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12

Abstract

13 Due to increasing penetration of wind energy in the recent times, wind farmers tend to generate
14 increasing amount of energy out of wind farms. In order to achieve the target, many wind farms
15 are operated with a layout design of numerous turbines placed close to each other in a limited land
16 area leading to greater energy losses due to ‘wake effects’. Moreover, these turbines need to satisfy
17 many other constraints such as topological constraints, minimum allowable capacity factors, inter-
18 turbine distances, noise constraints etc. Thus, the problem of placing wind turbines in a farm to
19 maximize the overall produced energy while satisfying all constraints is highly constrained and
20 complex. Existing methods to solve the turbine placement problem typically assume knowledge
21 about the total number of turbines to be placed in the farm. However, in reality, wind farm
22 developers often have little or no information about the best number of turbines to be placed in a
23 farm. This study proposes a novel hybrid optimization methodology to simultaneously determine
24 the optimum total number of turbines to be placed in a wind farm along with their optimal
25 locations. The proposed hybrid methodology is a combination of probabilistic genetic algorithms
26 and deterministic gradient based optimization methods. Application of the proposed method on
27 representative case studies yields higher Annual Energy Production (AEP) than the results found
28 by using two of the existing methods.

29 **Keywords:** Wind energy; systems engineering; micro-siting optimization; genetic algorithms;
30 gradient based optimization; hybrid techniques

31

32

33 1. INTRODUCTION

34 Wind energy has turned out to be a promising alternative energy source in order to compete with
35 the depleting conventional sources. Due to its wide-scale availability, low cost and environment
36 friendly operation, the idea of utilizing wind power at a massive scale has become a primary focus
37 in the power industry, government policies and academic research [1-3]. According to the Global
38 Wind Energy Council (GWEC) [4], the global cumulative installed wind capacity has increased
39 from 6100 MW to 318,105MW in the last two decades and is expected to reach 1100GW over the
40 next five years (~12% of electricity supply of the world). The standard systems engineering
41 approach of capturing the potential wind energy in a farm is to place wind turbines at optimal
42 locations, known as micro-siting, and thereby tapping the maximum energy out of it. The problem
43 of micro-siting optimization is not trivial due to various challenges involved in problem formulation
44 and development of solution methodology. The challenges related to problem formulation appear
45 while handling different kind of constraints such as inter-turbine distance, topology, overall
46 capacity factor, longevity of turbine life, turbine noise, consideration of turbine wakes etc. While
47 dealing with above constraints, micro-siting problems often lead to mixed integer nonlinear
48 programming (MINLP) formulations for which the methodologies which can guarantee the global
49 solution are yet to be developed. Moreover, the fact that the predictions of the commercial softwares
50 [5-7] for designing the layout of turbines in a wind farm till date are still not up to the mark [2] and
51 human intervention is required to reduce the installation and operational costs, shows the scope of
52 improvement in this field both in terms of development of methodologies for efficient problem
53 formulation and solution technique.

54 A huge amount of work has been done in the area of micro-siting over the past two decades [8-
55 10], where binary-coded Genetic Algorithms (GAs) have been used to maximize the net Annual
56 Energy Production (AEP) with less installation cost over fixed number of turbines in a wind farm.
57 Mossetti's [8] work showed the effectiveness of GA for solving such problems. The results for
58 different wind conditions shown in this work were improved later by Grady [9] by considering a
59 higher population size and number of generations thus allowing candidate solutions to have
60 sufficient time to converge. In the study of Emami and Noghreh [10], the conflict of AEP and the
61 cost involved in the project was expressed in the form of weighted sum of these two objectives and
62 better results were found for certain set of weight values in the objective function. These studies
63 consider a farm of regular shape (rectangle) that can be sub-divided into several cells of the size of
64 five times the rotor diameter of the turbines. Assuming only one turbine can be accommodated in
65 each of these cells, these formulations ensure the turbines are placed sufficiently away from one
66 another to avoid wake effects. Mittal [11] reduced these cell sizes by 40 times and shown the
67 effectiveness of the approach by improving the earlier results [8-9] substantially. Wan et al. [12]
68 used real coded GA to solve the positioning problem of fixed number of turbines and obtained better
69 results as compared to the work of Grady [9]. Mora et al. [13] proposed variable length
70 chromosomes in GA to handle different types of turbines in micro-siting and developed novel
71 crossover and mutation operators to handle these chromosomes of different lengths. Gonzalez et al.
72 [14] proposed another variable length codification in an efficient GA setup to optimize the layout
73 of turbines by calculating net yearly income obtained by selling net energy produced by each turbine
74 considering various kinds of energy losses. The step of codification represents each of the
75 chromosomes as different layouts, where the length of chromosome is driven by the total number
76 of turbines in a farm and information related to turbine attributes is also coded. Apart from GA,

77 other evolutionary techniques such as Imperialist Competitive Algorithm [15], Strength Pareto
78 Evolutionary Algorithm (SPEA) [16], Ant Colony Optimization [17], Particle Filtering Approach
79 [18], Particle Swarm Optimization [3] etc. were used to deal with the optimal placement of turbines
80 in a wind farm layout and solve different single or multi-objective optimization formulations. In
81 another multi-objective formulation, Kwong et al. [19] considered the maximization of AEP and
82 minimization of the noise level for a fixed number of turbines in a wind farm. Zhang et al. [20]
83 presented Constrained Programming and Mixed Integer Programming models to maximize the total
84 farm-level energy produced for simple to complex wind scenarios. Currently, several commercial
85 software programs are available addressing the problem of wind farm layout and design. The most
86 widely used is WASP [5], which offers modules that allow assessing wind behavior in complex
87 terrain using computational fluid dynamics (CFD). It helps to develop wind farm design by
88 considering previously obtained wind climate observations and wake effect is calculated using
89 Katic model [21]. Windfarmer [6] optimizes the layout using Reynolds Average Navier-Stokes
90 (RANS) based CFD model. It considers uncertainty, noise, and electrical infrastructure as additional
91 aspects. WindPro [7] designs the layout by sequentially adding the wind turbines at positions with
92 maximum available energy while optimizing the net AEP of a farm.

93 Most of the existing models and software packages solve the micro-siting problem assuming the
94 total number of turbines in a wind farm is fixed i.e. the rated power capacity of a wind farm is
95 known and the goal here is to find out the turbine locations. In this case, the problem is a nonlinear
96 programming problem, where turbine locations are the only decision variables. Under different
97 circumstances, either the rated power capacity has been driven by certain business decisions or it
98 has been arrived at based on past experiences of the experts. There are issues with either of these
99 approaches. If the rated capacity is higher than the optimal rated capacity (which is unknown and

100 needs to be found out), the rated capacity will be misleading and will never be realizable. On the
101 other hand, if the rated capacity is lower than the optimal value, the purpose of tapping the full
102 potential of wind energy can be jeopardized. However, the optimal rated capacity can be found by
103 formulating an optimization problem which can calculate the total number of turbines that can be
104 placed in a farm layout as well as their locations. A common practice observed in many practical
105 installations is to erect as many turbines as possible in a wind farm ignoring the wake effect and
106 thereby generating an inefficient as well as sub-optimal micro-siting plan. It is, therefore, more
107 realistic to find out the optimal total number of turbines as well as their locations simultaneously
108 while performing micro-siting in presence of several other constraints.

109 Though some of earlier studies address this issue of simultaneous determination of optimal total
110 number and locations of turbines in a wind farm, a severe compromise has been made in terms of
111 assuming the locations of the turbines only at fixed locations. For example, a wind farm is divided
112 into certain number of cells and the center of the cell is assumed to be the only location of a turbine
113 in that cell. No additional constraint for tackling the inter turbine distance has been considered;
114 instead the size of each of these cells is assumed to be some integer times (e.g. five times) the rotor
115 diameter of the turbine. Simultaneous determination of optimal total number and locations of
116 turbines in a wind farm for an objective, say maximization of AEP, involves both binary (“yes / no”
117 decisions for turbines at several locations) and continuous variables (turbine coordinates) and leads
118 to mixed integer (non)linear programming (MINLP) formulations. Assuming the total number of
119 turbines to be installed is N_f and the whole farm area under study is divided into N_{cell} units, the
120 possible number of distinct solutions that has to be considered during optimization can be given by
121 equation (1) [22].

$$N_{sol} = \binom{N_{cell}}{N_t} = \frac{N_{cell}!}{N_t!(N_{cell} - N_t)!} \quad (1)$$

123 The size of the problem and thereby the complexity increase with the increase in number of cells in
 124 the search space (the case of division of the wind farm into finer grids) and the problem size could
 125 be unmanageable after a certain extent of granularity in the grid / cell size. Recently, Chen et al.
 126 [23] adopted a mix of real and binary coded GA to solve this problem where each layout is
 127 represented by a triplet of a fictitious number (N_f number for each of them) of x, y coordinates and
 128 binary variables. Depending on the number of '1's present in the N_f binaries, the total number of
 129 turbines in a layout is calculated whereas their corresponding x and y coordinates are their
 130 respective locations in the layout. Since the total number of turbines is not known here, several
 131 optimization runs with different values of N_f are recommended. The amount of complexity involved
 132 in this formulation can be guessed from the estimates of number of solutions to be considered from
 133 (1) since the real values of the coordinates can assume any value within the given bounds. In another
 134 study, Kulkarni and Mittal [24] developed a novel heuristic approach, where the optimal number of
 135 turbines and their optimal locations can be found out simultaneously in order to maximize the net
 136 AEP and minimize the wake losses in a wind farm. It suffers from the drawback of other grid-based
 137 methods: since all candidate turbine-locations lie on the grid, possibly better locations lying
 138 between grid-points can never be chosen. Moreover, refining the grid resolution to better represent
 139 the wind farm area may make the problem computationally very demanding. Another limitation of
 140 this approach is that the performance of the algorithm is driven by the selection of the starting
 141 solution. To overcome these limitations, a novel hybrid methodology has been proposed in this
 142 work which makes use of a bi-level optimization formulation. GA has been used in the first level
 143 to determine the number of turbines out of certain number of possible candidate locations (a discrete

144 formulation) whereas a classical optimization technique improves those locations in the second
145 level assuming the number of turbines in the layout as obtained from the first level are fixed (a
146 continuous formulation). This study additionally considers the presence of other constraints such
147 as inter turbine distance, overall capacity factor, presence of wake in energy calculations etc. The
148 rest of the paper has been organized as follows. Section 2 describes the optimization problem
149 formulation while AEP calculation and functioning of the wake model are discussed in section 3.
150 Section 4 explains the heuristic methodology followed by the brief description of the proposed
151 hybrid optimization methodology in section 5. Finally, the results of different representative case
152 studies and the conclusions are presented in the sections 6 and 7 respectively.

