

66/2016

MENTAL LOGIC AND ITS DIFFICULTIES WITH DISJUNCTION

Miguel López-Astorga

University of Talca milopez at utalca cl

Abstract

The mental logic theory does not accept the disjunction introduction rule of standard propositional calculus as a natural schema of the human mind. In this way, the problem that I want to show in this paper is that, however, that theory does admit another much more complex schema in which the mentioned rule must be used as a previous step. So, I try to argue that this is a very important problem that the mental logic theory needs to solve, and claim that another rival theory, the mental models theory, does not have these difficulties.

Keywords: disjunction; mental logic; mental models; semantic possibilities

López-Astorga, Miguel. 2016. Mental logic and its difficulties with disjunction. *Círculo de Lingüística Aplicada a la Comunicación* 66, 195-209. http://www.ucm.es/info/circulo/no66/lopez.pdf http://revistas.ucm.es/index.php/CLAC http://dx.doi.org/10.5209/CLAC.52772

© 2016 Miguel López-Astorga *Círculo de Lingüística Aplicada a la Comunicación (clac)* Universidad Complutense de Madrid. ISSN 1576-4737. http://www.ucm.es/info/circulo

Contents

1. Introduction 196

2. The disjunction introduction rule and its problems 197

3. The Core Schema 2 of the mental logic theory 198

4. Braine and O'Brien's Schema 2 and the disjunction introduction rule 200

5. Schema 2, the disjunction introduction rule, and the mental models theory 201

6. Conclusions 206

References 207

1. Introduction

There is a basic rule in standard propositional calculus that is said to be problematic from a cognitive point of view. That rule is the one of disjunction introduction, and can be formally expressed as follows:

p / Ergo p v q Where 'v' stands for disjunction.

The rule is problematic because the literature on cognitive science shows that people do not often use it. Thus, this circumstance has led to think that human reasoning does not work in accordance with the requirements of calculi such as that of Gentzen (1935). In fact, not even the current theories stating that the human mind applies logical rules when it reasons admit the disjunction introduction rule or claim that reasoning is based on standard logic. One of these theories is the mental logic theory (e.g., Braine & O'Brien, 1998a; O'Brien, 2009, 2014; O'Brien & Li, 2013; O'Brien & Manfrinati, 2010). However, this theory has a difficulty in this regard. While it rejects the disjunction rule, it admits, at the same time, another formal schema that supposes that rule. That schema is considered to be natural on the human mind and is by far more complex than the disjunction introduction rule. And, in addition, as said, it needs this later rule to be applied, since it can even be thought that the disjunction introduction rule is an indispensable previous step that must be fulfilled to apply the schema.

To show this is the main goal of this paper. To achieve it, I will firstly comment on the problems of the disjunction introduction rule in more details. Secondly, I will describe the mentioned schema proposed by the mental logic theory. And then, I will give arguments in favor of the idea that the schema cannot work if the disjunction introduction rule is not accepted. Nevertheless, before finishing, I will also explain that there is another rival theory, the mental models theory (e.g., Johnson-Laird, 2006, 2010, 2012, 2015; Johnson-Laird, Khemlani, & Goodwin, 2015; Khemlani, Orenes, & Johnson-Laird, 2012, 2014; Orenes & Johnson-Laird, 2012), which can account for most of the results in reasoning tasks related to disjunction and does not have difficulties as serious as those of the mental logic theory. I hence will begin with the disjunction rule.

2. The disjunction introduction rule and its problems

The problems related to this rule are clear in the literature, and a very relevant fact in this regard is that, as mentioned, the mental logic theory does not admit it. Indeed, although the mental logic theory is a framework claiming that there is a logic on the human mind and that human thought is essentially formal and syntactic, it does not accept all of the rules of standard logic, but only those that empirical evidence shows that people actually use. Thus, in Braine and O'Brien (1998b, pp. 80-81) a table (Table 6.1) presenting all of the natural schemata used by human beings is included, and the disjunction introduction rule is not one of them. The reason is evident: the experiments reveal that individuals do not always apply it. And this is explained in details in other papers and works authored by proponents of the theory, for example, in Braine, Reiser, and Rumain (1998, pp. 120-121).

