same length and width, but 4 fg thick, are affixed on the copper diagonal rules by nails (*mismār*). [This clearly promotes the stability of the instrument and decreases its wear.] The diagonal rules that are placed along the meridian and east-west lines are kept immobile by stones and plaster.

Side Rules: Four other rules, called *Side Rules* (*misṭarah-i ḍilʿ*), with dimensions $10\frac{2}{3}$ cb (length) × 4 fg (width) × 2 fg or 1/4 cb (thickness) join at the ends of the diagonal rules to make a square, in which there are four right isosceles triangles where the half of each diagonal rules form their legs ($\sqrt{7.5^2 + 7.5^2} \cong 10\frac{2}{3}$). The height of the side rules is 1/2 cb, kept in place by stones and plaster.



(a)

Table 3: (a) Length of triangle segment *CD* vs. angle *h* (solid), and Crd(*h*) (dashed).

$\overline{CD}$	h	$\overline{CD}$	h	$\overline{CD}$	h
01	00° 40′ 59.″73	15	12° 07′ 10.″16	29	27° 26′ 21.″56
02	01° 22′ 58.″10	16	13° 04′ 55.″91	30	28° 40′ 30.″18
03	02° 05′ 56.″46	17	14° 03′ 55.″74	31	29° 55′ 35.″70
04	02° 49′ 56.″15	18	15° 04′ 9.″91	32	31° 11′ 34.″87
05	03° 34′ 58.″47	19	16° 05′ 38.″49	33	32° 28′ 24.″10
06	04° 21′ 4.″74	20	17° 08′ 21.″38	34	33° 45′ 59.″54
07	05° 08′ 16.″20	21	18° 12′ 18.″26	35	35° 04′ 17.″09
08	05° 56′ 34.″10	22	19° 17′ 28.″60	36	36° 23′ 12.″40
09	06° 45′ 59.″61	23	20° 23′ 51.″63	37	37° 42′ 40.″86
10	07° 36′ 33.″85	24	21° 31′ 26.″37	38	39° 02′ 37.″68
11	08° 28′ 17.″88	25	22° 40′ 11.″53	39	40° 22′ 57.″88
12	09° 21′ 12.″69	26	23° 50′ 5.″59	40	41° 43′ 36.″36
13	10° 15′ 19.″17	27	25° 01′ 6.″73	41	43° 04′ 27.″84
14	11° 10′ 38.″10	28	26° 13′ 12.″86	42	44° 25′ 27.″00

(b)

Table 3: (b) Angle values *h* for a measured value $\overline{CD}$ (for $\overline{BC} = 60$).

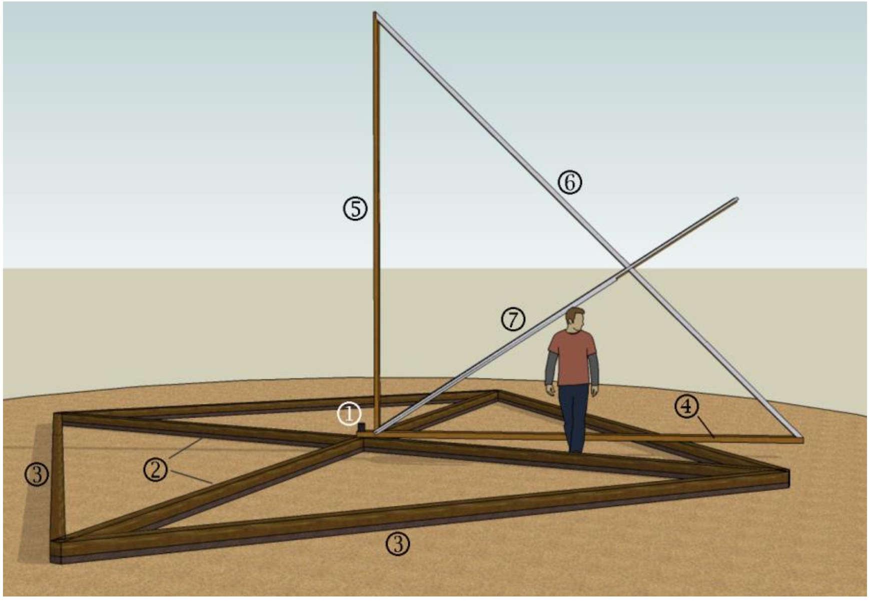


Figure 6: (a) Virtual reconstruction of Instrument #2: (1) Iron Shaft (2) Diagonal Rules (3) Side Rule (4) Azimuth Rule (5) Perpendicular Rule (6) Chord Rule (7) Alidade.

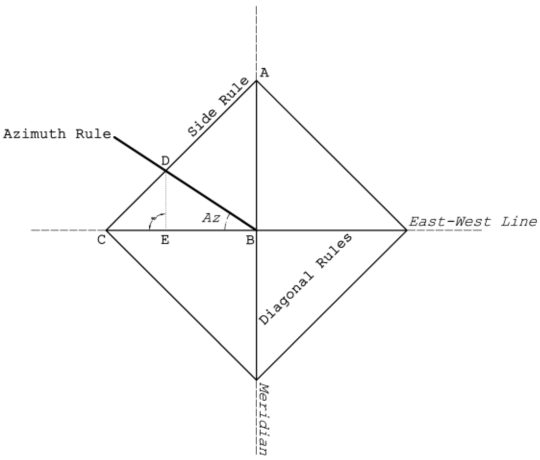


Figure 6: (b) Finding the Azimuth.

Second configuration: For simplicity, our author suggests removing the diagonal rules. In this case, instead of the round cylindrical iron shaft, a round wooden cylinder with 1/2 cb in diameter is first placed into a cavity. In a second step, a round cylindrical iron shaft with 2 fg in diameter is placed in its center (it is unclear why the diameter of the iron shaft has been changed). It is clear that the side rules are coplanar to the top surface of the wooden cylinder.

B. Upper Part

There are four other rules of copper or brass, named *Azimuth Rule*, *Perpendicular Rule*, *Chord Rule* and the *Alidade Rule*; the first three make a right isosceles triangle which is perpendicular to the horizon.

Azimuth Rule: with dimensions ≈ 8 cb (half the diameter of the abovementioned square, or 7.5 cb=*R*, + 0.5 cb) × 2.5 fg (width) × 2.5 fg (thickness) which has a hole to place it on the round cylindrical iron shaft at one of its ends. The distance between this hole and the joint of the azimuth and perpendicular rules is 1/4 cb.

Perpendicular Rule: with the same length *R*, but narrower and thinner.

Chord Rule: with length of $\sqrt{2}$ × height of the perpendicular *R* or of azimuth *R*, or almost 11 cb.

Alidade Rule: length and width the same as for the perpendicular rule and 1 fg thick. As we have seen in instrument #1, one third of the alidade height in respect to the bisecting line of its width is removed [in other words, one sixth of the alidade] to make a fiducial edge in order to improve the reading of degrees.

2.2.2. Graduation and application

Each of the side and Chord Rules are divided like the triangle sides of Instrument #1 (namely 2545 parts from each end to the midway of the Chord Rule).

To determine the altitude of a celestial body, the perpendicular triangle is moved and placed in the plane of a Great Circle passing through the zenith, the nadir and the celestial body. Then, the alidade is moved so that the light of the observed celestial body shines through its sights. To calculate the celestial object's altitude *h* or zenith distance *z*, the methods of instrument #1 are applied (In order to understand the procedure for finding the angle of azimuth by this instrument, see Figure 6b and compare it with Figure 5).

2.2.3. Comments

Our author gives definite sizes and dimensions for most of the rules, and instructions regarding the materials to use for the construction. However, the

2.5 fg wide Azimuth Rule will only fit the second configuration with its thinner shaft. Our reconstruction (Figure 6a) presents the first configuration with an Azimuth Rule of 4 fg in width.

We were doubtful of the usability of this huge instrument. If made of copper, the mass of a 4 fg × 2.5 fg × 8 cb bar of solid copper is about 364 kg (with 2.5 fg width: 227 kg). The perpendicular rule (no thickness given explicitly) of (assumed) 2fg × 2 fg × 7.5 cb adds 137 kg, and the Chord Rule of 2fg × 2 fg × 7.5 $\sqrt{2}$ cb another 193 kg. An alidade of only 1 fg × 1 fg × 7.5 cb, with 1/6 of it removed at the end to provide a fiducial edge, would add another 28 kg, but should be even heavier if the dimensions of the rules are followed strictly. In total, this gives a moving mass of more than 700 kg (or with the azimuth rule width of 2.5 fg: almost 600 kg)! It seems clear that this instrument could only be used to measure the altitude of a celestial object in a preselected azimuth, like the meridian or east-west vertical, set by several strong assistants. For high altitudes, a ladder was clearly required to read the scale value.

2.3. Instrument #3

This instrument can measure both azimuth and altitude. Contradicting the straight-rule concept, it uses a graduated ring for reading azimuths.

2.3.1. Configuration

Great Circle: A copper and brass circle with an arbitrary (but as large as possible) diameter (radius *r*), 3 fg in width and 2 fg in thickness, is installed 0.5 cb [sic! see below] above the ground, and secured by stones and plaster. Quadrant lines are drawn on the outer surface (*maḥdab*) of the circle along the meridian and east–west lines. Each quadrant is divided into degrees and minutes, namely 90 × 60 = 5400 min (Figure 7a).

Great Cylinder: In the circle's center there is a pit of 1/4 cb in diameter, where a round wooden cylinder (named "great cylinder", *usṭuwānih-i kabīr*) is placed. In the center of the upper surface of the wooden cylinder, there is a cavity with dimensions 2 fg (diameter) × 1/4 cb (depth) for a round cylindrical iron shaft [similar to the second configuration in instrument #2]. The upper surface of the wooden cylinder is covered by an iron plate to reduce wear, kept in place with nails. It is clear that both the upper surfaces of the iron plate and of the Great Circle are coplanar.

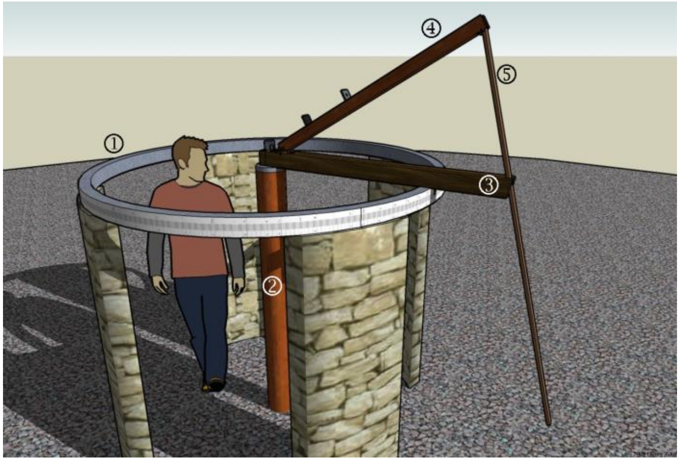


Figure 7: (a) Virtual reconstruction of Instrument #3: (1) Great Circle (2) Great Cylinder (3) Azimuth Rule (4) Alidade (5) Chord Rule. –

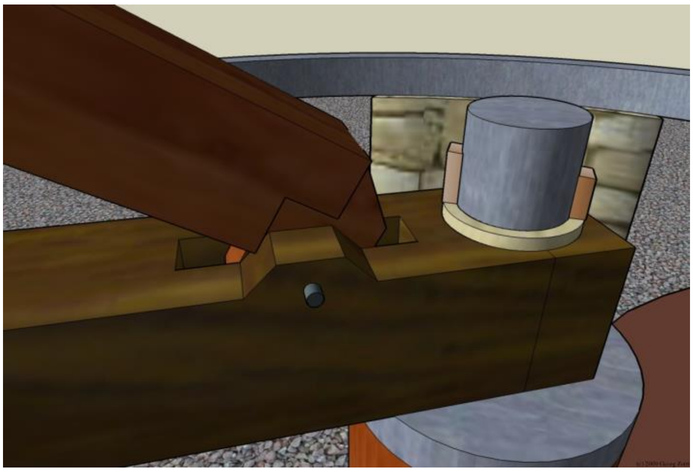


Figure 7: (b) The connection of the Azimuth Rule to the Great Cylinder and the connection of the Alidade to the Azimuth Rule.

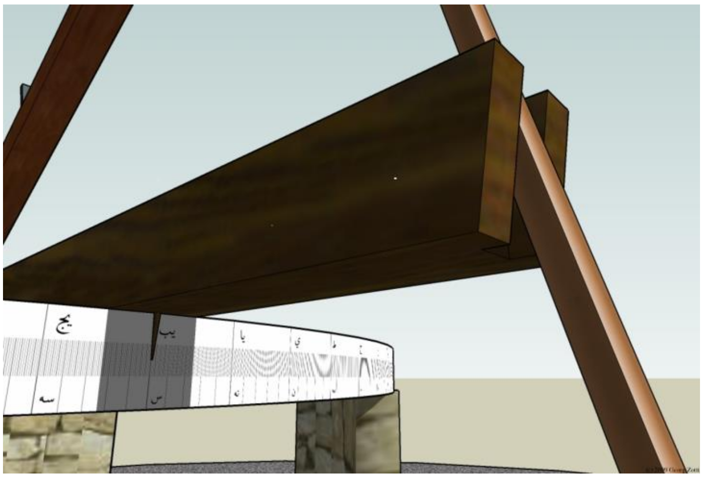


Figure 7: (c) Sharp Nail attached to the lower surface of the Azimuth Rule, indicating the angle of azimuth.

Figure 7: (d) Alidade and its sights.

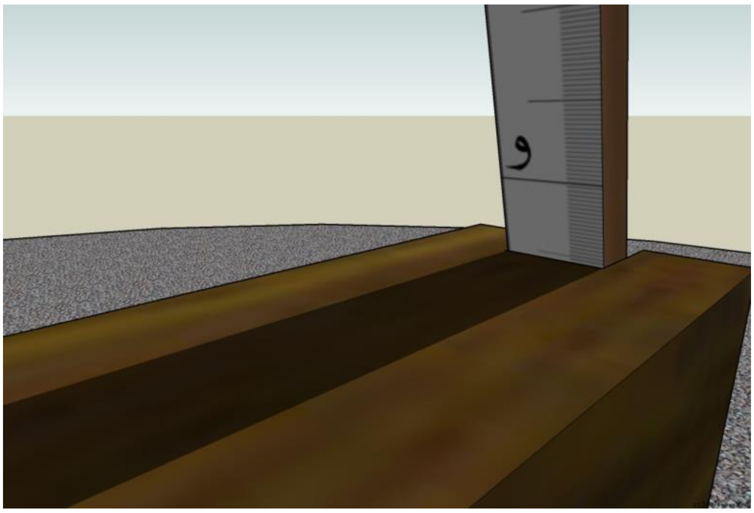


Figure 7: (e) Chord Rule in the split of the Azimuth Rule.

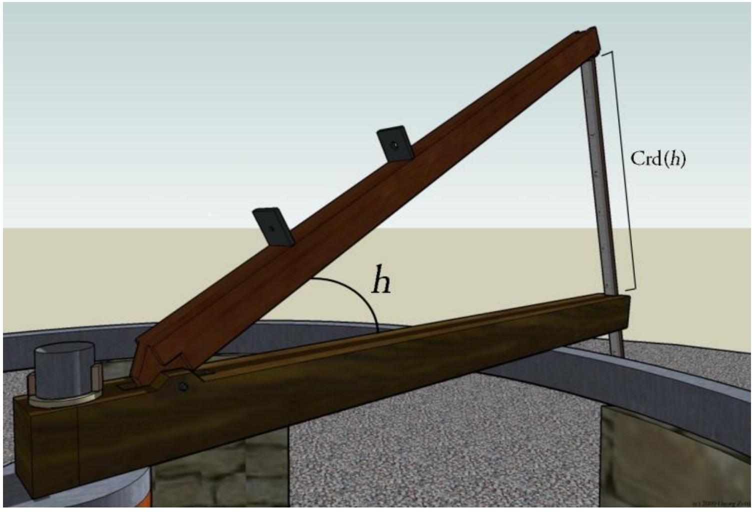


Figure 7: (f) The Measurement of Altitude.

Rules:

Azimuth Rule: A rule of teak wood or copper, parallel to the horizon, with dimensions "larger than the radius of the copper/brass ring" (length *l*) × 3 fg (depth) × 3 fg (thickness), with a hole (2 fg) on the inner extremity so that the iron shaft passes into it. It is secured here by a wedge (*faras*) and washer/ring (*ḥalqah*) as in an astrolabe (Figure 7b). In the outer extremity, at a distance approximately 0.5 cb from its outer edge, there is a pointed nail (*mismār-i muḥaddadah*) sliding along the outer surface of the Great Circle to count the degrees shown in it (Figure 7c). On the Azimuth Rule, at a distance of 1 sh from the central hole, there is a rectangular cavity where the Alidade Rule is attached [in #2, this distance was 1/4 cb] here by a nail through the sides of the square cavity.

Alidade Rule: It is 1 sh shorter than the Azimuth Rule. We make its inner extremity with a lengthy protrusion to fit inside the square cavity of the Azimuth Rule (attached by a nail) so that the alidade can freely move along the Azimuth Rule. It is clear that the distances between the outer extremities of both the rules from the joint place are equal. On the Alidade Rule there are two sights (*hadafa*) which are placed at least 1 hd from each other (Figure 7d).

In the outer extremities of both the rules are rectangular splits for joining the Chord Rule.

Chord Rule with length $\sqrt{2}$ times the distance between the joint place of the Azimuth and Alidade Rules and their outer extremities, namely $\sqrt{2}(l - 1\,\mathrm{sh}) \times 1\,\mathrm{fg}$ (width) × 1/2 fg (depth). It is jointed with the Alidade Rule in the split of its outer extremity by a nail and can freely move in the plane of the Alidade and Azimuth Rules. Its outer mobile extremity is placed at the split of the Azimuth Rule (Figure 7e).

2.3.2. Graduations and Applications

"The Chord Rule is divided into the same parts as the Azimuth Rule." Of course, it is clear that the Azimuth Rule is never divided (or graduated). Our author's intention is to instruct that if the Azimuth Rule is divided into 60 equal parts, the length of each part will be used as a unit for graduating the Chord Rule. As we have seen above, the length of the Chord Rule was 85/60 $\approx \sqrt{2}$ times that of the distance between the outer extremities of the two other rules from the square cavity on the Azimuth Rule.

To observe, the alidade is moved so that a star's light is seen through both the sights. As far as we have seen, the outer extremity of the Chord Rule allows that it moves freely in the split of the Azimuth Rule. The distance on the Chord Rule between the outer ends of the two other rules is Crd h (Figure 7f). The azimuth is read directly off from the graduated limb of the Great Circle.

2.3.3. Comments

Despite the original attempt to present only instruments built from straight bars, one crucial component of this instrument is the azimuth ring.

The elevation part of this instrument is in principle a version of Ptolemy's parallax instrument tilted 90°. A crucial dimension, the circle diameter, is not given, but it must be large. The Azimuth Rule is about 1/2 cb longer than the circle radius. However, if built with the height as given in the manuscript, it would be strictly limited to the observation of altitudes close to the zenith: for lower altitudes, the Chord Rule will slide along the rectangular slit downwards and will finally hit the ground (Ex.: for an assumed rule length of 2 cb and 0.5 cb height of the Great Circle, the maximum zenith angle is less than 25°.)

If we assume a copying error or omission, namely *nīm* (0.5) instead of *duw wa nīm* (2.5), and accept a height of the ring of 2.5 cb instead of 0.5 cb, we would find a very usable instrument (Figure 7a). The height of the ring is now only slightly less than shoulder height. However, to provide a Chord Rule which would not collide with the ground for low altitudes, this height (plus the Azimuth Rule thickness) must be at least equal to the chord length. This limits the Azimuth Rule length to about 2 cb and the ring diameter to 3 cb, as shown in our reconstruction. On the usability side, reading the scale value at the split where the Chord Rule slides along the Azimuth Rule is straighforward and does not require a ladder. Any larger instrument would require a higher elevation of the azimuth ring, a step for the observer, or possibly a ditch for the Chord Rule, or would be limited to the zenith area.

The small size shown in the reconstruction suggests that the instrument could be used by a single scholar, aided by an assistant standing outside the ring and reading the values from the scales. Made of copper, however, the alidade mass for 3 fg × 3 fg × (2 cb – 1 sh) would be almost 70 kg, again requiring at least another strong assistant; using teak, the estimated mass would be around 1/10 of this figure.

2.4. Instrument #4

2.4.1. Configuration (Figure 8a)

Meridian and East-West Rule: Two rules made of teak wood with arbitrary (but as long as possible) length *l* and 4 fg wide and 1 hd thick are connected perpendicularly in their midways. Their width bisector lines are placed along the meridian and east-west lines. There is a hole 3 fg in diameter at their intersection.

Shaft: A round iron shaft 4 cb in length × 3 fg in diameter is placed in the hole perpendicular to the horizon and is strengthened here by tin and lead.

Half-Diameter Rule and Alidade: Two equal rules of brass or copper with dimensions 1/2 *l* (= *h*) × 2 fg (width) × 2 fg (thickness) are linked at one of their ends (upper end) like a compass in order to be mobile. The back of one of them has rings of a diameter slightly larger than the iron shaft which is placed in them. This rule is called *half diameter rule* (*niṣf-i quṭr*) to represent half of the [celestial sphere's] diameter [in the text literally "Meridian's diameter" (*niṣf al-nahār quṭr*)]. The other rule is the *alidade* with two sights on its upper surface.

Chord Rule: At the lower end of the half diameter rule, there is a cavity where one of the ends of a third rule is linked, namely the *Chord Rule*, with an iron nail. In addition, there is a split in the lower end of the alidade for the Chord Rule. It is clear that the Chord rule's length is approx. $\sqrt{2}$, or 85/60 times, greater than the half diameter rule/alidade, and it is accordingly divided into the parts of the chord of a quadrant, or 85^p.

Azimuth Rules: Two rules of teak wood, equal to and graduated like the Chord Rule, are placed on the upper surfaces of the meridian and east-west rules, at equal distances from the instrument's center, one of them in north-eastern direction and the other in south-western direction, and pivoting on the east-west rule, one on the east end and another on the west end. Therefore, the Azimuth Rules have one fixed head and one movable head. [Note that they can move freely, so the movable head of each can easily meet the two ends of the meridian rule. Therefore, only one Azimuth Rule is needed to measure the azimuth angle in each half of the horizon: one for the eastern half and the other for the western half.]

Supportive Rules: Four rules with 1 cb in length are joined with the meridian and east-west rules to make a square that supports the Chord Rule.

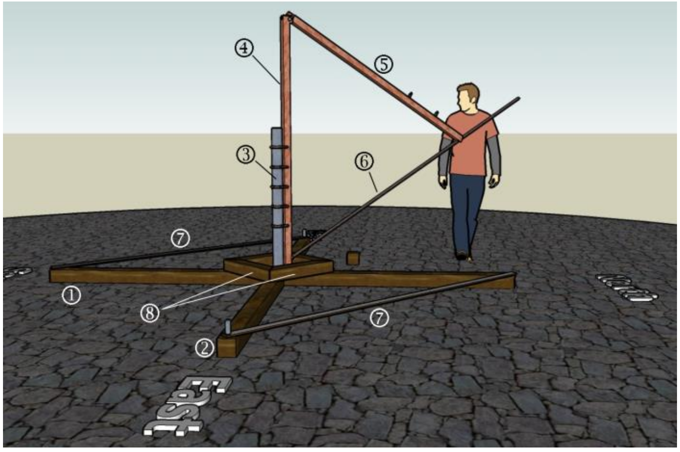


Figure 8: (a) Virtual reconstruction of Instrument #4: (1) Meridian Rule (2) East–West Rule (3) Iron Shaft (4) Half-Diameter Rule (5) Alidade (6) Chord Rule (7) Azimuth Rules (8) Supportive Rules.

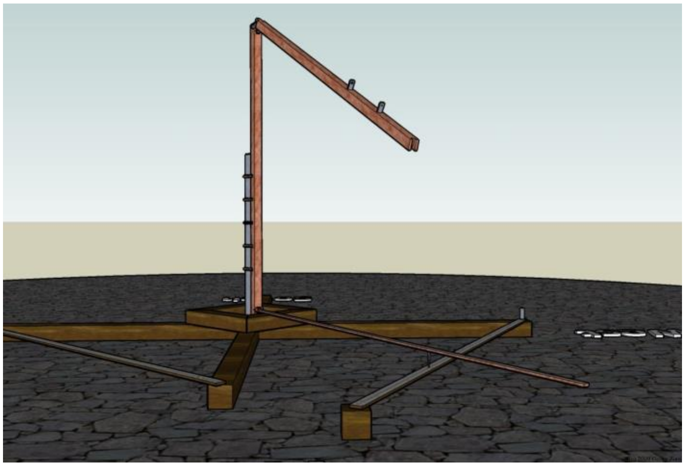


Figure 8: (b) Measuring azimuth (lateral view); the alidade is still shown in an elevated position, but would usually hang down vertically. The wooden block supporting the Azimuth Rule (with the chord rule indicating azimuth by a nail) is not documented, but is a functional requirement.

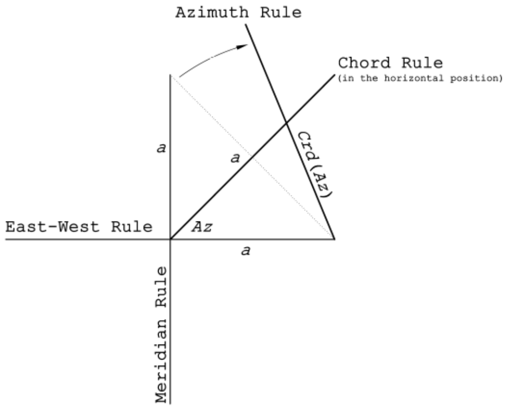


Figure 8: (c) Measuring the azimuth (top view).