153 **2. PROBLEM FORMULATION**

154 The development of mathematical model for wind farm micro-siting is limited to certain
155 assumptions. These assumptions can be modified or even removed as and when needed. The
156 assumptions are described as follows:

157 *Assumption 1:* Wind turbine locations are described by a Cartesian coordinate system (x_i, y_i) ,
158 $i = 1, \dots, N$, where N is the number of turbines.

159 *Assumption 2:* Wind turbines are assumed to have uniform specifications in terms of rated power,
160 rotor diameter, hub-height etc.

161 *Assumption 3:* A widely used Jensen wake model [25] is used to calculate the velocity deficit due
162 to wake effects.

163 *Assumption 4:* As widely used in literature, the wind speed is assumed to follow a two parameter

164 Weibull distribution [26] $C_v(u, A, k) = 1 - e^{-\left(\frac{u}{A}\right)^k}$, where A is the scale parameter, k is the shape
 165 parameter and $C_v(\cdot)$ is the cumulative Weibull distribution function.

166 *Assumption 5:* Power and thrust coefficient curves [30] for a Vestas-V52 850 kW turbine are used
 167 to evaluate the power and coefficient of thrust for corresponding wind-speeds (as shown in Fig. 1).

168 Mathematically, the problem can be represented as:

169 *Objective Function:*
$$\text{Max}_{N_t} \text{Max}_{x_i, y_i} \sum_1^{N_t} AEP(x_i, y_i)$$

170 *Constraints:*
$$g_j(x_i, y_i) \leq 0, \quad j=1, \dots, Nc; \quad i=1, \dots, N_t$$

(2)

$$lb \leq (x_i, y_i) \leq ub$$

171 Here, in equation (2) [24], N_t is the total number of turbines, is taken as a *upper level* decision
 172 variable and (x_i, y_i) , the location co-ordinates of these turbines, are considered as a *lower level*
 173 decision variables. Nc denotes the number of constraints, whereas the geographical boundary limits
 174 are depicted by lb and ub . For a regular square farm of 500×500 m² considered here, the lb and
 175 ub for x_i and y_i can be 0 and 500, respectively.

176 The inequalities $g_j(x_i, y_i) \leq 0$, represent the following constraints:

- 177 i) Inter turbine distance (ITD), which is kept greater than or equal to 3 times the rotor
 178 diameter of the turbines.

179
$$g_1(x_i, y_i) = n_{space} * D - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq 0,$$

$j=i, j; j=1, \dots, N_t$

(3)

180 ii) Overall capacity factor (OCF), which is kept to be higher than the specified limit for it

$$181 \quad g_2(x_i, y_i) = OCF^{\text{lim}} - \frac{\sum_{N_t} AEP(x_i, y_i)}{(8766) * N_t * \text{Pr}} \quad (4)$$

182 In the above equations (3 and 4), D is the rotor diameter of turbine, n_{space} is the minimum
 183 allowable distance between two turbines which is assumed to equal to 3, OCF^{lim} is the selected
 184 limit of allowable capacity factor which is assumed to be 20% and Pr is the rated power (850kW)
 185 of a wind turbine. These inequality constraints are explained briefly in section 5. The above problem
 186 belongs to the class of mixed integer nonlinear programming problems (MINLP) that are generally
 187 very hard (NP-hard) to solve due to the combinatorial complexity involved.

188 3. AEP CALCULATION AND WAKE MODELING

189 3.1. AEP Calculation

190 To calculate the net energy produced accurately, the spatial and temporal distribution of wind
 191 resource must be known which is generally expressed in terms of wind resource grid (WRG) that
 192 stores information about Weibull parameters at a given location. The net AEP (kWh) at a given
 193 location of wind farm can be expressed as [26]:

$$194 \quad AEP = (8766) \sum_{i=1}^{\text{directions}} \sum_{j=1}^{\text{speed}} \sum_{k=1}^{\text{turbines}} \text{Frequency}_{ijk} \text{Power}_{ijk} \quad (5)$$

195 where, Frequency_{ijk} is the frequency or probability of wind coming from direction i , with wind
 196 speed j on to the turbine k , and similar terminology holds for Power_{ijk} in kilowatts (KW).

197 Practically, the above formula can be approximated as [16]:

$$AEP = (8766) \sum_{i=1}^{360^\circ} \sum_{j=1}^{u_{\max}} Pwr(\theta_i, u_j) p(\theta_i) p(u_j) \Delta\theta_i \Delta u_j \quad (6)$$

where $p(\theta_i)$ and $p(u_j)$ determine the probability that the wind blows in direction θ_i at speed u_j and are obtained from WRG data. Depending on whether a turbine is affected by wake and the number of upstream turbines generating the wake, the reduced speed at the turbine affected by wake is calculated. The corresponding power $Pwr(\theta_i, u_j)$ for that particular speed can be calculated using the turbine power curve (see Fig.1). The two-parameter Weibull distribution is used to calculate the $p(u_j)$ by using equations (7) and (8) [30].

$$W_{cum}(u, A, k) = 1 - e^{-\left(\frac{u}{A}\right)^k} \quad (7)$$

$$p(u_j) = W_{cum}\left(u_j + \frac{u_{step}}{2}, A, k\right) - W_{cum}\left(u_j - \frac{u_{step}}{2}, A, k\right) \quad (8)$$

Where W_{cum} is the cumulative probability distribution and $p(\theta_i)$ is extracted from parameter f given in WRG for a particular location.

3.2. Wake model and calculation

In a wind farm, different turbines interact with each other due to wake effects that upstream turbines create on downstream turbines. Among various wake models reported in the literature, a widely accepted Jensen wake model [25] has been adopted here. An expression for the reduced wind-speed of downwind turbines due to wake-effects can be expressed as follows:

$$\Delta u_{ij} = u_o \left(1 - \sqrt{1 - C_T}\right) \left(\frac{R_o}{R_o + k_w * d_{ij}}\right)^2 \left(\frac{A_{ij}}{A_j}\right) \quad (9)$$

The following nomenclature is followed in the above equation assuming i and j as upwind and downwind turbines, respectively.

Δu_{ij} : Reduction in the wind speed on turbine j due to the turbine i,

u_o : Free stream wind speed,

C_T : Coefficient of thrust (Fig. 1),

R_o : Rotor radius,

k_w : wake decay constant for Jensen model,

d_{ij} : Distance between upstream and downstream turbines (see Fig. 2),

A_{ij} : Overlapped area [3] varies depending on type of wake effect on downwind turbine and

A_j : Downwind turbine area.

Fig. 2 depicts the variation in distance calculation due to three types of wake effects. Here, R_r is the rotor radius of downwind turbine and R_{ij} is the wake radius created by an upwind turbine on the downwind. Depending on the area overlapped, the distance between two turbines d_{ij} is calculated.

In reality, a downwind turbine may be under the influence of multiple upwind turbines. In that case, equation (9) can be modified as follows:

$$U_j = u_o \left(1 - \sqrt{\sum_{k=1, k \neq j}^{N_{upwind}} (\Delta u_{ij})^2} \right) \quad (10)$$

where, U_j is the effective wind-speed at turbine j while accounting for all wake effects and N_{upwind} is the number of upwind turbines. Speed deficit, Δu_{ij} in equation (10), is a function of location coordinates as well as wind directions [15 and 24].

4. HEURISTIC APPROACH

In the heuristics methodology of Kulkarni and Mittal [24], the given square layout is divided into a fine grid and the points where the grid lines cross each other can be considered as possible turbine locations. Subsequently, turbines are placed in these possible locations one by one starting with the point where the gross AEP is maximum. The subsequent turbines are placed at locations where AEP will be the best and none of the constraints such as ITD, OCF will most likely be violated. The algorithm is implemented as follows. In the first step, a point is selected based on the gross AEP and added to the accepted turbine location matrix (M). In the next step, other locations surrounding the accepted location and violating other constraints are discarded and are added up to the rejected turbine location matrix (V). The left over locations are next updated as available locations. Now, the next turbine can again be added at the location that shows highest gross AEP value in the map and no constraint violation among all available locations. This way of adding turbines is continued till the search on all possible candidate locations is exhausted. Fig. 3 shows the schematic view of this methodology. It can be seen that the matrices M and V are updated at each iteration. In this fashion, the total number of turbines and their respective locations can be found out in one shot. As explained earlier, the above mentioned heuristic approach [24] of determining the optimal number as well as location of turbines in a farm layout has a drawback of

252 lack of continuity i.e. the turbines can only have certain available locations for the optimal
253 placement. This is because the heuristic algorithm discretizes the given geographical boundary
254 into finite number of grid-points, and the grid-cross sites act as the only possible locations for
255 candidate turbines. Therefore, the turbines can be placed only in those available locations leaving
256 the scope of any other nearby points to be one of the optimal points. Also, the heuristic
257 methodology lacks the stability, since outcomes can be different depending on the selection of the
258 starting point. Due to lack of stability, it might be difficult for wind farm developers to decide on
259 which starting point to start the search process of locating the turbines using the heuristic approach
260 and this shows that the practical application of this approach could be limited. However, the results
261 generated by the heuristic approach can be used as an intelligent initial guess to other
262 methodologies.

