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Domain-wall motion in random potential and hysteresis modeling

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Two different approaches to hysteresis modeling are compared using a common ground based on energy relations, defined in terms of dissipated and stored energy. Using the Preisach model and assuming that magnetization is mainly due to domain-wall motion, one can derive the expression of magnetization along a major loop typical of the Jiles–Atherton model and then extend its validity to cases where mean-field effects and reversible contributions are present. © 1998 American Institute of Physics. [S0021-8979(98)39311-1]

I. INTRODUCTION

The Preisach1 and Jiles–Atherton2 models are two widely used approaches to the description of magnetic hysteresis. They have been applied to a wide range of static and dynamic conditions, ranging from the solution of circuits containing hysteretic components to microstructural analysis.7–5 Their physical significance can be best appreciated when modeling features are reduced to fundamental energetic aspects, and this result is particularly helpful in the clarification of the relations between different approaches to hysteresis modeling. In this paper, we show that by referring to fundamental energy relations, which can be used to describe the models in terms of stored and dissipated energy, one is able to derive the fundamental expression of magnetization laws of the Jiles–Atherton model by applying a physically meaningful set of assumptions to the Preisach model.

When work is performed by external sources on a system displaying hysteresis, part of the energy is stored and part is dissipated. This energy balance can be described using a model which takes into account a minimum set of relevant physical quantities. Considering, for example, a homogeneous magnetic system and assuming, for the sake of simplicity, that the bulk magnetization $M$ is aligned to the homogeneous magnetic system and assuming, for the sake of simplicity, that the bulk magnetization $M$ is aligned to the applied magnetic-field $H_a$, the general expression for the balance between stored and dissipated energy can be expressed as

$$-\mu_0 MdH_a = dg + \delta Q,$$  \hspace{1cm} \text{(1)}

where $dg$ corresponds to the change in free energy per unit volume and $\delta Q$ is the dissipated energy term. In this paper, we show how one can express the terms of Eq. (1) with quantities easily recognized in each of the modeling schemes, and furthermore, how this can be used as a common reference for the quantitative comparison of the Jiles–Atherton and Preisach models.

II. JILES–ATHERTON MODEL

In the case of the Jiles–Atherton model, it is assumed that the free-energy term $dg$ of Eq. (1) can be expressed in terms of the anhysteretic curve $M_{an}(H_a)$; the idea is that the energy supplied coincides with the change in magnetostatic energy in the absence of hysteresis:

$$dg = -\mu_0 M_{an}dH_a.$$  \hspace{1cm} \text{(2)}

The other term of Eq. (1), $\delta Q$, corresponding to the dissipated energy, will be the difference between the energy supplied and the change in magnetostatic energy. This can be taken to be proportional to the change in magnetization,

$$\delta Q = \mu_0 kdM,$$  \hspace{1cm} \text{(3)}

since $dM$ can be thought to be proportional to the number of pinning sites seen by a moving domain wall; and each pinning event giving rise to Barkhausen jump, when integrated over the entire specimen, will produce a dissipation contribution proportional to the pinning site density $k$. The dissipation is always positive, that is, there can be loss of energy only, and therefore, in Eq. (3) and the subsequent analysis it is implicitly assumed that $dM$ is positive. The equations can easily be modified to take into account a negative $dM$. Using these assumptions, Eq. (1) becomes then,

$$-\mu_0 MdH_a = -\mu_0 M_{an}dH_a + \mu_0 kdM,$$  \hspace{1cm} \text{(4)}

which can be directly written as

$$\frac{dM}{dH_a} = \frac{M_{an}(H_a) - M(H_a)}{k},$$  \hspace{1cm} \text{(5)}

which is the simplest expression of the basic magnetization law of the Jiles–Atherton model$^2$ in the absence of an internal coupling field.