There is no doubt that the works on this issue are numerous. Nevertheless, maybe one of them especially relevant is that of Orenes and Johnson-Laird (2012). Many examples of reasoning problems with thematic content in which the disjunction introduction rule is involved are to be found in that paper. One of them is as follows:

"Gorka tried the jam. Does it follow that Gorka tried 'the chocolate cake' or he tried the jam?" (Orenes & Johnson-Laird, 2012, p. 375).

Obviously, most of the participants in tasks such as this one responded negatively and could not accept that the fact that 'Gorka tried the jam' is true implies that the fact that 'Gorka tried 'the chocolate cake' or he tried the jam' is true too. But a very important finding of Orenes and Johnson-Laird's experiments is that there are certain cases in which this kind of problem is correctly solved. Which those particular cases are have been studied by López-Astorga 2015a) too, who has also analyzed to what extent the mental logic theory can explain these phenomena and the additional assumptions that this later theory needs to accept (and the modifications that it needs to make) to account for them. Nonetheless, beyond these particular problems of the mental logic theory, what is truly relevant for this paper is that, as said, while the theory explicitly rejects the disjunction introduction rule, in truth, one of its natural schemata reveals that that rule is, at the same time, presupposed by it, and that the mental logic theory can only be considered as a correct framework if it solves this problem and acknowledges that human beings can really understand the sense of the disjunction introduction rule in a natural way. The next section shows which that particular schema of the mental logic theory is, explains it, and reviews it.

3. The Core Schema 2 of the mental logic theory

As indicated, the mental logic theory is not about standard propositional calculus. There are many differences between the former and the latter. On the one hand, as also said, all of the rules of the latter are not admitted by the former, but other differences are deeper. For example, the principle of explosion (*Ex Contradictione Quodlibet Sequitur*), which, as it is well known, is absolutely valid in Gentzen's (1935) calculus, is not accepted by the mental logic theory, and the reason is the usual one in this theory: experiments show that people do not reason following it. Of course, this can lead one to think that the mental logic theory proposes a logic that, at least in a sense, appears to be linked to systems such as that of Bolzano (1837), but it can also lead one to other problems that need to be solved. For instance, it can be thought that the theory needs to clarify why, beyond the empirical evidence, the principle of explosion is not theoretically valid in its system of schemata (see, for a discussion on these points and the relationship between the mental logic theory and logics such as the Aristotelian one, López-Astorga, 2016).

However, perhaps the most important difference for the aims of this paper is that the schemata of the mental logic theory can be of several kinds. One of these kinds is the Core Schemata set. The Core Schemata are schemata that, according to the proponents of the theory, people use and apply whenever they can. And one of these Core Schemata, which is Schema 2 in Braine and O'Brien (1998b, p. 80, Table 6.1), is this one:

Obviously, we can use symbols and give this schema a logical form that transforms it into an inference with formulae in standard calculus (it have already been said that all of the schemata admitted by the mental logic theory are valid in standard propositional calculus). That logical form can be the following:

 $(p_1 v...v p_n) \rightarrow q; p_i / Ergo q$

On first thought, this seems a very complex inference. Nevertheless, Braine and O'Brien (1998b, p. 80, Table 6.1) inform that their studies reveal that its percentage of mistakes is 0%, which means that, given premises such as $[(p_1 v...v p_n) -> q]$ and $[p_i]$, individuals practically always derive [q]. Thus, from empirical results such as these ones, the adherents of the mental logic theory claim that there is a 'syntax of thought' (Braine & O'Brien, 1998c), and that the formal structures of inferences lead us to draw particular conclusions, even if such structures are as complex as that of Schema 2 of the theory (for an analysis of this schema and discussions in this regard, see also, e.g., López-Astorga, 2015b).

Nonetheless, as mentioned, there is a question that the mental logic theory needs to answer: if the syntax of thought allows applying Schema 2 where possible, why does not that syntax enable to apply the disjunction introduction rule too? And this question needs to be responded because, as also said, Schema 2 implies the disjunction introduction rule. The next section shows this point.

