The Toledan Tables in Castilian: Excerpts of the Planetary Equations

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Abstract

A recently recovered bifolio consisting of four pages contains excerpts of tables for the equations of the planets with headings in Castilian. Analysis of its contents, both astronomical and terminological, indicates that in all likelihood these excerpts represent parts of the Toledan Tables, and offer the first evidence for a copy of them in Castilian.

The General Archive of Navarre preserves an uncatalogued bifolio, once used in the binding of a book belonging to the shoemakers' and tailors' guild of Pamplona, Navarre, probably in the 16th century. The handwriting of this bifolio seems from the 14th century, although it has been dated to the 14th or 15th centuries, as indicated by Fernando Serrano, the first scholar to call attention to this bifolio, in his short and accurate description of it.[1] The bifolio contains valuable astronomical material: excerpts of tables for the planetary equations with titles and headings in Castilian. It will be argued here that these excerpts provide the first evidence for a copy of the Toledan Tables in Castilian.

[1] See Serrano 2009, pp. 549–552.

1. The four pages in the two folios contain a fragmentary copy of the planetary equations for Saturn, Venus, and Mercury. Two pages display entries for Saturn for arguments from 45;30º to 135;0º, one page for Venus from 135;30º to 180º, and another page for Mercury from 0;30º to 45;0º. We first note that Mercury comes after Venus; thus, the order given to the planets in this set of tables is Saturn, Jupiter, Mars, Venus, and Mercury, in accordance with the vast majority of medieval astronomical tables. Second, we note the use of signs of 30º, which distances this set from many analogous tables in the Alfonsine corpus, where signs of 60º are often used. Third, the argument is given at intervals of a half degree, which to our knowledge is unprecedented in any set of planetary equations in the West, and therefore makes the table for each planet twice as long as the most complete sets of tables for the same purpose. To be sure, this bifolio manages to fit 45º of the argument on each page, from which it follows that the entries for each planet should have filled four pages. If we consider the opening page for Saturn to have been on folio 1v (most probably after the tables for the equation of the Moon), the two pages preserved for Saturn should correspond to f. 2r–v, the one for Venus to f. 9r, and the one for Mercury to f. 9v. Thus, 16 out of the presumed 20 pages for the tables of planetary equations are missing.

The titles on the four preserved pages read *Complimiento de la tabla del ygoalamiento de Saturnus* (twice), *Complimiento de la tabla del ygoalamiento de Venus* and *Tabla del ygoalamiento de Mercurius*, respectively, and for each of them two different inks (red and brown) have been used (see Figures 1 and 2).

The argument is presented in two columns. The first column is written in blue and the entries correspond to signs 1–5 from 0;30º to 30;0º for each sign, and the second column is in red and displays their complement, signs 6–11 from 0;0º to 29;30º. For each value of the argument we are given five other columns, and we will refer to them as c_3, ..., c_7.

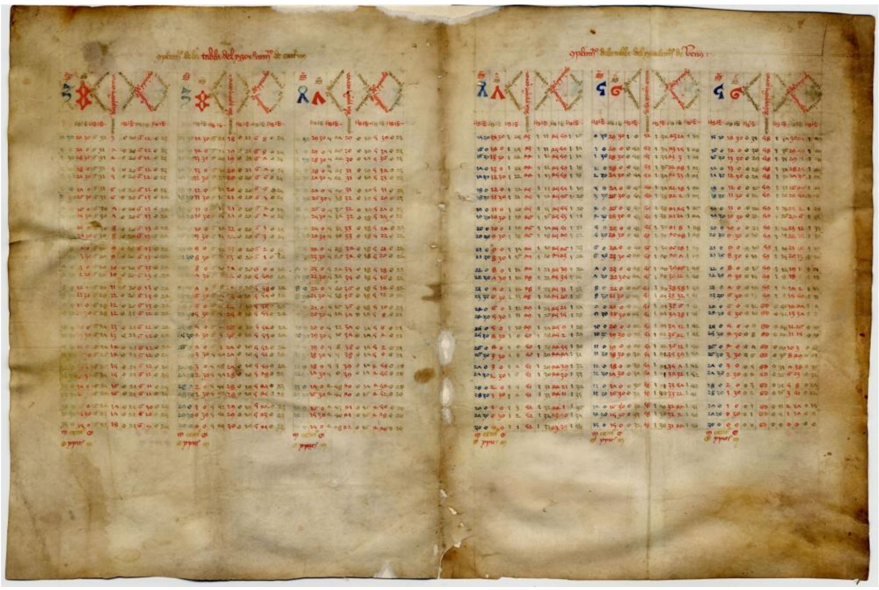

Figure 1. Fragments of the planetary equations for Saturn (left) and Venus (right)