III. PREISACH MODEL

In the Preisach model, hysteresis is described starting from the hypothesis that a free-energy profile characterized by multiple local minima and metastable states can be decomposed into a set of many elementary bistable contributions. Each bistable unit can occupy one of two states, which we shall call (+) and (−) states and is characterized by two
fields $h_c$ and $h_a$, respectively proportional to the height of the barrier separating the (+) and (−) states and to the energy difference of the (+) and (−) states. The basic relation of the magnetization in the Preisach model is

$$M = 2M_s \int_0^\infty dh_c \int_0^{b(h_c)} dh_a \, p(h_c, h_a),$$

(6)

obtained by integration on the plane defined by the ensemble of bistable units. The integration upper limit $b(h_c)$ consists of a chain of segments of alternating slope $db/dh_c = +1$ and $db/dh_a = -1$ generated by the past field history. This line defines the partition of the Preisach plane in only one (+) and one (−) region. In the case of the saturation loop branch with peak-field $H_a$, $b(h_c)$ becomes simply $H_a - h_c$, a segment of slope $db/dh_a = -1$. The physical meaning of the Preisach model emerges clearly when one considers the properties of domain walls (DW) moving in this complex energy profile, which is rich in metastable states. It has been proved that the hysteresis properties of a DW moving in a Wiener-like pinning field profile can be described by the Preisach hysteresis model, where the Preisach distribution $p(h_c, h_a)$, weighting the elementary contributions, is proportional to

$$p(h_c, h_a) \propto \exp(-h_c/k),$$

(7)

independent of $h_a$ where $k$ describes the statistical properties of the pinning field. This result pertains to indefinite wall motion and contains no description of magnetic saturation. Saturation can then be taken into account by modifying Eq. (7) into

$$p(h_c, h_a) \propto u(h_c) \nu(h_a),$$

(8)

where the $\nu$ function is an integrable even function of $h_a$. This generalization corresponds again to the physical picture where stored energy (represented by the $h_a$ variable) and dissipated energy (represented by $h_c$) can be factorized as

$$u(h_c) \nu(h_a)$$

for a proper description of the system.

In the case of a saturation loop branch, by applying Eq. (8) to Eq. (6) for the calculation of the saturation loop branch, one will obtain a relation identical to the Jiles–Atherton hysteresis model where the integral of $\nu(h_a)$ coincides with the anhysteretic magnetization process, and it can be obtained for any value of applied-field $H_a$ by applying an oscillating field history with decreasing peak amplitude in order to demagnetize the material in the bias field $H_a$. The anhysteretic curve is closely connected with the function $\nu(h_a)$. In fact, the upper integration limit $b(h_c)$ associated with the anhysteretic state assumes the particularly simple form $b(h_c) = H_a$ when the demagnetization is performed using an arbitrarily large number of decreasing steps. In this case, Eq. (8) can be written as

$$M_{an}(H_a) = 2M_s \int_0^\infty dh_c \int_0^{H_a} dh_a \, u(h_c) \nu(h_a),$$

(9)

and if the dissipation term is normalized so that

$$\int_0^\infty dh_c \, u(h_c) = 1,$$

(10)

the anhysteretic magnetization can simply be expressed as

$$M_{an}(H_a) = 2M_s \int_0^{H_a} dh_a \nu(h_a),$$

(11)

and $\nu(h_a)$ must, therefore, be the derivative of $M_{an}$ with respect to $h_a$ rescaled by some constant. This result follows directly from the chosen factorization Eq. (7) of the Preisach distribution. Considering now the expression for magnetization of Eq. (6), one obtains an expression for $dM$,

$$\frac{dM}{dH_a} = 2M_s \int_0^\infty dh_c \frac{\exp(-h_c/k)}{k} \nu(H_a - h_c),$$

(12)

that integrated by parts gives

$$\frac{dM}{dH_a} = \frac{1}{k} [M_{an}(H_a) - M(H_a)],$$

(13)

which demonstrates the actual equivalence between the Preisach and Jiles–Atherton models in the calculation of a saturation branch, under the hypothesis of DW dominance and saturation approach according to an anhysteretic law. The result in Eq. (13) can be used to clarify the equivalence through the direct comparison of the terms appearing in Eq. (1) derived from the two models, and to this end we will write Eq. (13) for later use as

$$\mu_o k dM = \mu_o M_{an} dH_a - \mu_o M dH_a.$$

(14)

