

Distinguished Preduals of Spaces of Holomorphic Functions

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ABSTRACT. For U open in a locally convex space E it is shown in [13] that there is a complete locally convex space $G(U)$ such that $G(U)' = (\mathcal{H}(U), \tau_\delta)$. We will show that when U is balanced there is an \mathcal{L} -absolute decomposition for $G(U)$ in terms of the preduals of the spaces of homogeneous polynomials. For U balanced open in a Fréchet space we investigate necessary and sufficient conditions for $(\mathcal{H}(U), \tau_\delta)$ to be equal to $G(U)'_b$.

1. INTRODUCTION

Let U be an open subset of a locally convex space E over \mathbb{C} and let $\mathcal{H}(U)$ be the space of holomorphic functions from U into \mathbb{C} . We will denote by τ_c the compact-open topology on $\mathcal{H}(U)$. A semi-norm p on $\mathcal{H}(U)$ is said to be *ported* by the compact subset K of U if for each open $V, K \subset V \subset U$, there is $C_V > 0$ such that

$$p(f) \leq C_V \|f\|_V$$

for all $f \in \mathcal{H}(U)$. The τ_ω -topology on $\mathcal{H}(U)$ is the topology generated by all semi-norms ported by compact subsets of U .

If K is a compact subset of E we denote by $\mathcal{H}(K)$ the space of holomorphic germs on K . The τ_c and τ_ω topologies are defined by

$$(\mathcal{H}(K), \tau_o) = \lim_{\substack{\rightarrow \\ K \subset U}} (\mathcal{H}(U), \tau_o)$$

and

$$(\mathcal{H}(K), \tau_\omega) = \lim_{\substack{\rightarrow \\ K \subset U}} (\mathcal{H}(U), \tau_\omega)$$

Let U be an open subset of a locally convex space E . We say that a seminorm p on $\mathcal{H}(U)$ is τ_δ -continuous, if for each countable increasing open cover $\{U_n\}_n$ of U there is an integer n_o and $C > 0$ such that

$$p(f) \leq C \|f\|_{U_{n_o}}$$

for every f in $\mathcal{H}(U)$. The τ_δ -topology on $\mathcal{H}(U)$ is the topology generated by all τ_δ continuous semi-norms.

In [10], Mazet shows that there is a locally convex space $G(U)$ and a holomorphic map δ_U from U into $G(U)$ with the following universal property: Given any complete locally convex space F and $f \in \mathcal{H}(U, F)$ there is a unique $T_f \in \mathcal{L}(G(U), F)$ such that $f = T_f \circ \delta_U$. In particular if we take $F = \mathbb{C}$, we see that $G(U)$ is a predual of $\mathcal{H}(U)$. Mujica and Nachbin [13] give a new proof of this theorem and show that $G(U)$ is also a topological predual and the inductive dual of $G(U), G(U)'_i$, is equal to $(\mathcal{H}(U), \tau_\delta)$. In §2 we show that the spaces $(P(^n E), \tau_\omega)$ and $(\mathcal{H}(K), \tau_\omega)$ also have topological preduals which we denote by $Q(^n E)$ and $G(K)$ respectively. We show that the spaces $\{Q(^n E)\}_n$ are an \mathcal{L} -absolute decomposition for $G(U)$ when U is balanced and therefore many of the topological properties of $G(U)$ can be obtained from the topological properties of $Q(^n E)$.

In the final section we assume U is a balanced open subset of a Fréchet space. We show that we can construct $(\mathcal{H}(U), \tau_o)'_b$ from the $G(K)$'s, and use this result to show that $G(U)'_b = (\mathcal{H}(U), \tau_\delta)$ if and only if $(\mathcal{H}(U), \tau_\delta)$ is the bidual of $(\mathcal{H}(U), \tau_o)$.

If E is a locally convex space and n a positive integer, $\widehat{\otimes}_{n,\pi} E$ will denote the n -fold tensor product of E with itself completed with respect to and endowed with the π or projective topology. We denote by $\widehat{\otimes}_{s,n,\pi} E$ the completion of the subspace generated by the symmetric tensors.