2.4.2. Graduation and Applications

The half diameter rule is rotated and the alidade moved until the light from the celestial object shines through the alidade's sights. Then the Chord Rule is placed in the split at the lower end of the alidade. Finally, the part of the Chord Rule between these two rules is the chord of the zenith distance of that celestial object (Figure 8a).

In the following step, (without rotating the half-diameter rule) the Chord Rule is placed parallel to the horizon on one of the Azimuth Rules (Figure 8b, c).

There is a mark/indicator (*miqyās*, lit. "scale") in the lower surface of the Chord Rule for counting azimuth parts engraved on the Azimuth Rule. The height of the indicator is 3^p of the 60^p of the half diameter rule, so that the Chord Rule placed parallel to the horizon is higher than the Azimuth Rule by $(3/60) \times h = (3/60) \times (1/2)\ l$.

The Azimuth Rule is moved until its free end is placed tangentially to the nail pointing downward close to the Chord Rule's 60-part mark. [This mark is not necessarily exactly at the 60-part mark, but at a length equal to these 60 parts measured from the central axis.] On the Azimuth Rule, the mark shows the chord of the celestial object's azimuth (Figure 8c).

2.4.3. Comments

Again, the author gives the instruction to select the base length "as long as possible" and then gives the dimensions for the thickness of the rules, from which we must estimate usable sizes.

Although our author says nothing about the dimensions of the Chord Rule, its width and thickness are clearly less than the two abovementioned rules.

The Alidade Rule has to be lifted high for low altitudes. Its length can only be estimated. The mass of a copper bar of 2 fg × 2 fg as described is about 18 kg per cubit of length. It seems obvious, from the point of view of usability, that the alidade length cannot have been much more than 3 cb; this seems reasonable for use by a standing observer, supported by one strong assistant. A longer alidade would require a ladder for the observer at low altitudes, and probably several assistants to lift it. Of course, if built from teak wood, much longer rules could be handled.

The exact placement of the Supportive Rules is not given, but their role, as our author clearly noted, is to act as support for the Chord Rule when it is laid in its horizontal position, circulating in a surface parallel to the horizon, as shown in Figure 8a; thus they would have been placed on top of the base cross. Some support for the free end of the Azimuth Rule is also required, as shown.

Given the potentially greater base length, this instrument can provide the altitude and azimuth of a celestial object with higher accuracy than instrument #3, although the computation of the azimuth will require some effort.

2.5. Instrument #5

Although not explicitly mentioned, instruments #5 and #6 form a pair, which can be used for different altitude ranges. The previous instruments used long pieces (*misṭarah*) of wood and brass or copper, but instruments #5 and #6 used a *Khayṭ*, a catgut or copper wire, for measuring.

2.5.1. Configuration (Figures 10 and 11)

Pillar: A cylindrical wall or a wall with a rectangular base, made of sun-dried bricks, with arbitrary (but substantial) height *H*.

Cone: On top of it, a cone is erected and covered by iron or copper until it is round. Its height is one sh.

Top Ring: On top of the cone, there is a small ring of iron, whose area is slightly smaller than that of a silver coin (*diram*).

Shaft: On top of the cone, there is a shaft (*miqyās*) [probably of iron]. The shaft is *kawkabah*-shaped,[87] so the Top Ring cannot fall off the Shaft. The Top Ring can move around the Shaft.

First Khayṭ: A thin but long *khayṭ* (wire) of copper or catgut (*rūdkish*) attached to the Top Ring.

Indian Circle: We draw an Indian circle (*dā'irih-i hindī*) on the ground, and we deduce the meridian line, one of the diameters of the Indian circle. (The other diameter is of course the east-west line, but our author does not say whether the east-west line is drawn within the Indian Circle). The diameter of this circle is arbitrary but as large as possible.[88] The Indian circle is made of stones and plaster, and it is completely circular.

The Pillar is erected whether it is in the center of the Indian circle or not. Nevertheless, the line bisector of [the width of] the Pillar should be perpendicular to the meridian.[89]

At the base of the Pillar, under the ground, there is a hidden thickness (*thakhn-i nahānī*) of firm wood or iron.[90]

Base Ring: Around the base of the Pillar there is a circular groove of 1/2 fg (depth) × 1/2 fg (width), into which a circle of iron is fixed so that it can freely move around the Pillar.[91]

Second Khayṭ: A thin but long *khayṭ* attached to the Base Ring which is called the *Shadow khayṭ* (*khayṭ-i ẓill*).[92]

Third Khayṭ: Another long *khayṭ* is placed on the meridian line and near the base of the Pillar and kept in place here with a nail and a ring. This *khayṭ* represents the meridian line.[93]

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[87] The Arabic term *kawkab* means star, but here apparently the Persian *kawkabah* is meant, a staff with an incurved head. We can assume this form here. Originally, it was also a representative symbol of a staff with incurved head with an iron ball hanging from it, carried ahead of the king.

[88] In our reconstruction, the circle encloses the pillar at the distance of the pillar's height.

[89] This instruction is surprising, because the shaft around which the Top Ring rotates appears to be in the central axis of the Indian Circle. Maybe our author means that with this instrument, it is only necessary that the central Pillar is placed on the meridian line.

[90] Its purpose is not given, but it was probably to increase the stability of the instrument. See comments.

[91] It is not completely clear whether the circle is attached to the pillar as shown, or in some undescribed way to the ground (maybe to the "hidden thickness" just mentioned), where it could also encircle a non-circular pillar. The instrument's principle would not change if this circle were fixed to the ground; a rectangular pillar could be used with such an arrangement.

[92] *ẓill* can mean either *shadow* or *tangent*. In this case, *shadow* appears more appropriate (see comments).

[93] This description may be misleading. See comments.

Alidade: This is 1/4 cb long and has two vanes. On the end nearer the apex of the Pillar, there is a hole to join it to the first *khayṭ*. On the back of the alidade, corresponding to the eye sight, there is a ring through which a fourth *khayṭ* passes, the *Alidade khayṭ*, which has a conical plummet at its other end.

Graduated Rule: A rule made of wood: if we assume the height of the Pillar is *H*, this rule can be *H*/2, *H*/3, or *H*/10. In the first case, it is divided into 30 parts, in the second case, it is divided into 20 parts, and in the third case, it is divided into six.

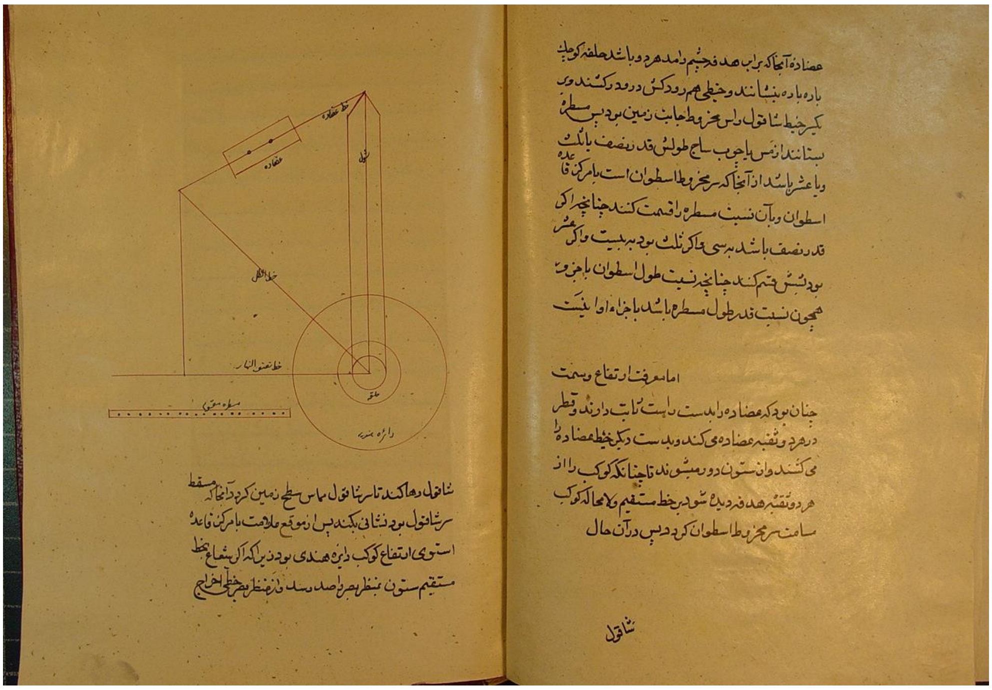


Figure 9: The instrument #5 as illustrated in MS. S. Note that this instrument cannot actually be built following this schematic configuration. The fiducial triangle's vertical edge must be carried along the pillar wall.

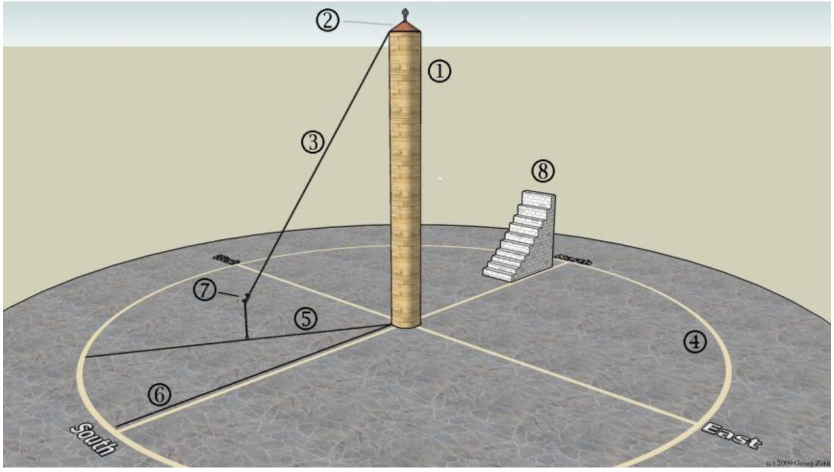


Figure 10: Instrument #5: (1) Pillar (2) Cone (3) First *Khayṭ* (4) Indian Circle (5) Second (Shadow) *Khayṭ* (6) Third *Khayṭ* (7) Alidade (8) Steps. A fiducial triangle is formed by the point where the First *Khayṭ* bends around the top cone, the ring on the Base Ring where Second and Third *Khayṭ* are attached, and the alidade endpoint, where the plumb line goes down to the ground.

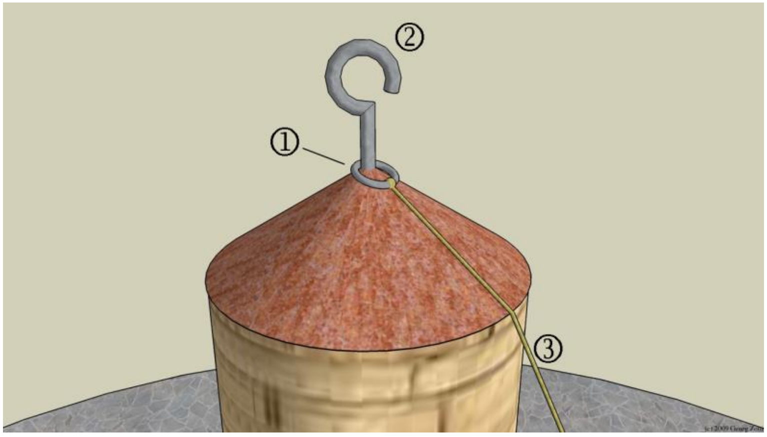
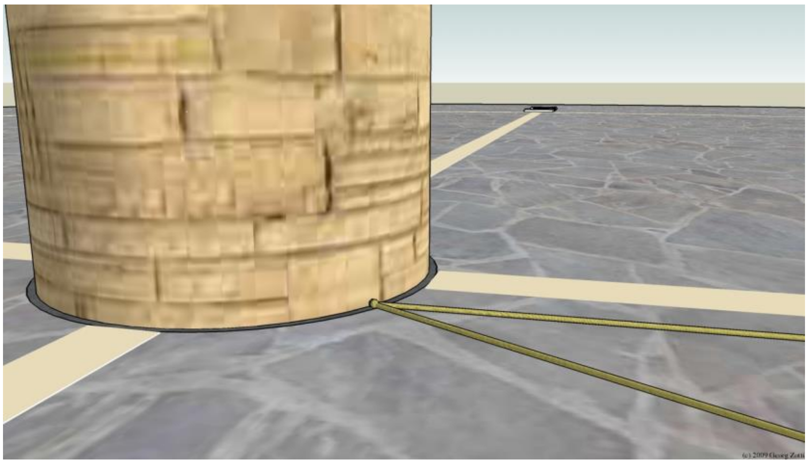
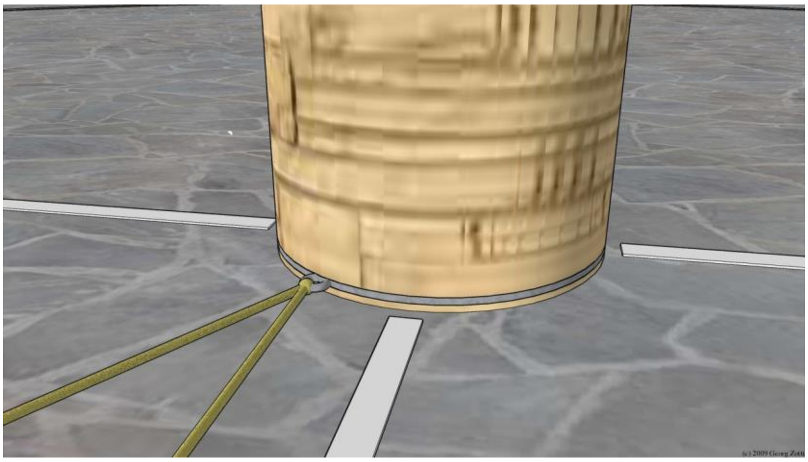


Figure 11: Instrument #5. (a) Top of the pillar, showing (1) Top Ring, (2) *Kawkabah*-shaped shaft, and (3) the *Khayṭ* bending at the roof corner.



(b)



(c)

Figures 11: (b and c) Base Ring, where on a small ring the two *Khayṭ*s for measurements are attached. The moving ring was either in the ground next to the pillar, or around its base.

2.5.2. Applications

First method

To measure the altitude, first we hold the alidade in the right hand, we then stretch the First *Khayṭ* with the other hand and move away from the pillar so that we can see a given star through the two sights. Then, we release the plummet until it touches the ground to leave a mark. Then the distance l_2 from the mark to the pillar's foot is the *umbra recta* (cotangent) of the altitude, h, of that given star. We place the Shadow *Khayṭ* (Second *Khayṭ*) on this distance and measure it with the Graduated Rule[94].

To measure the azimuth, we stretch the Meridian *Khayṭ* (Third *Khayṭ*) so that it coincides completely with the meridian line and never departs from it.[95] The angle between the Shadow *Khayṭ* and the Meridian *Khayṭ* is the co-azimuth ($Az' = 90° - Az$).[96] To measure it, we produce a line N perpendicular to the Meridian *Khayṭ* through the place marked by the plummet and measure it with the Graduated Rule. The result is the Sine of co-azimuth [counted from the meridian], but based on the length l_2 of the fiducial triangle (OP in Figure 12a). Thus,

$$\mathrm{Sin}\,(Az') = 60\left(\frac{N}{l_2}\right)$$

To test the accuracy of this procedure (Figure 12a), we stretch the Shadow *Khayṭ* to the plummet mark P, and then we assume an arbitrary point S on the Shadow *Khayṭ*. Then we stretch the Meridian *Khayṭ* on [parallel to] the meridian and we produce a perpendicular line from the arbitrary point [S] to the Meridian *Khayṭ*. Therefore, we have a triangle made of three sides: as the first side a part m of the Meridian *Khayṭ*, as the second side the perpendicular line n, and as the third side the part s of the Shadow *Khayṭ*. We measure the second and third sides with the Graduated Rule [and validate:]

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[94] This is not exact, because one has to take the correction due to the height p of the observer's eye into account. The correct relation must be expressed as below (see Figure 12b): $\cot(h)=l_2/(H-p)$. Our author neglects this correction here, but considers a similar one in the measurement of azimuth (below).

[95] In our reconstruction, we understand this instruction to create a true meridian line for the current azimuth setting, *i.e.*, attaching the Meridian *Khayṭ* from the same small ring (*zirih*) on the Base Ring where the Shadow *Khayṭ* is attached outwards parallel to the meridian line drawn on the ground in the instrument's centerline. The literal description in the text, where the Meridian *Khayṭ* is directly attached to the Pillar, presents obvious problems.

[96] Remember that azimuths are counted from east/west in this treatise. In modern use, this angle is the azimuth.

$$\mathrm{Sin}\,(Az') = 60\left(\frac{n}{s}\right) \rightarrow \frac{n}{s} = \frac{N}{l_2}$$

To test the altitude, if

Umbra recta of the altitude (= length along the Shadow *Khayṭ* l_2)

$= \sqrt{(\text{part of Meridian Khayt } M)^2 + (\text{perpendicular line } N)^2}$

then the altitude is correct.[97]

Second Method

If the length of the First Khayṭ is equal to the height of the pillar [$L = H$], then the Shadow Khayṭ will be the chord line. [In this case, the plumb is not used.]

Ladder/Steps: At the end of this section, our author adds that we make a ladder of stone and plaster from the base of the pillar until the altitude of a star with any zenith distance (even for stars close to the horizon) is known by it.[98]

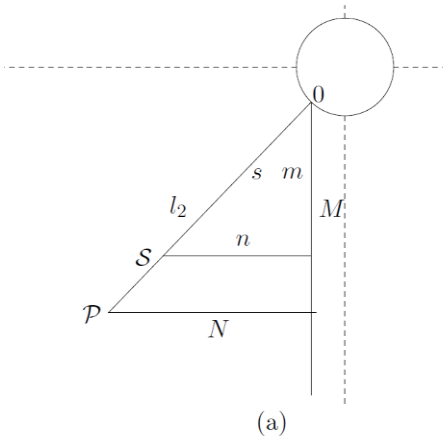
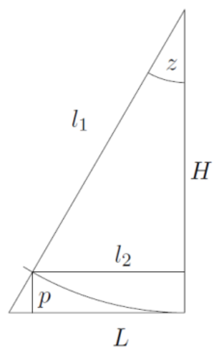


Figure 12: Using instrument #5: (a) Top View: Testing accuracy with similar triangles.

[97] Such a validation using similar triangles appears to make sense only if the original distances are longer than the longest available measuring rule.

[98] According to our understanding, the instrument has a lower limit for altitude: see comments.



(b)

Figure 12: (b) Side View: Angles and lengths for sines and tangents.

2.5.3. Comments

The description of this instrument is very confusing and leaves many details open, and we had considerable trouble in developing an instrument that both fits the description and provides correct and usable measurement results. The sketch in the manuscript (Figure 9) shows the First *Khayṭ* attached to the top center of the pillar, and the Second *Khayṭ* to the base of the central axis, which is however obviously buried in the stones of the pillar, preventing a physical implementation of this idea. No physical (or proper virtual) reconstruction can follow this illustration, but we propose a usable, working instrument based on an interpretation that does not contradict the textual description except for one detail, the attachment point of the Meridian *Khayṭ* (Figure 10).

Our pillar shown here is 10 cb high and 1 cb in diameter, though neither dimension is explicitly given. The key idea of this instrument as we interpret it, although not described in the treatise and even contradicting the original sketch, appears to be that the fiducial triangle does *not* use the central axis of the pillar, but a vertical line that is formed by the point where the First *Khayṭ* bends around the corner of the flat conic roof on top of the pillar (Figure 11a), and the small ring (*zirih*) on the Base Ring, where the Shadow *Khayṭ*, and quite likely also the Meridian *Khayṭ*, are attached (Figure 11(b)(c)). The roof's tilting angle therefore also defines the minimum altitude of observable objects. The only dimensional instruction found in the treatise is a roof height of 1 sh, which would form the low angle shown with our (assumed) pillar diameter of 1 cb. If the pillar is made thinner, the roof would be steeper. As soon as the First *Khayṭ* does not bend around the edge of the cone but is directly stretched from the central shaft on top of the pillar (for our dimensions, these are altitudes below about 33.7°), any measurements would be erroneous. We cannot assume that an obviously erroneous instrument would have been used, so the flat cone appears to be a required component, and its diameter must be identical to the Base Ring's diameter.

The Meridian *Khayṭ* is described literally as being attached to a fixed point on the pillar's base on the meridian line. In this case it appears pointless to have another movable *khayṭ*, because the meridian line permanently drawn on the ground would fulfil the same purpose. Also, the measurements and calculation of azimuth would have to take the radius of the pillar (or, for a rectangular pillar, the radius of the Base Ring) into account. To be usable as simply as described, it appears likely that this *khayṭ* was also attached to the Base Ring on the same point as the Shadow *Khayṭ*, so that a "local coordinate system" is cleverly carried around the pillar.

The originally assumed name ("Tangent *Khayṭ*") of the Shadow *Khayṭ* and description of its usage are misleading. The term *ẓill* means both *shadow* and *tangent*, and we understand that *shadow* is the better translation here, probably given because this *Khayṭ* always follows the pillar's shadow on the ground as seen from the observed object. However, a true connection with the (co-)tangent function could also be found, if the length p of the Alidade *Khayṭ* (plummet line) is measured (see Figure 12b), as described in Section 2.5.2.

Nor is it clear whether the alidade was fixed on the lower end of a First *Khayṭ* of length $l_1 = H = 60^p$, or was sliding along the lower end of a *Khayṭ* of undefined length. In the first case, the assumed solution concerning the length of the Shadow *Khayṭ* ($l_2 = \text{Cos}(h) = \text{Sin}(z) = l_1 \cos(h) = l_1 \sin(z)$) is correct, but it was not possible to measure objects around the zenith, where the eye of the observer would have to be below ground. It may be significant here that our author uses only the cotangent function. If the alidade were sliding along the First *Khayṭ*, the abovementioned solution $l_2 = \text{Cot}(h) = \text{Tan}(z) = (H-p)\cot(h) = (H-p)\tan(z)$ (cf. n. 94) after measuring p could have been applied. Regarding the treatise, the latter solution is more acceptable.

The Base Ring is shown to run on the ground in a 1/2 fg deep groove just adjacent to the pillar (Fig. 11b). This arrangement also allows in principle the case of a rectangular pillar. However, in this case the ring must be fastened to the ground, or else it can easily be pulled up and out of its described 1/2 fg deep groove. (An iron ring of 1 cb diameter and 1/2 fg thickness weighs less than 2 kg.) The "hidden thickness" mentioned in the description just before the Base Ring might also indicate a buried structure holding the Base Ring, instead of giving additional support to the pillar. On a circular pillar, it appears possible that the ring was mounted on the base of the pillar itself (Fig. 11c), so it could be securely fixed in the vertical position of the ground plane. In any case, the conical roof's lower circular rim must be of a diameter identical to the Base Ring to provide a usable instrument with a vertical edge.

Regarding stairs or a ladder, it seems possible that simple stones or bricks were used as steps up to moderate heights, but any higher device could hardly be called practical for fast removal to another place. We show a staircase in the meridian only, where it appears to make the most sense. Its position and shape are governed by the need to look through the alidade and lower the plumb onto the ground. Of course, for lower altitudes (*i.e.*, when the First *Khayṭ* must be lifted higher), errors caused by flexion and slack of the First *Khayṭ* will increase. For altitudes lower than the angle of the top cone, the instrument was however unusable as shown above; for these cases, instrument #6, which we will now describe, was the natural complement.

2.6. Instrument #6

2.6.1. Configuration

Indian Circle: On an open field or on a wide, round and extensive ground, we select a platform which we make firm by stones and plaster. We carefully draw an Indian Circle, Meridian and East–West Lines.

Shaft: A relatively thin cylindrical shaft with a hemispherical head and a length of >1 fg. There is a small ring on the head of the shaft, which can rotate freely. We place the shaft perpendicular to the center of the Indian Circle.

Alidade: An alidade with length >1 sh with two sights on one of its surfaces. There is a small ring on the lower surface of the alidade in a point opposite to the observer's sight.

Wires: Two wires (*khayṭ*) made of copper or catgut (*rūdkish*), one from the small ring on the shaft to the small ring of the alidade, and another hanging from the small ring of the alidade onto the ground, which has a plummet on its lower end.

Ruler: If the length of the first *Khayṭ* (from shaft to alidade) is *L*, the length of the ruler is 1/2 *L* or 1/3 *L* or 1/10 *L*. We graduate it into parts so that the whole length of the first wire is assumed to be 60 parts (namely 30, 20 or six parts for the three instances, as in instrument #5).

Pulpit An auxiliary chair as an observing pulpit.

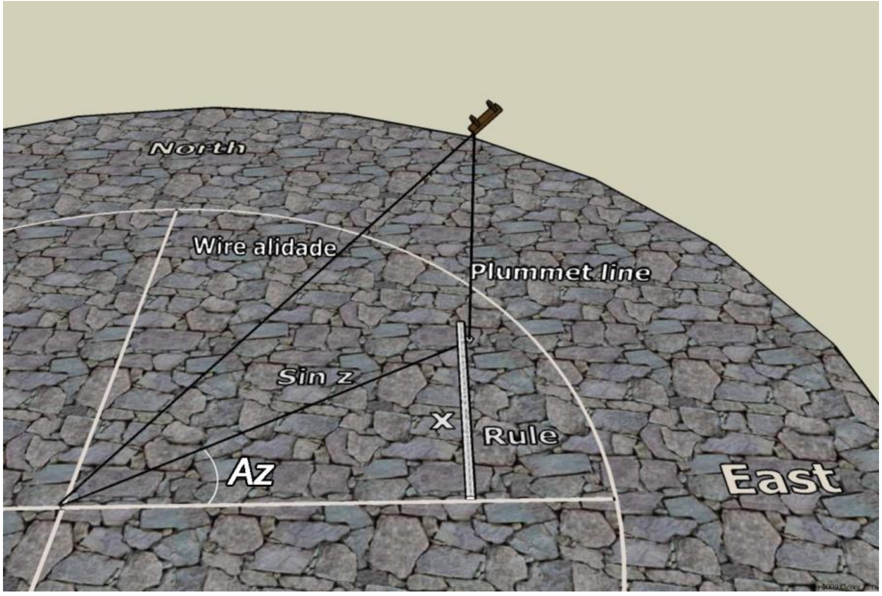


Figure 13: Virtual reconstruction of Instrument #6.

