POD-Galerkin reduced-order models for real-time surgical simulation

Hay, Alexander; Audet, Martin; Marcotte, Jean-Philippe; Hétu, Jean-Francois

POD-GALERKIN REDUCED-ORDER MODELS FOR REAL-TIME SURGICAL SIMULATION

A. Hay*, M. Audet, J.-P. Marcotte and J.-F. Hétu

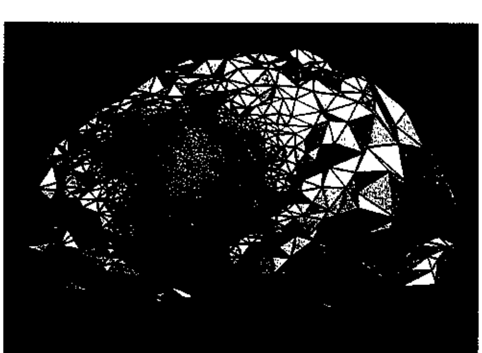
Industrial Materials Institute,
National Research Council Canada,
Boucherville, Québec, Canada.

*presenter contact information: alexander.hay@imi.cnrc-nrc.gc.ca, 450-641-5422

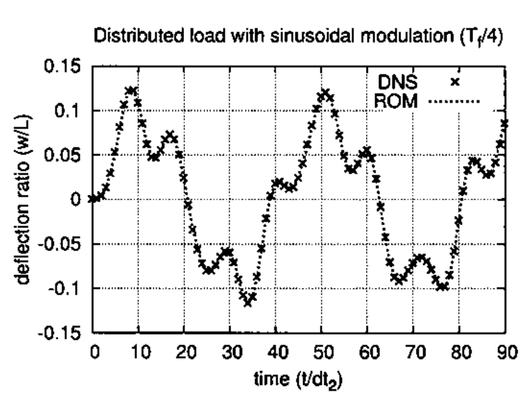
We are interested in this work with real-time simulation of surgical operations with a particular emphasis on brain oncology simulation. It is part of a broader initiative to provide surgeons with a Virtual Reality (VR) rehearsal system.

The constitutive behavior of brain tissues is not yet fully characterized by rheologists but there are well-known to be nonlinear and time-dependent due to their ability to undergo large deformations. While Direct Numerical Simulation (DNS) of such problems has long been mastered given the wide range of applications in mechanical engineering, they are computationally demanding. By far, the most popular and general approach is the Finite-Element Method (FEM) which results in a large set of equations in the unknown displacements. It allows for simulation of complex geometries and loading configurations (see Fig. 1(a)). A major limitation of such procedures, especially for nonlinear problems, is that they require a significant computational time that make them unsuitable for real-rime integration. Real-time surgical simulation gives rise to even greater cpu times restriction since numerical solutions are required at haptic force-feedback rates (15 Hz to 500 Hz).

To make many of this problem tractable, reduced-order modeling is a good candidate to minimize the simulation requirements. It aims at reducing the mathematical complexity of such problems while preserving physical modeling. It is based on building a low-dimensional subspace well-adapted to represent the set of solutions to the particular problem at hand. The Proper Orthogonal Decomposition (POD) is the prevailing method for generating such subspaces because it can readily be applied to any problem regardless of the physics and the governing equations involved. The POD and its variants are also known as Karhunen–Loève Expansions (KLE), Principal Component Analysis (PCA), and Empirical Orthogonal Functions (EOF).



(a) Typical grid for brain oncology simulation



(b) Comparison between DNS-FEM and ROM

We present POD-Galerkin Reduced-Order Models (ROM) for which systems can be precomputed prior to time integration leading to integration cost independent of geometric complexity. This feature is the key enabler to real-time simulation. Efficient runtime evaluation of internal forces vectors and stiffness matrices makes possible the use of both implicit time integrators and high-frequency damping which in turns guarantee model stability. We show that low-dimensional approximations comprised of 10 to 100 POD modes allow for real-time simulation with only a limited lost of accuracy compared to FEM analysis (see Fig. 1(b)). However, as will be presented, offline model training and building steps must be done with care. Furthermore, they are as computationally demanding as FEM simulation driving the need for efficient parallel implementation.