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Adaptive Piezoelectric Vibration Control With Synchronized Switching

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1 Introduction

Traditional approaches to passive vibration control include the attachment of viscoelastic materials and mechanical vibration absorbers [1–3]. Piezoelectric materials, which have the ability to convert mechanical energy into electrical energy and vice versa, are often used in active and passive vibration control applications [4]. When the piezoelectric material is strained, a charge develops across the element and energy is dissipated as current flows through an external electrical network or shunt.

In one embodiment, the piezoelectric element is connected to a resistor-inductor shunt and attached to the surface of a vibrating beam [5,6]. The capacitance of the material couples with the network to form a resonant circuit. By tuning the natural frequency of the circuit to match the beam’s targeted modal frequency, the current and energy dissipation are maximized. The resonant shunt technique enables attenuations of 15–25 dB at resonance, depending on the inherent structural damping [6,7]. However, inductances on the order of 10–100 H can be required for control of vibration below 250 Hz, an attribute that is problematic for applications constrained by physical space [7]. Furthermore, a resonant shunt can become mistuned by environmental effects, variation in the system’s mass or stiffness, temperature fluctuations, loosening of bolted connections, and the formation of cracks [8–12].

As a semi-passive control approach, the present technique is an extension of the synchronized [13,14] or pulse [7,15] switching methods. In this view, the piezoelectric element is switched to the resonant shunt at the instant of the beam’s maximum modal displacements, and the switch remains closed for half of the shunt’s period. As a result, the system is stiffened over the ensuing motion until the switch reopens, at which point the energy stored in the piezoelectric element is dissipated through the shunt. A bandpass filter isolates the response of a particular vibration mode [7,15], and switching is optimized when the filter’s center frequency approximately matches the structure’s natural frequency. However, should that frequency change over time, performance is degraded by the filter’s non-ideal phase response [16]. Conventional synchronized switching control therefore lacks adaptability for situations in which either the structure’s or the excitation’s frequency evolves in time.

In what follows, an implementation of synchronized switching is examined for applications where the dynamics or the excitation varies slowly in time. A continuous model of the mechanical system captures with fidelity higher-order modal content and is more accurate than the single degree of freedom approximations [7,13,14]. An adaptive controller is implemented in a manner that reduces the large inductance requirement that is typically associated with resonant shunts [17–19]. Even with minimal a priori knowledge of the system’s parameters, the controller measures response and converges to the filter’s optimal center frequency and adapts to minimize vibration amplitude.

2 Synchronized Switching Vibration Control

2.1 System Modeling. Figure 1 illustrates a Euler–Bernoulli cantilever beam of length \( l_1 \) that is subjected to base excitation \( u_b(t) \). The beam’s absolute motion is given as \( u(t) = u_b(t) + u(x,t) \), where \( u(x,t) \) measures displacement relative to the base. Piezoelectric elements are attached near the base and extend over region \((l_1,l_2)\). The beam \( b \) and each piezoelectric element \( p \) are rectangular with cross-sectional areas \( A_p = w_p h_p \) and second moments of area \( I_p = w_p h_p^3 / 12 \), where \( w_p \) and \( h_p \) denote widths and thicknesses, and...
Fig. 1 Model of a cantilever beam and attached piezoelectric elements that is subjected to base excitation

Fig. 2 Schematic of the piezoelectric elements that are switched to a resonant shunt

\[ K_{ij} = \int_0^{l_b} E I (x) \phi_i^T \phi_j^T \, dx \] (8)

Elements \( i \) of the excitation and electromechanical coupling vectors are

\[ f_i = -u_i \int_0^{l_b} \rho A(x) \phi_i \, dx \] (9)

\[ \Theta_i = \left( \frac{w_0 (h_b + h_p) E_p d_{31} V_p}{C} \right) \left( \delta'(x - l_2) - \delta'(x - l_1) \right) \] (10)

When the switch of Fig. 2 is set to the open state, the applied charge on the piezoelectric elements is constant \( q_{a0} = 0 \). When the switch is closed, the elements are connected to the shunt having resistance \( R \) and inductance \( L \), and the charge then satisfies

\[ L \ddot{q}_a + R \dot{q}_a + \frac{1}{C} q_a = \Theta^T y \] (11)