2.6.2. Application

To calculate the altitude of a given star, we raise the alidade with both hands so that we observe the star through both sights (Figure 13). At this moment, the first wire must be stretched fully. We release the plummet until it touches the ground. The distance from the small ring of the alidade to the plummet is the Sine of the star's altitude, which is measured with the ruler. If the star is very high on the horizon, we use the pulpit.

We mark the location of the plummet on the ground. Then, we place the first wire on the East–West Line so that it is stretched fully as a straight line. Then, we draw a perpendicular line from the mark of the plummet on the ground to the East–West Line. We measure it with the ruler (x, the Sine of Azimuth in parts of the Sine of co-altitude, *i.e.*, zenith distance z). Therefore, to calculate the azimuth (Az), we have:

$$\frac{x}{\mathrm{Sin}\, z} = \frac{\mathrm{Sin}\, Az}{\mathrm{Sin}\, 90°}$$

where Az is measured from East or West respectively.

2.6.3. Comments

The instrument is remarkably simple. However, its application may be error-prone: the alidade must be parallel to, and extend, the wire. With a small ring where the wire is attached to the alidade, there is the danger that the user will not align the alidade exactly with the wire, but will tilt it slightly, so the altitude measured will be wrong. Also, long wires may stretch and cause problems because of their slack.

Note that Al-ᶜUrḍī explicitly rejected the use of threads or ropes at least to measure the chord length of a parallactic rule, as their length is non-constant and depends on their tension.[99]

2.7. Instrument #7

2.7.1. Configuration (Figure 14)

Square: A square made of four rules that have equal and parallel sides. On two ends of the upper side, at the corners [between the upper rule and two perpendicular sides], there are two circular holes.

Each side is divided into 60 parts, and each part is also divided into minutes and seconds. [Although this is written in the text, it is clear that there is no need for the upper side to be graduated. In the figure available in the treatise, only two perpendicular sides have been graduated (Figure 15a).] The divisions [of the two vertical rules] are counted from the holes.

Alidade Rule: The Alidade Rule of $\sqrt{2}$ times the length of the other rules has a hole with the same diameter as the hole of the upper sides of the square, placed on the end of the surface (*nahāyat-i saṭḥ-i*) of the alidade. The alidade has two sights on the upper side as we find in the figure in the treatise.

The alidade is installed on the square and is kept in the hole by a washer (*fals*), ring (*ḥalqah*) and axis (*quṭb*) only for the time of observation, depending on the direction, north or south, in which a given star is seen; or we use two alidades, which can both remain installed on the square.

This instrument is installed in the plane of the meridian so that two of its sides are parallel, and the other two sides are perpendicular to the horizon.

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99 [Seemann 1929, p. 107].

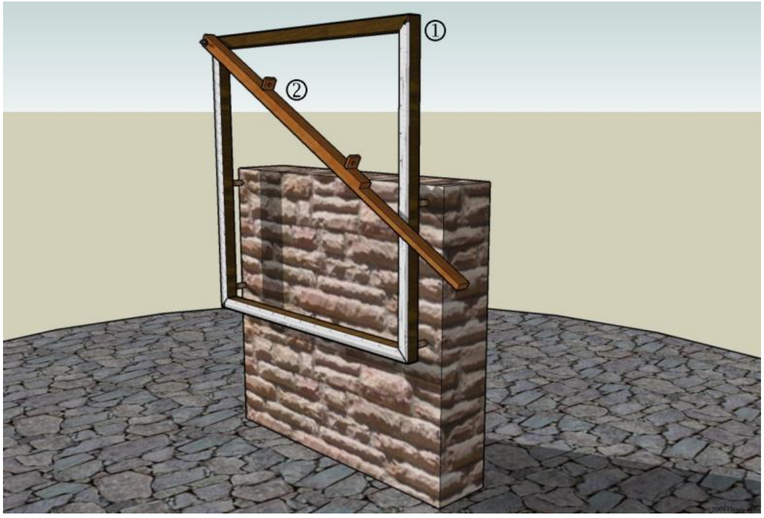

Figure 14: Virtual reconstruction of Instrument #7: (1) Square (2) Alidade.

2.7.2. Applications

This instrument is used to determine the obliquity of the ecliptic from the equator (*mayl-i kullī*), the latitude of the place [of observation], and the maximum altitude of a given star. We move the alidade in the direction of a given star so that the star is seen through both sights. If the free end of the alidade is placed on one of the two rules which are perpendicular on the horizon, then the Tangent of the altitude will be obtained. And if it is placed on the one [lower] of the two rules which are parallel to the horizon, the Tangent of the co-altitude/Cotangent of the altitude will be obtained.

2.7.3. Comments

In this instrument, the base of divisions of the Tangent is 60 (*'ajzā*) instead of 12 (*aṣābiʿ*) or 6.5 (or 7) (*'aqdām*) which were customary in the Islamic period. As al-Bīrūnī (973–1048) says in *Ifrād al-maqāl*, Kūshīyār al-Jīlī used 60 parts as the base to calculate the Tangent. Al-Būzajānī considered the radius of the trigonometric circle as one.[100]

This instrument is practically identical to the *Quadratum Geometricum* of Georg von Peuerbach (1423–1461) (See Section 1.4). The differences in the latter are that it was not fixed in the meridian, but mobile, and was equipped with a device for vertical alignment similar to the one described by Ptolemy

[100] [Bīrūnī 1948, pp. pp. 42–43]; [Bīrūnī 1976, Vol. 1, p. 99].

for his parallactic rule (*Almagest* V, 12), and its graduation on the two sides opposite its singular alidade was counted up to 1200 (Figure 15b).

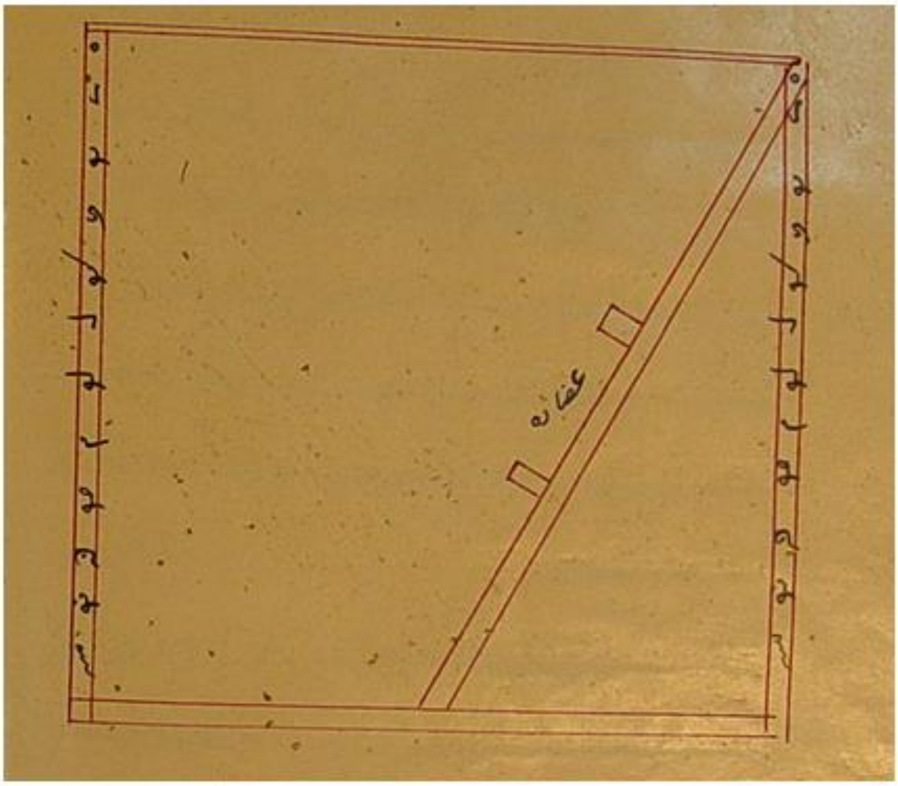

Figure 15: (a) Instrument #7, early 14th ct. AD (S: fol. 36v)

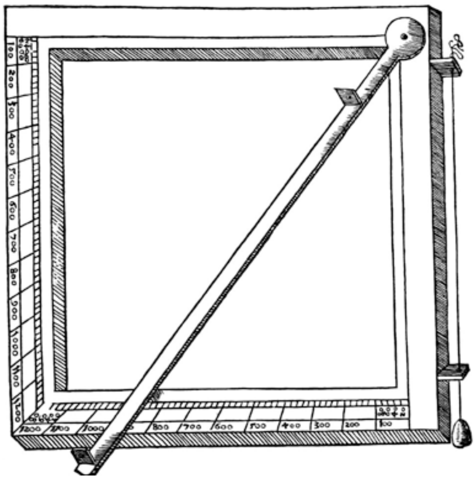

Figure 15: (b) Peuerbach's *Quadratum Geometricum*, mid 15th ct. AD [Schöner 1544, fol. 62v]

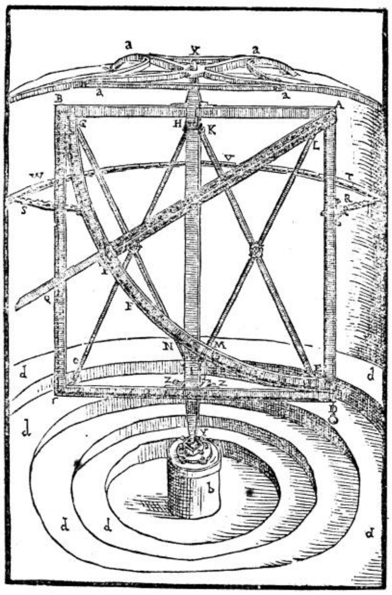

Figure 15: (c) A movable model by Tycho Brahe [Brahe 1602, fol. B3|B4] [Barrettus 1666, Proleg., p. cxvii]

Several samples of Peuerbach's instrument survived, *e.g.*, the ones fabricated by Christoph Schissler in 1569, now in the Staatliche

Kunstsammlungen Dresden. We also find a gigantic, movable model of it in Tycho Brahe's Observatory in Uraniborg, probably built by Hans Crol (d. 30 Nov 1591)[101] in ca. 1588 AD.[102] This was one of Tycho's four great instruments essential to solar observations (Figure 15c).

We do not know whether Peuerbach knew of this instrument or whether he made an independent re-invention. The instrument may have not been completely new to either author: Lelgemann[103] describes a reconstruction of a *Skiotherikós Gnomon* (shadow frame) as an antique precursor of the Geometric Square, for use as (terrestrial) trigonometric survey instrument. Also, Al-Bīrūni used a similar instrument for terrestrial surveys.[104]

2.8. Instrument #8

"This instrument provides the altitude and the azimuth of a given star with complete accuracy and certainty. Using this instrument, all observations that were known through the observational instruments, as well as several other matters, are known and witnessed. This instrument is preferable to other observational instruments for the four reasons mentioned earlier."

– **S**: fols. 37v–38r (Four reasons mentioned in Part I, Section 3.)

2.8.1. Configuration (Figure 16)

First Square: A square is made of four equal rules of copper with dimensions arbitrary in length, but "it will certainly be more correct if the length is taken as long as possible", × 4 fg in width × 2 fg in height. The length of each rule is divided into two halves. Each division mark coincides with one of the four points displaying North, South, East or West. The width of each rule is divided into three parts. Therefore, we have three bands, where the upper [outer] band is divided into 60 parts from any of the four points to its end. The middle and lower [inner] bands are divided, respectively, into minutes and seconds. Therefore, there are 120 parts on the upper band of each rule. This square is placed in a wooden framework.

We determine the Meridian and East–West lines. We place the square parallel to the horizon and keep it in place here by stones and plaster. The height of the square from the ground is [1] cubit.[105]

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[101] [Christianson 1999, pp. 170f].
[102] [Thoren 1991, p. 178].
[103] [Lelgemann 2005, pp. 238–247].
[104] [Bīrūnī 1962, pp. 210–212].
[105] Unfortunately, the exact value is omitted in the text; we assume that it is "one cubit" because, in Persian and Arabic, sometimes "one" is not written.

Iron Shaft: There is a cavity in the center of the instrument [*i.e.*, the intersection of the meridian line with East–West Line] with dimensions 3 fg (diameter) × 0.5 cb (height) where a round cylindrical shaft of iron with dimensions 3 fg (diameter) × 3 cb is fixed perpendicular to the horizon with tin and lead.

Second Square: Another square is made of four equal rules of copper so that the length of each side of it is half as long as the diagonal of the first square. On the [outer] surface of one of its sides, there are 40 (*chihil*) round rings [probably a scribal error for 4, *chahār*] with equal separation from each other. The diameter of each ring is slightly greater than the diameter of the iron shaft. When the Second Square rotates, its lower side will touch the upper surface of the First Square.

On the intersection of the side which has rings with the lower side of the second square, there is a hole. The opposite sides are divided into 60 parts and their minutes and seconds, as far as possible, from the corner opposite the hole (*muqābil-i zāwiyih-i thuqbih*), namely from the outer-upper corner. [The latter statement is dubious. As we will see in the Applications, the two remaining sides of the second square have to be divided from the outer-lower or inner-upper corner, respectively, to the outer-upper corner.]

Alidade Rule The Alidade Rule is $\sqrt{2}$ times as long as each side of the second square, or [$\sqrt{2}\times$] half the length of each diagonal of the first square.[106] The alidade has two sights. On one of its ends [its inner end], there is a hole with dimensions equal to the second square's hole. The alidade's hole is at the very end of its length [namely, as with instrument #7, the hole of the alidade is placed at the end of the surface (*nahāyat-i saṭḥ-i*) of the alidade].

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[106] *i.e.*, also as long as the side of the first square.

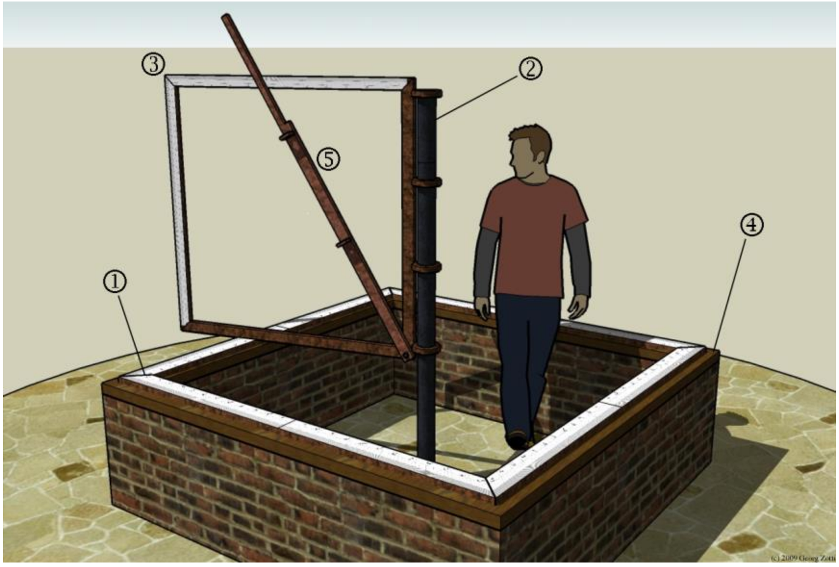

Figure 16: Virtual reconstruction of Instrument #8: (1) First Square (2) Iron Shaft (3) Second square (4) Wooden Framework (5) Alidade.

2.8.2. Applications

We move the second square in the direction of a given star so that it is seen through both of the sights. The tangent of the altitude, or the co-altitude/zenith distance, will be seen on the graduated sides of the second square.

The lower side of the second square will [on the first square] indicate the tangent of the azimuth or co-azimuth.

2.8.3. Comments

Our author devotes special importance to this instrument, described at the beginning of this section.

Whereas instrument #7 was lacking in comparison to Peuerbach's *Quadratum Geometricum*, this instrument now surpasses it for astronomical observations, allowing the measurement also of the azimuth. This combination is similar to instruments #1 and #2, where a pure meridian instrument is described first, followed by a rotating extension. Using viable dimensions, this smaller instrument is easier to handle than #2. We estimate the upper frame with 2 cb × 2 fg × 2 fg bars, still giving about 160 kg of moving mass for the frame with the alidade.

2.9. Instrument #9

2.9.1. Configuration (Figure 17)

Double pillar: after determining the meridian line, we place two pieces of teak wood parallel to each other and perpendicular to the horizon, so that the distance between them is 4 fg, and the meridian passes between them.

Alidade: its dimensions are slightly less than 4 fg in width and 2 fg in thickness, and its length is less than one of the two abovementioned rules. One of its ends is linked to the double pillar by a pin (axis) which is parallel to the horizon, so that its other end is 0.5 cb higher than the ground [probably a scribal error or omission, see comment. 1.5?] and the alidade can move freely. There are two sights on the alidade.

Chord Rule: the rule whose length is $85/60 \approx \sqrt{2}$ times as long as the alidade's, 2 fg wide and 1 fg thick.

On the lower end of the alidade, there is a split in which the Chord Rule is placed. It is kept there by a pin (axis) so that it can move freely about the split in both directions. The other end of the Chord Rule, which is in contact with the two parallel rules at the site where the lower end of the Alidade is placed between two pillars, is free. The Chord Rule is divided into the parts of chord of a quadrant. The graduation begins at the joint place between the alidade and the Chord Rule (the place of the axis). There is a small ring on the alidade end with the halving split to which we attach a rope of catgut (*Khayṭī az rūdkish*) from which the arch of cotton-beating is made.[107]

Another pillar: with height equal to the height of the double pillar, it is perpendicular to the meridian line so that the distance between it and the double pillar is more than the alidade's length. On its apex, there is a pulley wheel (*bakrah*). The abovementioned rope passes through this wheel.

2.9.2. Applications

This instrument is used to determine the maximum altitude of a given star. We pull the rope [lifting the alidade] until a given star is seen through both sights of the alidade. The distance between the halving split end of the alidade and the other end of the Chord Rule[108] is the chord of the zenith distance (co-altitude) of that star.

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[107] In Ancient Iran, a method for cotton-beating was to use a big arch with the very strong and elastic catgut made from intestine (*rūdih*) of sheep.

[108] This description is inexact: we should draw a mark on the two parallel rules where the lower end of the alidade swings through. At the moment of observation, we move the Chord Rule right below this mark, and then the distance between the head of the alidade and this mark will be the chord of the zenith distance of the star.

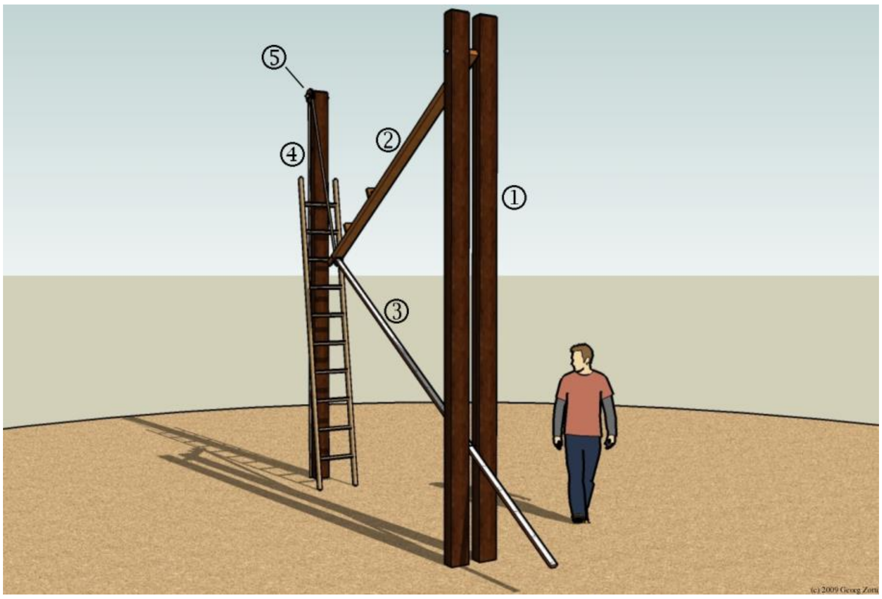

Figure 17: Instrument #9. (1) Double Pillar (2) Alidade (3) Chord Rule (4) Single Pillar (5) Pulley. The form of the ladder is not documented.

2.9.3. Comments

This is obviously another variant of Ptolemy's parallax instrument (*Almagest* V, 12). The interesting differences are the full-length Chord Rule and the pulley (both already described by al-ᶜUrḍī, who mounted it to an arm extending from a wall) and the mounting of the Chord Rule on the alidade. The latter however makes it necessary to raise the point where the loose end of the chord goes through the double pillar: for a Chord Rule of length $\sqrt{2}$ of the base length, the chord will point 1/4 of the base length below this contact point at zenith distance $z \approx 41.41°$, and may collide with the ground if it is not high enough[109]. We choose 5 cb as base length, as this size is identical to the similar instrument described by al-ᶜUrḍī, which, however, has the chord attached to the base of the double pillar. Literally the text says "the lower end of the alidade is 1/2 cb above ground", which would allow no more than 2 cb base length. If the instrument had been built with this length, we no longer

[109] In an instrument of base length 1, when measuring zenith angle z, the chord rule of length $\sqrt{2}$ extends below the contact point by $y = \sin \frac{z}{2} . (\sqrt{2} - \mathrm{crd}\ z)$. The maximum extent is where $\frac{d}{dz} \sin \frac{z}{2} . (\sqrt{2} - 2 \sin \frac{z}{2}) = 0$, thus $z = 2 \arcsin \frac{\sqrt{2}}{4} = 41.41°$, where this amount is $y = 1/4$.

need the pulley wheel machinery intended to aid in the lifting. In his treatise *Description of the Observational Instruments*, Al-Kāshī clearly states that the base length l should *not* be less than 2.5 cb.[110] In the *Almagest*, $l \geq 4\,\text{cb}$ is given.[111] Assuming another scribal error, and a true elevation of 1.5 cb, we can solve the problem; otherwise a trench of almost 1 cb depth for the Chord Rule must be assumed just south of the double pillar, although there is no mention of this in the text. So, the double pillar may have been as high as 7 cb, base length 5 cb, and must have had the chord contact point at a height of 1.5 cb.

2.10. Instrument #10

2.10.1. Configuration (Figure 18a)

Sine and Versine Rules: Two rules of copper with parallel surfaces of equal lengths, but one is slightly wider and thicker.

At one of the ends of the larger rule, there is a hole with dimensions equal to the width and thickness of the smaller rule, so that the latter is perpendicular to the larger one, which can move along the length of the smaller one. The upper end of the larger rule is connected with the head [end] of the smaller rule that is further from the hole, made of catgut (*zih*), so that it [the catgut] is always above the smaller rule's surface, and the larger movable rule will not come out from its hole.

The movable larger rule is the Sine Rule, and the stationary rule is the Versine Rule, or arrow (*sahm;* Ver α = 60 – Cos α).

Alidade Rule: A third rule, the Alidade Rule, has dimensions (width and thickness) equal to the Versine Rule. One of its ends is linked to one of the ends of the Versine Rule like a pair of compasses. These rules have equal length.[112] The sights on the alidade must be attached on the side, as our author says, on the "outer surface" (*Saṭḥ-i Khārij*), of the alidade, or else the Sine Rule will block the target sight.

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[110] Al-Kāshī, *Sharḥ*, **S**: fol. 9v; **M**: p. 31; T: fols. 115v–116v: fol. 115v; [Kennedy 1961, p. 99].

[111] [Toomer 1998, p. 244]; [Arabic *Almagest*, fol. 71v]. However, 1 Greek cb has the value of 463.2 mm.

[112] Their conceptual lengths are the same. Of course, the physical construction requires that the Versine Rule is slightly longer in order to traverse the Sine Rule.

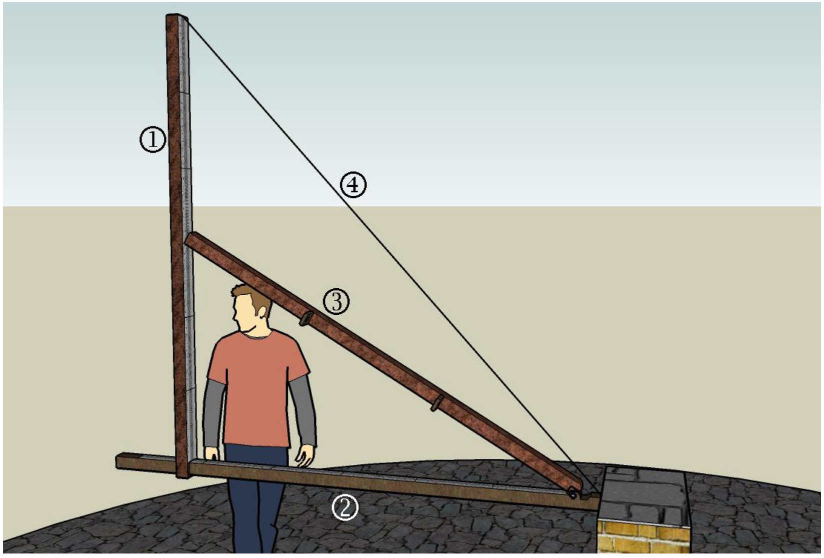


Figure 18: (a) Virtual reconstruction of Instrument #10: (1) Sine Rule (2) Versine Rule (3) Alidade (4) Catgut.

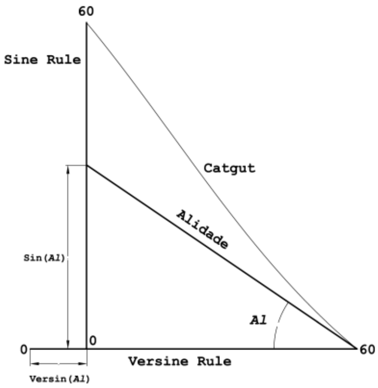


Figure 18: (b) Application.