Figure 2. Fragments of the planetary equations for Mercury (left) and Saturn (right)

The heading of column c_3 is *ygoalamiento del çentro*, that is, the equation of center. The entries are given in degrees and minutes. The fragment for Saturn includes the maximum, 6;31°, in the range 2s 30;0°–3s 3;30° of the argument, and we note that the equation of center for 3s 4° (= 94°) is 6;30°, which differs from the value 6;31° in tables for the same purpose in the most common sets of tables in the Ptolemaic tradition: the *Handy Tables*, the zij of al-Battānī, the Toledan Tables, and the Parisian Alfonsine Tables.[2] The entries for Saturn and Mercury are the same as those for the four sets of tables just mentioned. As for Venus, although the maximum does not appear in the extant excerpt, the rest of the entries follow the pattern of the zij of al-Battānī and the Toledan Tables, where a maximum value of 1;59° was used, and thus the Parisian Alfonsine Tables and the *Handy Tables*, with maximum equations of center of 2;10° and 2;24°, respectively, can be ruled out.

Column c_4 gives the minutes of proportion to be used for interpolation when the planet is not at mean distance. The heading reads *minutos de la proporcion lexos* or *minutos de la proporcion çerca*, depending on the part of the table where the planet is near apogee (when the argument is 0°) or near perigee (when the argument is 180°).

The heading of column c_5 is *torçimiento lexos*, that is, the subtractive difference to be applied to the equation of anomaly, presented in c_6, when the planet is near apogee. The entries are given in degrees and minutes.

Column c_6 gives the equation of anomaly at mean distance, in degrees and minutes. Its heading reads *ygoalamiento del ypiçiclo*, which is consistent with the term used for the equation of center. Note also that the name given to the variable makes no use of the Castilian equivalent to the most common Latin term *argumentum*, sometimes meaning "anomaly" and sometimes "argument", but refers to the fact that it is the correction to be applied to a variable, the anomaly, measured on the epicycle.

The heading of column c_7 is *torçimiento çerca*, and corresponds to the additive difference to be applied to the equation of anomaly when the planet is near perigee. The entries are given in degrees and minutes.

The structure of the tables in the bifolio follows the same pattern as that in other sets of tables in the *Handy Tables* tradition for determining the equations of the planets and thus their true positions, mentioned above. In this ap-proach, the true longitude of a planet, λ, results from adding the

[2] For the *Handy Tables*, see Stahlman 1959, pp. 295–324; for the zij of al-Battānī, see Nallino 1903–1907, pp. 108–137, and Paris, Bibliothèque de l'Arsenal, MS 8322, ff. 66v–81r ; for the Toledan Tables, see Pedersen 2002, pp. 1259–1307, and Toomer 1968, pp. 60–67; and for the Parisian Alfonsine Tables, see Ratdolt 1483, ff. e7r–g5v, and Poulle 1984, pp. 154–183.

longitude of its apogee at a certain time, $\lambda(A)$; the true center of the planet, κ; and a correc-tion $c(\alpha, \overline{\kappa})$ depending both on the true anomaly and the mean center of the planet. The true center is computed by adding algebraically the equation of center (c_3, *ygoalamiento del çentro*) to the mean center ($\overline{\kappa}$, not displayed in this bifolio)

$$\kappa = \overline{\kappa} + c_3(\overline{\kappa}),$$

where $c_3(\overline{\kappa})$ can be positive (when $180^o \leq \overline{\kappa} \leq 360^o$) or negative (when $0^o \leq \overline{\kappa} \leq 180^o$). To compute the correction $c(\alpha, \overline{\kappa})$ one first determines the true anomaly, α, from the mean anomaly ($\overline{\alpha}$, not displayed in this bifolio),

$$\alpha = \overline{\alpha} - c_3(\overline{\kappa}).$$

When the planet is near apogee, at maximum distance from the observer, the correction $c(\alpha, \overline{\kappa})$ is obtained from c_6 (*ygoalamiento del ypiçiclo*), c_4 (*minutos de la proporcion lexos*), and c_5 (*torçimiento lexos*) by means of the expression:

$$c(\alpha, \overline{\kappa}) = c_6(\alpha) - c_4(\overline{\kappa}) \cdot c_5(\alpha).$$

Similarly, when the planet is near perigee, at minimum distance from the observer, the correction $c(\alpha, \overline{\kappa})$ is obtained from c_6 (*ygoalamiento del ypiçiclo*), c_4 (*minutos de la proporcion çerca*), and c_7 (*torçimiento çerca*):

$$c(\alpha, \overline{\kappa}) = c_6(\alpha) + c_4(\overline{\kappa}) \cdot c_7(\alpha).$$

The tables preserved in this bifolio are therefore perfectly inserted in the tradition of the Ptolemaic tables for the equations for the planets.

