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START-UP OF FLOW THROUGH A 4:1:4 CONSTRICTION IN A TUBE USING THE ROUSE-CCR TUBE MODEL FOR LINEAR ENTANGLED POLYMERS WITH FINITE EXTENSIBILITY

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ABSTRACT

The nonlinear Rouse-CCR tube model for linear entangled polymers [1], which avoid decoupling approximation between stretch and orientation, is studied. The so-called Rolie-Poly model is further modified to account for finite extensibility of polymer chains. The underlying model is used to analyze, by means of a time-dependent three-dimensional finite element method, the kinematics, pressure drop and stability of the start-up flow through a 4:1:4 constriction in a tube. The matrix-logarithm-based formulation of the conformation tensor, σ, introduced by Fattal and Kupferman [2], which guaranties positivity of σ, is used. A fixed-point algorithm is used to iterate between the solution of the momentum and constitutive equations at each time step, thereby making the overall algorithm coupled in a segregated manner. The stabilized discrete elastic viscous stress splitting (DEVSS-G) method and stabilized finite element methods (GLS, SUPG) are used to carry out three-dimensional time-dependent simulations. Computations are conducted using our parallel computation framework. The practical utility and effectiveness of the proposed numerical scheme is demonstrated by solving fully three-dimensional constriction flow.

MODEL

In order to account for finite extensibility of polymer chains into the original Rolie-Poly equation, we require that, in the absence of any other mechanisms, the trace of the original equation leads to the relaxation for the stretch similar to the MLD model. The resulting model, which accounts for finite extensibility, is written in the form

$$\frac{\partial \boldsymbol{\sigma}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\sigma} = \boldsymbol{L} \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot \boldsymbol{L}^T + \boldsymbol{f}(\boldsymbol{\sigma}), \qquad (1)$$

where $\boldsymbol{L} = \nabla \boldsymbol{u}^T$ is the transpose of velocity gradient tensor and the tensor function, f, is now given by

$$f(\sigma) = -\frac{1}{\tau_d}(\sigma - \delta) - \frac{2}{\tau_{\mathrm{R}}} k_s(\lambda)\left(1 - \sqrt{\frac{3}{tr\sigma}}\right)\left(\sigma + \beta\left(\frac{tr\sigma}{3}\right)^{\delta}(\sigma - \delta)\right). \tag{2}$$

Here τ_d is the fixed-tube disengagement time or reptation time, τ_R is the longest Rouse time or stretch time, β is the CCR coefficient analogous to the coefficient introduced by Marrucci in his original CCR paper, δ a negative power which can be obtained by fitting to the full theory, $\lambda = \sqrt{tr\sigma/3}$ is the chain stretch ratio and $k_s(\lambda)$ is the nonlinearity of the spring coefficient accounting for the finite extensibility of polymer chains.

STAR-TUP OF FLOW THROUGH AN AXISYMMETRIC 4:1:4 CONSTRICTION

Starting from rest and using time-independent (static) inlet boundary conditions, we obtained results through a true three-dimensional transient development. The temporal evolution of the stretch and the vortex are shown in Fig. 1. There is a clear evidence of a lip vortex growth (t = 0.3 s and 0.5 s), stretching from re-entrant corner.

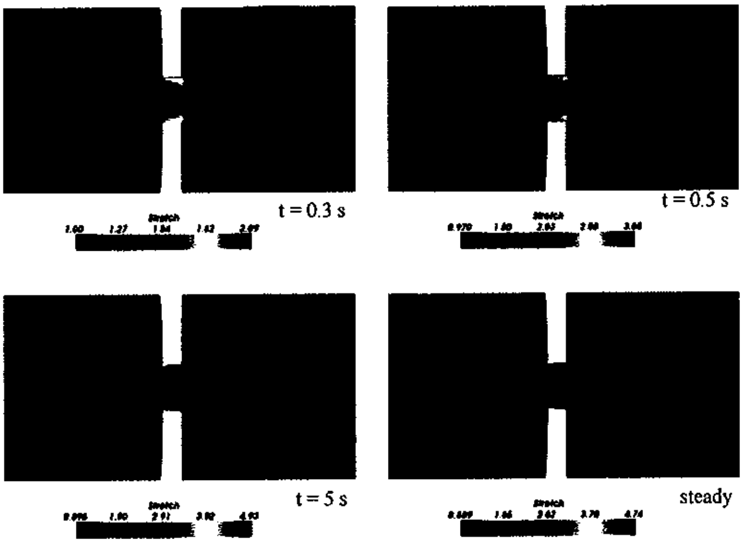

Fig. 1. Temporal evolution of the stretch ratio and vortex during the start-up flow.

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