2.10.2. Graduation and Application

The Sine and Versine Rules are divided into 60 parts, and each part is divided into minutes and seconds.[113] The origin of the graduation in the Sine Rule is

[113] "Minutes and seconds" can again only mean a division into meaningful smaller units.

the hole, and in the Versine Rule it is the end that is related [*i.e.*, closer when the Versine Rule is pulled almost out] to the hole. This instrument is installed in the meridian plane so that the Sine Rule is perpendicular to the horizon and the Versine Rule is parallel to it.

We move the alidade until a given star is seen through both its sights, then the Sine Rule is brought onto the alidade until they touch (Figure 18b). The distance on the Sine Rule from the alidade to the hole of the Sine Rule is the Sine of the star's altitude. The distance from the catgut join on the end of the Versine Rule, which is linked to the Sine Rule, is the Sine of the co-altitude (or the Cosine of the altitude) of the star. [It is clear that the remaining distance on the Versine Rule, i.e. the part that protrudes through the hole, is the Versine of altitude (Ver $h = 60 - \text{Cos } h$).]

2.10.3. Comments

No dimensions of the rules are presented in the text. From the description of instrument #11 we also estimate a length for the rule of 3 cb, so that the "minute" divisions are about 0.55 mm apart. Based on this length and the dimensions for the rules found earlier in this text, an alidade and a Versine Rule of 1 fg × 1 fg × 3 cb weigh about 14 kg each, the Sine Rule, with an estimated 1.5 fg × 1.5 fg × 3 cb, would weigh about 31 kg.

The author does not state how the instrument is installed, and just instructs that it be placed on the "meridian's surface" (*Saṭḥ-i Niṣf al-Nahār*). It should be noted that this is stated only in the case of Instruments # 1 and #7, both of which are installed, above the ground, on a wall. So it would be reasonable to assume that the instrument was likewise installed higher than the horizon, *e.g.*, on a wall. Around 1.5 cb seems to be a usable height. Given the sliding Sine Rule and the purpose of the catgut to prevent the Sine Rule from falling off the Versine rule, the only fixed point can be the end of the Versine Rule where the alidade is attached. This solution (Figure 18b), however, would face mechanical and deformation problems; a support for the other end would be required, which would render the catgut unnecessary to prevent the Sine Rule from leaving the *sahm*.

Possibly, however, this description just presents the basic thoughts for the next instrument, and was never actually built.

2.11. Instrument #11

2.11.1. Configuration (Figure 19)

Meridian and East–West Rules: These are two rules of teak wood or copper. We draw a line on the surfaces of them bisecting their widths. These rules intersect each other so that the two abovementioned lines are perpendicular to each other. We place them on the meridian and east–west lines so that the lines bisecting their widths coincide with them. We keep these structures in place by means of stones and *sārūj* (a plaster of lime, ashes, and sand).

Shaft of Iron: At the intersection of these rules we make a cavity in the ground, 2 fg in diameter and 1 sh in depth. A very round iron shaft is kept in place here, perpendicular to the horizon, by lead and tin.

Supportive Rules: Four parallelepiped rules of teak wood with a length of 1 cb are attached to these two rules so that they provide a square which supports the instrument.

Versine Rule: A very round copper rule, 2.5 fg thick, with a round hole on one of its ends equal to the diameter of the iron shaft linked to it by a ring and washer. The distance from its central hole (*thuqbih-i quṭb*) to its outer end is equal to the distance from the central hole of the two abovementioned rules to their outer ends.

Sine Rule: A parallelepiped rule of copper of the same length as the Versine Rule, 2 fg wide and 0.5 fg thick.

Pipe: There is a pipe (*anbūbih*) perpendicular to the lower end of the Sine Rule, 3 fg in length and thickness (diameter) equal to the thickness of the Versine Rule. We place the Versine Rule inside the pipe so that it can move along the Versine Rule, which is then perpendicular to the Sine Rule.

Alidade: A parallelepiped rule of copper of the same length as one of the Sine and Versine Rules (its dimensions are probably similar to the Versine Rule, as in Instrument #10), is linked to the outer end of the Versine Rule like a pair of compasses. There are two sights on it. [In the figure in the treatise (Fig. 4b), there are several sights.]

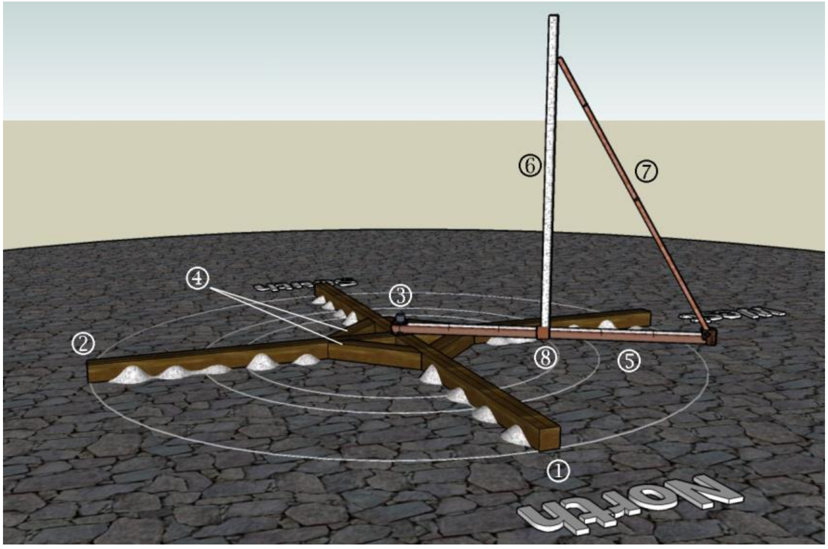


Figure 19: Virtual reconstruction of Instrument #11. (1) Meridian Rule (2) East-West Rule (3) Iron Shaft (4) Supportive Rules (5) Versine Rule (6) Sine Rule (7) Alidade (8) Pipe. To measure the altitude, the Sine Rule was shifted to touch the alidade (cf. Figures 18a and 18b).

2.11.2. Graduations and Applications

On the Versine and Sine Rules there are three bands: one graduated into 60 parts, and two are divided into minutes and seconds [*i.e.*, smaller parts].

To determine the altitude, we target a given star through the alidade's sights, and then we slide the Sine Rule [along the Versine Rule, and in vertical position] onto the alidade until they touch (Figure 19) and the bisector lines of the widths of the Sine and Versine Rules are perpendicular to each other (see Figure 18b).

To determine the azimuth, we hold the Versine Rule in its current place [*i.e.*, the place of observation] and rotate the Sine Rule around the Versine Rule so that their width bisector lines are [still] perpendicular to each other and the Sine Rule touches the East–West Rule (Figure 20a). We slide the Sine Rule onto the outer end of the East–West Rule (Figure 20b). At this point,

$\overline{VW} = \mathrm{Sin}A$ Sine of the Azimuth of the observed object, *jayb-i samt-i irtifāʿ*

$\overline{OV} = \mathrm{Sin}(90° - A)$ Cosine of the Azimuth of the observed object, *jayb-i samt-i tamām-i irtifāʿ*

2.11.3. Comments

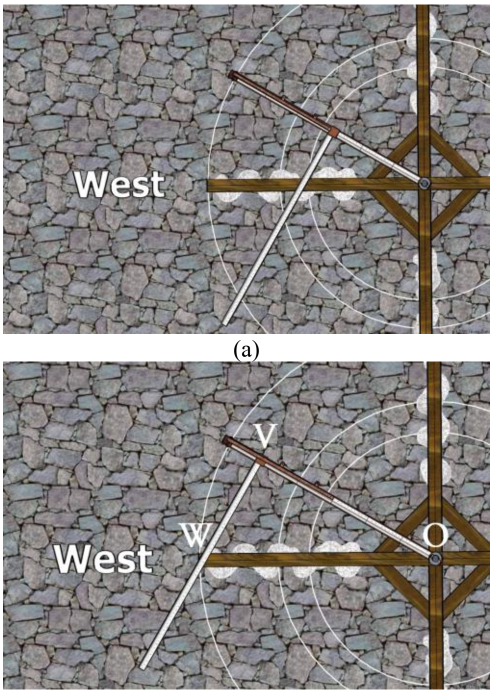
In the treatise, three separate figures are devoted to this instrument, more than for any other: **S**: fols. 41v., 42r., and 44r (top), which show, respectively, the upper part, the foundation and the general shape of the instrument.

With this construction, objects close to the horizon cannot be observed, because the pipe will collide with the central axis, and thus the alidade cannot be moved lower than about 15 degrees when it should still touch the Sine Rule. The observation of objects very close to the zenith may also be impossible due to a collision between the pipe and the Sine Rule with the alidade joint.

The original drawing (S: fol. 44r above, Figure 4b) shows three concentric circles labeled as having semi-diameters of 1.6 cb, 2 cb and 3 cb (Figures 20a, and 20b), and with the largest one having the same radius as the rule lengths, from which we assume this length of 3 cb for Versine, Sine and Alidade Rules. The mass of a copper alidade of 1 fg × 1 fg × 3 cb is about 14 kg, which also seems reasonable.

The purpose of these circles is not documented. The circle of 1.6 cb radius would indicate an altitude angle of $h = 62° 11'$, or a culminating declination of $\delta = 9° 31'$, while the circle with 2 cb radius indicates $h = 70° 31'$, or a culminating declination of $\delta = 17° 52'$, equivalent to ecliptic longitudes of ♈24°/ ♍6° or ♉20°/♌10° respectively; however, the purpose of these dates is unclear.

The description on the reading of the azimuth probably hides a clever detail of construction, and would only be fully correct if the Sine Rule were labelled from the axis of the Versine Rule. However, on the physical instrument, the scale on the Sine Rule starts on top of the Versine Rule, to be used with the alidade. With the dimensions described, this distance is 1.25 fg = 34.64 mm. On a 3 cb instrument, one unit on the Sine scale (1/60) is 33.25 mm. To read the azimuth, the unit count read on the outer end of the East-West Rule had to be reduced by 62.5', or roughly 1^p, to find the correct result. If the Versine Rule was only 2.4fg in diameter, this offset would have been exactly 1^p.



(a)

(b)

Figure 20: Instrument #11: To determine the azimuth, the Sine Rule was laid on the ground (a) and shifted to the end of the East-West Rule (b).

2.12. Instrument #12

The description of this instrument in our MSS appears to be somewhat misplaced in the text. It is placed after the date of copy (**S**, fol. 49r.–49v.), and seems to be an appendix to the treatise. Nevertheless, two figures, one apparently intended to illustrate the description and the other depicting the base of it, were drawn five folios earlier (fol. 44r) below the last sketch of instrument #11 (Fig. 4b), thus dispelling the doubt that it might not be one of our treatise's instruments. Strangely, the figure rather seems to depict al-ᶜUrḍī's dioptra for eclipse observation. But our author speaks of 12 instruments, and this number will only be completed if we count this instrument.

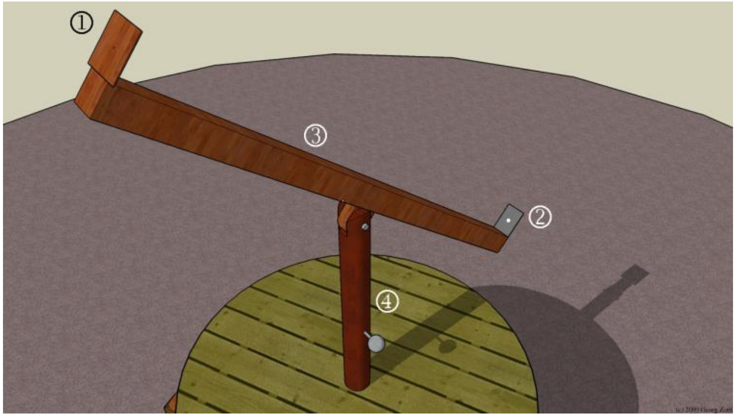


Figure 21: Virtual reconstruction of Instrument #12: (1) Great Pinnula (2) Small Pinnula (3) Rule (4) Base/Support: the support construction is shown following al-ᶜUrḍī's description of the dioptra.

2.12.1. Configuration:

2.12.1.1. Upper Part [Figure at S: fol. 44r, middle]:

The exact translation of our author's name given to the upper part of Instrument #12 is as follows:

"The rule with which by the image of the ray (*ᶜaks-i shuᶜāᶜ*) the radial magnitude of the eclipsed sun is found.[114]

"From a straight and rigid piece of wood, we make a rule like the astrolabe's alidade. There are pinnulae (*lubnih*) at both of its ends. One of them is 4 fg wide, and the other one is 2 fg wider [total: 6 fg]. On the center of the greater pinnula, there is an exactly round hole (*thuqbah*).

[114] The use of this long qualitative description instead of a specific term such as Dioptra may be due to the fact that the term *διόπτρά* had apparently not been translated or entered into Arabic. In the initial sentences of *Almagest*, V, 14 in which Ptolemy says: "We too constructed the kind of *dioptra* which Hipparchus described, which uses a four-cubit rod" [Toomer 1998, p. 251–2] [Heiberg 1899, p. 417]. Isḥāq-Thābit's Arabic translation reads: "We too constructed the *Miqyās* which Hipparchus made, with a four-cubit rod/rule, *Misṭara*." [Arabic *Almagest*, fol. 74r] In Arabic and Persian, *Miqyās* ("scale", "measuring tool") is a general term without a specialized usage; for example, as we have seen up to now, in Ghāzān's treatise, it would have been applied to name different things. (To mention just Instrument #4, both the Central Iron Shaft and the Chord Rule's Nail Pointer are called "*Miqyās*".) Therefore, it seems that the Islamic medieval astronomers, following the linguistic style of the Arabic *Almagest*, appealed to qualitative practical descriptions such as "the rule by which …" to name the Dioptra-shaped instruments.

Around the center of the smaller sight, which is aligned with the center of the greater sight, we draw a circle whose radius is equal to the apparent radius of the sun."

2.12.1.2. Base of the Instrument:

According to the figure illustrating the base of the instrument (S: 44r, bottom; Fig. 4b), it consists of (1) a cross-shaped base, *Ṣalīb*, in the middle of which (2) a central axis has been erected. (3) Twelve "supportive" wires in the figure, all called *Daᶜāma*, hold and support the central axis, three on each side connecting the ends of each branch of the cross-shaped base with three different heights of the central axis. The supporting wires seem to provide a strong base for the instrument, but the figure is very sketchy and there is no textual description with details of the construction. (For Figure 21 a base adopted from Al-ᶜUrḍī's description of the support of his own dioptra has been used.)

2.12.2. Graduations and Application

The exact translation of our author's statements concerning the application of Instrument #12 is as follows:

"To draw this circle, two days before the eclipse, we place this instrument facing the sun so that the size of [the circle of] the sunlight coming through the hole of the great pinnula and shining on the small pinnula is revealed. Then, the circle around the center of the smaller sight is drawn with the dimensions of the illuminated circle.

Then we divide this circle's diameter into 12 equal parts, revealing the digits of the diameter [of the eclipsed sun] (*'aṣābiᶜ-i quṭr*).

Then the circle's circumference is divided into 12 parts, revealing the digits of the area [of the eclipsed sun] (*'aṣābiᶜ-i jirm*).

We draw semidiameter lines from the center of the circle onto the parts of the circle's circumference.[115] We draw circles on the digits of the diameter, by which the digits of the area [of the eclipsed sun] are made clear. [Note that after drawing the circles only the corresponding arcs will remain on the circle presenting the solar disk on the small pinnula.] If we need high accuracy, we

[115] The text does not say whether this partition should be regular. The dotted radial lines in Figure 22 show the necessary angles for both systems of counting the magnitude of the eclipse.

will divide the digits of the diameter and the ones of the circumference into minutes.

On the day of the eclipse, we place the sights in line with the sun and wait for the appearance of a slight shadow, like a fly's wing. This time is the beginning of the eclipse. Using the astrolabe or *shīsha-i sāʿat* (lit. "time-glass"; water- or sand-clock?),[116] we determine the time of start and end of the eclipse. We wait while the shadow is increasing and until it grows no further and begins to decrease. By the darkness of the circle of the diameter of the sun, the digits of the diameter and of the area are revealed. By the times given by clepsydra or determined by the altitude of the sun, the times of the beginning and the end of the eclipse are found. When the darkness vanishes from the light circle, this is the time of complete luminosity [end of eclipse]."

2.12.3. Comments

This instrument is apparently intended to replace the antique dioptra. An early dioptra seems already to have been described by Archimedes (3rd c. B.C.) in his *Sandreckoner*.[117] Ptolemy used a dioptra originally described by Hipparchus, four cubits long.[118] This dioptra has a fixed pinnula (the lower pinnula), on which there is a hole for sighting, and a movable one (outer pinnula), which is placed in front of the sun. The solar/lunar angular diameter is calculated based on the movable pinnula's width and the distance between the two pinnulas.

The application of the classical dioptra was to determine the apparent angular diameter of the sun and the moon. Like the other medieval scholars, our author noticed that Ptolemy had said nothing on its construction, but that his successors had. For instance, in his commentary on Book V of *Almagest*, Pappus of Alexandria presented a description of this instrument. Proclus described it slightly differently from Pappus' account.[119] Heron of Alexandria also promoted the dioptra by constructing two types (vertical and horizontal).[120] Not all of them added more details especially regarding the use of this instrument to determine the eclipsed diameter or the area of the sun or

[116] Wābkanawī uses the Persian term *Pangān* in the otherwise similar paragraph in his own *Zīj*, which refers to the clepsydra. *Pangān* was originally a simple inflow clepsydra; cf. [Mozaffari 2013a, p. 256, note 80]. About the clepsydra used in the Maragha observatory, cf. [Mozaffari 2013a, pp. 256–257].

[117] [Heath 1897, pp. 221–232]; see also: [Shapiro 1975, pp. 75–83].

[118] 4 cb = 185.28 cm in his case: 1 Greek fg = 19.3 mm; thus 1 cb = 46.32 cm; *Almagest* (V, 14), [Heiberg 1898, Vol. I, Part 1, p. 417]; [Toomer 1998, p. 56].

[119] [Goldstein 1987, pp. 174–175].

[120] [M. J. T. Lewis 2001, pp. 41–42 and pp. 51f].

the moon by either drawing a circle on the lower pinnula (*e.g.*, our treatise) or by using a circular plate on the lower pinnula (like al-ᶜUrḍī). This quantity is usually obtained by calculations.[121] In the ancient and the early Islamic period, the astronomers estimated it with the naked eye without using an instrument of any kind, and then applied their own estimates to check the results of their calculations.[122]

In his treatise, al-ᶜUrḍī presented an addition to the ancient dioptra for determining the eclipsed diameter of the sun or the moon[123]. As with the ancient dioptra, he uses a movable pinnula and a fixed one, but there is a conical hole on each of them. The angular diameter of sun and moon is calculated based on the width of the hole on the outer pinnula and the distance between the pinnulae. For him, however, the most important application of the instrument is the measurement of the eclipsed diameter/area of the sun and the moon. To do this, he uses two circular brass plates (*mir'āt*), one for each type of eclipse. Before the eclipse, the upper pinnula is shifted until the luminary of interest exactly fills its visible diameter. The value of its visible diameter is read on a scale. During the eclipse, the respective brass aperture is brought in front of the entrance of the upper pinnula to cover the bright part of the luminary.[124] In any case, the instrument requires the user to look directly through the pinnulae, which is known to be dangerous in case of solar observations. Also, when using a device with two conical holes, a movable pinnula with a graduated scale and additional apertures seems unnecessarily complicated.

Our author here presents a new instrument that fulfills the same purpose, *i.e.*, the measurement of solar eclipses, but it is significantly easier to produce, and does no harm to the eyes.

The upper pinnula is described as 2 fg larger than the lower one, most likely to provide good shading for the lower projection screen.

Our author attributes the instrument under discussion to al-Ṭūsī (1201–1274 AD), but we have not found anything on this in his works. Only in his *Exposition of the Almagest* did al-Ṭūsī note, referring to the dioptra described in the *Almagest*, that "it is possible that errors occur [in the calculation of the

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121 *Almagest*, VI, 7

122 [Stephenson and Said 1991]; [Said and Stephenson 1991]; [Said and Stephenson 1996]; [Said and Stephenson 1997].

123 [Seemann 1929, pp. 61–71].

124 Seemann [1929, p. 66f] notes however that this description is not completely clear, since no mechanism for measuring the amount of shift of the aperture is explicitly described.

apparent diameter], if the length of the rule was much longer than the width of the sight."[125]

In his *Zīj*, Wābkanawī described this same instrument and called it "one of the marvels of the observational works" (*min jumlih gharā'ib-i aᶜmāl-i raṣadī*).[126] The details he gives are the same as in our treatise. He worked in the same period as the most important royal astronomer at Ghāzān's court, and from this we can reason that the astronomers of that era were aware of this instrument and that it was a new device; in his *Zīj*, Wābkanawī usually called his own innovations "the marvel."

It is important that this arrangement makes the instrument a pinhole image device, and it is also quite similar to the instrument described at least two decades later by Levi ben Gerson (1288–1344 AD).[127] In fact, we may consider instrument #12 as the link between the antique Dioptra and the instruments that were used as Camera Obscura.[128]

Most of the information on the basic principle of the construction and early use of the pinhole device is derived from the *Kitāb al-manāẓir* of Ibn al-Haytham (Alhazen, ca. 1038 AD).[129] The application of pinhole images in the astronomical observation, esp. for the eclipsed diameter/surface of the luminaries, was known in the West at least from 1187 AD onwards, as it is mentioned by Roger of Hereford. He was followed by figures such as William of Saint-Cloud (ca. 1292 AD), Levi ben Gerson (Gersonides, 1288-1344 AD), Henry of Hesse (1325-1397 AD), Leonardo da Vinci, Tycho Brahe, Johannes Kepler, *etc.* Nevertheless, Levi has hitherto been known as the first person to construct a single instrument in the form of a pinhole device for astronomical purposes.[130] But it is evident that Wābkanawī or the writer of our treatise preceded Levi in this field by about two decades, although there is no evidence of any relation between them.

In *Almagest* VI, 7, Ptolemy describes the relation between eclipse size given in 12ths of solar diameters and size given in 12ths of the visible disk

[125] [al-Ṭūsī, *Taḥrīr al-Majisṭī*, fol. 37v].
[126] [Wābkanawī, *Zīj*, Book IV, Sec. 15, Ch. 8; A: fols. 159r–159v, B: fols. 92r–92v].
[127] Cf. [Mancha 1992, p. 293]. Note that this instrument of Levi is different from his Jacob's Staff.
[128] E.g., [Kepler 1604, Ch. IX]; see also: [Sigismondi and Fraschetti 2001, pp. 380–385].
[129] [Sabra, 1989, pp. 90–91].
[130] For a history of the progress of using pinhole images in astronomy, see: [Mancha 1992]; [Goldstein 1987]; esp. for Da Vinci, see: [Weltman 1986].

area, sizes which our author also mentions. We give a more modern expression[131] as follows:

We write the eclipse size in solar diameters as $s = [0 \dots 12]$, and in visible disk area as $a = [0 \dots 12]$. Solar disk diameter equals 12^p, so radius $R_s = 6$. The lunar disk radius R_m is typically slightly larger than 6 for a total eclipse (Ptolemy derived $12^p20'$ lunar diameter in mean distance for his table). The distance between the disk centers of sun and moon is then

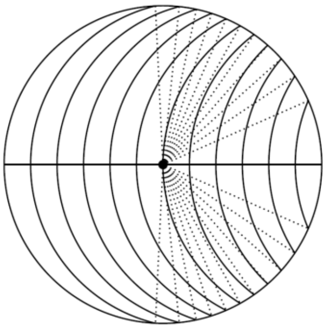
$$d = R_m + \frac{R_s}{12}(12 - 2s)$$

Then, the covered surface area of the sun is

$$A = R_m^2 \arccos\left(\frac{d^2 + R_m^2 - R_s^2}{2dR_m}\right) + R_s^2 \arccos\left(\frac{d^2 + R_s^2 - R_m^2}{2dR_s}\right) - \frac{1}{2}\sqrt{(-d + R_m + R_s)(d + R_m - R_s)(d - R_m + R_s)(d + R_m + R_s)}$$

where the result of the arccos is expressed in radians, and which must be normalized to $a = 12A/R_s^2\pi$.

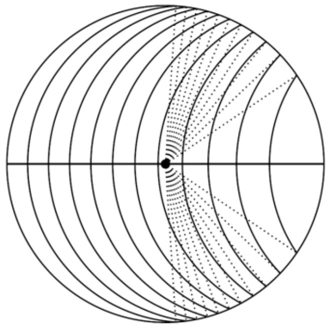
There is no linear, easy solution for $a(s)$ or $s(a)$. Table 4 provides numerical values for full 12ths in cases $R_m = R_s = 6^p$ and $R_m = 6^p10'$, and Figure 22 shows these sizes for 12ths of diameter and area, respectively, for $R_m = 6^p$.



(a)

Figure 22: (a) Eclipse sizes shown in the 12ths of diameter s

[131] Weisstein: Circle-Circle Intersection. MathWorld–A Wolfram Web Resource.



(b)

Figure 22: (b) Eclipse sizes shown in the 12ths of area *a.* The purpose of the radial lines described in the text is not evident.

3. Concluding Remarks

This study was originally intended as an overview (including the translation of certain phrases, whenever necessary) of an anonymous Persian treatise on the observational instruments of the second period of the Maragha Observatory, the construction of which was proposed during the reign of Ghāzān Khān and which was most probably built at that time.

We have created virtual reconstructions of the instruments following the text as closely as possible. We have found several inconsistencies which must be copying errors in all the extant copies of the treatise accessible to us. With a few corrections, the instruments could be shown to work. It appears that some instruments, if made of copper, would have been barely usable due to the large size required to achieve a satisfactory degree of accuracy. One instrument, #12, seems to be the first pinhole device specifically described for solar eclipse observations. On the other hand, the author disregards the recommendations of his precursor, al-ᶜUrḍī, not to use ropes for measuring lengths.