2. In order to have a sense of the scientific language used in the tables in this bifolio, we have compared the few technical terms appearing in the headings and the titles with the equivalent terms in other astronomical texts in Castilian. We have chosen four texts of about the same period, among the few texts devoted to astronomical tables written in Castilian language. The first two are directly related to the group of astronomers working for Alfonso, king of Castile and León (second half of the 13th century): the unique copy of the canons of the Castilian Alfonsine Tables (Madrid, Biblioteca Nacional, MS 3306) and the Castilian version of the zij of al-Battānī by an unnamed collaborator of the king (Paris, Bibliothèque de l'Arsenal, MS 8322). The other two texts are translations into Castilian from Latin and Hebrew, respectively, made towards the end of the 15th century: John of Saxony's canons to the Parisian Alfonsine Tables, translated anonymously (El Escorial, Biblioteca del Monasterio, MS T-III-29, ff. 120r–169v), and Abraham

Zacut's *ha-Ḥibbur ha-gadol* in the translation made by Juan de Salaya, profe-ssor at the University of Salamanca, in 1481 (Salamanca, Biblioteca de la Universidad, MS 2–163). The information is presented in Table 1.

From Table 1 it is clear that the technical terminology in this bifolio is unre-lated to that in any other Castilian astronomical text, in particular to the two texts produced by the collaborators of King Alfonso: the Castilian Alfonsine Tables, and the Castilian version of the zij of al-Battānī.[3]

3. The tables for the planetary equations set up by Ptolemy in *Almagest* XI.11 were expanded and slightly modified in Ptolemy's *Handy Tables*. In medieval sets of tables in the Ptolemaic tradition, such as the zij of al-Battānī and the Toledan Tables, the planetary tables were reproduced from the *Handy Tables* with no changes, except for Venus, for which both zijes use a different parameter for the equation of center, 1;59º, as mentioned above. In the Parisian version of the Alfonsine Tables there are new parameters for the equations of center of Jupiter and Venus, whereas for the equation of anoma-ly of Mars the basic parameter was only changed very slightly.[4]

The entries preserved in this bifolio agree with those in the zij of al-Battānī and the Toledan Tables. These two sets display entries which are the same but for variant readings, and both give the argument at intervals of 1º, as in the *Handy Tables*. There is, however, a noticeable difference at first sight between them regarding presentation: most copies of the Toledan Tab-les have an extra column for the first station, not found in the only extant co-py in Arabic and the only known copy in Castilian of al-Battānī's zij. But this difference in presentation is not a characteristic of either set of tables, for there are counterexamples in both ways: copies and adaptations of the Tole-dan Tables exist where the stations are treated separately, and there are tables based on the zij of al-Battānī which include a column for the first station in the table for the planetary equations.[5]

[3] Note, however, that some of these terms, including *torçimiento* and *ygoalamiento*, are alrea-dy found in Alfonsine texts, with different spellings and meanings than those used here; see Kasten and Nitti 2002.

[4] For some historical values and excerpts of tables for the planetary equations, see Chabás and Goldstein 2012, pp. 73–81.

[5] See e. g. Pedersen 2002, pp. 1265 and 1307, and d'Alverny *et al.* 2009, pp. 306–321, for the adaptation of the Toledan Tables made by Raymond of Marseilles to the latitude of his city. On the other hand, the tables of Abraham bar Ḥiyya, largely drawn from those by al-Battānī al-though arranged for the Hebrew calendar, display the first station together with the planetary equations (Goldstein 2011, p. 139 and personal communication).

As mentioned above, the presentation of the tables in this bifolio also differs from that in any other previous sets of tables in the number of entries, for on each page we find entries for 45º of the argument (at intervals of 0;30º) whereas the Toledan Tables and the zij of al-Battānī only display on each page entries for 30º of the argument (at intervals of 1º). In particular, this presentation has not been found in any text from the medieval Iberian Peninsula, whether in Arabic, Hebrew, Latin, Castilian, or Catalan.

4. In the West, the zij of al-Battānī circulated among Andalusian astronomers, and was used by Abraham bar Ḥiyya (d. *ca.* 1140) to draw up his own tables in Hebrew.[6] Al-Battānī's work was translated into Latin at least twice in the 12th century in the northeastern part of the Iberian Peninsula, although one of the translations is apparently lost. It was also translated into Castilian by the astronomers working under the patronage of King Alfonso in the late 13th century;[7] the text is preserved in Paris, in the manuscript mentioned above. Altogether, the number of extant manuscripts directly representing the zij of al-Battānī is very limited.