263 **5. HYBRID METHODOLOGY**

264 To overcome the drawbacks in the heuristic approach, a novel hybrid methodology is
265 developed to determine the optimal number and location of turbines, simultaneously. The proposed
266 hybrid approach is a combination of probabilistic GAs and deterministic gradient search based
267 methods. The problem of simultaneous determination of optimal number and layout of turbines is
268 decomposed into two sub-problems that can be solved in sequence. In the first step, the regular
269 square wind farm is converted into a finite number of grid points and the optimal turbine number
270 and locations are simultaneously determined from a selected finite number of possible locations
271 (grid cross points) through GAs. In the second step, the turbine number is fixed at the value obtained
272 in the first step and the turbine co-ordinates are improved through classical gradient-based
273 optimization techniques. The first sub-problem solves an integer programming problem over the
274 possible turbine locations (the grid cross-points) through binary variables 0 and 1 signifying

275 absence and presence of turbine at different locations, respectively. Based on number of possible
276 locations, the total number of binary variables are determined. The second sub-problem is a
277 continuous nonlinear programming problem where the total number of turbines is fixed, as
278 determined in the first step, and the focus is on determining optimal turbine coordinates given the
279 total number of turbines. The proposed hybrid methodology can start the search procedure using
280 one of the feasible heuristic outcomes [24] as initial guess and the cycle between evolutionary and
281 gradient approach (Fig. 4) is continued until a predefined termination criteria is met. The proposed
282 hybrid methodology comprises five important components.

283 *5.1. Feasible initial guesses through heuristics*

284 GA needs an initial population which can be generated randomly as well as using the afore-
285 mentioned heuristics (section 4). It can help the algorithm to converge faster if feasible initial
286 guesses can be provided as compared to starting with different random infeasible guesses
287 especially when the search space is huge. Hence, different feasible layouts with different starting
288 points in the heuristic algorithm can be used as initial population of GA.

289 *5.2. Grid Formation*

290 The square ($500 \times 500 \text{ m}^2$) wind farm is converted into a finite number of grid-points (7×7)
291 leading to 49 possible locations for turbines. Though grids are formed for both approaches, grid
292 resolution of heuristic approach and hybrid methodology are not necessarily the same. So, the final
293 solutions of the heuristic approach may not belong to the set of grid points of the GA. After
294 obtaining a heuristic outcome (say 8 turbines can be feasibly located), the starting matrix of
295 candidate turbine location in GA is formed by adding these 8 locations to 49 grid cross points
296 when there is no points common between them. Using these 57 locations, a *location index array*

297 with unique index for each location is formed (Fig. 5). Each location can be represented by 0 or 1
 298 depending on the absence or presence of turbines in that location, respectively (*binary array*).

299 *5.3. Evolutionary Algorithm*

300 An elitist version of binary coded genetic algorithm has been used here. Evolutionary algorithm
 301 is a combination of several steps which is described in Fig. 6.

302 Step I (Initialization):

303 First, an initialization matrix of $n_{pop} \times 57$ size has been formed (as shown in Fig 7) where one of
 304 the chromosomes would be a feasible heuristic outcome and others are generated randomly.
 305 Different GA parameters can be found in Table 1.

306 Step II (Modified Function Evaluation):

307 The constrained optimization problem has been converted into an unconstrained optimization
 308 problem in order to reduce the complexity of constraint handling in GA. The constraints are first
 309 normalized and added to the objective function to form a modified unconstrained objective
 310 function that can be represented as:

$$311 \quad \text{Modified obj. : } \underset{N_t}{\text{Max}} \underset{x_i, y_i}{\text{Max}} \sum_1^{N_t} AEP(x_i, y_i) - \text{NormConstraints} \quad (11)$$

312 Here, *NormConstraints* is a summation of all inequality constraints that are normalized to represent
 313 them into a scale of similar order of magnitude. As our main objective is to maximize both the
 314 number of turbines as well as the net AEP, *NormConstraints* are subtracted from the objective
 315 function to obtain the modified objective function. In this way, when a particular constraint is
 316 violated, the amount of normalized constraint violation is subtracted from the objective function

317 to lower the value of the modified objective function. Objective function is not modified when a
 318 particular solution is feasible. These constraints are explicitly defined as:

319 Inter turbine Distance (ITD): In order to lessen the wake loss and alleviate the fatigue loads,
 320 enough spacing can be provided between two turbines and the constraint in a normalized form can
 321 be represented as:

$$322 \quad g_1(x_i, y_i) = \max \left[0, \left(\frac{n_{space} * D}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} - 1 \right) \right] \quad (12)$$

323 where, (x_i, y_i) and (x_j, y_j) denotes the location coordinates of upwind and downwind turbines,
 324 n_{space} is the minimum distance between two turbines (taken here as 3) and D is the rotor diameter
 325 of a turbine (considered here as 52m) (Table 1).

326 Overall Capacity Factor (OCF): Due to various factors such as wind speed reduction, varying
 327 wind direction etc., the overall farm capacity is generally lower than the defined theoretical
 328 capacity. This constraint is defined in order to measure the wind farm performance. In normalized
 329 form it is expressed as:

$$330 \quad g_2(x_i, y_i) = \max \left[0, \left(OCF^{lim} / \left(\frac{\sum_{N_t} AEP(x_i, y_i)}{(8766) * N_t * Pr} \right) - 1 \right) \right] \quad (13)$$

331 The calculated farm capacity is kept greater than a selected limit value of OCF, called OCF^{lim} ,
 332 which is taken as 20% in this study.

333 Topological Constraints: This constraint is added only to ascertain that the turbines locations lie
 334 inside the given geographical boundary and expressed as:

$$335 \quad g_3(x_i, y_i) = \frac{1}{x_{\max}} \sum_1^{N_t} \max(-x_i, x_i - x_{\max}, 0) + \frac{1}{y_{\max}} \sum_1^{N_t} \max(-y_i, y_i - y_{\max}, 0) \leq 0 \quad (14)$$

336 Here x_{\max} and y_{\max} are the maximum value on x-axis and y-axis, respectively. In this study, both
 337 of these bounds equal to 500 that are given as the geographical limits of square wind farm.

338 After modifying the objective function for each chromosome, the corresponding modified function
 339 value is calculated and stored in the initialization matrix as an additional column.

340 Step III (Crossover and mutation):

341 The current population (called as ‘parents’) undergoes the cross-over and mutation [28] with
 342 defined parameters (Table 1) to generate a new set of solutions (called the ‘children’). Following
 343 the elitist strategy, both these populations are merged together ($2 n_{pop}$) and tournament selection
 344 is used to obtain the better chromosomes (n_{pop}) among them. Next the initialization matrix is
 345 updated and the process continues till the convergence is attained.

346 *5.4. Gradient Based Approach*

347 Though GA can solve the problem of optimal number and location of turbines simultaneously,
 348 it performs a search for certain number of fixed locations (grid cross points). If GA is employed
 349 to solve the problem with finer grids, the size of the problem (number of binaries) increases with
 350 increase in number of grid cross sites, thereby making the GA runs computationally more
 351 expensive. The first sub-problem involving GA should, therefore, be solved for a relatively coarser
 352 grid which can later be fine-tuned by solving the second sub-problem over the continuous x-y

353 coordinate space. Finally, GA declares the chromosome with the maximum modified function
354 value among all generations as the final solution. The final GA outcome of a feasible layout is next
355 passed as an initial guess to a gradient based solver. A well-known constrained nonlinear
356 optimization routine of MATLAB®, ‘fmincon’, (Table 1), has been utilized for this purpose. In
357 this step, the only decision variables are location coordinates of the turbines keeping the total
358 number of turbines as constant and the search is performed between the upper and lower bounds
359 of regular square boundary. Since a continuous optimization problem is solved in this step, it
360 searches for coordinates in addition to the points present on the grid for which further improved
361 AEP can be obtained.

362 *5.5. Grid Increment*

363 As mentioned in the section above, the outcomes of the gradient based search method can bring
364 in coordinates that may not be present in the set of grid cross sites that GA uses. As the last step
365 in the hybrid approach, these additional coordinates are added into the candidate location matrix
366 and the index matrix is updated accordingly. This is done to provide more coordinate locations to
367 be searched for GA in the next turn. For example, if the number of old locations were 57 and
368 gradient search provided (say 10) new locations as outcome, the new index array will have total
369 67 locations which are uniquely indexed (Fig 8). After an updated index matrix is obtained, GA
370 run is performed again using the new index array. Further, the outcome of GA is passed as a
371 starting point to gradient based approach and the cycle is continued until a stabilized AEP is
372 obtained as well as the location coordinates for three consecutive iterations are not changed.

373 As mentioned in the beginning of the section, the previous five steps are part of the elitist
374 genetic algorithm (EGA). Few steps are modified in the above approach, called modified EGA
375 (MEGA) as described below, to improve the execution time as well as efficiency of the algorithm.