Diameter size s	$R_m=6^p$ Area size a	Center distance d	$R_m=6^p$ 10' Area size a	Center distance d
0	0	12	0	12.167
1	0.342	11	0.344	11.167
2	0.955	10	0.962	10.167
2.053			1	10.114
2.063	1	9.937		
3	1.732	9	1.744	9.167
3.295			2	8.871
3.312	2	8.688		
4	2.629	8	2.649	8.167
4.360			3	7.806
4.384	3	7.616		
5	3.622	7	3.650	7.167
5.331			4	6.835
5.360	4	6.640		
6	4.692	6	4.729	6.167
6.242			5	5.925
6.277	5	5.723		
7	5.823	5	5.871	5.167
7.110			6	5.056
7.152	6	4.848		
7.949			7	4.218
7.998	7	4.002		
8	7.003	4	7.062	4.167
8.766			8	3.401
8.820	8	3.179		
9	8.220	3	8.291	3.167
9.566			9	2.600
9.628	9	2.372		
10	9.465	2	9.547	2.167
10.357			10	1.810
10.425	10	1.575		
11	10.728	1	10.819	1.167
11.143			11	1.024
11.214	11	0.786		
12	12	0	12	0.167

Table 4: Eclipse size in 12ths of solar diameter and 12ths of solar disk area, for $R_m = R_s = 6^p$ and $R_m = 6^p$ 10', respectively.

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Appendix 1: Quotes from Rashīd al-Dīn in Section I–1

First Quote:

«دوم روز [= یکشنبه، ۱۶رمضان/پنجم ژوئن] به تماشای رصد رفت و در همهٔ اعمال و آلات آن نظر فرمود و تمامت را به تأنّی تمام بدید و از کیفیّت هریک سؤال فرمود و با وجود مشکلی دقایق آنْ اکثر فهم کرد و فرمود که در جنب گنبد عالی و ابواب البرّ شم [در تبریز] رصدی سازند مخصوص به چند عمل. و کیفیّت آن اعمال را به تقریری واضح بیان فرمود بر وجهی که حاضران حکما متعجّب ماندند از حسن استنباط او، چه چنان عمل در هیچ عهدی نکرده‌اند. و حکما گفتند: "ساختن آن به غایت متعذّر باشد." ایشان را تعلیم و ارشاد کرد تا به امعان نظر در آن شروع نمودند و بر وفق تعلیم او به اتمام رسانیدند و ایشان و جملهٔ مهندسان ماهر متفّق‌اند که مثل آن کس نساخته و نشناخته.»

Second Quote:

«چون به کرّات به رصد مراغه رفت و شرح آن آلتها پرسید و از کیفیّت آن تفحّص نمود و یاد گرفت، بر کلیّات آن وقوف دارد، چنانکه، در این وقت، آنچه به وضع و عمارت رصد [= رصدخانهٔ مراغه] تعلّق دارد، از طبع خویش فرمود تا بساختند. و به جهت اعتبار دور آفتاب گنبدی هم از طبع خویش بنا فرمود و با منجمّان تقریر کرد. و تمامت گفتند که هرچند چنین آلتی هرگز ندیده‌ایم، لیکن معقول است. و در رصد که در جنب ابواب البّر تبریز است شکل گنبدی ساخته‌اند که آن معانی در آن درج است، چنانکه مشاهده می‌کنند.»

Appendix 2: Rukn al-Dīn Āmulī's *Zīj-i jāmi'-i Būsa'īdī*, Prologue:

MS. T: Iran, University of Tehran, no. 2558, fols. 1v–16v: 1v–2r.
MS. P: Iran, Parliament, no. 183/1, fols. 1v–29v: 1v.

/P: ۱پ/ /T: ۱پ/ «[...] در سنهٔ اثنین واربعین ثمان مائه هجریّه [۸۴۲ق.]، در شیراز، جماعتی از احبّاء [...] التماس می‌نمودند که زیجی تصنیف باید کرد که [...] محصول اعمالش موافق باشد به محاسبهٔ رصد[۱] ایلخانی نه به اعمال *زیج ایلخانی*. از آنکه خواجه – قدس سرّه– را در تصنیف *زیج ایلخانی* غلطی چند واقع شده بود – چنانچه مشهور است – و وصیّت فرموده که خواجه اصیل الدّین به اتّفاق افضل المتأخّرین مولانا قطب الدّین علّامه تغییر جداول زیج کرده،[۲] اصلاح فرمایند. و چون حضرت خواجه – قدّس سرّه – خطبهٔ زیج ایلخانی نوشت و اسامی علماء رصد را در آنجا ذکر کرده به جوار حق[۳] پیوست و به اسم جناب مولوی التفات نفرمود،[۴] از این سبب جناب مولوی به اصلاح جداول مشغول نشد[۵] تا که به التماس خواجه اصیل الدّین در حواشی زیج چند کلمه[۶] که نقل کرده بود[۷] نوشت که چون اوساط کواکب از جدول بردارند: سی دقیقه بر وسط قمر /T: ۲ر/ زیاده کنند؛ و هفت دقیقه بر مرکز زحل افزوده؛ یک درجه و سی و شش دقیقه از مرکز مشتری نقصان کنند و یک درجه[۸] و بیست و یک دقیقه بر خاصّهٔ مشتری افزایند؛ و یک درجه و نیم بر مرکز مرّیخ[۹] افزوده، از مرکز زهره نقصان کنند. و به آفتاب و عطارد التفات نفرمود.[۱۰] و چون علماء رصد بعد از وفات خواجه[۱۱] تا سی سال نشستند که دور زحل به اتمام رسید و هر یک مثل اثیرالدّین ابهری و محیی‌الدّین مغربی و نجم الدّین دبیران[۱۲] و فخرالدّین اخلاطی تصنیفات زیج و اقلیدس و مجسطی کردند، معلوم شد که کمابیش سه دقیقه از مرکز آفتاب نقصان می‌باید کرد[۱۳] تا محسوب اعمال طالع سال عالم و خسوف و کسوف و قرانات و احتراقات موافق مرئی باشد. و آنچه در این روزگار عمل می‌کردند آن است که سی دقیقه بر وسط قمر زیاده کرده، آفتاب و باقی کواکب را به حال خود می‌گذاشتند تا در طالع سال عالم قریب یک نصف برجی تفاوت افتاد،[۱۴] [و] به نسبت با زیج شاهی، در نوروز سلطانی یک شبانه‌روز تقدیم[۱۵] واقع شد و چون این معنی شهرت تمام یافت و اکابر روزگار بدین احوال مطّلع شدند در تاریخ سنهٔ ثلثین و ثمان مائه هجریّه [۸۳۰ق.]، سلطان سعید شهید امیرزاده الغ بیگ – انارالله برهانه – در سمرقند بنیاد عمارت رصد کرد ...»

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(۱) T : + ~~جدید~~ (۲) T : را (۳) P : + جلّ وعلاء (۴) P : ننمود و (۵) P : نگشت (۶) P : – کلمه (۷) P : بودند (۸) P : «و سی و شش دقیقه از مرکز مشتری نقصان کنند و یک درجه» در هامش (۹) T : «مرّیخ» در هامش (۱۰) P : ننمود (۱۱) P : + کلمه‌ای که خوانا نیست. (۱۲) T : دبرانی (۱۳) P : باید کردن (۱۴) P : + و به نسبت آن سه دقیقه (۱۵) T : تقویم

Appendix 3: *Risāla al-Ghāzāniyya fī 'l-ālāt al-raṣādiyya*

MS. S: Iran, Sipahsālār Library, no. 555D, fol. 15v–49v.
MS. P: Iran, Library of Parliament, no. 791, pp. 29–97.
MS. M: Iran, Malik National Library, no. 3536, pp. 41–56.
Note: since S and P are identical, the variants mentioned for S are also the case with P.

M: 41
P: 29
S: 15v

بسم الله الرّحمن الرّحيم وبه نستعين[1]

الحمدلله ربّ العالمين والصّلوة والسّلام على خير خلقه، محمّد، وآله أجمعين الطّيّبين الطّاهرين.

امّا بعد، چون معلوم همهٔ عقلا و اهل دانش است که شرف انسان و تفضيل او[2] بر ديگر حيوانات به مزيد عقل است تا به آن واسطه علم او[3] به حقيقت اشياء حاصل شود. و هر علم که موضع او شريفتر است، آن علم نيز شريفتر است از ديگر علوم؛ چنانکه علم نجوم که موضع [او] فلکيات و اجرام [M: 42] علوی‌اند و ايشان اشرف اجسام‌اند از جهت علوّ منزلت و دوام و ثبات ايشان که هميشه بر يک // حال باشند، و يقين علم ايشان که تمامت مبرهن است به براهين هندسی و حسابی؛ پس، علم نجوم نيز اشرف علوم اشياء باشد و او را ادراک نمی‌توان کرد الّا به آلات رصد تا به واسطهٔ آن به براهين هندسی،[4] هيأت و شکل افلاک و ستارگان معلوم گردد، و اوضاع و حرکات و ابعاد و مقادير اجرام ايشان معلوم شود، و به واسطهٔ آنْ شکل کرهٔ زمين و مقدار او نيز معلوم گردد.

P: 30
S: 16r

و آلات رصد که مشهور و مستعمل است// و اعتقاد متقدّمان و متأخّران بر آن است، آن پنج آلت است که در «مجسطی» مذکور است؛ هر يک مخصوص‌اند به مهمّی:

صفت [آلت] «حلقتان»[5]

P: 31
S: 16v
M: 43

که در سطحِ دايرهٔ // نصف‌النّهار نصب کنند[6] و آن سطحِ حلقه را که در سطح نصف‌النّهار// باشد به اجزائی که ممکن گردد،[7] قسمت کنند و در داخل اينْ حلقه‌ای ديگر متحرّک باشد و بر[و] دو هدفه متقاطر نشانده بود از جهت معرفت ميل کلّی و عرض بلد و غايت ارتفاع کوکب را.

صفت[8] آلت «لِبنه»

P: 32
S: 17r

که متأخّران آن را «ربع» گويند. آن را نيز در سطح// نصف‌النّهار نصب کنند از جهت معرفت همين مطالب.

صفت[9] آلت[10] «حلقهٔ نحاسيه»

M: 44

که صاحب «مجسطی» در مقالهٔ ثالثه // آورده است که نصب کرده بود در شهر اسکندريه، فی «الرّواق المعروف بالمربّع» [و] «[ال]مَلعَب»[11]، در سطح معدّل‌النّهار، چنانچه[12] سطحِ حلقهْ مايل بود از سمت‌الرّأس به قدر عرض بلد، تا وقت حلول شمس به نقطهٔ اعتدالين معيّن گردد.

(1) SM: + رسالة فی معرفة صفة آلات الرّصد من إملاء ملک الحکماء ورئيس المهندسين ودستور الرّاصدين، جمشيد بن محمود غياث الملّة والدّين الکاشانی. (2) S: آن (3) SM: + را (4) SM: + و (5) S: عنوان ندارد؛ M: حلقهٔ نحاسيه (6) SM: + و آنْ سطحِ حلقه را که در سطح نصف‌النّهار نصب کنند (تکرار جملهٔ پسين با کاربرد فعل جملهٔ پيشين که هر دو نسخه به همين صورت آمده.) (7) S: + و (8) S: ندارد. (9) S: ندارد. (10) S: ندارد. (11) SM: مکعب (12) SM: + در

و نصب کردن این آلت در سطح معدّل‌النّهار بعد از تحقیقِ عرضِ بلد صعوبت عظیم دارد و چون مدّتی که گذرد بعد از نصب کردن، این حلقه میل به جانبی می‌کند از جهت ثقل آلت و از سطح معدّل‌النّهار به درمی‌آید چنانچه// صاحب «مجسطی» را [اتّفاق] افتاده فی[1] الملعب الاسکندریة که در (P: 33, S: 17v) یکْ نقطهٔ[2] استواءْ دو نوبتْ ضوء انداخت در داخل حلقه.

صفت[3] آلت «ذات[4] الحلق»

(M: 45) که معظم آلات رصد است و آن در زمان ثاون اسکندرانی[5]// نُه حلقه بود و از عهد بطلمیوس به شش حلقه، و[6] این قرار گرفته: اوّل، حلقهٔ نصف‌النّهار است که محیط بر پنج حلقهٔ دیگر است؛ و دوم، حلقهٔ مارّه به اقطاب اربعه است؛ و سیُم،[7] حلقهٔ منطقةالبروج است که سطح او با سطح حلقهٔ مارّه به اقطاب اربعه متقاطع است بر زوایای قایمه؛ و چهارم، حلقهٔ عرضیّهٔ خارجیّه است؛// و پنجم، حلقهٔ (P: 34, S: 18r) عرضیّهٔ داخله است که هر دو بر قطب‌البروج گذشته باشند؛ و ششم، حلقهٔ صغیره است که داخل همهٔ حلقه‌ها باشد و برو دو هدفه نشانده باشند متقاطر یکدیگر.

و از این آلتِ ذات‌الحلقْ بعد از مشقّت و رنج بسیارْ بُعد میان کوکب معلوم و کوکب مجهول معلوم می‌شود به تقریب. و به واسطهٔ آنْ[8] موضعِ کوکبِ مجهول معلوم می‌گردد از فلکْ‌البروج. و آن نیز [در] صورتی بود که کوکبِ معلوم‌الموضع، که مقیس علیه[9] می‌گذرد،// مقاطع گردد با (M: 46, P: 35, S: 18v) منطقةالبروج به غیر قوایم[10] و طول و عرضِ کوکبِ مطلوبْ معلوم// نشود به تحقیق.

صفت آلت «ذات الشّعبتین»[11]

و آن سه[12] مسطرهٔ مستقیم باشد: یکی قوایم بود بر سطح افق و در سطح دایرهٔ نصف‌النّهار باشد و دو مسطره دیگر برو ترکیب کرده.

از این آلتْ غایتِ ارتفاعِ کواکب معلوم می‌گردد از دایرهٔ نصف‌النّهار که بیشتر از سی درجه باشد، و آن نیز به تقریب؛ بدان سبب که مسطرهٔ ثالثهٔ این آلت، که مقدار وتر زاویه ازو معلوم می‌شود، مؤثّر حقیقت زاویه نیست. و صاحب «مجسطی» این آلت را مخصوص داشته است از جهت معرفت اختلاف منظر قمر را در دایرهٔ ارتفاع، و معرفت غایت عرض قمر را. و کسوفات و ابعاد و اجرام[13] (M: 47) مبنی بر اختلاف // منظر قمر است و بر غایت عرض.

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چون نظر کرده آمد در این آلات ارصاد، بعد از مؤونت[14] و اخراجات بسیار و سعی در

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(1) SM: + الرّواق (2) M: ندارد. (3) S: ندارد. (4) S: ندارد. (5) S: اسکندری (6) M: ر (7) M: سیوم (8) S: او (9) («مقیس علیه» به معنی «اصل» است؛ ستاره‌ای که موقعیّت آن مشخّص است برای اندازه‌گیری وضعیّت ستاره‌ای دیگر مرجع یا اصل قرار می‌گیرد.) (10) S: فوایم (11) S: عنوان ندارد. (12) S: «و آن سه» ندارد. (13) M: + «و مقادیر اجرام» (14) S: مؤنت؛ M: مونت

روزگار دراز، مطالب از این آلاتْ اکثر به تقریبی حاصل می‌آید،[1] بدان سبب که این آلات اکثر حلقه
و قسی و دوایرند. اگر آلات کوچک می‌سازند، اجزاء او را به دقایق و ثوانی اعتبار نمی‌توان کرد، و[2]
مطالب به تحقیق و تدقیق معلوم نمی‌شود و به تقریب حاصل می‌آید. و اگر بزرگ می‌سازند، ممکن
نیست که استداره او را کما ینبغی به جای[3] توان آورد و فساد هر مطلب[4] بیشتر از علّت اوّل حاصل
می‌شود.

P: 36 و بندهٔ کمینهٔ کمالْ سالهاست که به دعاگویی دولت همایون پادشاه عالم، ایلخان // اعظم[5]
S: 19r شهنشاه روی زمین // سلطان غازان خان - خلّد الله ملکه و دوام علی العالمین ظلّه - مشغول است،
مدّتی[6] در این فکر بود و طالب آنکه آلات رصدی دست دهد که بی رنجی و مشقّتی[7] به تحقیق و
تدقیق مطالب ارصاد ازو حاصل شود تا به دولت پادشاه عالم - أدام الله ظلّه - دوازده نوع آلت رصد[8]
روی نمود که پیش از این هیچ کس از متقدّمان و متأخّران [را] مثل چنین آلات دست نداد و میسّر
نگشت که تمامت مطالب ارصاد از این آلات معلوم می‌شود به اندک مؤونت[9] و سعی به تحقیق و
تدقیق تمام، زیرا که این آلات تمامت مساطر و خطوط مستقیم‌اند و هر چند که دراز باشد، ممکن است
M: 48 که استقامت او را به جای[10] آورند // و به اجزاء و کسورْ قسمت و[11] اعتبار کنند تا مطالب[12] [به
تحقیق] و تدقیق معلوم می‌شود و چندین مهمّات دیگر از این آلات معلوم می‌شود[13] که ممکن نیست
که به آلات پنجگانه قدما معلوم گردد - چنانچه شرح داده آید - و این رساله را «رسالةُ الغازانیةُ فی
الآلات الرّصدیةُ»[14] نام نهادیم تا به دولت پادشاه جهان – خلّد الله ملکه – طالبان این علم از او فایده
می‌گیرند[15] و دولت و کامرانی او می‌خواهند –انّه ولیّ الإجابة.

این رساله را بر دو قسم نهادیم:
قسم اوّل: در ذکر صنعت آلات رصد و معرفت عمل از او؛
P: 37 قسم دویم: در استخراج موضع کواکب در طول و عرض از فلک // البروج.

(1) S: می‌یابد (2) M: ندارد. (3) M: جایی (4) SM: مطالب (5) M: + سه (6) S: مدّت (7) M: مشقیتی
(8) S: رصدی (9) S: مؤنت M: مونت (10) M: جا (11) S: قسمت و = او (12) S: + او (13) S: «و
چندین مهمّات دیکر از این آلات معلوم می‌شود» ندارد. (14) S: الرّصد (15) M: می‌کرند

قسم اوّل

S: 19v

در صنعت آلات // رصد و معرفت عمل از او

و این دوازده آلت[1] است:

[1] [آلت مثلّث]

که [از او] غایت ارتفاع کوکب از دایرهٔ نصف‌النّهار معلوم می‌گردد، به تحقیق و تدقیقِ تمامْ و به غایت پسندیده باشد از جهت معرفت غایت میل فلک‌البروج و معرفت عرض بلد را.

[پیکربندی]

دو مسطره بستانند که متوازی السّطوح باشند، و در[2] طول و عرض و عمق متساوی باشند، از مس یا از آهن یا از چوب ساج. و در دو طرفْ مسطره را چنان ترکیب کنند که اتّصال ایشان[3] بر M: 49 P: 38 S: 20r زاویهٔ[4] قایمه باشد و آن به گونیای[5] معمارین// بتوان دانست یا به ارسال شاقول. و مسطره‌ای دیگر بستانند که از این هر دو مسطرهْ طولش زیاده باشد؛ چنانچه نسبت طول او با طول یکی از دو مسطره همچون نسبت وتر ربع دایره گردد [با نصفِ قطرِ دایره].

پس، این قاعدهٔ مثلث را به خطّ مستقیم عرضْ او را منصّف گردانند و از آنجا که منتصف این خطّ باشد از طولِ مسطرهْ ثقبهٔ مستدیر بکنند. و مسطرهٔ دیگر بستانند که طول [او] نصفِ مسطرهٔ قاعدهٔ مثلث و در عرضِ و عمقْ کمتر از این مسطره‌ها بود که اضلاع مثلث‌اند و عرض این مسطره را[6] به خطّ مستقیم منصّف گردانند و قدر ثلث طول این مسطره را[7] از خطّ منتصف بردارند از یک سر، و طرف او M: 50 را حادّه سازند از جهت تقسیم // اجزاء را و این مسطره به جای[8] عضاده باشد. در[و] دو لبنه متساوی از مس یا از چوب یا از برنج بنشانند، چنانچه خطّ مستقیم که منصّف عرض مسطره است با خطّی که منصّف عرض لبنتین است متقاطع بر زاویهٔ[9] قایمه باشند و به بعدی معیّن از خطّ لبنهْ دو ثقبهٔ مستدیر بکنند و یک سر مسطرهٔ عضاده را به خطّ منصّف عرض به بُعد نصفِ عرضِ مسطرهٔ قاعدهٔ مثلث، ثقبهٔ P: 39 S: 20v مدوّر بکنند به قدر ثقبه‌ای که در وسط قاعدهٔ // مثلث است. پس، عضاده را به قطب و حلقه و فلس محکم کنند در ثقبهٔ مثلّث، چنانچه در // اسطرلاب بود.

پس، هریک از ساق[های] مثلث[10] به خطّ مستقیم به طرف عرض به سه بخش کنند و هر ساق[11] مثلث را به طول منصّف کنند تا از نصفیْ اجزایِ[12] تمامِ ارتفاعِ [کوکب] معلوم گردد و آن در وضع بود که اتصّال هر دو ساق به زاویهٔ قایمه باشد و از نصفِ دیگرْ ارتفاعِ آن [کوکبْ از] افق معلوم

(1) SM: آلات (2) S:+ هر (3) S: ندارد. (4) M: زوایه (5) SM: بکونیآ (6) S: ندارد. (7) S: ندارد. (8) M: بجایی (9) M: زاوایه (10) S: مثلثه (11) SM: ساقی (12) M: اجزآء

شود. و هر یک از آن قسم را که در وسط باشد قسمت کنند به اجزای[1] جیب ثمن دایره، که مب که له[2] [42;25,35=p] است. و یک بخش دیگر را که زیر این قسم باشد به حسب دقایق این اجزاء قسمت کنند و بر قسمی که بر بالای این هر دو قسم است، قسمت پنج پنج جزو بکنند و نام آن اجزاها M: 51 بنویسند به رقم و ابتدای قسمت و کتابت هر دو ساقی مثلث از آنجا کنند که زوایای مثلث است.//

پس، خطّ نصف النّهار را[3] اخراج کنند. و دیواری از سنگ و آجر و گچ برآورند بر خطّ نصف النّهار و[4] مثلثی دیگر از چوب محکم بتراشند و میانه آن را حفر کنند، چنانچه این مثلث را در آن بتوان نشاندن و [به] مسمار محکم گردانند و سرهای[5] مسمار را هموار سازند.

پس، این مثلث را در دیوار بنشانند، چنانکه قاعدهٔ مثلث موازی سطح افق باشد. و سطح مثلث P: 40 در سطح // نصف‌النّهار بود. و طریق او آن است که دو شاقول مخروطی شکل را بر دو خیط S: 21r رود[ه]کش[6] یا مس‌کش فرو آویزند، چنانچه رأس مخروط // سوی سطح زمین باشد. و باید از آنجا که خیط است تا سر مخروط هر یک مثل یکدیگر باشند در طول؛ پس، هر یک شاقول را بر طرف قاعدهٔ مثلث درآویزند آنجا که دو زاویه است؛ پس، هر گاه که سر هر دو مخروط مماس خطّ نصف‌النّهار شود، قاعدهٔ مثلث موازی سطح افق گردد و سطح مثلث در سطح نصف النّهار باشد.

[**کاربرد**]

M: 52 عمل ازو چنان باشد که هرگاه عضاده بر جزء مبدأ[7] قسمت نهند، یعنی آنجا [که] سر عضاده P: 41 منصّف زاویهٔ قایمه گردد، از این مثلثْ// دو مثلث حاصل آید چنانکه خطّ منصّف عضاده قایم مقام S: 21v خطّی[8] باشد که منصّف قاعدهٔ مثلث شود[9] و مثلث را دو مثلث قایم الزّاویه سازند: از یک مثلث ارتفاع کواکبی معلوم گردد که در جانب جنوب سمت‌رأس باشند. هرگاه که ارتفاع کوکبی[10] مطلوب بود، عضاده را تحریک می‌دهند[11] تا چندانکه[12] کوکب از دو ثقبهٔ هدفه دیده شود.

P: 42 S: 22r پس، در آن حالْ // مثلّثی حادث گردد // که یک ضلع او خطّی باشد موهوم[13] که منصّف زاویهٔ قایمه و قاعدهٔ مثلّث باشد و مقدار نصف قطر دایره باشد. و ضلع دیگر او از اجزای[14] ساق مثلث بود که مقسوم است. [پس،] آن نیز معلوم گردد و زاویهٔ نصف[15] قایمه که خطّ منصّف – که عمود M: 53 است بر قاعدهٔ مثلث – احداث کرده است، معلوم بود. // پس، زاویهٔ تمام ارتفاع که در این مثلث حاصل آمده باشد، معلوم شود،[16] اگر زاویه در نصف ساق مثلث بود که جانب زاویهٔ قایمه است. و زاویهٔ ارتفاع معلوم شود،[17] اگر در نصف [ساق مثلث] بود که جانب زاویهٔ نصف قایمه است.