We refer the reader to [6] for further reading on infinite dimensional holomorphy and to [9] for further reading on locally convex spaces.

2. PREDUALS OF HOMOGENEOUS POLYNOMIALS AND SPACES OF GERMS

Just as the space of holomorphic functions on each open subset of a locally convex space E has a predual, the space of n -homogeneous polynomials on E , for each integer n , and the space of holomorphic germs on each compact subset K of E will also have a predual. In fact, for n -homogeneous polynomials, by taking $Q(^nE)$ to be the space of all linear forms on $P(^nE)$ which when restricted to each locally bounded set is τ_o -continuous, the proof of Theorem 2.1 of [13] is easily adapted to show the following:

Proposition 1. *Let E be a locally convex space, then for each positive integer n , there is a complete locally convex space $Q(^nE)$ and an n -homogeneous polynomial $\delta_n \in P(^nE, Q(^nE))$ with the property that given any complete locally convex space F and any $P \in P(^nE, F)$ there is a unique $L_p \in \mathcal{L}(Q(^nE), F)$ such that $P = L_p \circ \delta_n$.*

This result has previously been proved by Mujica, [12], for Banach spaces and Ryan in [14] with $Q(^nE)$ replaced by $\widehat{\otimes}_{s,n,\pi} E$. By the uniqueness of L_p it will follow that $Q(^nE)$ is topologically isomorphic to $\widehat{\otimes}_{s,n,\pi} E$.

Let us define $G(K)$ to be the space of linear maps from $\mathcal{H}(K)$ to \mathbb{C} which are τ_o -continuous on each set of holomorphic germs which are defined and uniformly bounded on some neighbourhood of K . Applying Theorem 1.1 of [13] to the inductive limit $(\mathcal{H}(K), \tau_\omega) = \lim_{\substack{\rightarrow \\ K \subset V}} \mathcal{H}^\infty(V, \|\cdot\|_V)$

we get:

Proposition 2. *Let K be a compact subset of a locally convex space E , then $G(K)' = (\mathcal{H}(K), \tau_\omega)$.*

This result had been previously proved by Mujica [11] for E a Fréchet space. In [11], Mujica points out that in this special case $G(K) = (\mathcal{H}(K), \tau_o)'_b$.

The concept of \mathcal{S} -absolute decomposition ([5]) allows us to obtain topological properties of $(\mathcal{H}(U), \tau)$ from the corresponding properties of $(P(^nE), \tau)$, $\tau = \tau_o, \tau_w$ or τ_δ and U balanced. To show that $\{Q(^nE)\}_n$ is an \mathcal{S} -absolute decomposition for $G(U)$ we require the following lemma. (See Proposition 3.15 of [6] for a related result.) We let $\mathcal{S} = \{(\alpha_n) \in \mathbb{C}^{\mathbb{N}} :$

$$\limsup_{n \rightarrow \infty} |\alpha_n|^{\frac{1}{n}} \leq 1\}.$$

Lemma 3. *Let U a balanced open subset of a locally convex space E , $(\alpha_n)_n \in \mathcal{S}$ and $\{f_\beta\}_\beta$ be a family of functions in $\mathcal{H}(U)$ uniformly bounded on some neighbourhood of a compact balanced set K . Then there is an $M > 0$ such that*

$$\sum_{n=0}^{\infty} |\alpha_n| \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_V \leq M$$

for every β and some neighbourhood V of K .

For U open in any locally convex space E it can be shown that, for each n , the map $\tilde{\phi} : Q(^nE) \rightarrow G(U)$ defined by $\tilde{\phi}(f) = \phi\left(\frac{\hat{d}^n f(0)}{n!}\right)$ for $\phi \in Q(^nE)$, $f \in \mathcal{H}(U)$ identifies $Q(^nE)$ with a closed subspace of $G(U)$.