The switch is closed only at the instants of maximum modal displacement, and it remains closed for time

\[ \tau = \pi \sqrt{LC} \] (12)

namely, half of the electrical sub-system’s period [7]. Equations (6) and (11) couple as the system

\[
\begin{bmatrix}
M & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{q}_a \\
\ddot{q}_a \\
0^T R \dot{q}_a \\
0^T
\end{bmatrix}
+ \begin{bmatrix}
K_{oc} - \Theta^T & L \\
0 & R \\
-\Theta^T & 0
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{q}_a \\
\ddot{q}_a \\
\dot{q}_a \\
0
\end{bmatrix}
= \begin{bmatrix}
f \\
0
\end{bmatrix}
\]

of order \( m = n + 1 \), where \( K^\text{oc} = K + C \Theta \Theta^T \) is the so-called open-circuit stiffness matrix [24]. In short, the electromechanical system’s response is governed by Eq. (6) when the switch is open (termed state \( S^{(0)} \)), subject to the constraint of constant charge, and by Eq. (13) when the switch is closed (state \( S^{(1)} \)). Energy is dissipated when the switch is closed as current flows through the shunt, which adds damping to the system.

2.2 Piecewise State Response. In each state, the beam’s and circuit’s responses are linear and determined through modal analysis. The eigenvectors of state \( k \) are determined from Eqs. (6) or (13) and arranged in the form

\[ \mathbf{b}^{(k)} = [\psi_1^{(k)} | \psi_2^{(k)} | \cdots | \psi_p^{(k)}] \] (14)

where \( r = n \) modes are used in discretization during \( S^{(0)} \) and \( r = m \) during \( S^{(1)} \). By introducing the linear transformation

\[ \mathbf{z}^{(k)} = \mathbf{b}^{(k)} \mathbf{\eta}^{(k)} \] (15)

the responses \( \mathbf{z}^{(0)} = \mathbf{y} \) and \( \mathbf{z}^{(1)} = [\mathbf{y}^T \dot{q}_{a0}^T] \) in each state are described by the set \( \mathbf{\eta}^{(k)} \) of coordinates arranged in column vector \( \mathbf{\eta}^{(k)} \). Each coordinate satisfies
Table 1  Properties of parameters used in simulations.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric density</td>
<td>$p_v$</td>
<td>$7.87 \times 10^3$ kg/m$^3$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_p$</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Length</td>
<td>$l_b$</td>
<td>20.95 cm</td>
</tr>
<tr>
<td>Width</td>
<td>$w_b$</td>
<td>1.59 cm</td>
</tr>
<tr>
<td>Thickness</td>
<td>$h_b$</td>
<td>0.32 cm</td>
</tr>
<tr>
<td>Damping ratios ($i=1,2,\ldots,n$)</td>
<td>$\xi_i$</td>
<td>2.5%</td>
</tr>
<tr>
<td>Number of basis functions</td>
<td>$n$</td>
<td>10</td>
</tr>
<tr>
<td>Piezoelectric elements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric density</td>
<td>$p_p$</td>
<td>$7.80 \times 10^3$ kg/m$^3$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E_p$</td>
<td>66 GPa</td>
</tr>
<tr>
<td>Position</td>
<td>$l_1$</td>
<td>0.32 cm</td>
</tr>
<tr>
<td>Position</td>
<td>$l_2$</td>
<td>6.67 cm</td>
</tr>
<tr>
<td>Width</td>
<td>$w_p$</td>
<td>1.59 cm</td>
</tr>
<tr>
<td>Thickness</td>
<td>$h_p$</td>
<td>0.13 cm</td>
</tr>
<tr>
<td>Constant</td>
<td>$d_{31}$</td>
<td>$-1.75 \times 10^{-10}$ m/V</td>
</tr>
<tr>
<td>Total capacitance</td>
<td>$C$</td>
<td>24.02 nF</td>
</tr>
<tr>
<td>Shunt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance</td>
<td>$R$</td>
<td>2.2 kΩ</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L$</td>
<td>2.0 H</td>
</tr>
<tr>
<td>Natural frequency</td>
<td>$\Omega$</td>
<td>$4.33 \times 10^3$ rad/s</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>$\xi_m$</td>
<td>12%</td>
</tr>
</tbody>
</table>

$$\eta_i^{(k)} + 2\xi_i^{(k)} \omega_i^{(k)} \eta_i^{(k)} + \omega_i^{(k)} \eta_i^{(k)} = y_i^{(k)} F^{(k)}$$  (16)

with the damping ratio in state $k$ denoted by $\xi_i^{(k)}$ (Table 1). The loading terms in the switch’s two states are $F^{(0)}=f+\Theta q_a$ and $F^{(1)}=[f0]^T$, respectively.