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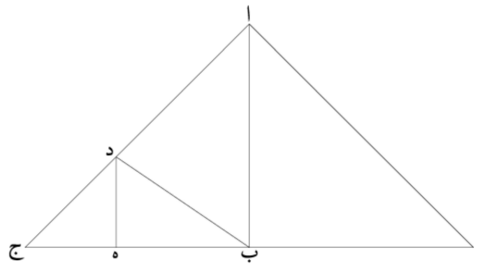
(1) M: باجزآء (2) SM: مثا له له (در نسخهٔ م در بالای «له» اخیر نوشته: ثانیه.) (3) M: ندارد. (4) M: دو (5) M: سرهاء (6) S: روکش (7) S: مبدء؛ M: مبدآ (8) S: خط (9) S: «مثلث شود» ندارد. (10) S: کوکب (11) M: + «و ساق» (12) S: انکه‌جدا؛ M: جدانکه (13) M: موموم (14) M: اجزائی (15) SM: نصف‌النّهار (16) SM: + و (17) SM: + و

برهانش:

مثلث اب ج[1] متساوی الساقین و زاویهٔ ا قایمه و زاویهٔ ج نصف قایمه و خطّ ب د عضاده که مارّه است از بصر راصد به مرکز کوکب و احداث کرده است مثلث ج ب د. و مطلوب در این مثلثْ زاویهٔ ج ب د است، پس از زاویه د عمود ده اخراج کردیم بر خطّ ب ج. پس، در مثلث د ج ه نسبت جیب زاویه د که نصف قایمه است با ضلع ده همچون[2] نسبت جیب زاویه قایمه است بر ضلع د جِ معلوم؛ پس، ده معلوم شد. و مربّع او را از مربّع د ج نقصان کردیم و جذر باقی گرفتیم، ج ه معلوم گشت. و خطّ ب ج که نصف قطر است معلوم. P: 43 S: 22v // پس، ب جِ باقی معلوم گشت و مربّع ب ه ده معلوم؛ پس، ب د[3] معلوم، و // جیب زاویهٔ ب نسبت ضلع ده است به ضلع ب د،[4] پس زاویهٔ ب معلوم شد؛ او ارتفاع کوکب باشد و اگر ما[5] زاویهٔ ج ب د را جزء جزء[6] فرض کنیم و زاویهٔ ج، که نصف قایمه است، هرگز[7] متغیّر نمی‌گردد، زاویهٔ ب د ج از قایمتین معلوم شود؛ مثلاً، نسبت جیب زاویهٔ ب اجزاء مفروض با ضلع[8] د ج همچون نسبت جیب زاویهٔ د[9] بود با ضلع ب ج، که نصف قطر است. پس، M: 54 مقدار د[10] معلوم شود. همچنین جمیع اجزاء ج ا معلوم // کنیم و جزء با ثمن دور و در جدول بنهیم، چنانچه وترِ جیب را نهاده‌اند، و آن جدول را «جدول اجزاء وتر ربع دایره» خوانیم.[11] پس، هرگاه که ما را از این آلت از خطّ ا ج مقداری معلوم شود، از آن جدولْ قوسِ او معلوم کنیم. آنْ مقدارِ زاویه ج ب د باشد. و اگر عضاده را قسمت کنند به شصت جزو؛ پس، از آنجا تقاطع اوست[12] با خطّ ساق مثلث، مقداری از عضاده معلوم شود. پس، هر گاه که جزو ثمن دایره که [مب که له] [=42;25,35p] است بر آن اجزاءْ منحط قسمت کنند تمام[13] زاویهٔ ارتفاع معلوم [می‌شود]؛ مثلاً، در مثلث ج ب د نسبت جیب زاویهٔ ج است به ضلع[14] ب د معلوم. پس، زاویهٔ د معلوم شود و زاویهٔ باقی از قایمتین معلوم شود. و امّا طریق بنشستن P: 44 S: 23r // به تحقیق ثوابتر از این باشد؛ زیرا که اجزای[15] عضادهٔ نصب قوس اندک می‌افتد و تدقیق نمی‌توان کردن اجزاء عضاده را. // و اجزاء ساق مثلث نصب قوس بسیارتر می‌افتد، ابتدای اجزای[16] آن را معلوم نمی‌توان کرد.

(1) اب ج = اب ن (2) S: «نسبت جیب زاویه د که نصف قایمه است با ضلع ده همچون» ندارد. (3) ب د = ب ج (4) SM: جیب زاویهٔ ب نسبت ضلع ده است به ضلع ب د = نسبت جیب زاویهٔ ب با ضلع ب د (5) S: با (6) SM: جزؤ جزؤ (7) M: هرکه (8) M: «ب اجزاء مفروض با ضلع ندارد» (9) د = د ج S: «همچون نسبت جیب زاویهٔ د ج» ندارد. (10) د = د ج (11) S: خانیم (12) S: او است (13) SM: تام (14) SM: اضلاع (15و16) M: اجزاء



[Figure 1] [Drawn based on the explanations given in the text]

[2] [آلت کامله]

که او شامل جمیع آلات است. از او ارتفاع کواکب معلوم می‌گردد در جمیع اجزاء[1] با سمت.

و این آلت تفضیل و ترجیح دارد بر جمیع آلات به چهار وجه:

وجه اوّل آنکه، آلاتی که مشهور است و مستعمل و اعتقاد متقدّمان و متأخّران بر آن است[2] هر M: 55 یک مخصوص‌اند به مهمّی[3] – چنانچه پیش از این// گفته آمد. و در این آلت جمیع مطالب آن آلات ممکن نگردد – چنانچه بعد از این شرح داده شود.

وجه دوم[4] آنکه، مؤونت و اخراجات[5] و سعی و شغل در این آلت کمتر از جمیع آلات قدماست؛

وجه سوم[6] آنکه، مطالب از آن آلات به تحقیق و تدقیق معلوم نمی‌گردد. بدان سبب که جمیع آن آلاتْ قسی و حلقه‌اند؛ اگر کوچک می‌سازند، قسمت او به دقایق و ثوانیْ اعتبار نمی‌توان کرد و مطالب به تقریب[7] حاصل می‌شود؛ اگر بزرگ می‌سازند، ممکن نیست که استدارهٔ او را کما ینبغی به جای توان آورد. پس فساد و خلل او زیاده از فایدهٔ[8] او می‌گردد. و در این آلت مسطره‌های مستقیم P: 45 است و خطوط مستقیم. و هرچند که دراز باشد ممکن است استقامت // او[9] به جای آوردن بی‌خللی و رنجی؛

S: 23v M: 56 P: 46 S: 24r وجه چهارم // آنکه، در آن آلاتْ قسی معلوم می‌شود و در این[10] // اجزائی که معلوم می‌گردد که حصّهٔ قوس او کمتر از او باشد بیشتر اوقات. پس، به تحقیق و تدقیقْ قوس معلوم می‌شود.

و بهترین آلات رصد که هست «آلت سمتیه» است که استاد آن فاضل ابوالعبّاس لوکری استنباط کرده است. و در او نیز این سه علّت آخرین موجود است. پس، معلوم شد که این «آلت کامله» بهترینِ آلات رصد است و شامل[11] همه است و ترجیح دارد بر دیگر آلات رصد.[12]

[**پیکربندی بخش پایهٔ ابزار–1**]

اوّل سطح زمین را مستوی کنند به سعی که ممکن گردد. و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب اخراج کنند. و باید که این[13] سطح را به سنگ و گچ محکم کرده باشند.

پس، بر نقطه‌ای که تقاطع خطّ نصف‌النّهار با خطّ مشرق‌ومغرب باشد دایره بکشند که قطر او سه اصبع[14] باشد. و آن مقدار را حفر کنند که عمق او نیم گز باشد. و باید که آنجا که این دایره حفر کرده باشند بر سنگی یا چوبی به غایتْ محکم باشد.

(1) M: اوضاع (2) SM: این نسبت (3) S: بهمین؛ M: بهمی (4) S: دویم (5) (به معنی «هزینه‌ها») M: اجزاجات (6) S: سیوم (7) M: تقریبی (8) S: قاعده (9) S: ندارد. (10) M: + صورت (11) M: شامله (12) S: «و شامل ... رصد» ندارد. (13) M: + را (14) S: ذرع؛ M: عرض

پس، مقیاس مستدیر اسطوانی شکل[1] بستانند از آهن که غلظ او مقدارِ دایرهٔ محفوره[2] باشد و طولش یک گز باشد. پس، آن مقیاس را در آن حفر به قلعی یا سرب محکم گردانند.

پس، دو مسطرهٔ متساوی مستقیم متساوی السّطوح از مس یا از چوب ساج بستانند که طول هر یک پانزده گز باشد و عرض چهار اصبع و عمق // دو اصبع، اگر مسطره از مس[3] باشد، و ربع گزی، P: 47 S: 24v [اگر] از چوب بود. و سطوح ایشان را مستوی گردانند و خطّ منصّف عرض از طول هر دو بکشند.

پس، هر دو مسطره را بر منتصفِ[4] هر دوْ متقاطع[5] گردانند به زاویهٔ قایمه، چنانچه منصّف هر دو خطّ که قایم [بر] عرض هر دو مسطره‌اند بر یکدیگر عمود باشند. و این هر دوْ مسطرهٔ قطر بُوَند. و آنجا که نقطهٔ تقاطع هر دو منصّف[6] است ثقبهٔ مستدیر بکنند که قطر او سه اصبع باشد.

پس، اگر این مسطره را قطر از مس باشد، دو مسطره[7] از چوب که طول و عرض ایشان مثل طول و عرض مسطرهٔ قطر بود و عمق هر یک چهار اصبع باشد، به مسمار محکم کنند و سرهای مسمار هموار[8] کنند و مسطره چوب را به قدر ثقبه سوراخ کنند. پس، مقیاس را در ثقبه بنشانند،[9] چنانچه خطّ منصّف عرض مسطرهٔ قطر منطبق باشد بر خطّ نصف‌النّهار.[10] پس، این مسطره را به سنگ و گچ محکم گردانند، چنانچه موازی افق باشد.

پس، چهار مسطره متساوی بستانند که طول هر یک ده گز و دو ثلث گزی باشد؛ یعنی، نسبت طول مسطرهٔ قطر با طول هر یک همچون باشد که طول قطر دایره[11] است با طول او که وتر ربع دایره P: 48 S: 25r است. و عرض و عمق این چهار مسطره،[12] // عرض و عمق هر دو مسطرهٔ قطر باشد.

و مسطره‌های چهارگانه ترکیب کنند با هر دو مسطرهٔ[13] قطر، چنانکه هر یکی از این مسطره‌های چهارگانه، وتر زاویهٔ قایمه گردند. و از[14] ترکیب این مسطره‌ها[15] مربّعی متساوی الأضلاع حادث شود که در او چهار مثلّث متساویْ‌ساقین قایمْ‌زاویه بود. و باید که هر زاویه از این مربّع متساوی‌الأضلاع بر نقطهٔ مشرق و مغرب و بر شمال و جنوب بود.[16]

[پیکربندی بخش پایهٔ ابزار-2]

و اگر خواهند که اختصار کنند به چهار مسطره که اضلاع مربّع متساوی‌الأضلاع است بی دو مسطرهٔ قطر: سطحی را از زمین[17] به سنگ و گچ محکم گردانند. و به هر بعدی که خواهند دایرهٔ هندی برکشند. و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب اخراج کنند به محیط دایره. و آنجا که تقاطع این هر دو خطّ باشد به زاویهٔ قایمه در مرکز دایره، چنانکه گفته شد، ثقبهٔ مستدیر بکنند که قطرش نیم گز باشد.

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(1) S: «شکل» مکرّر (2) M: یحفوره (3) (دراینجا نسخهٔ M ناتمام می‌ماند.) (4) S: منصف (5) S: تقاطع (6) S: منتصف (7) S: مسطرهای (8) S: مسوار همواره (9) S: ثقبه را در مقیاس بنشانند (10) S: + منطبق باشد (11) S: قطر با طول دایره (12) S: «عرض و عمق این چهار مسطره» مکرّر (13) S: (کاتب نوشته «مسطرهای» و سپس بر «های» خط کشیده و در بالا «ۀ» افزوده.) (14) S: آن (15) S: مسطرهای (16) S: + هر زاویه (17) S: + را

P: 49
S: 25v

و مقیاس استوانی قایم گردانند از چوب محکم که غلظ او قدر حفر بود و ارتفاعش از سطح زمین مساوی ارتفاع مربّع باشد. و بر مرکزِ رأسْ اسطوانهٔ دیگر از آهن قایم کنند که قطر او دو اصبع[1] باشد. پس، چهار مسطره را به هم ترکیب کنند تا مربّع متساوی‌الأضلاع حادث شود که چهار زاویه بر چهار نقطهٔ معین باشد، چنانچه خطّ مشرق‌ومغرب و خطّ // نصف‌النّهار // چهار زاویهٔ مربّع را تنصیف کنند. پس، این مربّع را به سنگ و گچ محکم کنند، چنانچه سطوح اضلاع مربّع موازی سطح افق باشد. و باید که بلندی مربّع از سطح زمین قدر نیم گز بیشتر باشد، مساوی سطح اسطوانهٔ کبیر. پس، هر ضلعی از اضلاع مربّع بر این اجزاء رسند و آن اقسام دیگر را به دقایق اجزاء و ششْ ششِ درجات، چنانچه رسم آلات آن است که قسمت کنند.[2]

[پیکربندی بخش بالایی ابزار]

پس، مسطره بستانند از مس یا برنج متوازی به سطوح که طول [آن] بیشتر از نصف قطر مربّع باشد به قدر نیم گز یا بیشتر. و بر[3] یک سر این مسطرهْ مدوّر شکلی کنند از جهت ثقبه را، همچون عضادهٔ اسطرلاب. و باید که آنجا که نهایت سطح مسطره است تا مرکز ثقبهٔ عضاده به خطّ مستقیم بود. و عرض و عمق این مسطره دو اصبع و نیم باشد و این مسطره، «مسطرهٔ سمت» بود.

پس، به بعد ربع گزی از ثقبهٔ عضادهْ مسطرهٔ دیگر بر او عمود کنند، چنانچه خطّ محیط سطح این مسطرهٔ عمود قایم بود. و طول این [مسطرهْ مساوی طول] مسطرهٔ سمت باشد از آنجا که تقاطع هر دو مسطره است با طرف مسطرهٔ سمت[5]. و عرض و عمق او کمتر از عرض و عمق مسطرهٔ سمت بود.

P: 50
S: 26r

و مسطره‌ای دیگر بستانند هم از مس و برنج. طول او بیشتر از طول این هر دو مسطره باشد، چنانچه نسبت طول او با طول هر یکی همچون نسبت وترِ ربعِ [دایره] بود // با نصفِ قطرِ دایره. و عرض و عمق او مثلِ مسطرهٔ عمود باشد. و این «مسطرهٔ وتر» بود. او را با طرف هر دو مسطره ترکیب کنند تا مثلّثی متساوی‌ساقین قایم‌زاویه حاصل آید. و مسطرهٔ وتر آن [را] – چنانچه در آلت مثلّث گفته آمد[5] – قسمت کنند [به اجزاء،] و اجزاء را به دقایق و کسوری که ممکن بود.

پس، مسطره‌ای دیگر بستانند از مس یا برنج که طول و عرض [او مثل طول و عرض هر یک از دو] ساق مثلّث باشد. و عمق او قدر اصبعی بود. و بر[6] یک طرف ثقبهٔ مستدیر بکنند که جای قطب بود. و از سر دیگرِ مسطره از خطّ منصّف[7] عرضْ قدر ثلث طول عضاده بردارند از جهت شمردن اجزاء را. و این مسطره به جای عضاده بود. وآنجا که زاویهٔ قایمهٔ مثلّث است، ثقبهٔ مستدیر بکنند، مثل ثقبهٔ عضاده،

(1) S: اصابع (2) (عبارت مغلق است؛ امّا آشکارا منظور مؤلّف آن است که هر ضلع مربّع را در طول به سه قسمت تقسیم می‌کنند و یک بخش را به ازای هر ۶°، بخش میانی را به ازای هر ۱° و بخش آخر را بر حسب دقایق – تا جایی که مقدور است – مدرّج می‌سازند.) (3) S: هر (4) S: مسطره است با طرف مسطرهٔ سمت = است مسطره و سمت (5) S: آید (6) S: و بر = وتر (7) S: منتصف

و مسطرهٔ عضاده را به این ثقبه به قطب و حلقه و فرس محکم کنند. و دو هدفه در عضاده بنشانند، چنانچه ثقبهٔ هر دو بر خطّ منصّف [عرض] عضاده بود.

پس، ثقبهٔ مسطرهٔ سمت را در مقیاس اسطوانی، که در مرکز دایرهٔ هندی قایم است، بنهند، چنانچه مسطرهٔ سمت بر اضلاع مربّع باشد، به شرط آنکه مسطرهٔ سمت بر اضلاعْ موازی سطح افق بود. پس، مقیاس اسطوانی [را درونِ] ثقبه بکنند. و به حلقه و فلس محکم گردانند مثلّث را، چنانچه هر وقت که مسطرهٔ سمت را تحریک دهند، مثلّث که بر اوست[1] حرکت کند و آن طرفِ مسطرهٔ سمتْ [بر]
P: 51 S: 26v
مرکز[2] [ابزار] دایره احداث کند // که مرکز او طریق معرفت دایرهٔ هندی باشد.

[کاربرد]

[معرفت ارتفاعِ] کوکب و سمت ارتفاع [او] در وقت مطلوب چنان باشد که مسطره سمت [را]
P: 52 S: 27r
تحریک می‌دهند سوی // کوکبی که ارتفاع او مطلوب بود. و مسطره عضادهٔ مثلّث را تحریک می‌دهند تا چندانکه کوکب از هر دو ثقبه دیده شود. پس، در آلت مثلّثْ مثلّثی حادث گردد: یک ضلع او ساق مثلّث بود، که مثل نصف قطر دایره بود؛ و یک ضلع دیگر او از مسطرهٔ وتر [که از او] اجزاء معلوم گردد. و یک زاویه که نصف قایمه است دایم بر یک حال باشد. پس، زاویهٔ ارتفاع معلوم گردد – چنانچه پیش از این در آلت مثلّث بیان کردیم. و همچنین در ضلع آلت مربّع از آنجا که مسطره سمت باشد اجزاء معلوم گردد. و یک ضلع دیگر نصف قطر مربّع است، معلوم، که او مثل نصف قطر دایره است. و زاویهٔ قایمه را که تنصیف کرده از خطّ مشرق‌ومغرب، معلوم. پس، زاویه‌ای که در مرکز دایره حادث گشته باشد[3] معلوم شود. آنْ سمت ارتفاع کوکب باشد.

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(1) S: برو سمت (2) S: آن طرفِ مسطرهٔ سمتْ [بر] مرکزِ = آن طرف مرکز مسطرهٔ سمت (3) S: باشند

[3]

که ارتفاع کوکب در نصف‌النّهار و غیر نصف‌النّهار در جمیع جهات[1] معلوم گردد با سمت ارتفاع.

[پیکربندی]

سطحی از زمین را مستوی کنند. و به ساروج و گچ محکم گردانند. و آن قدر که ممکن باشد، سطح او را مستوی کنند. و حلقه بفرمایند از مس یا برنج. و هر چند که بزرگتر باشد برتر بود. و عرض او سه اصبع بود و عمق او دو اصبع. و باید که روی حلقه بسیار هموار و راست باشد. پس، روی محدب P: 53 S: 27v حلقه را خطوط ارباع بیرون آورند. و حلقه را موازیِ // سطح افق به گچ و سنگ محکم گردانند، چنانچه بلندی حلقه از سطحْ قُرب [دو و؟] نیم گز باشد و خطوط ارباع حلقه بر خطّ نصف‌النّهار و خطّ مشرق‌ومغرب بود. پس، هر ربع از سطح محدب حلقه را قسمت کنند در درجات و دقایق بکنند – چنانچه رسم است.

P: 54 S: 28r پس، به مرکز حلقه، آنجا که نقطهٔ تقاطع خطّ نصف‌النّهار با[2] خطّ مشرق‌ومغرب باشد، // دایره‌ای بکشند که قطر او ربع گزی باشد. پس، اسطوانه‌ای بستانند از چوب محکم به غایت مستدیر که غلظ او قدر حفر دایره باشد. و بر یک سر[3] اسطوانه، آنجا که مرکز رأس اسطوانه بود، دایره‌ای بکشند که قطر او دو اصبع باشد. و آن را حفر کنند، چنانکه عمق او ربع گزی گردد. پس، اسطوانهٔ آهن را در آن حفر محکم کنند، چنانچه عمود باشد بر سطح رأس اسطوانه. و آن سطح رأس اسطوانهٔ کبیر را در آهن گیرند و به مسمار محکم کنند و سرهای مسمار هموار کنند تا در وقت عملْ سطح رأس اسطوانهٔ کبیر فرسوده نشود. پس، اسطوانهٔ کبیر را در آن حفر، که مرکز حلقه است، محکم گردانند، چنانچه ارتفاع او مساوی ارتفاع حلقه باشد از سطح زمین که اگر دایره‌ای توهّم کنند که بر سطح حلقه گذرد، بر[4] سطحِ رأسِ اسطوانهٔ کبیر نیز بگذرد.

پس، مسطره‌ای بستانند از چوب ساج یا از مس، طولش زیاده از نصفِ قطرِ حلقه و عرض و عمقِ او سه اصبع. و بر یک طرف این مسطرهْ ثقبه بکنند مستدیرْ به قدر اسطوانهٔ آهن که عمود است بر سطح رأس اسطوانهٔ کبیر. و ثقبهٔ مسطرهٔ را در آن عمود به حلقه و فلس[5] محکم کنند تا مسطره، همچون عضاده، در محیط حلقه بگردد. از جانب زیر مسطرهْ آنجا که مماس محیط حلقه است مسمار کوچک P: 55 S: 28v حادّالرّأس فرو برند تا اجزاء سطح // محدب // حلقه را بشمارند در وقت عمل سمت.

(1) S: جهة (2) S: تا (3) S: سر یک (4) S: پس (5) S: فلسه

و باید که از آنجا که مسمار است تا سر مسطره که خارج حلقه باشد قریب نیم گز باشد.

پس، مسطره‌ای دیگر بستانند مثل مسطرهٔ اوّل در عرض و عمق[1] و طولش قدری کمتر. و عرض به خطّ مستقیم منصّف گردانند. و یک سر او را از جانبین چنان سازند که زایدهٔ طولانی حاصل آید. و در مسطرهٔ اوّل از آنجا که ثقبهٔ قطب است با فرسْ[2] بُعدِ شبریْ حفرهٔ مربّع طولانی [به اندازهٔ زایدهٔ مسطرهٔ ثانی سازند و] مسطرهٔ ثانی در او[3] بنشانند. پس، هر دو مسطره را به هم ترکیب کنند. و در پهلوی حفرْ سوراخی مستدیر بکنند چنانچه با زایدهٔ مسطرهٔ ثانی بگذرد از جانب پهلوی دیگر. و به مسماری محکم کنند، چنانچه مسطرهٔ ثانی در او حرکت کند، به شرط آنکه چنان نباشد [...][4]. و باید که از آنجا که پیوند هر دو مسطره است تا سر هر دو مسطره که خارج حلقه است مساوی باشد. پس، در مسطرهٔ [ثانی]، بر خطّ منصّف [عرض آن]، دو هدفه بنشانند. بُعد میان هر دو یک شبر کمابیش[5] بود.

و مسطرهٔ ثالثه بستانند، پس، زیاده از طول هر دو مسطره و عرضش یک اصبع و عمقش نصف اصبع. و او را قسمت کنند به اجزاء مسطرهٔ اولی که جزو طول[6] باشد؛ یعنی، نسبت طولش[7] با طول
P: 56
S: 29r
[مسطرهٔ] اوّل همچون نسبت اجزاء تربیع باشد با جزو. پس، هر دو طرف مسطرهٔ اوّل // و ثانی // که خارج حلقه است، آنجا که خطّ منصّف است، هر دو را بشکافانند، [چنانچه هر دو شکاف] متساوی باشند. و مسطرهٔ ثالثه را در شقّ مسطرهٔ اولی و ثانی بنشانانند. و طرف آخرِ مسطرهٔ ثالثه را در شقِّ مسطرهٔ ثانی به مسمار محکم کنند. و هر دو طرف شقِّ مسطرهٔ ثانی به مسماری بگذرانند تا در وقت حرکتْ مسطرهٔ ثالثه را در شقّ مسطره [ء اوّل] حرکت دهند[8]. و بایست[9] از شقّ او این است.

[کاربرد]

امّا معرفت ارتفاع و سمت ارتفاع کوکب چنان باشد که مسطرهٔ اولی را حرکت می‌دهند بر سطح حلقهٔ جانب کوکب. پس مسطرهٔ ثانی را نیز حرکت می‌دهند. و از هدفه نظر می‌کنند تا چندانکه کوکب از هر دو ثقبه دیده شود. پس زاویه‌ای که حادث گردد از مسطرهٔ اولی و ثانی، وتر ارتفاع بود. و از سطح محدب حلقه مابین [خطّ] مشرق‌ومغرب و مابین مسمار رقیق که مرکوز است در مسطرهٔ اولی، سمت ارتفاع کوکب باشد.

(1) S: + مسطرهٔ ثالث مقسوم (2) S: فرسه (3) S: دور (4) (به نظر می‌رسد در اینجا عبارتی جا افتاده است.)
(5) S: کمان بیش (6) S: اول (7) S: اولش (8) S: کنند (9) S: بلت

[4]

از او ارتفاع کواکب در جمیع جوانب معلوم کرده می‌شود.

[پیکربندی]

تا سطح زمین مستوی گردانند. و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب را اخراج کنند.

پس، دو مسطرهٔ مستقیم متساوی بستانند از چوب ساج. طولش هرچند که بیشتر باشد، بهتر (P: 57 S: 29v) است. و عرض هر یک چهار اصبع و عمق شِبری. و هر دو مسطره را در منتصف بر زوایای قایمه // متقاطع گردانند. // و خطّ مستقیم به منتصف عرض هر دو مسطره بکشند. و هر دو مسطره را موازی سطح افق محکم کنند، چنانچه خطّ منصّف [عرض] یک مسطره بر خطّ مشرق‌ومغرب منطبق باشد و خطّ منصّف [عرض] مسطرهٔ دیگر بر خطّ[1] نصف‌النّهار منطبق بود. پس، آنجا که تقاطع منتصف هر دو (P: 58 S: 30r) مسطره باشد، ثقبهٔ مستدیر بکنند که قطرش سه اصبع باشد // و عمق او نیم گز بیشتر بود.

پس، عمود بستانند از آهنِ به غایتْ مستقیمِ اسطوانی شکل که سطبری او قدر این ثقبه باشد که در وسط هر دو مسطره است و طولش چهار گز باشد. پس، این عمود را در آن ثقبه محکم گردانند به قلعی و سرب[2]. و باید که قایم باشد بر[3] سطح افق.