Proposition 4. *Let U be a balanced open subset of a locally convex space E ; then $\{Q(^nE)\}_n$ is an \mathcal{S} -absolute decomposition for $G(U)$.*

Proof. Let $B = \{f_\beta\}$ be a family of locally bounded function in $\mathcal{H}(U)$. Recall that the topology on $G(U)$ is the topology of uniform convergence on locally bounded subsets of $\mathcal{H}(U)$. As

$$(1, 2^2, \dots, n^2, \dots) \in \mathcal{S}$$

it follows by Lemma 3 that for each x we can choose a neighbourhood V_x of Γ_x , the balanced hull of $\{x\}$, such that

$$\sup_{\beta} \sum_{n=0}^{\infty} n^2 \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_{V_x} = M_x < \infty.$$

Therefore for every m and every β we have,

$$\left\| m^2 \sum_{n=m}^{\infty} \frac{\hat{d}^n f_{\beta}(0)}{n!} \right\|_{V_x} \leq \sum_{n=m}^{\infty} n^2 \left\| \frac{\hat{d}^n f_{\beta}(0)}{n!} \right\|_{V_x} \leq M_x.$$

Thus the set

$$\tilde{B} = \left\{ m^2 \sum_{n=m}^{\infty} \frac{\hat{d}^n f_{\beta}(0)}{n!} \right\}_{m, \beta}$$

is locally bounded.

For $\phi \in G(U)$, let $\phi_n = \phi|_{P^n(E)}$. For each semi-norm α let $U_{\alpha} = \{x \in E: \alpha(x) \leq 1\}$ and $B_{\alpha}^n = \{P \in P^n(E): \|P\|_{U_{\alpha}} \leq 1\}$. Then B_{α}^n is a locally bounded set of holomorphic functions on $\mathcal{H}(U)$ and $\phi_n|_{B_{\alpha}^n} = \phi|_{B_{\alpha}^n}$ is τ_{α} -continuous and $\phi_n \in Q^n(E)$. Since ϕ is τ_{α} -continuous on \tilde{B} and the Taylor series expansion of f_{β} about 0 converges to f_{β} in the τ_{α} -topology we have that,

$$\begin{aligned} \left\| \phi - \sum_{k=0}^{m-1} \phi_k \right\|_{\tilde{B}} &= \sup_{\beta} \left| \left(\phi - \sum_{k=0}^{m-1} \phi_k \right) (f_{\beta}) \right| \\ &= \sup_{\beta} \left| \phi \left(\sum_{n=m}^{\infty} \frac{\hat{d}^n f_{\beta}(0)}{n!} \right) \right| \leq \frac{1}{m^2} \|\phi\|_{\tilde{B}} \rightarrow 0 \end{aligned}$$

as $m \rightarrow \infty$. Thus $\phi = \sum_{n=0}^{\infty} \phi_n$ in $G(U)$.

As

$$(\alpha_1, 2^2\alpha_2, \dots, n^2\alpha_n, \dots) \in \mathcal{S}$$

for $(\alpha_n)_n \in \mathcal{S}$, it follows by Lemma 3 that for every $x \in U$, we can find $N_x > 0$ such that

$$\sup_{\beta} \sum_{n=0}^{\infty} n^2 \alpha_n \left\| \frac{\hat{d}^n f_{\beta}(0)}{n!} \right\|_{W_x} \leq N_x$$

for some neighbourhood W_x of Γ_x . In particular the set

$$B' = \left\{ n^2 \alpha_n \frac{\hat{d}^n f_{\beta}(0)}{n!} \right\}_{n, \beta}$$

is locally bounded. Let $\phi_\eta \rightarrow 0$ in $G(U)$. As every locally bounded subset of $P({}^nE)$ is locally bounded in $\mathcal{H}(U)$ and

$$\|(\phi_\eta)_n\|_{\left\{\frac{\hat{d}^n f_\beta(0)}{n!}\right\}_\beta} = \sup_\beta \left| \phi_\eta \left(\frac{\hat{d}^n f_\beta(0)}{n!} \right) \right| \leq \frac{1}{n^2 \alpha_n} \|\phi_\eta\|_{B'},$$