376 *5.6. Modified Approach*

377 As computation of AEP is found to be the most time expensive step in the algorithm, avenues
378 were sought that can save significant computation in terms of computing AEP selectively. In EGA,
379 AEP was calculated for all the chromosomes in a population. As opposed to that, AEP was
380 calculated for only a part of the population in MEGA. The whole population is partitioned into
381 several sections based on the criteria of constraint satisfaction. If carefully watched, the constraint
382 ITD does not involve AEP computation. ITD computation for the entire population, is, therefore,
383 allowed. Chromosomes in the population for which this constraint (ITD) is unsatisfied, are
384 assigned a flag (say flag 1). Rest of the chromosomes, which satisfies ITD, are further checked
385 their satisfaction of the other constraint, say OCF. Since computation of OCF also involves
386 evaluation of AEP, no more AEP function calls can be saved. However, for implementing another
387 tournament selection based better constraint handling scheme, the population is further classified
388 into different categories. Chromosomes that satisfy the constraint OCF are flagged as 3 and rest of
389 the chromosomes which does not satisfy OCF are flagged as 2. From the above classification, it is
390 clear that the feasible chromosomes are flagged as 3 and chromosomes with other flags violate
391 either of the constraint. This classifies the entire population into feasible and infeasible solutions.
392 While conducting the tournament selection next, chromosome with flag 3 is always allowed to win
393 over chromosomes with any other flag, when two of such chromosomes are picked up randomly.
394 Upon comparison between chromosomes with flag 1 (violating ITD) and flag 2 (violating OCF),
395 one of the chromosomes is picked up randomly. If both the chromosomes with flag 3 are picked,
396 the chromosome with better AEP wins, whereas in case of both the flag to be 1 or 2, the
397 chromosomes with lesser constraint violation is chosen.

398 6. RESULTS AND DISCUSSIONS

399 The optimal total number of turbines and their locations have been determined while
400 maximizing the net AEP in a wind farm under several constraints such as inter turbine distance,
401 overall capacity factor and the effect of wakes on the turbines. A hybrid methodology is proposed
402 to overcome the drawbacks of recently developed heuristic approach [24] to solve this problem.
403 The proposed methodology utilizes the merits of probabilistic GA and deterministic gradient based
404 approach to solve this problem. Due to the presence of wake effects, the energy terrain of the
405 problem becomes extremely nonlinear with the gradual addition of turbines into the wind farm.
406 Fig. 9 depicts evolution of the complex and non-linear energy terrain as turbines are successively
407 added to the search space. This set of figures has been generated in this fashion: first, the given
408 layout is discretized into fine grids (say, 101×101 as discussed in section 4). To see the energy
409 terrain in presence of say $n+1$ turbines, n turbines are placed at certain known locations and the
410 location of the last turbine is varied one by one in leftover available discretized locations and the
411 value of net AEP is captured and depicted through surface-contour plots. For example, Fig. 9a
412 shows the net AEP terrain for two turbines – here the energy surface is generated by keeping the
413 location of one turbine fixed and varying the location of the other turbine across all other locations
414 except the location of the first turbine. As the number of turbines is increased, the complex
415 distribution of net AEP (appearance of several local optima) and increase in non-linear behavior
416 of the problem can be observed from Figs 9b and 9c. In the hybrid methodology suggested here,
417 the classical optimization technique provides the ability to find the optimum more precisely once
418 the near global basin is identified by GA.

419 As discussed in the following paragraphs, three different case studies with different wind
420 conditions were considered for micro-siting optimization using the proposed hybrid optimization

421 method. These case studies differ from one another in terms of the gross AEP distributions over
422 the given geographical boundary. Information regarding these case studies is provided in Table 2.
423 In every case study, the outcome from the heuristic algorithm [24] (H0) is added as one of the
424 chromosomes in the initial population for MEGA and rest of the chromosomes are created
425 randomly. The outcome of MEGA (A1) is passed as an initial guess to the gradient based approach
426 (B1) which improves locations of the turbines further with better net AEP. This cycle between
427 MEGA and gradient-based approach is continued until the change in AEP between two
428 consecutive runs is less than a predefined tolerance. All reported simulations are performed on
429 Intel® Xeon® CPU E5-2690 0 @ 2.90GHz (2 processors) 128 GB RAM machine.

430 *6.1. Case 1: Type – I Gross AEP distribution*

431 The first case study is about a wind farm with complex and non-uniform distribution of wind
432 speed and Gross AEP. It has been assumed in this case that the wind is flowing in uncertain
433 direction at uncertain speed at every location (as shown in Fig. 10). It has been found that heuristic
434 approach [24] on this energy distribution map is able to place 3 turbines while proposed hybrid
435 methodology is able to place 4 turbines with ~44% improvement in AEP as presented in Table 3.
436 The justification of hybrid approach for micro-siting is clearly seen from the results as both the
437 algorithms i.e. MEGA and ‘fmincon’ are observed to contribute in the net AEP improvement.
438 Improvement in AEP for cases when total number of turbines is fixed (e.g. see the improvement
439 in AEP values from cycle 1 Gradient to cycle 2 MEGA) can be attributed to the detection of better
440 turbine locations. Fig. 11 shows the final superimposed accepted coordinates and number of
441 turbines (black cross markers) on the gross AEP contour plot obtained for the given boundary. As
442 can be observed, the algorithm manages to place only a few turbines in the given layout. The
443 optimal placement of turbines towards one of the boundaries can be attributed to the higher AEP

444 values available along that boundary. This particular case has been generated in such a way that
445 the parameter A of Weibull distribution (see assumption 4 in section 2) has zero values in all
446 locations in the given layout except having some nonzero values along the mentioned boundary.
447 More than four turbines are not possible to be placed along that boundary due to violation of ITD
448 constraints.

449 *6.2. Case II: Type – II Gross AEP distribution*

450 In this case study, a uniform distribution of gross AEP is considered across all locations except
451 one location where the wind speed is considered to be higher (Fig. 12). In Fig 12, the location with
452 a higher wind speed is represented by a bump whereas other locations with a negligible amount of
453 gross AEP variation are represented by a flat surface. It has been found that heuristic algorithm is
454 able to place 8 turbines under these wind conditions whereas the proposed hybrid methodology is
455 able to place 12 turbines with ~51% improved in net AEP (Table 4). As can be observed, both
456 MEGA and ‘fmincon’ efficiently increase the total turbine number and / or the AEP. Fig. 13a,
457 shows the final accepted 12 turbine locations (black cross markers) superimposed on Type –II
458 gross AEP contour plot for the given area. One of the challenging parts of the problem to handle
459 increasing number of binary variables as more number of cycles are completed is also visible from
460 this example (last column of Table 4), which has been successfully handled by GA.

461 *6.3. Case III: Type – III Gross AEP distribution*

462 In this case study, Gross AEP distribution is generated as combination of previous two case
463 studies. Here, the complex Gross AEP distribution is considered in such a way that a particular
464 location in the wind farm gets a higher wind speed and rest of the locations have a disturbed, non-
465 uniform wind flow (see Fig. 14). It has been found that heuristic algorithm alone can place 9

466 turbines in the wind farm whereas the proposed hybrid methodology can place 12 turbines with
467 ~30.25% improvement in the net AEP (Table 5). Though the final number of turbines is the same
468 as the previous case study (Case II), the locations for the turbines are different (Fig. 13). Fig. 13b
469 depicts the final turbine locations superimposed on gross AEP contour plot of Type –III.

470 For all three case studies, the net AEP generated (case 1: 865.95 kWh, case 2: 2054.43 kWh
471 and case 3: 2058.81 kWh) are individually better than the net AEP values obtained by the heuristic
472 algorithm working alone for them. At the same time, it can be observed that AEP improvement by
473 MEGA is further improved by ‘fmincon’ until convergence, thus establishing the importance of a
474 hybrid algorithm. In other words, using MEGA or ‘fmincon’ alone will not yield the best possible
475 AEP. Moreover, the MEGA approach is observed to work to the extent of twice as fast as the EGA
476 algorithm for the test cases discussed. Table 6 shows the examples of savings in function
477 evaluation during the calculation of expensive AEP function for each of the cases discussed earlier
478 (23, 52 and 39 % for case1, case 2 and case 3, respectively) which makes MEGA approach more
479 efficient than EGA. As evident, the fastness in obtaining the solution is due to the time saving in
480 the expensive AEP calculation.

481 *6.4. Case IV: A benchmark case study*

482 In order to validate the proposed hybrid optimization approach, a popular case study [9] of uniform
483 wind direction at a speed of 12m/s (see Fig. 16) has been considered next. In this case, similar
484 characteristics of wind farm, wind turbines, power curve and wake model as given in [8] are used
485 (see Table 7) and micro-siting has been performed on this layout using the hybrid optimization
486 approach. The main objective is to minimize the ratio ($COST/P_{tot}$) i.e. attain the maximum energy
487 throughput (P_{tot}) at minimum COST, while satisfying the ITD constraint of 5D or 200m to

488 minimize the wake effects [9]. Here, COST and P_{tot} is given by equations 15 and 16 and the
 489 constraint of ITD (equations 3 and 12) is modified with a new value of n_{space} of 5D [8, 9 and 12].

$$490 \quad COST = N \left(\frac{2}{3} + \frac{1}{3} e^{-0.00174N^2} \right) \quad (15)$$

$$491 \quad P_{tot} = \sum_{s=1}^{states} \sum_{k=1}^N Power_{sk} Frequency_{sk} \quad (16)$$

492 Here, N is the total number of turbines, states are the wind conditions in terms of direction and
 493 speed for a particular case study as given in [8], $Power$ is defined by power curve [23] and
 494 $Frequency$ is the occurrence of wind at a particular state [9 and 23]. After evolving the MEGA for
 495 2100 generations over 150 population size, the outcome of the hybrid algorithm is compared with
 496 other existing methods [8, 9, 12, and 23] and the results are presented in Table 8. It has been found
 497 that the hybrid approach is able to place more number of turbines inside the layout as compared to
 498 the previous approaches [8, 9 and 12] with better ratio of (COST/ P_{tot}) (Table 8) and the results are
 499 quite close to the same obtained by Chen et al. [23]. Figure 17 shows the different layouts of
 500 turbines obtained by various optimization methodologies for this case study. Table 9 shows the
 501 improvement in AEP and convergence of the hybrid algorithm as it marches through the different
 502 cycles.