2.3 Mapping Across States. The sets of modal coordinates are mapped across the switching discontinuity at the instants of the switch’s opening and closing. Figure 3 illustrates the mapping between the two sets of coordinates. One cycle of the beam tip’s resonant response is shown in Fig. 3(a) during which time the piezoelectric elements are twice switched to the resonant shunt. The corresponding phase trajectories of $y_1^{(0)}$ and $y_1^{(1)}$ are shown in Fig. 3(b). When the switch opens or closes, higher modes such as the second vibration modes $y_2^{(0)}$ and $y_2^{(1)}$ are also excited (Fig. 3(c)). The switch closure period as given in Eq. (12) can be adjusted to minimize such spillover. One cycle of the beam’s response proceeds as follows:

- Vibration occurs in $S^{(0)}$ until the tip modal velocity vanishes at $t_k$ (point 1 in Fig. 3).
- The coordinates $y$ and the charge $q_a$ are mapped across closure.
- At $t_k$, the switch closes and vibration occurs in $S^{(1)}$ until $t_{k+1}$ (point 2).
- The coordinates $y$ and the charge $q_a$ are mapped across opening.
- The cycle repeats for points 3 and 4 in Fig. 3.

For the specific closure time $t_k$, the displacement, velocity, and charge beforehand ($t_k^+$) match those afterwards ($t_k^-$). Current $q_a$ evolves only when the switch is closed, and at the instant of closure,

$$u^{(0)}(x,t_k^+) = u^{(1)}(x,t_k^-)$$

$$q_a^{(0)}(t_k^+) = q_a^{(1)}(t_k^-)$$

Similarly, at the switch’s time $t_{k+1}$ of opening, the states in $S^{(0)}$ and $S^{(1)}$ are related by

$$u^{(0)}(x,t_{k+1}^-) = u^{(0)}(x,t_{k+1}^-)$$

$$q_a^{(0)}(t_{k+1}^-) = q_a^{(0)}(t_{k+1}^-)$$

The continuity conditions (17) relate the two sets of modal coordinates through

$$\eta^{(1)}(t_k^-) = [T^{01} \quad Y^{01}] \begin{bmatrix} \eta^{(0)}(t_k^-) \\ q_a^{(0)}(t_k^-) \end{bmatrix}$$

(19)

as the state at closure is mapped to $S^{(1)}$ through the transition matrices

$$T^{01} = b^{(1)^T} [M \quad 0] b^{(0)}$$

(20)

$$Y^{01} = b^{(1)^T} [0 \quad L] b^{(0)}$$

(21)

Matrix $T^{01}$ maps both the modal displacement and velocity, and since there is no current flow at $t_k^+$, $q_a^{(0)}(t_k^-)$ is not included in the $\eta^{(1)}$ mapping. The companion matrix $T^{00}=[T^{01}]^T$ maps coordinates from $S^{(1)}$ to $S^{(0)}$ as the switch opens. Additionally, the loading term in $S^{(0)}$ becomes

$$F^{(0)}(x,t_{k+1}^-) = f + \Theta q_a^{(1)}(t_{k+1}^-)$$

(22)

3  Adaptive Control Using Fuzzy Logic

The efficacy of synchronized switching is reduced when the system’s parameters change with time, and in that situation, the filter can be tuned online to improve performance. The adaptive controller measures vibration amplitude and adjusts the filter so...
that maximum attenuation, as defined by a performance index, is attained. Fuzzy logic offers an effective approach for the self-tuning process.

### 3.1 Performance Index

The controller’s performance index scales with the beam’s tip vibration amplitude through

\[ J = \frac{1}{u_{\text{rms}}} \]  

(23)

where

\[ u_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\bar{u}(b_i,t_{i})^2} \]  

(24)

is the root-mean-square (rms) velocity, \( \bar{u}(b_i,t_{i}) \) is the measured tip velocity at instant \( t_{i} \), and \( N \) is the number of measurements. The index varies as a function of the filter’s center frequency as shown in Fig. 4 for \( J \) measured over 56–112 Hz. In that instance, the optimal \( J \) develops at \( f_0=81 \) Hz, which corresponds to the beam’s fundamental frequency. The controller maintains that condition by continuously adjusting the center frequency based upon the values of \( J \) and \( \Delta J \), the latter being the difference between the current and the previous measurements. The control approach is based on general fuzzy logic design strategy [25] in which a rule base is imposed for various combinations of \( J \) and \( \Delta J \), and those rules are combined in a weighted average for decision-making purposes.