However, its relevance in Latin Europe stems from the fact that it was one of the main sources for the Toledan Tables. These were probably composed in Arabic in Toledo in the late 11th century by a group of astronomers led by Ibn Ṣāᶜid.[8] As indicated previously, the tables for the planetary equations in the Toledan Tables are identical to those in al-Battānī's zij. This is not an isolated case, for other tables in the Toledan Tables are directly taken from the zij of al-Battānī, or from previous sets of tables. The Toledan Tables, of which the Arabic original is not extant, was the most widely used set of tables in Latin for more than two centuries until they began to be superseded by the Parisian Alfonsine Tables in the course of the 14th century. An indicator of the wide circulation of the Toledan Tables is the fact that more than a hundred copies of them have been preserved, mostly from the late 13th century and the 14th centuries.[9] Moreover, the Toledan Tables were adapted to the Christian calendar and the geographical coordinates of different localities in Western Europe, including Marseilles, Toulouse, Novara, Hereford, among others. All this material circulated almost exclusively via manuscripts in La-

[6] Millás 1959.
[7] For the impact of the zij of al-Battānī in the Iberian Pensinsula, see Samsó 2005, pp. 79–80.
[8] Samsó 2011, pp. 147–152, 482–484.
[9] Pedersen 2002, pp. 11–23.

tin, but the Toledan Tables are also preserved in other languages, including Greek (*ca.* 1330) and French (1271).[10]

In the context of the astronomical work carried out in the 14th century in Europe, the tables for the planetary equations that belonged to the zij of al-Battānī, and ultimately to previous sources, were in all likelihood seen as a part of the Toledan Tables, *tout court.* And in this sense, the excerpts in the bifolio preserved in Navarre provide the first evidence for a copy of the Toledan Tables in Castilian.

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[10] For the Greek version see Pingree 1979; for the French version see Boudet and Husson 2012.

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Table 1: Technical terminology in Castilian

	Bifolio (G. A. of Navarre)	*Castilian Alfonsine Tables* (Madrid, MS 3306)[11]	Castilian version of the zij of al-Battānī (Arsenal, MS 8322)[12]
1	çentro	çentro (14:2) / centro (1:13)	çenptro / centro (p. 162)
2	Mercurius	Mercurio (18:22)	Mercurio (f. 78v)
3	minutos dela proporçion	minutos proporçionales (17:10)	menudos menguados (f. 66v) menudos annadidos (f. 68r)
4	Saturnus	Saturno (22:2)	Saturno (f. 66v)
5	tabla	tabla (0:16)	tabla (f. 28v)
6	torçimiento çerca	longura mas çerca (18:14)	longura çercana (f. 66v)
7	torçimiento lexos	longura mas luenga (18:14)	longura mas luenne (f. 66v)
8	Venus	Venus (32:22)	Venus (f. 75v)
9	ygoalamiento del çentro	equaçion del çentro (18:6)	eguacion del argumento (f. 66v)
10	ygoalamiento del ypiçiclo	equaçion del argumento (18:14)	la longura mediana (f. 66v)
11	ypiçiclo	epiçiclo (18:14) / epeciclo (32:70) / piçiclo (17:15)	–

	Castilian version of John of Saxony's canons[13]	Castilian version of Zacut's *Ḥibbur*[14]
1	çentro (p. 64)	çentro (p. 156) / centro (p. 204)
2	Mercurio (p. 47)	Mercurio (p. 156)
3	minutos proporçionales (p. 65)	–
4	Saturno (p. 63)	Saturno (p. 156)
5	tabla (p. 34)	tabla (p. 151)
6	longura mas çercana (p. 67)	–
7	longura mas luenga (p. 65)	–
8	Venus (p. 47)	Venus (p. 156)
9	equaçion del çentro (p. 64)	egualaçion del çentro (p. 204)
10	equaçion del argumento (p. 65)	egualaçion del argumento (p. 204) / equaçion del argumento (p. 210)
11	epiçyclo / epiçiclo (p. 64)	epiçiculo (p. 169)

[11] For each term, its first occurrence is indicated by two numbers separated by a colon, referring to chapter and sentence, respectively. For a glossary of technical terms in this manuscript see Chabás and Goldstein 2003, pp. 95–133.

[12] A comprehensive study of the terminology of this text is found in Bossong 1978. The numbers in parenthesis correspond to the folio in the manuscript or the page in Bossong's monograph.

[13] Martínez Gázquez 1989 edited the translation into Castilian of the canons composed by John of Saxony in 1327. The numbers in parenthesis correspond to the page of this edition.

[14] For a transcription of this uniquely preserved text, see Cantera Burgos 1931. The page numbers refer to Cantera's publication.