پس، دو مسطرهٔ دیگر بستانند متساوی از مس یا برنج، که طول هر یک نصف او در مسطرهٔ اوّل باشد، و عرض و عمق هر یک دو اصبع بود. پس، یک طرف هر دو را با هم ترکیب کنند چنانچه ترکیب پرگار باشد و متحرّک بود. یکی از آن دو مسطره را بر ظهر او سه یا چهار حلقهٔ به غایت مستدیر محکم کنند که فراخی هر حلقه مثل غلظ عمود حدیدی باشد اندک زیاده. و حلقه‌ها را در عمود کنند، چنانچه آن مسطره‌ای که در آن حلقه باشد قایم بود بر سطح افق و متحرّک باشد بر گرد عمود حدیدی. و این مسطره به جای نصف[4] قطر باشد. و آن مسطرهٔ دیگر، مسطرهٔ عضاده بود. در او [دو] هدفه بنشانند که ثقبهٔ هر دو بر خطّ[5] مستقیم منصّف [عرض] مسطره باشد.

پس،[6] آنجا که سر[7] آخر مسطرهٔ نصف قطر است که مماس سطح زمین است، حفر طولا[نی] به مسمار آهن بدوزند، چنانچه [مسطرهٔ وتر] متحرّک باشد در او. اینْ مسطرهٔ وتر بود. باید نسبت طول او (P: 59 S: 30v) با طول مسطرهٔ نصف // قطر همچون // نسبت وتر قایمه باشد با نصف قطر. و این مسطره را به اجزاء وتر ربع دایره قسمت کنند. و بر طرف دیگر مسطرهٔ عضاده بر خطّ منصّف شقّی بکنند به قدر مسطرهٔ وتر، به شرط آنکه طول مسطرهٔ نصف قطر و مسطرهٔ عضاده متساوی باشد و نسبت طول مسطرهٔ وتر به طول هر

(1) S: خطّ در (2) S: اسرب (3) S: هر (4) S: نصف‌النّهار (5) S: بر خطّ = بخطّ (6) (به نظر می‌رسد در ابتدای شرح مسطرهٔ وتر، عبار(ا)تی از دید کاتب مغفول مانده باشد.) (7) S: مر

دوْ نسبت مذکور بود.

پس، در آن دو مسطره از چوب ساج که بر خطّ مشرق‌ومغرب نهاده است، از خطّ منصّف از هر دو جانب او به قدر طول مسطرهٔ نصف قطر دو[1] [مقیاس] محکم و قایم کنند، چنانکه یکی بر نقطهٔ مشرق باشد و دیگر[2] بر نقطهٔ مغرب. و بر سر هر مقیاسْ مسطرهٔ مستقیم محکم کنند، چنانچه متحرّک باشد از جوانب. و طول هر یک مسطرهٔ مساوی طول مسطرهٔ وتر بود. و هر یک از آن مسطره را به اجزای وتر ربع دایره قسمت کنند. آن[3] هر دوْ مسطرهٔ سمت باشد.

و مسطرهٔ وتر را به بُعد سه جزو از قاعدهٔ مسطرهٔ نصف قطر، بر آن سطح که جانب سطح زمین باشد، بر منصّف او، مقیاسی دقیق محکم گردانند. با آنْ اجزای[4] دو مسطره را که بر خطّ مشرق‌ومغرب است، می‌شمارد.

P: 60
S: 31r

و چهار مسطرهٔ دیگر بستانند که طول هر یک گزی باشد تقریباً. و آنجا که تقاطع // هر دو مسطره [است] که بر خطّ مشرق[ومغرب] // و خطّ نصف‌النّهارند، ترکیب کنند، چنانچه مربّعی حاصل آید تا حافظ مسطرهٔ وتر بود که آنْ موازات افق بگردد.

[**کاربرد**]

امّا عمل از او چنان بود که مسطرهٔ نصف قطر [را] تحریک می‌دهند تا چندانکه کوکب از هر دو ثقبهٔ هدفه دیده می‌شود. آن مقدار که از مسطرهٔ وتر میان سر هر دو مسطره بود وتر تمام ارتفاع کوکب باشد. آن را در[5] جدول وترْ قوس کنند تا تمام ارتفاع کوکب معلوم شود.

پس، مسطرهٔ وتر را بجنبانیده، چنانچه موازی افق گردد. و مسطرهٔ سمت را، که بر نقطهٔ مشرق یا مغرب کرده است، جانب مسطرهٔ وتر آرند. از آنجا که مقیاس است تا[6] نقطهٔ مشرق یا مغرب از مسطرهٔ سمتْ وترِ سمت باشد. قوس اوْ سمتِ ارتفاع بود.

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(1) S: در (2) S: «از هر دو جانب ... باشد و دیگر» مکرّر (3) S: از (4) S: اجزای آن (5) S: + دو (6) S: با

[5]

که ازو ارتفاع کواکب معلوم گردد در جمیع جهات با سمتِ ارتفاع.

[پیکربندی]

اسطوانی راست بستانند از خشت متن.[1] بایست[2] اسطوان شکل یا متوازی بر اضلاع. و هرچند که طولش بیشتر باشد، بهتر باشد. پس یک برِ او را به قدر شِبْری بر شکل مخروط صنوبری راست کنند. و آن مخروط را در آهن یا در مس درگیرند تا استواء گردد. و حلقهٔ کوچک مستدیر از آهن بستانند، // چنانچه به قدر درمی کمتر بود، بر سر مخروط بنهند. و مقیاس دقیق راست بر سر مخروط فروبرند محکم. و باید که سر مقیاس مدوّر[3] باشد بر شکل کوکبه تا حلقه از سر مقیاس به در نیاید و متحرّک باشد بر مقیاس.

P: 61
S: 31v

پس، خیطی[4] باریک بستانند مس‌کش یا رود[ه]کش. و یک سر او را بر حلقه محکم[5] کنند و باید که خیط[6] دراز باشد.

پس، در میدانی یا در موضعی که او را وسعت کافی[7] فضایی باشد و سطح او هموار بود، در موضعی از زمین، دایرهٔ هندی بکشند. و هرچند که دایره بزرگتر باشد، بهتر بود. و خطّ نصف‌النّهار اخراج کنند به محیط دایره. و باید آنجا که دایره بود، // زمین او را به سنگ و گچ محکم کرده باشند.

P: 62
S: 32r

استون[8] را قایم گردانند بر مرکز دایره یا بر غیر مرکز دایره، چنانچه [خطّ] مستقیم که بر منتصف ستون باشد، عمود بود بر خطّ نصف‌النّهار. و باید آنجا که قاعدهٔ ستون است، ثخن نهانی باشد از چوب محکم یا از آهن.

پس، از آنجا که قاعدهٔ استون[9] است، بر گرد او، حفر مستدیر بر شکل حلقه بزنند،[10] چنانکه عرض و عمق [او] قدرِ نیم اصبع باشد. و حلقه بستانند از آهن به غایت مستدیر. و سطبری آن از آن حفر مستدیر قدر[ی] کمتر بود.

خیط هم از روده‌کش دراز در حلقه محکم کنند. و آن خیط را «خیط ظلّ» می‌خوانیم. پس آن حلقه را در آن حفر که بر گرد قاعدهٔ اسطوان است، درگیرند. و[11] حلقه را محکم و راست کنند، چنانچه حلقه به آسانی بر گرد قاعدهٔ اسطوان دور کند و از حفر به در نیاید.

و خیط دیگر دراز بر حلقه و زرهٔ کوچک ببندند. و زره را آنجا که قاعدهٔ اسطوان است، به خطّ نصف‌النّهار فروکوبند. آنْ «خیطِ[12] نصف‌النّهار» باشد.

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(1) («متن» به معنای «استوار»، «زمین درشت و بلند»، «جای بلند و سخت»، «قلعه و حصار» و ... است؛ در اینجا به معنای خشتی محکم است از آن نوع که در ساخت قلاع یا بناهای محکم به کار می‌آمده.) (2) S: بایت (3) S: بدو (4) S: خطی (5) S: خط (6) S: «محکم» مکرّر (7) S: کاهی (8) («اُستون» همان سُتون است (مخفّف آن: اُسْتُن). اسطوان/اسطوانه نیز معرّب آن است. در اینجا مؤلّف تقریباً از همهٔ این واژگانِ معادل استفاده کرده است.) (9) S: استوان (10) S: بزند (11) S: + هر (12) S: خط

P: 63
S: 32v

پس، عضاده بستانند بقدر ربع گزی. و در[و] دو لبنه بنشانند. و لبنه‌ها را متقاطر یکدیگر سـوراخ مستدیر کنند. و بر یک سر عضاده، آنجا که خطّ منصّف که به طول عـضاده قـایم اسـت، سـوراخ دقیـق بکنند. و آن خطّ رو[ده]کش را که بر سر مخروط اسطوان است، در آن ثقبـه درکـشند. // و از ظهـر // عضاده، آنجا که برابر هدفهٔ چشم راصد هر دو باشـد، حلقـهٔ کوچـک بـاره بـاره بنـشانند. و خیطـی هـم رود[ه]کش درو درکشند. و بر یک سر خیطْ شاقولِ رأسْ مخروط°[1] جانبِ زمین بود.

پس مسطره بستانند از مس یا چوب ساج. طولش قدرِ نصف یا ثلث یا عشر باشد از آنجا که سر مخروط اسطوان است تا مرکز قاعدهٔ اسطوان. و به آن نسبتْ مسطره را قسمت کننـد، چنانچـه اگـر قـدر نصف باشد، به سی، و اگر ثلث بود، به بیست، و اگر عشر بود به شش قـسم کننـد، چنانچـه نـسبت طـول اسطوان با جزو همچون نسبت قدر طول مسطره باشد به اجزاء او. [شکل] این است:[2]

[کاربرد: روش یکم]

P: 64
S: 33r

P: 65
S: 33v

امّا معرفت ارتفاع و سمت چنان بود که عضاده را به دست راست ثابت دارنـد. و نظـر[3] در هـر دو ثقبهٔ عضاده مـی‌کننـد. و بـه دسـت دیگـر خـیط عـضاده را مـی‌کـشند و از سـتون دور مـی‌شـوند تـا چندانکه[4] کوکب[5] از هر دو ثقبهٔ هدفه دیده شـود بـر خـطّ مـستقیم. و لامحالـهْ کوکـبْ مـسامت سـر مخروط اسطوان گردد. پس در آن حال // شاقول رها کنند تا سر شاقول مماس زمـین گـردد. آنجـا کـه مسقط سر شاقول بود، نشانی بکنند. پس، از موضع علامت تـا مرکـز قاعـدهٔ دایـرهٔ هنـدی، ظـلّ مـستوی ارتفاع کوکب[6] بود، زیرا که اگر شعاع به خطّ مستقیم ستون به منظرِ بصرِ راصـد رسـد و از منظـرِ بـصر خطّی اخراج // کنیم موازی خطّ [نصف‌النّهار]، مساویِ این خطّ باشد. و آن خطّ، ظلّ ثانیِ [تمامِ] ارتفاع بود. پس خیط ظلّ را می‌کشند به موقع علامت، به مدّ تمام. آن مقدار از خیط، ظلّ مـستوی ارتفـاع بـود. آن را با مسطرهٔ مقسوم تطبیق [می‌کنند] تا اجزاء معلوم گردد از ظلّ. قوس او ارتفاع کوکب باشد.

پس، خیط نصف‌النّهار را بر خطّ نصف‌النّهار، که در دایرهٔ هندی اخراج کـرده باشـند، بنهنـد. و می‌کشند به مدّ تمام، به شرط آنکه خیط از خطّ نصف‌النّهار جدا نشود تا زاویه‌ای حادث شـود از آن دو خیط و مرکز قاعدهٔ ستون.[7] آن زاویه، تمامِ سمتِ ارتفاع بود. پس، از نقطـهٔ علامـت، کـه موقـع شـاقول است در وقت رصد، عمودی اخراج کنند بر خطّ نصف‌النّهار. آن مقدار جیبِ زاویه باشـد. آن را تطبیـق با مسطرهٔ مقسوم کنند تا اجزاء [خیطِ] ظلّ [معلوم شود]. پس، با هر واسـطهْ اجـزاء معلـوم گـردد کـه [بـه حسبِ] اجزاء خیط ظلّ بود که وتر قایمه است، آن اجزاءْ جیبِ تمامِ سمتِ کوکب باشد.

(1) (شاقولی که سر آن به شکل مخروط باشد یا رأس مخروطی شاقول) (2) (این تنهـا جـای رسـاله اسـت کـه مؤلّف مستقیماً به شکل ارجاع می‌دهد.) (3) S: قطر (4) S: چنانکه (5) S: + را (6) S: قاعدهٔ دایرهٔ هندی، ظلّ مستوی ارتفاع کوکب = قاعدهٔ استوی ارتفاع کوکب دایـرهٔ هنـدی (7) S: تـا زاویـه‌ای حـادث شـود از آن دو خیط و مرکز قاعدهٔ ستون = با زاویهٔ حادث و دو مرکز قاعدهٔ ستون

[آزمون درستی نتایج]

امّا امتحان صحّت سمت و[1] ارتفاع چنان باشد که از آن سمت خیط ظلّ را می‌کشند تا آنجا که علامت موقع شاقول است به مدّ تمام. پس، از آن خیط ظلّ هرجا که خواهند نقطه‌ای تعیین کنند از سطح زمین. و خیط نصف‌النّهار را نیز می‌کشند چنانچه منطبق باشد بر خطّ نصف‌النّهار که داخل دایرهٔ هندی است. پس، از نقطهٔ معیّن[2] عمودی اخراج کنند به خیط نصف// النّهار تا مثلثی حادث گردد که یک [ضلع] او از خیط نصف‌النّهار باشد، و ضلع ثانی از عمود بود، و ضلع ثالث او از خیط ظلّ بود. پس، به مسطرهٔ مقسوم ضلع[3] ثانی و ثالث را اجزاء معلوم کنند. به واسطهٔ آن دو مقدار، ضلع ثانی را مقداری معلوم کنند به حسب آنکه ضلع ثالث وتر زاویهٔ قایمه باشد.[4] آن مقدار، جیبِ تمامِ سمتِ ارتفاع بود. اگر موافق اوّل باشد، عمل ازو صحیح بود.

P: 66
S: 34r

امّا امتحانِ صحّت ارتفاع: مثلّثی که اوّل حادث شده باشد از اخراج عمود از موقع علامت، که مسقط شاقول است در زمان رصد، به خیطِ نصف‌النّهار. پس، آن ضلع که آنِ عمود است و آن ضلع دیگر که آنِ خطّ نصف‌النّهار است، به مسطرهٔ مقسومْ اجزاءْ معلوم کنند. و هریک را مربع کنند. و جذر مجموع بستانند. اگر موافق ظلّ ثانیِ ارتفاع باشد، صحیح بود ارتفاع.

[کاربرد: روش دوم]

و اگر ستونی چنانکه گفته آمد، به جمیع شرایط بر خیط نصف‌النّهار قایم کنند، چنانچه خطّ منصّف ستون عمود باشد بر خطّ نصف‌النّهار. و خیط عضاده را به قدر طول ستون بستانند. و بر طرف او عضاده‌ای محکم ببندند. و کوکب را رصد کنند تا مرکز جرم کوکب از هر دو ثقبهٔ هدفهٔ عضاده دیده شود. پس خیط[5] ظلّ، که در این // آلتْ خیط[6] وتر است، بکشند به طرف عضاده // تا خیط وتر چون خطّ مستقیم گردد. آن مقدار از خیط وتر، وترِ تمامِ ارتفاعِ کوکب باشد. و در آن حال شاقول فرو [اندازند] و از مسقط شاقولْ عمود اخراج کنند بر خطّ نصف‌النّهار و، چنانچه گفته آمد،[7] جیبِ تمامِ سمت معلوم کنند.

P: 67
S: 34v

و این آلت در کواکبی که در دایرهٔ نصف‌النّهار را اخراج کند به درازی هرچه تمامتر. پس، بر آن خیط[8] هر چند بعدی حاصل می‌شود از قاعدهٔ ستون نردبان پایه می‌سازند از گچ و سنگ تا ارتفاع کوکب به هر بعدی از سمت‌رأس معلوم گردد تا به حدّی که کوکب به افق باشد.

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(1) S: + خطّ (2) S: تعیین (3) S: ظلع (4) S: ضلع ثالث وتر زاویهٔ قائمه باشد = ضلع ثالث باشد که وتر زاویهٔ قائمه است (5) S: خطّ (6) S: و خطّ (7) S: آید (8) S: خط

[6]

که ارتفاع کوکب با سمت در جمیع اوضاع حاصل می‌شود به تحقیق.

در میدانی یا زمینی که واسع و عریض بود و مسطَح، موضعی را از زمین به گچ و سنگ وساروج محکم کنند و دایرهٔ هندی به احتیاط هرچه تمامتر به درآرند و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب اخراج کنند.

پس، مقیاس مدوّر که غلظ او قدرِ کم باشد و طولش یک اصبع زیادت[1] و سرش مدوّر باشد [بستانند]. و حلقه‌ای کوچک را بر مرکز دایرهٔ هندی [نهند]. و مقیاس را بر مرکز محکم کنند، چنانچه قایم بود.

پس، خیطی[2] رود[ه]کش بر حلقهٔ مقیاس محکم ببندند، چنانکه حلقه بر گرد مقیاس دور کند.

P: 68 S: 35r پس، // بر سرِ دیگر خیطْ عضاده بندند که // طولش قدر شبری کمابیش بود، چنا[نـ]ـچه] در وقت مدّ خیط، خطّ منصّف [عرض] عضاده با خیطْ یک خطّ مستقیم شود.

پس، مسطره بستانند قدر نصف یا ثلث یا ربع یا عشر آن خیط. و مسطره را قسمت کنند به جیب اجزائی که تمامت طول خیط جزو باشد. و حلقهٔ کوچک با زره در ظهر[3] عضادهٔ محاذی ثقبه‌ای که بر بصر راصد است محکم کنند. و خیط و شاقول بر صفت مذکور در حلقه گذرانند.[4]

پس، چون خواهند که کوکب را ارتفاع معلوم کنند، عضاده به هر دو دست محکم بگیرند و مستقیم می‌دارند و ثقبهٔ هدفه بر چشم نهند و روی جانب کوکب نهند. و نظر می‌کنند تا وقتی که خیط[5] سخت کشیده شود تا چون خطّ[6] مستقیم گردد و کوکب از هر دو ثقبه دیده شود. و اگر کوکب از افق مرتفع گشته باشد، کرسی بر شکل منبر زیرپای نهد تا مرتفع شود.

پس، چون کوکب دیده شود از ثقبه، در آن حال، شاقول رها کنند تا رأس شاقول مماس زمین گردد. پس، از آنجا که [سرِ] خیط است تا رأس شاقولْ جیبِ ارتفاع کوکب باشد. و آن سطح زمین را از آنجا که مسقط شاقول است، نشان کنند. پس، آن مقدار را از خیط شاقول که جیب ارتفاع است با[7] مسطرهٔ مقسوم تطبیق کنند تا اجزاءِ جیب معلوم // گردد. و قوسِ آنْ ارتفاعِ کوکب بود.

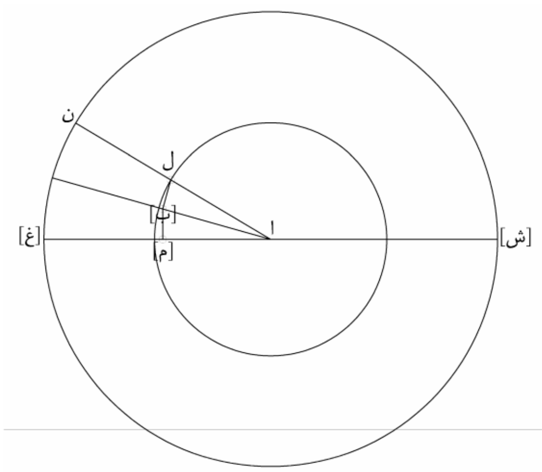
P: 69 S: 35v پس، همان خیط[8] را که بر حلقه و // مقیاس بسته باشند، بر خطّ مشرق یا مغرب نهند، که داخل دایرهٔ هندی است، و می‌کشند خیط را به مدّ تمام به شرط آنکه از محاذات خارج نگردد. پس، [از] آن نقطهٔ علامت که مسقط شاقول است، بر این خطّ عمود اخراج کنند و آن را با خیط دیگر برابر کنند.

(1) (جمله اندکی ناواضح است، ولی محتملاً منظور مؤلّف آن بوده که ضخامت مقیاس کوچکتر از 1 اصبع و ارتفاع آن بزرگتر از 1 اصبع است.) (2) S: خطی (3) S: ظهرهٔ (4) S:گذرانیده (5) S: خیطی (6) S: خیط (7) S: تا (8) S: خط

آنْ جیب سمت ارتفاع بود. آن را با مسطره مقسوم تطبیق کنند تا[1] اجزاء معلوم گردد به حسب اجزایِ تمامِ ارتفاع. و به واسطهٔ آن اجزاء، آن مقدار باقیِ اجزائی که معلوم گردد، به آن نسبت که آن خطّ که تمام ارتفاع استْ وتر زاویهٔ قایمه است، س [= 60] جزو باشد.

مثال: نصف ظاهر از فلک [غ‌ن‌ش] و کوکبی که ارتفاع او مطلوب است نقطهٔ [ن] و دایرهٔ هندی که موضع [بر] نقطهٔ [ا] است، [که وسطِ خطِّ] مشرق و مغرب است و موضع مقیاس او. خطّ مستقیم که خیط با عضاده است [ال]. و خیط شاقول در وقت رصد [ل‌ب]. و خطّ مستقیم موهوم از قاعدهٔ مقیاس تا[2] مسقط شاقول [اب]. پس، زاویهٔ [ل‌اب] ارتفاع کوکب است، یعنی خطّ [ل‌ب] جیب زاویهٔ [ارتفاع است که] عمود است بر نقطهٔ [ب]. و [اب] جیب تمام[3] او [= ارتفاع]. خطّ باریک که وتر زاویهٔ قایمه است، [ل‌ب] بود. و خطّ که بر خیط مشرق‌ومغرب منطبق کردیم به استقامتْ [اغ]. // P: 70 پس، از نقطهٔ [ب]، [که] مسقط شاقول بود، عمود اخراج کردیم بر خطّ [اغ]. این خطّ جیبِ زاویه را // S: 36r سمت بود[4] به آن اجزاء که از جیب تمام ارتفاع کوکب است، [که این] معلوم است به تطبیق مسطرهٔ مقسوم که برابر [چند] اجزاء است، امّا به آن مقدار که از وتر زاویهٔ قایمه است که باشد چند بود، معلوم کردیم سمت ارتفاع کوکب در زمان مطلوب.

(1) S: با (2) S: با (3) S: «تمام» مکرّر (4) (یعنی: خطِّ جیبِ زاویهٔ سمت بود.)



[Figure 2] [Reconstructed based on the figure drawn in the manuscripts: S: fol. 36r]

[7]

از جهت معرفت میل کلّی را و عرض بلد را و غایت ارتفاع کوکب را به غایت تحقیق و تدقیق.

از چهار مسطرهٔ متساویِ متوازی الأضلاع که از مس باشد، مربّعی سازند متساوی بر اضلاع قایم الزّوایا.

P: 71 S: 36v پس، این آلت را نصب کنند در سطح نصف‌النّهار، چنانچه دو // ضلع که مقابل هم باشند // موازی سطح افق بود و دو ضلع دیگرِ مقابلْ عمود باشند بر سطحِ افق.(1)

پس، دو طرف ضلع بالایین که موازی افق است، آنجا که زاویهٔ [قایمه] است، ثقبهٔ(2) مدوّر بکنند. و هر ضلعی را به شصت جزو قسمت کنند. و آن اجزاء را به دقایق و ثوانی که ممکن گردد. و ابتدای قسمت از جانب هر دو ثقبه کنند که بر زاویه است.

پس، [مسطره‌ای] بستانند که نسبت طولش با طول هر ضلعی همچون نسبت وتر قایمه است با
P: 72 S: 37r نصف قطر. و بر یک طرف مسطرهٔ ثقبهٔ مدوّر بکنند به قدر آن ثقبه // که بر طرف // ضلع مربّع است. و باید که این ثقبه را مرکز(3) بر آن خطّ مستقیم باشد که نهایت سطح این مسطره است. و بر اینْ دو هدفه بنشانند - چنانچه رسم است - و این مسطرهٔ عضاده باشد.

[کاربرد]

امّا عمل از آن چنان باشد که [اگر] ارتفاع کوکبی مطلوب باشد، عضاده را بر ثقبه‌ای که بر ضلع مربّع است، موافق جهتی که کوکب باشد از سمت‌رأس، به قطب و حلقه و فلس محکم گردانند، اگر عضاده واحد باشد؛ و اگر در هر ثقبهْ عضاده باشد، عضادهٔ آن ضلع مقسوم مربّع(4) را که سوی کوکب باشد، تحریک می‌دهند، چندانکه(5) کوکب از هر دو ثقبه دیده شود.

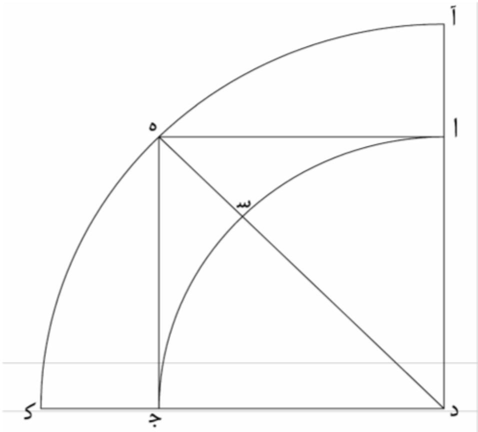
پس، آنجا که طرف عضاده باشد از ضلع(6) مقسومِ مربّعْ ظلّ اوّل ارتفاع افتاده باشد. و ظلّ تمام ارتفاع بود، اگر ارتفاع در ضلع تمام ارتفاع افتاده باشد. قوس او ارتفاع کوکب باشد یا(7) تمام ارتفاعش.