4. Braine and O'Brien's Schema 2 and the disjunction introduction rule

It appears to be absolutely clear that Schema 2 provided by Braine and O'Brien (1998b) is, as claimed by López-Astorga, a version of *Modus Ponendo Ponens* "with disjunctions embedded into the antecedent of the conditional" (López-Astorga, 2015b, p. 147). As it is well known, *Modus Ponendo Ponens* was proposed by Chrysippus of Soli (Diogenes Laërtius, *Vitae Philosophorum* 7, 80), and it is admitted as valid by several theoretical frameworks, including purely logical approaches, and reasoning theories. Its formal structure is also known:

p -> q; p / Ergo q

But it has a characteristic that, in principle, can seem to be problematic: its first premise is a conditional. This can be a difficulty because, as it is well known as well, the controversies on the conditional began in ancient times (e.g., Sextus Empiricus, in texts such as *Pyrrhoneae Hypotyposes* or *Adversus Mathematicos*, distinguishes four different interpretations of the conditional, including that of Philo of Megara, that of Diodorus Cronus, and that of Chrysippus of Soli), and they somehow continue nowadays (e.g., standard logic assumes the material interpretation, i.e., that of Philo of Megara, and the mental logic theory rejects that same interpretation). Nevertheless, these discussions have no an influence on the arguments that I will offer, since what is important here is to note that *Modus Ponendo Ponens*, whose initial version is also assumed as another Core Schema by the mental logic theory (it is Core Schema 7 in Braine & O'Brien, 1998b, p. 80, Table 6.1), only enables to derive the consequent of the conditional if the antecedent is true.

If this is so, it is evident that it is only possible to draw [q] from $[(p_1 v...v p_n) \rightarrow q]$, i.e., from the first premise of Schema 2, if $[p_1 v...v p_n]$ is true. But the only information provided by the second premise is that $[p_i]$ is true, which means that individuals practically always (remember that the percentage of errors of Schema 2 is 0%) understand that the fact that $[p_i]$ is true implies that $[p_1 v...v p_i v...v p_n]$ is also true, which is to say that $[p_1 v...v p_n]$ is also true, which in turn allows deducing [q]. There is no doubt that the mental process from $[p_i]$ to $[p_1 v...v p_n]$ is an application of the disjunction introduction rule. So, it is necessary to explain why, if individuals seem to understand the nature of the logical disjunction, they do not often solve adequately reasoning tasks with the structure [p / Ergo p v q]. After all, this later structure is much simpler than Schema 2 of the mental logic theory, and, if the latter is usually easy for individuals, the former should be so as well.

In my view, this is an actual problem at foundation of the mental logic theory that cannot be solved without a substantial change in its basic theses. Therefore, maybe it is worth looking for the solution to these difficulties in other theoretical frameworks.

5. Schema 2, the disjunction introduction rule, and the mental models theory

A possible solution can be that offered by a theory mentioned above: the mental models theory. To this theory, what happens is that the human mind does not reason by virtue of logical forms, but of the consideration of semantic possibilities. Thus, given that a sentence such as [p v q] admits the semantic possibility of [p] being false and [q] being true, [p v q] cannot be concluded from [p], unless the meanings of [p] and [q] do not allow that semantic possibility. This is the case of, for example, a sentence such as "...Pedro tried 'the chocolate cake' or he tried the dessert?" (Orenes & Johnson-Laird, 2012, p. 375). A sentence such as this one can be drawn from another such as "Pedro tried the dessert" (Orenes & Johnson-Laird, 2012, p. 375), and the reason is that it is not possible that Pedro tries 'the chocolate cake' and he does not try the dessert, because 'the chocolate cake' is a dessert.

Orenes and Johnson-Laird (2012) carried out experiments whose results were consistent with these ideas, and, as indicated, López-Astorga (2015a) analyzes whether or not the mental logic theory can also explain these phenomena. López-Astorga's (2015a) thesis seems to be that the mental logic theory can only account for them arguing that disjunctions such as that of Pedro, 'the chocolate cake,' and the dessert are not actual disjunctions, but only apparent disjunctions, since the meanings of their disjuncts do not enable that either of the two disjuncts can happen without the other one (López-Astorga, 2015a, p. 147). But this solution implies that, in general, people do not use the disjunction introduction rule, and this idea is in conflict with the fact that, according to the theory, individuals tend to use Schema 2. Therefore, it cannot be said that that solution solves the problem that is being addressed in this paper. And this regardless of the fact that it imposes a great challenge to mental logic theory, which would have to

supplement its framework with a procedure or algorithm that enabled to recover true logical forms from sentences that appear to have others. As far as this later point is concerned, López-Astorga's (2015a, pp. 146-147) suggestion seems to be the analysis of the real truth tables of the sentences, but this task can be really difficult for the human working memory, and, at the moment, it cannot be said that an algorithm of that kind has been found (Johnson-Laird et al., 2015, p. 202).

