IMPOSED $\omega$-$k$ MAGNETOMETER AND DIELECTROMETER

APPLICATIONS

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INTRODUCTION

Dispersive properties (i.e., frequency dependent properties) of multiple layered media cannot be measured with conventional multiple-frequency techniques, because such techniques cannot uniquely distinguish between the impedance transitions caused by property variations with frequency and those caused by spatial property variations. In addition, at a single frequency, conventional eddy current sensor techniques can measure only the conductivity-thickness product for a thin metal layer or coating.

Spatial distributions of dispersive properties can be measured with new Meandering Winding Magnetometers (MWMs) for conducting and magnetic media and with Interdigital Electrode Dielectrometers (IDEDs) for insulating and dielectric media. In addition, MWMs can measure the conductivity and thickness independently for a thin coating at a single frequency. Potential MWM applications that require these new capabilities are the characterization of metal-matrix composites, detection of hidden corrosion near fasteners in aging aircraft, characterization of graphite fiber composites, and measurement of percent carbon content distributions near the surface of carburized steel parts. Potential applications for IDEDs include characterization of ceramics, composites, lubricants, and epoxies.

For imposed $\omega$-$k$ magnetometers, the depth of penetration, $\Delta$, is dependent on both the angular frequency, $\omega=2\pi f$, and the wavenumber, $k=2\pi/\lambda$, while for dielectrometers the depth of penetration depends on the wavenumber only. These sensors provide the capability to vary the field depth of penetration into the material under test without changing the input source frequency. This is accomplished by adjusting the spatial wavelength of the winding or electrode pattern. The application of these sensors to the direct measurement

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1 Also, President, JENTEK Sensors, Inc. 335 Arboretum Way, Burlington MA.
of geometric (layer thickness, and proximity) and physical properties (conductivity, complex permeability, complex permittivity) has been demonstrated [1,2,3,4,5,6]. The focus of ongoing research has included investigations into the measurement of dependent properties. For example, temperature measurement with MWMs and moisture front tracking with IDEDs are described in the last section.

THEORY

Over a significant frequency range, the behavior of MWMs and IDEDs can be modeled as magnetoquasistatic (MQS) and electroquasistatic (EQS), respectively. To be considered quasistatic at a prescribed frequency, the dimensions of the sensor must be much smaller than the distance traveled by an electromagnetic wave during an excitation period. In the MQS regime, the magnetic diffusion time is large compared to the charge relaxation time; while in the EQS regime, the reverse is true. For MQS, the relevant potential is the magnetic vector potential, \( A_z \); while for EQS, it is the electric potential, \( \phi \) [8].

Schematics of an MWM and an IDED are provided in Figures 1 and 2. These sensor constructs impose periodic spatial distributions of the magnetic vector potential (for the MWM), and electric potential (for the IDED) with the spatial wavelength, \( \lambda \), indicated in the figures. For the MWM, a current, \( i_1 \), is applied to the primary winding with a prescribed angular frequency, \( \omega \), and the voltage, \( v_2 \), is measured at the terminals of the secondary. The magnitude and phase of the transinductance, \( v_2 / j \omega i_1 \), is then used to determine the properties of the material under test (MUT). For the IDED a voltage, \( V_D \), is applied to the primary electrode and the voltage, \( v_S \), is measured at the terminals of the secondary. For the IDED the gain, \( v_S / V_D \), is used to determine the properties of the MUT.

![Fig. 1. Schematic of Meandering Winding Magnetometer (MWM). The material under test covers the entire sensor footprint, and has conductivity, \( \sigma \), and permeability, \( \mu \), that are functions of the x coordinate.](image)
Fig. 2. Schematic of Interdigital Electrode Dielectrometer (IDED). The material under test covers the entire sensor footprint, and has complex permittivity, $\varepsilon^*$, that is a function of the x coordinate.

Field Depth of Penetration

The depth of penetration into an MUT for a spatially sinusoidal potential is given by

$$\Delta = \frac{2}{\text{Re}(\gamma)}$$

where

$$\gamma = \sqrt{k^2 + j\omega \mu \sigma} = \sqrt{(2\pi\lambda)^2 + 2j\delta^2}$$

for an MWM

and $\delta = \sqrt{2/\omega \mu \sigma}$ is the skin depth, and $\mu$ and $\sigma$ are the permeability and conductivity of the MUT. For an IDED, $\gamma = k = 2\pi/\lambda$.

The wavenumber, $k$, corresponds to the fundamental Fourier mode. To accurately predict the sensor response for the winding and electrode patterns illustrated in Figures 1 and 2, it is necessary to account for the higher spatial harmonics (Fourier modes) as well. To accomplish this, the magnetic vector potential, $A(x,y)$, for the magnetometer and the electric potential, $\phi(x,y)$, for the dielectrometer are expressed as
\[ A_z(x,y) = \text{Re} \{ \hat{A}_z(x,y)e^{j\omega t} \} \quad ; \quad \Phi(x,y) = \text{Re} \{ \hat{\Phi}(x,y)e^{j\omega t} \} \] (3)

where

\[ \hat{A}_z(x,y) = \sum_{n=0}^{\infty} \hat{A}_{zn}(x)\cos(k_n y) \quad ; \quad \hat{\Phi}(x,y) = \sum_{n=0}^{\infty} \hat{\Phi}_n(x)\cos(k_n y) \] (4)

Fig. 3. Schematic representation of a media with P layers. The field quantities for the magnetoquasistatic (MQS) regime are indicated.