رُبعِ فلکْ قوسِ آ ک. و خطّ د ا و د ج نصف قطر دایرهْ متصّل‌اند به قوسِ ربعِ آ ه. و قوس آ ه ثمن آ د د آ(8). و خطّ ا ه و ه ج خطّ مماس که عمودند بر نصف قطر. و خطّ [د ه] خطّی که از مرکز بیرون آمده است و به نقطهٔ ه که ثمن دور است بگذشته و قطع کرده خطّ مماس. پس، خطّ ا ه و ه جْ ظلّ
P: 73 S: 37v قوسِ ا س و س ج(9) باشد، یعنی ظلّ ثمن دور که // مثل نصف قطر دایره باشد.

و خطّ [د ه] عضاده است در وقت رصد کوکب و قطع کرده خطّ ا ه مماس را. پس، خطّ ا د(10)

(1) S: + بود؛ «و دو ضلع دیگر مقابل عمود باشند بر سطح افق» مکرّر (2) S: زاویهٔ (3) S: «مرکز را ثقبه» (4) S: «آن ضلع مقسوم مربّع عضاده» (5) S: چندکه (6) S: ظلع (7) S: تا (8) (درمتن: اددو. نقطهٔ و در شکل وجود ندارد. دراینجا منظور آن است که قوس آ ه ثمن تمام دایرهٔ فلک است. تمام دایرهٔ فلک را به آ د د آ نشان داده‌ایم. در اینجا رساله دو شکل دارد؛ اوّلی در حاشیه و دومی در متن. گرچه عبارات متن واضح است، امّا نمادهای حرفی بر روی شکلها با متن همخوانی ندارد، لیکن شکل دوم تطابق بیشتری با متن دارد.) (9) S: اهح (10) S: اح

ظلّ قوس اس باشد. وهوالمطلوب.



[Figure 3] [Reconstructed based on the figure drawn in the manuscripts: S: fol. 37v]

[8]

که ارتفاع کوکب با سمت ارتفاع از این آلت معلوم می‌شود به غایت استقصا[1] و تحقیق. و P: 74 S: 38r جمیع[2] مطالب که از آلات ارصاد معلوم گردد، از این آلت دانسته // می‌شود // و محقّق با چندین مطالب دیگر. و این آلت ترجیح و تفضیل دارد بر آلات ارصاد قدما از چهار وجه مذکور.

[پیکربندی]

چهار مسطرهٔ متساوی بستانند از مس که عرض هریک چهار اصبع باشد و عمق دو اصبع و طول هرچند بیشتر باشد، به تحقیق صوابتر بود. و از این چهار مسطرهٔ مربّعی سازند متساوی بر اضلاع قایم.

پس، هر ضلع او را از طول منصّف گردانند به خطّ مستقیم تا نقاط [اربعه] تعیین[3] شود؛ یعنی، مشرق و مغرب و جنوب و شمال. و عرض هر ضلعی به سه قسم کنند از تقسیم اجزاء. پس، یک قسم از اقسام سه‌گانه را که بالای همه بود [به] شصت جزو بخش کنند. و ابتدای قسمت از نقاط اربعه کنند، چنانچه هر ضلعی از اضلاع مربّع به صدوبیست جزو شود. و یک قسم دیگر که در وسط است، اقسام دقایق اجزاء کنند. و قسم آخر را اقسام ثوانی[4] که ممکن[5] گردد.

پس، سطح از زمین، بر وجه مذکور، محکم و مستوی گردانند. و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب به غایت صحّت اخراج کنند. و این مربّع را در محدّه چوب گیرند. وآن را به گچ و سنگ محکم کنند موازی سطح افق، چنانکه خطوط منصّف طول اضلاع مربّعْ هریک، که ابتدای P: 75 S: 38v قسمت‌اند، بر خطّی باشد از خطّ مشرق‌ومغرب و خطّ نصف‌النّهار – چنانچه نقاط اربعه // باشد. و باید که ارتفاع مربّع از // سطح زمین قدر [یک] گز باشد.

پس، در داخل مربّع آنجا که تقاطع خطّ نصف‌النّهار با خطّ مشرق‌ومغرب است، دایره‌ای بکشند که قطرش سه اصبع باشد. و آن را حفر کنند، چنانچه نیم‌گزْ عمقِ او گردد.

و عمودی بستانند از آهن به غایت مستوی و اسطوانی شکل. غلظ او قدرِ آن حفر بود که در داخل مربّع است. و طولش سه گز باشد.

P: 76 S: 39r پس، این عمود حدیدی را // در آن // حفر محکم گردانند به قلعی و سرب، چنانچه عمود باشد بر سطح افق.

پس، از چهار مسطرهٔ متساویْ از مسْ مربّعی دیگر سازند متساوی بر اضلاعْ قایم الزّوایا، چنانچه طول هر ضلعی از اضلاع متساوی نصف قطر مربّع اوّل [باشد].

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(1) S: استقضا (2) S: جمع (3) S: یقین (4) S: توالی (5) S: امکان

و بر یک ضلع مربّع ثانی، چهار(1) حلقهٔ مستدیر محکم کنند، چنانچه بعد میان حلقه‌ها متساوی باشد. و مرکز هر حلقه بر محاذی سطح ضلع مربّع ثانی باشد. فراخی داخل هر حلقه قدر غلظ عمود حدیدی بود اندک زاید، چنانچه حلقه‌ها در عمود حدیدی(2) دور کند. پس، حلقه‌ها در عمود حدیدی کنند، چنانچه ضلعی که درو حلقه کنند با ضلع(3) مقابل او عمود باشند بر سطح افق. و آن دو ضلعِ باقیِ مربّعِ ثانی موازی(4) سطح افق باشد، چنانچه هر وقت مربّع ثانی را تحریک دهند، ضلع اسفل او بر سطح اضلاع مربّع اوّل دور می‌کند و مماس او می‌شود.

پس، آنجا که اتصّال ضلع مربّع ثانی که درو(5) حلقه است با ضلع دیگر از جانبِ نشیب(6) [است] ثقبهٔ مدوّر بکنند. و آن دو ضلع دیگرْ هریکْ [به] شصت جزو قسمت کنند. و هر جزوی به دقایق و ثوانی که ممکن بود. و ابتدای قسمت از زاویه‌ای کنند که مقابل زاویهٔ ثقبه است بر [اضلاع دیگر].

P: 77
S: 39v

مسطره‌ای دیگر بستانند که نسبت طول او با طول // هر ضلعی از اضلاع مربّع ثانی همچون نسبت وتر ربع دایره // باشد با نصف قطر. و در[و] دو هدفه – چنانچه شرط است – بنشانند. و این مسطرهٔ عضاده بود. و بر یک سر او ثقبه‌ای [کنند متساوی با آن ثقبه‌ای] که بر زاویهٔ مربّع ثانی است، به شرط آنکه مرکز ثقبه بر خطّ مستقیم که منصّف(7) سطح عضاده [است،] باشد. پس، عضاده را به قطب و حلقه و فرس(8) در ثقبهٔ مربّع ثانی محکم کنند.

[**کاربرد**]

امّا عمل ازو چنان بود که مربّع ثانی را تحریک می‌دهند جانب کوکب [تا] از هر دو ثقبهٔ هدفه مرئی گردد. پس، آنجا که عضاده باشد بر ضلعِ مقسومِ مربّعِ ثانی، ظلٍّ ارتفاع یا(9) ظلٍّ تمامِ ارتفاع بود. قوس او، ارتفاع بود یا(10) تمام قوس او. و آنجا که ضلع مربّع ثانی باشد بر ضلع(11) مربّعِ مقسومِ اوّل، ظلّ سمت بود یا(12) ظلّ تمام قوس او. [تمام قوس سمت] معلوم گردد یا سمت معلوم کنند.

(1) S: چهل (2) S: عمودی بود (3) S: ضلعی (4) S: موافق (5) S: دور (6) S: شیب (7) S: نهایت (8) S: فرسه (9و10) S: با (11) S: ظلع (12) S: با

[9]

[پیکربندی]

بر سطح از زمین خطّ نصف‌النّهار اخراج [کنند].

پس، دو[1] مسطرهٔ مستقیم از چوب ساج عمود کنند از سطح زمین، چنانچه بعد میان هر دو چهار اصبع باشد؛ [و] موازی هم باشند؛ و خطّ نصف‌النّهار بر منتصف بُعد هر دو بگذرد.

P: 78
S: 40r

پس، مسطره‌ای دیگر بستانند [که طول آن] کمتر از طول هر دو عمود باشد. و عرضش چهار اصبع بود اندکی کمتر. و عمق او دو اصبع[2] // بود. و این مسطرهٔ عضاده باشد.

پس، یک سر او را بر بعدی که میان // هر دو عمود است بنهند و به مسمار مستقیم بگذرانند از هر دو طرف عمود، چنانچه مسمارْ موازی سطح افق باشد و مسطرهٔ عضاده متحرّک باشد میان هر دو عمود، به شرط آنکه سرِ دیگرِ مسطرهٔ عضادهْ مرتفع باشد از سطح زمینْ نیم گز تقریباً. و در[و] دو هدفه بنشانند - چنانچه شرط است.

پس، مسطره‌ای دیگر بستانند که عرض او دو اصبع بود و عمق اصبع و [نسبت] طولش با مسطرهٔ عضاده نسبت[3] وتر قایمه باشد با نصف قطر. و اینْ مسطرهٔ وتر باشد.

پس، بر سر مسطرهٔ عضاده، که او تخته‌ای است میان دو عمود، شقّی بکنند مثل عرض و عمق مسطرهٔ وتر. و مسطرهٔ وتر را در آن شقّ بنهند. و هر دو طرف[4] شقّ را به مسمار درآرند، چنانچه مسطرهٔ وتر دوخته نگردد و متحرّک باشد در میان شقّ. پس، یک طرف مسطرهٔ وتر را در آن شقّ بنهند و هر دو طرف[5] شقّ را به مسمار بدوزند، چنانچه مسطرهٔ وتر دو جهته بگردد و متحرّک باشد در میان شقّ.

پس، یک طرف [مسطرهٔ] وتر را آنجا که مماس هر دو ستون است با مسطرهٔ عضاده، که تخته‌ای است [میان دو عمود]، مسمار ندوزند، چنانچه متحرّک باشد درو.

P: 79
S: 40v

پس، مسطرهٔ [وتر] را به اجزاء وتر ربع دایره قسمت کنند و ابتدا[ی قسمت] از آنجا [کنند] // که به مسمار مربوط است به هر دو مسطره[6]. پس حلقه و زره کوچک بر آن سر مسطرهٔ عضاده که شقّ // دارد محکم کنند. و خیطی از رود[ه]کش بر حلقه محکم کنند.

P: 80
S: 41r

پس، ستونی دیگر که طولش سیِ[6] آن دو ستون باشد [بستانند]. و بر یک سر او بکره ببندند. و ستون را قایم کنند بر خطّ نصف‌النّهار، چنانچه // بعد میان قاعدهٔ او و میان هر دو ستون اوّل بیشتر باشد // از طول مسطرهٔ عضاده. پس، آن خیط را در بکره بگذرانند.

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(1) (در نسخهٔ س کاتب نوشته «خطّ» و سپس بر روی آن خطّ کشیده است.) (2) S: «دو اصبع» مکرّر (3) S: بسته (4و5) S: طرف هر دو (6) S: ستون (7) (به معنی: مثل، مانند)

[**کاربرد**]

و چون خواهند که غایت ارتفاع کوکب معلوم کنند، خیط را می‌کشند به تدریج. و نظر در ثقبهٔ عضاده می‌کنند تا وقتی که کوکب دیده شود از هر دو ثقبهٔ هدفه.

پس، بُعدی که میان طرف شقِّ سرِ مسطرهٔ عضاده باشد و میان رأس مسطرهٔ وتر که مربوط است با هر دو عمود، وتر تمام ارتفاع بود.

از این آلتْ غایت ارتفاع کوکب معلوم می‌شود از دایرهٔ نصف‌النّهار.

[10]

[پیکربندی]

[د]و مسطره بستانند از مس متوازی السّطوح و در طول متساوی. و عرض و عمق یکی از این دو مسطره قدری بیشتر از دیگری باشد.

پس، بر یک طرف آن مسطره که عرض و عمق او بیشتر است حفر طولانی کنند به قدر عرض و عمق آن مسطرهٔ دیگر.

پس، آن مسطره را در آن حفر کنند، به شرط آنکه مسطرهٔ محفورْ قایم باشد بر آن مسطرهٔ دیگر و متحرّک باشد در جمیع طول او.

P: 81

پس، به خطّ مستقیم منتصف عرض هر دو مسطره را بگذرانند، چنانچه خطّ هر دو مسطره عمود باشند بر یکدیگر. و آن «مسطرهٔ جیب» باشد و مسطرهٔ دیگر، که متحرّک است در حفر، «مسطرهٔ سهم» بود. باید که بر یک سر // مسطرهٔ زهی گرفته باشند تا مرتفع باشد از سطح، و مسطره[1] از حفر بیرون نیاید.

S: 41v

پس، // مسطره‌ای دیگر بستانند در طول و عرض و عمق متساوی مسطرهٔ سهم. و یک طرف او را با طرف دیگر مسطرهٔ سهم ترکیب کنند همچون ترکیب پرگار تا برو متحرّک باشد. بر سطح خارج او، دو هدفه متساوی متقاطر می‌نشانند - چنانچه رسم است. و این «مسطرهٔ عضاده» باشد.

P: 82
S: 42r

و باید که این هر سه مسطره در طول // متساوی باشند.

پس، هر یک از مسطرهٔ جیب و سهم را قسمت به شصت جزو متساوی کنند، و هر جزو را به اجزائی که ممکن گردد. و ابتدای قسمت از آنجا کنند که حفر است در مسطره جیب؛ و در مسطرهٔ سهم از آنجا که نهایت طرفی که متصّل است به حفر.

پس، این آلت را در سطح نصف‌النّهار نصب کنند، چنانچه مسطرهٔ سهم موازی افق باشد و مسطرهٔ جیب عمود باشد بر سطح افق.

[کاربرد]

P: 83
S: 42v

امّا [عمل] ازو چنان باشد که مسطره عضاده را تحریک می‌دهند تا وقتی که کوکب مرصود از هر دو ثقبهٔ هدفه مرئی گردد. پس، مسطرهٔ // جیب را جانب مسطرهٔ عضاده [می‌آورند] تا[2] طرف مسطرهٔ [عضاده] مماسِ سطحِ مسطرهٔ جیب گردد. پس، از آنجا که طرف مسطرهٔ عضاده است تا حفر،

(1) S: مسطره و (2) S: با

1 از اجزای مسطرهٔ [جیب]، جیبِ ارتفاع باشد. از آنجا که حفر(1) است تا(2) طرف مسطرهٔ سهم کـه متـصّل است به مسطرهٔ عضاده، [از اجزای مسطرهٔ سهم]، جیبِ تمامِ ارتفاع باشد.

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(1) S: چوب (2) S: با

[11]

که ارتفاع کوکب[1] با سمت ِ]ارتفاع از این آلت] معلوم می‌شود در جمیع اوضاع به تحقیق.

[**پیکربندی**]

سطحی از زمین را مستوی گردانند. و خطّ نصف‌النّهار و خطّ مشرق‌ومغرب اخراج کنند به غایت صحّت.

پس، دو مسطره بستانند از چوب ساج یا از مس، متوازی‌السّطوح. و یک سطح ایشان را از عرض به خطّ مستقیمْ منصَّف گردانند. و هر دو را در منتصف متقاطع گردانند، چنانچه خطّ منصِّف [عرض] هر دو مسطره بر یکدیگر عمود باشند.

پس، هر دو مسطره را بر خطّ نصف‌النّهار و بر خطّ مشرق‌ومغرب نصب کنند موازی سطح افق، چنانچه خطّ مستقیم که منصّف عرض [هر دو] مسطره‌اند، منطبق گردند به خطّ نصف‌النّهار و به خطّ مشرق‌ومغرب. و به گچ و ساروج محکم کنند.

پس، [بر] نقطه‌ای که تقاطع[2] خطوط منصّف عرض مسطره‌هاست، حفر مستدیر بکنند، چنانچه قطر او دو اصبع باشد و عمق او شبری تقریباً.

P: 84
S: 42r

و عمود بستانند از آهن به غایت مستدیر. غلظ او قدر حفر مستدیر. محکم گردانند [او را در آن حفر] به قلعی و سرب، به شرط آنکه عمود // باشد بر سطح افق.

و چهار مسطرهٔ متوازی‌السّطوح بستانند متساوی، که طول هر یک گزی باشد تقریباً. و آنجا که اتصّال هر دو مسطرهٔ اوّل[3] است، ترکیب کنند تا مربّعی حاصل آید که حافظ آلت باشد[4] در وقت عمل.

پس، مسطره‌ای بستانند از مس به غایت مستدیر که غلظ او دو اصبع و نیم باشد. و بر یک طرف او سوراخ مستدیر بکنند به قدر عمود حدیدی. و این مسطره را به این[5] [عمود] حدیدی به حلقه و فلس محکم گردانند. و باید که این مسطره از آنجا که ثقبهٔ قطب است تا طرف او متساوی باشند از آنجا که ثقبهٔ عمود حدیدی است با طرف آن دو مسطره اوّل. و این «مسطرهٔ سهم» باشد. و سطح او را به خطّ مستقیم ارباع کنند.

پس، مسطرهٔ دیگر بستانند از مس متوازی السّطوح. در طول متساوی مسطرهٔ سهم باشد و عرضش دو اصبع و عمقش نیم اصبع. و بر یک طرف او انبوبهٔ به غایت مستدیر، که طولش سه اصبع باشد تقریباً

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(1) S: آلت (2) S: متقاطع (2) S: او (3) S: + و (4) S: بیان

و فراخی او به قدر غلظ مسطرهٔ سهم بود، ترکیب کنند، چنانچه مسطره بر انبوبه قایم باشد. پس، انبوبه در مسطرهٔ سهم بکنند، چنانچه انبوبه در جمیع طول مسطرهٔ سهم[1] متحرّک باشد، به حکم آنکه هر دو مسطره بر یکدیگر عمود و قایم باشند. و این «مسطرهٔ جیب» باشد.

P: 85 S: 43v پس، [بر] طرف دیگر مسطرهٔ سهم که مقابل // طرف عمود حدیدی است، مسطره‌ای دیگر از مس متوازی‌السّطوح ترکیب کنند، همچون ترکیب پرگار. و طولش مثل آن مسطره باشد. و بر سطح خارج آنْ دو هدفهٔ متساوی متقاطر یکدیگر محکم گردانند. و این «مسطرهٔ[2] عضاده» باشد.

پس، هر یک از آن مسطرهٔ سهم و جیب [را] آنجا که خطّ منصّف عرض است، به شصت[3] جزو قسمت کنند و هر جزوی را به اجزائی که ممکن گردد.

[کاربرد]

و چون خواهند که ارتفاع کوکب با سمت ارتفاع معلوم کنند از این آلت، مسطرهٔ عضاده را تحریک دهند جانب عضاده تا چندانکه طرف مسطرهٔ عضاده مماس سطح مسطرهٔ جیب گردد و خطّ مستقیم که منصّف عرض هر دو مسطره‌اند، بر یکدیگر قایم شوند. آن مقدار از مسطرهْ جیبِ ارتفاعِ کوکب باشد. و از آنجا که مسطرهٔ جیب است تا آنجا که اتّصال مسطرهٔ سهم با مسطرهٔ عضاده است از مسطرهٔ سهمْ جیب تمام ارتفاع کوکب باشد.

پس، مسطرهْ محکم دارند به جای خویش، و مسطرهٔ جیب را موازی افق گردانند و تحریک می‌دهند تا چندانکه[4] طرف مسطره‌ای که منطبق [بر] خطّ مشرق‌ومغرب است، مماس سطح مسطرهٔ جیب گردد، به شرط آنکه مسطرهٔ سهم ثابت باشد و خطوط مستقیم که منصّفِ عرض هر دو مسطره‌اند، P: 86 S: 44r قایم باشند بر یکدیگر. از آنجا که طرف مسطره که بر خطّ مشرق‌ومغرب منطبق است با طرف مسطرهٔ // جیب که عمود است بر مسطره سهمْ از [مسطرهٔ] جیبْ جیب سمت ارتفاع بود و از مسطره سهمْ جیب سمت تمام ارتفاع بود.

خاتمه

(1) S: + «بکنند چنانچه انبوبه» (2) S: + را (کاتب بر آن خطّ کشیده است.) (3) S: سه (4) S: چنانکه

در معرفت امتحان اعمال که در ضمن این رساله مبرهن است.

جدول

/P: 87; S: 44v/ جدول ظلّ ثانی و یسمّی بالظّل المستوی ایضاً

/P: 88; S: 45r/ [جدول] میل اوّل و میل ثانی

/P: 89; S: 45v/ جدول ظلّ معکوس

/P: 90; S: 46r/ [جدول] الجیب

/P: 91; S: 46v/ جدول سهم قوس نصف دور

/P: 92; S: 47r/ مطالع البروج بخطّ الأستوا؛ مطالع البروج فی موضع الرّصد، مراغه؛ مطالع البروج باوّل الجدی

/P: 93; S: 47v/ هذا الجدول السّمت الّذی لهذه العروض لکلّ خمس ربع ربع دائرۀ الأفق

/P: 94; S: 48r/ [سپید و بدون نوشته]

P: 95 S: 48v // جدولی چند از زیج خانی به جهت تصحیح آن [اعمال] که در ضمن این رساله مذکور است، تا به وقت حاجت رجوع به زیجات نشود:

آنچه در جداول این رساله مندرج است: - ظلّ مستوی از اصباع و اقدام؛ - ظلّ معکوس؛ و - میل اوّل و ثانی؛ و - جیب و سهم؛ - و مطالع خطّ استوا و مطالع فلک مستقیم من اوّل الجدی و مطالع بلد رصد، که مراغه باشد و عرضش [لز ک] [= 37;20°]؛ و – تعدیل النّهار خطّ استوا (؟) و تعدیل
P: 96 S: 49r النّهار // مراغه؛ و – ساعات النّهار خطّ استوا (؟) و ساعات النّهار مراغه؛ و – سموت ربع دایره به تضعیف پنج پنج و دوایر مقنطرات، که محتاجٌ الیه این کتاب است، از جداول مرسوم گشت.

اتمام تحریر این سواد و پیشین روز پنج‌شنبه بیست و سیم ماه جمادی الثّانی سنۀ 94[2]1.

من افادات استاد البشر وعقل الحادی عشر قدماء حکما[1] در کیفیّت رصد کسوف افادهٔ وجوه فرموده‌اند که [ابزاری که] اقدم می‌نماید و به صحّت اقرب است، «ذات الشّعبتین» است که حکیم عظیم، بطلمیوس، در «مجسطی» اشاره به آن فرموده و متعرّض کیفیّت صنعت آن نشده و متأخّرون به وجوه کیفیّت صنعتش بیان نموده‌اند.

مسطره‌ای که به عکس شعاع کمیّت مقدار منکسف معلوم گردد: طریق صنعتش

که از چوبی راست که تغییر و اعوجاج‌پذیر نباشد، مسطره‌ای سازند مثل عضادهٔ اسطرلاب که بر دو طرف آن دو لبنه باشد. عرض یکی قریب چهار انگشت و دیگر ازو اوسع به قدر دو انگشت. و در میان لبنهٔ بزرگ ثقبه‌ای دقیق باشد مستدیر. و در لبنهٔ کوچکتر [بر] مسامت مرکز آن ثقبه، نقطه‌ای تعیین باید کرد که بر آن نقطهْ دایره رسم نمایند به بُعدِ نصف قطر صفحهٔ آفتاب؛ طریقش آنکه: لبنهٔ بزرگتر را دو روز پیش از کسوف محاذی آفتاب سازند و نظر کنند که شعاع آفتاب از لبنهٔ بزرگتر چقدر از لبنهٔ خوردتر منوّر// می‌گرداند و لامحالهٔ قدرِ منوّرْ دایره خواهد// بود. پس، [بر] نقطهٔ مسامت مشارالیها به بعد نصف قطر دایرهٔ مضیّهْ دایره‌ای رسم کنند.

P: 97
S: 49v

پس، قطر آن دایره را به دوازده قسم متساوی تقسیم نمایند که از اقسام اصابع قطر باشد و همچنان محیط آن دایره را به دوازده قسم منقسم سازند که از آن اصابع جرم معلوم شود. و از اقسام محیطْ انصاف اقطار را به مرکز دایره اخراج نمایند. و بر اقسام [اصابع] قطرْ دوایر رسم کنند تا اصابع جرم ظاهر شود. و اگر زیاده تدقیق خواهند، هر یک از اصابع قطر و اصابع جرم را به دقایق قسمت کنند.

[**طریق کاربردش**]

پس، در روز کسوف، آن دو لبنه را به دست گیرند[2] مسامت آفتاب و مترصّد باشند تا ظلّ ضعیف مثل پر مگس بر کنارهٔ شعاع پیدا شود. و آن زمان ابتدای کسوف باشد. به اسطرلاب در آن حال از شعاع آفتاب بدانند تا ساعات بدو کسوف معلوم شود و اگر در اوّل طلوع آفتاب شیشهٔ ساعت وضع[3] نموده باشد، ساعت بدو کسوف به وجهی دیگر معلوم شود[4] و مصحَّح گردد. و مترصّد باشد تا سیاهی به غایت رسد که زیاده نشود و میل به نقصان کند. از مقدار سیاهی دایرهٔ نور آفتابْ کمیّت اصابع قطر و جرم معلوم شود و از ساعات و ارتفاعْ کمیّت ساعات بدو کسوف و غایتش ظاهر شود. و چون سیاهی از دایرهٔ نور زایل شود، وقت تمام انجلاء باشد. والله أعلم.

(1) (منظور نصیرالدّین طوسی است) (2) S: بدستور (3) S: وضوع (4) S: «و اگر ... معلوم شود» مکرّر