### 3.2 Fuzzification

Membership functions quantify the relative magnitudes of the \( J \) and \( \Delta J \) measurements and guide decision-making for prospective changes to the filter’s selection. Linguistic descriptors characterize the values of \( J \) as being “small” or “large.” The sign of \( \Delta J \) is “pos” for a positive change or “neg” for a negative change, and the magnitude is “zero,” “small,” “medium,” or “large.” For instance, a large positive change in \( J \) is designated to be within 5% of the greatest measurement value, and \( J_{\text{max}} \) is chosen to be an excursion beyond 50% of \( J_{\text{max}} \). The ordinates in Figs. 5(a) and 5(b) are certainty values assigned to \( J \) and \( \Delta J \); these values vary between zero and unity, and they capture the certainty of a measured value falling in a particular bin. In terms of piecewise-linear membership functions, the certainty \( \mu \) is determined by

\[ \mu_{\text{small}}(J) = \begin{cases} 1 - 0.95J/\max & 0 \leq J \leq 0.95J_{\text{max}} \\ 0 & 0.95J_{\text{max}} \leq J \end{cases} \]  

(25)

\[ \mu_{\text{large}}(J) = \begin{cases} 0.95J/\max & 0 \leq J \leq 0.95J_{\text{max}} \\ 1 & 0.95J_{\text{max}} \leq J \end{cases} \]  

(26)

Likewise, the certainty of \( \Delta J \) is found from

\[ \mu_{\text{negl}}(\Delta J) = \begin{cases} 1 & \Delta J \leq - \Delta J_{\text{max}} \\ -3\Delta J/\Delta J_{\text{max}} + 3 & - \Delta J_{\text{max}} \leq \Delta J \leq - \frac{2}{3} \Delta J_{\text{max}} \\ 0 & - \frac{2}{3} \Delta J_{\text{max}} \leq \Delta J \end{cases} \]  

(27)

\[ \mu_{\text{negmed}}(\Delta J) = \begin{cases} 0 & \Delta J \leq - \Delta J_{\text{max}} \\ -3\Delta J/\Delta J_{\text{max}} + 3 & - \Delta J_{\text{max}} \leq \Delta J \leq - \frac{2}{3} \Delta J_{\text{max}} \\ -3\Delta J/\Delta J_{\text{max}} + 1 - \frac{2}{3} \Delta J_{\text{max}} & - \frac{2}{3} \Delta J_{\text{max}} \leq \Delta J \leq - \frac{1}{3} \Delta J_{\text{max}} \\ 0 & - \frac{1}{3} \Delta J_{\text{max}} \leq \Delta J \end{cases} \]  

(28)

\[ \mu_{\text{neglarge}}(\Delta J) = \begin{cases} 0 & \Delta J \leq - \frac{2}{3} \Delta J_{\text{max}} \\ 3\Delta J/\Delta J_{\text{max}} + 2 & - \frac{2}{3} \Delta J_{\text{max}} \leq \Delta J \leq - \frac{1}{3} \Delta J_{\text{max}} \\ -3\Delta J/\Delta J_{\text{max}} - 1 + \frac{2}{3} \Delta J_{\text{max}} & - \frac{1}{3} \Delta J_{\text{max}} \leq \Delta J \leq 0 \\ 0 & 0 \leq \Delta J \end{cases} \]  

(29)
various combinations of $J$ toward the optimal frequency is shifted substantially in order to trend the response from measurements similar to those in Fig. 4. Alternatively, when $J$ is small and $\Delta J$ is “possml,” the controller imposes a small change in the condition. The linguistic descriptors that classify the corresponding transfer function $G(s)$ is:

$$G(s) = \frac{s(\omega_0/Q)}{(s^2 + s(\omega_0/Q) + \omega_0^2)}$$

where $H$ is the output gain at $\omega = \omega_0$, $s$ is the complex Laplace variable, and $Q$ is the filter’s quality factor. The center frequency was set between 56–112 Hz using an external clock circuit. The switch (Maxim 4690) connected the piezoelectric element and the active inductor.
ity, the performance index, and the center frequency as the beam was excited at resonance, with and without action of the controller. The beam was driven without control (open-switch) until \( t = 1.5 \) s. The controller was then initialized through \( t = 8 \) s while sweeping coarsely through the filter’s range of center frequencies in order to record the largest \( J \) and \( \Delta J \) values achieved. Since the beam was driven as resonance, \( J \) was not bandpass filtered to measure the modal velocity. After \( t = 8 \) s, the controller continuously measured \( J \) and adapted through the logic algorithm while maximizing \( J \). That condition occurred with a center frequency of approximately 81 Hz, and at steady state, the root-mean-square velocity of the beam’s tip was reduced during simulation by 83%, compared with the experimental reduction of 77%. The controller minimized the response amplitude through synchronized switching without explicit knowledge of the beam’s natural frequency or the excitation frequency.