Multiple Layered Media Representation

Figure 3 provides a schematic representation of a media with P layers. The influence of the multiple layered media on the applied potentials is represented by the surface inductance density, \( L_n \) [1,2], and the surface capacitance density, \( C_n \) [5], for the MQS and EQS regimes, respectively. These Fourier amplitudes are defined as follows:

\[ \hat{\phi}^{(j)}_n = \frac{k^2 \hat{A}^{(j)}_{zn}}{\hat{H}^{(j)}_{yn}} \quad ; \quad \hat{c}^{(j)}_n = \frac{\hat{\Phi}^{(j)}_{xn}}{\hat{\Phi}^{(j)}_n} \] (5)
where
\[ \hat{H}_{yn}^{(j)} = n^{th} \text{ Fourier amplitude for the magnetic field component in the y direction just above the j}^{th} \text{ surface}, \]
and
\[ \hat{D}_{xn}^{(j)} = n^{th} \text{ Fourier amplitude for the x directed electric displacement flux density just above the j}^{th} \text{ surface}. \]

The field laws within each layer are then solved using "jump" (or continuity) conditions relating field quantities above and below the surfaces to determine the distribution of the field quantities within the MUT as a function of the x coordinate [1,2,5,9]. In Figure 3, the field quantities for the MQS regime are indicated.

**Multiple Wavenumber Interrogations**

For eddy current type measurements, the multiple spatial wavenumber approach provides information similar to that provided by conventional multiple temporal frequency techniques. However, with the multiple wavenumber approach, multiple impedance measurements are provided at a single input source frequency. This permits the determination of spatial property variations at a single frequency. This is important for heterogeneous and dispersive media.

For example, the spatial profile of a material's complex permittivity can be uniquely deduced using the response of multiple-wavelength interdigital-electrode structures. The deduction of inhomogeneities in the material properties can also be achieved from temporal frequency response information, but this process cannot be implemented without assuming that the frequency dispersion of the material is known. On the other hand, if the material properties are not dispersive, the spatial profile of the complex permittivity cannot be uniquely determined unless the charge relaxation times of the layers are distinct [10].

**Sensor Response Prediction and Numerical Inverse Problem Solution**

Given the surface inductance (or capacitance) density for the MUT, the sensor response is computed using the subdomain method of weighted residuals to numerically solve a one dimensional problem in the winding (or electrode) plane, x=0, and to determine the sensor terminal response for a given set of MUT properties [1,2,5].

The estimation of unknown MUT properties ("numerical inverse problem") has been demonstrated using several methods, including (1) multiple frequency nonlinear least squares [2], (2) single frequency secant method for complex permeability [1,2] and complex permittivity [5], and (3) single frequency measurement grids [1,2,5]. A demonstration of the third approach is provided in the first example of the next section.
EXAMPLE APPLICATIONS

Temperature Measurement, Using a Meandering Winding Magnetometer

In this first example, original results are presented for surface temperature tracking for an aluminum plate as it cools from an elevated temperature to room temperature. Figure 4 shows an example of a measurement grid used to independently estimate the electrical conductivity, \( \sigma \), and the lift-off, \( h \), for a thick aluminum plate located above the plane of the sensor. For this example the number of layers is two. Layer 1 is the aluminum plate and layer 2 is the air-gap with thickness, \( h \), between the sensor winding plane and the surface of the plate. The plate was heated in an oven to approximately 90°C and then allowed to cool. Thermocouples attached to the plate surface were used to measure temperature.

As shown in Figure 4, the lift-off height of the plate varied somewhat randomly over a range of about 40 \( \mu m \) during the experiment. Automatic compensation for lift-off is demonstrated in Figure 5, where the conductivity is plotted as a function of time. The conductivity is tracked consistently and well within a measurement error of approximately 8%. The experimentally determined relationship between conductivity and temperature for the aluminum plate was then used to convert the conductivity tracking data to temperature data. Also, comparison of the temperature dynamics with those predicted by a heat transfer model permitted noncontact estimation of the convection coefficient.

![Diagram](image-url)

Fig. 4. MWM measurement grid for conductivity and lift-off measurement for a thick metal plate.
Moisture Front Tracking using a Multiple Wavenumber Interdigital Electrode Dielectrometer

In this second example, results are reviewed from an example of moisture front tracking in pressboard [7]. A three wavenumber approach is used to track the diffusion of moisture into the pressboard. Figure 6a shows a schematic of the experimental set up. At the beginning of the experiment, the pressboard is relatively dry and wet nitrogen gas (N₂ + H₂O vapor) is placed in contact with the pressboard surface. Figure 6b shows the variation in the moisture front position as water diffuses into the pressboard from the gas. The sensor has three sensing regions with wavelengths of 5mm, 2.5mm, and 1mm. First, a phase and magnitude transition is observed in the response of the 5mm sensor at about 1 hour into the experiment. Then at approximately 9 hours the 2.5mm wavelength sensor goes through a similar transition. Finally, at about 13 hours the moisture front has moved to within the sensing range of the 1mm sensor. These time lag measurements provide an estimate of the diffusion coefficient for water in pressboard.

![Graph](image)

Fig. 5. Experimental data demonstrating conductivity tracking as a function of time with automatic lift-off compensation, using the MWM.

SUMMARY

Imposed ω-k magnetometry and dielectrometry provide significant enhancements beyond the capabilities of conventional sensing approaches. Demonstrated measurements indicate the capability to improve material characterization for many high priority NDE applications in the aerospace, materials processing, electronics, and automotive industries.
Fig. 6. (a) Schematic of IDED measurement of moisture front position, $X_{mf}$, as a function of time, and (b) experimental Xmf estimates using a three wavelength IDED ($\lambda=1\text{mm}, 2.5\text{mm}, \text{and } 5\text{mm}$).

REFERENCES