$(\phi_\eta)_n \rightarrow 0$ in $Q({}^nE)$ for every n . This shows that $\{Q({}^nE)\}_n$ is a Schauder decomposition for $G(U)$. For $\sum_{n=0}^\infty \phi_n \in G(U)$ and $(\alpha_n)_n \in \mathcal{S}$,

$$\begin{aligned} \left\| \sum_{n=k}^\infty \alpha_n \phi_n \right\|_B &\leq \sum_{n=k}^\infty |\alpha_n| \|\phi_n\|_B \\ &= \sum_{n=k}^\infty \sup_\beta \left| \phi \left(\alpha_n \frac{\hat{d}^n f_\beta(0)}{n!} \right) \right| \\ &= \sum_{n=k}^\infty \frac{1}{n^2} \sup_\beta \left| \phi \left(n^2 \alpha_n \frac{\hat{d}^n f_\beta(0)}{n!} \right) \right| \\ &\leq \|\phi\|_{B'} \sum_{n=k}^\infty \frac{1}{n^2}. \end{aligned}$$

This shows that $\{Q({}^nE)\}_n$ is an \mathcal{S} -decomposition for $G(U)$, and taking $k=0$, we see that the decomposition is \mathcal{S} -absolute. \square

In a way similar to that in which each $Q({}^nE)$ can be identified with a closed subspace of $G(U)$ it can be shown that each $Q({}^nE)$ can be identified with a closed subspace of $G(K)$ for K a compact subset of E and E any locally convex space. By a modification of Proposition 4 we have:

Proposition 5. *Let K be a balanced compact subset of a locally convex space E , then $\{Q({}^nE)\}_n$ is an \mathcal{S} -absolute decomposition for $G(K)$.*

Corollary 6. *If E is a Fréchet space then $Q({}^nE) = (P({}^nE), \tau_o)'_b$.*

Proof. Both $\{Q({}^nE)\}_n$ and $\{(P({}^nE), \tau_o)'_b\}_n$ are \mathcal{S} -absolute decompositions for $G(K) = (\mathcal{H}(K), \tau_o)'_b$. \square

3. DISTINGUISHED PREDUALS OF SPACES OF HOLOMORPHIC FUNCTIONS

In this section we investigate conditions for $G(U)'_b$ to be equal to $(\mathcal{H}(U), \tau_\delta)$. We begin by relating $(\mathcal{H}(U), \tau_\delta)'_b$ to the $G(K)$'s when U is a balanced open subset of a Fréchet space. We denote by $R_{K,U}$ the map from $\mathcal{H}(U)$ to $\mathcal{H}(K)$ which assigns to each f in $\mathcal{H}(U)$ its restriction to K .

Proposition 7. *Let U be a balanced open subset of a Fréchet space E , then $(\mathcal{H}(U), \tau_\delta)'_b = \lim_{\substack{\vec{K \subset U} \\ K \text{ balanced}}} G(K)$.*

Proof. Since $(\mathcal{H}(U), \tau_\delta)$ and $(\mathcal{H}(K), \tau_\delta)$ are semi-Montel, we have that $(\mathcal{H}(U), \tau_\delta)'_b = (\mathcal{H}(U), \tau_\delta)'_\tau$ and $(\mathcal{H}(K), \tau_\delta)'_\tau = (\mathcal{H}(K), \tau_\delta)'_b = G(K)$, where τ denotes the Mackey topology. If K is a balanced compact subset of U then the Taylor series of f about 0 converges to f in $(\mathcal{H}(K), \tau_\delta)$. Since $\mathcal{H}(U)$ contains all polynomials, $R_{K,U}((\mathcal{H}(U), \tau_\delta))$ is dense in $(\mathcal{H}(K), \tau_\delta)$. Therefore the inductive limit

$$(\mathcal{H}(U), \tau_\delta) = \lim_{\substack{\vec{K \subset U} \\ K \text{ balanced}}} (\mathcal{H}(K), \tau_\delta)$$

is reduced. Therefore by IV.4.4 of [15] we have that

$$(\mathcal{H}(U), \tau_\delta)'_b = \lim_{\substack{\vec{K \subset U} \\ K \text{ balanced}}} (\mathcal{H}(K), \tau_\delta)'_b = \lim_{\substack{\vec{K \subset U} \\ K \text{ balanced}}} G(K).$$