503 Motivated by the approach adopted by Chen et al. [23], which shows improvement in the
 504 above case study, where a mix of real and binary coded GA has been used to tackle binary and real
 505 variables simultaneously, micro-siting for wind layout with conditions as presented in case 2 has
 506 been carried out next. Here a generic GA code has been developed (named as RBGA) where the

507 total number of turbines are fixed to any assumed value (e.g. $N_f = 20$) and the turbine coordinates
508 of these N_f turbines are represented as real variables and existence of turbines in those N_f locations
509 are represented as binary variables (zero for absence and one for presence). This leads to a total of
510 60 decision variables where 40 such variables are x-y coordinates of turbines and 20 variables are
511 binary numbers. Using SBX and polynomial mutation operators for real coded GA [28] and similar
512 operators as MEGA for binary variables, RBGA has been developed. This approach uses binary
513 tournament selection and elitist strategy as adopted in NSGA II [28]. As the MEGA component of
514 the hybrid approach ran for 6 cycles each with 50 population and 150 generations (see Table 4) to
515 generate the final results for case 2, RBGA has been allowed to run for a similar number of
516 generations ($150 \times 6 = 900$) with population 50. RBGA could place 9 turbines altogether in the
517 given layout and Fig. 15 shows the final superimposed accepted coordinates and number of
518 turbines (black cross markers) on the gross AEP contour plot. This is slightly better than the results
519 of the heuristic approach (~ 23.16 % improvement in AEP over heuristic approach) which could
520 place 8 turbines in the same layout; however, this is inferior to the results of the hybrid approach
521 which could place 12 turbines in the same layout (~ 22.61 % improvement in AEP over RBGA
522 approach). This shows the superiority of this hybrid approach over two of the existing approaches
523 [23, 24] in the literature.

524 7. CONCLUSIONS

525 Simultaneous maximization of total number of turbines and the net AEP has been carried out
526 for a given wind farm using a novel hybrid optimization strategy. The presence of various types
527 of constraints such as inter turbine distance, overall capacity factor and wake effects have also
528 been considered while conducting the above mentioned micro-siting study. Binary decisions
529 depicting the presence or the absence of turbines across several grid cells in the given regular wind

530 farm and the continuous nature of coordinate variables make the formulation a complicated mixed
531 integer nonlinear programming problem. The proposed hybrid methodology is based on the
532 decomposition of the decision variable set into real and binary parts and utilizes the merits of both
533 GA and gradient based approaches to solve this NP-hard MINLP problem. The first sub-problem
534 solves the optimal number and location problem together for selected number of possible locations
535 using GA whereas the second sub-problem improves the coordinates over the continuous
536 coordinate space by keeping the total number of turbines fixed as obtained by the first sub-problem.
537 The proposed methodology is applied to three different wind farm conditions and it has been
538 shown that the proposed methodology works better (~44%, ~51% and ~30% improvement in the
539 net AEP over the heuristics approach) than two of the existing approaches in the literature. This
540 solution methodology can not only help the wind farm developers to find out turbine locations
541 optimally in a given wind farm but also find out the maximum number of turbines that can be
542 optimally fitted in the wind farm simultaneously.

543

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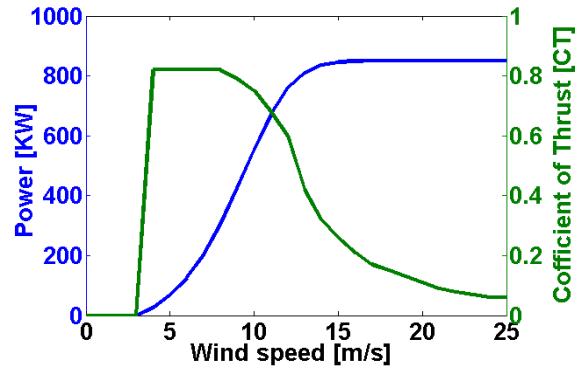
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Figures

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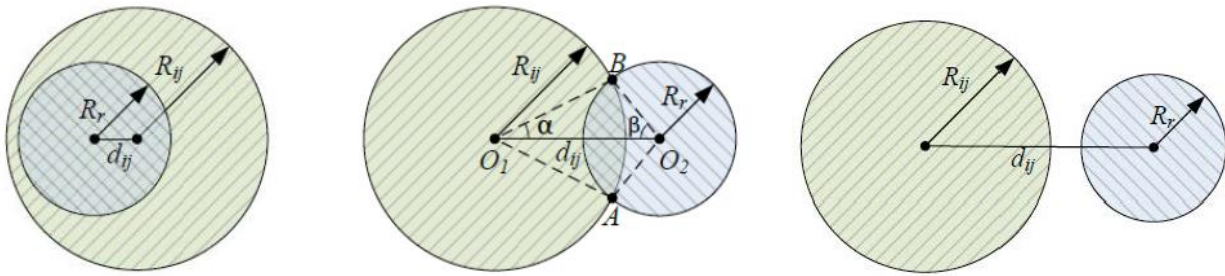
Figure 1: Power and C_T curve for Vestas-V52 850 kW [29]

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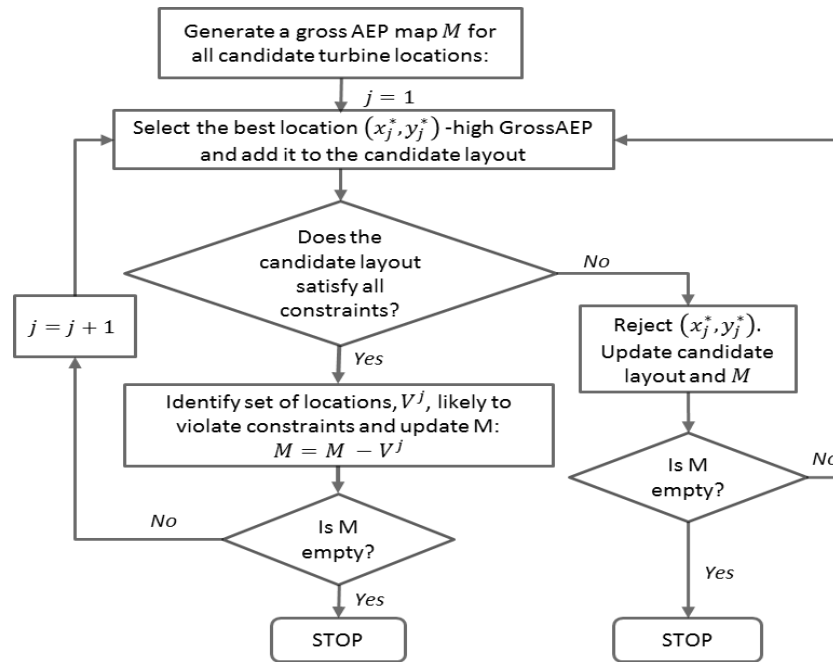
(a) $d_{ij} \leq R_{ij} - R_r$ (b) $R_{ij} - R_r < d_{ij} < R_{ij} + R_r$ (c) $d_{ij} \geq R_{ij} + R_r$

555 **Figure 2** : Schematic view of affected area of turbines while considering wake effects under 3
556 situations (a) full wake or complete wake, (b) partially wake, (c) out of wake [27].

557 Note : To be reproduced in color on the Web and in black-and-white in print.

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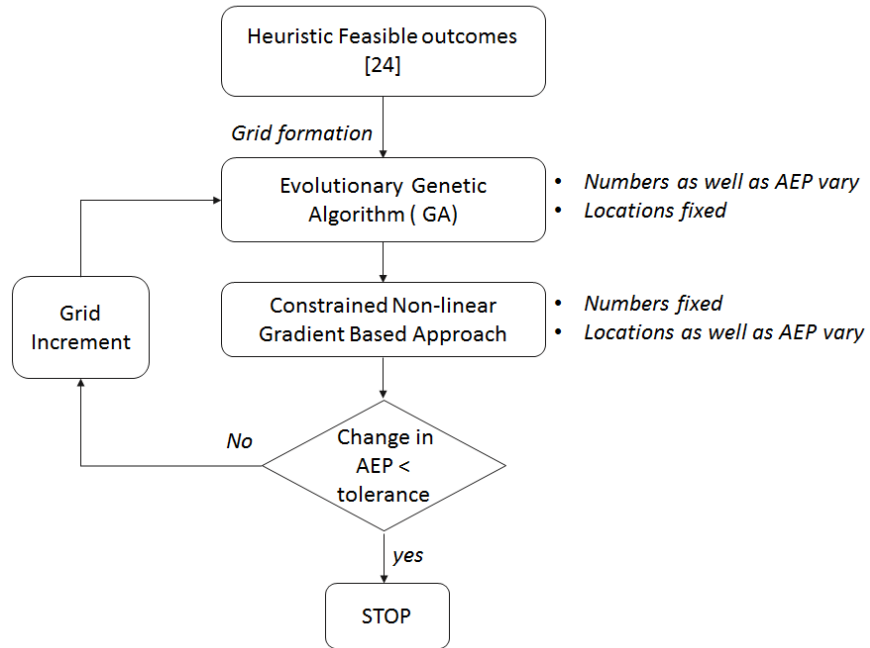
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Figure 3: Flowchart of Heuristic approach [24]