So, given all of these difficulties, the view of the mental models theory about disjunction appears to be more appropriate and clearer. This later theory proposes that the full set of semantic models of an exclusive disjunction such as 'p or q' is

p $\neg q$ $\neg p$ q Where '¬' stands for denial.

Obviously, the first model represents a situation in which the first disjunct is true and the second one is false, and the second model denotes a scenario in which the first disjunct is false and the second one is true. But "a mental model has a structure that corresponds to the known structure of what it represents" (Johnson-Laird, 2012, p. 136), which means that maybe a sentence such as that of Gorka, 'the chocolate cake,' and the jam is better represented without using the letters 'p' and 'q', and as follows:

Chocolate cake \neg (Jam) \neg (Chocolate cake) Jam

Of course, this is so assuming that the disjunction is exclusive. If it were considered to be inclusive, a third model would have to be added, i.e., a model indicating that Gorka tried both 'the chocolate cake' and the jam. However, what is important to note now is that, as indicated, given those models and the information that 'Gorka tried the jam,' it cannot be deduced that 'Gorka tried 'the chocolate cake' or he tried the jam,' since in the first of the models Gorka did not try the jam, which is in conflict with the datum that 'Gorka tried the jam.'

Nevertheless, because the models are semantic and their meanings are relevant as well, the case of the second example indicated (i.e., that of Pedro, the dessert, and 'the chocolate cake') is different. From what has been said above, it can be deduced that, if the disjunction were considered to be exclusive, only one model would be possible:

 \neg (Chocolate cake) Dessert

The other model (Pedro tries 'the chocolate cake' and he does not try the dessert) would not be possible because, as also mentioned, 'the chocolate cake' is a dessert. Therefore, as Orenes & Johnson-Laird (2012) experimentally checked and as indicated as well, individuals do tend to derive a sentence such as 'Pedro tried 'the chocolate cake' or he tried the dessert' form a sentence such as 'Pedro tried the dessert.'

Evidently, this is a simpler explanation than the one that the mental logic theory can offer, even assuming the challenge proposed by López-Astorga (2015a), since, as it can be seen, no logical forms recovery process is necessary. Nonetheless, to address Schema 2 of the mental logic theory, it is also required to take into account the semantic possibilities that the mental models theory assigns to the conditional. The full set of models of a conditional such as 'if p then q' is the following:

р	q
$\neg p$	q
$\neg p$	$\neg q$

But, as said, this is the full set of models, and the theory claims that people do not always detect all of them. In particular, the first one is almost always detected. However, the other two of them can only be identified if the individual makes further cognitive effort. From this point of view, it can be thought that, given a sentence with a structure as difficult as Schema 2, people will tend to consider only the first model, which, in the case of that schema, would be akin to this one:

 $p_1 \text{ or } \dots \text{ or } p_n \qquad q$

Nonetheless, because this model includes a disjunction in one of its clauses (p_1 or...or p_n), it can be said that it is not totally deployed. If we assume that the disjunction is exclusive and that n = 2, the actual models would be two:

$$\begin{array}{ccc} p_1 & \neg p_2 & q \\ \neg p_1 & p_2 & q \end{array}$$

Evidently, as in the previous cases, if we assumed that the disjunction is inclusive, one more model would be necessary (i.e., a model in which p_1 , p_2 , and q were all true). But the point is that these later models can be further simplified. As mentioned, the models indicated above for disjunction are also the elements of its full set, which means that,

given a disjunction such as 'p or q', the models identified by individuals can omit the denials and (if interpreted as exclusive) be just

р

q

And this in turn means that the two totally deployed models of Schema 2 can also be simplified in this way:

 p_1

q p₂ q

Thus, it is absolutely clear that, given a sentence such as 'if p_1 or p_2 then q' and, for example, ' p_2 ', 'q' can be easily derived. And this is so because in the only scenario in which ' p_2 ' is true (the second one), 'q' is true too.

So, the advantages of the mental models theory over the mental logic theory are evident. On the one hand, it can explain why individuals most of the time do not apply the disjunction introduction rule and in which cases they do use it. On the other hand, the mental models theory does not have problems such as those of the mental logic theory with regard to Schema 2. Under the framework of the former, the inferences with structures similar to that schema do not imply the use of the disjunction introduction rule, since, as accounted for, the mental processes are neither syntactic nor based on formal rules, but they only consider semantic possibilities.