□

Lemma 8. *Let U be a balanced open subset of a Fréchet space E , then $G(U)'_b = ((\mathcal{H}(U), \tau_\delta)'_b)'_b$.*

Proof. It follows by Grothendieck's Completeness Theorem, Theorem 3.11.1 of [9], that $G(U)$ is the completion of $(\mathcal{H}(U), \tau_\delta)'_b$. Therefore, $((\mathcal{H}(U), \tau_\delta)'_b)'_b = G(U)'_b = \mathcal{H}(U)$. To show that these two topologies agree

it is sufficient to prove that each bounded subset of $G(U)$ is contained in the closure, in $G(U)$, of a bounded subset of $(\mathcal{H}(U, \tau_o)'_b)$. Let B be a bounded subset of $G(U)$. Define \tilde{B} by

$$\tilde{B} := \left\{ \sum_{n=0}^m \phi_n : \sum_{n=0}^{\infty} \phi_n \in B, m \in \mathbb{N} \right\},$$

where $\phi_n = \phi|_{P(^nE)}$. By the proof of Proposition 3.13 of [6], \tilde{B} is bounded and as $\phi_n \in Q(^nE) = (P(^nE), \tau_o)'_b$, $\tilde{B} \subset (\mathcal{H}(U, \tau_o)'_b)$. Since $\sum_{n=0}^m \phi_n \rightarrow \phi$ on the locally bounded (and therefore τ_o -bounded) subsets of $\mathcal{H}(U)$, B is contained in the closure of \tilde{B} . This completes the proof. \square

The space $G(U)$ is not in general a Fréchet space when U is an open subset of a Fréchet space E . However, our next result shows that $G(U)$ behaves very like a Fréchet space vis-a-vis necessary and sufficient conditions needed in order to show it is distinguished. We will denote by τ_d the topology on $\mathcal{H}(U)$ defined by $(\mathcal{H}(U), \tau_d) = G(U)'_b$.

Theorem 9. *Let U be a balanced open subset of a Fréchet space E , then the following are equivalent:*

- (a) $G(U)'_b = G(U)'_i = (\mathcal{H}(U), \tau_\delta)$,
- (b) $G(U)$ is distinguished ($G(U)'_b$ is barrelled),
- (c) $G(U)'_b$ is infrabarrelled,
- (d) $G(U)'_b$ is bornological.

Proof. Since $(\mathcal{H}(U), \tau_\delta)$ is barrelled and bornological we see that (a) will imply (b), (c) and (d).

It follows from Lemma 8 and the fact that $G(U)'_i = (\mathcal{H}(U), \tau_\delta)$, that τ_d is a topology on $\mathcal{H}(U)$ which satisfies $\tau_o \leq \tau_d \leq \tau_\delta$. It now follows from Corollary 3 of [1] that (b) implies (a). Since $G(U)$ is barrelled, see Theorem 4.4 of [13], it follows by Theorem 3.6.1. of [9], that $G(U)'_b$ is quasi-complete and so (b) is equivalent to (c). Finally, we note that (d) always implies (c). \square

Conditions (a) and (b) of the above Theorem are also true in the case where U is a balanced open subset of a DF space where we replace Corollary 3 of [1] by Corollary 5 of [1].

From Lemma 8 and Theorem 9 we see that if U a balanced open subset of a Fréchet space E , then $G(U)'_b = (\mathcal{H}(U), \tau_\delta)'_b$ if and only if the bidual of $(\mathcal{H}(U), \tau_\alpha)$ is equal to $(\mathcal{H}(U), \tau_\delta)$. The question of when $((\mathcal{H}(U), \tau_\alpha)'_b)'_b$ is equal to $(\mathcal{H}(U), \tau_\delta)$, was investigated by Dineen and Isidro in [8]. There they proved the following Proposition.