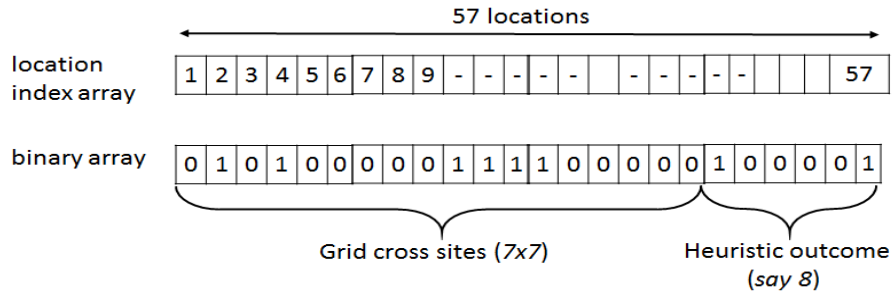
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Figure 4: Schematic Representation of Hybrid Methodology

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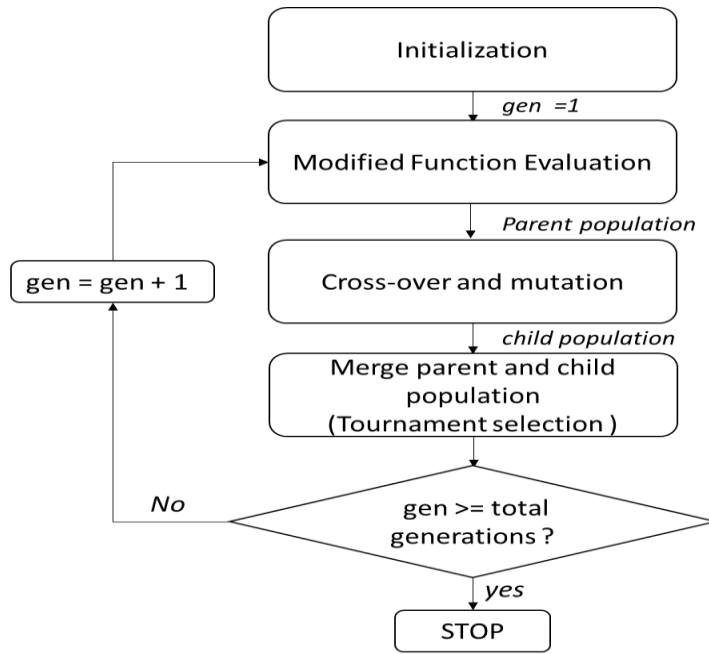


566

567 **Figure 5:** Binary array and location index array at grid formation step.

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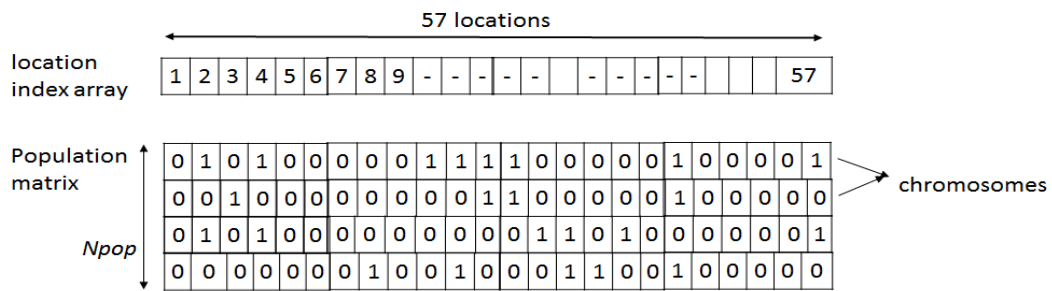
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Figure 6: Flowchart of Evolutionary Algorithm

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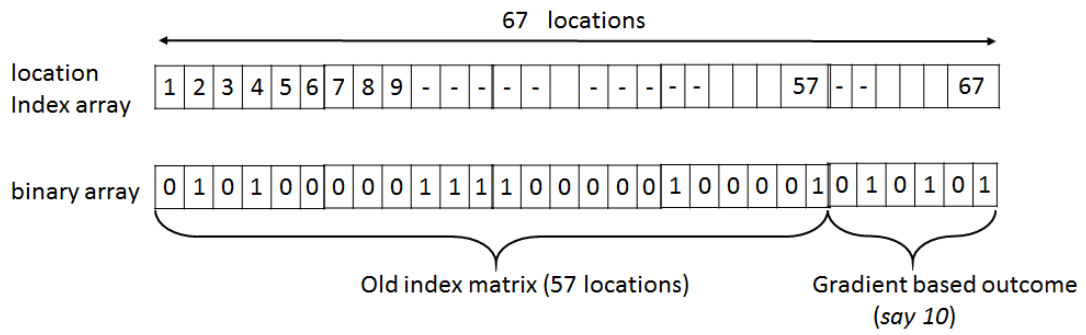
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Figure 7: Population matrix formed at initialization step.

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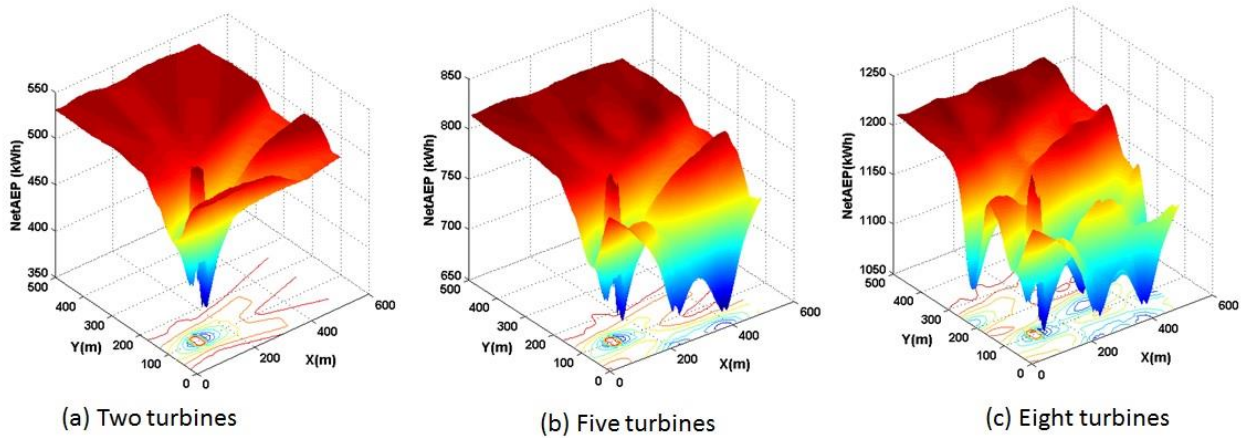


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579 **Figure 8:** Formation of new index matrix after adding gradient outcomes in grid increment step.

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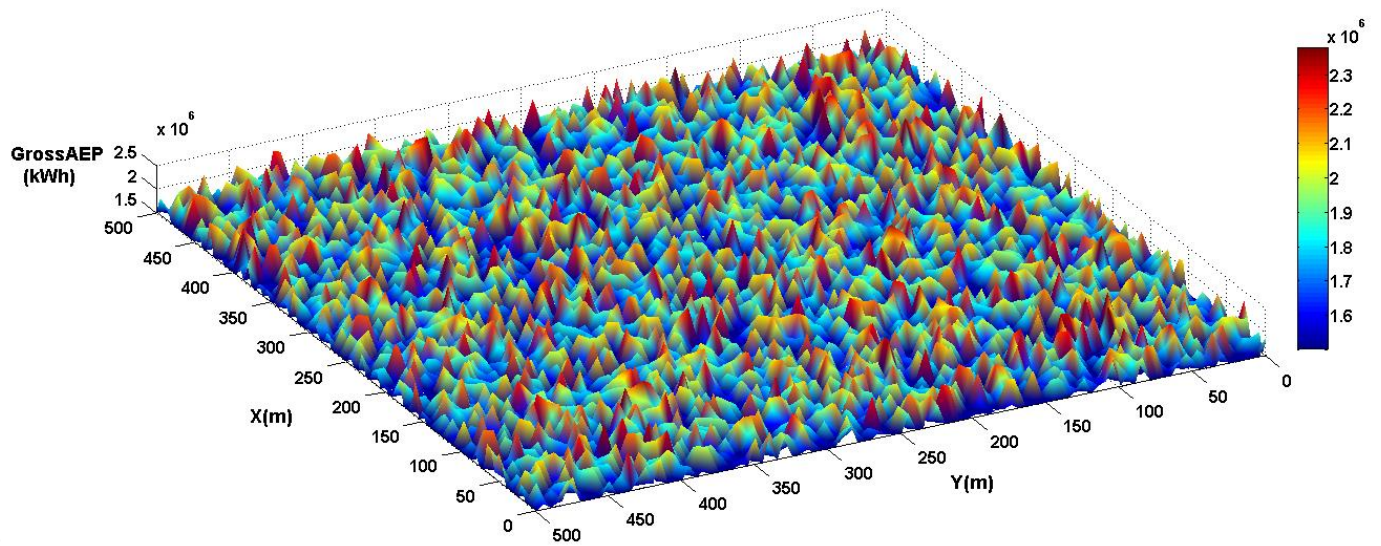
583 **Figure 9:** Surface contour plot showing the distribution of net AEP over the given layout for 3
584 situations (a) two turbines (b) Five turbines (c) Eight turbines.