Nevertheless, a possible objection against the previous arguments could be that raised by López-Astorga (2015b). As reminded by him, the main theses of the mental models theory imply "that the inferences that refer to more models are more difficult than those that need less models" (López-Astorga, 2015b, p. 148). Thus, *Modus Ponendo Ponens* (or, if preferred, Schema 7 of the mental logic theory) should be less difficult than Schema 2, since the latter requires more models to be applied. However, in his view, there are certain facts that appear to prove that that is not the case. For example, in Table 6.1 in Braine and O'Brien (1998b, p. 80), it is indicated that, while *Modus Ponendo Ponens* has a percentage of mistakes of 2%, that of Schema 2 is, as said, 0%. This is clearly a problem for the mental models theory, because *Modus Ponendo Ponens* only needs one model to be detected: the first one of the full set of the conditional, that is, the one that is often identified without effort:

q

р

This model and the information that 'p' happens, i.e., the information provided by the second premise of *Modus Ponendo Ponens*, are enough to derive 'q'. And this is so because, in the only identified model, 'q' is always true when 'p' is also so.

Nonetheless, Schema 2 needs at least two models in the easiest and simplest case. As said, such models are the following:

p₁ q p₂ q

So, the data presented by Braine and O'Brien (1998b) seem to be in conflict with the predictions of the mental models theory. And, in López-Astorga's opinion, the problem could be even bigger if we considered cases in which the disjunction included in the if-clause of the first premise of Schema 2 had more than two disjuncts (López-Astorga, 2015b, pp. 149-150).

Nevertheless, I think that it would not be hard to respond to an objection such as this one. Firstly, the differences between the percentages of errors of *Modus Ponendo Ponens* and Schema 2 do not appear to be very significant (as said, the percentages are 0% and 2%). Secondly, those percentages, as indicated by Braine and O'Brien (1998b), come from the experiments carried out by Braine et al. (1998), and, in those experiments, the arguments matching Schema 2 that were used only had two disjuncts in the antecedent of the conditional. An example is this one:

"If there is either a D or a J, then there's not a Q

There is a D

? There is a Q?" (Braine et al., 1998, p. 139).

Therefore, it can be thought that, if the experimental tasks had included more disjuncts in the antecedents of their conditionals, maybe the percentages of mistakes would have been higher, and that hence Braine et al.'s results do not clearly undermine the theses and predictions of the mental models theory.

In any case, what does be true is that perhaps the mental models theory needs to go any further into this issue and shows how the models are deployed on the human mind in a more systematic way. Nonetheless, in my view, this is not a task hard to do. The main assumptions of the mental models theory are, for example, absolutely consistent with the idea that, given a sentence with a structure as complex as that of the first premise of Schema 2, it is of course possible that, because working memory is limited, people only think about its models after knowing additional information (in this case, the information offered by the second premise), and that that information leads them to focus only on the relevant model, i.e., the model in which the information of the second premise is involved. In other words, this means that it is possible that, given a sentence such as 'if p_1 or p_2 then q,' individuals consider it to be so difficult and only reflect on its models after knowing, for example, ' p_2 ', which would lead them to think about only a model such as this one:

p₂ q

And this model in turn would lead them to conclude 'q.' Anyway, as stated, this is a point of the mental models theory that needs more exploration.

6. Conclusions

But this paper has shown that the mental logic theory is the one that has more difficulties to solve. Besides problems such as that of the identification of logical forms, which are to be found in the literature and have been mentioned above, its theses appear to include an important contradiction. On the one hand, it rejects the disjunction introduction rule. On the other hand, that rule seems to be absolutely necessary to use one of its Core Schemata: Schema 2.

As commented on, the mental models theory, in principle, does not have this kind of problems. It can explain not only why people tend not to use the disjunction introduction rule, but also why there are certain cases in which that rule does be applied. In addition, this later theory can also account for why, in spite of the fact that individuals do not often use the mentioned rule, they do be able to make inferences involving sentences such as the conditional corresponding to Schema 2 of the mental logic theory.

It is true that, as indicated, the mental models theory has its little difficulties too. However, as also explained, they do not seem to be so important and, probably, can be easy solved. Therefore, Ockham's razor leads us to the mental models theory at the moment, since it appears to better account for the problems related to disjunction in human reasoning. And this makes us doubt the existence of the syntax of thought claimed by the mental logic theory and seriously consider the idea that human inferential activity works by means of semantic analyses of possibilities.