Proposition 10. (Dineen-Isidro) *Let U be a balanced open subset of a locally convex space E , then $((\mathcal{H}(U), \tau_\alpha)'_b)'_b = (\mathcal{H}(U), \tau_\delta)$ if and only if $(\mathcal{H}(U), \tau_\delta)$ has a basis of absolutely convex τ_α -closed neighbourhoods of 0.*

Thus, in the case where U is a balanced open subset of a Fréchet space E , we see that the sufficient condition given in [13] Theorem 1.1 for $G(U)'_b$ to be equal to $(\mathcal{H}(U), \tau_\delta)$ is in fact also necessary.

As a complemented subspace of a distinguished space is distinguished, and $Q(^n E)$ is complemented in $G(U)$ for every integer n , we see that a necessary condition for τ_α to equal τ_δ is that each $Q(^n E)$ is distinguished. This gives us a means of obtaining Fréchet spaces E such that $G(U)'_b \neq G(U)'_i$ for any balanced open subset U of E . We begin with the observation that $Q(^1 E) = E$, and therefore we have that $G(U)'_b \neq (\mathcal{H}(U), \tau_\delta)$ for any balanced open subset of a non-distinguished Fréchet space.

Taskinen, [16], constructs a Fréchet-Montel space F_α , such that $Q(^2 F_\alpha) = F_\alpha \widehat{\otimes}_{s,\pi} F_\alpha$ is not distinguished. Thus we see that $G(U)'_b \neq (\mathcal{H}(U), \tau_\delta)$ for any balanced open subset U of F_α . This also means that there are Fréchet-Montel spaces such that $G(U)$ is not distinguished.

To apply Theorem 9 and obtain examples of open subsets of Fréchet spaces where $G(U)'_b = (\mathcal{H}(U), \tau_\delta)$ is very difficult. This is because it is hard to show that an arbitrary locally convex space is distinguished. A necessary condition for τ_α to be equal to τ_δ is that each $Q(^n E)$ is distinguished. When this holds we have the following Proposition showing that τ_α is finer than τ_δ .

Proposition 11. *Let E be a Fréchet space such that $Q(^n E)$ is distinguished for every integer n , then, for every balanced open set U in E , τ_α is finer than τ_δ on $\mathcal{H}(U)$.*

Proof. For every balanced compact subset K of E , $G(K)$ is a Fréchet space, and it follows by Proposition 2 of [3], that $G(K)$ is distinguished. Therefore $G(K)'_i = G(K)'_b$, for every balanced compact set K of E . Hence

$$(\mathcal{H}(U), \tau_\omega) = \lim_{\substack{\leftarrow \\ K \subset U}} (\mathcal{H}(K), \tau_\omega) = \lim_{\substack{\leftarrow \\ K \subset U}} G(K)'_i = \lim_{\substack{\leftarrow \\ K \subset U}} G(K)'_b$$

By the definition of projective limit, $\lim_{\substack{\leftarrow \\ K \subset U}} G(K)'_b$ is weaker than

$$(\lim_{\substack{\leftarrow \\ K \subset U}} G(K))'_b = ((\mathcal{H}(U), \tau_\omega)'_b)'_b = G(U)'_b.$$

□

If E is a Banach space, $Q^n(E)$ will also be a Banach space for every integer n . In particular, each $Q^n(E)$ is distinguished and therefore by Proposition 11 we have that $\tau_\omega \leq \tau_d \leq \tau_\delta$ on $\mathcal{H}(U)$ for every balanced open subset U of E . Therefore if we know that $\tau_\omega = \tau_\delta$ on $\mathcal{H}(U)$, we can conclude that $G(U)'_b = (\mathcal{H}(U), \tau_\delta)$. For examples of Banach spaces and products of Banach spaces with nuclear spaces with this property we refer to [4,2,7].

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