Figures 8–10 demonstrate the controller’s adaptation characteristics. In Figs. 8 and 9, at \( t = 8 \) s, the logic algorithm was initialized at the lower and upper bounds of the filter’s center frequency range, respectively. Those conditions represent worst-case scenarios with the controller being initialized well away from its optimal setting. In each case, the controller subsequently converged to the proper frequency and reduced the beam’s response amplitude. Initially, with the center frequency poorly placed, the controller provided insufficient attenuation, but at \( t = 10 \) s, the system converged as desired.

In Fig. 10, the controller adapted to changes for both the natural frequency and the excitation frequency. At \( t = 30 \) s, the beam’s fundamental frequency was shifted to 71 Hz by the addition of mass to the cantilever’s tip, and it was excited at the new resonant frequency. The controller adapted to those changes and reached the condition of maximum attenuation within 5 s. As shown in Table 4, the simulated and measured root-mean-square velocities of the beam under adaptive control were reduced by 31% and 35%, respectively, as compared with conventional synchronized switching using nonadaptive control. At \( t = 60 \) s in Fig. 10, the excitation and natural frequencies were returned to their original...
values, and the controller again tracked those changes. When compared with nonadaptive synchronized switching, the present controller provided greater vibration attenuation and exhibited adaptation to changes in the beam’s natural frequency and the excitation frequency.

5 Conclusion

Conventional synchronized switching utilizes a bandpass filter to isolate and attenuate the response of a particular vibration mode. However, the filter in that case possesses undesirable phase characteristics that can degrade performance if the system’s natural frequencies or the excitation frequency should change in time. An adaptive controller was developed in order to adjust to such changes by using a fuzzy logic algorithm. The controller maximizes attenuation using synchronized switching by measuring the velocity and adjusting the filter’s frequency. When compared with traditional synchronized switching control, the adaptive controller reduced the root-mean-square tip velocity by over 30% in both simulation and experiments. In addition, synchronized switching can attenuate several vibration modes of a system [15], and the adaptive controller could be developed to provide multimodal control. In such a design, multiple filters would be used to target the desired resonances, and the adaptive controller could be varied to adjust these filters in real time based on a similar performance index. In summary, the contributions of this work are as follows:

- A continuous model was developed to simulate vibration of the cantilever beam with attached piezoelectric elements. The state-to-state modal analysis method maps generalized coordinates between the open- and closed-switch states. This model captures higher-order response components more accurately than previous single degree of freedom approximations.
- The controller optimizes the performance of synchronized switching while having minimal a priori knowledge of the system. The primary requirement for implementation is that the targeted mode lie within the lower and upper bounds of the filter’s frequency range.
- The controller adapts to environmental changes such as variations in the beam’s mass and stiffness and the excitation frequency. Furthermore, the controller is compact in size as a result of its low inductance and computation requirements.

Acknowledgment

This research was sponsored by the Naval Nuclear Propulsion Fellowship Program.

References


Table 4 The beam’s steady state tip velocity for different controllers during the natural frequency and excitation change.

<table>
<thead>
<tr>
<th>Control type</th>
<th>Measured (%)</th>
<th>Simulated (%)</th>
<th>Reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open switch (no control)</td>
<td>52.3</td>
<td>53.8</td>
<td>-</td>
</tr>
<tr>
<td>Closed switch</td>
<td>45.7</td>
<td>39.4</td>
<td>62.6</td>
</tr>
<tr>
<td>Synchronized switching</td>
<td>27.3</td>
<td>17.4</td>
<td>67.7</td>
</tr>
<tr>
<td>Adaptive control</td>
<td>17.7</td>
<td>12.0</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Fig. 9 Beam tip response with fuzzy control implementation and suboptimal placement of the filter’s initial condition (f_0 = 112 Hz): (a) simulated and (b) measured tip velocity of the beam with control (light) and without (dark), (c) simulated (•••) and measured (––) evolution of the performance index, and (d) simulated (•••) and measured (––) evolution of the filter’s center frequency.

Fig. 10 Beam tip response with fuzzy control implementation and on-line filter adaptation to impose stepwise changes 81 Hz–71 Hz–81 Hz in the structure’s fundamental frequency: (a) simulated and (b) measured tip velocity of the beam with control (light) and without (dark), (c) simulated (•••) and measured (––) evolution of the performance index, and (d) simulated (•••) and measured (––) evolution of the filter’s center frequency.