It has also been shown in Ref. 7 that through the Preisach operator of Eq. (6) one can obtain an expression for the terms $dQ$ and $g$ appearing in Eq. (1):
where

\[
\frac{dQ}{\mu_0 Ho dM - \mu_0 X dH_a}.
\]

Finally, Eqs. (14) and (17) can be combined with Eq. (1) to obtain an expression for \( dQ \) :

\[
dQ = -\mu_0 M_{an} dH_a + \mu_0 X dH_a,
\]

and this result, when compared with Eqs. (2) and (3) shows that both the Preisach and Jiles–Atherton models are fully consistent with the energy balance expressed in Eq. (1), even though a deviation term is observed.

### A. Irreversible and reversible magnetization

In Eq. (13) it has been shown that the expressions for magnetization derived in the Preisach framework can be used to obtain an expression for magnetization along the major loop as in the Jiles–Atherton model. The general expression for total magnetization is usually given as the sum of two contributions \( M_{irr} \) and \( M_{rev} \). Equations (6) and (8) refer to the irreversible part only, so that

\[
M_{irr}(H_a) = \frac{2 M_s}{k} \int_0^\infty dh_a \int_0 h_a \exp(-h_a/k) \nu(h_a),
\]

and the Jiles–Atherton relation (5) becomes

\[
\frac{dM_{irr}(H_a)}{dH_a} = \frac{1}{k} [M_{an}(H_a) - M_{irr}(H_a)].
\]

Reversible magnetization processes can be quite naturally introduced in the Preisach model by association with elementary hysteresis loops with no energy dissipation, e.g., where \( h_a = 0 \). These objects will be distributed along the \( h_a \) axis and they can be defined using Dirac’s delta function \( \delta(h_a) \). The Preisach distribution referring to total magnetization is then the sum of two parts, both multiplied by the same function \( \nu(h_a) \),

\[
[(1 - c) \nu(h_a) + \nu(h_a)] \nu(h_a),
\]

where \( \delta(h_a) \) is the Dirac function.

Using Eq. (21) and referring to the general expression for magnetization in Eq. (6) and (9), where one can substitute \( p(h_a) \) with \( \delta(h_a) \), the reversible contribution to magnetization can be written as

\[
M_{rev}(H_a) = \frac{2 M_s}{k} \int_0^\infty dh_a \int_0 h_a \delta(h_a) \nu(h_a)
\]

\[
= c M_{an}(H_a),
\]

having made use of Eq. (13). With this result, total magnetization can be written both for the Preisach and the Jiles–Atherton models as

\[
M = [(1 - c) dM_{irr} + c dM_{an}].
\]

### B. Mean-field contributions

Wishing to express the same quantities in the presence of a mean-field interaction, one has to distinguish the applied-field \( H_a \) from the internal-field \( H_{eff} \), which is equal to the sum of the applied field and a coupling term proportional to magnetization through a coefficient \( \alpha \neq 0 \):

\[
H_{eff} = H_a + \alpha M,
\]

taking the derivative of \( H_a \) with respect to internal-field \( H_{eff} \) one obtains,

\[
\frac{dH_a}{dH_{eff}} = 1 - \alpha \frac{dM}{dH_{eff}},
\]

which can be used to write the derivative of \( M \) with respect to \( H_a \) :

\[
\frac{dM}{dH_{eff}} = \frac{dM}{dH_a} \frac{dH_a}{dH_{eff}} = \frac{dM}{dH_a} \left( 1 - \frac{dM}{dH_{eff}} \right).
\]

Now one is able to write Eq. (20) using \( H_{eff} \) instead of \( H_a \),

\[
\frac{dM(H_{eff})}{dH_{eff}} = \frac{1}{k} [M_{an}(H_{eff}) - M_{irr}(H_{eff})],
\]

and transform it to a derivative with respect to \( H_a \):

\[
\frac{dM(H_a)}{dH_a} = \frac{M_{an}(H_a) - M_{irr}(H_a)}{k - \alpha (M_{an}(H_a) - M_{irr}(H_a))},
\]

which gives the Jiles–Atherton law for irreversible magnetization.

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