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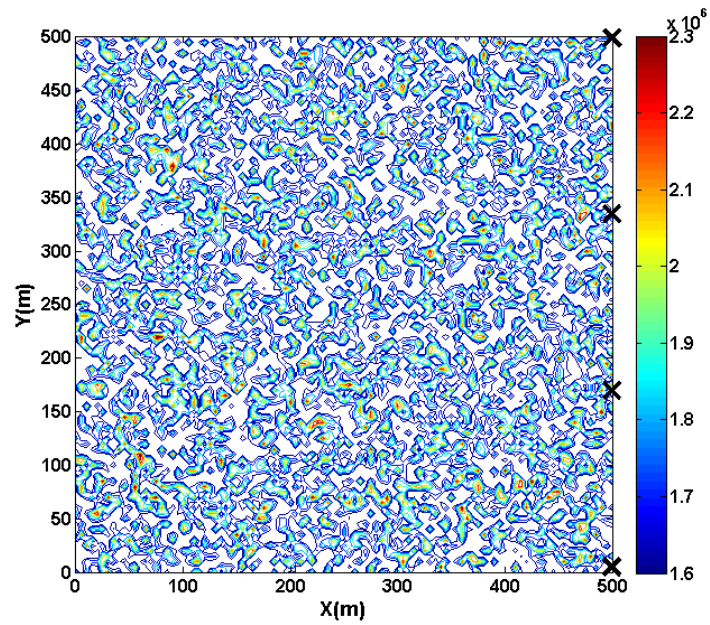
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589 **Figure 10:** Type –I Gross AEP distribution over a given boundary.

590 Note : To be reproduced in color on the Web and in black-and-white in print.

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594 **Figure 11:** Accepted turbines locations (black cross markers) superimposed on Type – I Gross

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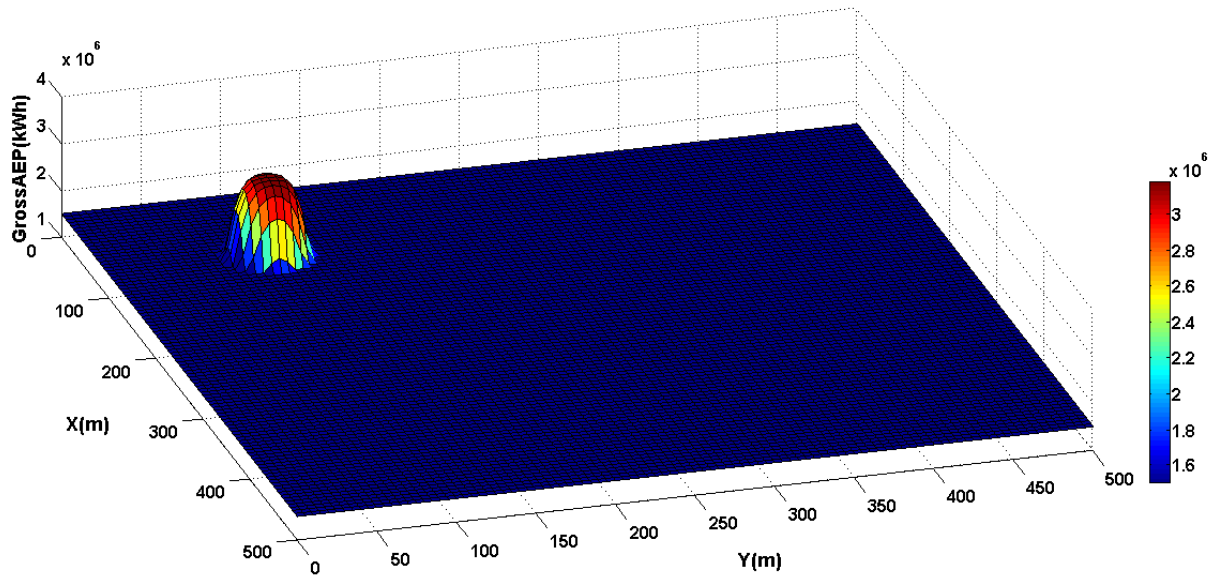
AEP contour plot of wind farm

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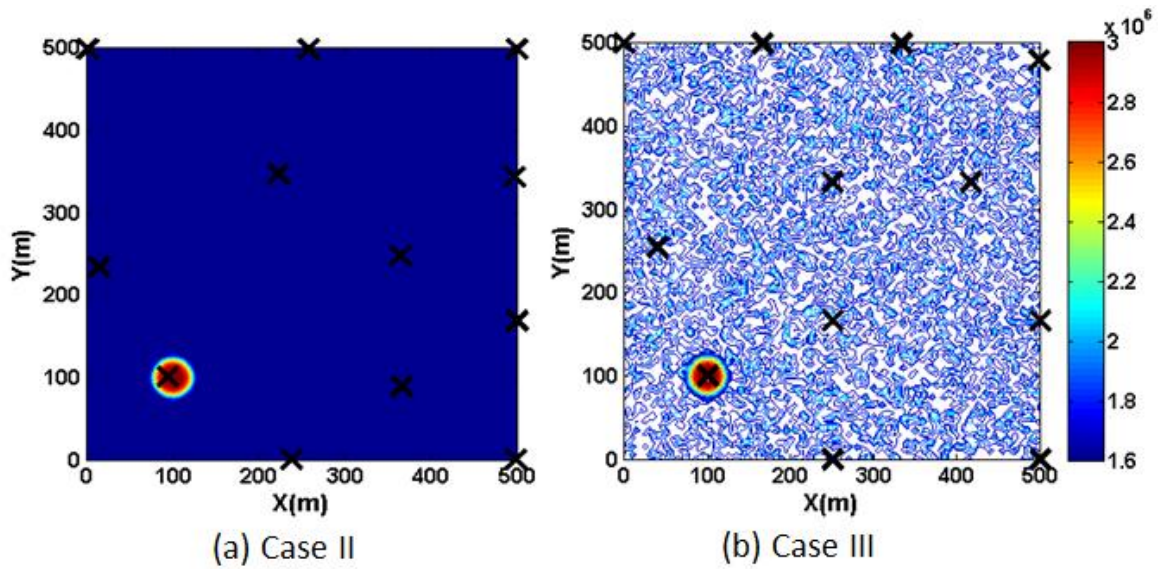
Figure 12: Type –II Gross AEP distribution over a given boundary

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605 **Figure 13:** Comparison of accepted turbines (black cross markers) superimposed on (a) Case II

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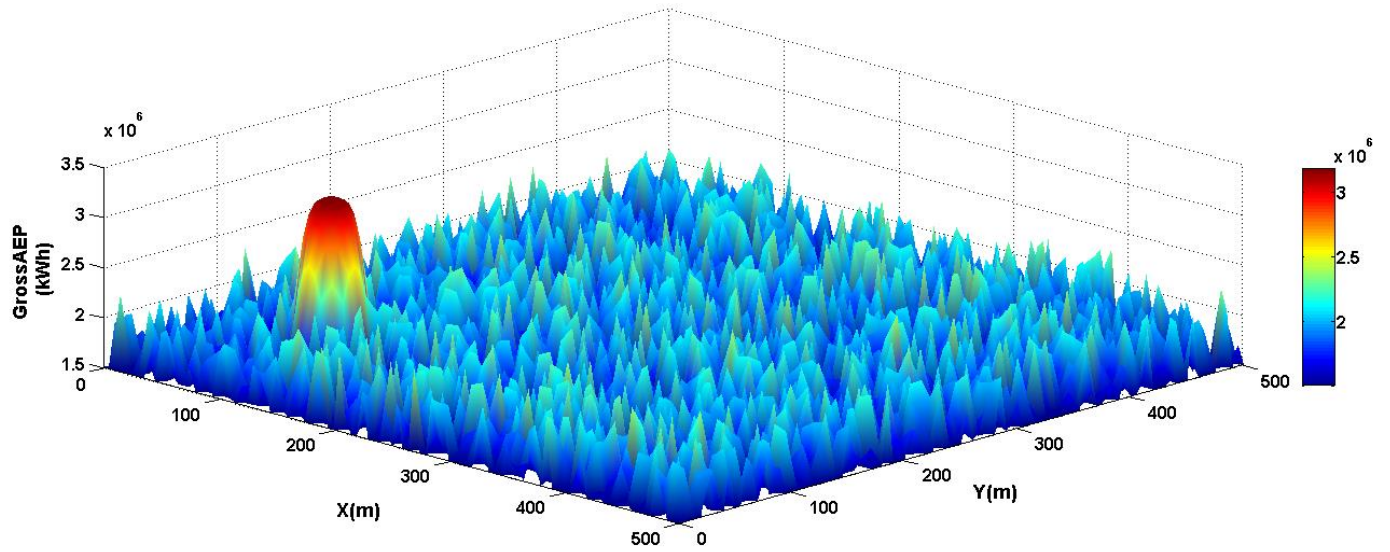
(Type II Gross AEP plot) and (b) Case III (Type – III Gross AEP plot) of a wind farm.