And this is so because, while the difficulties of the mental logic theory seem to be essential and related to its more important theses, those of the mental models theory appear to be only little details that need to be qualified. As also indicated, this later task does not seem to be very hard, and it can be easily made with further research. In fact, I have proposed a possible solution here for the problem of the number of models that the arguments with the structure of Schema 2 need. It is very possible that working memory only considers the strictly necessary models to make inferences, especially when the sentences are as complex as the conditional of that schema. Maybe further experimental work carried out by the proponents of the mental models theory could clarify to a greater extent this issue.

References

Bolzano, B. (1837). Wissenschaftslehre. Sulzbach, Germany: Seidel Buchhandlung.

- Braine, M. D. S. & O'Brien, D. P. (Eds.) (1998a). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Braine, M. D. S. & O'Brien, D. P. (1998b). The theory of mental-propositional logic:
 Description and illustration. In M. D. S. Braine & D. P. O'Brien (Eds.), *Mental Logic* (pp. 79-89). Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Braine, M. D. S. & O'Brien, D. P. (1998c). How to investigate mental logic and the syntax of thought. In M. D. S. Braine & D. P. O'Brien (Eds.), *Mental Logic* (pp. 45-61). Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Braine, M. D. S., Reiser, B. J., & Rumain, B. (1998). Evidence for the theory: Predicting the difficulty of propositional logic inference problems. In M. D. S.
 Braine & D. P. O'Brien (Eds.), *Mental Logic* (pp. 91-144). Mahwah, NJ: Laurence Erlbaum Associates, Inc., Publishers.
- Gentzen, G. (1935). Untersuchungen über das logische Schließen I. Mathematische Zeitschrift, 39, 176-210.

- Johnson-Laird, P. N. (2006). How We Reason. Oxford, UK: Oxford University Press.
- Johnson-Laird, P. N. (2010). Against logical form. *Psychologica Belgica*, 5(3/4), 193-221.
- Johnson-Laird, P. N. (2012). Inference with mental models. In K. J. Holyoak & R. G. Morrison (Eds.), *The Oxford Handbook of Thinking and Reasoning* (pp. 134-145). New York, NY: Oxford University Press.
- Johnson-Laird, P. N. (2015). How to improve thinking. In R. Wegerif, L. Li, & J. C. Kaufman (Eds.), *The Routledge International Handbook of Research on Teaching Thinking* (pp. 80-91). Abingdon, UK, & New York, NY: Routledge.
- Johnson-Laird, P. N., Khemlani, S., & Goodwin, G. P. (2015). Logic, probability, and human reasoning. *Trends in Cognitive Sciences*, 19(4), 201-214.
- Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2012). Negation: A theory of its meaning, representation, and inference. *Psychological Review*, *109*(4), 646-678.
- Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2014). The negation of conjunctions, conditionals, and disjunctions. *Acta Psychologica*, 151, 1-7.
- López-Astorga, M. (2015a). The disjunction introduction rule: Syntactic and semantic considerations. *Pragmalingüística*, 23, 141-149.
- López-Astorga, M. (2015b). The case of the conditionals with disjunctions embedded into their antecedents: Mental logic versus semantic models. *Universum*, 30(2), 143-152.
- López-Astorga, M. (2016). The principle of explosion: Aristotle versus the current syntactic theories. *Schole*, *10*(1), 40-49.
- O'Brien, D. P. (2009). Human reasoning includes a mental logic. *Behavioral and Brain Sciences*, 32, 96-97.
- O'Brien, D. P. (2014). Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013). *Universum*, 29(2), 221-235.
- O'Brien, D. P. & Li, S. (2013). Mental logic theory: A paradigmatic case of empirical research on the language of thought and inferential role semantics. *Journal of Foreign Languages*, *36*(6), 27-41.

- O'Brien, D. P. & Manfrinati, A. (2010). The mental logic theory of conditional proposition. In M. Oaksford & N. Chater (Eds.), *Cognition and conditionals: Probability and Logic in Human Thinking* (pp. 39-54). Oxford, UK: Oxford University Press.
- Orenes, I. & Johnson-Laird, P. N. (2012). Logic, models, and paradoxical inferences. Mind & Language, 27(4), 357-377

Received: February 24, 2015

Accepted: May 18, 2016

Published: May 30, 2016