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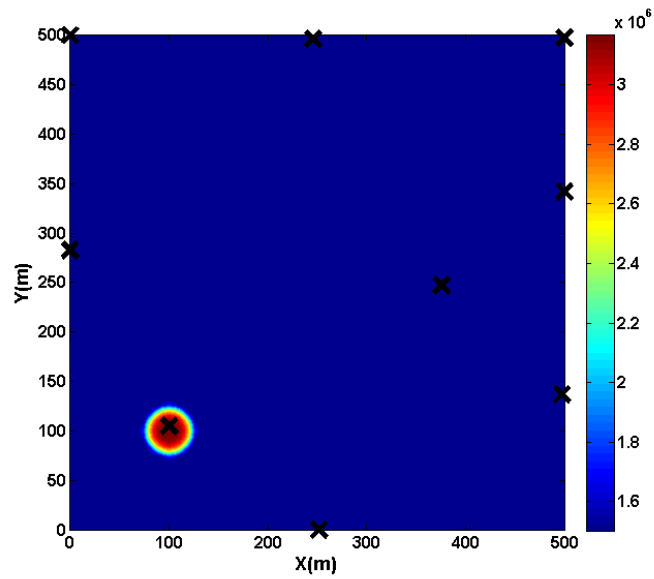
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611 **Figure 14:** Type –III Gross AEP distribution over a given boundary

612 Note : To be reproduced in color on the Web and in black-and-white in print.

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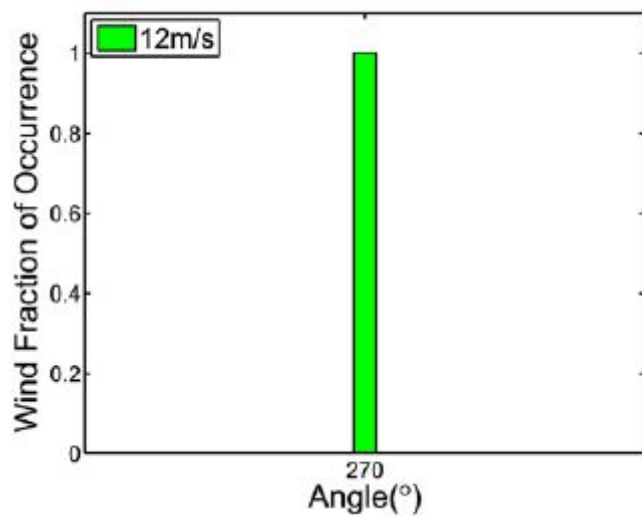


615

616 **Figure 15:** Accepted turbine locations (black cross markers) from binary- real coded GA
617 superimposed on Type – II Gross AEP contour plot.

618 Note : To be reproduced in color on the Web and in black-and-white in print.

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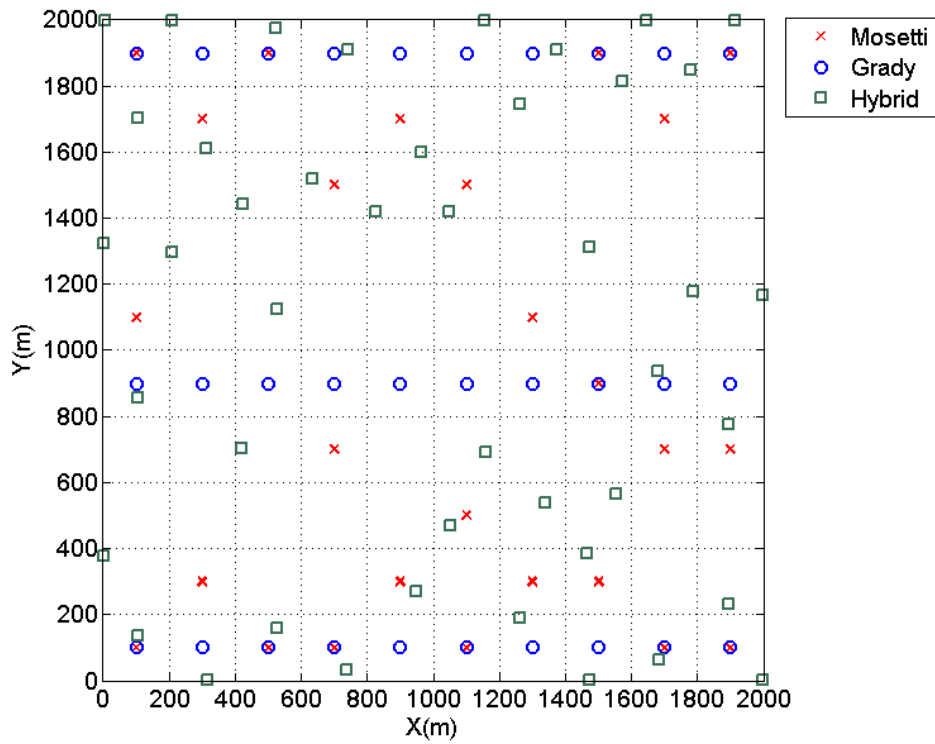
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Figure 16 : Uniform wind distribution for case IV.

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Note : To be reproduced in color on the Web and in black-and-white in print.

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626 **Figure 17** : Accepted turbine layouts for various methodologies (shown by different markers)
 627 applied on Case study IV.
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629 Note : To be reproduced in color on the Web and in black-and-white in print.

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Tables

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Table 1: Parameters used in GA and Gradient based approach

Genetic Algorithm (MEGA) specifications	
Algorithm Type	Elitist-Tournament selection
Number of Population (n_{pop})	50
Number of Generations (n_{gen})	150
Crossover Probability	0.80
Crossover Type	Uniform
Mutation Probability	0.01
Gradient Based solver	
Solver	fmincon MATLAB®
Algorithm	Interior Point

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635

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Table 2: Wind farm, wind turbine and wake model specifications [24]

637

Wind farm Information	
Farm area (m ²)	500 × 500
Wind turbine specifications	
Turbine Type	Vestas V52-850 KW [29]
Turbine Rated Capacity (Pr) (kW)	850
Turbine Diameter (m)	52
Wake model Information	
Model	Jensen [25]
Jensen Constant (k_w)	0.075

638

Table 3: Outcome of hybrid methodology case 1

Cycle	Algorithm	Outcome	Number of turbines / feasible locations	AEP (Kwh)	Number of binaries
1	Heuristic [24]	H0	3	599.70	52
	MEGA	A1	4	626.67	
	Gradient	B1	4	651.96	
2	MEGA	A2	4	859.96	55
	Gradient	B2	4	865.95	
3	MEGA	A3	4	865.95	59
	Gradient	B3	4	865.95	

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Table 4: Outcome of hybrid methodology case II.

Cycle	Algorithm	Outcome	Number of turbines / feasible locations	AEP (Kwh)	Number of binaries
1	Heuristic [24]	H0	8	1360.00	57
	MEGA	A1	10	1712.28	
	Gradient	B1	10	1765.33	
2	MEGA	A2	10	1789.96	67
	Gradient	B2	10	1803.83	
3	MEGA	A3	11	1921.74	77
	Gradient	B3	11	1941.28	
4	MEGA	A4	11	1941.54	88
	Gradient	B4	11	1943.26	
5	MEGA	A5	12	2046.92	99
	Gradient	B5	12	2054.29	
6	MEGA	A6	12	2054.35	111
	Gradient	B6	12	2054.43	

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Table 5: Outcome of hybrid methodology case III.

Cycle	Algorithm	Outcome	Number of turbines / feasible locations	AEP (Kwh)	Number of binaries
1	Heuristic [24]	H0	9	1580.58	58
	MEGA	A1	9	1605.24	
	Gradient	B1	9	1607.71	
2	MEGA	A2	11	1978.64	62
	Gradient	B2	11	1978.64	
3	MEGA	A3	11	1996.68	66
	Gradient	B3	11	1996.68	
4	MEGA	A4	11	2058.81	67
	Gradient	B4	11	2058.81	

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Table 6: Savings in expensive function evaluation by MEGA approach over EGA approach

Cycle	Function calls by EGA	Function calls saved by MEGA	% saved	Overall saved per case study
Case 1: Type – I Gross AEP distribution				
1	7550	1896	25.11	22.75
2	7550	1580	20.92	
3	7550	1679	22.23	
Case 2: Type – II Gross AEP distribution				
1	7550	3811	50.47	51.85
2	7550	3356	44.45	
3	7550	3549	47.00	
4	7550	3940	52.18	
5	7550	4307	57.04	
6	7550	4526	59.94	
Case 3: Type – III Gross AEP distribution				
1	7550	2899	38.39	38.85
2	7550	2783	36.86	
3	7550	3033	40.17	
4	7550	3018	39.97	

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Table 7: Wind farm, wind turbine and wake model characteristics [8] for case IV.

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Wind farm Information	
Farm area (m ²)	2000 × 2000
Wind turbine specifications	
Turbine Diameter (m)	40
Turbine Rated Power (Pr) (kW)	630
Hub Height (Z) (m)	60
Coefficient of Thrust (C_T)	0.88
Surface Roughness (Z_0) (m)	0.3
Wake model Information	
Model	Jensen [25]
Jensen Constant (k_w)	0.0944

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Table 8: Comparison of various methodologies with present study for Case study IV.

	Mosetti et al. [8]	Grady et al. [9]	Wan et al. [12]	Present study	Chen et al. [23]
COST/ P_{tot}	0.0016197	0.0015436	0.0014475	0.0014386	0.0013456
Total Power (P_{tot}) (kW)	12352	14310	15262	20742.54	22624.3
Number of turbines	26	30	30	44	45

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Table 9: Outcome of hybrid methodology on case IV.

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Cycle	Algorithm	Outcome	Number of turbines / feasible locations	Fitness ratio(COST/P_{tot})
1	MEGA	A1	41	0.0014579
	Gradient	B1	41	0.0014505
2	MEGA	A2	43	0.0014496
	Gradient	B2	43	0.0014491
3	MEGA	A3	43	0.0014491
	Gradient	B3	43	0.0014470
4	MEGA	A4	43	0.0014470
	Gradient	B4	43	0.0014450
5	MEGA	A5	42	0.0014435
	Gradient	B5	42	0.0014428
6	MEGA	A6	42	0.0014428
	Gradient	B6	42	0.0014423
7	MEGA	A7	44	0.0014403
	Gradient	B7	44	0.0014386
8	MEGA	A8	44	0.0014386
	Gradient	B8	44	0.0014386

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